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Fundamental Theorem on the Relationship between Trade Balances in Value Added and Gross Terms: Amendment

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Abstract

This paper amends Kuboniwa (2014). We present a proof of a fundamental theorem on the relationship between trade balances in value added and gross terms in a general model with many countries and many sectors: the total sum of a country's trade balances with all other countries measured in value added equals that in gross terms. This theorem implies that the total sum of differentials between balances in value added and those in gross terms equals zero. Using an aggregated World Input-Output data (WIOD) of Groningen University with eight countries (BRICs, the USA, the EU, Japan and the rest of the world (ROW)) and 20 sectors for 2010, we show an empirical evidence of the theorem.

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Key words: trade in value added, gross trade, input-output

Fundamental theorem on the relationship between trade balances in value added and gross terms

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1. Introduction

Facing the development of intermediate goods trade, Johnson and Noguera (2012) and WTO and IDE JETRO (2011) addressed the new concept of trade in value added (TiVA) in place of conventional trade in gross terms. WTO and OECD also provided empirical results based on some international input-output tables. The global trade network captured and generated by TiVA is called global value chains (GVC). The new concept of value added exports from an origin country to a destination country is defined as the origin county's value added induced by the destination country's final demand, excluding intermediate goods exports, for the world. In this paper, using a general framework, we prove a fundamental theorem on the identity between the total sum of a country's trade balances in value added and that in gross terms (conventional net exports). This identity also implies that the total sum of differentials between balances in value added and those in gross terms equals zero. Employing an aggregated World Input-Output data (WIOD) of Groningen University with eight countries (BRICs, the USA, the EU, Japan and the rest of the world (ROW)) and 20 sectors for 2010, we show an empirical evidence of the theorem.

2. Model and definition of TiVA

2.1. Model

Following Isard (1951), WTO and IDE (2011), and Johnson and Noriega (2012), we reproduce an inter-country multi-sector model in a general framework.[1]

We assume there are r,s =1, 2, …, R countries (areas or regions) each of which produces and inputs $r(i)$, $s(j)$=1,2, .., n products. We assume the classical Leontief open input-output model with fixed input coefficients and final demand for each country. In this model each sector produces a single commodity without joint production. We regard the last country R as the rest of the world (ROW). We consider an international input-output system not in physical terms but in *value terms*.

We denote: $\boldsymbol{A}_{rs}= (a_{r(i)s(j)})$ $(n\times n)$: country r's export coefficient matrix to country s or country s's import coefficient matrix from country r if $r\neq s$, and country r's input coefficient matrix of domestically produced intermediate goods if s=r; $\boldsymbol{Y}_r=[Y_{r(i)}]$ $(n\times 1)$: country r's final demand vector in an international input-output table; $\widetilde{\boldsymbol{Y}}_r = [\widetilde{\boldsymbol{Y}}_{r(i)}](n\times 1)$: country r's final demand vector, including exports of intermediate goods, in each country's input-output system; $\boldsymbol{Y}_{rs}= [Y_{r(i)s}]$ $(n\times 1)$: country s's final demand vector for country r $(n\times 1)$ or country r's final goods export vector to country s if $r\neq s$; $\boldsymbol{F}_s = [\boldsymbol{Y}_{rs}]$ $((n\times R)\times 1)$: country s's final demand vector for all countries; $\boldsymbol{X}_r= [X_{r(i)}]$ $(n\times 1)$: country r's output vector ; $\boldsymbol{X}= [\boldsymbol{X}_r]$ $((n\times R)\times 1)$: an overall output vector ; $\boldsymbol{I}$: an $(n\times R)$ dimensional identity matrix; $\boldsymbol{I}_n$: an n dimensional identity matrix. We assume that non-negative matrixes $\boldsymbol{A}$ and $\boldsymbol{A}_{rr}$ are productive.

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[1] The model below is essentially equivalent to models presented by Johnson and Noriega (2012) except for our explicit exposition of a dual price system associated with an input-output system.

Denoting $\boldsymbol{X}^*$ as the equilibrium output vector, the global equilibrium (market clearing) condition for an Isard type of non-competitive inter-country multi-sector input-output table in value terms can be written as:

$$\boldsymbol{X}^* = \boldsymbol{A}\boldsymbol{X}^* + \boldsymbol{Y};\ \boldsymbol{X}^* = \boldsymbol{B}\boldsymbol{Y}, \text{where } \boldsymbol{B} = (\boldsymbol{I}-\boldsymbol{A})^{-1}, \tag{1}$$

where

$$\boldsymbol{A} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1s} \dots & A_{1R} \\ \dots & \dots & \dots & \dots & \dots \\ A_{r1} & A_{r1} & \dots & A_{rs} \dots & A_{rR} \\ \dots & \dots & \dots & \dots & \dots \\ A_{R1} & A_{R2} & \dots & A_{Rs} \dots & A_{RR} \end{bmatrix},$$

$$\boldsymbol{B} = (\boldsymbol{I}-\boldsymbol{A})^{-1} = \begin{bmatrix} \boldsymbol{B}_{11} & \boldsymbol{B}_{12} & \dots & \boldsymbol{B}_{1s} & \boldsymbol{B}_{1R} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{B}_{r1} & \boldsymbol{B}_{r1} & \dots & \boldsymbol{B}_{rs} & \boldsymbol{B}_{rR} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{B}_{R1} & \boldsymbol{B}_{R2} & \dots & \boldsymbol{B}_{Rs} & \boldsymbol{B}_{RR} \end{bmatrix},$$

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{Y}_1 \\ \dots \\ \boldsymbol{Y}_r \\ \dots \\ \boldsymbol{Y}_R \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{11} \\ \dots \\ \boldsymbol{Y}_{r1} \\ \dots \\ \boldsymbol{Y}_{R1} \end{bmatrix} + \dots + \begin{bmatrix} \boldsymbol{Y}_{1s} \\ \dots \\ \boldsymbol{Y}_{rs} \\ \dots \\ \boldsymbol{Y}_{Rs} \end{bmatrix} + \dots + \begin{bmatrix} \boldsymbol{Y}_{1R} \\ \dots \\ \boldsymbol{Y}_{rR} \\ \dots \\ \boldsymbol{Y}_{RR} \end{bmatrix} = \boldsymbol{F}_1 + \dots + \boldsymbol{F}_s + \dots + \boldsymbol{F}_R; \boldsymbol{X} = \begin{bmatrix} X_1 \\ \dots \\ X_r \\ \dots \\ X_R \end{bmatrix}.$$

Overall output $\boldsymbol{X}^*_{*s}$ and country *r*'s output $\boldsymbol{X}^*_{r*s}$, induced by a fixed destination country $*s$'s final demand $\boldsymbol{F}_{*s}$, are given by

$$\boldsymbol{X}^*_{*s} = \boldsymbol{A}\boldsymbol{X}^*_{*s} + \boldsymbol{F}_{*s} = (\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{F}_{*s};\ \boldsymbol{X}^*_{r*s} = \Sigma_k \boldsymbol{A}_{rk}\boldsymbol{X}^*_{k*s} + \boldsymbol{Y}_{r*s}\,. \tag{2}$$

This equation is essential for the definition of trade in value added.

By definitions of $\boldsymbol{F}_s$ and $\boldsymbol{Y}_{rs}$ we have

$$\boldsymbol{X}^* = \Sigma_s \boldsymbol{X}^*_{*s};\ X^*_{r*s} = \Sigma_i X^*_{r(i)*s}\,. \tag{3}$$

Country *r*'s gross exports to country *s*, $\boldsymbol{E}_{rs}$ are given by $\boldsymbol{E}_{rs} = \boldsymbol{A}_{rs}\boldsymbol{X}^*{}_s + \boldsymbol{Y}_{rs}\ (s \neq r)$. Hence, the local equilibrium (market clearing) condition that each country must satisfy is given by

$$\boldsymbol{X}^*_r = (\boldsymbol{I}_n - \boldsymbol{A}_{rr})^{-1}\widetilde{\boldsymbol{Y}}_r, = (\boldsymbol{I}_n - \boldsymbol{A}_{rr})^{-1}\left(\Sigma_{s\neq r}\boldsymbol{E}_{rs} + \boldsymbol{Y}_{rr}\right). \tag{4}$$

This can also be written as $\boldsymbol{X}^*_r = \boldsymbol{B}^r\widetilde{\boldsymbol{Y}}_r$, where $\boldsymbol{B}^r = (\boldsymbol{I}_n - \boldsymbol{A}_{rr})^{-1}$. It is noteworthy to learn that global equilibrium and local equilibria are simultaneously satisfied for the international input-output system.

2.2 Price system and definition of trade in value added

Let us define country r's i-th value added ratio as $v_{r(i)} = V_{r(i)}/X_{r(i)}$ where $V_{r(i)}$ is country r's i-th value added. Country r's value added ratio vector and the overall value added vector are $\boldsymbol{v}_r = (v_{r(i)})(1 \times n)$ and $\boldsymbol{v} = (\boldsymbol{v}_r)(1 \times (n \times R))$ respectively. Then, by virtue of definitions of input coefficients and value added ratios, we have

$$\boldsymbol{u} = \boldsymbol{uA} + \boldsymbol{v};\ \boldsymbol{u}_n = \boldsymbol{u}_n \Sigma_k \boldsymbol{A}_{kr} + \boldsymbol{v}_r\ . \tag{5}$$

Therefore, value added ratios are given by

$$\boldsymbol{v} = \boldsymbol{u}(\boldsymbol{I} - \boldsymbol{A});\ \boldsymbol{v}_r = \boldsymbol{u}_n(\boldsymbol{I}_n - \Sigma_k \boldsymbol{A}_{kr})\ . \tag{6}$$

where $\boldsymbol{u} =$ (1,1,…,1) $(1\times(n\times R))$ and $\boldsymbol{u}_n = (1,1,\dots,1)$ $(1\times n)$ are aggregation vectors of unities. That is to say, the price vector associated with an input-output system in value terms always equals an aggregation vector.

The new concept of value added trade is defined as follows.

Definition 1. The new concept of value added exports and trade balance: Johnson and Noguera (2012), and WTO and IDE (2011)

Country r's value added exports to country s are defined as $\widehat{\boldsymbol{V}}_r \boldsymbol{X}^*_{rs}$ where $\widehat{\boldsymbol{V}}_r = diag\left\{v_{r(1)}, \dots, v_{r(n)}\right\}$ $(n\times n)$. The total value added exports of origin country r to destination country s amounts to $\boldsymbol{u}_n \widehat{\boldsymbol{V}}_r \boldsymbol{X}^*_{rs} = \boldsymbol{v}_r \boldsymbol{X}^*_{rs}$. Country r's value added trade balance with country s is then

$$T^{va}_{rs} = \boldsymbol{u}_n \widehat{\boldsymbol{V}}_r \boldsymbol{X}^*_{rs} - \boldsymbol{u}_n \widehat{\boldsymbol{V}}_s \boldsymbol{X}^*_{sr} = \boldsymbol{v}_r \boldsymbol{X}^*_{rs} - \boldsymbol{v}_s \boldsymbol{X}^*_{sr}\ . \tag{7}$$

Country r's gross trade balance with country s is

$$T^{g}_{rs} = \boldsymbol{u}_n(\boldsymbol{E}_{rs} - \boldsymbol{E}_{sr}) = \boldsymbol{u}_n(\boldsymbol{A}_{rs}\boldsymbol{X}^*{}_s + \boldsymbol{Y}_{rs}) - \boldsymbol{u}_n(\boldsymbol{A}_{sr}\boldsymbol{X}^*{}_r + \boldsymbol{Y}_{sr})\ .\ (s \neq r) \tag{8}$$

Based on Definition 1, Johnson and Noguera (2012), and WTO and IDE (2011) tried to demonstrate empirical results of the relationship between value added trade balances and gross trade balances. However, rather surprisingly, they did not report any theoretical result and implication of this relationship mainly due to the complexity of equations.

2.3. Fundamental theorem

Following Johnson and Noguera (2012, § 2.2.3), we consider a simple but important case with two countries (r, s =1,2) and multi-sectors ($r(i)$, $s(j)$=1,2,…,n). Then we have

$$\boldsymbol{X}_1^* = \boldsymbol{X}_{11}^* + \boldsymbol{X}_{12}^*;\ \boldsymbol{X}_{12}^* = \boldsymbol{X}_1^* - \boldsymbol{X}_{11}^* \text{ and } \boldsymbol{X}_2^* = \boldsymbol{X}_{21}^* + \boldsymbol{X}_{22}^*;\ \boldsymbol{X}_{21}^* = \boldsymbol{X}_2^* - \boldsymbol{X}_{22}^*.$$

Equation (4) can be written as

$$\boldsymbol{X}_1^* = (\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}(\boldsymbol{E}_{12} + \boldsymbol{Y}_{11}) \text{ and } \boldsymbol{X}_2^* = (\boldsymbol{I}_n - \boldsymbol{A}_{22})^{-1}(\boldsymbol{E}_{21} + \boldsymbol{Y}_{22}).$$

Using equations (2) and (4), we have

$$\boldsymbol{X}_{11}^* = \boldsymbol{A}_{11}\boldsymbol{X}_{11}^* + \boldsymbol{A}_{12}\boldsymbol{X}_{21}^* + \boldsymbol{Y}_{11} = (\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}(\boldsymbol{A}_{12}\boldsymbol{X}_{21}^* + \boldsymbol{Y}_{11}).$$

$$\boldsymbol{X}_{22}^* = \boldsymbol{A}_{21}\boldsymbol{X}_{12}^* + \boldsymbol{A}_{22}\boldsymbol{X}_{22}^* + \boldsymbol{Y}_{22} = (\boldsymbol{I}_n - \boldsymbol{A}_{22})^{-1}(\boldsymbol{A}_{21}\boldsymbol{X}_{12}^* + \boldsymbol{Y}_{22})$$

Therefore, we have

$$\boldsymbol{X}_{12}^* = \boldsymbol{X}_1^* - \boldsymbol{X}_{11}^* = (\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}(\boldsymbol{E}_{12} - \boldsymbol{A}_{12}\boldsymbol{X}_{21}^*),$$

$$\boldsymbol{X}_{21}^* = \boldsymbol{X}_2^* - \boldsymbol{X}_{22}^* = (\boldsymbol{I}_n - \boldsymbol{A}_{22})^{-1}(\boldsymbol{E}_{21} - \boldsymbol{A}_{21}\boldsymbol{X}_{12}^*).$$

When we add imports $\boldsymbol{A}_{21}\boldsymbol{X}_{12}^*$ induced by output transfer $\boldsymbol{X}_{12}^*$ to value added exports, in view of equations (4) and (5) and we have

$$\boldsymbol{v}_1\boldsymbol{X}_{12}^* + \boldsymbol{u}_n\boldsymbol{A}_{21}\boldsymbol{X}_{12}^* = \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{11} - \boldsymbol{A}_{21})\boldsymbol{X}_{12}^* + \boldsymbol{u}_n\boldsymbol{A}_{21}\boldsymbol{X}_{12}^*\ .$$

$$= \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{11})\boldsymbol{X}_{12}^* = \boldsymbol{u}_n(\boldsymbol{E}_{12} - \boldsymbol{A}_{12}\boldsymbol{X}_{21}^*).$$

Similarly, by virtue of $\boldsymbol{v}_2 = \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{12} - \boldsymbol{A}_{22})$, we have

$$\boldsymbol{v}_2\boldsymbol{X}_{21}^* + \boldsymbol{u}_n\boldsymbol{A}_{12}\boldsymbol{X}_{21}^* = \boldsymbol{u}_n(\boldsymbol{E}_{21} - \boldsymbol{A}_{21}\boldsymbol{X}_{12}^*).$$

Hence, we arrive at the following important result:

$$\begin{aligned} T_{12}^{va} &= \boldsymbol{v}_1\boldsymbol{X}_{12}^* - \boldsymbol{v}_2\boldsymbol{X}_{21}^* \\ &= \boldsymbol{u}_n(\boldsymbol{E}_{12} - \boldsymbol{A}_{12}\boldsymbol{X}_{21}^*) - \boldsymbol{u}_n\boldsymbol{A}_{21}\boldsymbol{X}_{12}^* - \boldsymbol{u}_n(\boldsymbol{E}_{21} - \boldsymbol{A}_{21}\boldsymbol{X}_{12}^*) + \boldsymbol{u}_n\boldsymbol{A}_{12}\boldsymbol{X}_{21}^* \\ &= \boldsymbol{u}_n(\boldsymbol{E}_{12} - \boldsymbol{E}_{21}) = T_{12}^{g}\,. \end{aligned}$$

We can generalize this result for the case with many countries and many sectors.

Fundamental theorem Identity between the total sum of a country's trade balances with many countries in value added and that in gross terms: Stehrer (2012), Benedetto (2012) and Kuboniwa (2014). For $s \neq r$

$$T_{r1}^{va} + T_{r2}^{va} + \cdots + T_{rs}^{va} + \cdots + T_{rR}^{va} = T_{r1}^{g} + T_{r2}^{g} + \cdots + T_{rs}^{g} + \cdots + T_{rR}^{g}; \quad (9)$$

$$(T_{r1}^{va} - T_{r1}^{g}) + (T_{r2}^{va} - T_{r2}^{g}) + \cdots + (T_{rs}^{va} - T_{rs}^{g}) + \cdots + (T_{rR}^{va} - T_{rR}^{g}) = 0 \quad (10)$$

Proof

We consider origin country 1's trade with destination countries 2, 3, …,*s*,… *R* (*r*=1; *s*=2, 3, …,*R*) without loss of generality. Then, by virtue of equations (1) to (5) and the definition of $\boldsymbol{E}_{rs} = \boldsymbol{A}_{rs}\boldsymbol{X}_s^* + \boldsymbol{Y}_{rs}$ $(s \neq r)$, we have

$$\begin{aligned}
& T_{12}^{va} + T_{13}^{va} + \cdots + \cdots + T_{1R}^{va} \\
&= \boldsymbol{v}_1(\boldsymbol{X}_{12}^* + \boldsymbol{X}_{13}^* + \cdots + \boldsymbol{X}_{1R}^*) - (\boldsymbol{v}_2\boldsymbol{X}_{21}^* + \boldsymbol{v}_3\boldsymbol{X}_{31}^* + \cdots + \boldsymbol{v}_R\boldsymbol{X}_{R1}^*) \\
&= \boldsymbol{v}_1\boldsymbol{X}_1^* - \boldsymbol{v}_1\boldsymbol{X}_{11}^* - (\boldsymbol{v}_2\boldsymbol{X}_{21}^* + \boldsymbol{v}_3\boldsymbol{X}_{31}^* + \cdots + \boldsymbol{v}_R\boldsymbol{X}_{R1}^*) \\
&= \boldsymbol{v}_1\boldsymbol{X}_1^* - \boldsymbol{v}\boldsymbol{B}\boldsymbol{F}_1 = \boldsymbol{v}_1\boldsymbol{X}_1^* - \boldsymbol{u}(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{B}\boldsymbol{F}_1 = \boldsymbol{v}_1\boldsymbol{X}_1^* - \boldsymbol{u}\boldsymbol{F}_1 \\
&= \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{11})\boldsymbol{B}^1\widetilde{\boldsymbol{Y}}_1 - \boldsymbol{u}_n\Sigma_{k\neq 1}\boldsymbol{A}_{k1}\boldsymbol{X}_1^* - \boldsymbol{u}\boldsymbol{F}_1 \\
&= \boldsymbol{u}_n(\Sigma_{s\neq 1}\boldsymbol{E}_{1s} + \boldsymbol{Y}_{11}) - \boldsymbol{u}_n(\Sigma_{s\neq 1}\boldsymbol{E}_{s1} - \Sigma_{s\neq 1}\boldsymbol{Y}_{s1}) - \boldsymbol{u}_n\,\Sigma_s\boldsymbol{Y}_{s1} \\
&= \boldsymbol{u}_n(\Sigma_{s\neq 1}\boldsymbol{E}_{1s} - \Sigma_{s\neq 1}\boldsymbol{E}_{s1}) \\
&= T_{12}^{g} + T_{13}^{g} + \cdots + \cdots + T_{1R}^{g}.
\end{aligned}$$

Q.E.D.

The first three lines of the above expansion are in principle equivalent to the mathematical exposition in Stehrer (2012, p.4) whereas he did not provide an explicit exposition of the local equilibrium condition of equation (3). Although Johnson and Noguera (2012) explicitly considered both global equilibrium and local equilibria, they did not provide any theoretical proposition of the relationship between trade in value added and gross trade. Benedetto (2012) reached an important insight shown by Kuboniwa (2014), while he did not provide his definition of value added trade or any mathematical exposition of his insight. Furthermore, his empirical results without international input-output data were incompatible with the recent research to which he referred.[2] Kuboniwa (2014) provided a mathematical proof and an empirical result for the case with two countries and many sectors. A country's trade balances with its

[2] The importance of Stehrer (2012) and Benedetto (2012) was suggested by William Powers's helpful comment on Kuboniwa (2014).

partners can always be summarized by that with one aggregate partner including the rest of the world. He also suggested the zero-sum relation of differentials between a country's trade balances with many countries in value added and gross terms. In addition, he demonstrated empirical results for the case with three countries (China, the USA and ROW) and many sectors. However, the paper did not provide a general mathematical proof of his suggestion and empirical results.

The above theorem is "fundamental" in twofold senses. First, this theorem clearly links trade in value added with conventional gross trade. Sectoral trade balances in value added differ from those in gross concept, depending upon sectoral value added ratios and international input-output relations within the macro identity shown by the theorem. Second, a country's GDP on the expenditure side, which incorporates a conventional trade balance as an essential element, is free from so called double accounting problems of gross trade. The theorem ensures that the paradigm shift from gross to value added trade does not change the GDP concept on the expenditure side at all.

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3. Empirical results

We employ an aggregated version of WIOD (World Input-Output Database) of Groningen University (Timmer *et al.*, 2012) with eight countries (BRICs, the USA, the EU, Japan and the rest of the world (ROW)) and 20 sectors for 2010. In WIOD, there are several vectors including net tax on products and international transport margins, which are not distributed to intermediate quadrant or value added one. We aggregate these undistributed vectors into a single dummy vector. We define country r's i-th dummy ratio as $d_{r(i)} = D_{r(i)}/X_{r(i)}$ where $D_{r(i)}$ is country r's i-th dummy value. Country r's dummy ratio vector and the overall dummy vector are $\boldsymbol{d}_r = (d_{r(i)})(1 \times n)$ and $\boldsymbol{d} = (\boldsymbol{d}_r)(1 \times (n \times R))$ respectively. Then, by virtue of definitions of input coefficients and value added ratios, equation (5) can now be rewritten as

$$\boldsymbol{u} = \boldsymbol{u}\boldsymbol{A} + \boldsymbol{d} + \boldsymbol{v};\ \boldsymbol{u}_n = \boldsymbol{u}_n \Sigma_k \boldsymbol{A}_{kr} + \boldsymbol{d}_r + \boldsymbol{v}_r\ . \qquad (5')$$

Accordingly, in the world with many countries and many sectors, equation (9) (r=1) is rewritten as

$$(T_{12}^{va} + \boldsymbol{d}_1\boldsymbol{X}_{12}^{*} - \boldsymbol{d}_2\boldsymbol{X}_{21}^{*}) + \cdots + (T_{1s}^{va} + \boldsymbol{d}_1\boldsymbol{X}_{1s}^{*} - \boldsymbol{d}_s\boldsymbol{X}_{s1}^{*}) + \cdots + (T_{1R}^{va} + \boldsymbol{d}_1\boldsymbol{X}_{1R}^{*} - \boldsymbol{d}_R\boldsymbol{X}_{R1}^{*})$$
$$= T_{12}^{g} + T_{13}^{g} + \cdots + \cdots + T_{1R}^{g}. \qquad (9')\ .$$

We call the terms of the left-hand side of this equation, $(T_{1s}^{va} + \boldsymbol{d}_1\boldsymbol{X}_{1s}^{*} - \boldsymbol{d}_s\boldsymbol{X}_{s1}^{*})$, as the value added trade balance adjusted for the dummy sector.

Tables 1 and 2 show our empirical result for eight countries by country and sector for 2010 when China is our main origin country (r=China). Fig. 1 summarizes this result by country. As shown by Table 1, the total sum of China's trade balances (adjusted for dummy sector) with eight countries in value added, 310.1 bln US$, exactly equals that in gross terms. Our theorem is clearly justified. As can be seen from Table 2 and Fig.1, China's trade balances with the EU, Russia, India and the USA are 35%, 28%, 23% and 21% smaller respectively when measured in value added (before inclusion of dummy sector) whereas those with Japan, ROW and Brazil are 23%, 125%, 636% larger respectively. Trade balance differentials for the EU, Russia and the USA with the paradigm shift from gross to value added are -42 bln US$, -6 bln US$ and -48 bln US$ respectively. The largest absolute differential size is recorded by the USA, followed by the EU. It is noteworthy that the relative differential change for the EU is much higher than that for the USA although a reduction of the USA-China imbalance with the paradigm shift has often be discussed in the literature. China's trade imbalance with Japan is improved by more than 20% with the paradigm shift. China's trade balance with ROW is dramatically improved from an imbalance of 91 bln US$ to a positive balance of 23 bln US$ with the paradigm shift, which cancels out most of decreases in trade balances with the paradigm shift. China's trade balance change for Brazil with the paradigm shift is huge due to the negligible gross balance of 0.4 bln US$.

Let us look at empirical results by sector for 2010. China's trade balances of agriculture and mining are larger when measured in value added. China's largest import partner of mining (including crude oil and gas) is ROW, followed by Brazil and Russia.

The trade imbalances of mining with ROW, Brazil and Russia are largely improved by, 33%, 78% and 31%, respectively, when measured in value added. The trade balance of food except for Japan is also larger with the paradigm shift. The trade balance of textile except for India is much smaller with the paradigm shift due to textile's low value added ratio. The trade balances of the wooden products and the pulp and paper except for Japan are larger when measured in value added. The trade balance of chemicals except for Brazil and India is larger with the paradigm shift. The trade balance of oil products except for Brazil and Russia is larger when measured in value added. The gross trade balances of oil products with Brazil and Russia are rather small (0.1 bln US$ and -0.5 bln US$ respectively) in comparison with the case of ROW of -18 bln US$. The trade balance of rubber products except for Japan and Russia is larger when measured in value added. The trade balance of non-metallic minerals except for Japan and the USA are smaller with the paradigm shift. The trade balance of metals is larger without exceptions when measured in value added. The trade balances of industrial machinery with Japan, the EU and ROW are much larger when measured in value added. The trade balance of electrical equipment except for Japan is much smaller with the paradigm shift due to the electrical equipment's low value added ratio. The trade imbalances of transport equipment with major import partners (Japan, the EU and the USA) are much smaller with the paradigm shift due to generally low value added ratios of auto production in any country. The balances of the trade and transport except for Brazil and Russia are smaller when measured in value added. Directions of trade balance changes for China's typical export sectors with low value added ratios such as textile and electrical equipment are not uniform across countries although exceptions cannot change the dominant directions.

Table 1. China's trade balances with destinations of BRICs, the USA, the EU, Japan and ROW

(in billion US$)

	Gross trade balance								Value added trade balance							
	Destination								Destination							
	Brazil	India	Japan	EU	Russia	USA	ROW	World	Brazil	India	Japan	EU	Russia	USA	ROW	World
Agriculture	-7.4	-2.4	1.9	0.9	-0.3	-8.9	-22.3	-38.6	-3.1	0.7	10.0	18.8	4.7	15.2	13.5	59.8
Mining	-11.6	-1.7	0.8	0.0	-10.1	-0.1	-218.5	-241.1	-2.5	0.9	5.5	11.9	-6.7	13.1	-150.3	-128.1
Food	-1.3	-0.3	7.8	1.9	0.5	3.3	-3.8	8.1	-0.1	0.7	3.6	5.1	1.1	5.5	6.0	21.8
Textile	2.0	1.5	18.0	46.6	24.4	45.7	81.8	220.2	1.0	1.7	7.8	21.0	10.0	21.9	30.7	94.0
Wooden products	0.0	0.1	1.3	1.8	-0.3	1.6	-0.4	4.1	0.1	1.1	0.8	1.8	0.0	2.8	1.7	8.4
Pulp and paper	-1.0	0.2	-0.3	-1.6	-0.7	-1.1	-3.2	-7.6	-0.2	0.6	-0.6	0.8	0.1	2.3	1.3	4.3
Chemicals	2.0	4.2	-8.1	-5.6	-0.9	2.5	-45.5	-51.3	1.0	2.5	-1.6	5.9	1.3	9.8	1.6	20.5
Oil products	0.1	0.1	-1.2	-0.3	-0.5	-0.2	-17.9	-19.9	0.0	0.4	-1.1	2.6	-0.6	1.6	-0.3	2.5
Rubber products	0.7	1.2	-0.7	6.9	0.2	9.8	14.7	32.8	0.5	1.1	-0.5	3.8	0.6	6.9	7.3	19.7
Non-metallic minerals	0.3	1.1	-0.5	2.9	0.4	3.2	6.1	13.5	0.2	0.7	-0.2	2.3	0.3	3.3	4.2	10.7
Metals	0.3	0.7	-10.6	3.8	-1.0	7.3	-0.1	0.5	0.8	2.6	-6.2	5.8	0.4	17.0	14.3	34.8
Industrial machinery	2.2	6.8	-15.1	-26.0	3.6	14.7	5.5	-8.2	1.0	2.4	-5.6	-8.7	1.4	6.8	8.4	5.7
Electrical equipment	12.9	8.1	-10.3	87.3	7.4	131.8	17.9	255.1	3.4	2.7	-9.5	15.6	2.3	22.7	6.1	43.3
Transport equipment	0.3	2.2	-14.4	-16.8	2.3	-1.8	33.5	5.3	0.3	0.8	-5.7	-2.9	0.9	2.1	10.5	5.9
Other manufacturing	0.3	20.8	1.2	7.2	0.5	14.8	-0.7	44.1	0.2	6.9	0.8	3.1	0.3	6.6	1.6	19.5
Utilities	0.0	0.0	0.1	-0.6	0.0	0.2	-0.4	-0.7	0.3	1.3	0.1	4.5	0.4	8.3	7.7	22.6
Construction	0.0	0.0	0.0	0.4	0.0	0.0	1.7	2.1	-0.1	-0.1	-1.1	-3.1	-0.1	-0.4	-0.7	-5.5
Trade and transport	0.6	0.9	14.4	30.3	-4.0	-1.2	61.7	102.8	0.7	3.6	0.4	16.3	-3.3	24.0	44.1	85.8
Other services	-0.1	0.6	2.9	-19.0	-0.5	18.4	-0.7	1.5	-0.6	3.4	-6.3	-26.0	2.2	17.3	16.1	6.1
Public administration	0.0	0.0	0.0	-0.9	0.0	-10.9	-0.8	-12.6	-0.1	0.0	-0.3	-1.4	-0.4	-5.1	-1.1	-8.3
Total excl. dummy	0.4	44.1	-12.6	119.4	20.9	229.2	-91.3	310.1	2.8	33.8	-9.8	77.1	15.0	181.6	23.0	323.6
Dummy sector									-1.7	-0.5	0.4	-12.9	-0.8	4.4	-2.4	-13.5
Total	0.4	44.1	-12.6	119.4	20.9	229.2	-91.3	310.1	1.1	33.3	-9.3	64.2	14.2	186.1	20.5	310.1

Source: Author's calculation based on an aggregated WIOD for 2010.

Table 2. China 's trade balance differentials between value added and gross terms

	Trade balance differential (in billion US$)								differential : gross balance (%)							
	Destination								Destination							
	Brazil	India	Japan	EU	Russia	USA	ROW	World	Brazil	India	Japan	EU	Russia	USA	ROW	World
Agriculture	4.4	3.1	8.1	17.8	5.0	24.1	35.8	98.3	58.6	130.8	436.8	*	*	270.4	160.6	255.0
Mining	9.1	2.6	4.7	12.0	3.3	13.2	68.2	113.0	78.4	152.4	603.8	*	33.0	*	31.2	46.9
Food	1.2	1.0	-4.3	3.1	0.6	2.2	9.8	13.7	94.4	311.8	-54.4	163.0	129.8	64.4	255.9	168.8
Textile	-1.0	0.1	-10.2	-25.7	-14.4	-23.8	-51.0	-126.1	-52.0	7.5	-56.7	-55.1	-59.1	-52.1	-62.4	-57.3
Wooden products	0.1	1.0	-0.5	0.0	0.3	1.1	2.1	4.3	325.6	894.9	-35.2	1.5	112.2	69.5	506.4	105.3
Pulp and paper	0.7	0.4	-0.3	2.4	0.8	3.3	4.5	11.9	76.5	211.0	-129.5	148.3	121.4	307.5	142.8	156.8
Chemicals	-1.0	-1.7	6.5	11.5	2.2	7.3	47.1	71.9	-50.2	-40.2	80.2	205.9	241.4	290.7	103.6	140.0
Oil products	-0.1	0.3	0.1	2.8	-0.1	1.8	17.6	22.4	-89.2	276.5	9.8	*	-21.6	786.9	98.2	112.6
Rubber products	-0.3	-0.1	0.2	-3.1	0.4	-2.9	-7.4	-13.2	-35.8	-9.9	22.8	-45.0	173.3	-29.3	-50.4	-40.2
Non-metallic minerals	0.0	-0.4	0.2	-0.6	-0.1	0.2	-1.9	-2.7	-14.7	-39.7	50.3	-21.7	-24.0	5.2	-31.6	-20.2
Metals	0.5	1.9	4.4	2.0	1.4	9.7	14.3	34.2	183.5	253.9	41.5	52.8	145.6	132.0	*	*
Industrial machinery	-1.3	-4.4	9.5	17.3	-2.1	-7.9	2.9	13.9	-56.5	-64.9	62.8	66.5	-59.6	-54.0	53.2	169.5
Electrical equipment	-9.5	-5.4	0.8	-71.7	-5.1	-109.1	-11.9	-211.8	-73.4	-66.5	7.5	-82.1	-68.7	-82.8	-66.1	-83.0
Transport equipment	-0.1	-1.3	8.6	13.8	-1.4	3.9	-23.0	0.6	-19.1	-61.0	60.2	82.4	-61.4	216.0	-68.6	12.1
Other manufacturing	-0.1	-13.9	-0.4	-4.1	-0.2	-8.2	2.3	-24.6	-41.0	-66.7	-33.0	-56.5	-47.5	-55.5	335.6	-55.7
Utilities	0.3	1.3	0.1	5.2	0.4	8.1	8.0	23.3	*	*	169.9	856.3	*	*	*	*
Construction	-0.1	-0.1	-1.1	-3.4	0.0	-0.4	-2.4	-7.6			**	-941.4	-433.0		-139.9	-366.1
Trade and transport	0.1	2.6	-14.0	-14.1	0.7	25.2	-17.5	-16.9	22.2		-97.2	-46.4	17.7	*	-28.4	-16.5
Other services	-0.5	2.8	-9.2	-7.0	2.8	-1.1	16.8	4.5	-502.9	501.0	-318.5	-36.8	504.5	-6.0	*	295.6
Public administration	0.0	0.0	-0.3	-0.6	-0.4	5.7	-0.3	4.2	-664.1		**	-63.3	**	52.8	-33.2	33.7
Total excl. dummy	2.4	-10.3	2.8	-42.2	-5.9	-47.6	114.3	13.5	635.6	-23.4	22.5	-35.4	-28.3	-20.8	125.1	4.4
Dummy sector	-1.7	-0.5	0.4	-12.9	-0.8	4.4	-2.4	-13.5								
Total	0.7	-10.8	3.3	-55.2	-6.7	-43.1	111.8	0.0	184.5	-24.5	26.0	-46.2	-32.2	-18.8	122.5	0.0

Source: Author's calculation based on an aggregated WIOD for 2010.

Differential-gross balance ratio is calculated as (-1)×differential/gross balance×100 if gross balance < 0 so that we should make the ratio meaningful.

(*) denotes the differential to gross balance ratio > 1000%. (**) denotes the ratio < -1000%.

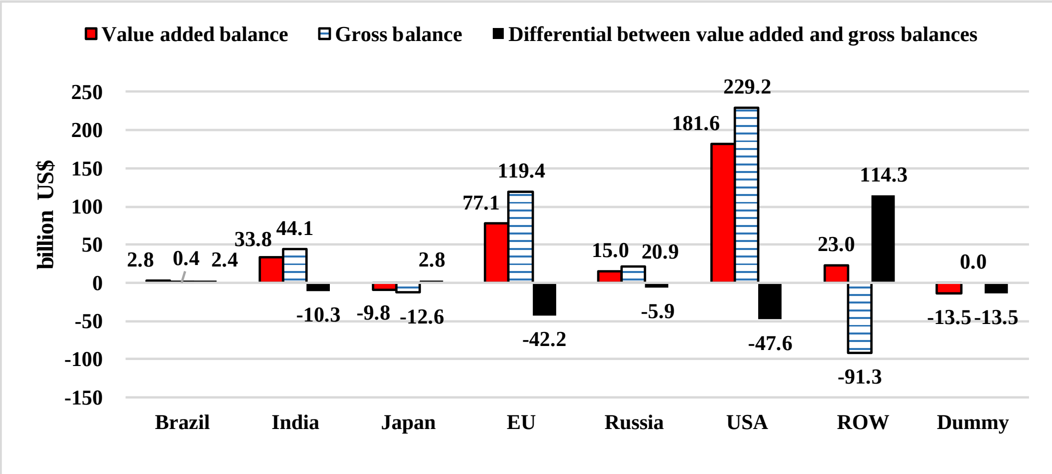

Fig. 1. China's trade balances with BRICs, the EU, the USA, Japan and ROW: 2010

4. Concluding remarks

Growing intermediate goods trade in the world needs further developments of theoretical and empirical investigations in international trade. Responding to this task, Kuboniwa (2014) tried to further develop Jonhson and Noguera (2012)'s theoretical and empirical studies on their new concept of trade in value added. Following Stehrer (2012), Benedetto (2012) and Kuboniwa (2014), we proved theoretically and empirically that, in the many countries and many sectors world, the total sum of a country's trade balance with its partners in the value added equals that in the gross terms. This theorem led us to the fact that in world with the many countries and many sectors the differential of a country's (e.g., China) balances with its partners (e.g., the EU and the USA) in value added and gross terms must be offset by the differentials of the country's balances with other partners (e.g., ROW). In this paper we only amended Kuboniwa (2014) for the new concept of trade in value added, considering a general theoretical framework and the case with eight countries and many sectors.

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