FACTOR TAXATION AND LABOR SUPPLY IN A DYNAMIC ONE–SECTOR GROWTH MODEL

By

Been-Lon Chen*
Academia Sinica and University of Tsukuba

and

Huei-Yu Chiu
National Taiwan University

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Abstract

This paper studies a class of AK-type growth models with public capital stock and elastic labor supply. The government taxes both factor incomes and conduct expenditure. To rationalize the taxation, government expenditure affects the productivity of private sectors. It shows the existence of a unique balanced-growth path, near which there is only a transitional dynamic path leading the economy toward it. It finds that while a higher capital tax rate reduces economic growth in the short run, its long-term growth effect is ambiguous, and this long-term growth effect remains ambiguous even if the level of tax rate is larger than the degree of government externality. However, a higher labor tax rate always lowers economic growth both in the short run and in the long run, despite the existence of productive government taxation. Our finding that labor taxation is always detrimental, while capital taxation may be beneficial, is in contradict to conventional wisdom that labor taxation is better than capital taxation, from the economic growth point of view.

Key words: taxation, infrastructure, economic growth, transitional dynamics
JEL classification: E6, O4.

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* Correspondence author. 1-1-1 Tennodai, Institute of Social Sciences, University of Tsukuba, Tsukuba, Ibaraki, Japan 305-8571. Email: bchen@ieas.econ.sinica.edu.tw. Phone: (0298)536132; Fax: (0298)536616; (After 2/28/2002, Institute of Economics, Academia Sinica, 128 Academic Rd, Sec.2, Taipei 11529, Taiwan. Same email).

I. Introduction

Since the interest in economic growth research has been ignited once again in the late 1980s, much attention has been paid on the long-term effects of economic policies. Among the many lines of research examining economic policies, is the growth effect from taxation.[1] Conventional wisdom modeled labor income taxation as better than capital income taxation from the growth point of view, and in extreme cases, set a zero tax rate for the capital income, raising all revenues from the labor income.[2] However, as the labor employment and capital are complementary, it is difficult to imagine that the taxation of labor income will not affect capital accumulation and thereby, discourage long-run economic growth.

The purpose of this paper is to examine the growth effect of labor income and capital income taxation. In order to isolate the labor employment factor from other considerations, this paper will not introduce either the human capital accumulation, or the learning-by-doing via the labor employment. When labor is in the form of a reproducible human capital, it is less different from physical capital. Moreover, in order to rationalize the government taxation, we will introduce productive government expenditure.

When tax policies are examined, existing literature usually includes distortionary taxation for the lump-sum transfer purpose, without assigning a proper motive for the government expenditure. One exception is the Barro model (1990) that explicitly specifies the purpose of taxation. Barro is a Ak-type model without the accumulation of government expenditure. As a result, his model can only examine the steady-state effects of capital taxation policy. Later, Futagami, Morita and Shibata (1993) extend Barro (1990) by considering the accumulation of government expenditure. As a result, they can envisage the transitional dynamics of tax policies. Yet, in these two models, capital is the only variable private input. Only the capital income is taxed. The one-sector growth models of Barro (1990) and Futagami, Morita and Shibata (1993) provide a basic framework for our purposes.

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[1] Contributions include Barro (1990), Lucas (1990), and Rebelo (1991), among others.

[2] See, for examples, Judd (1985), Chamley (1986), Lucas (1990) and Jones, Manuelli and Rossi (1997).

A major departure of our work from Barro (1990) and Futagami, Morita and Shibata (1993) is to include household elastic labor with government collection of labor income taxes, in addition to capital income taxes. We will examine both the short term and the long term effects of individual taxation policies upon labor supply and economic growth.

When the labor supply is elastic, we will find an alleviation of detrimental effects of capital taxation upon long term economic growth, even if the capital tax rate is larger than the degree of the government externality. On the other hand, we will find that a larger labor tax will always discourage economic growth both in the short run and in the long run. This latter result may be surprising, as labor is not the engine of growth in our model. In this model, there is neither the learning-by-doing effect nor the accumulation of human capital. As a result, a larger labor tax rate which diminishes economic growth is neither based upon reductions in learning-by-doing effect, nor via reductions in accumulation of human capital.

Reasons for these effects of taxation are as follows. A larger capital tax rate policy actually *raises labor demand* and via *elastic labor* conditions, which may raise labor employment in the long run. A higher capital tax rate also increases government expenditure in the long run. When these indirect effects are strong enough, they could negate the taxes' detrimental effect, thereby spurring healthy economic growth and physical capital accumulation. However, a larger labor income tax rate policy directly *reduce*s *the labor supply*, leaving labor employment in equilibrium, which in turn reduces the amount of tax revenues for the government infrastructure, under plausible parameter conditions. As both the labor employment and the government infrastructure complement the marginal productivity of capital, capital accumulation is also discouraged, leading to lower economic growth in equilibrium.

Compared with existing literature, Turnovsky (2000) is the most compatible to our work in that it examines the effects of factor taxation and other changes within an AK-type model. Two major deviations differentiate our work from Turnovsky. First, it is the flow of government expenditure, not the stock, that affects the private sector in Turnovsky. As a result, his model is intrinsically static and unable tot analyze the short-run dynamical and transitional growth effects from factor taxation. Second, Turnovsky has the effects of factor taxation examined with tax revenues rebated in a lump-sum fashion, whereas we investigate the effects with the productive government expenditure. Since labor employment and government expenditure both complement capital productivity, which is the engine of economic growth, the growth effects of factor taxation are different.[3]

Elastic labor supply is considered among the existing endogenous growth models developed by Lucas (1990), Rebelo (1991), Jones, Manuelli and Rossi (1993), Stokey and Rebelo (1995), and Kim (1998).[4] Several factors differentiate our model from theirs. First, our model is a one-sector model, not a multiple-sector model. Second, while their government expenditure is just a lump-sum transfer, it is presented as productive in stead in our model. Finally, their main concern is on quantitative growth effects of tax reform, rather than focusing on the analytical effects from different factor taxation. Also in contrast to our model, a labor income tax policy is considered beneficial to economic growth in Lucas (1990) and Manuelli and Rossi (1993), with only sector taxes, and not input taxes, modeled in Rebelo (1991) and Stokey and Rebelo (1995).

There are other related papers. Benhabib and Perli (1994) emphasize elastic labor, like in our study. Yet, they indicate it as very plausible to obtain local indeterminancy by a Lucas (1988)- type model, when considering elastic labor supply. In their paper , effects of taxation are not discussed nor examined. Caballé (1998) and Lin (1998) also investigate the effect of factor taxation on economic growth. In an overlapping-generations model with inelastic labor supply, Caballé (1998) constructs a formula for a threshold, above which zero-taxes on capital income (i.e., the income of the old) delivers faster economic growth, and below which taxing capital income leads to increased growth. Like our study, Caballe also believes that capital taxation may not have long term negative effects, but we render

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[3] For example, as an income tax rate increases together with the productive government expenditure, the RR locus in Figure 2 of Turnovsky (2000) shifts upwards. As a result, a larger labor income tax rate is probably growth enhancing.

[4] Although Rebelo (1991), Jones, Manuelli and Rossi (1993), and Stokey and Rebelo (1995) present several models, only a comparison with the model of elastic labor supply is made, as we compare them with our work below.

differences regarding the mechanisms which induce the end result. In another overlapping-generations model with human capital accumulation and inelastic labor supply, Lin (1998) finds that the positive or negative effect of labor taxation on economic growth, depends on whether savings are positively or negatively related to the labor tax rate. However, the growth effect of labor taxation in our model is entirely different from that of Lin.

Finally, King and Rebelo (1990) Bond, Wang and Yip (1994) and Mino (1996), and Milesi-Ferretti and Roubini (1998) also analyze the growth effect of taxation. These works are two-sector models and, with the exception of Milesi-Ferretti and Roubini (1998), all focus on inelastic labor supply. All these papers extend that higher tax rates hurt economic growth, no matter whether they are in the form of sectoral taxes, as in King and Rebelo (1990) or in the form of factor taxation, as in Bond, Wang and Yip (1994), Mino (1996) and Milesi-Ferretti and Roubini (1998). The main reason is that while government expenditure is neutral, taxation on labor, in the form of human capital, discourages the accumulation of human capital. As a result, labor taxation always diminishes economic growth. Barro and Sala-i-Martin (1992) also study the growth effect of taxation in several models with inelastic labor supply. They focus on which of the lump-sum taxation and the income taxation is the better way in financing government expenditure. They do not differentiate from different input income taxation.

In addition to this Section, the organization of this paper is as follows. While Section II sets up the model, Section III analyzes the balanced-growth and transitional dynamic paths of the model in equilibrium. Section IV examines the short-run and long-run growth effect of factor taxation policies. Section V concludes the paper.

II. A Basic Model

Our basic model draws on Barro (1990). Consider an economy populated by households and firms. Time (indexed by *t*) is continuous. There exists a continuum of infinite-lived representative households. There is no population growth, and the size of population is normalized to be unity. There exists a continuum of representative firms and each firm is endowed with a production technology.

Additionally, there is a government.

1. Household's Problem

The representative household is assumed to possess a discounted lifetime utility of the following form:[5]

$$\int_0^{\infty} e^{-\rho t}\,[\ln c(t) - \frac{l(t)^{1-\theta}}{1-\theta}]\,dt, \quad \rho > 0,\ \theta < 0,$$

in which $\rho>0$ is the instantaneous time-preference rate, and $\theta < 0$ is the reciprocal of the intertemporal elasticity of substitution for working/leisure. We assume $\theta < 0$ so that the marginal disutility of working increases in labor employment. The function $c(t)$ is the instantaneous private consumption expenditure in t, and $l(t)$ is the instantaneous labor supply. The intertemporal elasticity of substitution for consumption is set to 1, in order to guarantee the existence of a balanced growth path in the steady state.

Each representative household is endowed with one unit of labor in every period and supplies a fraction $l(t)$ of labor to work. The market wage rate is $w(t)$. A household also has wealth/capital $k(t)$ accumulated from the past. They lend capital to producers at the market interest rate of $r(t)$. Both earners of wage income and wealth/capital income must pay taxes, and their tax rates are τ_l and τ_k, respectively. Disposable income that is not consumed in each period will accumulate as wealth/capital in the next period. As a result, each representative household possesses the following budget constraint:

$$\dot{k} = (1-\tau_l)\,w(t)l(t) + (1-\tau_k)\,r(t)\,k(t) - c(t), \tag{1}$$

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[5] The same functional form is used by Benhabib and Peril (1994). An alternative instantaneous utility functional form that is consistent with a balanced growth path is $\frac{c(t)\, e^{\left(-\frac{l^{1-\theta}(t)}{1-\theta}\right)^{1-\sigma}}}{1-\sigma}$, where σ is the reciprocal of the intertemporal elasticity of substitution for consumption. This alternative form generates the same results.

where a dot notation over a variable denotes the time derivative of that variable.

The representative household chooses the consumption flow $c(t)$, the labor supply $l(t)$, and the wealth/capital accumulation $k(t)$ over time, in order to maximize its total discounted present value of lifetime utility, subject to budget constraint in (1). To solve the dynamic optimization problem, we define a present-value Hamiltonian equation and derive the following first-order conditions:

$$\frac{\dot{c}}{c} = (1-\tau_k)r(t) - \rho, \qquad \textbf{(2a)}$$

where $\lambda(t)$ is the shadow price of the wealth/capital in t.

$$l(t)^{-2} = \frac{(1-\tau_l)w(t)}{c(t)}, \qquad \textbf{(2b)}$$

$$\lim_{t\to\infty} e^{\rho t}\lambda(t)k(t) = 0, \qquad \textbf{(2c)}$$

Equation (2a) equates the period's marginal utility of consumption, and next period's marginal utility of consumption, resulting from the savings from this period. This condition determines the optimal trade-off between the consumption flow and the accumulation of capital. While (2b) equates marginal disutility of labor supply, and net marginal revenue of labor supply, in order to determine the flow of labor supply,[6] (2c) is the transversality condition that guarantees the market value of the capital stock to be eventually bounded.[7]

2. Producer's Problem

As in Barro (1990), the production technology is affected by the productive government infrastructure expenditure. The government expenditure have public-goods properties. The difference in our setting from that of Barro (1990) is that the stock of government infrastructure, and not the flow

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[6] In deriving (2b), we have used the condition that $c(t)=e^{-\rho t}/\lambda(t)$.

[7] To be precise, the transversality conditions also requires $\lim_{t\to\infty} e^{-\rho t}\lambda(t)g(t) = 0$. As $g(t)$ is linear in $k(t)$ in steady state, (2c) is sufficient to guarantee transversality conditions.

of government expenditures, affects private production in our model. This setup is also implemented in Futagami, Morita and Shibata (1993). The production of a representative firm is assumed to take the following Cobb-Douglas form:

$$y(t) = A\,k(t)^{1-\beta}\,l(t)^{\beta}\,g(t)^{\beta}, \tag{3}$$

in which *y(t)* is the instantaneous output per capita, *k(t)* is the instantaneous capital stock per capita, and *g(t)* is the per capita stock of government infrastructural services in *t*. The parameter β captures the degree of externality to which the government infrastructure affects private production, and $A>0$ is a productivity parameter summarizing the level of technology. The functional form of production technology ensures that the problem of profit maximization faced by each firm is concave and well-defined. Without loss of generality, we assume no depreciation of capital. Firms are assumed to be competitive in the goods and inputs markets.

Facing a given market rental rate, a given market wage rate, and the stock of government infrastructure, each representative producer, under endowed production technology in (3), determines demand for capital stock rental and demand for labor services in each period, in order to maximize its periodic profit flows. The necessary conditions of the optimization lead to the following two input demand schedules:

$$r(t) = A(1-\beta)k(t)^{-\beta}\,l(t)^{\beta}\,g(t)^{\beta}, \tag{4a}$$

$$w(t) = A\beta k(t)^{1-\beta}\,l(t)^{\beta-1}\,g(t)^{\beta}, \tag{4b}$$

in which (4a) equates the rental rate of capital stock and the marginal productivity of capital stock, whereas (4b) equates the wage rate and the marginal productivity of labor employment.

3. The Government's Problem

The government behaves passively in this model. It collects both the labor income taxes and the capital income taxes in each period, and then spends the total amount of tax revenues in accumulating public infrastructure stock. Like the capital stock, we assume no depreciation for the stock of government infrastructure. As a consequence, the government tax revenues become the net accumulation of government infrastructure stock, which evolves in the following manner:

$$\dot{g} = \tau_l w(t) l(t) + \tau_k r(t) k(t). \tag{5}$$

To simplify the analysis, we assume flat tax rates. Moreover, the optimal tax rates are not investigated, as the model is not analytically tractable. A quantitative assessment is not the focus of this paper.

III. Equilibrium

In equilibrium, the commodity market and the two input markets must be clearly identified in each period. The capital market equilibrium condition can be obtained by substituting the demand for capital stock in (4a) into (2a):

$$\frac{\dot{c}}{c(t)} = (1-\tau_k)A(1-\beta)\left(\frac{g(t)}{k(t)}\right)^{\beta} l(t)^{\beta} - \rho. \tag{6}$$

Similarly, substituting the labor demand in (4b) into the labor supply in (2b) yields the labor market clearing condition:

$$l(t)^{1-2-\beta} = (1-\tau_l)A\beta\left(\frac{g(t)}{k(t)}\right)^{\beta}\left(\frac{c(t)}{k(t)}\right)^{-1}. \tag{7}$$

The commodity marker is automatically satisfied if we combine the household budget constraint (1) and the government budget constraint (5), together with (4a), (4b) and (3).

We are now ready to define the equilibrium.

Definition: *A perfect foresight equilibrium (PFE) is a tuple* {*r(t), l(t), w(t)/k(t), y(t)/k(t), c(t)/k(t)*

$g(t)/k(t)$ $\dot{c}/c(t)$, $\dot{k}/k(t)$, $\dot{g}/g(t)\}$ *such that:*

(i) the representative household budget (1) satisfies;

(ii) the representative household optimizations (2a)-(2b) and transversality condition (2c)satisfy;

(iii) the technology (3) and the optimization of producers (4a)-(4b) satisfy;

(iv) the government budget (5) balances;

(v) the capital market (6) and the labor market (7) clear.

To solve the equilibrium with perpetual growth, we begin by transforming the equilibrium economic system. Define $x(t) \equiv \frac{c(t)}{k(t)}$, $z(t) \equiv \frac{g(t)}{k(t)}$, $\tau \equiv \beta\tau_l + (1-\beta)\tau_k$, and $D \equiv [A\beta(1-\tau_l)]^{\frac{1}{1-\alpha-\beta}}$. Then, (7) can be rewritten as:

$$l(t) = D\left(\frac{z(t)^{\beta}}{x(t)}\right)^{\frac{1}{1-\alpha-\beta}}. \qquad \textbf{(8)}$$

Next, we divide both sides of (1) by $k(t)$, and both sides of (5) by $g(t)$, and then substitute (4a) and (4b) into these two resulting equations. We proceed in taking a difference of these two equations, together with (8), to yield:

$$\frac{\dot{z}}{z(t)} \equiv \frac{\dot{g}}{g(t)} - \frac{\dot{k}}{k(t)} = A\tau D^{\beta}\frac{1}{z(t)}\left(\frac{z(t)^{(1-\alpha)}}{x(t)}\right)^{\frac{\beta}{1-\alpha-\beta}} - A(1-\tau)D^{\beta}\left(\frac{z(t)^{(1-\alpha)}}{x(t)}\right)^{\frac{\beta}{1-\alpha-\beta}} + x(t). \qquad \textbf{(9)}$$

Equation (9) characterizes the evolution in government infrastructure difference and in capital accumulation.

Finally, dividing both sides of (1) by $k(t)$, together with the relationships in (4a) and (4b), and then taking a difference between (6) and the resulting (1), together with (8), leads to:

$$\frac{\dot{x}}{x(t)} \equiv \frac{\dot{c}}{c(t)} - \frac{\dot{k}}{k(t)} = -A\beta(1-\tau_l)D^{\beta}\left(\frac{z(t)^{1-\alpha}}{x(t)}\right)^{\frac{\beta}{1-\alpha-\beta}} + x(t) - \rho. \qquad \textbf{(10)}$$

Equation (10) describes the evolution in household consumption difference and in capital accumulation choice.

With these transformations, the economic system is recursive and easy to solve. Equations (9) and (10) can be used to solve equilibrium $z(t)$ and $x(t)$, and after substituting the resulting equilibrium $z(t)$ and $x(t)$ into (8), gets equilibrium $l(t)$. After solving for these three variables, we can substitute $z(t)$ and $l(t)$ into (4a), (4b) and (3) to obtain $r(t)$, $w(t)/k(t)$ and $y(t)/k(t)$. We can also substitute $r(t)$ into (2a), substitute $w(t)/k(t)$, $l(t)$ and $x(t)$ into (1), and substitute $r(t)$, $w(t)/k(t)$, $l(t)$ and $z(t)$ into (5) to obtain $\dot{c}/c(t)$, $\dot{k}/k(t)$ and $\dot{g}/g(t)$ respectively. Therefore, all the endogenous variables are solved in equilibrium.

We now start with the solution of the economic system in the steady state.

1. Steady State

Definition: A steady state is a balanced growth path (BGP) of a PFE under which r, l, w/k, y/k, c/k and g/k are constant, and $\dot{c}/c$, $\dot{k}/k$ and $\dot{g}/g$ are all constant and equal over time.

In solving for BGP, we begin with the economic system in (9) and (10). Since $\frac{\dot{c}}{c}$, $\frac{\dot{k}}{k}$ and $\frac{\dot{g}}{g}$ are constant and equal along the BGP, it must be that $\frac{\dot{x}}{x} = 0$ and $\frac{\dot{z}}{z} = 0$ along the BGP. While the relationship in (10) under $\frac{\dot{x}}{x} = 0$ can be written as:

$$A[A\beta(1-\tau_l)]^{\frac{\beta}{1-\alpha-\beta}}\,\beta(1-\tau_l)\,z^{\frac{(1-\alpha)\beta}{1-\alpha-\beta}} = x^{\frac{1-\alpha}{1-\alpha-\beta}} - \rho x^{\frac{\beta}{1-\alpha-\beta}}, \tag{11}$$

the relationship in (9) under $\frac{\dot{z}}{z} = 0$ can be expressed as:

$$A[A\beta(1-\tau_l)]^{\frac{\beta}{1-\alpha-\beta}}\left(1-(\frac{1}{z}+1)[\beta\tau_l+(1-\beta)\tau_k]\right) z^{\frac{(1-\alpha)\beta}{1-\alpha-\beta}} = x^{\frac{1-\alpha}{1-\alpha-\beta}}. \tag{12}$$

For convenience, we will call the relationship in (11) as the CK (consumption- capital evolution) locus in the (*z*, *x*) plane, and that in (12), the GK (government-capital evolution) locus. To guarantee non-negative, steady-state values of *x* and *z*, the right-hand side of (11) and the left-hand side of (12) must be positive. Denote the minimal value of *x* as $x_{\min}$ and that of *z* as $z_{\min}$. Then, it is necessary to require:

Condition NS: (Nondegenerate Steady State) $x_{\min} > \left(2\&\frac{\$}{1\&2}\right)\mathrm{D} > \mathrm{D}$, and $z_{\min} > \frac{\mathrm{J}}{1\&\mathrm{J}}$.

While the requirement $x_{\min} > \mathrm{D}$ warrants the right-hand side of (11) positive, the condition $z_{\min} > \frac{\mathrm{J}}{1\&\mathrm{J}}$ guarantees the left-hand side of (12) positive. The requirement $2\&\frac{\$}{1\&2} > 1$ is to make sure that the 1-2-$ of the power in (11) and (12) is positive.

From examining the two loci in the (*z*, *x*) plane, they are very nonlinear. The CK locus intersects the vertical axis at *x*=D, and is upward-slopping and concave. Similarly, the GK locus intersects the horizontal axis at *z*=J/(1-J), and is upward-slopping and concave. See Figure 1.

The intuition for a positive-sloped CK locus is that, when the government infrastructure-capital ratio increases, output will increase. Under given tax rates, both the growth rates of consumption and capital increase. Since the effect on the capital growth rate dominates that of consumption, the consumption-capital ratio decreases. In order to return to steady state, the consumption-capital ratio must increase, thereby reducing the growth rate of consumption and raising the growth rate of capital. The reason for a positive-sloped GK locus is that, a larger government infrastructure-capital ratio increases both the growth rates of government infrastructure and that of capital stock for given tax rates, thereby lowering the ratio of government infrastructure-capital over time. In the steady state, the consumption-capital ratio needs to increase in order to (i) raise the growth rate of government infrastructure, and (ii) decrease the growth rate of capital stock, leading the government infrastructure-capital ratio back to a constant level.

Given the positive slope and the non-linearity and concavity of the CK and the GK loci, the two loci may not intersect, or may intersect more than once. To see their intersection, rewriting (11) as $A\rho^{\beta}\left(\frac{z^{1-\alpha}}{x}\right)^{\frac{\beta}{1-\alpha-\beta}} = \frac{x-\rho}{\beta(1-\tau_l)}$, and substituting this into (12) to obtain $z = \frac{\beta\tau_l+(1-\beta)\tau_k}{1-\beta-(1-\beta)\tau_k}\frac{x-\rho}{x-\Phi\rho}$, and then substituting this z expression back into (11) yields:

$$A\rho^{\beta}\left[\frac{\beta\tau_l+(1-\beta)\tau_k}{1-\beta-(1-\beta)\tau_k}\right]^{\frac{\beta(1-\alpha)}{1-\alpha-\beta}}\left[\frac{x^*-\rho}{x^*-\Phi\rho}\right]^{\frac{\beta(1-\alpha)}{1-\alpha-\beta}} x^{\frac{-\beta}{1-\alpha-\beta}} = \frac{(x^*-\rho)}{\beta(1-\tau_l)}, \qquad \textbf{(13)}$$

where $\Phi \equiv \frac{1-\beta\tau_l-(1-\beta)\tau_k}{1-\beta-(1-\beta)\tau_k} > 1$.

Under Condition NS, the left-hand side of the above expression is negative for $x \in (\rho, (1-\tau)\rho/[(1-\beta)(1-\tau)])$, and is positive and monotonically decreasing in x from infinity for $x > (1-\tau)\rho/[(1-\beta)(1-\tau)]$, whereas the right-hand side is positive for $x > \rho$, and is monotonically increasing in x. We denote them as the LHS locus and the RHS locus, respectively, as illustrate in Figure 2.[8] The unique value $x^* > (1-\tau)\rho/[(1-\beta)(1-\tau)]$ is therefore determined by Point E in the figure. In light of this, the GK locus must intersect the CK locus only once, as illustrated by Point E in Figure 1.

[Insert Figures 1 and 2 here]

The intersection of the GK and the CK loci uniquely determines the level of z^* and x^* in the steady-state equilibrium. See Point E and the corresponding z^* and x^* in Figure 1. After deriving the steady-state x^* and z^*, all other equilibrium values of endogenous variables in steady state can be obtained. We can substitute x^* and z^* into (8) to solve for l^*. Then, we substitute these values into (4a), (4b), (3), (2a), (1) and (5) to obtain the steady-state value of r^*, $(w/k)^*$, $(y/k)^*$, $(\dot{c}/c)^*$, $(\dot{k}/k)^*$, and $(\dot{g}/g)^*$.

The equilibrium BGP in the model is thus completely solved. In particular, the balanced rate of economic growth in (2a) is $\left(\frac{\dot{c}}{c}\right)^* = (1-\tau_k)r^* - \rho$.

Denote $r_{\min} \equiv A(1-\beta)z_{\min}^{\beta}\left(\frac{A(1-\tau_l)\beta z_{\min}^{\beta}}{x_{\max}}\right)^{\frac{\beta}{1-\alpha-\beta}}$, where $x_{\max}$ is the possible maximal value of x in the BGP. It suffices to consider:

[8] For clarity, we do not present the negative part of LHS locus for $x \in (\rho, (1-\tau)\rho/[(1-\beta)(1-\tau)])$ in Figure 2.

Condition NG: (Nondegenerate Growth) $(1-\tau_k)r_{\min} > \rho$.

Condition NG is sufficient (not necessary) to guarantee a nondegenerate economic growth rate for the BGP. This condition affirms if *A* is large enough.

Summarizing the above results, we obtain:

Theorem: (Existence and Uniqueness of Steady State)   *Under Conditions NS and NG, there exists a unique balanced-growth path in equilibrium.*

2. Transitional Dynamics

We now solve the transitional dynamics of economic system in the neighborhood of a balanced growth path. We linearize the system in (9) and (10) around the steady-state point (z^*, x^*) to obtain:

$$\begin{pmatrix} \dot{z} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} z-z^* \\ x-x^* \end{pmatrix}, \qquad \textbf{(14)}$$

where $a_{11} = -\frac{\tau x^*}{z^*-(1+z^*)\tau} - \frac{(1-\theta)\beta}{1-\theta-\beta}x^* < 0$, $a_{12} = \frac{(1-\theta)}{1-\theta-\beta}z^* > 0$, $a_{21} = -\frac{(1-\theta)\beta}{1-\theta-\beta}\frac{(x^*-\rho)x^*}{z^*} < 0$, and $a_{22} = \frac{(1-\theta)x^*-\beta\rho}{1-\theta-\beta} > 0$.

The local dynamic properties are characterized by two eigenvalues of the Jacobian matrix in (14). Denote *J* the Jacobian matrix, and λ_1 and λ_2 the two eigenvalues. Then, λ_1 and λ_2 satisfy:

$$\begin{cases} \lambda_1+\lambda_2 = Trace(J) = a_{11}+a_{22} = -A\tau\rho^{\beta}\frac{1}{z^*}\left(\frac{z^{*\,1-\theta}}{x^*}\right)^{\frac{\beta}{1-\theta-\beta}} - \frac{\beta}{1-\theta\beta}\rho < 0, \\ \lambda_1\lambda_2 = Det(J) = a_{11}a_{22}-a_{21}a_{12} = -\frac{\tau}{z^*-(1+z^*)\tau}\frac{(1-\theta)x^*-\beta\rho}{1-\theta-\beta} - \frac{(1-\theta)\beta\rho}{1-\theta-\beta} < 0. \end{cases}$$

The result in the determinant of the Jacobian matrix asserts exactly the following condition regarding the relative slopes of the GK locus and the CK locus; that is,

$$\left.\frac{\partial x}{\partial z}\right|_{\dot{z}=0;\, GK\ locus} \equiv \frac{-a_{11}}{a_{12}} \;>\; \left.\frac{\partial x}{\partial z}\right|_{\dot{x}=0;\, CK\ locus} \equiv \frac{-a_{21}}{a_{22}}.$$

In light of this, the economic system is saddle-path stable, and there is only a locally unique transitional dynamic path in equilibrium, leading the economy toward the steady state. The slope of the unique, transitional dynamic path is positive, as illustrate in Figure 1.

Summarizing the above results, we have:

Proposition 1. *Under Conditions NS and NG, there exists only a unique transitional dynamic path leading the economy toward the unique balanced-growth path.*

IV. Effects of Tax Policies

We now characterize the PFE by conducting comparative-static effects of the changes in tax policies and the intertemporal elasticity of substitution for labor supply on the consumption-capital ratio, the government infrastructure-capital ratio, the labor employment, the wage-capital ratio, the interest rate and the economic growth rate. In particular, we are interested to see the effect of the two tax rates on labor employment, the interest rate, and the economic growth rate in the short run and in the long run. We start with the capital taxation.

1. Capital Taxation

When the capital income tax rate is raised, the CK locus is not affected, as its detrimental effect on consumption growth and capital accumulation exactly cancels each other. The GK locus, on the other hand, shifts rightwards, as a larger capital tax rate increases government tax revenues and expenditure, and reduces disposable income and capital accumulation. (See the G′K′ locus in Figure 3).

[Insert Figure 3 here]

As the result of a larger capital tax rate, the consumption-capital ratio will increase instantaneously. Intuitively, a larger capital tax discourages savings and encourages consumption, thus inducing a larger consumption-capital ratio. A larger consumption reduces marginal utility of consumption, which also reduces the shadow price of wealth and thus, lowers marginal revenue of labor supply. As a result, the labor supply is decreased, reducing labor employment in equilibrium. Since labor input complements marginal productivity of physical capital, a lower labor employment therefore, reduces the interest rate. A higher capital income tax rate and a lower interest rate, both have direct detrimental effects on economic growth. When the labor employment is reduced, the marginal productivity of labor input is increased, allowing the wage-capital ratio to increase in the short run. In the short run, both capital stock and the government infrastructure stock are not affected, leaving the government infrastructure-capital ratio unchanged.[9] (See the summary of effects in the upper panel in Table 1.)

[Insert Table 1 here]

Over time, the smaller amount of savings under a high capital tax rate will reduce physical capital accumulation, which will further increase the consumption-capital ratio. A higher capital tax rate, on the other hand, increases government expenditure, which in turn increases the government infrastructure-capital ratio over time. While a higher stock of government infrastructure increases marginal productivity of labor, a lower stock of physical capital reduces marginal productivity of labor. The effect on labor demand over time is thus ambiguous, indicating that the effect on labor employment change is also ambiguous in the steady state. While a larger stock of the government infrastructure increases marginal productivity of capital, the ambiguous change in the labor employment will have an ambiguous effect over marginal productivity of capital. However, the positive effect of larger government infrastructure always dominates the ambiguous effect via the ambiguous labor

[9] More specifically, the short run here means the instantaneous run, where only flow variables change while stock variables remain the same.

employment. Therefore, the interest rate always increases in the steady state. To see this, substitute l^* in (8) into (4a) to yield:

$$r^* = A(1-\beta)\delta^{\beta}\left(\frac{z^{*\,1-\theta}}{x^*}\right)^{\frac{\beta}{1-\theta-\beta}}. \tag{15}$$

Since the CK locus does not change under a higher capital tax rate, the change in the interest rate must occur along the CK locus of $A\delta^{\beta}\left(\frac{z^{*\,1-\theta}}{x^*}\right)^{\frac{\beta}{1-\theta-\beta}} = \frac{x^*-\delta}{\beta(1-\tau_l)}$. Therefore, the interest rate in (15) will become:

$$r^* = \frac{(1-\beta)(x^*-\delta)}{\beta(1-\tau_l)}. \tag{16}$$

Since a larger τ_k leads to a larger x^* in steady state, marginal productivity of capital and thus interest rate, must also be larger in steady state. Although the growth rate of capital is raised as marginal productivity of capital increases, it is directly discouraged by a higher capital tax rate. As a consequence, the long-run change in economic growth is ambiguous.

Proposition 2. (Capital Taxation) *Under Conditions NS and NG, while a larger capital income tax rate reduces labor employment, interest rate and economic growth in the short run, it raises interest rate, with ambiguous effects on labor employment and economic growth in the long run.*

While most existing works find a negative, long-run growth effect of capital taxation, we obtain an ambiguous long-run growth effect. This resulting difference is because we include productive government expenditure in the model. In Barro (1990) and Futagami, Morita and Shibata (1993) where productive government expenditure is considered, the growth effect of capital taxation is negative, when the tax rate is evaluated at the degree of government expenditure externality. Yet, it is not the case in our model. In order to see this, we derive the effect of a higher capital tax rate on the output growth rate in steady state, evaluating both capital income tax rate and labor income tax rate at the government externality, i.e., $\tau_l=\tau_k=\tau=\beta$. We find that the growth effect remains ambiguous.[10] It should be noted that, when θ goes to $-\infty$, then taxing both capital and labor tax at the degree of government externality is optimal, from the economic growth point of view. Therefore, the consideration of elastic labor supply alters the otherwise held property.

2. Labor Income Tax

When labor income tax rate is raised, both the GK and the CK loci shift downwards, with the intersection of the CK locus and the vertical axis remaining unchanged, and a higher intersection of the GK locus and the horizontal axis. The reason for the downward shift of the GK locus is the same as that for a higher capital income tax rate as discussed above. The CK locus shifts downward because a higher labor income tax rate reduces disposable income, and thus also the savings and growth rate of capital stock. In steady state, the consumption-capital ratio needs to decrease to render an increase in growth rate of consumption, in order to bring the consumption-capital ratio to a constant steady-state level.

Moreover, it can be shown that the downward shift of the CK locus is higher than the GK locus when $\beta < ½$.[11] (See the $C'K'$ and the $G'K'$ loci in Figure 4.) As an AK technology, capital should be broadly interpreted as an amalgam of physical and human capital. Therefore, $\beta < ½$ is very plausible.[12]

[Insert Figure 4 here]

As a result of a higher labor tax rate, the consumption-capital ratio is reduced instantaneously. The labor supply is also reduced instantaneously under $\theta < 0$. Intuitively, when the labor tax rate increases, it has a direct negative effect on the incentive for labor supply and thus, for labor employment. A higher labor tax rate directly reduces the disposable income. Moreover, when the

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[10] See Appendix 1 for the derivation.

[11] See Appendix 2 for a derivation.

[12] Turnovsky (2000), for example, sets β=.08 when calibrating his AK model.

labor supply is reduced, the disposable income is reduced even further. A lower disposable income begins to curtail consumption, decreasing the consumption-capital ratio. A lower labor employment will reduce marginal productivity of capital and thus also the interest rate in the short run, thereby discouraging economic growth in the short run. Yet, the wage-capital ratio increases in the short run as marginal productivity of labor increases, because of lower labor employment.

While a higher labor income tax rate directly increases government expenditure, the reduction in labor employment decreases government expenditure.[13] As the indirect effect via the reduction in labor supply dominates the direct effect, government expenditure is reduced over time, thereby lowering the government infrastructure-capital ratio. Due to the fact that not only labor employment but also government infrastructure is decreased, marginal productivity of capital diminishes even further, compared to that of the short run. Therefore, interest rate is lower further in steady state, which reduces economic growth in the long run. The change in wage-capital ratio is ambiguous in steady state, due to negative effects from reduced government infrastructure-capital ratio, offset by positive effects of reduced labor employment.

Summarizing the growth effect, we obtain:

Proposition 3. (Labor Taxation) *Under Conditions NS and NG, a higher labor income tax rate reduces interest rate and economic growth in the short run, and reduces even further in the long run. It also reduces labor employment both in the short run and in the long run.*

It is interesting to compare the above growth effect with existing studies. Taxation on human capital has been found detrimental to economic growth. See, for example, Bond, Wang and Yip (1996) and Mino (1996). In these models, an economic growth reduction occurs mainly because the taxation on human capital discourages human capital accumulation, which is the engine of economic

[13] More specifically, a drop in labor employment along with a decrease in interest rate due to lower labor employment, together reduces government expenditure. We will explain this linkage between lower employment and lower interest rate below.

growth. In our model, capital accumulation is the engine of growth, but raising the tax rate on labor input that is not the engine of growth, always deters economic growth. Moreover, even though government expenditure is productive in the model, labor taxation always unambiguously reduces economic growth. These are surprising results. These results also indicate that applying zero taxes on capital and government revenues raised entirely from labor income, do not necessarily enhance growth. The reasons for these results are as follows. Labor tax rate directly reduces labor employment and indirectly tightens government expenditure. The reduction in both of these two factors lowers marginal productivity of capital. As a consequence, the accumulation of capital is reduced, thereby decreasing economic growth.

3. Intertemporal Elasticity of Labor Supply

Since elastic labor supply is an important feature of this study, it is interesting to examine the effect from a larger intertemporal elasticity of labor supply; i.e., a larger 2 or a smaller *2*. When the intertemporal elasticity of labor supply becomes larger, both the GK and the CK loci shift leftward and upward, with an unchanged starting point at the horizontal and vertical axes, respectively. Intuition for this shift is that a larger intertemporal elasticity of labor supply tends to reduce labor supply, under a given tax rate and thus a given net wage rate. This decreases tax revenues and government expenditure, making both the GK and the CK loci to shift leftward. Under Condition NS, the GK locus shifts upward more than the CK locus.[14] See the new $G^{N}K^{N}$ and the $C^{N}K^{N}$ loci in Figure 5.

As a result of the shifts, both the labor supply and consumption-capital ratio increase instantaneously. Intuitively, a larger intertemporal elasticity of labor reduces marginal disutility of labor supply, which raises the incentive to supply labor under a given wage rate, labor tax rate, and capital shadow price. In equilibrium, the labor employment is larger. A larger labor employment increases output, disposable income and consumption, therefore increasing the consumption-capital ratio. A larger labor employment increases marginal productivity of capital, moving interest rate higher

[14] See Appendix 3 for a derivation.

and wage-capital ratio lower. A larger productivity of capital and thus higher interest rate, increases rate of economic growth in the short run. (See the summary results in the upper panel of Table 1.)

Over time, a larger output from a larger labor supply will enhance the accumulation of capital and increase government expenditure. Yet, the accumulation of government expenditure is smaller than that of capital. Consequently, the government infrastructure-capital ratio decreases over time. A smaller government expenditure-capital ratio reduces marginal productivity of labor and thus labor supply, which also reduces disposable income and thus, the consumption-capital ratio, over time until the steady state E_3. Nevertheless, the new steady-state labor supply and consumption-capital ratio remain higher than their original level. Since labor employment and government infrastructures both enhance marginal efficiency of capital, the interest rate increases. Consequently, economic growth also increases. The wage-capital ratio is ambiguous as a larger labor supply and a larger stock of capital reduce wages, while a larger government infrastructure supports an increase.

Summarizing the results, we obtain:

Proposition 4. (Intertemporal Elasticity) *Under Conditions NS and NG, a larger intertemporal elasticity of labor supply increases labor supply, interest rate, and economic growth, both in the short run and in the long run.*

V. Concluding Remarks

This main objective of this paper is to examine the growth effect of factor taxation. In order to rationalize the taxation, we allow for the productive government expenditure. In order to consider both capital taxation and labor taxation, we allow households to decide their savings behavior and labor supply. To isolate the labor supply decision from other factors, we do not consider the human capital or learning-by-doing of labor employment. We have shown that, while a larger capital taxation reduces economic growth in the short run, its long-run growth effect is ambiguous. This long-run growth effect remains ambiguous even if tax rates are larger than degree of government externality. We also find that regardless of the level of labor tax rate, a larger labor taxation always lowers economic growth, both in the short run and in the long run, despite the existence of productive government taxation. The above two results arise mainly from elastic labor supply, and the complementarity of physical capital with labor employment and government expenditure. We also find that a larger intertemporal elasticity of substitution for labor supply, enhances economic growth in the short run, but probably reduces economic growth in the long run when the intertemporal elasticity of labor supply is large.

Existing wisdom considers labor taxation as better than capital taxation, from the economic growth point of view. Although we do not analyze the optimal factor taxation, our results indicate that labor taxation is always detrimental while capital taxation may be better, which suggest that for economic growth, labor taxation is not always better than capital taxation especially when the response of labor supply is taken into consideration.

There are possible extensions of the model. A natural extension is the welfare analysis of different factor taxation. This extension will be of course, very difficult, given that the expectations on households' consumption, labor supply choices, producers' capital demand, and labor demand behavior, affect the choices of tax rates. Therefore, some kinds of simplification need to be made. The analysis of this extension is even more complicated if the issues of time inconsistency are taken into account. Would the new governments in the future choose to continue the taxation policies enacted by the current government? Another extension is to consider a public production sector. In stead of buying final goods from the market, the government uses a production technology to produce the public goods. In order to produce the public goods, the public sector uses both capital and labor inputs. It would be interesting to contemplate the growth effects of factor taxation under this framework in the future.

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Appendix (Not Intended for Publication)

1. Derivation of a higher capital income tax rate's effect upon economic growth, evaluating capital income tax rate and labor income tax rate, both at the degree of government externality.

Substituting (15) into (2a), and differentiating it with respect to capital tax rate and evaluating at $\tau_k=\tau_l=\tau=\beta$ yields:

$$\left.\frac{d\gamma_c}{d\tau_k}\right|_{\tau_k=\tau_l=\beta} = \frac{x^*-\rho}{\beta}\left(-1+\frac{(1-\sigma)(1-\beta+\beta^2)x^*(x^*-M\rho)}{[(1-\sigma)(1-\beta)-\beta]x^*(x^*-M\rho)+\beta(x^*-\rho)(x^*-M\rho)+\beta^2x^*(1-\sigma)(x^*-M\rho)}\right) \quad \textbf{(A1)}$$

with $M=\frac{1}{1-\beta}$, and $\left.\frac{d\gamma_c}{d\tau_k}\right|_{\tau_k=\tau_l=\beta}\begin{pmatrix}>\\<\end{pmatrix}0 \quad if \quad x^*\begin{pmatrix}\in\\ \notin\end{pmatrix}(x_1, x_2)$,

where $x_1=\frac{(2-\beta-\sigma)-\sqrt{(2-\beta-\sigma)^2-4(1-\sigma)(1-\beta)^2\beta}}{2(1-\sigma)(1-\beta)^2}\rho$ and $x_2=\frac{(2-\beta-\sigma)+\sqrt{(2-\beta-\sigma)^2-4(1-\sigma)(1-\beta)^2\beta}}{2(1-\sigma)(1-\beta)^2}\rho$.

Therefore, starting from $\tau_k=\tau_l=\tau=\beta$, a higher capital income tax rate may not decrease the rate of economic growth. It should be noted that, when σ goes to $-\infty$, then $\left.\frac{d\gamma_c}{d\tau_k}\right|_{\tau_k=\tau_l=\beta}=0$. That is, when the labor supply is inelastic, taxing both capital and labor tax at the degree of government externality is optimal, from the economic growth point of view.

2. Derivation of the downward shift of the CK and GK loci, under a higher labor income tax rate.

Totally differentiating (11) with respect to x and τ_l, given the level of z, yields:

$$\left.\frac{\partial x}{\partial \tau_l}\right|_{CK,\ z\ given} = -\frac{\frac{\alpha}{1-\gamma-\alpha}\frac{1}{1-\tau_l}+\frac{1}{1-\tau_l}}{\frac{1}{x-\delta}+\frac{\alpha}{1-\gamma-\alpha}\frac{1}{x}} < 0, \qquad \textbf{(A2)}$$

and extending similarly for (12) leads to:

$$\left.\frac{\partial x}{\partial \tau_l}\right|_{GK,\ z\ given} = -\frac{\frac{\alpha}{1-\gamma-\alpha}\frac{1}{1-\tau_l}+\frac{(\frac{1}{z}+1)\alpha}{1-(\frac{1}{z}+1)\tau}}{\frac{1-\gamma}{1-\gamma-\alpha}\frac{1}{x}} < 0. \qquad \textbf{(A3)}$$

When $\left.\frac{\partial x}{\partial \tau_l}\right|_{CK,\ z\ given} < \left.\frac{\partial x}{\partial \tau_l}\right|_{GK,\ z\ given}$, then the CK locus shifts downwards, more than the GK locus. If the condition holds, from (A3) and (A2), we derive:

$$\frac{1-\gamma}{1-\gamma-\alpha}\left(\frac{1}{1-\tau_l}-\frac{(\frac{1}{z}+1)\alpha}{1-(\frac{1}{z}+1)\tau}\right) > \frac{\delta}{x-\delta}\left(\frac{\alpha}{1-\gamma-\alpha}\frac{1}{1-\tau_l}+\frac{(\frac{1}{z}+1)\alpha}{1-(\frac{1}{z}+1)\tau}\right). \qquad \textbf{(A4)}$$

The sufficient condition for (A4) is to have (i), the term outside the parenthesis, on the left-hand side, larger than the term outside the parenthesis, on the right-hand side, and (ii) the term inside the parenthesis, on the left-hand side, larger than the term inside the parenthesis, on the right-hand side. Since (i) affirms $x > [2-\frac{\alpha}{1-\gamma}]\delta$, it is automatically satisfied under Condition NS. By denoting $s = \alpha+(1-\alpha)(\tau_l+\tau_k)-\frac{2\alpha^2(1-\tau_l)}{1-\gamma}$, the condition in (ii) is reduced to:

$$z_{\min} > \frac{s}{1\&s}. \qquad \textbf{(A5)}$$

As long as \$<1/2, Condition NS sufficiently guarantees the meeting of (ii).

3. Derivation of the effects of a lower intertemporal elasticity of substitution.

Taking the logarithm of (11) and (12), and differentiating them with respect to 2 and *x*, yields the shifts of Locus CK and Locus GK, respectively, as follows:

$$\left.\frac{dx}{d2}\right|_{CK,\ z\ fixed.} \ ' \ \frac{(x^{(}\&D)x^{(}\$\,lnl^{(}}{x^{(}(1\&2)\&D\$}>0. \qquad \textbf{(A6)}$$

$$\left.\frac{dx}{d2}\right|_{GK,\ z\ fixed.} \ ' \ \frac{(x^{(}\&D)x^{(}\$\,lnl^{(}}{x^{(}(1\&2)\&D(1\&2)}>0. \qquad \textbf{(A7)}$$

First, both Loci shift upwards as $x_{\min}$ > D and 1-2 > \$ under Condition NS. Second, the above two expressions differ only for the denominators. As 1-2 > \$ under Condition NS, the denominator in (A6) is larger than that in (A7), implying that the upward shift the GK locus (A7) is larger than the CK locus (A6).

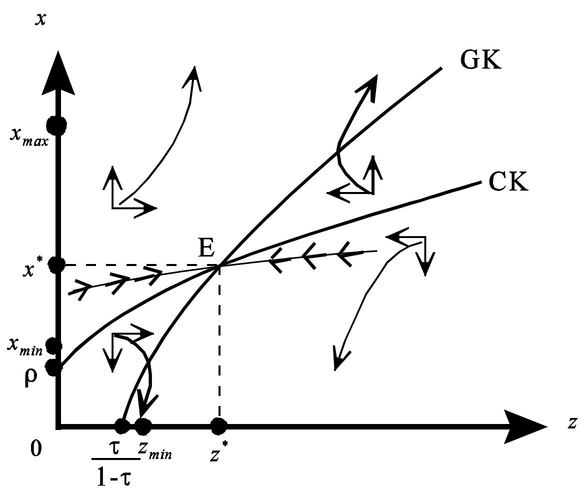

Figure 1. Steady State and Transitional Dynamics

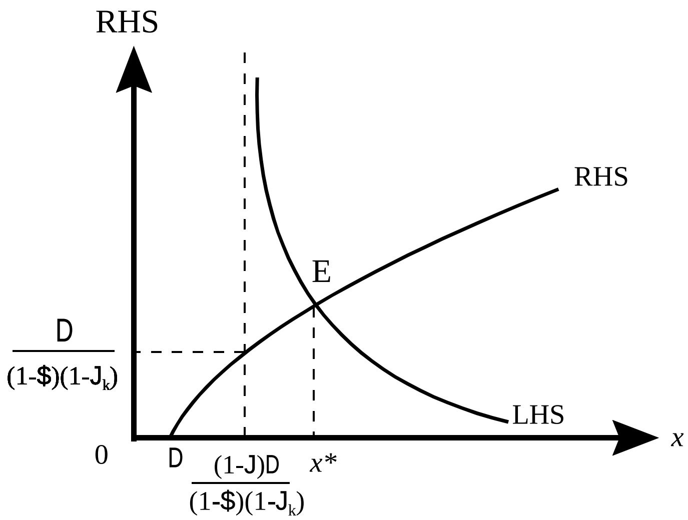

Figure 2: Determination of Steady State for x

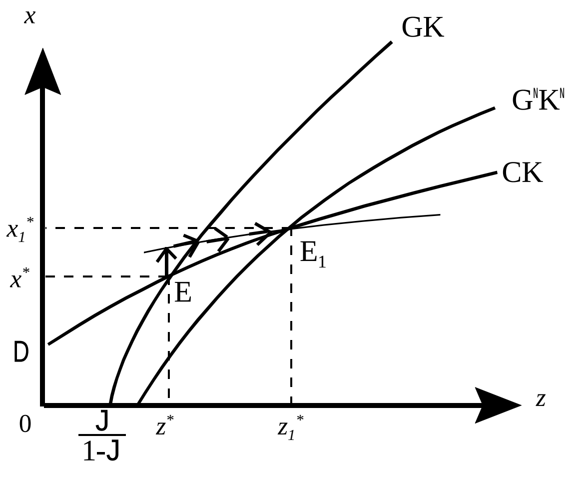

Figure 3. Effect of A Higher Capital Tax Rate

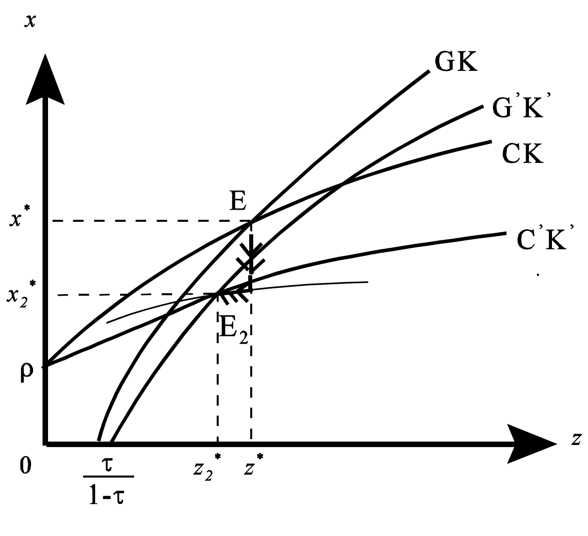

Figure 4. Effect of A Higher Labor Tax Rate

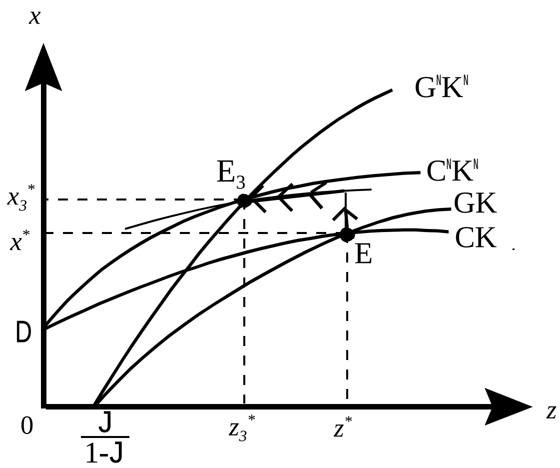

Figure 5. Effect from A Derceasing Intertemporal
Elasticity of Substitution

Table 1: Comparative-Static Effects

Short-Run Effects	$x(0)$	$z(0)$	$l(0)$	$r(0)$	$\frac{w(0)}{k(0)}$	$\frac{\dot{y}}{y(0)}$
J_k	%	0	&	&	%	&
J_l	&	0	-	-	%	-
2	%	0	%	%	&	%
Long-Run Effects	$x*$	$z*$	$l*$	$r*$	$\frac{w^{(}}{k^{(}}$	$(\frac{\dot{y}}{y})^{(}$
J_k	%	%	?	%	?	?
J_l	&	-	-	&	?	&
2	%	&	%	%	?	%