Fall 2008, Hitotsubashi University

Monetary Economics 1

(Corporate Finance)

LECTURE 2

Basic Theory of Interest and Project Valuation

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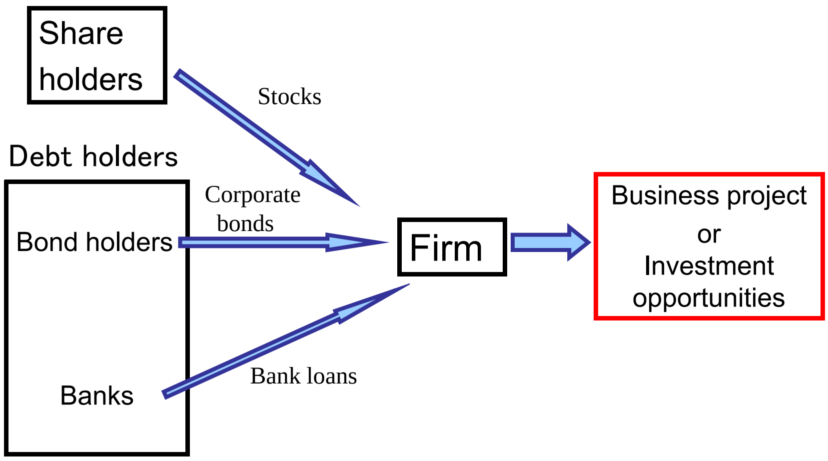
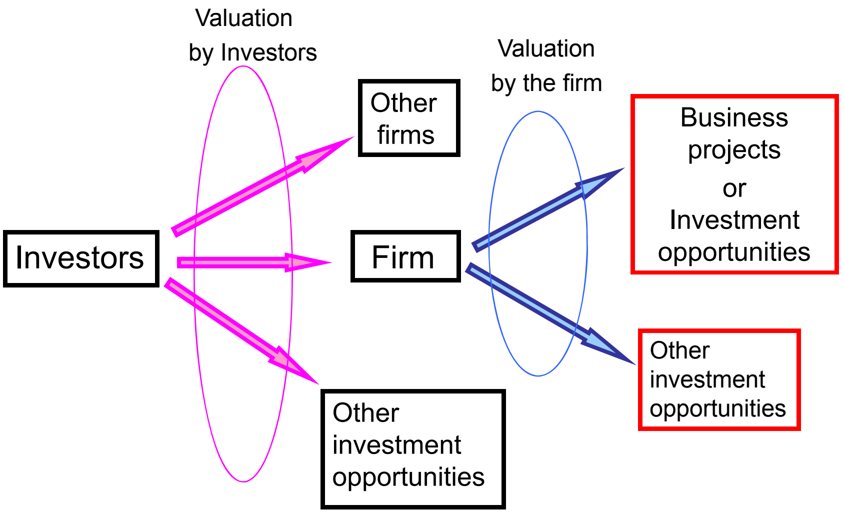
Investors





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How should we determine the value of the investment opportunity?

- **Present-value principle**
  - Or **discounted present value**

- “Discount” future earnings and costs.
- Evaluate them in the values today.

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Investing to a project or buying government bonds?

- How much will you pay for the claim (=bond) that pays ***$1*** after one year?
  - Suppose people are willing to pay ***$Y*** for the bond.
- There is the project that pays out ***$X*** after one year.
- Investing to this project is same as buying ***X*** units of the bonds.

Discounting

- Buying ***X*** units of bonds costs ***$XY*** today.
- Hence, the project should cost ***$XY*** too.

- Alternatively, let $D=1/Y$ and $1+d=D$.
- Then the project should costs: $X/(1+d)$.
  - "D" is called a "discount factor."
  - "d" is called a "discount rate."

PART 1

BASIC THEORY OF INTEREST

Basic theory of interest (1)

- The case of bank deposit
- Deposit **$100** today.
  - Interest rate is **1%** ⇒ **r = 1% = 0.01**
  - After one year: $101 in bank account (**= $100 x 1.01**)

Basic theory of interest (2)

- The case of government (riskless) bond
- The claim for receiving **$100** after one year from today (the maturity of the bond is one year)
  - Face value: $P_1 = \$100$
  - Bond price: $P_0 = \$99.01$
  - Interest rate: $r = P_1 / P_0 - 1 = 0.01$

Basic theory of interest (3)

- Algebraic representation
  - Bonk deposit: $(1+r)P_0=P_1$
  - Bond: $P_1/P_0=1+r$ or $(P_1-P_0)/P_0=r$

- P_0, P_1, r: If two of them were given, the remaining is automatically determined.
- "Bond price" and "interest rate" have a one-on-one relationship.

Basic theory of interest (4)

- Economic interpretation
  - Face value P_1 is given and fixed
  - Suppose bond price P_0 goes up.
  - Today's price of "P_1 yen in future" goes up.
  - Interest rate goes down.
- Interest rate: relative price of future cash in terms of today's money

Two views on bond / loan market

	Issuing bond, bank borrowing	Financial asset
What is traded	Funds	Payoff at maturity
Demand side	Firm	Investor
Supply side	Investor / creditor	Firm
Price	Interest rate	Bond price

Basic theory of interest multi-period case (1)

- Compounding
- $P_T = P_0(1+r_1)(1+r_2)(1+r_3)\ldots(1+r_T)$

- Suppose r_i = constant for $i=1, 2, \ldots, T$
- $P_T = P_0(1+r)^T$ or $P_0 = P_T/(1+r)^T$

Basic theory of interest multi-period case (2)

- Coupon payment: C_t
- One period case: $P_0 = (P_1 + C_1)/(1+r)$

- Multi-period case

$$P_0 = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots\ldots + \frac{P_T}{(1+r)^T}$$

Notes on compounding

- Convenient approximation
  - Ln(A)= Natural log of A
  - If the absolute value of x was very small, $Ln(1+x) \sim x$
  - $Ln(XY) = Ln(X) + Ln(Y)$
- Let $P_2 = P_0(1+r_1)(1+r_2)$ and $P_2/P_0 = 1+R$
  - $Ln((1+r_1)(1+r_2)) = Ln(1+r_1)+Ln(1+r_2) \sim r_1+r_2$
  - Thus $R \sim r_1+r_2$

PART 2

VALUATION BY DISCOUNTING CASH FLOW (DCF)

Present value principle: general formula

$$P_0 = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + ... + \frac{P_T + C_T}{(1+r)^T}$$

$$But, P_T = \frac{P_{T+1} + C_{T+1}}{1+r}$$

$$P_0 = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + ... + \frac{C_T}{(1+r)^T} + \frac{P_{T+1} + C_{T+1}}{(1+r)^{T+1}}$$

$$\Rightarrow P_0 = \sum_{j=1}^{\infty} \frac{C_j}{(1+r)^j}$$

What is "C_t"? What is "T"?

- If stock: C_t is dividend payments
- If real estate: C_t is rent payments
- If bond: C_t is coupon payments

- Terminal period, T
- If bond: T is finite (e.g. T =1, 5, .., 10 years)
- Stocks and real estates: T is infinite

Some useful formula (1)

- Annuity
  - financial asset that pays constant amount every period: $C = C_1 = C_2 = C_3 = ... = C_T$
  - Discount rate is constant: r
- Perpetuity: $T \rightarrow$ infinity

$$P_0 = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + ...$$

$$\Rightarrow P_0 = \frac{C}{r}$$

Some useful formula (2)

- Growing perpetuity/Growing Gordon formula
  - C grows every period at the rate of "g": $C_1 = (1+g)C_0$

$$P_0 = \frac{C}{1+r} + \frac{(1+g)C}{(1+r)^2} + \frac{(1+g)^2 C}{(1+r)^3} + ...$$

$$\Rightarrow P_0 = \frac{C}{r-g}$$

Discounting future cash-flows of the business project

- Suppose a project generates cash flow stream, $C_0, C_1, C_2, \ldots\ldots, C_T$.
  - C_t = "Period t sales" - "Period t costs"
- We discount future cash flows by the government bond interest rate.

Comparison with the investment to government bond

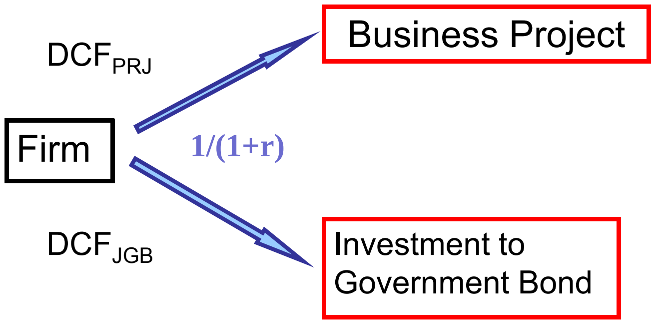
- Discounted cash flow (DCF) of the project

$$DCF_{PRJ} = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots\ldots$$

- Investment to JGB : $C_0 = -P_0,\ C_1 = P_1 = (1+r)P_0$

$$DCF_{JGB} = -P_0 + \frac{P_1}{1+r} = -P_0 + \frac{(1+r)P_0}{1+r} = 0$$

Take "investment to JGB" as a benchmark



Valuation by present-value principle

- If $DCF_{JGB}<DCF_{PRJ}$, then the firm should invest to the project.
- But, always $DCF_{JGB}=0$. Thus, the firm should invest to the project only if $DCF_{PRJ}>0$.
- DCF_{PRJ} is the present-value of the project when it is evaluated with appropriate discount rate r.

Important preservations

- In our discussions so far, we have completely ignored any "risk."
- If there is any risk, we have to assume risk neutrality.
- "Risk neutral" = investors care only expected payoffs, do not care its variance.
- Otherwise, we have to explicitly incorporate "the price of risk" or "risk premium" into the analysis. --- We will do this latter.

Comparing multiple projects

Choosing one of mutually exclusive projects.

	0	1	2		Total
Project 1	-7	11	12.1		16.10
Project 2	-1	22	-12.1		8.90
Project 3	-5	44	24.2		63.20
Project 4	-1	11	0		10.00

Different discount rates and present value

		Cash flow		Discount rate	PV
	0	1	2	**10%**	
Project 1	-7	11	12.1	0.10	13
Project 2	-1	22	-12.1	0.10	9
Project 3	-5	44	24.2	0.10	55
Project 4	-1	11	0	0.10	9

	Cash flow			Discount rate	PV
	0	1	2	**20%**	
Project 1	-7	11	12.1	0.20	10.56944
Project 2	-1	22	-12.1	0.20	8.930556
Project 3	-5	44	24.2	0.20	48.47222
Project 4	-1	11	0	0.20	8.166667

	Cash flow			Discount rate	PV
	0	1	2	**30%**	
Project 1	-7	11	12.1	0.30	8.621302
Project 2	-1	22	-12.1	0.30	8.763314
Project 3	-5	44	24.2	0.30	43.16568
Project 4	-1	11	0	0.30	7.461538

PART 3

VALUATION BY ARBITRAGE

Arbitrage and present-value relations

- No arbitrage condition = law of one price
  - hamburger + cola + potato
  - The set of three and buying them individually should cost the same.
- Application of "No arbitrage condition"
- Using zero-coupon bond price to price coupon bonds.
- The price of zero-coupon bond that matures at time $T =$ The price of cash in period T .

An application: Pricing coupon bonds

- Face value: *$100*
  Maturity: 36 months
  Coupon: *$5* yen coupon payment at *12* months and *24* months later.

- Data (Face value = **100 thousand**)
  - Zero-coupon bond price (T=12 months): *97.5*
  - Zero-coupon bond price (T=24 months): *94.3*
  - Zero-coupon bond price (T=36 months): *90.7*

Replicating payoffs of the coupon bond using zero-coupon bonds

- $Y_{12}=5$, $Y_{24}=5$, $Y_{36}=100$

- Zero-coupon bond (T=12): **0.05** units
- Zero-coupon bond (T=24): **0.05** units
- Zero-coupon bond (T=36): **1**unit
- This synthetic coupon bond has exactly same payoff pattern.

Theoretical value of coupon bond price

- 97.5x0.05 + 94.3x0.05 + 90.7x1

=4.875+4.715+90.7

=100.29

- Theoretical value: $100.29

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A digression

- How risk-free interest rate will be determined?
- Simple answer: “Demand and supply”
- Demand and supply of what?

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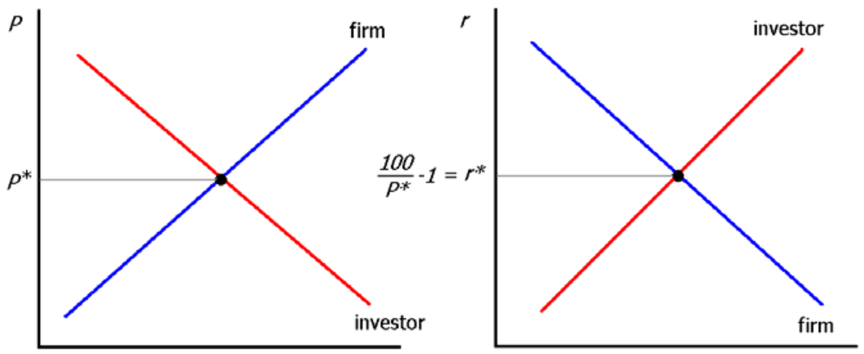
Remember…

There are two views on bond / loan market

	Issuing bond, bank borrowing	Financial asset
What is traded	Funds	Payoff at maturity
Demand side	Firm	Investor
Supply side	Investor / creditor	Firm
Price	Interest rate	Bond price

Bond market equilibrium:

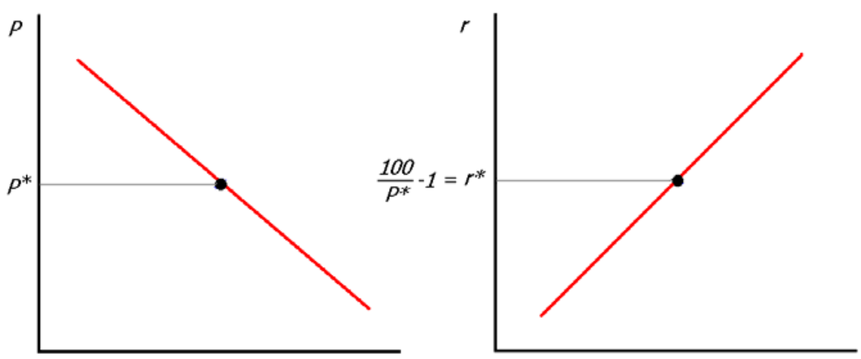
Equilibrium bond price and interest rate



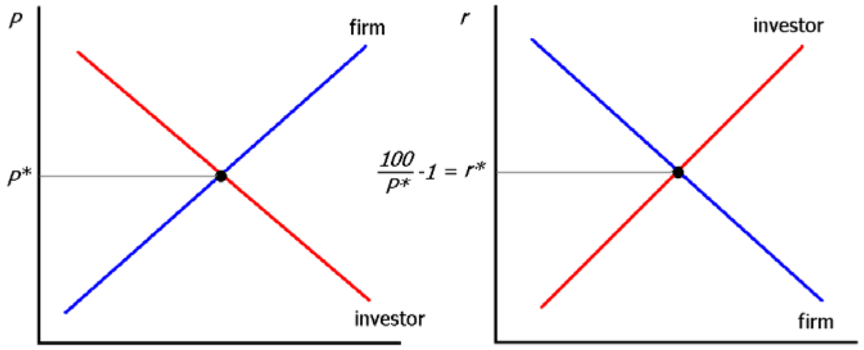
Relation between bond price and its interest rate

- Face value P_1 is given and fixed
  - Suppose bond price P_0 goes up.
  - Today's price of "P_1 yen in future" goes up.
  - Interest rate goes down.
- Interest rate: relative price of future cash in terms of today's money

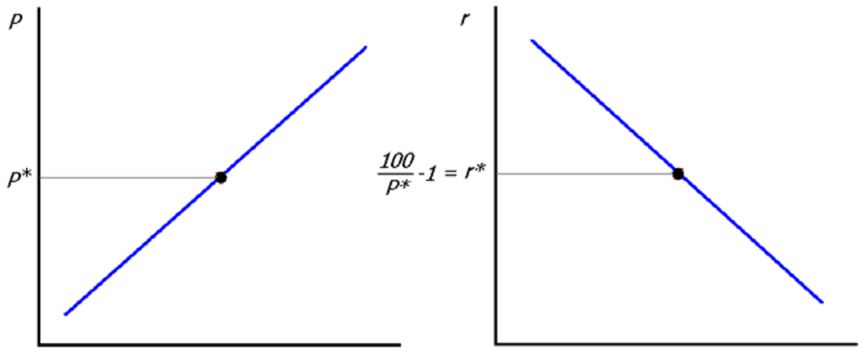
Demand for bond = supply of loans



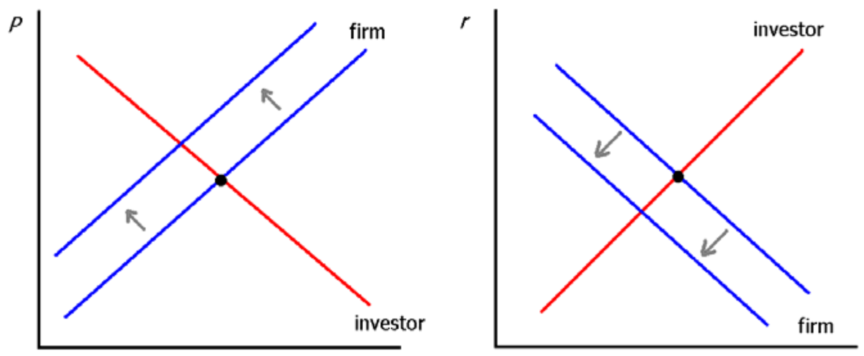
Demand for bond by lenders



Supply of bond = demand for loans



Eg.Expected profitability falls → Bond price will increases



Factors shift supply curve for bonds

- Corporate tax on profit
- Tax subsidies for investment
- Expected inflation
- Government borrowing

Factors shift demand curve for bonds

- Investors wealth
- Expected returns on bonds
- Expected returns on other assets
- Riskiness of bonds relative to other assets
- Expected inflation
- Liquidity of bonds relative to other assets

Methodology of arbitrage pricing

- You have an asset (or business project) that you would like to price. Check the payoff pattern of your asset.
- Replicate the payoff pattern of the asset using existing assets. Construct the replicating portfolio.
- The valuation of an asset must be equal to the value of assets used to construct the replicating portfolio.

Use tracking portfolio to value investment projects

- Find some asset or combination of assets that perfectly tracks the cash flows of the investment project.
- Apply no-arbitrage condition.
- The price of a tracking portfolio is the value of the investment project.

Limit of tracking portfolio approach

- For example, oil price can be used to price oil well.
- However, there will not be a perfectly replicating portfolio in practice. There are always tracking errors.
- If there are significant tracking errors, use asset pricing models.
  → Theme of the next lecture.

Arbitrage: Formal definition

- An arbitrage opportunity is:
  - A riskless profit opportunity that any individual who prefers more wealth to less will exploit.
- Riskless profit opportunity
  - The opportunity never loses money today or tomorrow.
  - The opportunity sometimes makes money today or tomorrow.
- No-arbitrage condition: there is no arbitrage opportunity

Examples of arbitrage pricing Currency forward contract

- Currency forward contract
  - Investment in one-year JGB: r_{JGB}
  - Make an investment to US treasury bill (r_{Tbill}) by spot FX rate (E). Use forward contract (F) to hedge FX risk.
  - Both investment opportunities are riskless, hence must yield same rate of return.
  - $(1+r_{JGB})=(1+r_{Tbill})(E/\mathrm{F})$
  - $r_{JGB} = r_{Tbill} + e - f$
- Various derivative products such as B-S formula.

Problem of tracking portfolio approach

- For example, oil price can be used to price oil well.
- However, there will not be a perfectly replicating portfolio in practice. There are always tracking errors.
  – when is tracking error relevant for pricing?
- If there are tracking errors, use asset pricing models.