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International Corporate Finance I

LECTURE 5 and 6

Valuing Risky Investment Project

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Evaluating risky investment projects

- Alternative approaches:
  - Use tracking portfolios to value investment projects.
  - Use risk-adjusted discount rate value investment projects.
  - Actually, they are the same.
- Effects of leverage
  - Calculating WACC

Use tracking portfolio to value investment projects

- Find some asset or combination of assets that perfectly tracks the cash flows of the investment project.
- Apply no-arbitrage condition.
- The price of a tracking portfolio is the value of the investment project.

Arbitrage: Formal definition

- An arbitrage opportunity is:
  - A riskless profit opportunity that any individual who prefers more wealth to less will exploit.
- "Riskless" profit opportunity
  - The opportunity never loses money today or tomorrow.
  - The opportunity sometimes makes money today or tomorrow.
- No-arbitrage condition: there is no arbitrage opportunity

Examples of arbitrage trading (1)

- Suppose it was possible to buy the value set (hamberger+drink+potato) at the fast food restaurant and sell them individually at the ICU campus without a cost.
- Individually, items (hamberger=100, cola=200, potato=150) cost 450 as a total. Buying them as the value set costs only 430. So there would be 20 yen profit per trading.

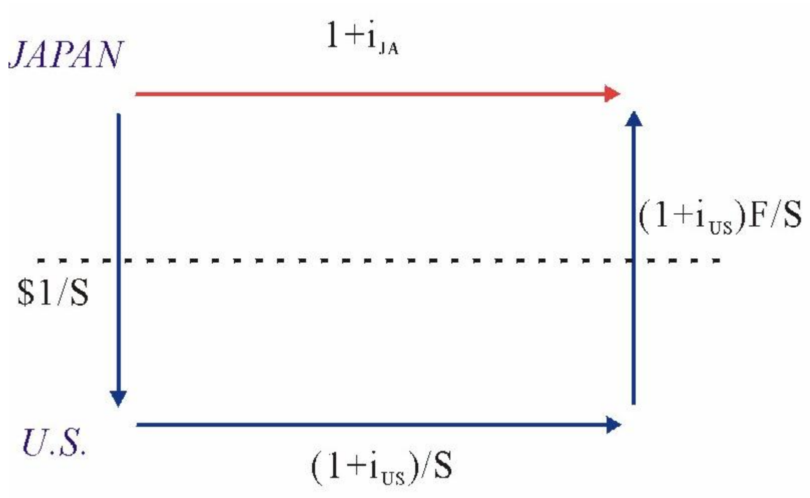
Examples of arbitrage trading (2): Currency forward contract

- Two investment opportunities
  - Invest 100 thousand yen to one-year JGB
  - Convert 100 thousand yen to dollar and invest to one-year US treasury bill.
- If the FX had been fixed, both investment opportunities are riskless. So $i_{JA}=i_{US}$
- Even with flexible FX rate, we can use the forward contract and perfectly hedge the FX risk.

1. Convert 100 thousand yen in your hand to US dollar at the spot market for the rate of \$1＝¥*S*. You have 100/*S* thousand US dollar now.

2. Invest 100/*S* thousand dollar to US treasury bill today. After one year you will receive $(1+ i_{US})(100 \text{ thousand}/S)$ dollar.

3. You sell $(1+ i_{US})(100 \text{ thousand}/S)$ dollar today at the forward market, so that you will be able to convert your investment back to Japanese yen <u>without any risk</u>. Let *F* be the FX rate for the forward contract. Then after one-year, you'll have $F(1+ i_{US})(100\text{thousand}/S)$ yen in your hand.



Covered interest parity

- Both invest opportunities do not include any uncertainty. So they are supposed to yield the same payoff.
  - Investment to JGB
  - Investment to US treasury bill, FX rate risk perfectly hedged.

- Rates of return from two investment opportunities must be same.
  - → $1 + i_{JA} = (1 + i_{US})(F/S)$
  - → Approximately, $i_{JA} = i_{US} + fd$ where $fd = (F-S)/S$

Comparison method

- Find a ***comparison*** firm whose cash flows closely track the cash flows of the project of your interest.
- For simplicity, we assume both the project and a comparison firm are financed solely by equities, no debts.
  - Latter, we discuss why this assumption is important.
- Compare expected return of the project and a comparison firm.
- If $E[r_{project}] \geq E[r_{comp.}]$, you should invest to the project.

Problem of tracking portfolio approach

- Maybe there is no close comparison firm.
- Tracking errors can be very large.

- If there are tracking errors, use asset pricing models.
- Use any asset pricing model to calculate risk-adjusted discount rate or "risk premium".

Use CAPM to determine risk-adjusted discount.

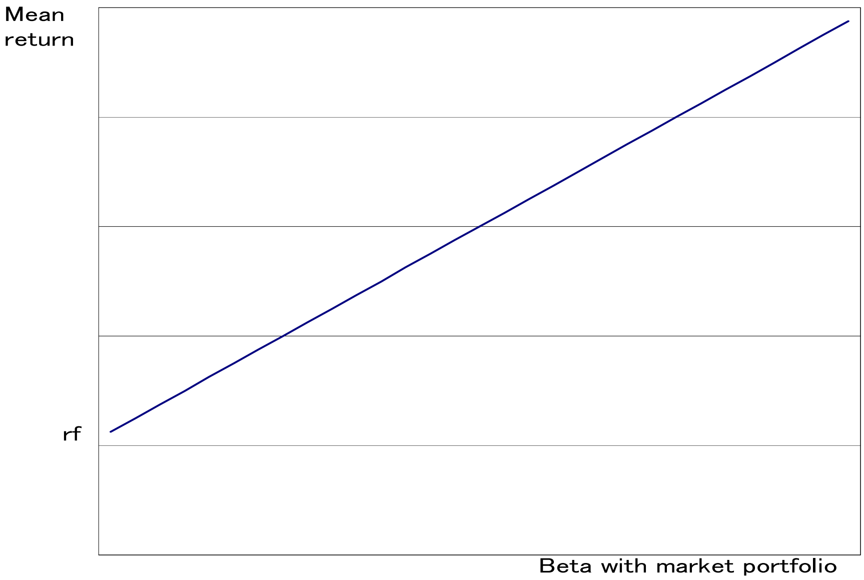
- The Capital Asset Pricing Model (CAPM): A Primer
- $E[r_i] - r_f = \beta_i (E[r_M] - r_f)$
  - r_i: return of asset i; r_f: return of riskless asset
  - r_M : return of market portfolio
  - $\beta_i = Cov(r_i, r_M)/Var(r_M)$

  - Please do read about the derivation of CAPM by yourself.

Interpretation of CAPM

$$E[r_i]- r_f = \beta_i (E [r_M] - r_f)$$

- Relation between expected excess returns of asset i and market portfolio.
- Higher the covariance with market portfolio, higher the expected excess return.
- What is relevant for valuation is covariance risk, not variance.
- If correlation is zero, no matter how large the variance was, $E[r_i]- r_f = 0$.



Implementation of CAPM

- $E[r_i]- r_f = \beta_i (E [r_M] - r_f) + \varepsilon_i$
- Regression of "excess return of asset *i*" on "excess return of market portfolio"
  - No constant term.
  - $Y_i = \beta_i X_M + e_i$

Why is the market portfolio a benchmark?

- By investing to many different assets, one can diversify risks.
- The risks that will disappear by diversification are not be priced, i.e. risk premium is zero.
- Only covariance with most diversified portfolio matters.
- Most diversified portfolio = the market portfolio

A digression: What is risk premium?

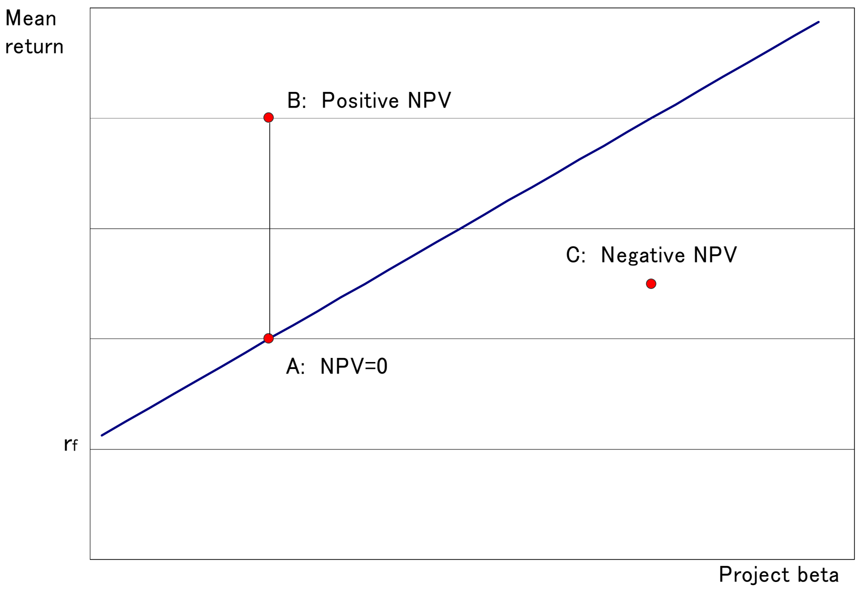
- Riskless asset: r_f
- Risky asset: r_i E$[r_i]$, Var(r_i)
- Var$(r_i) > 0 \rightarrow r_i > r_f$
- The condition that both riskless and risky assets are invested:
  $U(W \cdot r_f) = E[U(W \cdot r_i)]$
- Let "q" be the premium required for a risky asset over riskless asset return so that the condition above is satisfied:
  $E[r_i] = r_f + q$

Determination of risk premium by CAPM

- Volatility of security i
  ＝"β_i × market risk " + "idiosyncratic risk"
- $E[r_i]$- r_f ＝ Risk premium of security i
  ＝ β_i × "risk premium of market risk"
  ＝ β_i (E $[r_M] - r_f$)
- The project should be invested if and only if:
  $E[r_{project}] \geq E[r_{comp.}]$
  $= r_f + \beta_{comp.}(E[r_M] - r_f)$

Graphical expression

- If the project was located on or at the left-side of SML, the investment project should be accepted.
- Given the estimated project beta, expected return is sufficiently large.



Risk-adjusted discount rate

- Forecast future cash flow： $\mathrm{E}[CF]$
- Estimate the project's beta： β
- Use β to calculate risk premium
- Discount forecasted future cash flow $\mathrm{E}[CF]$ by risk-adjusted discount rate

$$PV = \frac{\mathrm{E}[CF]}{1 + r_f + \beta(R_M - r_f)}$$

Important points

- Tracking portfolio approach is implicit in risk-adjusted discount rate method.

- We assumed full equity finance, i.e. no leverage, in the discussions so far.

The effect of leverage

- In general comparison firms are financed by both equities and debts (leverage).
- In such a case, beta of comparison firm's business is different from the firm's equity beta.

- We still make an important simplifying assumption:
  - Modigliani=Miller's theorem (MM theorem) is satisfied so that the financial structure does not affect the firm's value.

Equity vs debt

- Cost of debt financing＝interest rate payment on debt

- "Firm's value" minus "firm's debt value" ＝ Value of firm's equities

- Cost of equity financing: Equity's expected return (interest rate + risk premium)
- Risk premium is required so that equity holders will bear the volatility of stock prices.

Calculating WACC : simple case

- WACC=Weighted average cost of capital
- “Cost of equity financing” x “the amount financed by equities”
- “Cost of debt financing” x “the amount financed by debts”
- Summing them up, then divide by total amount = WACC

Algebraic derivation (1)

- The firm’s total value: V
- The firm’s future cash flow: CF
- The firm’s cost of capital (= expected return from its business): r_{Firm}
- The firm’s total value: $V = CF / r_{Firm}$
- Capital structure of the firm: $V=E+D$
- $r_{Firm} = CF/V = CF/(E+D)$

Algebraic derivation (2)

- Equity holders' future cash flow: CF- r_{Debt}D
- So, E=(CF-r_{Debt}D)/ r_{Stock}
- CF= r_{Stock}E + r_{Debt}D
  - LHS: Future cash flow from the firm's business
  - RHS: The amount required to payout for stock holders and debt holders.

WACC: general formula

- Divide both LHS and RHS by V=E+D
- Firm's cost of capital =
  "Ratio of equity finance" x "Equity cost of capital"
  + "Ratio of debt finance" x "interest rate on debt"

$$r_{Firm} = \left(\frac{E}{E+D}\right) r_{Stock} + \left(\frac{D}{E+D}\right) r_{Debt}$$

The role of MM theorem

- We assumed MM is satisfied:
  - MM says the firm's value is not affected by its financial structure.
  - So r_{Firm} is constant.
- r_{Stock} and r_{Debt} are determined in market and can be observed.
- We should use r_{Firm} to evaluate investment project, but r_{Firm} cannot be observed directly.

Firm's capital structure

- Capital structure on the balance sheet (stock value)
- Contingent claims for the firm's profits
- Simple case: Equity vs Debt
  - Preferred stock
  - Subordinate debt
  - Convertible bonds, warrants

Equity vs Debt

- Cost of capital for debt ＝ Interest rate on corporate bond and/or bank borrowing
- Firm's value－Debts ＝ Equity value
- Equity cost of capital：risk-free rate plus equity premium
- Equity premium = Additional cost the firm has to pay so that investors will bear the volatility of equities.

Modigliani-Miller theorem

- Foundation of Modern Corporate Finance
- Referred as "MM" hereafter
- Corporate Finance Doesn't Matter!

Exactly, MM theorem says

- Proposition 1
  - Capital structure is irrelevant as long as the firm's investment decisions are taken as given.

Little more precisely,

- Proposition 1: If
  - No taxes/transaction costs
  - Efficient Capital Markets
  - No bankruptcy costs
- Then, the value of the firm is independent from its capital structure.
  - e.g., debt-equity ratio, dividend policy, and etc.

Example: Tsukuba Corporation, no debt
Share price is 1 million yen
Total value 100 million yen (100 shares)

	GREAT	GOOD	OK	BAD
EBIT	20	15	10	5
#of shares	100	100	100	100
EPS	0.2	0.15	0.10	0.05
ROE	20%	15%	10%	5%

Scenario 1
Tsukuba Co. repurchased a half of its own shares by borrowing： interest rate 10%

	GREAT	GOOD	OK	BAD
EBIT	20	15	10	5
Interest	5	5	5	5
EAIT	15	10	5	0
#of shares	50	50	50	50
EPS	0.3	0.2	0.1	0
ROE	30%	20%	10%	0%

Scenario 2:

Investor X borrowed 0.1million yen at interest rate=10% and bought one share of Tsukuba Co.

	GREAT	GOOD	OK	BAD
EBIT	20	15	10	5
Share Payoff	0.4	0.3	0.2	0.1
Interest	0.1	0.1	0.1	0.1
Net profits	0.3	0.2	0.1	0

ROE is same for Scenario 1 and 2

- Two ways to understand proposition 1
- Value additivity
- Arbitrage

- Cost of capital is independent from the amount of the firm's leverage.

Implications for cost of capital
(Discount rate for the firm's business)

$$r_c = \left(\frac{E}{E+D}\right) r_E + \left(\frac{D}{E+D}\right) i$$

- Firm's cost of capital =
  "Ratio of equity finance" x "Equity cost of capital"
  + "Ratio of debt finance" x "interest rate on debt"

Timberland Co.'s cost of capital

- Timberland Co.'s debt and equity

	Book value		Market value	
	Amount	Shares	Amount	Shares
Debt	100.0	27.3%	100.0	16.5%
Equity	266.2	72.7%	505.7	83.5%
Total	366.2	100.0%	605.7	100.0%

Timberland Co.'s debt

- Yield of Timberland Co.'s corporate bonds： 7%
- Exemption of corporate tax for interest rate payment
- Corporate tax rate: 35%
- $(1-0.35)\times 0.07 = 0.046$
- $r_{debt} = (1 - \tau) \times r$

Timberland Co.'s equity

- Use CAPM to calculate equity premium
- r_{stock} = JGB rate + risk premium
  = JGB rate
  + β × risk premium on market portfolio
- β =1.49
- r_{stock} = 5.1%+1.49×7.5% = 16.3%

Timberland Co.'s WACC

	Amount	Shares	After tax cost	Weighted cost
Debt	100.0	16.5%	4.6%	0.8%
Equity	505.7	83.5%	16.3%	13.6%
			WACC	**14.4%**

Cost of equity financing ＞ cost of debt financing

- Is it possible to decrease the firm's cost of capital by increasing debt financing and decreasing equity financing?
- (If MM holds) **No!**
- When capital structure changes, riskiness of equity will change too, i.e. β will change.
- As long as MM theorem holds, the firm's cost of capital will not change.

$$\sigma_{Firm} = \left(\frac{D}{D+E}\right)\sigma_{Debt} + \left(\frac{E}{D+E}\right)\sigma_{Stock}$$

$$\beta_{Firm} = \left(\frac{D}{D+E}\right)\beta_{Debt} + \left(\frac{E}{D+E}\right)\beta_{Stock}$$

But, beta for debt is zero. So reversing these relations, we have

$$\sigma_{Stock} = \left(1+\frac{D}{E}\right)\sigma_{Firm}$$

$$\beta_{Stock} = \left(1+\frac{D}{E}\right)\beta_{Firm}$$

Relaxing the assumptions of MM

- Exemption of corporate tax on interest rate payment
- 100% debt financing is optimal

- The possibility of bankruptcy
- 100% equity financing is optimal

With bankruptcy risk

- Debt ratio: D/(D+E)
- D/(D+E) → 0 $\beta_{\text{Stock}} \to \beta_{\text{Firm}}$
- D/(D+E) → 1： Beyond the threshold level, the probability of bankruptcy will be positive and beta for debt will be positive.

$$\beta_{Firm} = \left(\frac{D}{D+E}\right)\beta_{Debt} + \left(\frac{E}{D+E}\right)\beta_{Stock}$$

$$\beta_{Stock} = \left(1+\frac{D}{E}\right)\beta_{Firm} + \left(\frac{D}{E}\right)\beta_{Debt}$$