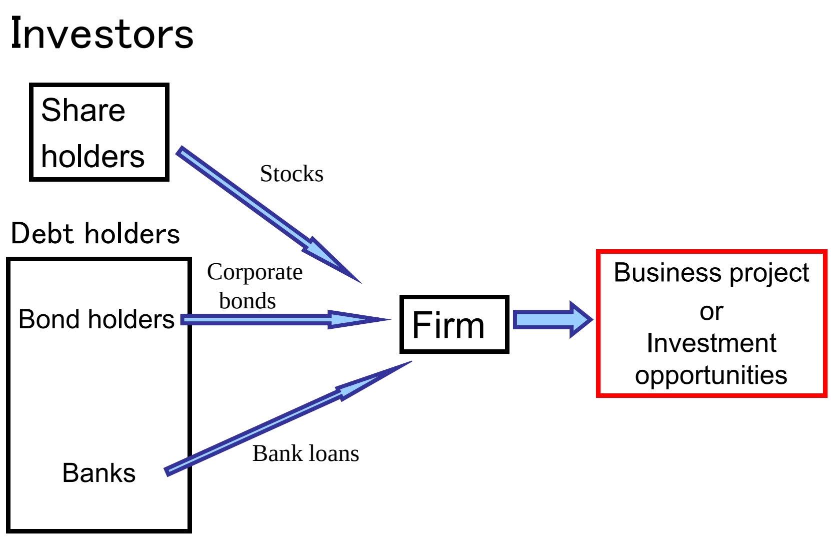
Fall 2008
International Corporate Finance I

LECTURE 3 & 4

Basic Theory of Interest and Project Valuation

Tokuo Iwaisako
HITOTSUBASHI UNIVERSITY

Part 1: Firm's investment decision making and present value principle



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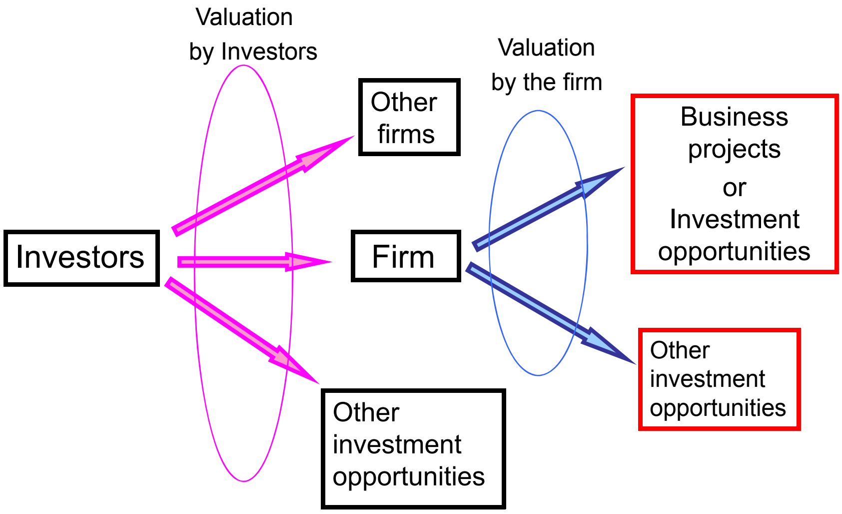

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Valuation of business project(s): Examples

- Business project that requires $1 million initial investment
  - $1.2 million profits in one year
  - $1.5 million profits in five years
  - Should the firm invest to such investment opportunities?

- Mutually exclusive projects
  - $1.2 million profits in one year
  - $1.5 million profits in five years
  - To which project should the firm invest?

How should we determine the value of the investment opportunity?

- **Present-value principle**
  - Or **discounted present value**

- "Discount" future earnings and costs.
- Evaluate them in the values today.

Investing to a project or buying government bonds?

- How much will you pay for the claim (=bond) that pays ***$1*** after one year?
  - Suppose people are willing to pay ***$Y*** for the bond.
- There is the project that pays out ***$X*** after one year.
- Investing to this project is same as buying ***X*** units of the bonds.

Discounting

- Buying ***X*** units of bonds costs ***$XY*** today.
- Hence, the project should cost ***$XY*** too.

- Alternatively, let ***D=1/Y*** and ***1+d=D***.
- Then the project should costs: ***X/(1+d)***.
  - "***D***" is called a "discount factor."
  - "***d***" is called a "discount rate."

Part 2: Basic Theory of Interest

Basic theory of interest (1)

- The case of bank deposit
- Deposit **$100** today.
  - Interest rate is **1%** ⇒ **r = 1% = 0.01**
  - After one year: $101 in bank account (**= $100 x 1.01**)

Basic theory of interest (2)

- The case of government (riskless) bond
- The claim for receiving **$100** after one year from today (the maturity of the bond is one year)
  - Face value: $P_1 = \$100$
  - Bond price: $P_0 = \$99.01$
  - Interest rate: $r = P_1 / P_0 - 1 = 0.01$

Basic theory of interest (3)

- Algebraic representation
  - Bonk deposit: $(1+r)P_0=P_1$
  - Bond: $P_1/P_0=1+r$ or $(P_1-P_0)/P_0=r$

- P_0, P_1, r: If two of them were given, the remaining is automatically determined.
- "Bond price" and "interest rate" have a one-on-one relationship.

Basic theory of interest (4)

- Economic interpretation
  - Face value $\mathbf{P_1}$ is given and fixed
  - Suppose bond price $\mathbf{P_0}$ goes up.
  - Today's price of "$\mathbf{P_1}$ yen in future" goes up.
  - Interest rate goes down.
- Interest rate: relative price of future cash in terms of today's money

Two views on bond / loan market

	Issuing bond, bank borrowing	Financial asset
What is traded	Funds	Payoff at maturity
Demand side	Firm	Investor
Supply side	Investor / creditor	Firm
Price	Interest rate	Bond price

Basic theory of interest multi-period case (1)

- Compounding
- $P_T = P_0(1+r_1)(1+r_2)(1+r_3)\ldots(1+r_T)$

- Suppose r_i = constant for $i=1, 2, \ldots, T$
- $P_T = P_0(1+r)^T$ or $P_0 = P_T/(1+r)^T$

Basic theory of interest multi-period case (2)

- Coupon payment: C_t
- One period case: $P_0 = (P_1 + C_1)/(1+r)$

- Multi-period case

$$P_0 = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots\ldots + \frac{P_T}{(1+r)^T}$$

Notes on compounding

- Convenient approximation
  - Ln(A)= Natural log of A
  - If the absolute value of x was very small, Ln(1+x)~x
  - Ln(XY) = Ln(X) + Ln(Y)
- Let $P_2= P_0(1+r_1)(1+r_2)$ and $P_2/P_0 = 1+R$
  - $Ln((1+r_1)(1+r_2)) = Ln(1+r_1)+Ln(1+r_2) \sim r_1+r_2$
  - Thus $R \sim r_1+r_2$

8/28/2008 ICU08_lecture3: Present Value 17

Part 3: Back to investment problem

8/28/2008 ICU08_lecture3: Present Value 18

Present value principle: general formula

$$P_0 = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \ldots.. + \frac{P_T + C_T}{(1+r)^T}$$

$$But, P_T = \frac{P_{T+1} + C_{T+1}}{1+r}$$

$$P_0 = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \ldots.. + \frac{C_T}{(1+r)^T} + \frac{P_{T+1} + C_{T+1}}{(1+r)^{T+1}}$$

$$\Rightarrow P_0 = \sum_{j=1}^{\infty} \frac{C_j}{(1+r)^j}$$

What is "C_t"? What is "T"?

- If stock: C_t is dividend payments
- If real estate: C_t is rent payments
- If bond: C_t is coupon payments

- Terminal period, T
- If bond: T is finite (e.g. T =1, 5, .., 10 years)
- Stocks and real estates: T is infinite

Some useful formula (1)

- Annuity
  - financial asset that pays constant amount every period: $C = C_1 = C_2 = C_3 = \ldots\ldots = C_T$
  - Discount rate is constant: r
- Perpetuity: $T \rightarrow$ infinity

$$P_0 = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots\ldots$$

$$\Rightarrow P_0 = \frac{C}{r}$$

8/28/2008 ICU08_lecture3: Present Value 21

Some useful formula (2)

- Growing perpetuity/Growing Gordon formula
  - C grows every period at the rate of "g": $C_1 = (1+g)C_0$

$$P_0 = \frac{C}{1+r} + \frac{(1+g)C}{(1+r)^2} + \frac{(1+g)^2 C}{(1+r)^3} + \ldots\ldots$$

$$\Rightarrow P_0 = \frac{C}{r-g}$$

8/28/2008 ICU08_lecture3: Present Value 22

Discounting future cash-flows of the business project

- Suppose a project generates cash flow stream, $C_0, C_1, C_2, \ldots\ldots, C_T$.
  - C_t = "Period t sales" - "Period t costs"

- We discount future cash flows by the government bond interest rate.

Comparison with the investment to government bond

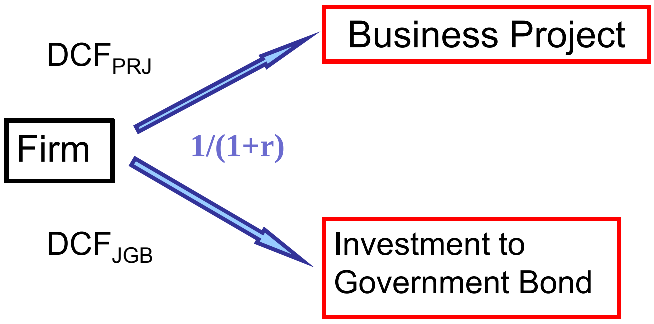
- Discounted cash flow (DCF) of the project

$$DCF_{PRJ} = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots..$$

- Investment to JGB: $C_0 = -P_0,\ C_1 = P_1 = (1+r)P_0$

$$DCF_{JGB} = -P_0 + \frac{P_1}{1+r} = -P_0 + \frac{(1+r)P_0}{1+r} = 0$$

Take “investment to JGB” as a benchmark



Valuation by present-value principle

- If $DCF_{JGB}<DCF_{PRJ}$, then the firm should invest to the project.
- But, always $DCF_{JGB}=0$. Thus, the firm should invest to the project only if $DCF_{PRJ}>0$.
- DCF_{PRJ} is the present-value of the project when it is evaluated with appropriate discount rate r.

Important preservations

- In our discussions so far, we have completely ignored any “risk.”
- If there is any risk, we have to assume risk neutrality.
- “Risk neutral” = investors care only expected payoffs, do not care its variance.
- Otherwise, we have to explicitly incorporate “the price of risk” or “risk premium” into the analysis. --- We will do this latter.

Comparing multiple projects

Choosing one of mutually exclusive projects.

	0	1	2		Total
Project 1	-7	11	12.1		16.10
Project 2	-1	22	-12.1		8.90
Project 3	-5	44	24.2		63.20
Project 4	-1	11	0		10.00

Different discount rates and present value

		Cash flow		Discount rate		PV
	0	1	2		**10%**	
Project 1	-7	11	12.1	0.10		13
Project 2	-1	22	-12.1	0.10		9
Project 3	-5	44	24.2	0.10		55
Project 4	-1	11	0	0.10		9

	Cash flow			Discount rate		PV
	0	1	2		**20%**	
Project 1	-7	11	12.1		0.20	10.56944
Project 2	-1	22	-12.1		0.20	8.930556
Project 3	-5	44	24.2		0.20	48.47222
Project 4	-1	11	0		0.20	8.166667

	Cash flow			Discount rate		PV
	0	1	2		**30%**	
Project 1	-7	11	12.1		0.30	8.621302
Project 2	-1	22	-12.1		0.30	8.763314
Project 3	-5	44	24.2		0.30	43.16568
Project 4	-1	11	0		0.30	7.461538

Part 4: Valuation by arbitrage

Arbitrage and present-value relations

- No arbitrage condition = law of one price
  - hamburger + cola + potato
  - The set of three and buying them individually should cost the same.

- Application of "No arbitrage condition"
- Using zero-coupon bond price to price coupon bonds.
- The price of zero-coupon bond that matures at time $T =$ The price of cash in period T .

An application: Pricing coupon bonds

- Face value: *$100*
  Maturity: 36 months
  Coupon: *$5* yen coupon payment at *12* months and *24* months later.

- Data (Face value = **100 thousand**)
  - Zero-coupon bond price ($T=12$ months): *97.5*
  - Zero-coupon bond price ($T=24$ months): *94.3*
  - Zero-coupon bond price ($T=36$ months): *90.7*

Replicating payoffs of the coupon bond using zero-coupon bonds

- $Y_{12}=5, Y_{24}=5, Y_{36}=100$

- Zero-coupon bond ($T=12$): **0.05** units
- Zero-coupon bond ($T=24$): **0.05** units
- Zero-coupon bond ($T=36$): 1unit
- This synthetic coupon bond has exactly same payoff pattern.

Theoretical value of coupon bond price

- **97.5x0.05 + 94.3x0.05 + 90.7x1**

  =4.875+4.715+90.7

  =100.29

- Theoretical value: **$100.29**

A digression

- How risk-free interest rate will be determined?
- Simple answer: “Demand and supply”
- Demand and supply of what?

Remember…

There are two views on bond / loan market

	Issuing bond, bank borrowing	Financial asset
What is traded	Funds	Payoff at maturity
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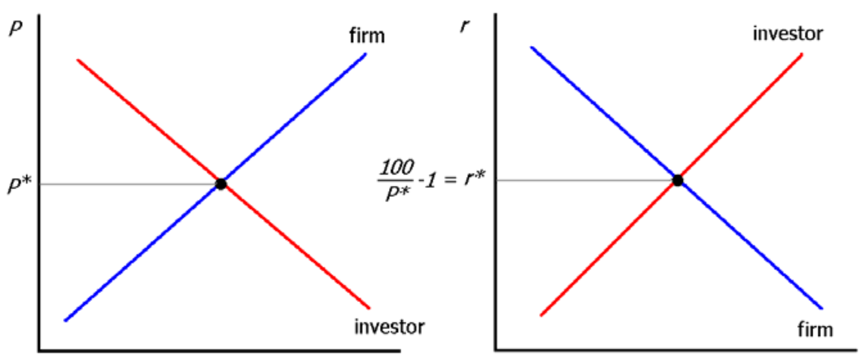
Bond market equilibrium:

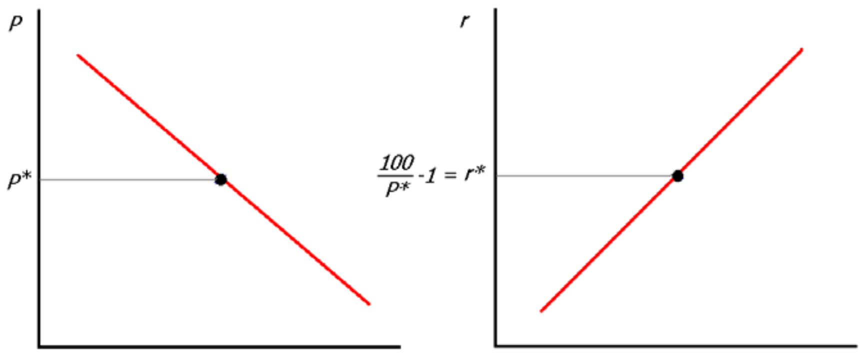
Equilibrium bond price and interest rate



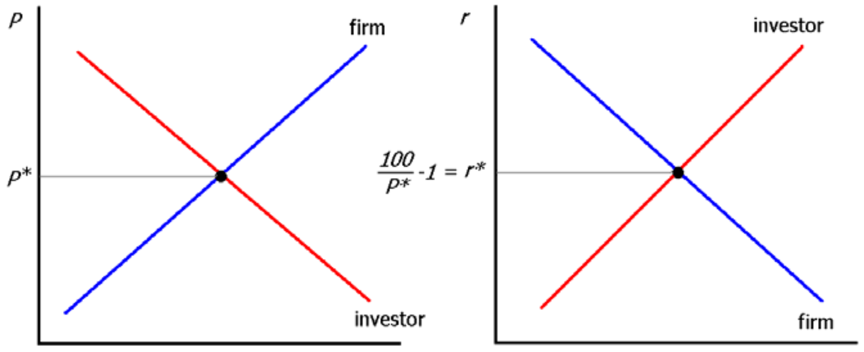
Relation between bond price and its interest rate

- Face value P_1 is given and fixed
  - Suppose bond price P_0 goes up.
  - Today's price of "P_1 yen in future" goes up.
  - Interest rate goes down.
- Interest rate: relative price of future cash in terms of today's money

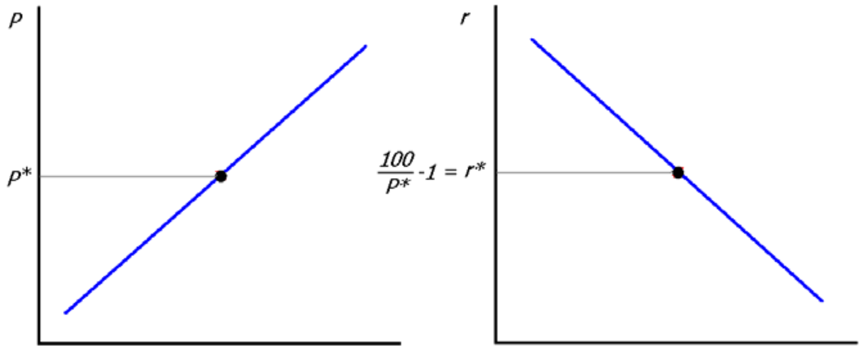
Demand for bond = supply of loans



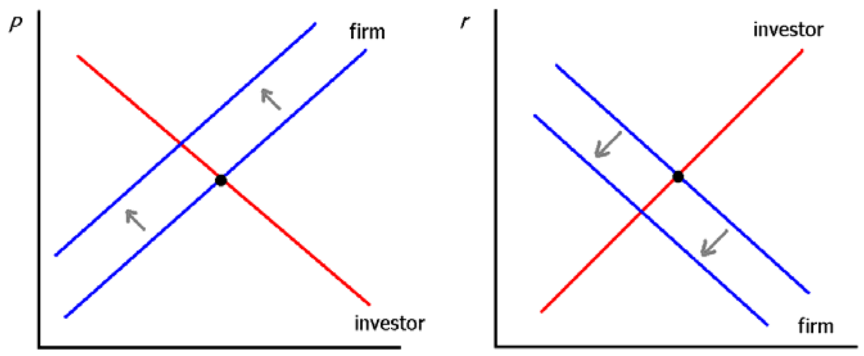
Demand for bond by lenders



Supply of bond = demand for loans



Eg.Expected profitability falls → Bond price will increases



Factors shift supply curve for bonds

- Corporate tax on profit
- Tax subsidies for investment
- Expected inflation
- Government borrowing

Factors shift demand curve for bonds

- Investors wealth
- Expected returns on bonds
- Expected returns on other assets
- Riskiness of bonds relative to other assets
- Expected inflation
- Liquidity of bonds relative to other assets

Methodology of arbitrage pricing

- You have an asset (or business project) that you would like to price. Check the payoff pattern of your asset.
- Replicate the payoff pattern of the asset using existing assets. Construct the replicating portfolio.
- The valuation of an asset must be equal to the value of assets used to construct the replicating portfolio.

Use tracking portfolio to value investment projects

- Find some asset or combination of assets that perfectly tracks the cash flows of the investment project.
- Apply no-arbitrage condition.
- The price of a tracking portfolio is the value of the investment project.

Limit of tracking portfolio approach

- For example, oil price can be used to price oil well.
- However, there will not be a perfectly replicating portfolio in practice. There are always tracking errors.
- If there are significant tracking errors, use asset pricing models.
  → Theme of the next lecture.