

# Infrequent Changes of Policy Target: Stop-Go Monetary Policy under Knightian Uncertainty

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## Abstract

In many countries, the monetary policy instrument remains unchanged for a long period and shows infrequent responses to exogenous shocks. The purpose of this paper is to provide a new explanation on why the central bank's policy instrument remains so unchanged. In the following analysis, we explore how Knightian uncertainty affects optimal monetary policy. We apply the Choquet expected decision theory to a new Keynesian model. A main result is that the policymaker may frequently keep the interest rate unchanged even when exogenous shocks change output gaps and inflation rates. This happens because a change of the interest rate increases uncertainty for the policymaker when structural parameters are not known. To the extent that the policymaker has uncertainty aversion, it can therefore be optimal for the policymaker to maintain an unchanged policy stance for some significant periods and to make discontinuous changes of the target rate following a Taylor rule. Our analysis departs from previous studies in that we determine an optimal monetary policy rule that allows time-variant feedback parameters instead of restricting ourselves to time-invariant feedback parameters. This leads to an optimal stop-go policy rule that sometimes responds to output and inflation gaps but sometimes does not.

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## Introduction

In monetary economics, it has widely been discussed what policy rules central banks follow. A growing number of studies advocate a variety of monetary policy rules that can lead to good performance. In particular, many argue that macroeconomic stabilization should be implemented through a “Taylor rule” in which interest rates are adjusted in response to output gap and inflation rate. However, when we look at high-frequency data, the policy instrument remains unchanged for a long period and shows infrequent responses to frequent exogenous shocks. Figure 1 plots daily data of targeted federal fund rates from January 2001 to December 2006. It is easy to see that the changes of the targeted federal fund rates were rare throughout the period. Since the Federal Reserve’s Trading Desk keeps the federal funds rate near a target set by the Federal Open Market Committee (FOMC), this implies that the baseline of the U.S. short-term interest rate changed very infrequently.<sup>1</sup>

One of the reasons why the changes of the targeted federal fund rates were so rare is that the FOMC meeting is usually held only eight times a year. It is the FOMC that decides some discontinuous jumps of the targeted rates. However, except in 2001 and 2005, the FOMC decided not to change the target rate in most of the meetings (see [Table 1](#)). Infrequent FOMC meetings would not be enough to explain less frequent changes of the targeted rates. Similar infrequent policy changes can be observed for the other central banks that face different environments. For example, [Table 2](#) summarizes the number of monetary policy decisions and the number of decisions with no policy change in the Bank of Japan, the European Central Bank, and the Bank of England from 1999 to 2006. It is easy to see that these central banks changed the targeted policy instruments much less frequently than the Federal Reserve Board throughout the period.

Why do the central banks decide not to change the policy targets so frequently? The purpose of this paper is to provide a new explanation on why the central bank’s policy instrument remains so unchanged under parameter uncertainty. In the following analysis, we explore how Knightian uncertainty affects optimal monetary policy in a new Keynesian model. The decision-making theory we use in the analysis is that of expected utility under a nonadditive probability measure, that is, the Choquet expected model, developed by Gilboa (1987) and Schmeidler (1989).<sup>2</sup> We apply the Choquet expected decision theory to a new Keynesian model. A main result is that the

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<sup>1</sup> The realized federal fund rates that are called “effective federal fund rates” show some daily fluctuations over time. However, they only show small fluctuations around the targeted rates.

<sup>2</sup> Based on the Gilboa-Schmeidler’s axioms, studies such as Epstein and Wang (1994), and Mukerji and Tallon (2004) incorporate Knightian uncertainty in economic models.

policymaker may frequently keep the interest rate unchanged even when exogenous shocks change output gaps and inflation rates. This happens because a change of the interest rate increases uncertainty for the policymaker when the structural parameters are not well known. To the extent that the policymaker has uncertainty aversion, it can therefore be optimal for the policymaker to maintain an unchanged policy stance for some significant periods and to make discontinuous changes of the target rate following a Taylor rule.

In previous literature, there are a large number of studies that focused on model uncertainty and the performance of policy rules across different models. Brainard (1967) is a seminal study that explored how the policymaker's optimal rule is altered when faced with parameter uncertainty. McCallum (1988) has argued for evaluating policy proposals in a variety of economic models as a means of assessing their robustness. Using five macroeconomic models, Levin, Wieland, and Williams (2003) identify the robust rules that respond to the inflation forecast and the output gap but that incorporate a substantial degree of policy inertia. Using a new Keynesian model, Giannoni and Woodford (2003a, 2003b) have analyzed policy rules that are robust to misspecification of the disturbance process of a known model, while Kimura and Kurozumi (2003) and Levin and Williams (2003b) have focused on whether parameter uncertainty leads to more cautious or more aggressive policy responses to shocks when the effects of structural parameters on the loss function are taken into account. However, since none of these studies considered Knightian uncertainty, the model uncertainty has never lead to the conclusion that it is optimal for the policymaker to keep the interest rate unchanged without responding to inflation and output gaps.

Several recent studies explored "robust optimal policy rules" under a version of Knightian uncertainty, which are designed to be robust in the sense of minimizing the worst case scenario when the policymaker believes that the true model is in a neighborhood of a given reference model. These studies include Hansen and Sargent (2003), Onatski and Stock (2002), Tetlow and von zur Muehlen (2001), and Giannoni (2006). Walsh (2004) has argued that optimal monetary policy under Hansen-Sargent framework is equivalent to that of Giannoni and Woodford where the optimal policy rule becomes less aggressive under uncertainty. In contrast, Onatski and Stock argued that the minimax approach of robust control provides robust monetary policies that are more aggressive than the optimal policies absent model uncertainty.<sup>3</sup> However, unlike ours, none of these studies has reached a conclusion that the optimal policy is to keep the policy instrument unchanged for some periods.

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<sup>3</sup> Giannoni (2002, 2006) supports this under more general environments.

Our analysis departs from these previous studies in two important ways. First, instead of restricting ourselves to time-invariant feedback parameters, we determine an optimal monetary policy rule that allows time-variant feedback parameters. This leads to an optimal stop-go policy rule that sometimes responds to output and inflation gaps but sometimes does not. Second, we assume that the policymaker can observe all relevant macro variables before making the policy decision. The assumption implies that the policymaker faces no uncertainty on these variables only without policy changes. Under the assumption, uncertainty on structural parameters becomes crucial in deriving infrequent policy changes under Knightian uncertainty or in a robust control framework.

Our result is similar to that of Dow and Werlang (1992) in that a player chooses the status quo under Knightian uncertainty. Dow and Werlang provide a simple example where the optimal portfolio choice can be the status quo under Knightian uncertainty. However, given that the policy changes are rare, it deserves to pay a special attention to see why the central banks prefer the status quo under Knightian uncertainty. Central bankers have multiple objectives and confront a variety of economic circumstances. They know that their actions have significant impacts on the economy, but the timing, magnitude, and channels of those impacts are not fully understood. They, in contrast, have a concern that their reputation would deteriorate dramatically if their actions have wrong impacts on the economy. Under the circumstances, it may become desirable for the central banks not to change the policy targets when the parameter uncertainty makes the impacts uncertain enough.

In macroeconomics, it was almost a conventional wisdom that central banks implement monetary policy in a gradual fashion (see, for example, Blinder [1997]). Many researchers claim that this gradualism is due to 'optimal cautiousness', although some others suggest alternative interpretations (see, for example, Rudebusch [2005]). Interest-rate smoothing or monetary policy inertia is, however, different from monetary policy with infrequent changes and some discontinuous jumps. When using low frequency data, the two types of monetary policies may be observationally equivalent. But their macroeconomic implications will be different at least in the short-run and may be so even in the long-run. It is practically very important to pay a special attention to macroeconomic consequences of the stop-go policy that changes the policy instrument infrequently.

The paper proceeds as follows. Section 2 sets up the basic model and section 3 explains the policy objectives. Sections 4 and 5 derive the optimal monetary rules with and without policy changes. Section 6 shows some results of stochastic

simulation and section 7 checks their robustness. Section 8 summarizes our main results and refers to their implications.

## 1. The Basic Model

Our basic model follows a simple new Keynesian model:

$$(1) \quad x_t = x_{t+1}^e - \alpha (i_t - \pi_{t+1}^e) + u_t,$$

$$(2) \quad \pi_t = \beta \pi_{t+1}^e + k_t x_t + e_t,$$

where  $x_t$  = the gap between actual output and the flexible-price equilibrium output level,  $i_t$  = the nominal interest rate,  $\pi_t$  = the inflation rate,  $u_t$  = a demand disturbance, and  $e_t$  = a supply shock. The variable with superscript  $e$ , such as  $x_{t+1}^e$  and  $\pi_{t+1}^e$ , denotes the private agents' expectations.

Equation (1) is the Euler condition from the representative household's consumption decision, while equation (2) is a new Keynesian Phillips curve. Subscript  $t$  denotes time period. All variables are expressed as log deviations from the steady state. Although there is some arbitrariness in the information structure, the model is standard. We, however, impose an additional assumption that the parameter  $k_t$  follows a binomial distribution that takes either  $k_1$  or  $k_2$ , where  $k_1 > k_2$ . The parameter  $k_t$  captures both the impacts of a change in real marginal cost on inflation and the co-movement of real marginal cost and the output gap. In literature, there exists large disagreement on an appropriate value of  $k_t$ . When the time interval is one quarter, McCallum and Nelson (2000) report that the empirical evidence is consistent with a value of  $k_t$  in the range [0.01, 0.05]. Roberts (1995) show that the coefficient on the output gap is about 0.3 by annual data, which implies a value for  $k_t$  is 0.075 by quarterly data. Jensen (2002) uses a baseline value of  $k_t = 0.1$ , while Walsh (2003) uses 0.05. In the following analysis, we assume that the random parameter  $k_t$  is independently and identically distributed over time.

In our model, the following timing of events is assumed in period  $t$ . At the beginning of period  $t$ , the private agents form their expectations  $x_{t+1}^e$  and  $\pi_{t+1}^e$ . When they form the expectations, shocks in period  $t$  have not occurred yet, so that the expectations are based on the information available at the end of period  $t-1$ . After the private agents formed  $x_{t+1}^e$  and  $\pi_{t+1}^e$ , innovations to  $u_t$  and  $e_t$  as well as the parameter value of  $k_t$  are realized. However, the policymaker cannot observe the realized innovations directly. Instead, he (or she) observes the realized values of  $x_t$  and  $\pi_t$  that

reflect the innovations. The policymaker decides the nominal interest rate  $i_t$  based on the updated information. It is, however, noteworthy that the updated information only includes the realized values of  $x_t$  and  $\pi_t$  before the nominal interest rate is changed. There still exists uncertainty on what value was realized for  $\pi_t$  when the nominal interest rate is changed in period  $t$ . The period  $t$  ends after the policymaker decided the nominal interest rate  $i_t$ . At the end of the period, both the private agents and the policymaker observe the realized innovations to  $u_t$  and  $e_t$  directly, which are reflected only in their decision makings after period  $t+1$ .

**Figure 2** summarizes the timing of the events in period  $t$ . A key assumption in the timing is that the policymaker faces a significant uncertainty on the effects of the nominal interest rate change on  $\pi_t$ . To distinct the state before the policy change from that after the policy change, we define  $x_t^o$  and  $\pi_t^o$  respectively as the realized values of  $x_t$  and  $\pi_t$  before the nominal interest rate is determined. By definition, it holds that

$$(3) \quad x_t^o = x_{t+1}^e - \alpha (i_{t-1} - \pi_{t+1}^e) + u_t,$$

$$(4) \quad \pi_t^o = \beta \pi_{t+1}^e + k_t x_t^o + e_t.$$

Equations (1)-(4) therefore lead to:

$$(5) \quad x_t = x_t^o - \alpha \Delta i_t,$$

$$(6) \quad \pi_t = \pi_t^o - \alpha k_t \Delta i_t.$$

where  $x_t$  and  $\pi_t$  denote the realized values of  $x_t$  and  $\pi_t$  after the nominal interest rate is determined.

Equations (5) and (6) determine the equilibrium values of  $x_t$  and  $\pi_t$  in our model. It is noteworthy that these equations hold for more general models. For example, we can show that our model leads to (5) and (6) even when we include a variety of lagged variables,  $x_{t-1}, x_{t-2}, \dots, \pi_{t-1}, \pi_{t-2}, \dots$ , in the right-hand sides of (1) and (2).

From (5), the policymaker can infer the exact value of  $x_t$  when observing  $x_t^o$ . In addition, from (4), the observation of  $x_t^o$  and  $\pi_t^o$  will make the estimate of  $k_t$  more precise. However, to the extent that the supply shock  $e_t$  is stochastic, the policymaker cannot identify the exact value of  $k_t$ . Given the prior distribution of  $k_t$ , the Bayesian policymaker deduces its posterior distribution when observing  $x_t^o$  and  $\pi_t^o$ . In the following analysis, the Bayesian policymaker is supposed to have the posterior distribution that  $k_t$  is  $k_1$  with probability  $v_t$  and  $k_2$  with probability  $1-v_t$ . It is noteworthy that even if the policymaker's prior distribution of  $k_t$  is time-invariant, the

posterior probability  $v_t$  is time-variant because  $\pi_t^0$  changes over time.

## 2. The Policy Objectives

The policymaker chooses its policy instrument so as to achieve the policy objective. We suppose that the objective of the policymaker is to set the nominal interest rate at each point of time so as to minimize the “expected” value of the following loss function:

$$(7) \quad L_t = \lambda(x_t - x^*)^2 + (\pi_t - \pi^*)^2,$$

In the loss function, loss depends on deviations of output gap and inflation from their targets  $x^*$  and  $\pi^*$ . An exogenous parameter  $\lambda$  is positive and is treated as independent of the specification of the structural equations. We assume that the loss depends only on the current output gap and inflation deviations. We imposed the assumption for analytical tractability. However, to the extent that the policymaker minimizes (7) in each period, minimizing the present discounted value will have only second-order welfare effects under reasonable environments. In addition, we can justify the myopic loss function when the private agents’ expectations are independent of previous policy changes or when they are uncertain enough for the policymaker (see Appendix for their examples).

What makes the following analysis distinctive from the standard minimization problem is that we characterize the expected loss minimization of the policymaker by the Choquet expectation. To distinguish it from standard expectation operator, we defined the Choquet expectation operator by  $E_t^Q$ . Having aversion to Knightian uncertainty, the policymaker chooses its policy instrument  $\Delta i_t$  so as to minimize  $E_t^Q L_t$ . More general representation of the Choquet expectation is extensively discussed in Schmeidler (1989).<sup>4</sup> But, since the parameter  $k_t$  follows a binomial distribution that takes either  $k_1$  or  $k_2$  ( $k_1 > k_2$ ), the loss function is simply written as

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<sup>4</sup> For example, define  $L(k_t) \equiv \lambda(x_t^0 - \alpha \Delta i_t - x^*)^2 + (\pi_t^0 - \alpha k_t \Delta i_t - \pi^*)^2$ . If the random variable  $k_t$  takes  $n$  alternative values,  $k_1, k_2, \dots, k_n$ , such that  $0 \leq L(k_1) \leq L(k_2) \leq \dots \leq L(k_n)$ , the Choquet expectation is expressed as  $E_t^Q L(k_t) = \sum_{i=1}^{n-1} [L(k_i) - L(k_{i+1})] \theta(\cup_{j=1}^i k_j) + L(k_n)$ , where  $\theta(\cdot)$  is a convex probability capacity (or a convex non-additive probability function).

$$\begin{aligned}
(8) \quad E_t^Q L_t &= \lambda(x_t^0 - \alpha \Delta i_t - x^*)^2 + v_t(1-\varepsilon) (\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 \\
&\quad + \{1-v_t(1-\varepsilon)\} (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2 \\
&\quad \text{when } (\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 < (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2, \\
&= \lambda(x_t^0 - \alpha \Delta i_t - x^*)^2 + \{1-(1-v_t)(1-\varepsilon)\} (\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 \\
&\quad + (1-v_t)(1-\varepsilon) (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2 \\
&\quad \text{when } (\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 > (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2.
\end{aligned}$$

where  $v_t$  is the original posterior probability that  $k_t = k_1$  (so that  $1-v_t$  is the original posterior probability that  $k_t = k_2$ ). A parameter  $\varepsilon$  ( $> 0$ ) denotes the degree of  $\varepsilon$ -contamination in the Choquet expectation. Since the Choquet expectation puts more weight on the worst outcome,  $v_t$  is contaminated to be smaller when  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 < (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2$ , so is  $1-v_t$  when  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 > (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2$  in the Choquet expectation.

The problem the policymaker faces is similar to that of a Bayesian statistician who is confronted with “uncertainty” in a posterior distribution of  $k_t$ . One procedure that the Bayesian statistician often follows under Knightian uncertainty is to introduce a set of posteriors obtained by “contaminating” original posteriors. The loss function (8) follows this procedure. When  $\varepsilon = 0$ , the problem is degenerated to the traditional expected loss minimization problem. When  $\varepsilon = 1$ , the problem is degenerated to the classical mini-max problem where the policymaker minimizes only the worst case scenario. An increase in  $\varepsilon$  implies that the policymaker becomes less certain that the original posterior distribution is true distribution. Thus, an increase in  $\varepsilon$  can be interpreted as an increase in Knightian uncertainty.

If  $\varepsilon = 0$ , the first-order condition  $\partial E_t^Q L_t / \partial \Delta i_t = 0$  leads to

$$(9) \quad \Delta i_t = \frac{\lambda(x_t^0 - x^*) + \{v_t k_1 + (1-v_t)k_2\}(\pi_t^0 - \pi^*)}{\alpha[\lambda + \{v_t k_1^2 + (1-v_t)k_2^2\}]}$$

This simple monetary policy rule is similar to a Taylor rule in the sense that the nominal interest rate is adjusted in response to “output gap” and “inflation”. Because of uncertainty in  $k_t$ , variance of  $k_t$  appears in the denominator of the feedback rule. This reflects a version of Brainard’s effect where the policymaker’s optimal rule becomes less aggressive under parameter uncertainty. It is noteworthy that the rule does not depend on how expectations are formed nor what stochastic processes the exogenous shocks follow. However, “output gap” and “inflation” in (9) are those before the



central bank sets a new interest rate. In addition, unlike standard Taylor rules, the coefficient of lagged inflation is always equal to unity.<sup>5</sup>

### 3. The Optimal Monetary Rule that Removes Uncertainty

The policy rule (9) is no longer optimal when the policymaker has some aversion to uncertainty. One technical problem in deriving the optimal rule under uncertainty is that unless  $\varepsilon = 0$ , the “expected” loss function  $E_t^Q L_t$  is not differentiable. However, since  $E_t^Q L_t$  is convex in  $\Delta i_t$ , it is optimal for the central bank to set  $\Delta i_t = \Phi$  if and only if  $\partial E_t^Q L_t / \partial \Delta i_t \geq 0$  when  $\Delta i_t$  approaches to  $\Phi$  from above and  $\partial E_t^Q L_t / \partial \Delta i_t \leq 0$  when  $\Delta i_t$  approaches to  $\Phi$  from below. This leads to the following proposition.

**Proposition:** When  $\pi_t^0 > \pi^*$ , the central bank decides not to change the nominal interest rate if and only if

$$(10) \quad \frac{-\lambda(x_t^0 - x^*)}{\{1 - (1 - \nu_t)(1 - \varepsilon)\}k_1 + (1 - \nu_t)(1 - \varepsilon)k_2} \leq \pi_t^0 - \pi^* \leq \frac{-\lambda(x_t^0 - x^*)}{\nu_t(1 - \varepsilon)k_1 + \{1 - \nu_t(1 - \varepsilon)\}k_2}.$$

When  $\pi_t^0 < \pi^*$ , the central bank decides not to change the nominal interest rate if and only if

$$(11) \quad \frac{-\lambda(x_t^0 - x^*)}{\nu_t(1 - \varepsilon)k_1 + \{1 - \nu_t(1 - \varepsilon)\}k_2} \leq \pi_t^0 - \pi^* \leq \frac{-\lambda(x_t^0 - x^*)}{\{1 - (1 - \nu_t)(1 - \varepsilon)\}k_1 + (1 - \nu_t)(1 - \varepsilon)k_2}.$$

**Proof:** When  $\pi_t^0 > \pi^*$ ,  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 < (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2$  if  $\Delta i_t = +0$  and  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 > (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2$  if  $\Delta i_t = -0$ . Equation (8) therefore implies that

$$\left. \frac{\partial E_t^Q L_t}{\partial \Delta i_t} \right|_{\Delta i_t = +0} = -2\alpha\lambda(x_t^0 - x^*) - 2\alpha [\{1 - (1 - \nu_t)(1 - \varepsilon)\}k_1 + (1 - \nu_t)(1 - \varepsilon)k_2] (\pi_t^0 - \pi^*)^2, \text{ and}$$

$$\left. \frac{\partial E_t^Q L_t}{\partial \Delta i_t} \right|_{\Delta i_t = -0} = -2\alpha\lambda(x_t^0 - x^*) - 2\alpha [\nu_t(1 - \varepsilon)k_1 + \{1 - \nu_t(1 - \varepsilon)\}k_2] (\pi_t^0 - \pi^*)^2.$$

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<sup>5</sup> In previous literature, Levin, Wieland, and Williams (1999) provides strong support for rules in which the first-difference of the federal funds rate responds to output and inflation gaps.

Since it is optimal to set  $\Delta i_t = 0$  if and only if  $\left. \frac{\partial E_t^o L_t}{\partial \Delta i_t} \right|_{\Delta i_t=+0} \geq 0$  and  $\left. \frac{\partial E_t^o L_t}{\partial \Delta i_t} \right|_{\Delta i_t=-0} \leq 0$ , we

thus obtain the first part of the proposition. Similarly, we can derive the second part of the proposition. [Q.E.D.]

The above result suggests that the policymaker may keep the policy instrument unchanged even if the exogenous shocks change output gaps and inflation rates. In the absence of Knightian uncertainty,  $\varepsilon$  is equal to zero, so that there exists no measurable range of  $\pi_t^o - \pi^*$  that satisfies the above inequalities. However, when  $\varepsilon > 0$ , some measurable range of  $\pi_t^o - \pi^*$  satisfies the above inequalities. Given the parameters, the range is wider as  $\varepsilon$  is larger. The reason why the policymaker may choose  $\Delta i_t = 0$  is that  $x_t = x_t^o$  and  $\pi_t = \pi_t^o$  when  $\Delta i_t = 0$ , so that the policymaker faces no uncertainty when  $\Delta i_t = 0$ . To the extent that the policymaker has uncertainty aversion, it can therefore be optimal to set  $\Delta i_t = 0$  for some measurable range.

It is noteworthy that neither (10) nor (11) holds unless  $(x_t^o - x^*)(\pi_t^o - \pi^*) < 0$ . This implies that some conflict between output stability and inflation stability is an important source for the policymaker to keep the interest rate unchanged. For example, when  $x_t^o > x^*$  and  $\pi_t^o < \pi^*$ , lowering the interest rate achieves output stability but sacrifices inflation rate stability. The tradeoff leads to infrequent changes of the interest rate under uncertainty in our model.

Since it holds that  $(\pi_t^o - \alpha k_1 \Delta i_t - \pi^*)^2 = (\pi_t^o - \alpha k_2 \Delta i_t - \pi^*)^2 = [(k_1 - k_2)/(k_1 + k_2)]^2 (\pi_t^o - \pi^*)^2$  when  $\Delta i_t = (2/\alpha)(\pi_t^o - \pi^*)/(k_1 + k_2)$ , there also exists no uncertainty in the loss function when  $\Delta i_t = (2/\alpha)(\pi_t^o - \pi^*)/(k_1 + k_2)$ . This leads to the following corollary.

**Corollary:** When  $\pi_t^o > \pi^*$ , the central bank sets  $\Delta i_t = (2/\alpha)(\pi_t^o - \pi^*)/(k_1 + k_2)$  if and only if

$$(12) \quad \left[ 2 \frac{\lambda + \Omega_1}{k_1 + k_2} - \Psi_1 \right] (\pi_t^o - \pi^*) \leq \lambda (x_t^o - x^*) \leq \left[ 2 \frac{\lambda + \Omega_2}{k_1 + k_2} - \Psi_2 \right] (\pi_t^o - \pi^*),$$

where  $\Omega_1 \equiv v_t(1-\varepsilon) k_1^2 + \{1-v_t(1-\varepsilon)\} k_2^2$ ,  $\Omega_2 \equiv \{1-(1-v_t)(1-\varepsilon)\} k_1^2 + (1-v_t)(1-\varepsilon) k_2^2$ ,  $\Psi_1 \equiv v_t(1-\varepsilon) k_1 + \{1-v_t(1-\varepsilon)\} k_2$ , and  $\Psi_2 \equiv \{1-(1-v_t)(1-\varepsilon)\} k_1 + (1-v_t)(1-\varepsilon) k_2$ . Similarly, when  $\pi_t^o < \pi^*$ , it is optimal for the central bank to set  $\Delta i_t = (2/\alpha)(\pi_t^o - \pi^*)/(k_1 + k_2)$  if and only if

$$(13) \quad \left[ 2 \frac{\lambda + \Omega_2}{k_1 + k_2} - \Psi_2 \right] (\pi_t^0 - \pi^*) \leq \lambda (x_t^0 - x^*) \leq \left[ 2 \frac{\lambda + \Omega_1}{k_1 + k_2} - \Psi_1 \right] (\pi_t^0 - \pi^*).$$

Proof: When  $\pi_t^0 > \pi^*$ , it holds that

$$\begin{aligned} \left. \frac{\partial E_t^0 L_t}{\partial \Delta i_t} \right|_{\Delta i_t - A = +0} &= -2\alpha(\lambda (x_t^0 - x^*) + [v_t(1-\varepsilon) k_1 + \{1-v_t(1-\varepsilon)\} k_2] (\pi_t^0 - \pi^*)^2) \\ &\quad + 4\alpha(\lambda + [v_t(1-\varepsilon) k_1^2 + \{1-v_t(1-\varepsilon)\} k_2^2] (\pi_t^0 - \pi^*)/(k_1 + k_2)), \end{aligned}$$

$$\begin{aligned} \left. \frac{\partial E_t^0 L_t}{\partial \Delta i_t} \right|_{\Delta i_t - A = -0} &= -2\alpha(\lambda (x_t^0 - x^*) + [\{1-(1-v_t)(1-\varepsilon)\} k_1 + (1-v_t)(1-\varepsilon) k_2] (\pi_t^0 - \pi^*)^2) \\ &\quad + 4\alpha(\lambda + [\{1-(1-v_t)(1-\varepsilon)\} k_1^2 + (1-v_t)(1-\varepsilon) k_2^2] (\pi_t^0 - \pi^*)/(k_1 + k_2)), \end{aligned}$$

where  $A \equiv (2/\alpha)(\pi_t^0 - \pi^*)/(k_1 + k_2)$ . Since it is optimal to set  $\Delta i_t = A$  if and only if

$$\left. \frac{\partial E_t^0 L_t}{\partial \Delta i_t} \right|_{\Delta i_t - A = +0} \geq 0 \text{ and } \left. \frac{\partial E_t^0 L_t}{\partial \Delta i_t} \right|_{\Delta i_t - A = -0} \leq 0, \text{ we thus obtain the first part of the proposition.}$$

Similarly, we can derive the second part of the proposition. [Q.E.D.]

This corollary suggests that the under some circumstances, the policymaker may change the interest rate responding only to the gap between inflation rate and its target. This corollary is, however, an artifact from our specific assumption that the parameter  $k_t$  follows a binomial distribution. For example, if  $k_t$  follows a trinomial distribution that takes either  $k_1$ ,  $k_2$ , or  $k_3$ , the corollary no longer holds because  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 = (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2 = (\pi_t^0 - \alpha k_3 \Delta i_t - \pi^*)^2$  if and only if  $\Delta i_t = 0$ .

#### 4. The Optimal Monetary Rule with a Taylor Rule

Unless  $\Delta i_t$  is equal to either zero or  $(2/\alpha)(\pi_t^0 - \pi^*)/(k_1 + k_2)$ , it does not hold that  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 = (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2$ . Therefore, when neither (10)-(11) nor (12)-(13) hold, the first-order condition  $\partial E_t^0 L_t / \partial \Delta i_t = 0$  leads to the optimal monetary policy rule. In particular, when  $\pi_t^0 > \pi^*$ , it holds that  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 > (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2$  if  $\Delta i_t < 0$  or  $\Delta i_t > (2/\alpha)(\pi_t^0 - \pi^*)/(k_1 + k_2)$  and  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 < (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2$  if  $0 < \Delta i_t < (2/\alpha)(\pi_t^0 - \pi^*)/(k_1 + k_2)$ . When  $\pi_t^0 > \pi^*$  and neither (10)-(11) nor (12)-(13) hold, differentiating equation (8) thus leads to the optimal rule:

$$(14) \quad \Delta i_t = \frac{\lambda(x_t^0 - x^*) + [\nu_t(1-\varepsilon)k_1 + \{1-\nu_t(1-\varepsilon)\}k_2](\pi_t^0 - \pi^*)}{\alpha(\lambda + [\nu_t(1-\varepsilon)k_1^2 + \{1-\nu_t(1-\varepsilon)\}k_2^2])}$$

if  $0 < \Delta i_t < A$ , and

$$(15) \quad \Delta i_t = \frac{\lambda(x_t^0 - x^*) + [\{1-(1-\nu_t)(1-\varepsilon)\}k_1 + (1-\nu_t)(1-\varepsilon)k_2](\pi_t^0 - \pi^*)}{\alpha(\lambda + [\{1-(1-\nu_t)(1-\varepsilon)\}k_1^2 + (1-\nu_t)(1-\varepsilon)k_2^2])}$$

if  $\Delta i_t < 0$  or  $\Delta i_t > A$ , where  $A \equiv (2/\alpha)(\pi_t^0 - \pi^*)/(k_1+k_2)$ .

Similarly, when  $\pi_t < \pi^0$ , it holds that  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 > (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2$  if  $\Delta i_t > 0$  or  $\Delta i_t < (2/\alpha)(\pi_t - \pi^*)/(k_1+k_2)$  and  $(\pi_t^0 - \alpha k_1 \Delta i_t - \pi^*)^2 > (\pi_t^0 - \alpha k_2 \Delta i_t - \pi^*)^2$  if  $\Delta i_t < 0$  or  $\Delta i_t > (2/\alpha)(\pi_t - \pi^*)/(k_1+k_2)$ . Therefore, when  $\pi_t < \pi^0$  and neither (10)-(11) nor (12)-(13) hold, the optimal rule is (14) if  $\Delta i_t > 0$  or  $\Delta i_t < A$ , and (15) if  $\Delta i_t < 0$  or  $\Delta i_t > A$ .

Both of the monetary policy rules (14) and (15) are similar to a Taylor rule in the sense that the nominal interest rate is adjusted in response to “output gap” and “inflation”. They also reflect a version of Brainard’s effect where the policymaker’s optimal rule becomes less aggressive under parameter uncertainty. However, the elasticity of  $\Delta i_t$  to output gap and inflation rate depends on the degree of uncertainty aversion (that is,  $\varepsilon$ ) and differs between (14) and (15).<sup>6</sup> Consequently, the nominal interest rate shows different responses to “output gap” and “inflation” depending on whether  $\Delta i_t$  is positive or not and whether  $\Delta i_t$  is greater than  $A$  or not.

It is noteworthy that neither the rule (14) nor the rule (15) is optimal when (10)-(11) hold. This implies that the policymaker, who has uncertainty aversion, sometimes keeps the interest rate unchanged and sometimes implements discontinuous jumps of the interest rate. This type of stop-go policy is different from standard interest-rate smoothing or monetary policy inertia that was regarded as a conventional wisdom in macroeconomics. The macroeconomic consequences are also different because  $x_t$  and  $\pi_t$  take different values depending on whether  $i_t$  was changed or not.

## 6. Stochastic Simulations

The purpose of this section is to examine how the policymaker chooses the policy instrument  $i_t$  under Knightian uncertainty for some specific parameter values and

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<sup>6</sup> For example, since  $k_1 > k_2$ , the elasticity of  $\Delta i_t$  to output gap is larger in (14) than in (15).

stochastic shocks. Our model has three constant parameters:  $\alpha$ ,  $\beta$ ,  $\lambda$  and one random parameter:  $k_t$ . The discount factor  $\beta$  is set equal to 0.999, appropriate for interpreting the time interval as one month. We use the interest rate elasticity of the aggregate demand of  $\alpha = 0.5$ , which implies  $\alpha = 0.2$  by quarterly data, and a weight on output fluctuations of  $\lambda = 0.25$ . For the random parameter  $k_t$ , we set  $k_1 = 0.03$  and  $k_2 = 0.01$ , which implies  $k_1 = 0.12$  and  $k_2 = 0.04$  by quarterly data. Since all variables are expressed as log deviations from the steady state,  $x^*$  and  $\pi^*$  can take any sign when the policy target is different from the steady state. In the benchmark case, we set  $x^*$  to be zero but  $\pi^*$  to be -0.5. This implies that the policymaker is an aggressive inflation fighter.

As for the private agents' expectations, we assume the following backward-looking expectations:

$$\begin{aligned} x_{t+1}^e &= \theta_x x_{t-1} + (1-\theta_x) x_t^e, \\ \pi_{t+1}^e &= \theta_\pi \pi_{t-1} + (1-\theta_\pi) \pi_t^e. \end{aligned}$$

To the extent that  $0 < \theta_x < 1$  and  $0 < \theta_\pi < 1$ , the expectations are a version of adaptive expectations. Growing empirical studies support backward-looking adaptive expectations over forward-looking rational expectations. We set  $\theta_x = \theta_\pi = 0.5$  in the benchmark experiments.

As for the exogenous shocks, we suppose that both the demand disturbance  $u_t$  and the supply shock  $e_t$  follow a first-order autoregressive process:  $u_t = \rho_u u_{t-1} + \eta_t$  and  $e_t = \rho_e e_{t-1} + \omega_t$ . We set  $\rho_u = \rho_e = 0.8$  and assume that  $\eta_t \sim N(0, 0.01)$  and  $\omega_t \sim N(0, 0.05)$ . The assumption implies that supply shocks are more dominant than demand shocks in the economy. As for the prior distribution of  $k_t$ , we assume that  $k_t = k_1$  with probability 0.5 and  $k_t = k_2$  with probability 0.5. Given this prior distribution for  $k_t$ , the Bayes' theorem implies that

$$v_t \equiv \Pr(k_t = k_1 | \pi_t^o = \pi^o) = \frac{\Pr(\pi_t^o = \pi^o | k_t = k_1)}{\Pr(\pi_t^o = \pi^o | k_t = k_1) + \Pr(\pi_t^o = \pi^o | k_t = k_2)}.$$

Since  $\Pr(\pi_t^o = \pi^o | k_t = k_1) = \Pr(\omega_t = \pi^o - \beta \pi_{t+1}^e - k_1 x_t^o - \rho_e e_{t-1})$ , the observations of  $\pi_t^o$  and  $x_t^o$  and the predetermined values of  $\pi_{t+1}^e$  and  $e_{t-1}$  lead to the posterior probability  $v_t$ .

In the simulations, we randomly draw stochastic shocks following the above distributions for 460 periods and calculate the optimal monetary policy for the shocks.

We depict a series of optimal nominal interest rates when  $\varepsilon = 0.2$  and  $0$ . The interest rate for  $\varepsilon = 0.2$  is the optimal monetary policy under Knightian uncertainty and the interest rate for  $\varepsilon = 0$  is that without Knightian uncertainty. It is noteworthy that the policy instrument follows a version of Taylor rule when  $\varepsilon = 0$ . The comparison between the two interest rates will thus show how the optimal policy deviates from a Taylor rule when the policymaker has uncertainty aversion. As for the initial conditions, we set  $x_{t+1}^e = \pi_{t+1}^e = 0$  when  $t = 0$ . But to avoid the impacts of the arbitrary initial conditions, we show the series from  $t = 201$  to  $260$ , from  $t = 301$  to  $360$ , and from  $t = 401$  to  $460$ .

Figures 3-A, 3-B, and 3-C depict the interest rates for three alternative sixty-month periods. Since the realized shocks are different, each series shows different movements for sixty-month periods. However, the basic features are essentially the same among the three series. Comparing two interest rate series, we can see that uncertainty aversion of the policymaker does not affect medium-run and long-run movements of the interest rates. In other words, when using low frequency data such as quarterly or annual data, a version of Taylor rule holds approximately in our model. However, when focusing on high frequency data, the optimal interest rates deviate from the Taylor rule significantly because the changes of the interest rates become very infrequent when  $\varepsilon = 0.2$ . The status quo of the interest rate is most conspicuous in Figure 3-A. For example, the interest rate remained the same for 9 periods from  $t = 206$  to  $213$  and for 13 periods from  $t = 242$  to  $253$ . Even in Figures 3-B and 3-C, we can see that the interest rate remained the same for multi periods many times. In contrast, when  $\varepsilon = 0$ , we can observe continuous changes of the interest rates in response to output and inflation changes in all of the three figures. However, the interest rate often reacts more aggressively when  $\varepsilon = 0.2$  than when  $\varepsilon = 0$ .

## 7. Robustness Checks

### (1) The degree of uncertainty

The purpose of the following experiments is to investigate robustness of our basic results. In this subsection, we check how the degree of uncertainty will affect frequency of the interest rate changes. In our model,  $k_t$  is the only random structural parameter that changes over time. For the random parameter, we set  $k_1 = 0.03$  and  $k_2 = 0.01$  in our benchmark case. We examine how the interest rates will change when we use alternative combinations of  $(k_1, k_2) = (0.0225, 0.0175)$ ,  $(0.025, 0.015)$ , and  $(0.035, 0.005)$ . In all of the combinations, the expected value of  $k_t$  is equal to  $0.02$  in the prior

distribution. The experiment thus explores how mean-preserving spread will affect the interest rates.

**Figure 4** depicts the interest rates for these alternative sets of  $k_1$  and  $k_2$  from  $t = 201$  to 248. It is easy to see that the interest rates change most frequently when  $(k_1, k_2) = (0.0225, 0.0175)$  and least frequently when  $(k_1, k_2) = (0.035, 0.005)$ . This implies that mean-preserving spread of  $k_t$  will make the interest rates change less frequent. However, even when  $(k_1, k_2) = (0.0225, 0.0175)$ , we can still observe that the interest rates remained unchanged for several periods.

In contrast with  $k_t$ , the degree of  $\varepsilon$ -contamination changes the policymaker's aversion to uncertainty. Therefore, given the distribution of  $k_t$ , changes of  $\varepsilon$  will capture another type of uncertainty changes. For the degree of  $\varepsilon$ -contamination, we set  $\varepsilon = 0.2$  in our benchmark case. We examine how the interest rates will change when we use alternative values of  $\varepsilon = 0.5, 0.3, \text{ and } 0.1$ . **Figure 5** depicts the interest rates for these alternative values of  $\varepsilon$  from  $t = 201$  to 248. It is easy to see that the interest rates change very infrequently when  $\varepsilon = 0.5$  but frequently when  $\varepsilon = 0.1$ . This implies that an increase of uncertainty aversion will make the interest rates change less frequent. However, even when  $\varepsilon = 0.1$ , we can still observe that the interest rates remained unchanged for a few months.

In our model,  $\alpha$  is assumed to be constant over time. However, as you see in (6), the value of  $\alpha$  affects the degree of uncertainty about the effects of the interest rate on the inflation rate. In the benchmark case, we used the interest rate elasticity of the aggregate demand of  $\alpha = 0.5$ . We examine how the interest rates will change when we use alternative values of  $\alpha = 0.025, 0.075, \text{ and } 0.3$ . **Figure 6** depicts the interest rates for these alternative values of  $\alpha$ . We can see that the choice of  $\alpha$  does not make a significant difference for frequency of policy changes. However, interest rates are most volatile when  $\alpha = 0.025$  and least volatile when  $\alpha = 0.3$ . This happens because  $\alpha$  appears in the denominator of our Taylor rules (14) and (15). It reflects the fact that the policy becomes more aggressive when its impact is certain but weak.

## (2) The role of persistence

In this subsection, we investigate how the degree of persistence in exogenous shocks will affect frequency of the interest rate changes. In our model, both the demand disturbance  $u_t$  and the supply shock  $e_t$  follow a first-order autoregressive process:  $u_t = \rho_u u_{t-1} + \eta_t$  and  $e_t = \rho_e e_{t-1} + \omega_t$ . In the benchmark case, we set  $\rho_u = \rho_e = 0.8$  and assumed that  $\eta_t \sim N(0, 0.01)$  and  $\omega_t \sim N(0, 0.05)$ . We first explore how alternative values of  $\rho_u$  and  $\rho_e$  will affect the interest rates. Specifically, we examine how the interest rates

will change when we use alternative values of  $\rho_u = \rho_e = 0.9, 0.5,$  and  $0.2$ . **Figure 7** depicts the interest rates for four alternative sets of  $\rho_u$  and  $\rho_e$  from  $t = 201$  to  $248$ . It is easy to see that the interest rates tend to change less frequently when the shocks are more persistent (that is, when  $\rho_u = \rho_e = 0.9$ ) but more frequently when they are less persistent, especially when  $\rho_u = \rho_e = 0.2$ . This implies that more persistent exogenous shocks will make the interest rates change less probable. However, even when  $\rho_u = \rho_e = 0.2$ , we can still observe infrequent changes of the interest rates.

We next explore how the interest rates will change when the variances of the innovations  $\eta_t$  and  $\omega_t$  differ. When the innovations are more volatile, the exogenous shocks become less persistent. In the benchmark case, we assumed that supply shocks were more dominant than demand shocks in the economy. We examine how the interest rates will change when we use alternative combinations of variances for  $\eta_t$  and  $\omega_t$ , that is,  $(0.01, 0.01)$ ,  $(0.05, 0.01)$ , and  $(0.005, 0.025)$ . The combination  $(0.01, 0.01)$  corresponds to the case where supply and demand shocks are equally important, while the combination  $(0.05, 0.01)$  corresponds to the case where demand shocks are more dominant than supply shocks. The combination  $(0.005, 0.025)$  implies that supply shocks are more dominant than demand shocks but that both shocks are less volatile than those in the benchmark.

**Figure 8** depicts the interest rates for four alternative combinations of the variances from  $t = 201$  to  $248$ . When we use the combination  $(0.005, 0.025)$ , the interest rates show similar infrequent changes to those in the benchmark. This suggests that less volatile innovations do not affect likelihood of no policy change so much. However, when we use  $(0.05, 0.01)$  or  $(0.01, 0.01)$ , the interest rates change much more frequently than those in the benchmark. Although we still observe some infrequent changes when we use  $(0.01, 0.01)$ , unchanged policy stance becomes extremely rare when we use  $(0.05, 0.01)$ . This indicates that supply shocks need to be more volatile than demand shocks in our model to observe infrequent policy changes. This is because the estimate of  $k_t$  becomes more precise as supply shocks are less volatile. It is the supply shock  $e_t$  that makes the policymaker's estimate of  $k_t$  uncertain. Since the degree of uncertainty on the parameter  $k_t$  is a source of infrequent policy changes, infrequent policy changes disappear when demand shocks are dominant in the economy. However, this result comes from our simplified assumption that the only parameter uncertainty is that of  $k_t$  in our model. If we relax the assumption and allows parameter uncertainty of  $\alpha$ , we may observe infrequent policy changes under some environments even if demand shocks are volatile.

Given the degree of exogenous shock persistence, the shocks may have more



persistent impacts on the interest rates when the degree of expectation adjustment is slower. In our experiments, we assumed that  $x_{t+1}^e = \theta_x x_{t-1} + (1-\theta_x) x_t^e$  and  $\pi_{t+1}^e = \theta_\pi \pi_{t-1} + (1-\theta_\pi) \pi_t^e$ . The expectation adjustment is thus slower when  $\theta_x$  and  $\theta_\pi$  are smaller. In the benchmark case, we set  $\theta_x = \theta_\pi = 0.5$ . We explore how alternative values of  $\theta_x$  and  $\theta_\pi$  will affect the interest rates. Specifically, we examine how the interest rates will change when we use alternative values of  $\theta_x = \theta_\pi = 0.9, 0.3, \text{ and } 0.1$ . **Figure 9** depicts the interest rates for four alternative sets of  $\theta_x$  and  $\theta_\pi$  from  $t = 201$  to  $230$ . It is easy to see that the interest rates tend to change less frequently when the expectation adjustment is slower, especially when  $\theta_x = \theta_\pi = 0.1$ . This implies that the degree of sluggish expectation adjustment will make the interest rates change less probable. However, even when  $\theta_x = \theta_\pi = 0.9$ , we can still observe infrequent changes of the interest rates.

### (3) The policy objectives

In this subsection, we investigate how the choice of the policy objectives will affect frequency of the interest rate changes. There is some arbitrariness on the choice of the policy objective. In our model, the policy objective depends on two target variables  $x^*$  and  $\pi^*$  and on a weight on output fluctuations  $\lambda$ . In our benchmark case, we set  $x^*$  to be zero but  $\pi^*$  to be  $-0.5$ . For the inflation target, we here consider the other three cases:  $\pi^* = -1.0, 0, \text{ and } 0.5$ . These values respond to the cases where the policymaker's attitude towards for inflation is very conservative, neutral, and benevolent respectively. For simplicity, we keep setting  $x^*$  to be zero. **Figure 10** depicts the interest rates for four alternative values of  $\pi^*$  from  $t = 201$  to  $248$ . The change of  $\pi^*$  does not make big difference for the interest rates. But if we compare the series more carefully, we can see that the interest rates tend to change less frequently when  $\pi^* = -0.1$  and more frequent when  $\pi^* = 0$ . This implies that the interest rates change becomes less probable when the policymaker's desirable inflation rate is different from the steady state. However, even when  $\pi^* = 0$ , we can still observe very infrequent changes of the interest rates.

The interest rates are more sensitive to the choice of  $\lambda$ . In our benchmark case, we set  $\lambda = 0.25$ . We here consider the other three cases:  $\lambda = 0.1, 0.5, \text{ and } 0.75$ . **Figure 11** depicts the interest rates for four alternative values of  $\lambda$  from  $t = 201$  to  $230$ . It is easy to see that the interest rates tend to change more frequently when  $\lambda = 0.5$  and  $0.75$ . In particular, the interest rates change very frequently when  $\lambda = 0.75$ . In contrast, when  $\lambda = 0.1$ , we can observe very infrequent changes of the interest rates. This implies that the changes of the interest rates become unlikely when the policymaker's objective puts

less weight on output stability and more weight on inflation stability. The reason is that the impacts of monetary policy are ambiguous on  $\pi_t$  but unambiguous on  $x_t$  when only the parameter  $k_t$  is uncertain. Larger weight on inflation thus leads to less frequent policy changes.

## 8. Concluding Remarks

In this paper, we explored why the central bank's policy instrument remains so unchanged under uncertainty. Although infrequent policy changes have been widely observed in many central banks, they have not been taken into account in previous macro models. This is true even in previous studies that investigated optimal monetary policy under model uncertainty or robust optimal policy rules. A large number of studies agreed that there is clearly much uncertainty over policy multipliers. However, most previous studies concluded that, under certain conditions, multiplier uncertainty may make optimal policy more conservative but does not lead to a policy of "doing nothing". A key departure of our paper from these studies is the introduction of a stop-go monetary policy in a Knightian uncertainty or a robust control framework. This increases an incentive for the central bank to keep the policy instrument unchanged even when exogenous shocks change output gap and inflation rate.

For analytical simplicity, our model relies on several restrictive assumptions. For example, our model allows multiplier uncertainty only for the parameter  $k_t$  that follows a binomial distribution. Introducing uncertainty to the other parameters such as  $\alpha$  and allowing more general stochastic processes are straightforward extensions to our analysis. The extensions would lead to more complicated but more rich stop-go monetary policy rules that may be more relevant for the policy analysis.

Needless to say, our stop-go monetary policy is not the only explanation for why the central banks' policy changes are so infrequent. Infrequent decision-making meetings would be one reason why the policy target changes so infrequently. Infrequent observations of macroeconomic data could be another reason. However, as we discussed briefly in the introduction, policy changes are less frequent than what these institutional constraints predict, so that these explanations are not satisfactory. This paper will probably fill some gaps that the institutional constraints cannot explain. What we have not discussed in the paper but what seems important is a constraint that a unit the central banks change the target rate is 0.25 % point. It could be another source for infrequent policy changes but probably calls for another paper.

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**Table 1. The Number of FOMC Release Dates and No Policy Change Announcement**

	1991	1992	1993	1994	1995	1996	1997	1998
Number of FOMC meetings	18	12	8	9	8	7	8	9
Number of meetings without policy change	9	9	8	3	4	6	7	6

	1999	2000	2001	2002	2003	2004	2005	2006
Number of FOMC meetings	8	8	11	8	8	8	8	8
Number of meetings without policy change	5	5	0	7	7	3	0	4

**Table 2. The Number of Meetings and No Policy Change Announcement**

## (1) Bank of Japan (Monetary Policy Meetings)

	1999	2000	2001	2002	2003	2004	2005	2006
Number of Monetary Policy Meetings	19	18	17	16	16	16	15	14
Number of meetings without policy change	18	17	12	13	12	15	15	12

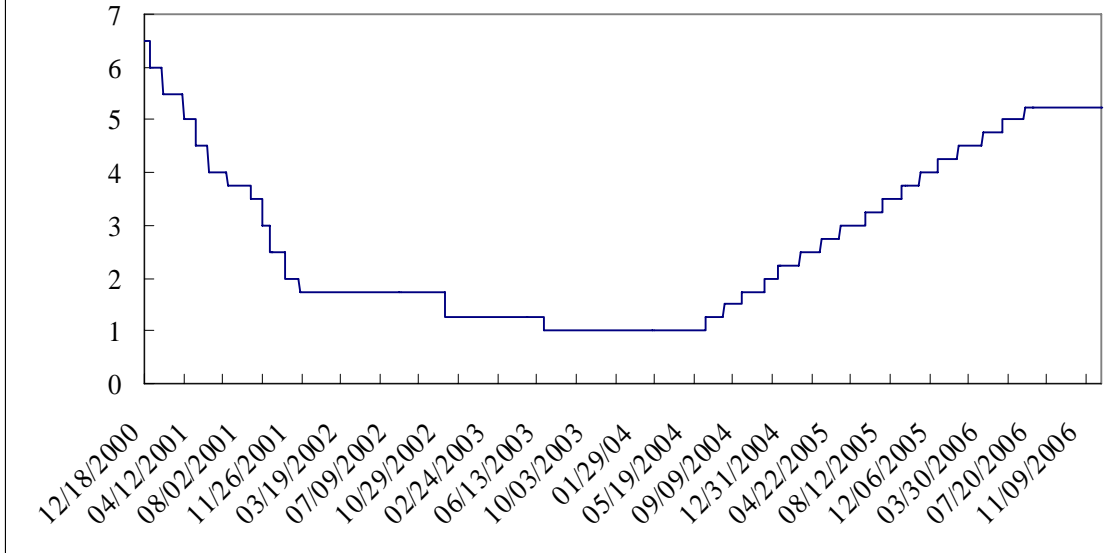
## (2) European Central Bank (The Governing Council)

	1999	2000	2001	2002	2003	2004	2005	2006
Number of Government Council meetings	24	24	24	12	12	12	12	12
Number of meetings without policy change	19	17	20	11	10	12	11	7

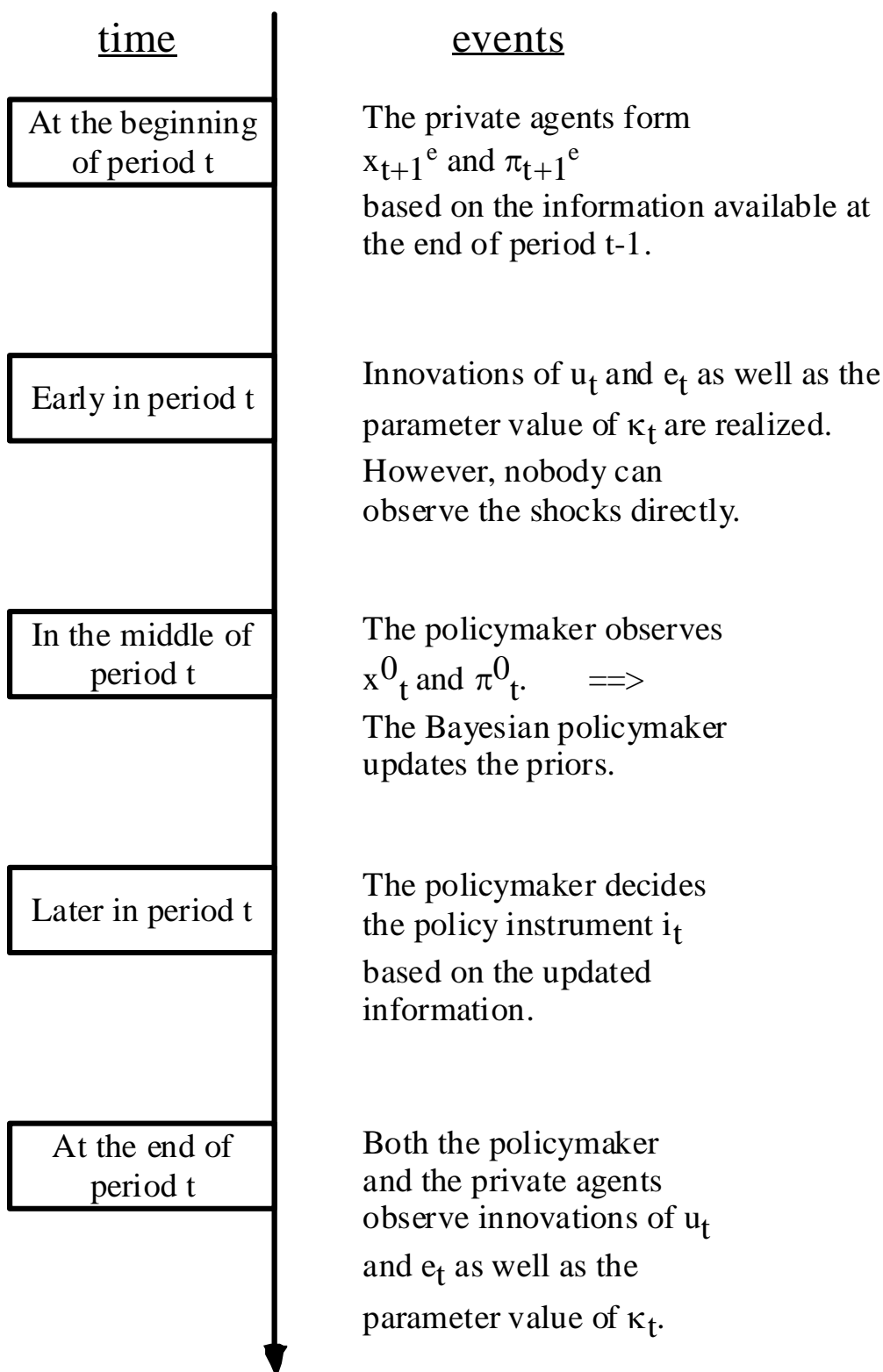
## (3) Bank of England (The Monetary Policy Committee)

	1999	2000	2001	2002	2003	2004	2005	2006
Number of MPC meetings	12	12	13	12	12	12	12	12
Number of meetings without policy change	6	10	6	12	9	8	11	10

**Figure 1. Federal Funds Target Rate:  
From Dec. 2000 to Dec. 2006**

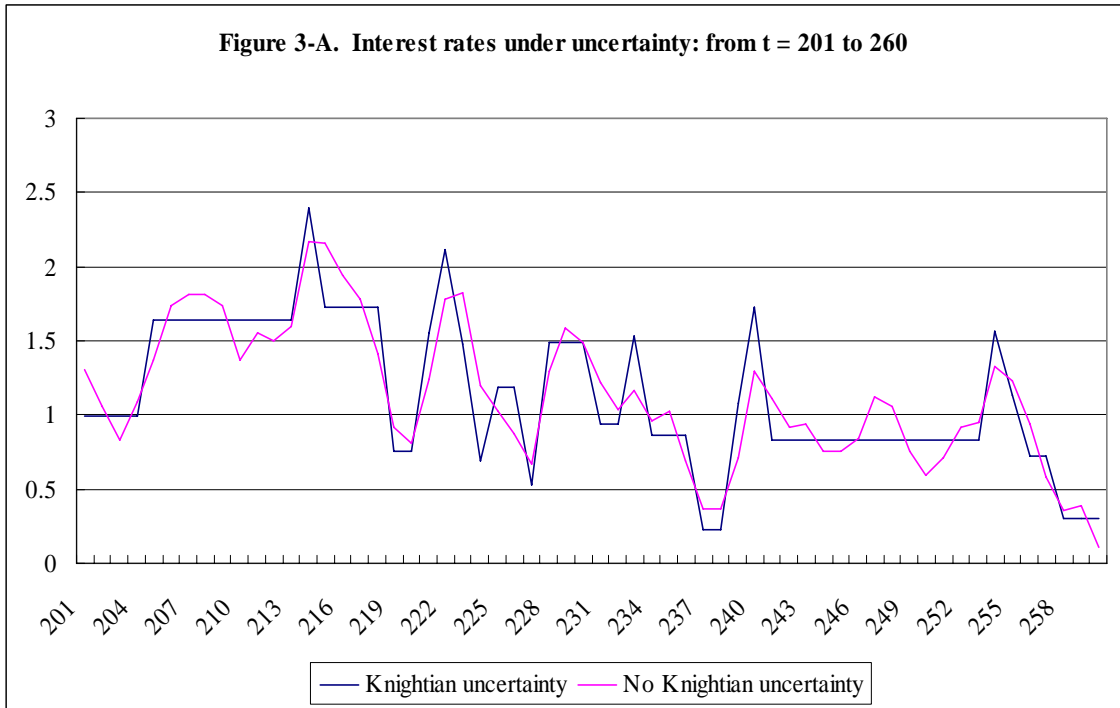


**Figure 2. The Timing of Events in Period t**





**Figure 3-A. Interest rates under uncertainty: from t = 201 to 260**



**Figure 3-B. Interest rates under uncertainty: from t = 301 to 360**

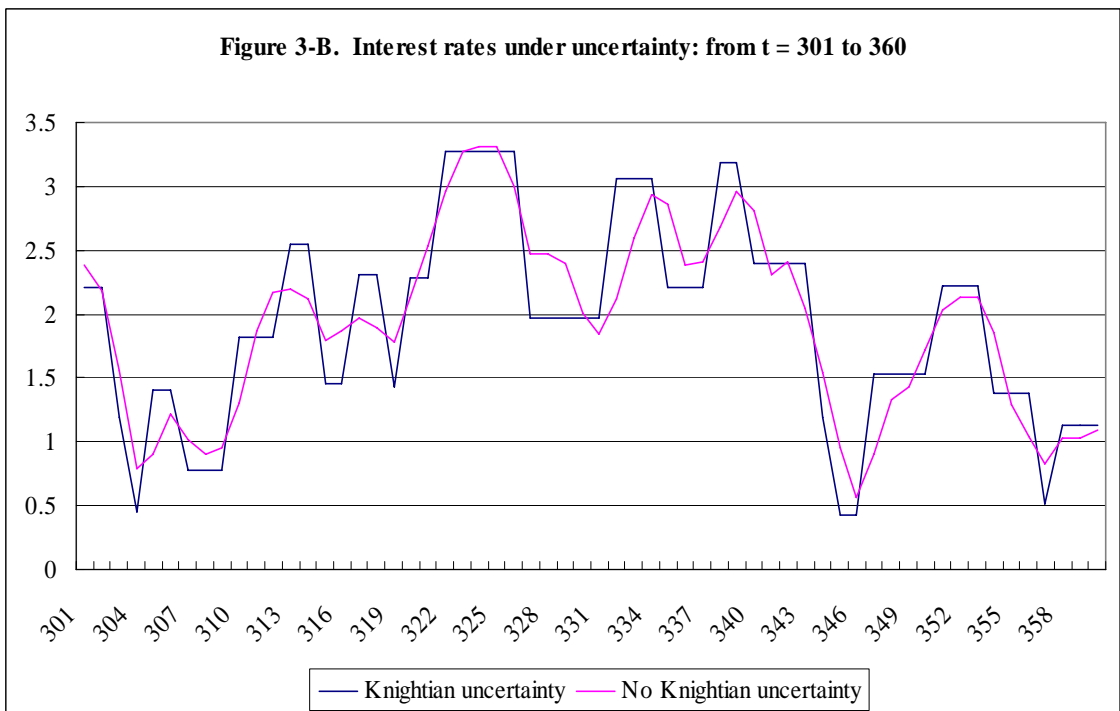
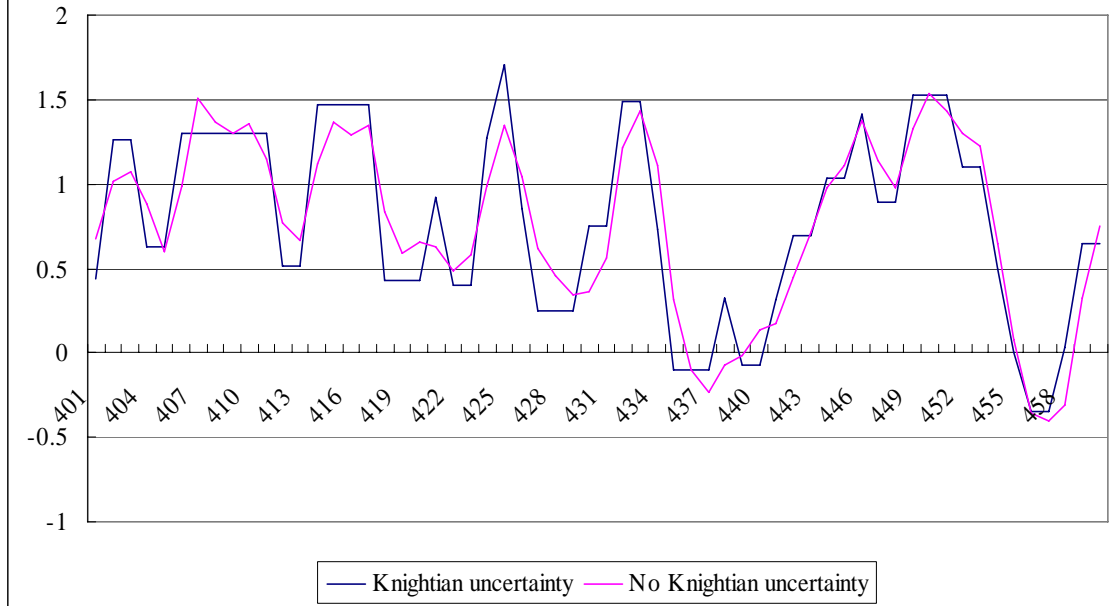
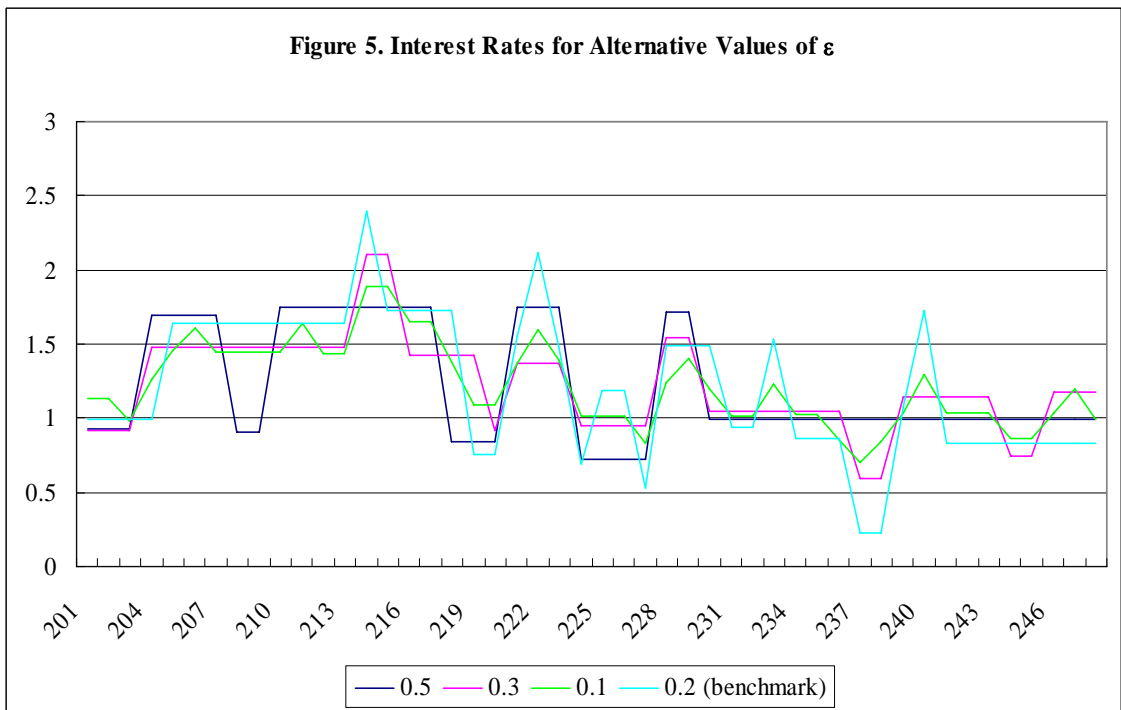
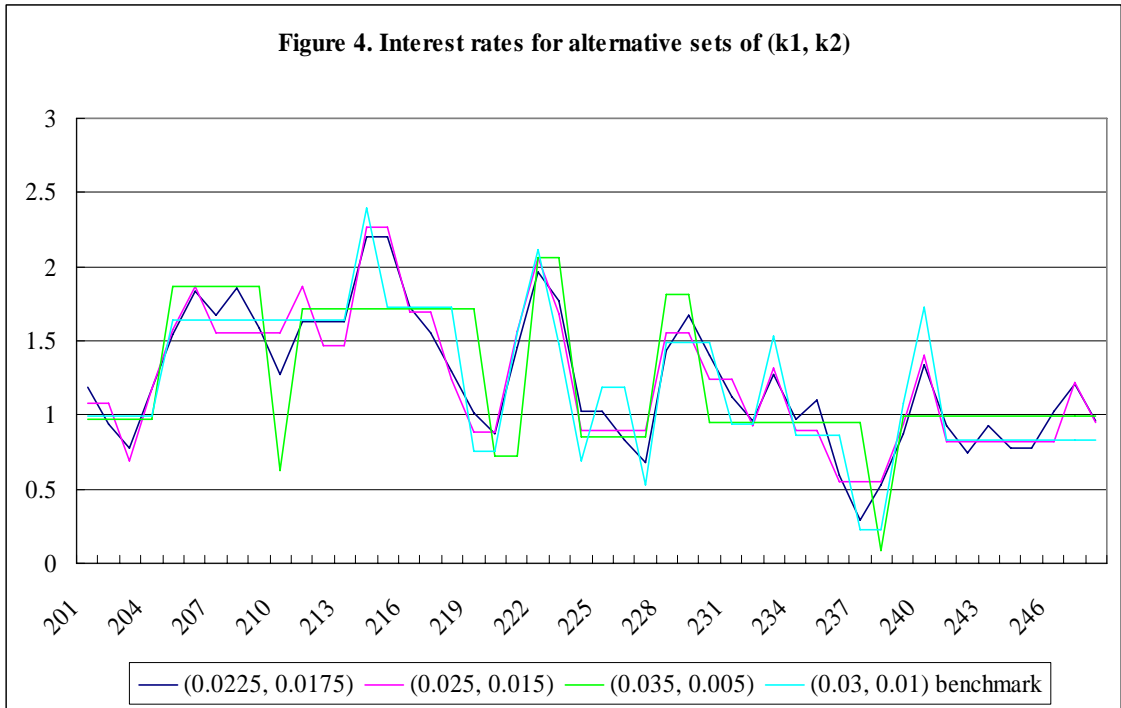
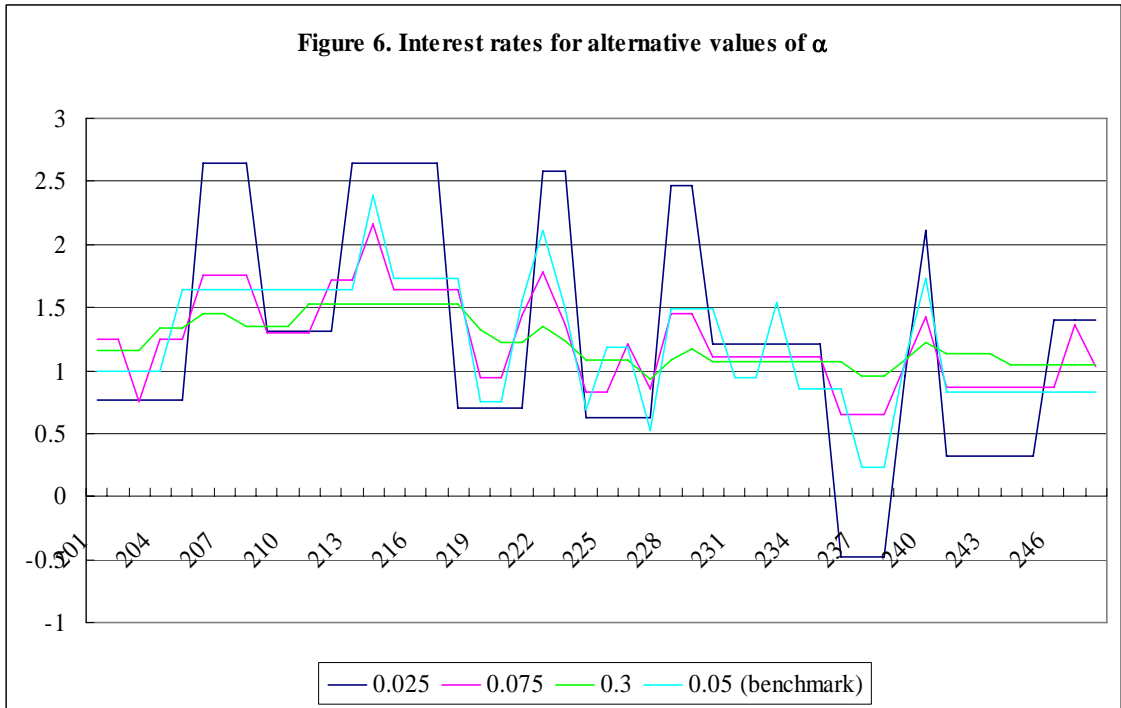


Figure 3-C. Interest rates under uncertainty: from t = 401 to 460





**Figure 6. Interest rates for alternative values of  $\alpha$**



**Figure 7. The effects of persistence of exogenous shocks:  $\rho$**

