# Choice, Opportunities, and Procedures: Collected Papers of Kotaro Suzumura

## Part IV Individual Rights and Social Welfare

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# Chapter 13
# On the Consistency of Libertarian Claims*

## 1 Introduction

If one social state is unanimously preferred to another, it is difficult to argue that the former state should not be socially chosen over the latter and, as a result, the claim that the collective choice rule we are interested in should be Pareto-inclusive has seldom been challenged. It may also be claimed that there are certain matters which are purely personal and our collective choice rule should be so designed that each person is empowered to decide what should be socially chosen, no matter what others may think, in choices over his personal matters. Sen [8] has shown that these two principles conflict, namely, there exists no Pareto-inclusive collective choice rule (with an unrestricted applicability) satisfying a mild libertarian claim.

Since the logical correctness of Sen's argument is beyond any doubt, we are forced to weaken either the Pareto rule or the libertarian claim in order to avoid this difficulty, unless we renounce the general applicability of our collective choice procedure. Although many subsequent contributions have modified libertarian claims in favour of the Pareto rule, one of the lessons Sen [8 and 9] has drawn from his *Paradox of a Paretian Liberal* is that a mechanical use of the Pareto rule (irrespective of the motivation behind people's preferences) seems unsound. In line with this observation, Sen [10, Section XI] has recently proposed a resolution of this paradox which restricts the use of the Pareto rule. We will succinctly reconstruct his resolution with some clarifications of the structure of his rights-assignment (in Section 2) and then show (in Section 3) that one of the Gibbard's paradoxes [4, Section 3] can be solved by essentially the same line of argument. This might be of some importance, because Sen's paradox and that of Gibbard are essentially different in nature. Suffice it to quote a passage from Gibbard [4, p.394]: "[Sen's libertarianism] guarantees each person a special voice on only one pair of alternatives, but the special voice is a strong one: the alternative he prefers is to be preferable, no matter

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what his other preferences. [Gibbard's libertarianism] guarantees each person a special voice on many pairs of alternatives ... but the voice is limited. The one he prefers is to be preferable if indeed he prefers its distinguishing feature unconditionally; otherwise his preference may be overridden."

In Section 4, we will examine the possibility of introducing information on interpersonal welfare comparisons into the conceptual framework. It will be shown that, if the rights-exercising is restricted by the Rawlsian maximin justice consideration (which is now available to us by the stronger informational basis we are working on), the modified libertarian claim is made compatible with the Pareto principle. This is in sharp contrast with Sen's [10, p.228] assertion that "for this class of impossibility results, introducing interpersonal comparisons is not much of a cure (in contrast with the impossibility results of the Arrow type)", which he has drawn from Kelly [6] *Impossibility of a Just Liberal*.

In Section 5, we will briefly summarize our conclusions. Some basic concepts and lemmas are put forward in the Appendix at the end of the chapter.

# 2 A Resolution of Sen's Paradox

**2.1.** Let $X$ be the set of all *conceivable* social states, and $N$ a set of $n$ individuals, each of whom has a preference ordering $R_i$ on $X$, together forming a *profile* of individual preference orderings. We say that $i \in N$ weakly prefers $x$ to $y$ iff $(x, y) \in R_i$. The strict preference relation corresponding to $R_i$ will be denoted by $P(R_i) : (x, y) \in P(R_i)$ iff

$$[(x, y) \in R_i \ \& \ (y, x) \notin R_i]$$

$K$ stands for the set of all non-empty *finite* subsets of $X$. (An intended interpretation is that each and every $S \in K$ is the set of available states.) A *collective choice rule* (CCR) is a method of choosing, for each profile, a *social choice function* (SCF) on $K$. Given a profile $(R_1, R_2, \dots, R_n)$, a CCR $F$ amalgamates this into an SCF:

$$C = F(R_1, R_2, \dots, R_n) \tag{1}$$

Given an $S \in K$, $C(S)$ represents the set of socially chosen states from $S$ when the profile $(R_1, R_2, \dots, R_n)$ prevails. We want our CCR to be generally applicable and Pareto-inclusive:

*Condition* **U** (*Unrestricted Domain*). The domain of CCR consists of all logically possible profiles of individual preference orderings.

*Condition* **P** (*Pareto Rule*). For all $x, y \in X, (x, y) \in \cap_{i \in N} P(R_i)$ implies

$$[x \in S \ \& \ y \in C(S)]$$

for no $S \in K$.

Our third requirement is that our CCR should respect some personal liberty. Letting $\Omega$ be the set of all non-empty subsets of $X \times X$ and denoting by $\Omega(n)$ the $n$-fold product of $\Omega$, the requirement in question is put as follows:

*Condition* **SL** (*Sen's Libertarian Claim*).[1] There exists a symmetric

$$D = (D_1, D_2, \ldots, D_n) \in \Omega(n)$$

such that, for each $i \in N$, $D_i$ contains at least one non-diagonal member and that:

$$(x, y) \in D_i \cap P(R_i) \Rightarrow [x \in S \ \&\ y \in C(S)] \text{ for no } S \in K. \tag{2}$$

$D_i$ is meant to be the set of all protected personal pairs of the individual $i$ and it will be referred to as $i$'s *protected sphere*. $D = (D_1, D_2, \ldots, D_n)$ will be called a *rights-assignment*. Using these terms the condition (2) may be interpreted that the individual $i$ can get his way in choices over his protected sphere irrespective of what others may think. We say that the rights-assignment $D = (D_1, D_2, \ldots, D_n)$ is symmetric if $(x, y) \in D_i \Leftrightarrow (y, x) \in D_i$ for all $i = 1, 2, \ldots, n$. In this case, if $i$ is allowed to impose his preference for $x$ against $y$, he can also impose his will for $y$ against $x$.

Taken in isolation, these requirements seem to be rather reasonable. A disturbing fact is that, given Condition U, Condition P and Condition SL cannot *simultaneously* be satisfied by *any* CCR. It is this *Paradox of a Paretian Liberal* established by Sen [9, pp.81-82 and 10, Theorem 7] which necessitates a closer examination of Condition P and Condition SL.

**2.2.** Let us begin with the Condition SL. Our first task is to introduce the concept of coherent rights-assignment, which goes as follows.

Let $D = (D_1, D_2, \ldots, D_n)$ be an $n$-tuple of subsets of $X \times X$. A *critical loop* in $D$ is a sequence of ordered pairs $\{(x^\mu, y^\mu)\}_{\mu=1}^{t}$ $(t \geq 2)$ such that (i) $(x^\mu, y^\mu) \in \cup_{i=1}^{n} D_i$ for all $\mu = 1, 2, \ldots, t$, and (ii) there exists no $i^* \in \{1, 2, \ldots, n\}$ such that $(x^\mu, y^\mu) \in D_{i^*}$ for all $\mu = 1, 2, \ldots, t$, and (iii) $x^1 = y^t$ and $x^\mu = y^{\mu-1}$ for all $\mu = 2, \ldots, t$.[2] We say that $D = (D_1, D_2, \ldots, D_n)$ is *coherent* iff there exists no critical loop in $D$.

From the analytical viewpoint, the importance of the concept of coherence in our present context stems from the following basic lemma, the proof of which will be given in the Appendix.

**Lemma 1.** *$D = (D_1, D_2, \ldots, D_n)$ is coherent iff, for every $n$-tuple of orderings,*

$$(R_1, R_2, \ldots, R_n),$$

*there exists an order-extension $R$ of each and every $D_i \cap R_i$ $(i = 1, 2, \ldots, n)$.*

It was Farrell [3] and Gibbard [4] who have shown that (i) if the rights-assignment $D$ is not coherent, Condition U and Condition SL conflict by themselves (without invoking Condition P), and that (ii) if $D$ is coherent, Condition U and Condition SL are compatible. It follows that we should restrict Condition SL by requiring $D$ to be coherent if

[1] Sen's condition of *minimal liberalism* is still weaker than this. In his formulation, $D_i$ may be empty for at most $n - 2$ individuals. See Sen [9, p.87].

[2] The concept of the critical loop is due to Farrell [3].

we wish to make it compatible with Condition U *and* some version of the Pareto rule. Furthermore an existential statement of Condition SL, though ideal for an impossibility exercise, is ill-suited for our present purpose. The doctored version of Condition SL we are going to work with is as follows.

Condition **CL** (*Coherent Libertarian Claim*)
For any coherent rights-assignment $D = (D_1, D_2 \ldots, D_n) \in \Omega(n)$, (2) holds for each $i \in N$.

**2.3.** Turn to Condition P. Let $(R_1, R_2, \ldots, R_n)$ be any given profile and let $R_i^*$ be a transitive subrelation of $R_i$ which individual $i$ wants to count in collective decision. Thus $(x, y) \in P(R_i)$ means that $i$ prefers $x$ to $y$ personally, while $(x, y) \in P(R_i^*)$ means that he wants his preference for $x$ over $y$ to count in social choice. The basic idea here is that "the guarantee of a minimal amount of personal liberty may require that certain parts of individual rankings should not count in some specific social choices, and in some cases even the persons in question may agree with this" (Sen [10, pp.237-238]). Armed with this important distinction, we now introduce a version of the *conditional* (strong) Pareto rule.

*Condition* **CP** (*Conditional Pareto Rule*). Let $R^* = \cap_{i \in N} R_i^*$. For all $x, y \in X$,

(a) $(x, y) \in R^* \Rightarrow [x \in S \setminus C(S)$ & $y \in C(S)]$ for no $S \in K$, and

(b) $(x, y) \in P(R^*) \Rightarrow [x \in S$ & $y \in C(S)]$ for no $S \in K$.

The efficacy of Condition CP as a resolvent of Sen's paradox depends squarely on the extent that $(R_1^*, R_2^*, \ldots, R_n^*)$ is restrictive vis-à-vis $(R_1, R_2, \ldots, R_n)$. For example, if $R_i^* = R_i$ for all $i \in N$, then the paradox clearly remains intact. If $R_i^* = \emptyset$ for all $i \in N$, however, Condition CP becomes vacuous and the paradox is "resolved". The problem really is to formalize a "reasonable" way of restricting $R_i$ into $R_i^*$ so as to avoid Sen's difficulty. Sen's [10, Section XI] proposal to this effect is now to be recapitulated. Let

$$D = (D_1, D_2, \ldots, D_n) \in \Omega(n)$$

be any coherent rights-assignment and let $(R_1, R_2, \ldots, R_n)$ be any given profile. Thanks to our Lemma 1 given above, we then have an ordering $R$ which subsumes each and every individual preference over respective protected spheres. There may well be multiple order-extensions, so that let $\mathscr{R}$ stand for the set of all such orderings. Let an individual $j \in N$ be called a *liberal* iff

$$R_j^* = R_j \cap R \text{ for some } R \in \mathscr{R}. \tag{3}$$

Namely an individual $j$ is liberal iff he claims only those parts of his preferences to count which are compatible with others' preferences over their respective protected spheres.

Some remarks on this basic concept might be in order. Firstly it should be emphasized that a liberal never drops his preferences over his own protected sphere, so that a liberal need not die a martyr for his faith in liberalism. Secondly a liberal need not really care very much how the order extension $R$ is constructed from $Q = \cup_{i \in N}(R_i \cap D_i)$. An "active" liberal would hold a clear idea of that part of his preference ordering which he wants to count in the collective decision. (Obviously he needs lots of information as to the structure of rights-assignment and the wishes of individuals.) A "passive" liberal, on the other hand, does not know his $R_i^*$; instead, he knows only his $R_i$ and that he knows he wants to be liberal. A well-informed umpire then comes in, who constructs an order-extension $R$ of $Q$ and thereby constrains the preference ordering of an individual who is wishing to be liberal. Our concept of a liberal admits both species.[3]

**2.4.** Now the theorem.

**Theorem 1** (Sen [10, Theorem 9]). *If there exists at least one liberal individual, a rational* CCR *which satisfies* **U, CL** *and* **CP** *exists.*

*Proof.* Let $N_1$ stand for the set of all liberal individuals. By assumption, $N_1$ is a non-empty subset of $N$. Let $D = (D_1, D_2, \ldots, D_n)$ be a given coherent rights-assignment and let $(R_1, R_2, \ldots, R_n)$ be a given profile. Letting $\mathscr{R}$ be the set of all order-extensions of $Q = \cup_{i \in N}(R_i \cap D_i)$, we define:

$$R_i^* = \begin{cases} R_i \cap R^i \text{ for some } R^i \in \mathscr{R} \text{ if } i \in N_1 \\ R_i \qquad\qquad\qquad\qquad\quad \text{otherwise.} \end{cases}$$

Denoting $R^* = \cap_{i \in N} R_i^*$ and $P = \cap_{i \in N_1} P(R^i)$ we define:

$$R_0 = \{(x, y) \in X \times X : (y, x) \notin P \cup P(R^*)\}. \tag{4}$$

Let us establish that $R_0$ is complete. Suppose that there are $x$ and $y$ in $X$ such that $(x, y) \notin R_0$ and $(y, x) \notin R_0$, so that we have $(x, y) \in P \cup P(R^*)$ and $(y, x) \in P \cup P(R^*)$. There are four possible cases to consider:

(i) $(x, y) \in P$ & $(y, x) \in P$,
(ii) $(x, y) \in P(R^*)$ & $(y, x) \in P(R^*)$,
(iii) $(x, y) \in P$ & $(y, x) \in P(R^*)$,

and

(iv) $(x, y) \in P(R^*)$ & $(y, x) \in P$.

The case (i) and the case (ii) contradict, respectively, the asymmetry of $P$ and that of $P(R^*)$. Take any $i_0 \in N_1$. Then we have $P \subset P(R^{i_0})$ and $P(R^*) \subset R_{i_0}^* \subset R^{i_0}$, so that the case (iii) and the case (iv) contradict the fact that $R^{i_0}$ is an ordering. Therefore $R_0$ must be complete.

Next we establish that:

$$P(R_0) = P \cup P(R^*). \tag{5}$$

If $(x, y) \in P(R_0)$, then $(y, x) \notin R_0$, which implies $(x, y) \in P \cup P(R^*)$ by definition. Therefore $P(R_0) \subset P \cup P(R^*)$. To show the converse, suppose there exists an ordered

[3] Thanks are due to Peter Hammond for his comment on this point.

pair $(x, y)$ such that $(x, y) \in P \cup P(R^*)$ but $(x, y) \notin P(R_0)$. But this contradicts the completeness of $R_0$ established above. Therefore (5) must be true.

Our next task is to establish the acyclicity of $R_0$. If there exists a $\{x^1, x^2, \ldots, x^t\} \in K$ such that $(x^\mu, x^{\mu+1}) \in P(R_0)$ $(\mu = 1, 2, \ldots, t-1)$ and $(x^t, x^1) \in P(R_0)$, we have a contradiction with the transitivity of $R_i$ $(i \in N_1)$ or that of $R^*$ thanks to the fact that $P(R^*) \subset R^i$ $(i \in N_1)$. This contradiction establishes the acyclicity of $R_0$. Now that $R_0$ is complete and acyclic,

$$C(S) = G(S, R_0)$$

for all $S \in K$ is a well-defined rational choice function by virtue of Lemma $2^*$ in the Appendix. Associating this $C$ with the given profile $(R_1, R_2, \ldots, R_n)$ we obtain a rational CCR.

What remains to be shown is that this CCR satisfies CP and CL.

In order to show that it satisfies CP(a), suppose that there exist $x$ and $y$ such that $(x, y) \in R^*$, $x \in S \backslash C(S)$ and $y \in C(S)$ for some $S \in K$. We will bring out a contradiction by showing that $(x, z) \in R_0$ for all $z \in S$. Take therefore any $z \in S$. Since $y \in C(S)$, we have $(z, y) \notin P \cup P(R^*)$, so that $(z, y) \notin P$ and $[(z, y) \notin R^*$ or $(y, z) \in R^*]$. Thanks to the definition of $P$, we have $(z, y) \notin P$ iff $(y, z) \in R^i$ for some $i \in N_1$. By assumption we have $(x, y) \in R^*$ so that we obtain $(x, y) \in R_i^* \subset R^i$ for this $i \in N_1$. $R^i$ being transitive, $(x, y) \in R^i$ and $(y, z) \in R^i$ yield $(x, z) \in R^i$. It then follows that $(z, x) \notin P(R^i)$, which implies that:

$$(z, x) \notin P. \tag{6}$$

Suppose now that $(z, y) \notin R^*$, which implies $(z, y) \notin R_i^*$ for some $i \in N$. If $i \in N \setminus N_1$, then we have $(y, z) \in P(R_i)$. As $(x, y) \in R_i$ follows from $(x, y) \in R^*$, we obtain $(x, z) \in P(R_i)$, namely, $(z, x) \notin R_i$. Therefore $(z, x) \notin R^*$. If $i \in N_1$, then $[(z, y) \notin R_i$ or $(z, y) \notin R^i]$, namely, $[(y, z) \in P(R_i)$ or $(y, z) \in P(R^i)]$ holds true, which implies $(x, z) \in P(R_i) \cup P(R^i)$ in view of $(x, y) \in R^*$. Therefore we again obtain $(z, x) \notin R^*$. Consider the case where $(y, z) \in R^*$. Coupled with $(x, y) \in R^*$ this implies that $(x, z) \in R^*$, hence $(z, x) \notin P(R^*)$. Therefore in every conceivable case we obtain that:

$$(z, x) \notin P(R^*). \tag{7}$$

It follows from (6) and (7) that $(z, x) \notin P \cup P(R^*)$, so that we have arrived at $(x, z) \in R_0$ as desired. The proof of CP(a) is thereby complete. Next CP(b). If there are $x$ and $y$ satisfying $(x, y) \in P(R^*)$, $x \in S$ and $y \in C(S)$ for some $S \in K$, we have $(y, x) \in R_0$ entailing $(x, y) \notin P \cup P(R^*)$. But this contradicts $(x, y) \in P(R^*)$.

Finally we show that our CCR satisfies the Condition CL. Suppose to the contrary that there are an $i \in N$ and $S \in K$ satisfying $(x, y) \in D_i \cap P(R_i)$, $x \in S$ and $y \in C(S)$. Then we obtain $(y, x) \in R_0$ entailing $(x, y) \notin P \cup P(R^*)$. By definition of $\mathscr{R}$, we have $P(Q) \subset \cap_{i \in N_1} P(R^i) = P$, so that if we can show

$$D_i \cap P(R_i) \subset P(Q), \tag{8}$$

we are home. (Because, then we have $(x, y) \in D_i \cap P(R_i) \subset P$, in contradiction with $(x, y) \notin P \cup P(R^*)$.) To show (8), let $(w, z) \in D_i \cap P(R_i)$. Clearly $(w, z) \in Q$, so that if $(w, z) \notin P(Q)$ then $(z, w) \in Q = \cup_{i \in N} Q_i$. Then there exists a $j \in N$ $(j \neq i)$ such that $(z, w) \in D_j \cap R_j$. If follows that $D$ contains a critical loop, a contradiction. $\|$

The gist of this resolution is very simple and intuitive. The Pareto principle is enforced only by unanimous agreement. Its use can therefore be vetoed by any one person and a liberal may well serve as a vetoer. Notice that a liberal is, by definition, one who always (for every profile) exercises the veto in favour of every expressed protected right, and of every consequence of all of these, and of further arbitrary additions.

**2.5.** It might help if we exemplify how this resolution works.

*Example* 1 (*Lady Chatterley's Case*, Sen [9, pp.80-81]). There is a single copy of *Lady Chatterley's Lover*. The set of social states consists of three alternatives: Mr $A$ (the prude) reading it, $(x)$, Mr $B$ (the lascivious) reading it, $(y)$, and no one reading it $(z)$. Mr $A$ prefers $z$ most, next $x$ (wishing thereby to take the hurt on himself) and lastly $y$ (for fear of the possible misbehaviour of Mr $B$), while Mr $B$ prefers $x$ most (in order to educate the reactionary Mr $A$), $y$ next and lastly $z$. Therefore $R_A = \Delta \cup \{(z, x), (x, y), (z, y)\}$ and $R_B = \Delta \cup \{(x, y), (y, z), (x, z)\}$, where $\Delta$ denotes the diagonal binary relation on the space in question. (In our present context, $\Delta = \{(x, x), (y, y), (z, z)\}$.) The protected sphere of Mr $A$ is $D_A = \{(x, z), (z, x)\}$ and that of Mr $B$ is $D_B = \{(y, z), (z, y)\}$. (Notice that this rights-assignment $D = (D_A, D_B)$ is coherent.) No Pareto-inclusive CCR can realize this rights-assignment, however.

In this case $Q_A = R_A \cap D_A = \{(z, x)\}$ and $Q_B = R_B \cap D_B = \{(y, z)\}$, so that $Q = \{(z, x), (y, z)\}$. The order-extension of this $Q$ is unique:

$$R = \Delta \cup \{(y, z), (z, x), (y, x)\}.$$

Suppose that Mr $A$ is liberal while Mr $B$ is not, so that $R_A^* = R \cap R_A = \Delta \cup \{(z, x)\}$ and $R_B^* = R_B$, entailing $R^* = \Delta$. By definition we then have

$$R_0 = \Delta \cup \{(y, z), (z, x), (y, x)\},$$

so that $G(\{x, y, z\}, R_0) = \{y\}$. Therefore our suggested solution for the Lady Chatterley's Case is: Give that copy to the lascivious. $\|$

A few remarks might be in order here. Firstly, in line with the statement of Sen's libertarian claim (SL), we supposed that $D_A$ and $D_B$ were symmetric in the Lady Chatterley's Case. Sen's paradox still works, however, even if $D_A = \{(z, x)\}$ and $D_B = \{(y, z)\}$. It is easy to verify that our resolution given above still applies without any change. Secondly, our solution to the Lady Chatterley's Case does not hinge on our supposing that it is Mr $A$ who is liberal. Mr $B$ being liberal leads us to the same solution. Is this a general feature of our solution procedure? To show that it is not, we put forward the following:

*Example* 2 (*Two Meddlers Case*, Blau [2]). There are two individuals, Mr $A$ and Mr $B$, and four distinct alternatives $x, y, z$ and $w$. The rights-assignment is $D_A = \{(x, y), (y, x)\}$

and $D_B = \{(z, w), (w, z)\}$. Mr $A$ prefers $w$ to $x$ to $y$ to $z$, while Mr $B$ prefers $y$ to $z$ to $w$ to $x$. Mr $A$ is meddlesome in that his preference over his protected pair is weaker (in the ordinal intensity sense) than his opposition to the other's preference over that individual's protected pair. Mr $B$ is also a meddler in this sense. In this case the unadulterated exercise of rights, coupled with the mechanical use of the Pareto rule, brings us into the impasse of social indecision.

Let us see how our solution procedure will fare in coping with this situation. It is easy to see that, in this Two Meddlers Case, $Q = \{(x, y), (z, w)\}$. There are multiple order-extensions of this $Q$, thirteen altogether, from which we pick out

$$R^\alpha = \Delta \cup \{(x, y), (x, z), (x, w), (y, z), (y, w), (z, w)\},$$

$$R^\beta = \Delta \cup \{(z, w), (z, x), (z, y), (w, x), (w, y), (x, y)\}, \text{ and}$$

$$R^\gamma = \Delta \cup \{(x, z), (z, x), (x, y), (x, w), (z, y), (z, w), (y, w)\}.$$

Depending on who is liberal and which order-extension is to be used, there are different solution schemes. Let the scheme where Mr $A$ is liberal with the order-extension $R^\alpha$ be denoted by $(A, \alpha)$. It is easy, if tedious, to verify that the solution in the scheme $(A, \alpha)$ is $\{x\}$, that in the scheme $(B, \beta)$ is $\{z\}$, and that in the scheme $(\{A, B\}, \gamma)$ is $\{x, z\}$. ‖

# 3 A Resolution of Gibbard's Paradox

**3.1.** We start with an observation that the Condition SL and the Condition CL share two important peculiarities. Firstly the rights-assignment in SL as well as that in CL is *independent* in the sense that, whenever $(x, y) \in D_i$ and $i$ strictly prefers $x$ to $y$, he can get his way *whatever his preference over* $X \setminus \{x, y\}$ *happens to be*. Secondly, apart from our *interpretation*, there is nothing in the *formal statement* of SL and CL which assures us that, whenever $(x, y) \in D_i$, the difference between $x$ and $y$ is $i$'s purely personal concern. Gibbard's [4] libertarian claim differs from SL and CL in these respects and, as a result, a paradox he arrived at is essentially different from that of Sen. We will show in this section that this different paradox can nevertheless be resolved along the similar line of reasoning as we used above.

**3.2.** The social state is now construed as a list of impersonal and personal features of the world. Let $X_0$ be the set of all impersonal features and $X_i$ the set of all personal features of individual $i \in N$. $X$, the set of all social states, is now represented as

$$X = X_0 \times X_1 \times \ldots \times X_n.$$

We assume that $X_0$ and $X_i$ $(i \in N)$ are finite with at least two elements each. Our notational convention is that, for each $i \in N$ and each $x = (x_0, x_1, \ldots, x_n) \in X$,

$$X_{)i(} = X_0 \times X_1 \times \ldots \times X_{i-1} \times X_{i+1} \times \ldots \times X_n$$

and $x_{)i(} = (x_0, x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$. Furthermore, if $x_i \in X_i$ and

$$z = (z_0, z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n) \in X_{)i(}, (x_i; z) = (z_0, z_1, \ldots, z_{i-1}, x_i, z_{i+1}, \ldots, z_n).$$

Finally we define $D'_i$ by:

$$D'_i = \{(x, y) \in X \times X : x_{)i(} = y_{)i(}\} \quad (i \in N).$$

Therefore if $(x, y) \in D'_i$, then $x$ and $y$ can possibly differ only in the specification of $i$'s personal feature.

**3.3.** Now Gibbard's [4, p.393] version of a libertarian claim.

*Condition* **GL** (*Gibbard's Libertarian Claim*). For each $i \in N$, if $(x, y) \in D'_i$ and $((x_i; z), (y_i; z)) \in P(R_i)$ for all $z \in X_{)i(}$, then $[x \in S \;\&\; y \in C(S)]$ for no $S \in K$.

In words, it is required that, if $x$ and $y$ differ only in $i$'s personal feature and if $i$ prefers $x_i$ *unconditionally* to $y_i$, then his personal choice should be socially respected. Gibbard [4, Theorem 2] has shown that there exists no CCR satisfying U, GL and P. Notice that $D' = (D'_1, D'_2, \ldots, D'_n) \in \Omega(n)$ is *not* independent and it is *not* coherent by construction.

**3.4.** As a first step in resolving Gibbard's dilemma, we show that a binary relation $Q'$ defined by

$$Q'_i = \{(x, y) \in D'_i : ((x_i; z), (y_i; z)) \in P(R_i) \text{ for all } z \in X_{)i(}\} \ (i \in N)$$

and

$$Q' = \cup_{i \in N} Q'_i$$

is consistent for any profile $(R_1, R_2, \ldots, R_n)$. Suppose to the contrary that there exists a $\{x^1, x^2, \ldots, x^t\} \in K$ such that $(x^1, x^2) \in P(Q'), (x^\mu, x^{\mu+1}) \in Q'$ for all $\mu = 2, \ldots, t-1$ and $(x^t, x^1) \in Q'$. Then there exists an $i \in N$ such that

$$(x^1, x^2) \in D'_i \tag{9}$$

and

$$((x^1_i; z), (x^2_i; z)) \in P(R_i) \text{ for all } z \in X_{)i(}. \tag{10}$$

Corresponding to the sequence $\{x^\mu\}^t_{\mu=1}$ define a sequence $\{x^\mu_*\}^t_{\mu=1}$ by

$$x^\mu_* = (x^\mu_i; x^1_{)i(}), \ (\mu = 1, 2, \ldots, t). \tag{11}$$

By virtue of Gibbard's lemma [4, p.396] we then have $(x^\mu_*, x^{\mu+1}_*) \in R_i$ $(\mu = 2, \ldots, t-1)$ and $(x^t_*, x^1_*) \in R_i$, while (10) entails that $(x^1_*, x^2_*) \in P(R_i)$. But this contradicts the transitivity of $R_i$. Now that $Q'$ is consistent, there exists an ordering $R'$ subsuming $Q'$

by virtue of Lemma 1* in the Appendix. Let $\mathscr{R}'$ be the set of all order-extensions of $Q'$. Call an individual $j \in N$ a G-*liberal* iff

$$R_j^* = R_j \cap R' \text{ for some } R' \in \mathscr{R}'. \tag{12}$$

In words $j$ is $G$-liberal iff he claims only those parts of his preferences to count which are compatible with others' unconditional preferences over their personal variations. Our remarks on the nature of a liberal individual presented in 2.3. also apply to a G-liberal individual as well.

Now the following theorem is true.

**Theorem 2.** *If there exists at least one* G-*liberal individual, a rational* CCR *satisfying* **U, GL** and **CP** exists.

A slight modification of the proof of Theorem 1 (replying $\mathscr{R}$ by $\mathscr{R}'$) establishes Theorem 2, the detail of which may safely be skipped.

**3.5.** Let us analyse an example and contrast our solution with that of Gibbard [4].

*Example* 3 (*Wall Colour Case*, Gibbard [4, pp.394-395]). There are two individuals, Mr $A$ and Mr $B$, and four alternative states, all of which are identical with respect to the impersonal features of the world. They differ only in the colour of their respective bedroom walls. Let these alternative states be $(w, w), (y, w), (w, y)$ and $(y, y)$, dropping for the sake of simplicity the coordinate of impersonal features. (The first coordinate designates the colour of Mr $A$'s walls and the second that of Mr $B$'s, $w$ and $y$ standing respectively for white and yellow.) In this case,

$$D'_A = \{((w, w), (y, w)), ((y, w), (w, w)), ((w, y), (y, y)), ((y, y), (w, y))\}$$

and

$$D'_B = \{((y, y), (y, w)), ((y, w), (y, y)), ((w, y), (w, w)), ((w, w), (w, y))\}.$$

It is cleat that $D' = (D'_A, D'_B)$ is not coherent. Suppose that their preferences are such that:

$$R_A : (w, w), (y, w), (w, y), (y, y)$$

and

$$R_B : (y, y), (y, w), (w, y), (w, w).$$

Namely Mr $A$ prefers $w$ to $y$ unconditionally and that he wants Mr $B$ to choose as he does. Mr $B$ in turn prefers $y$ to $w$ unconditionally and he wants Mr $A$ to choose as he does. In this case, the rights-exercising of Mr $A$ and Mr $B$, coupled with the naïve use of the Pareto rule, kicks out all alternatives from social choice and no CCR can be satisfactory if we stick to U, GL and P.

Now our solution procedure. Corresponding to the given profile $(R_A, R_B)$, we have $Q'_A = \{((w,w),(y,w)),((w,y),(y,y))\}$ and $Q'_B = \{((y,y),(y,w)),((w,y),(w,w))\}$. An order-extension $R'$ of $Q'_A \cup Q'_B$ is then given by:[4]

$$R' : (w,y),(y,y),(w,w),(y,w).$$

Suppose that Mr $A$ is G-liberal while Mr $B$ is not, so that

$$R^*_A = R' \cap R_A = \Delta \cup \{((w,y),(y,y)),((w,w),(y,w))\}$$

and $R^*_B = R_B$, yielding $R^* = \Delta$. We then have

$$R_0 = \Delta \cup \{((w,y),(y,y)),((w,y),(w,w)),((w,y),(y,w)), \\ ((y,y),(w,w)),((y,y),(y,w)),(w,w),(y,w))\},$$

so that $G(\{(w,w),(y,w),(w,y),(y,y)\},R_0) = \{(w,y)\}$. Therefore our solution is: Let people choose whatever colour they unconditionally prefer. (Our conclusion remains intact if Mr $B$ is G-liberal and Mr $A$ is not.) ||

Gibbard's [4, Section 4] way-out of his paradox is to make his libertarian claim *alienable* and goes typically as follows. Although Mr $A$ prefers $(w,w)$ to $(y,w)$, and could avoid $(y,w)$ by exercising his right to $(w,w)$ over $(y,w)$, Mr $B$ claims his right to $(w,y)$ over $(w,w)$, and Mr $A$ prefers $(y,w)$ to $(w,y)$. By exercising his right to avoid $(y,w)$, Mr $A$ ends up with what he likes no better, so that, Gibbard argues, his right to $(w,w)$ over $(y,w)$ is waived. By the same token Mr $B$'s right to $(y,y)$ over $(y,w)$ is waived. Following this reasoning we arrive at the conclusion that Gibbard's suggested social choice out of

$$\{(w,w),(y,w),(w,y),(y,y)\}$$

is $(y,w)$. It seems to us that this is a suggestion which is rather hard to swallow. Why on earth *should* people be assigned the colour of their bedroom walls which they unconditionally dislike?[5]

We have thus shown that restricting the use of the Pareto rule is a workable way-out of Gibbard's paradox and that, in some cases at least, it provides us with a more "reasonable" solution than Gibbard's resolution *via* the alienability of rights.

[4]There are two other order-extensions of $Q'_A \cup Q'_B$, namely:

$$R'^{\alpha} : (w,y),(w,w),(y,y),(y,w)$$
$$R'^{\beta} : (w,y),[(w,w),(y,y)],(y,w).$$

(In $R'^{\beta}$,$(w,w)$ and $(y,y)$ are deemed to be indifferent, so that they are put together by square brackets.) Nothing will be changed even if we use $R'^{\alpha}$ or $R'^{\beta}$ instead of $R'$ in the rest of our argument: $(w,y)$ will still be chosen.

[5]It is true that our way of solving the paradox ignores what Gibbard [4] has called "a strong libertarian tradition of free contract", according to which "a person's rights are his own to use or bargain away as he sees fit" [4, p.397]. This argument does not seem to deprive our resolution of its reasonableness in the present example, however.

# 4 Justice and Liberty: Interpersonal Welfare Comparisons

**4.1.** Back now to Sen's paradox in Section 2. The problem at hand is to find a way around the difficulty by making use of information on the interpersonal welfare comparisons. More explicitly we make use of the information available from "extended sympathy" (Arrow [1, p.114]), in the form of placing oneself in the position of another. In the literature there are assertions that this additional information does not provide us with a way-out of Sen's dilemma (Kelly [6] and Sen [10]). We will show, however, that if the rights-exercising is restricted by the maximin justice consideration along the line of Rawls [7] and Sen [9, Chapter 9], the *constrained* libertarian claim is made compatible with the Pareto rule, so that (as in the case of Arrovian impossibility theorems) the possibility of the interpersonal welfare comparisons does help us in circumventing Sen's paradox.

**4.2.** Interpersonal comparisons of the extended sympathy type are of the form: it is better to be an individual $i$ in state $x$ than to be an individual $j$ in state $y$. This is formally put by an ordering $\tilde{R}$ (to be called an *extended ordering*) on $X \times N$ with $(x, i) \in X \times N$ standing for being in the position of individual $i$ in social state $x$. We will work exclusively with the extended orderings satisfying Sen's *axiom of complete identity* (Sen [9, p.156]) in the sense that we assume that all individuals in the society share identical extended orderings. Needless to say this still allows each and every individual to have full freedom in judging social states placing himself in his own shoes, so that if we define

$$R_i = \{(x, y) \in X \times X : ((x, i), (y, i)) \in \tilde{R}\} \ (i \in N), \tag{13}$$

each $R_i$ is an ordering on $X$ and $(R_1, R_2, \ldots, R_n)$ is a profile (in the sense of Section 2) on which no restriction is placed. We are now concerned with a *generalized collective choice rule* (GCCR) which is a method of choosing, for each extended ordering, an SCF on $K$:[6]

$$C = \Psi(\tilde{R}). \tag{14}$$

The requirement of general applicability of our GCCR reads as follows.

*Condition* **GU** (*Unrestricted Domain*). The domain of GCCR consists of all logically possible extended orderings.

Notice that we can reinterpret Condition P, Condition SL and Condition CL as requirements on GCCR with the understanding that $R_i$ there now stands for (13).

**4.3.** Let $\Sigma$ be the set of all one-to-one correspondences between $N$ and $N$. Given an extended ordering $\tilde{R}$, the *maximin relation of justice* $M(\tilde{R})$ and the *Suppes' relation of justice* $J(\tilde{R})$ [11] are defined respectively by

[6]This is a *functional* CCR analogue of what Hammond [5] called the *generalized social welfare function.*

$$(x,y) \in M(\tilde{R}) \Leftrightarrow \exists k \in N : [\forall i \in N : ((x,i),(y,k)) \in \tilde{R}] \tag{15}$$

$$(x,y) \in J(\tilde{R}) \Leftrightarrow \exists \delta \in \Sigma : \begin{cases} \forall j \in N : ((x,j),(y,\delta(j))) \in \tilde{R} \\ \qquad \& \\ \exists k \in N : ((x,k),(y,\delta(k))) \in P(\tilde{R}). \end{cases} \tag{16}$$

In words, $x$ is more just than $y$ in the maximin sense iff it is no worse to be anyone in state $x$ than to be some specified individual in state $y$, while $x$ is more just than $y$ in Suppes' sense iff there exists a one-to-one transformation of $N$ into itself such that (a) being in state $x$ in someone's position is better than being in state $y$ in the position of the corresponding individual and (b) being in the position of each individual in $x$ is no worse than being the corresponding individual in $y$. It is known that, for each $\tilde{R}$, $M(\tilde{R})$, is an ordering on $X$ (Sen [9, Theorem 9*4]) and $J(\tilde{R})$ is an asymmetric and transitive relation on $X$ (Sen [9, Theorem 9*1]). Furthermore the following inclusions are true for each $\tilde{R}$:

$$\cap_{i \in N} P(R_i) \subset J(\tilde{R}) \subset M(\tilde{R}), \tag{17}$$

where $R_i$ is defined by (13).

**4.4.** Consider now the following requirement on GCCR.

*Condition* **SJ** (*Suppes' Justice Rule*). For all $x, y \in X$ if $(x,y) \in J(\tilde{R})$, then

$$[x \in S \ \& \ y \in C(S)]$$

for no $S \in K$.

A little reflection convinces us that there is no hope for our obtaining a GCCR satisfying GU, SJ and SL. (Suffice it to notice that Condition SJ implies Condition P by virtue of (17) and Condition GU implies Condition U (trivially rephrased as a condition on GCCR), while Sen's liberal paradox tells us that U,P and SL conflict.) Kelly [6, Theorem 3] has strengthened this observation in that even if we weaken SL so that (2) is constrained in such a way that

$$(x,y) \in D_i \cap P(R_i) \ \& \ (y,x) \notin J(\tilde{R}) \Rightarrow [x \in S \ \& \ y \in C(S)] \text{ for no } S \in K \tag{18}$$

we still cannot break the impasse. This is what he called the *Impossibility of a Just Liberal*.

**4.5.** The libertarian claim we are going to work with is a constrained version of the previous Condition CL.

*Condition* **ML** (*Maximin Libertarian Claim*). For any coherent rights-assignment $D = (D_1, D_2, \ldots, D_n) \in \Omega(n)$,

$$(x,y) \in D_i \cap P(R_i) \ \& \ (y,x) \notin M(\tilde{R}) \Rightarrow [x \in S \ \& \ y \in C(S)] \text{ for no } S \in K \tag{19}$$

holds for each $i \in N$.

In words, an individual $i$ can get his way for $x$ against $y$ if $(x, y)$ is his protected pair and $y$ is not more just than $x$ in the maximin sense. Therefore rights-exercising is restricted in ML by the maximin justice consideration.

The following theorem is true, which clearly contrasts with Kelly's impossibility theorem.

**Theorem 3.** *There exists a rational* GCCR *satisfying* **GU, ML** and **SJ**.

Before proving this proposition, we refer to a simple corollary thereof.

**Corollary**. *There exists a rational* GCCR *satisfying* **GU, ML** *and* **P**.

The message of this proposition is clear: The possibility of the interpersonal welfare comparisons does help us in finding a way around Sen's impossibility theorem as it helped us in avoiding Arrow's impossibility theorem. Kelly's and Sen's contrary statement is due to their insufficient use of the information which is actually available from the extended sympathy.

**4.6.** *Proof of Theorem* 3. Let $\tilde{R}$ be any given extended ordering and let $R_i$ $(i \in N)$ be defined by (13). $R_i$ being an ordering for all $i \in N$, there exists an order-extension $R$ of $Q = \cup_{i \in N}(R_i \cap D_i)$ as in 2.2. Let $\tilde{\mathscr{R}}$ be the set of all such orderings. Take an $R \in \tilde{\mathscr{R}}$ (which is fixed once and for all) and let $R_0$ be defined by

$$R_0 = \{(x, y) \in X \times X : (y, x) \notin J(\tilde{R}) \cup [P(R) \cap P(M(\tilde{R}))]\}. \tag{20}$$

We show that this $R_0$ is complete. Suppose to the contrary that $(x, y) \notin R_0$ and $(y, x) \notin R_0$ for some $x$ and $y$ in $X$. By definition we then have four cases to consider:

(i) $(x, y), (y, x) \in J(\tilde{R})$,

(ii) $(x, y), (y, x) \in P(R) \cap P(M(\tilde{R}))$,

(iii) $(x, y) \in J(\tilde{R})$ & $(x, y) \notin P(R) \cap P(M(\tilde{R}))$ &$(y, x) \notin J(\tilde{R})$
& $(y, x) \in P(R) \cap P(M(\tilde{R}))$

and

(iv) $(x, y) \notin J(\tilde{R})$ & $(x, y) \in P(R) \cap P(M(\tilde{R}))$ & $(y, x) \in J(\tilde{R})$
& $(y, x) \notin P(R) \cap P(M(\tilde{R}))$.

The case (i) and the case (ii) contradict, respectively, the asymmetry of $J(\tilde{R})$ and that of $P(R)$. The case (iii) cannot occur because $(x, y) \in J(\tilde{R})$ and (17) imply $(x, y) \in M(\tilde{R})$, while $(y,x) \in P(M(\tilde{R}))$ iff $(y,x) \in M(\tilde{R})$ & $(x,y) \notin M(\tilde{R})$. Similarly the case (iv) leads us to a contradiction. Therefore $R_0$ is complete. Next we show that

$$P(R_0) = J(\tilde{R}) \cup [P(R) \cap P(M\ \tilde{R})]. \tag{21}$$

If $(x,y) \in P(R_0)$, then $(y,x) \notin J(\tilde{R}) \cup [P(R) \cap P(M(\tilde{R}))]$ and

$$(x,y) \in J(\tilde{R}) \cup [P(R) \cap P(M(\tilde{R}))]$$

by definition, so that we have $P(R_0) \subset J(\tilde{R}) \cup [P(R) \cap P(M(\tilde{R}))]$. On the other hand, if there are $x$ and $y$ in $X$ such that $(x,y) \in J(\tilde{R}) \cup [P(R) \cap P(M(\tilde{R}))]$ and that $(x,y) \notin P(R_0)$, a contradiction with the completeness of $R_0$ ensues. Therefore (21) is true. Thirdly, we show the acyclicity of $R_0$. Suppose to the contrary that there exists a $\{x^1, x^2, \ldots, x^t\} \in K$ such that $(x^\mu, x^{\mu+1}) \in P(R_0)$ $(\mu = 1, 2, \ldots, t-1)$ and $(x^t, x^1) \in P(R_0)$. Noticing (17) and (21) this leads us to a contradiction either with the transitivity of $J(\tilde{R})$ or with that of $M(\tilde{R})$.

Now that $R_0$ is complete and acyclic, $C(S) = G(S, R_0)$ for all $S \in K$ is a well-defined rational choice function on $K$. Associating this $C$ with the given $\tilde{R}$, we obtain a rational GCCR. To show that this GCCR satisfies Condition SJ, suppose that there exist $x$ and $y$ in $X$ such that $(x,y) \in J(\tilde{R})$ and $[x \in S$ & $y \in C(S)]$ for some $S \in K$. We then have

$$(x,y) \notin J(R) \cup [P(R) \cap P(M(\tilde{R}))]$$

thanks to the construction of $C$, in contradiction with $(x,y) \in J(\tilde{R})$. Thus Condition SJ is satisfied. Condition ML is also satisfied. To see this, suppose that there are $x$ and $y$ in $X$ and $i \in N$ such that $(x,y) \in D_i \cap P(R_i), (y,x) \notin M(\tilde{R})$ and $[x \in S$ & $y \in C(S)]$ for some $S \in K$. We then have $(x,y) \notin J(\tilde{R}) \cup [P(R) \cap P(M(\tilde{R}))]$. On the other hand, we have $(x,y) \in D_i \cap P(R_i) \subset P(Q) \subset P(R)$, while $(y,x) \notin M(\tilde{R})$ implies $(x,y) \in P(M(\tilde{R}))$. Therefore we obtain $(x,y) \in P(R) \cap P(M(\tilde{R}))$, a contradiction. This completes our proof. ||

**4.7.** The simplest possible case which is of interest is provided by the following:

*Example* 4 (*Two Meddlers Case with Extended Sympathy*). This is the same as the Example 2 save for the fact that Mr $B$, alas, is physically handicapped and it is commonly reckoned that Mr $B$'s welfare is lower in whatever social state than that of Mr $A$ in any social state. Mr $A$ should realize, then, that by exercising his holy right the worse-off Mr $B$ would become worst-off of all and, as a socially conscious creature, Mr $A$ might refrain from exercising his right. Put formally, we may assume in this example that

$$\tilde{R} : (w, A), (x, A), (y, A), (z, A), (y, B), (z, B), (w, B), (x, B).$$

It follows therefore that

$$M(\tilde{R}) = \Delta \cup \{(y, x), (w, x), (z, x), (y, w), (z, w), (y, z)\}$$

and

$$J(\tilde{R}) = \{(w, x), (y, z)\}.$$

Let $S = \{x, y, z, w\}$. Thanks to the Pareto rule we have $x \notin C(S)$ and $z \notin C(S)$. We also have $w \notin C(S)$ because $(z, w) \in D_B \cap P(R_B)$ and $(w, z) \notin M(\tilde{R})$. Although we have $(x, y) \in D_A \cap P(R_A)$, his right for $x$ against $y$ is waived because $(y, x) \in M(\tilde{R})$. It follows that the social choice from $S$ is determinate and we have $C(S) = \{y\}$. $\|$

This example poses an interesting problem concerning the use of information in resolving social conflict. Recall that our resolution of the Two Meddlers Case in the Example 2 under the scheme $(A, \alpha)$ was $\{x\}$. It follows that if the interpersonal welfare comparison is possible but it is not made use of in resolving the conflict in question, the outcome might well be the worst possible one relative to the extended sympathy ordering! We may suggest that failure to make efficient use of available information could be extremely costly.

**4.8.** It should be clear that Gibbard's paradox can similarly be resolved along the same line of argument if we are armed with the stronger informational basis allowing interpersonal welfare comparisons.

# 5 Concluding Remarks

In this paper we have shown that if *either* certain parts of individual preferences are refrained from being counted in social choice (thereby constraining the applicability of the Pareto rule) *or* the individual's rights-exercising is constrained by the maximin justice considerations, a minimal amount of personal liberty in a Paretian society may be guaranteed. As a conclusion we may suggest that one of the prerequisites for a liberal Paretian society is to develop individual attitudes which respect and care for each other's liberty and well-being. From a slightly different angle, we may put the general implication of our analysis as follows. Just as we needed stronger informational basis (than what is compatible with the independence of irrelevant alternatives axiom) in circumventing Arrow's impossibility theorem, it is necessary to look beyond the set of individual preference orderings and to secure stronger informational basis for collective decision if we wish to be successful in reconciling the libertarian claim with the Paretian ethics.

# Appendix

**1.** Let $X$ be the set of all alternatives and let $K$ denote the set of all non-empty *finite* subsets of $X$. A binary relation $R$ is a subset of $X \times X$. The *asymmetric component* of $R$ is defined by

$$P(R) = \{(x, y) : (x, y) \in R \ \& \ (y, x) \notin R\}.$$

$R$ is said to be

(a) *complete* iff $(x, y) \in R$ or $(y, x) \in R$ for all $x$ and $y$ in $X$,
(b) *acyclic* iff there exists no $\{x^1, x^2 \ldots, x^t\} \in K$ such that $(x^\mu, x^{\mu+1}) \in P(R)$ for all $\mu = 1, 2, \ldots, t-1$ and $(x^t, x^1) \in P(R)$,
(c) *consistent* iff there exists no $\{x^1, x^2 \ldots, x^t\} \in K$ such that

$$(x^1, x^2) \in P(R), (x^\mu, x^{\mu+1}) \in R$$

for all $\mu = 2, \ldots, t-1$ and $(x^t, x^1) \in R$,
(d) *transitive* iff $(x, y) \in R$ and $(y, z) \in R$ imply $(x, z) \in R$ for all $x, y$ and $z$ in $X$,
(e) *asymmetric* iff $R = P(R)$, and
(f) an *ordering* iff it is complete and transitive.

Some clarifying comments on the concept of consistency might be in order. As is easily verified, the transitivity of $R$ implies the consistency thereof, while the consistency of $R$ implies its acyclicity. In each case, the converse is not true in general. In order to see this, let $R^1$ and $R^2$ be defined on $X = \{x, y, z\}$ by

$$R^1 = \{(x, y), (y, z), (z, y)\}$$

and

$$R^2 = \{(x, y), (y, z), (z, y), (x, z), (z, x)\}.$$

Then $R^1$ is consistent but not transitive, while $R^2$ is acyclic but not consistent. The difference between transitivity and consistency disappears, however, if $R$ happens to be complete. To verify this, suppose that $R$ is complete but not transitive. Then there exist $x, y$ and $z$ such that $(x, y) \in R$, $(y, z) \in R$ and $(x, z) \notin R$. $R$ being complete, we then have $(z, x) \in P(R)$, $(x, y) \in R$ and $(y, z) \in R$, so that $R$ is not consistent. Therefore *a complete $R$ is consistent iff it is transitive*.

**2.** It might help if we give an alternative formulation for acyclicity and consistency. For any two binary relations $R^1$ and $R^2$ on $X$ we define the *composition* of $R^1$ and $R^2$ by

$$R^1R^2 = \{(x, y) : (x, z) \in R^1 \ \& \ (z, y) \in R^2 \text{ for some } z \in X\}.$$

Given a binary relation $R$ we define a sequence $\{R^{(n)}\}_{n=1}^\infty$ of binary relations by

$$R^{(1)} = R, R^{(n)} = RR^{(n-1)} \ (n \geq 2).$$

The *transitive closure* of $R$ is then defined by

$$T(R) = \cup_{n=1}^\infty R^{(n)}.$$

In these terms $R$ is acyclic iff $T(P(R)) \cap \Delta = \emptyset$, while $R$ is consistent iff

$$P(R)T(R) \cap \Delta = \emptyset,$$

where $\Delta$ is the diagonal binary relation on $X$. Noticing that $T(P(R)) \subset P(R)T(R)$ it immediately follows that a consistent relation is acyclic.

**3.** Let $R^1$ and $R^2$ be two binary relations. We say that $R^2$ is an *extension* of $R^1$ iff (i) $R^1 \subset R^2$, and (ii) $P(R^1) \subset P(R^2)$. In this case we also say that $R^1$ is a *subrelation* of $R^2$.

If $R^2$ is an extension of $R^1$ and that $R^2$ is an ordering, we say that $R^2$ is an *order-extension* of $R^1$. In view of the subtle difference between transitivity and consistency, it seems quite intuitive that the following extension theorem is true, although it is non-trivial in its full generality.

**Lemma 1**$^*$ (Suzumura [13, Theorem 3]). *A binary relation $R$ has an order-extension iff $R$ consistent.*

This proposition generalizes Szpilrajn's basic theorem to the effect that *every quasi-ordering has an order-extension.*[7] It should be noticed that there may well exist multiple order-extensions of a given consistent binary relation.

**4.** Let $R$ be any binary relation on $X$. For any $S \in K$, the set of all *$R$-greatest points of $S$* is defined by:

$$G(S, R) = \{x : x \in S \ \& \ (x, y) \in R \text{ for all } y \in S\}.$$

The following neat result is important.

**Lemma 2**$^*$ (Sen [9, Lemma 1$^*l$]). *$G(S, R)$ is non-empty for all $S \in K$ iff $R$ is complete and acyclic.*

**5.** A choice function $C$ on $K$ maps any $S \in K$ into a non-empty subset $C(S)$ of $S$. Thanks to Lemma 2$^*$, a complete and acyclic binary relation $R$ generates a choice function $C$ on $K$ defined by

$$C(S) = G(S, R) \text{ for all } S \in K. \tag{1*}$$

Conversely a choice function $C$ on $K$ is said to be *rational* iff there exists a binary relation $R$ satisfying (1$^*$). In this case $R$ is called a *rationalization* of $C$. A rational choice function whose rationalization is an ordering is said to be *full-rational*. See Suzumura [12 & 14] for the characterization of rational and full-rational choice functions.

**6.** Just as we defined in the above the composition of binary relations, we may define the composition of protected spheres, which enables us to present a less loaded definition of the coherent rights-assignment. Namely the rights-assignment $D = (D_1, D_2, \ldots, D_n)$ is coherent iff

$$(x, x) \notin D_{i_1} D_{i_2} \ldots D_{i_k} \text{ for all } x \in X$$

for every non-constant sequence $\{i_1, i_2, \ldots, i_k\}$ with $i_\mu \in N$ $(\mu = 1, 2, \ldots, k)$.

**7.** Finally we prove our basic Lemma 1.

[7] See Szpilrajn [15]. As a matter of fact Szpilrajn was concerned with partial orderings (rather than quasi-orderings) but the proposition referred to is a simple corollary of his theorem. See also Arrow [1, p.64].

*Proof of Lemma 1.* Let $D = (D_1, D_2 \ldots, D_n)$ be coherent and take any $n$-tuple $(R_1, R_2, \ldots, R_n)$ of orderings. Define $Q$ by

$$Q_i = D_i \cap R_i \ (i = 1, 2, \ldots, n) \text{ and } Q = \cup_{i=1}^{n} Q_i. \tag{2*}$$

We show that $Q$ is a consistent binary relation. Suppose to the contrary that there exists $\{x^1, x^2, \ldots, x^t\} \in K$ such that $(x^1, x^2) \in P(Q), (x^\mu, x^{\mu+1}) \in Q$ for all $\mu = 2, \ldots, t-1$ and $(x^t, x^1) \in Q$. By definition, $(x^1, x^2) \in P(Q)$ iff $(x^1, x^2) \in Q_i$ for some $i$, and $(x^2, x^1) \notin Q_i$ for all $i$, so that we have $(x^1, x^2) \in P(Q_i)$ for some $i$. Therefore there exists an $i^\mu$ for each $\mu = 1, 2, \ldots, t$ such that $(x^1, x^2) \in P(Q_{i^1}), (x^\mu, x^{\mu+1}) \in Q_{i^\mu}$ $(\mu = 2, \ldots, t-1)$ and

$$(x^t, x^1) \in Q_{i^t}.$$

It follows that (i) $\{i^1, i^2, \ldots, i^t\}$ is not a singleton set, and (ii)

$$(x^\mu, x^{\mu+1}) \in D_{i^\mu} \ (\mu = 1, 2, , \ldots, t-1)$$

and $(x^t, x^1) \in D_{i^t}$. Therefore $D$ contains a critical loop, a contradiction. Now that $Q$ turns out to be consistent, there exists an order-extension $R$ of $Q$ by virtue of our Lemma 1*. By construction we have $Q_i \subset Q \subset R$ and $P(Q) \subset P(R)$. If we can show that

$$P(Q_i) \subset P(Q),$$

we are home. Suppose therefore that there exists $(x, y) \in P(Q_i)$ such that $(y, x) \in Q_{i'}$ for some $i'$. But $D$ then contains a critical loop, a contradiction.

To prove the converse, suppose to the contrary that $D$ is not coherent. Then we have a critical loop in $D$: $(x^1, x^2) \in D_{i^1}$, $(x^2, x^3) \in D_{i^2}, \ldots, (x^t, x^1) \in D_{i^t}$. Let $(R_1, R_2, \ldots, R_n)$ be an $n$-tuple of orderings satisfying:

$$(x^1, x^2) \in P(Q_{i^1}), (x^2, x^3) \in Q_{i^2}, \ldots, (x^t, x^1) \in Q_{i^t}. \tag{3*}$$

Since $\{i^1, i^2, \ldots, i^t\}$ cannot be a singleton set, there is no contradiction in supposing (3*). Let $R$ be an order-extension of $Q_i$ $(i = 1, 2, \ldots, n)$. Then (3*) implies that

$$(x^1, x^2) \in P(R), (x^2, x^3) \in R, \ldots, (x^t, x^1) \in R,$$

in contradiction with the transitivity of $R$. This completes the proof. $\|$

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# Chapter 14
# Liberal Paradox and the Voluntary Exchange of Rights-Exercising*

## 1 Introduction

Among many recent contributions on the logical (in)compatibility of the Paretian ethics and the libertarian claims, initiated by Sen's [11, Chapter 6*] theorem on the *impossibility of a Paretian liberal*, Gibbard's [3] analysis, which culminates in the edifice of the alienable rights system, deserves particular scrutiny. It tries, among other things, to call due attention to "a strong libertarian tradition of free contract," according to which "a person's rights are his to use or bargain away as he sees fit."[1] Searching examinations of the Gibbard's system have already been put forward by Karni [7], Kelly [8; 9, Chapter 9], Sen [12, Sect. IV] and Suzumura [14], but it seems to us that there remain many important points to be made on this interesting contribution. The purpose of this chapter is to point out that Gibbard's system of alienable rights in a revised version proposed by Kelly [8; 9 Chapter 9] represents a standard for individual liberty which cannot be met by any universal collective choice rule. That is to say, it is logically impossible to construct a collective choice rule with unrestricted domain, which realizes the Gibbard-Kelly system of alienable rights. This clearly contradicts Kelly's assertion to the effect that "[the revision] causes no significant changes in the theorems that make up Gibbard's libertarian claim."[2] This is unfortunate, since Kelly's revision seems to be rather persuasive. To the extent that Kelly's proposed revision is acceptable, therefore, the workability and reasonableness of Gibbard's scheme seem to be in serious doubt. At the very least, the edifice of the alienable rights system should be evaluated with this subtlety in mind. In passing, we will examine the possibility and limitation of resolving

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[1]Gibbard [3, p.397]. See also Barry [1, p.166].

[2]Kelly [8 ,p.144; 9, p.148].

the liberal paradox via the metarational exercising of rights a la Howard [4-6] in view of the similarity between the prisoners' dilemma and the Paretian liberal paradox, which was pointed out by Fine [2].

# 2 Gibbard's Consistent Libertarian Claim and Kelly's Revision Thereof

**2.1.** Let $N = \{1, 2, \ldots, n\}$ denote the finite set of individuals ($n \geq 2$) and let $X$ stand for the set of all conceivable social states. What we call a social state is a list of impersonal and personal features of the world. Letting $X_0$ and $X_i$ stand respectively for the set of all impersonal features of the world and the set of all personal features of the individual $i \in N$, the set of all social states, $X$, is now given by $X = X_0 \times (\Pi_{i \in N} X_i)$. It is assumed that $X_0$ and $X_i$ are finite with at least two elements each. $R_i$ denotes a weak preference (at least as good as) relation of the individual $i \in N$. We assume that $R_i$ is an ordering on $X$, being *complete* [for all $x$ and $y, (x, y) \in R_i$ and/or $(y, x) \in R_i$] and *transitive* [for all $x, y$ and $z, (x, y) \in R_i$ and $(y, z) \in R_i$ imply $(x, z) \in R_i$]. The strict preference relation $P(R_i)$ is defined as usual by $(x, y) \in P(R_i) \Leftrightarrow [(x, y) \in R_i \,\&\, (y, x) \notin R_i]$. The indifference relation $I(R_i)$ is defined by $(x, y) \in I(R_i) \Leftrightarrow [(x, y) \in R_i \,\&\, (y, x) \in R_i]$. A list $R = (R_1, R_2, \ldots, R_n)$ of individual weak preference orderings will be called a *profile*. A *collective choice rule* (CCR) is a function $F$ which represents a method of amalgamating each profile $R = (R_1, R_2, \ldots, R_n)$ into a *social choice function* $C$ on the family $\mathcal{S}$ of all finite nonempty subsets of $X : C = F(R)$. When an $S \in \mathcal{S}$ is specified as a set of realizable states, $C(S)$ denotes the nonempty set of socially chosen states. In what follows, we will be concerned with constructing a CCR which may amalgamate *every* logically possible profiles.

*Condition* **U** (*Unrestricted domain*). The domain of our CCR consists of all logically possible profiles.

**2.2.** As a matter of notational convention, we let $X_{)i(} = X_0 \times X_1 \times \ldots \times X_{i-1} \times X_{i+1} \times \ldots \times X_n$ and, for each $i \in N$ and each $x = (x_0, x_1, \ldots, x_n) \in X$, $x_{)i(} = (x_0, x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$. Furthermore, if $x_i \in X_i$ and $z = (z_0, z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n) \in X_{)i(}$, then $(x_i; z) = (z_0, z_1, \ldots, z_{i-1}, x_i, z_{i+1}, \ldots, z_n)$. We define $D_i$ by

$$D_i = \{(x, y) \in X \times X | x_{)i(} = y_{)i(}\} \qquad (i \in N).$$

Therefore if $(x, y) \in D_i$, $x$ and $y$ may possibly differ only in the specification of $i$'s personal feature. Call $D = (D_1, D_2, \ldots, D_n)$ the *Gibbardian rights system* or the *decomposable rights system*.

**2.3.** The CCR should also be so designed, a naive libertarian might claim, that it endows the special say to the $i$th individual over each and every pair $(x, y) \in D_i$ in the following sense:

*Condition* **GL(1)** (*Gibbard's first libertarian claim*). For every profile $R = (R_1, R_2, \ldots, R_n)$, every $i \in N$, and every $x, y \in X$, if $(x, y) \in D_i \cap P(R_i)$, then $[x \in S \Rightarrow y \notin C(S)]$ for all $S \in \mathcal{S}$, where $C = F(R)$.

An unfortunate fact is that *there exists no* CCR *which satisfies this naive libertarian claim together with Condition* U. This is Gibbard's first impossibility theorem [3, Theorem 1] on the libertarianism.

**2.4.** It is now time we formulate two versions of the Pareto rule.

*Condition* **EP** (*Exclusion Pareto*). For every profile $R = (R_1, R_2, \ldots, R_n)$ and every $x, y \in X$, if $(x, y) \in \cap_{i \in N} P(R_i)$, then $[x \in S \Rightarrow y \notin C(S)]$ for all $S \in \mathcal{S}$, where $C = F(R)$.

*Condition* **IP** (*Inclusion Pareto*). For every profile $R = (R_1, R_2, \ldots, R_n)$ and every $x, y \in X$, if $(x, y) \in \cap_{i \in N} P(R_i)$, then $[\{x \in S \;\&\; y \in C(S)\} \Rightarrow x \in C(S)]$ for all $S \in \mathcal{S}$, where $C = F(R)$.

Let $x$ and $y$ be such that everyone in the society strictly prefers $x$ to $y$. Then Condition EP requires that the CCR prohibit $y$ from being chosen from every environment where $x$ is available, while Condition IP requires that the CCR be such that $x$ is chosen from every environment where $x$ is available and $y$ is chosen. Clearly EP, which is a quite common formulation of the Pareto rule, is a stronger requirement on CCR than IP.[3]

**2.5.** Turn now to Gibbard's second libertarian claim, which goes as follows.

*Condition* **GL(2)** (*Gibbard's second libertarian claim*). For every profile $R = (R_1, R_2, \ldots, R_n)$, every $S \in \mathcal{S}$, every $i \in N$, and every $x, y \in X$, if $(x, y) \in D_i \cap P(R_i)$ and $((x_i; z_{)i(}), (y_i; z_{)i(})) \in P(R_i)$ for all $z_{)i(}$ such that $(x_i; z_{)i(}), (y_i; z_{)i(}) \in S$, then $[x \in S \Rightarrow y \notin C(S)]$, where $C = F(R)$.

In Condition GL(1), it is required that the CCR allow each and every individual to exercise his right $(x, y) \in D_i$ whenever he happens to prefer $x$ to $y$. In contrast, Condition GL(2) does not necessarily allow the de facto individual preferences to rule the roost: It is required in GL(2) that the CCR protect the right $(x, y) \in D_i$ only when the $i$th individual prefers the distinguishing feature $x_i$ of $x$ *unconditionally* to the corresponding feature $y_i$ of $y$. Clearly GL(2) represents a milder libertarian claim than GL(1). Nevertheless, *there exists no* CCR *which satisfies* GL(2), EP, *and* U. This is Gibbard's second impossibility

[3] It might be of some interest to present a concrete CCR which satisfies the Condition IP but does not satisfy the Condition EP. The simplest example is a rule which assigns to each and every profile $R = (R_1, R_2, \ldots, R_n)$ a choice function C such that $C(S) = S$ for all $S \in \mathcal{S}$. An intrinsically more interesting example is the *majority closure method* (Sen [13, pp. 56, 74]). Let $M_R$ be the simple majority relation corresponding to a given profile $R$. Take any $S \in \mathcal{S}$ and let $T(M_R \mid S)$ be the transitive closure of $M_R$ on $S$. A choice set $C_R^*(S)$ represents a subset of $S$ consisting of the $T(M_R \mid S)$-greatest points in $S$. The majority closure method is a CCR which assigns to each $R$ the choice function $C_R^*$. It is easy to see that this CCR satisfies IP but not EP.

theorem [3, Theorem 2] on the libertarianism. A slight generalization thereof is the following:

**Theorem 1**. *There exists no* CCR *which satisfies* **GL(2), IP** *and* **U**.

*Proof.* Suppose that there exists such a CCR. Take any $a_0 \in X_0$ and $a_i \in X_i$ ($i \in N\backslash\{1,2\}$) and fix them for the rest of this proof. Take $x_i, x_i' \in X_i, x_i \neq x_i'$, where $i = 1, 2$ and define

$$x^1 = (a_0, x_1, x_2, a_3, \dots, a_n),$$

$$x^2 = (a_0, x_1, x_2', a_3, \dots, a_n),$$

$$x^3 = (a_0, x_1', x_2, a_3, \dots, a_n),$$

and

$$x^4 = (a_0, x_1', x_2', a_3, \dots, a_n).$$

Let $S = \{x_1, x_2, x_3, x_4\} \in \mathcal{S}$ and let a profile $R = (R_1, R_2, \dots, R_n)$ be such that

$$R_1(S) : x^1, x^3, x^2, x^4,$$

$$R_2(S) : x^4, x^3, x^2, x^1,$$

and, for all $i \in N\backslash\{1,2\}$,

$$R_i(\{x^2, x^3\}) : x^3, x^2,$$

where $R_i(S)$ denotes the restriction of $R_i$ on the set $S$: $R_i(S) = R_i \cap (S \times S)$.[4] Clearly, then, $(x^1, x^3) \in D_1, (x^2, x^4) \in D_1$ and $(x^2, x^1) \in D_2$. No other individual has right over these states. Since individual 1 prefers $x_1$ to $x_1'$ unconditionally and individual 2 prefers $x_2'$ to $x_2$ unconditionally, Condition GL(2) implies that $x^3 \notin C(S), x^4 \notin C(S)$, and $x^1 \notin C(S)$ for a choice set $C(S)$ which corresponds to the specified $R$ and $S$. Suppose now $x^2 \in C(S)$. Then we have $(x^3, x^2) \in \cap_{i \in N} P(R_i), x^3 \in S$ and $x^2 \in C(S)$, so that Condition IP requires that $x^3 \in C(S)$, a contradiction. Therefore we must have $C(S) = \emptyset$, which negates the existence of a CCR satisfying GL(2), IP and U. ■

**2.6.** A salient common feature of GL(1) and GL(2) deserves particular mention: It is supposed that the $i$th individual's right is exercised in complete negligence of any repercussion from the rest of the society, guided solely by the individual rational calculus. From this viewpoint, the gist of the Gibbardian impossibility theorems mentioned so far may be interpreted as the *failure of the isolated rational rights-exercising.* An ingenious

[4]Preference orderings are written horizontally with the less preferred states appearing to the right of the more preferred states, indifferent states, if any, being put together by square brackets.

proposal crystallized in Gibbard's third libertarian claim is to make individual's libertarian rights *alienable* in cases where the exercise of one's libertarian rights brings him into a situation he likes no better than the situation that would otherwise have been brought about. Gibbard observes that, given an $R = (R_1, R_2, \ldots, R_n)$ and an $S \in \mathcal{S}$, an individual $i \in N$ has the will as well as the right to exclude $y$ from $C(S)$ if $x \in S$ and $(x, y) \in D_i \cap P(R_i)$, but his right for the pair $(x, y)$ had better be *waived* if there exists a sequence $\{y_1, y_2, \ldots, y_\lambda\}$ in $S$ such that

$$y_\lambda = x, (y, y_1) \in R_i \ \& \ y \neq y_1, \tag{1}$$

and

$$(\forall t \in \{1, 2, \ldots, \lambda - 1\})\colon (y_t, y_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N \setminus \{i\}} [D_j \cap P(R_j)]). \tag{2}$$

Let us define a subset $W_i(R \mid S)$ of $D_i$, to be called the *waiver set*, by $(x, y) \in W_i(R \mid S)$ iff (1) and (2) are true for some sequence $\{y_\mu\}_{\mu=1}^{\lambda}$ in $S$.

*Condition* **GL(3)** (*Gibbard's third libertarian claim*). For every profile $R = (R_1, R_2, \ldots, R_n)$, every $S \in \mathcal{S}$, every $i \in N$, and every $x, y \in X$, if $(x, y) \in D_i \cap P(R_i)$ and $(x, y) \notin W_i(R \mid S)$, then $[x \in S \Rightarrow y \notin C(S)]$, where $C = F(R)$.

Clearly this is a claim which differs essentially from GL(1) and GL(2) in that it is explicitly recognized that an individual's rights-exercising may induce unfavorable responses of the others which might well nullify the benefit for which the initial exercising was intended. Gibbard has shown that GL(3) represents a Pareto-consistent libertarian claim; i.e., *there exists a* CCR *which satisfies* GL(3), EP, *and* U.[5]

**2.7.** Kelly [8; 9, Chapter 9] claims to have found some "flaws" in Gibbard's definition of the rights-waiving rule and proposes two revisions thereof, the first of which goes as follows[6]. Let a profile $R = (R_1, R_2, \ldots, R_n)$ and a set of realizable states $S \in \mathcal{S}$ be given. An individual $i$ waives his right for $(x, y) \in D_i$, namely that $(x, y) \in W_i^*(R \mid S)$, iff there exists a sequence $\{y_1, y_2, \ldots, y_\lambda\}$ in $S$ satisfying

$$y_\lambda = x \ \& \ (y, y_1) \in P(R_i), \tag{3}$$

and (2).

*Condition* **KL(1)** (*Kelly's first libertarian claim*). For every profile $R = (R_1, R_2, \ldots, R_n)$, every $S \in \mathcal{S}$, every $i \in N$, and every $x, y \in X$, if $(x, y) \in D_i \cap P(R_i)$ and $(x, y) \notin W_i^*(R \mid S)$, then $[x \in S \Rightarrow y \notin C(S)]$, where $C = F(R)$.

The only difference between GL(3) and KL(1) lies in the contrast between (1) and (3). The reason behind this revision is that "in forcing the move from $y$ to $x$ by exercising

[5] Gibbard [3, Theorem 4].

[6] As a matter of fact, Kelly proposes the third revised version. For our purpose it is not necessary to get into this complicated proposal, however.

$[(x,y) \in D_i$, the individual $i]$ does not seem to have gotten into trouble if he is forced in the end to take a $y_1$ where he is indifferent between $y_1$ and $y$. Waiving might be appropriate for a cautious exerciser if $[(y, y_1) \in P(R_i)]$ for some [sequence $\{y_1, y_2, \ldots, y_\lambda\}$], but not if only $[(y, y_1) \in R_i$ as in (1)]" (Kelly [8, p.141; 9, pp.146-147]).

Going one step further, Kelly proposes his second revised libertarian claim. Suppose that a profile $R = (R_1, R_2, \ldots, R_n)$ and an $S \in \mathcal{S}$ are given. This time, an individual $i$ is supposed to waive his right for $(x, y) \in D_i$, namely that $(x, y) \in W_i^{**}(R \mid S)$ iff:

(a) There exists a sequence $\{y_1, y_2, \ldots, y_\lambda\}$ in $S$ such that

$$y_\lambda = x \ \& \ (y, y_1) \in P(R_i), \tag{4}$$

and

$$(\forall t \in \{1, 2, \ldots, \lambda - 1\})\colon (y_t, y_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N \setminus \{i\}} [D_j \cap P(R_j)]); \tag{5}$$

(b) For any sequence $\{z_1, z_2, \ldots, z_{\lambda^*}\}$ in $S$ such that

$$z_{\lambda^*} = y_1 \ \& \ (z_1, y) \in P(R_i), \tag{6}$$

and

$$(\forall t \in \{1, 2, \ldots, \lambda^* - 1\})\colon (z_t, z_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N} [D_j \cap P(R_j)]), \tag{7}$$

there exists correspondingly a sequence $\{w_1, w_2, \ldots, w_{\lambda^{**}}\}$ in $S$ such that

$$w_{\lambda^{**}} = z_1 \ \& \ (y, w_1) \in P(R_i), \tag{8}$$

and

$$(\forall t \in \{1, 2, \ldots, \lambda^{**} - 1\})\colon (w_t, w_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N \setminus \{i\}} [D_j \cap P(R_j)]). \tag{9}$$

*Condition* **KL(2)** (*Kelly's second libertarian claim*). For every profile $R = (R_1, R_2, \ldots, R_n)$, every $S \in \mathcal{S}$, every $i \in N$, and every $x, y \in X$, if $(x, y) \in D_i \cap P(R_i)$ and $(x, y) \notin W_i^{**}(R \mid S)$, then $[x \in S \Rightarrow y \notin C(S)]$, where $C = F(R)$.

The difference between KL(1) and KL(2) is the addition of (b) in the definition of $W_i^{**}(R \mid S)$, which says basically that any sequence $\{z_1, z_2, \ldots, z_{\lambda^*}\}$ which seems to repair in the eyes of the individual $i$ the damage caused upon him by a sequence $\{y_1, y_2, \ldots, y_\lambda\}$ will be made ineffective by some other, out-of-control sequence $\{w_1, w_2, \ldots, w_{\lambda^{**}}\}$.

# 3 Impossibility Theorems

**3.1.** Taken by themselves, these proposed revisions may seem to be fairly persuasive and, according to Kelly [8, p.144; 9, p.148], "it causes no significant changes in the theorems that make up Gibbard's libertarian claim." The truth is, however, that Kelly's reasonable-looking revisions to Gibbard's libertarian claim change it into a standard for individual liberty which cannot possibly be met, as the following impossibility theorems show.

**Theorem 2**. *There exists no* CCR *which satisfies* **KL(1)** *and* **U**.

**Theorem 3**. *There exists no* CCR *which satisfies* **KL(2)** *and* **U**.

To prove these theorems, note first that, for every profile $R = (R_1, R_2, \ldots, R_n)$ and every $S \in \mathcal{S}$, the following set-inclusions are true.

$$W_i^{**}(R \mid S) \subset W_i^{*}(R \mid S) \subset W_i(R \mid S) \tag{10}$$

for all $i \in N$. Clearly, then, KL(2) is a *stronger* libertarian claim than KL(1), so that we have only to prove Theorem 2, Theorem 3 being a corollary thereof.

*Proof of Theorem* 2. Suppose that $F$ is a CCR which satisfies KL(1) and U. Let $S = \{x^1, x^2, x^3, x^4\} \in \mathcal{S}$ be defined as in the proof of Theorem 1 and let a profile $R = (R_1, R_2, \ldots, R_n)$ be such that

$$R_1(S) : x^1, x^4, [x^2, x^3],$$
$$R_2(S) : x^3, x^2, [x^1, x^4].$$

There is no restriction on $R_i$ for $i \in N\backslash\{1, 2\}$ whatsoever. It is clear that $(x^1, x^3) \in D_1$, $(x^4, x^2) \in D_1$, $(x^3, x^4) \in D_2$ and $(x^2, x^1) \in D_2$. No other individual has right over these pairs of states. Consider the pair of states $(x^1, x^3) \in D_1 \cap P(R_1)$. The worst which could happen to individual 1 after his exercise of $(x^1, x^3) \in D_1$ is the counterexercise by 2 of $(x^2, x^1) \in D_2$ in view of $(x^2, x^1) \in P(R_2)$. (Note that there is no state in $S$ which strictly Pareto-dominates $x^1$.) Since $x^2$ and $x^3$ are indifferent to individual 1 and $x^2 \neq x^3$, GL(3) would let 1 waive his right over $(x^1, x^3)$, but KL(1) does allow 1 to exercise his right over $(x^1, x^3)$: $(x^1, x^3) \in W_1(R \mid S)\backslash W_1^*(R \mid S)$. Similar reasoning leads us to $(x^4, x^2) \in W_1(R \mid S)\backslash W_1^*(R \mid S), (x^3, x^4) \in W_2(R \mid S)\backslash W_2^*(R \mid S)$ and $(x^2, x^1) \in W_2(R \mid S)\backslash W_2^*(R \mid S)$. By virtue of the Condition KL(1) it then follows that $C(S) = \emptyset$, a contradiction. ■

**3.2.** Kelly's first revised libertarian claim thus brings back an impossibility. A fortiori, his second (and stronger) revised libertarian claim is inconsistent with the existence of a universal CCR. One may thereby be tempted to conclude that the system of alienable rights is something like a fragile glasswork which may be easily smashed to pieces while giving the last finish to it. To be fair, however, one should not forget to examine whether the finishing touch was an appropriate one.

Back, then, to the contrast between GL(3) and KL(1), i.e., the contrast between (1) and (3). Stipulation (3) was recommended in place of (1), because, in forcing the move from $y$ to $x$ by exercising $(x, y) \in D_i$, individual $i$ does not *lose* anything even if he is forced in the end to take a $y_1$ such that $(y, y_1) \in I(R_i)$ and $y \neq y_1$. Note, however, that he does not *gain* anything either. Note also that the rights-exercising in Gibbard's system places very heavy demands on the information gathering and processing[7], so that the rights-exercising would be unwise unless it yielded a positive gain. This argument, if accepted, would favor (1) rather than (3) and would necessitate the following modification of KL(2). Given a profile $R = (R_1, R_2, \ldots, R_n)$ and an $S \in \mathcal{S}$, define the waiver set $W_i^0(R \mid S)$ by $(x, y) \in W_i^0(R \mid S)$ iff:

(a) There exists a sequence $\{y_1, y_2, \ldots, y_\lambda\}$ in $S$ such that

$$y_\lambda = x, (y, y_1) \in R_i \ \&\ y \neq y_1, \tag{11}$$

and

$$(\forall t \in \{1, 2, \ldots, \lambda - 1\}) \colon (y_t, y_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N \setminus \{i\}} [D_j \cap P(R_j)]); \tag{12}$$

(b) For any sequence $\{z_1, z_2, \ldots, z_{\lambda^*}\}$ in $S$ such that

$$z_{\lambda^*} = y_1 \ \&\ [(z_1, y) \in P(R_i) \vee z_1 = y] \tag{13}$$

and

$$(\forall t \in \{1, 2, \ldots, \lambda^* - 1\}) \colon (z_t, z_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N} [D_j \cap P(R_j)]), \tag{14}$$

there exists correspondingly a sequence $\{w_1, w_2, \ldots, w_{\lambda^{**}}\}$ in $S$ such that

$$w_{\lambda^{**}} = z_1, (y, w_1) \in R_i \ \&\ y \neq w_1, \tag{15}$$

and

$$(\forall t \in \{1, 2, \ldots, \lambda^{**} - 1\}) \colon (w_t, w_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N \setminus \{i\}} [D_j \cap P(R_j)]), \tag{16}$$

for all $i \in N$. Utilizing this modified definition of the waiver set, we now put forward the following:

*Condition* **GKL** (*Gibbard-Kelly libertarian claim*). For every profile $R = (R_1, R_2, \ldots, R_n)$, every $S \in \mathcal{S}$, every $i \in N$, and every $x, y \in X$, if $(x, y) \in D_i \cap P(R_i)$ and $(x, y) \notin W_i^0(R \mid S)$, then $[x \in S \Rightarrow y \notin C(S)]$, where $C = F(R)$.

How does GKL fare in the context of universal CCRs? That it fares no better than KL(1) and KL(2) is the thrust of the next theorem.

**Theorem 4**. *There exists no* CCR *which satisfies* **GKL** *and* **U**.

*Proof.* Suppose that $F$ is an eligible CCR. Let $S = \{x^1, x^2, x^3, x^4\} \in \mathcal{S}$ and $R = (R_1, R_2, \ldots, R_n)$ be the same as in the proof of Theorem 2. Consider now the pair of states

[7] Kelly [8, p.141; 9, p.146].

$(x^2, x^3) \in D_1 \cap P(R_1)$. The worst situation which individual 1's exercise of $(x^1, x^3) \in D_1$ may induce is the counterexercise by 2 of $(x^2, x^1) \in D_2$ in view of $(x^2, x^1) \in P(R_2)$. Individual 1 may then exercise $(x^4, x^2) \in D_1 \cap P(R_1)$ to secure $x^4$, which 1 prefers to $x^3$. Is there any nullifying sequence? The worst which could happen to 1 is the exercise by 2 of $(x^3, x^4) \in D_2$, which does not require 1 to waive his right $(x^1, x^3) \in D_1$ according to the definition of $W_1^0(R \mid S)$.[8] Therefore GKL ensures that $x^3 \notin C(S)$. By the same token we may verify that

$$[(x^4, x^2) \in D_1 \cap P(R_1) \ \& \ (x^4, x^2) \notin W_1^0(R \mid S)] \Rightarrow x^2 \notin C(S),$$
$$[(x^2, x^1) \in D_2 \cap P(R_2) \ \& \ (x^2, x^1) \notin W_2^0(R \mid S)] \Rightarrow x^1 \notin C(S),$$

and

$$[(x^3, x^4) \in D_2 \cap P(R_2) \ \& \ (x^3, x^4) \notin W_2^0(R \mid S)] \Rightarrow x^4 \notin C(S),$$

so that we obtain $C(S) = \emptyset$, a contradiction. ■

**3.3.** If we examine the profile which we utilized in proving Theorems 2, 3, and 4, it turns out that *both* 1 and 2 are expressing preferences which are conditional on the other's selection of his personal feature: 1 prefers $x_1$ to $x_1'$ if 2 has $x_2$, while he prefers $x_1'$ to $x_1$ if 2's choice is $x_2'$ and *vice versa*. Probably it is too much to ask for the existence of a universal CCR which protects individual's mere conditional preferences. On reflection, we need only require the existence of a CCR which protects individual's libertarian rights so far as the relevant individual expresses unconditional preference for his personal features. Therefore, let $N(R \mid S)$ be the set of individuals having unconditional preferences, given a profile $R$ and an available set $S \in \mathcal{S}$, i.e., $i \in N(R \mid S)$ iff $(x, y) \in D_i \cap (S \times S) \cap P(R_i)$ always implies that $((x_i; z_{)i(}), (y_i; z_{)i(})) \in P(R_i)$ for all $z_{)i(}$ such that $(x_i; z_{)i(}) \in S$ and $(y_i; z_{)i(}) \in S$. The relevant waiver set will then have to be specified thus: $(x, y) \in D_i$ will be waived, i.e., $(x, y) \in W_i^{00}(R \mid S)$ iff *either* (i) $i \in N \backslash N(R \mid S)$, *or* (ii) the following conditions hold true:

(a) There exists a sequence $\{y_1, y_2, \ldots, y_\lambda\}$ in $S$ such that

$$y_\lambda = x, (y, y_1) \in R_i \ \& \ y \neq y_1,$$

[8] Two clarifications might be in order here. First, from $x^2$, something else may happen (besides 1 exercising his right $(x^4, x^2) \in D_1$) *if* the Pareto-dominance relation is weakened from $\cap_{i \in N} P(R_i)$ to $P(\cap_{i \in N} R_i)$. That is to say, *if* $(x^3, x^2) \in R_i$ for all $i \in N \backslash \{1, 2\}$ and *if* in (12), (14) and (16) all instances of $\cap_{i \in N} P(R_i)$ are replaced by $P(\cap_{i \in N} R_i)$, $x^3$ might be picked over $x^2$ by a Pareto-dominance. It is clear, however, that this possibility does not affect our conclusion that $(x^1, x^3) \notin W_1^0(R \mid S)$. Second, in arriving at the conclusion that $(x^1, x^3) \notin W_1^0(R \mid S)$, we have followed the "path" $x^3 \to x^1 \to x^2 \to x^4 \to x^3$ generated by the successive rights-exercising of 1 and 2. Namely, we started from $x^3$ and came back to $x^3$ again! It might be asked, why don't we let 1 waive his right $(x^1, x^3) \in D_1$ in this case? Put differently, why do we stipulate the condition $y \neq y_1$ in (11)? The reason is that there exists an important difference between (i) the travel from $x^3$ (via a sequence of states) to $x^3$ back again, and (ii) the travel from $x^3$ to an $x^* \in S$ such that $(x^3, x^*) \in I(R_1)$. In the former case, individual 1 comes back to $x^3$ *without losing his right over* $x^3$, while in the latter case, 1 may well be stuck at $x^*$ *without having any right over* $x^*$. This is also the reason why we modified (6) into (13).

and

$$(\forall t \in \{1, 2, \ldots, \lambda - 1\}) \colon (y_t, y_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N(R|S) \setminus \{i\}} [D_j \cap P(R_j)]);$$

(b) For any sequence $\{z_1, z_2, \ldots, z_{\lambda^*}\}$ in $S$ such that

$$z_{\lambda^*} = y_1 \ \& \ [(z_1, y) \in P(R_i) \vee z_1 = y]$$

and

$$(\forall t \in \{1, 2, \ldots, \lambda^* - 1\}) \colon (z_t, z_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N(R|S)} [D_j \cap P(R_j)]),$$

there exists correspondingly a sequence $\{w_1, w_2, \ldots, w_{\lambda^{**}}\}$ in $S$ such that

$$w_{\lambda^{**}} = z_1, (y, w_1) \in R_i \ \& \ y \neq w_1,$$

and

$$(\forall t \in \{1, 2, \ldots, \lambda^{**} - 1\}) \colon (w_t, w_{t+1}) \in (\cap_{j \in N} P(R_j)) \cup (\cup_{j \in N(R|S) \setminus \{i\}} [D_j \cap P(R_j)]),$$

Our final version of the libertarian claim in the spirit of Gibbard and Kelly goes as follows.

*Condition* **GKL**$^*$. For every profile $R = (R_1, R_2 \ldots, R_n)$, every $S \in \mathcal{S}$, every $i \in N$, and every $x, y \in X$, if $(x, y) \in D_i \cap P(R_i)$ and $(x, y) \notin W_i^{00}(R \mid S)$, then $[x \in S \Rightarrow y \notin C(S)]$, where $C = F(R)$.

Clearly **GKL**$^*$ is *weaker* than GL(2). Unfortunately this modest version of the libertarian claim still may not break the impasse if there are at least three individuals in the society.

**Theorem 5**. *Suppose than $n \geq 3$. Then there exists no* CCR *which satisfies* **GKL**$^*$, **IP** *and* **U**.

*Proof.* Suppose that an eligible CCR $F$ does exist. Take any $a_0 \in X_0$ and $a_i \in X_i$ $(i \in N \setminus \{1, 2, 3\})$ and fix them for the rest of this proof. Take $x_i, x'_i \in X_i, x_i \neq x'_i$ for $i \in \{1, 2, 3\}$ and define

$$x^0 = (a_0, x_1, x_2, x'_3, a_4, \ldots, a_n),$$

$$x^1 = (a_0, x_1, x_2, x_3, a_4, \ldots, a_n),$$

$$x^2 = (a_0, x_1, x'_2, x_3, a_4, \ldots, a_n),$$

$$x^3 = (a_0, x'_1, x_2, x_3, a_4, \ldots, a_n),$$

and

$$x^4 = (a_0, x_1', x_2', x_3, a_4, \dots, a_n).$$

Let $S = \{x^0, x^1, x^2, x^3, x^4\} \in \mathcal{S}$ and let a profile $R = (R_1, R_2, \dots, R_n)$ be such that

$$\begin{aligned} R_1(S) &: x^1, x^3, x^2, x^4, x^0, \\ R_2(S) &: [x^3, x^4], x^2, x^1, x^0, \\ R_3(S) &: x^3, x^2, x^0, x^4, x^1, \end{aligned}$$

and, for all $i \in N \backslash \{1, 2, 3\}$ :

$$R_i(\{x^0, x^2, x^3\}) : x^3, x^2, x^0.$$

By definition we see that $(x^1, x^3) \in D_1$, $(x^2, x^4) \in D_1$, $(x^3, x^4) \in D_2$, $(x^2, x^1) \in D_2$ and $(x^0, x^1) \in D_3$. Note that individual 1 prefers $x_1$ to $x_1'$ unconditionally, and individual 3 prefers $x_3'$ to $x_3$ unconditionally. On the other hand, individual 2 prefers $x_2'$ to $x_2$ if 1 chooses $x_1$, while he is indifferent between $x_2$ and $x_2'$ if 1 chooses $x_1'$ instead. In view of this conditional nature of 2's preferences, his rights-exercising will be made ineffective by a CCR subject to GKL*. Consider now the pair of states $(x^1, x^3) \in D_1 \cap P(R_1)$. The worst contingency which could be induced by the exercise of $(x^1, x^3) \in D_1$ is the counterexercise of $(x^0, x^1) \in D_3 \cap P(R_3)$. Since $(x^3, x^0) \in \cap_{i \in N} P(R_i)$ holds true, and there exists no nullifying sequence in $S$, we may conclude from GKL* that

$$[(x^1, x^3) \in D_1 \cap P(R_1) \ \& \ (x^1, x^3) \notin W_1^{00}(R \mid S)] \Rightarrow x^3 \notin C(S).$$

Similarly we may verify that

$$[(x^2, x^4) \in D_1 \cap P(R_1) \ \& \ (x^2, x^4) \notin W_1^{00}(R \mid S)] \Rightarrow x^4 \notin C(S)$$

and

$$[(x^0, x^1) \in D_3 \cap P(R_3) \ \& \ (x^0, x^1) \notin W_3^{00}(R \mid S)] \Rightarrow x^1 \notin C(S).$$

We may then invoke Condition IP (as we did in the proof of Theorem 1) to conclude that $x^2 \notin C(S)$ and $x^0 \notin C(S)$. It then follows that $C(S) = \emptyset$, a contradiction. ■

# 4 Metarational Exercising of Rights

**4.1.** Among the libertarian claims we have examined in the above, GL(3), KL(1), KL(2), GKL and GKL* differ substantially from GL(1) and GL(2) in that due attention is paid in the former category of claims to the fact that, in deciding whether to exercise one's right or not, one should take into considerations the others' response via their rights-exercising and/or Pareto-dominance. Notice, however, that it is commonly assumed in these claims that each individual, in predicting the others' response to his rights-exercising, presumes that *the others follow the naive rights-exercising rule without making* *any effort of prediction on their part.*[9] It might be asked, what can we make of the libertarian claims if we assume instead the complete mutual prediction?

**4.2.** The situation of complete mutual prediction may be modeled after the metagame theory of Howard [4-6]. For simplicity let there be only two individuals in the society and let an impersonal feature vector $s_0 \in X_0$ be fixed for the rest of this chapter. $S_1$ and $S_2$ denote the set of all available personal feature alternatives of individual 1 and that of individual 2, respectively. Given a profile $R = (R_1, R_2)$, we may construe the 4-tuple $\Gamma = (S_1, S_2; R_1, R_2)$ as a *basic game* played by 1 and 2. Each individual $k \in \{1, 2\}$ wishes to choose such an $s_k \in S_k$ as to yield, coupled with the fixed $s_0$ and the choice $s_{\sim k} \in S_{\sim k}$ of the other individual $\sim k$, a state $(s_0, s_k, s_{\sim k}) \in \{s_0\} \times S_1 \times S_2 \equiv S$ which "optimizes" his preference $R_k$ over $S$.[10] In a liberal society $k$ obviously lacks the power to regulate the choice by $\sim k$ of his personal feature, so that the best $k$ can do is to predict the choice by $\sim k$ and to form a *metastrategy* which is a function $g^k$ from $S_{\sim k}$ into $S_k$ such that, for each $s_{\sim k} \in S_{\sim k}$, $(s_0, g^k(s_{\sim k}), s_{\sim k}) \in S$ is the best state in $S$ with respect to $R_k$. In effect, then, $k$ is thinking in terms of the first-level metagame $k\Gamma = (G^k, S_{\sim k}; R_k, R_{\sim k})$, where $G^k$ denotes the set of all functions from $S_{\sim k}$ into $S_k$. In doing this, however, $k$ should be aware that $\sim k$ may be able to, and should rationally try to, predict $k$'s choice of his metastarategy. The subjective game of $k$ in the situation of complete mutual prediction is the second-level metagame $(\sim k)k\Gamma = (G^k, F^{\sim k}; R_k, R_{\sim k})$, where $F^{\sim k}$ denotes the set of all functions from $G^k$ into $S_{\sim k}$. We repeat for emphasis: The metagame $(\sim k)k\Gamma$ represents a model in which $\sim k$ is able to predict $k$'s choice in the metagame $k\Gamma$, while the metagame $k\Gamma$ represents a model in which $k$ is able to predict $\sim k$'s choice in the basic game $\Gamma$. Thus in the metagame $(\sim k)k\Gamma$ there is complete mutual prediction.

**4.3.** Let us say, following Howard, that an outcome $(g^k_*, f^{\sim k}_*)$ of the metagame $(\sim k)k\Gamma$ is *metarational for $k$ via $(\sim k)k\Gamma$* iff:

$$[(g^k_*(f^{\sim k}_*(g^k_*)), f^{\sim k}_*(g^k_*)), (g^k_*(f^{\sim k}_*(g^k_*)), f^{\sim k}_*(g^k))] \in R_k \tag{17}$$

for all $g^k \in G^k$. In this case the corresponding basic outcome, which is given by $(s_0, g^k_*(f^{\sim k}(g^k_*)), f^{\sim k}(g^k_*)) \in S$, will be called hereafter the *metarational basic outcome for $k$ via $(\sim k)k\Gamma$*. Let $M_k[(\sim k)k\Gamma]$ denote the set of all metarational basic outcomes for $k$ via $(\sim k)k\Gamma$. We may similarly define $M_{\sim k}[(\sim k)k\Gamma], M_k[k(\sim k)\Gamma]$, and $M_{\sim k}[k(\sim k)\Gamma]$. It is now time we introduce the concept of metasolutions. In our present context, two possibilities suggest themselves. First we have

$$M^*(\Gamma) = M_k[(\sim k)k\Gamma] \cap M_{\sim k}[k(\sim k)\Gamma], \tag{18}$$

which is appealing for the following reason. In general, "there will be a strong tendency for player $i$'s subjective metagame to be one in which he follows all the others last. For

[9] This point was duly stressed by Kelly. He has shown that a serious difficulty of *correctable miscalculation* emerges from this peculiarity of the Gibbardian rights-exercising rule. See Kelly [8; 9, Chapter 9].

[10] It is assumed here that the set of realizable states $S$ satisfies Kelly's [8; 9, Chapter 9] *agenda-closedness* and Seidl's [10] *technological separability.*

such a metagame is precisely one in which he can predict the others' basic strategies, they cannot predict his, and they are possibly able to predict his [metastrategy]. Indeed, if we add the condition that $i$ believes there is complete mutual prediction ..., then this precisely describes the complete metagames in which he follows all others last."[11] It then follows that, if $s^* \in M^*(\Gamma)$, then $s^*$ is a metarational basic outcome for *both* individuals via their *respective* "natural" subjective metagames with complete mutual prediction. The second metasolution concept of our concern is

$$M_*(\Gamma) = \{M_k[(\sim k)k\Gamma] \cap M_{\sim k}[(\sim k)k\Gamma]\} \cup \{M_k[k(\sim k)\Gamma] \cap M_{\sim k}[k(\sim k)\Gamma]\}. \qquad (19)$$

By definition, $s^* \in M_*(\Gamma)$ holds true iff $s^*$ is a metarational basic outcome for *both* individuals via a *common* metagame with complete mutual prediction. Remember that an $s^* \in M^*(\Gamma)$, however "natural" it may be, cannot be an equilibrium via voluntary bargaining or negotiation except as a result of some kind of misunderstanding: Each individual behaves as if he can predict the other's basic strategy while the other can at best predict his metastrategy choice, but they cannot both be correct. In contrast, a metasolution $s^* \in M_*(\Gamma)$ is the basic outcome which constitutes an equilibrium when both individuals have agreed on the same subjective metagame.

**4.4.** We now examine these metasolution concepts in the context of the liberal paradoxes.

Example 1. Let $\Gamma = (S_1, S_2; R_1, R_2)$ be such that $S_1 = \{s_1, s_1'\}, S_2 = \{s_2, s_2'\}$ and

$$R_1(S) : s^1, s^3, s^2, s^4,$$
$$R_2(S) : s^4, s^3, s^2, s^1,$$

where $s^1 = (s_0, s_1, s_2), s^2 = (s_0, s_1, s_2'), s^3 = (s_0, s_1', s_2)$ and $s^4 = (s_0, s_1', s\prime_2)$. Note that this profile is the same as the one used in the proof of Theorem 1. In the game-theoretic context, this situation is known as the *prisoners' dilemma.*[12] It is easy, if tedious, to verify for this game $\Gamma$ that $M_1[21\Gamma] = M_2[21\Gamma] = M_1[12\Gamma] = M_2[12\Gamma] = \{s^2, s^3\}$, so that we have:

$$M^*(\Gamma) = M_*(\Gamma) = \{s^2, s^3\}.$$

Let $O_R(S)$ be the set of Pareto-optimal states in $S$ when the profile $R$ prevails. Note, then, that $\{s^3\} = O_R(S) \cap M^*(\Gamma) = O_R(S) \cap M_*(\Gamma)$. Therefore it may duly be said that the metarational exercising of rights brings about a Pareto-optimal state in the case of prisoners' dilemma profile, thereby resolving the *Paretian liberal paradox.* ■

An important feature of this scheme should be stressed: Each individual need not know the other's preference in this scheme, since $M_k[(\sim k)k\Gamma]$ as well as $M_k[k(\sim k)\Gamma]$ are definable solely in terms of $k$'s own preferences. This is certainly a nice feature, but

[11] Howard [6, p.106].
[12] Fine [2] pointed out the similarity between the prisoners' dilemma and the liberal paradox.

the real force of this resolution scheme is revealed if we examine the next example which relates to the internal inconsistency of rights.[13]

Example 2. Let $\Gamma^*$ be the same as $\Gamma$ in the Example 1 save for the following specification of the profile $R^* = (R_1^*, R_2^*)$:

$$R_1^*(S) : [s^1, s^4], [s^2, s^3],$$
$$R_2^*(S) : [s^2, s^3], [s^1, s^4].$$

Consider the metagames $(\sim k)k\Gamma^*$ and $k(\sim k)\Gamma^*$. We may verify that $M_1[21\Gamma^*] = M_1[12\Gamma^*] = \{s^1, s^4\}$ and $M_2[21\Gamma^*] = M_2[12\Gamma^*] = \{s^2, s^3\}$.[14] It then follows that $M^*(\Gamma^*) = M_1[21\Gamma^*] \cap M_2[12\Gamma^*] = \emptyset$, and $M_*(\Gamma^*) = \{M_1[21\Gamma^*] \cap M_2[12\Gamma^*]\} \cup \{M_1[12\Gamma^*] \cap M_2[12\Gamma^*]\} = \emptyset$. Therefore both metasolution concepts fail to resolve the liberal paradox embodied in the game $\Gamma^*$. ■

**4.5.** There are two classes of liberal paradox which we should cope with: the *Paretian liberal paradox*, and the *paradox of the internal inconsistency of rights*. We have shown in the above that the metarational exercising of rights in the situation of complete mutual prediction may systematically resolve the former class of paradoxes, while the latter difficulty may not be resolved in this manner.

# 5 Concluding Remarks

In reviewing a number of contributions to the Paretian liberal paradox, Sen [12, p.224] argued that "[Kelly] identifies a number of difficulties with Gibbard's system [which are] essentially arising from problems in deciding *when* a right is useful for a person. ... Some of these difficulties are eliminated by modifications of the Gibbard system proposed by Kelly ... ." We have examined in this chapter Gibbard's system of alienable rights in the light of Kelly's proposed modifications thereof. It was shown that the difficulties identified by Kelly are more serious than Kelly and Sen have thought them to be, and they are *not* eliminated by Kelly's proposed modifications. We have also examined the possibility and limitation of resolving the liberal paradox via the metarational rights-exercising. In conclusion, it is hoped that the negative results reported in this chapter will serve to clarify the nature and stubbornness of the paradox of a Paretian liberal.

[13] See Gibbard's Wall-Colour Case [3, p.389] and Sen's Zubeida-Rehana Case [12, p.234]. In the game-theoretic literature, the game $\Gamma^*$ in Example 2 is called the game of Matching Pennies. See Howard [4, p.182].

[14] Howard's *Characterization Theorem for Metarational Outcomes* [6, pp.89-96] is useful in verifying these results.

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# Chapter 15
# Individual Rights Revisited*

## Introduction

Ever since the publication of Sen's [13; 14] seminal contributions, the subject of individual libertarian rights has attracted much attention from economists, and Sen's paradox of the impossibility of a Paretian liberal has inspired a large volume of literature which has spanned several disciplines including economics, philosophy and politics.[1] Not surprisingly, Sen's contributions have provoked controversies. One of the most fundamental questions raised in this context relates to the very formulation of the concept of libertarian rights due to Sen. It has been contended by Nozick [11], Bernholz [1], Gärdenfors [5] and Sugden [18], among others, that this formulation does not capture our intuitive notion of rights.[2] In an important paper, Sen [16] responded to some of these criticisms. This, however, led to further debates (see Sugden [19]). In this chapter we seek to highlight some important strands in this debate, and to achieve some clarification of the different and often incompatible views of individual rights which have generated this controversy.

It may be worth clarifying at the outset that our basic concern is the formulation of the notion of individual rights *per se*, and not the conflict between individual rights and the weak welfarisitic requirement of Pareto optimality. This conflict is a fundamental

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[1] For a partial survey of the literature, see Sen [15], Suzumura [20, Chapter 7] and Wringlesworth [22].

[2] See also Gibbard [8].

problem in the theory of social choice, and has rightly received much attention, thanks to the path-breaking analysis of Sen. It is our belief that this problem persists under virtually every plausible concept of individual rights that we can think of. However, we do not discuss this issue of incompatibility between individual rights and the Pareto principle, nor do we seek to provide a solution to this problem. Instead, we focus on the conceptual problem of formulating the notion of individual rights in the theory of social choice, and the extent to which our intuition about individual rights is captured by the different formulations in the literature.

The structure of this chapter is as follows. In Section I we discuss some of the main features of Sen's [14; 15; 16; 17] formulation of individual rights. In Section II we put forward an example, the analysis of which reveals that our intuitive notion about certain types of individual rights is not properly captured by Sen's formulation. In Section III we generalize our criticism of Sen's formulation of individual rights, and argue that the intuitive difficulties discussed in Section II affect his formulation in the context of most rights one can think of. In Section IV an alternative formulation of individual rights in terms of game forms is introduced, and its relative merit *vis-à-vis* Sen's formulation is considered. Section V concludes with a brief account of the historical background of our analysis.

# 1 Sen's Formulation of Individual Rights

The basic intuitive idea of individual rights, which Sen [14; 15] as well as many of his critics have sought to capture, can perhaps be best stated in the language of J. S. Mill [10, Book 5, Chapter 11]: "... there is a circle around every individual human being which no government, be it that of one, of a few, or of the many, ought to be permitted to overstep."

Thus, both Sen and his critics would agree with Mill that each individual should have a "recognized personal sphere" (RPS) with which the rest of the society should not be allowed to interfere. Differences, however, arise when they seek to provide a precise formulation of this personal sphere and of how the recognition of the personal spheres of different individuals should get reflected in the choices made by the society.

Sen's [14; 15] original formulation was in terms of social preferences. Recollect that the notion of social preferences, so widely used in the theory of social choice, can bear alternative interpretations, depending on the exact nature of the problem in which one is interested (see Sen [16]). In this chapter we shall concentrate on the normative problem of social choice, and Sen's notion of individual rights will be discussed exclusively in terms of social choice.

For our purpose, a social state will be interpreted as a complete description of all aspects of the society that may be considered relevant, such as the colour of individual $i$'s bedroom walls, the number of hospital beds available, the consumption of wheat by each individual and so on. Every individual is assumed to have a preference ordering over all social states.

The notion of an individual's RPS constitutes the intuitive foundation of Sen's [13;

14] condition of liberalism, which embodies his conception of individual rights. Recollect that individual $i$ is said to be *decisive* over $\{x, y\}$, where $x \neq y$, if $y$ (resp. $x$) will never be socially chosen when $x$ (resp. $y$) is available and $i$ strictly prefers $x$ (resp. $y$) to $y$ (resp. $x$). Sen's condition of liberalism can then be stated as follows:

(1.1) For every individual $i$, there exist distinct social alternatives $x$ and $y$ such that $i$ is decisive over $\{x, y\}$.

Implicit in (1.1) is the interpretation that the two alternatives, $x$ and $y$, differ only with respect to some aspect (e.g. the colour of $i$'s bedroom walls) which comes within $i$'s RPS; otherwise the condition (1.1) can hardly be associated with the notion of libertarian rights.

Sen's condition of liberalism can be interpreted in a somewhat weaker sense. Let us say that $i$ is *locally decisive* over $\{x, y\}$, where $x \neq y$, if $y$ (resp. $x$) will never be socially chosen when $x$ and $y$ are only two available social alternatives and $i$ strictly prefers $x$ (resp. $y$) to $y$ (resp. $x$). Then a weaker interpretation of Sen's condition can read as follows:

(1.2) For every individual $i$, there exist distinct social alternatives $x$ and $y$ such that $i$ is locally decisive over $\{x, y\}$.[3]

Thus, under Sen's conception of individual rights, individual $i$ has a right if (1.1) or (1.2) is satisfied.[4] For convenience, Sen's formulation of individual rights in general will be called formulation S, while the distinction between his formulation is terms of (1.1), (1.2) and similar other conditions will be indicated by using terms such as formulation S(1.1), formulation S(1,2) and so on.

Clearly, (1.1) is much stronger than (1.2). It can be easily seen that (1.1) follows from the conjunction of (1.2) and the following property of social "rationality":

(1.3) For all social alternatives $z$ and $w$, if $z$ is socially chosen in rejection of $w$ when $z$ and $w$ are the only available social alternatives, then the society should never choose $w$ when $z$ is available.

[3] As Sen [16] points out, the question of how the social choice is actually made, as distinct from the question of what alternative society chooses, may be of considerable importance in the context of individual rights. Thus, individual $i$, left to himself, may always choose vegetarian food. If, however, instead of $i$ choosing vegetarian food for himself, he is given vegetarian food by someone else, without his having a choice, it does seem to make a difference to his rights. This aspect is ignored in (1.1) and (1.2).

[4] Sen's condition of liberalism admits both these interpretations, since the condition is stated in terms of social preference rather than social choice, and the strict social preference for $x$ against $y$ can be translated into the terminology of social choice in at least two ways. It can be interpreted to mean that $y$ should not be chosen by the society in any choice situation where $x$ is available. Alternatively, it can be interpreted to mean that, when $x$ and $y$ are the only two available alternatives, society should choose $x$ and reject $y$. Depending on which of these two interpretations we adopt, we have either (1.1) or (1.2).

It is worth noting that Sen [16] explicitly adopts interpretation (1.1) in the context of the normative problem of social choice, and therefore we shall concentrate mainly on (1.1) in our subsequent discussion.

Note that (1.1) is stated in terms of the individual's preferences, but it is easy to formulate a counterpart of (1.1) in terms of the individual's (actual or hypothetical) choice. Consider the following assumption:

> (1.4) For all distinct social alternatives, $z$ and $w$, if individual $i$ has to choose from the available set $\{z, w\}$, then he would choose $z$ and reject $w$ if and only if he prefers $z$ to $w$.

This is a highly plausible assumption, especially when $z$ and $w$ are assumed to differ only with respect to something in $i$'s RPS. In the presence of (1.4), (1.1) becomes equivalent to the following:

> (1.5) For every individual $i$, there exist distinct social alternatives $x$ and $y$ such that, if $i$ would choose $x$ (resp. $y$) and reject $y$ (resp. $x$) when asked to choose from $\{x, y\}$, then $y$ (resp. $x$) should never emerge as the social choice when $x$ (resp. $y$) is available.

Sen [13; 14] has shown that, if the social choice procedure satisfies (1.1), then the procedure will sometimes fail to satisfy the Pareto principle.[5] Recollect that the Pareto principle requires that, if all individuals in the society strictly prefer a social alternative $z$ to another social alternative $w$, then $w$ should never emerge as a social outcome when $z$ is available. While this impossibility result is of crucial importance in several different ways, our basic concern is not with this impossibility result, but with formulation S of individual rights embodied in (1.1).

Before concluding this section, let us note several distinguished features of formulation S.

First, in both (1.1) and (1.2), it is an individual's preference over two *complete descriptions of the society* (i.e. over two social states), differing only with respect to that individual's RPS, which constrains social choice. In (1.2) the constraint is imposed "locally" on social choice when two complete descriptions of the society constitute the only feasible social alternatives. In contrast, the constraint on social choice is imposed "globally" in (1.1). (As we have noted earlier, it is possible to restate the stipulations regarding individual rights in terms of the individual's choice rather than the individual's preference. However, even when the formulation is in terms of the individual's choice, it refers to the individual's choice from among complete descriptions of the society, which constitute the different feasible social alternatives, just as the formulation in terms of the individual's preference refers to the individual's preference over complete descriptions of the society, which constitute the relevant social alternatives.)

Second, it is important to remember that Sen intended (1.1) to be only a *necessary* condition for individual $i$ to have a right; it was not meant to be a *necessary and sufficient*

[5] For this impossibility theorem, it is not even necessary to require that (1.1) is satisfied. It is sufficient if at least two individuals are decisive over one pair of social alternatives each. This weaker condition is called by Sen [13; 14] the condition of *minimal liberalism*.

condition which would capture the entire content of the notion of individual rights. This being the case, it would be quite unreasonable if, in evaluating formulation S of individual rights, one were to criticize it on the ground that it failed to capture some part of our intuition about individual rights, however important that part may happen to be. On the other hand, it would be a shortcoming of formulation S if it turns out to be inconsistent with some important aspects of our intuition about individual rights. In the following sections, we argue that formulation S suffers from this type of logical flaws.

Third, it is of interest to note that a somewhat stronger version of (1.1) has been suggested by Gibbard [7]. Pursuing Sen's concept of an individual's RPS, he stipulated the following condition:

(1.6) For every individual $i$ and for all distinct social alternatives $x$ and $y$, if $x$ and $y$ differ only with respect to something in $i$'s RPS, then $i$ is decisive over $\{x, y\}$.

Although (1.6) seems to be much stronger than (1.1), it is not clear why one should object to (1.6) if one is ready to accept (1.1). At the very least, they seem to have much in common as far as the underlying motivation goes.

# 2 Critique of Sen's Formulation: A Counter-Example

In this section, we consider a simple example to show that there is an important category of individual rights which do not lend themselves to formulation S(1.1). The right that we discuss is the right of an individual to choose the colour of his own shirt.[6] We shall later argue that this example has some special features which may not be shared by many other rights. However, we shall also argue that the intuitive problems that arise when we seek to formulate this right in terms of (1.1) arise in the context of a very wide range of rights, and are not at all dependent on these special features. Besides, there are several fundamental rights, such as the right to choose one's own religion, the right to believe and to profess one's belief in the theory of evolution and so on, which have a structure similar to that of our example.

Now the example. There are two individuals, the conformist (individual 1) and the non-conformist (individual 2). Each individual has the right to choose his shirt from among the shirts which he owns. Each individual has two shirts: white ($w$) and blue ($b$). The two individuals are completely ignorant of each other's preferences, and at the time of making his choice each individual is ignorant about the other's choice. Everything else being fixed, there are only four social states, $(w, w), (b, b), (w, b)$ and $(b, w)$, where $(w, b)$ denotes white shirt for 1 and blue shirt for 2, and similarly for other social states. The two individuals' preference orderings are as follows:

[6]This example is due originally to Gibbard [7], though he used it for an entirely different purpose.

| 1 | 2 |
|---|---|
| $(w, w)$ | $(b, w)$ |
| $(b, b)$ | $(w, b)$ |
| $(b, w)$ | $(w, w)$ |
| $(w, b)$ | $(b, b)$ |

In words, 1 would like to have $w$ rather than $b$ if 2's choice is $w$, whereas he would like to have $b$ rather than $w$ if 2's choice is $b$ instead. On the other hand, 2 would like to have $w$ rather than $b$ if 1's choice is $b$, while he would rather have $b$ if 1's choice is $w$.

If we want to formulate the right of each individual to choose his shirt in Sen's terms, then (1.1) must hold for $i = 1, 2$, so that, given our intuition about each individual's RPS in this case, at least one of the conditions (2.1) and (2.2) and also at least one of the conditions (2.3) and (2.4) must be satisfied:

(2.1) 1 is decisive over $\{(w, w), (b, w)\}$;

(2.2) 1 is decisive over $\{(b, b), (w, b)\}$;

(2.3) 2 is decisive over $\{(b, w), (b, b)\}$;

and

(2.4) 2 is decisive over $\{(w, b), (w, w)\}$.

We shall now argue that three serious intuitive difficulties arise in the context of formulation S(1.1) and some of its natural extensions. For lack of convenient names, we shall call these problems A, B and C, respectively. Note that problem A affects formulation S(1.1) as well as its extensions, whereas problems B and C affect the extensions of formulation S(1.1), but not necessarily formulation S(1.1) itself.

Problem A. Suppose that (2.1) holds. Let the two individuals freely choose their shirts without knowing anything about the other individual's choice and preferences. Suppose, given such ignorance, 1 follows the maximin principle and chooses $b$. Similarly, suppose 2 follows the maximin principle and chooses $w$. Then $(b, w)$ will emerge as the social outcome. This would, of course, be inconsistent with (2.1). However, given that $(b, w)$ arose from the two individuals' free choices of their respective shirts, very few people would be willing to say that there was a violation of the right of any individual to choose his shirt. The fact that each individual is free to choose his shirt without any external constraint seems to capture the entire intuitive content of our conception of the right under consideration. Even if, as a result of exercising this freedom of choice, (2.1) is violated, it does not justify us in saying that there has been a violation of the right of either individual.

In a similar way, for each of the conditions (2.2), (2.3) and (2.4), it is possible to specify the individuals' preference orderings in such a way that, given the maximin behaviour on the part of each individual, the free choice of the individuals would give rise to a social outcome that would contradict the condition under consideration. Since Sen's formulation implies that at least one of the four conditions, (2.1), (2.2), (2.3) and (2.4), must be satisfied, it is clear that there are cases where the Sen right of some individual will be violated, even though, from our intuitive point of view, there is no violation of any individual's right.

Note that the assumption of maximin behaviour under uncertainty is not crucial to our reasoning. However, given complete ignorance about each other's preferences, there is no compelling reason why the two individuals should not follow the maximin principle either. Indeed, given any rule of choice under uncertainty, we can modify the features of our example suitably to get the same inconsistency between formulation S(1.1) and our intuition about the right under consideration.

Problem B. Problem A arises irrespective of whether 1 is decisive over only one of $\{(w, w), (b, w)\}$ and $\{(b, b), (w, b)\}$, or over each of the two sets. However, given that the pair $(w, w)$ and $(b, w)$, as well as the pair $(b, b)$ and $(w, b)$, differs only with respect to the colour of individual 1's shrit, it is not clear why 1 should be decisive over one of the two-element sets, $\{(w, w), (b, w)\}$ and $\{(b, b), (w, b)\}$,but not the other. In fact, the appeal to the notion of 1's RPS may suggest that it would be more natural to assume that 1 is decisive over both the two-element sets. Presumably, it might have been such considerations that led Gibbard [7] and several other writers to replace (1.1) by the more stringent (1.6). However, if we assume that both (2.1) and (2.2) are satisfied, then we have another serious intuitive problem in addition to the one discussed earlier. Put differently, (2.1) and (2.2) together imply a type of power for individual 1 that is completely inconsistent with our intuition.

Consider the type of power that 1 enjoys under our intuitive conception of the right to choose his shirt. He can choose $b$, thereby securing that the final social outcome will not lie in the set $\{(w, w), (w, b)\}$. Alternatively, he can choose $w$, thereby securing that the final social outcome will not lie in the set $\{(b, w), (b, b)\}$. Thus, 1 has the power to ensure that the final social outcome will never lie in the set $\{(w, w), (w, b)\}$ and also the power to ensure that the final social outcome will never lie in the set $\{(b, w), (b, b)\}$. Under our intuitive concept of the right, however, there is no way in which 1 can secure that the final social outcome will never lie in the set $\{(b, w), (w, b)\}$. Yet this is exactly the power that 1 has under (2.1) and (2.2), given 1's preference ordering as specified earlier. Given that preference ordering for 1, (2.1) and (2.2) imply that neither $(b, w)$ nor $(w, b)$ must be the social outcome. Thus, in addition to creating problem A, (2.1) and (2.2) together give to 1 a *preference-based* power which runs strongly counter to our intuition.

Problem C. We have considered the anomalies that arise when, by strengthening formulation S(1.1) in a very natural fashion, we assume that both (2.1) and (2.2) hold. However, if one assumes both (2.1) and (2.2), then it would be natural to assume (2.3) and (2.4) as well. Indeed, why should we give 1 the Sen rights without giving 2 the corresponding Sen rights? Suppose that we do this, and consider the implications of assuming that all the four conditions are satisfied.

Suppose (2.1), (2.2), (2.3) and (2.4) are all satisfied and the two individuals' preferences are as specified earlier. Then it is easy to check that, irrespective of the final social outcome that emerges, one of the four conditions is bound to be violated. Thus, if we accept this very plausible extension of formulation S(1.1), we would have to accept the inevitability of the violation of someone's right. This is nothing other than an instance of the so-called Gibbard paradox. However, whatever may be the final outcome, it is not at all intuitively clear why we should say that anybody's right is violated; after all, every individual is choosing completely freely.

As we emphasized earlier, the entire intuitive content of the right in our example seems to consist of each individual's freedom to choose a blue shirt or a white shirt for himself. If each individual freely chooses one of the two shirts available to him, then some social outcome will emerge from this process of free choice. No matter what shirt each individual chooses, and no matter what social outcome finally emerges from their separate choices, there would be no violation of the right under consideration.[7]

Let us now examine the origin of all these intuitive difficulties in which formulation S(1.1) and its extensions seem to be enmeshed. Concentrating on formulation S(1.1), consider problem A. The root of this problem is to be found in the following. Under our intuitive conception of the right to choose one's own shirt, the individual enjoys the power to determine a particular *aspect* or *feature* (i.e. the colour of his own shirt) of the social alternative; and when he makes his choice with respect to this particular aspect, his choice imposes restriction on the final social outcome in so far as, in the final social outcome, that particular aspect must be exactly as he chose it to be. In contrast, formulation S(1.1) does not mention the individual's ability to determine a particular aspect of the social alternative. Instead, the constraint on social choice is linked to the individual's preference over some pair(s) of social states or complete descriptions of all aspects of the society.

[7]A referee maintained that, while this notion of freedom of choice may capture our intuition about the right involved in the example at hand, it may leave out some important aspects of other types of rights. For example, he pointed out that we have ignored the rights such as the "right to conform" and the "right to be different," which may be associated with Gibbard's example.

There are two points we would like to make in this context. First, our example is intended to be a counter-example to show that our intuition about a certain right, i.e. the right to choose one's own shirt, can come into direct conflict with formulation S(1.1). Since we are interested in providing a counter-example, we have not analysed other possible rights mentioned by the referee. Second, it is not clear that the "right to conform" and the "right to be different" (as distinct from the "right to choose one's own shirt" ) can be captured in terms of (1.1), either. To see this, it is necessary to interpret the "right to be different" carefully. When someone says in a liberal society that "I should have the right to be different in my own dress", we should not interpret this statement as a claim that he should have the power to ensure that he should dress differently from the rest of the society. Rather, we would interpret it as a claim that he has no obligation *not* to be different from the rest of society in his dress, and that nobody should penalize him if he manages to be different in his dress. (See Kanger and Kanger [9] for an illuminating formal discussion of the notion of the rights involving absence of obligations as distinct from the rights involving power.) Hence we would not normally say that the right of the person concerned to be different in his dress has been violated if only he was left free to choose his dress and to try to be different, even if he ended up finally with the same dress as the rest of society. His lack of foresight may be lamented, but not his lack of the right.

Of course, if each possible choice by an individual with respect to the aspect of the social state, coming within his RPS, was linked to exactly one social state, then there would be an obvious and tight connection between such choice by the individual and his preferences over the social states or complete descriptions of the society. However, in our example no such tight connection exists. Thus, the choice of $w$ by 1 can, depending on 2's choice, lead to either $(w, w)$ or $(w, b)$. In the absence of a tight link between the social states and the alternative options that the individual can choose with respect to the aspect of social states that falls within his RPS, there arises a tension between our intuition, which runs in terms of choice of such options, and formulation S(1.1), which runs in terms of the individuals' preferences over social states.

To see this clearly, consider what would happen if we had exactly two feasible alternatives, say $(w, w)$ and $(b, w)$, differing with respect to the colour of 1's shirt; in other words, 1 had two shirts, $w$ and $b$, but 2 had only one shirt, $w$. If information about the feasible set constitutes a part of common knowledge, then 1's preference over $\{(w, w), (b, w)\}$ will have an obvious link with 1's choice from the set of two options relating to the colour of his shirt: if (and only if) he strictly prefers $(w, w)$ to $(b, w)$, he will choose $(w, w)$ and reject $(b, w)$; similarly, if (and only if) he prefers $(b, w)$ to $(w, w)$, he will choose $(b, w)$ and reject $(w, w)$. Therefore, in this simple case where $(w, w)$ and $(b, w)$ are assumed to be the only two feasible alternatives, problem A cannot possibly arise, since there is no plausible way in which it can happen that 1 will choose a colour only to regret that choice in the light of the (trivial) choice made by 2.

However, when all the four alternatives, $(w, w), (w, b), (b, w)$ and $(b, b)$, are feasible, the choice of $w$ by 1 is no longer uniquely linked to a single outcome: instead, it is associated with two possible outcomes, $(w, w)$ and $(w, b)$. Each option of each individual is, in this fashion, linked to two possible social states. This is what gives rise to problem A. Note that 1 does not know whether, if he chooses $w$, $(w, w)$ or $(w, b)$ will finally arise as the social outcome, although he does know that neither $(b, w)$ nor $(b, b)$ can arise, given his choice of $w$. Likewise, 1 does not know whether, if he chooses $b$, $(b, w)$ or $(b, b)$ will emerge as the social outcome. Therefore, it is perfectly possible that he will choose $w$ even though he prefers $(b, w)$ to $(w, w)$. Similarly, it is perfectly possible for 2 to choose $b$. Thus, it is perfectly possible that the two individuals, through their free and unconstrained choice of a shirt, will end up with the final social outcome $(w, b)$ when there exists another feasible outcome $(b, b)$ where the colour of 2's shirt is the same as in $(w, b)$ but which 1 strictly prefers to $(w, b)$. Even if this happens, we do not see any reason whatsoever to say that 1's right has been violated.

We have shown that formulation S(1.1) runs into serious problems in our example. What about the much weaker formulation S(1.2)? It seems to us that no such problem arises for (1.2), provided that a knowledge assumption (specified below) is satisfied, and provided one makes the very plausible assumption to the following effect:

> (2.5) For any individual $i$, and for any social alternatives $z$ and $w$, if $z$ and $w$ are the only available social alternatives and if $i$ strictly prefers $z$ to $w$, $i$ will choose $z$ and reject $w$ whenever he is empowered to act on behalf of the society.

To substantiate this assertion, remember that (1.2) requires that, if $(w, b)$ and $(b, b)$ are the only two feasible social alternatives, and if 1 prefers $(w, b)$ to $(b, b)$, then $(b, b)$ must not be the social outcome. However, in this case each option available to 1 is linked with a unique social outcome. Then it is clear from our earlier reasoning that, given the very plausible assumption (2.5), and given that 1 knows the feasible set to be $\{(w, b), (b, b)\}$, there cannot be any conflict between our intuition about the right and formulation S(1.2).

So far we have assumed that the information as to what constitutes the feasible set of social alternatives is available to both individuals. What happens if the feasible set is really $\{(w, b), (b, b)\}$, but 1 does not know this, and mistakenly thinks that the feasible set is $\{(w, b), (b, b), (w, w), (b, w)\}$? This, of course, means that 1 believes that 2 has two options, $w$ and $b$, when actually 2 has only one option, $b$. Then 1 may adopt the maximin strategy of choosing a white shirt. In that case, $(w, b)$ will be the social outcome, and this will violate 1's right according to formulation S(1.2). However, given that 1 was free to choose whatever shirt he wanted, we would not want to say that 1's right has been violated in any way. Thus, if the knowledge assumption is relaxed, even S(1.2) may turn out to be inconsistent with our intuition about the right.

Although the counter-example we have considered so far is a representative specimen from a class of rights that many people would consider to be very basic, the specific right discussed has some special features which are not necessarily shared by many other rights.

First, under the right discussed in our example, the individual, through his *actual* choices, *directly* controls the matters coming within his RPS. Sen [16; 17] has argued very convincingly against viewing rights exclusively in terms of such direct control. If $i$ is a Hindu, it may be thought that $i$ has a right to be cremated after his death according to Hindu rituals. However, $i$ can hardly control this aspect of the social state directly through his actual choice, even though it may be conceded that it comes within his RPS. To accommodate such instances, however, we have only to broaden the framework so that matters relating to the RPS of the individual are determined either by the free *actual choice* of the individual concerned or by the choice of some other agent acting on behalf of the individual in accordance with the individual's *hypothetical choice*, i.e. the choice that the individual would make if he were in a position to choose.

Second, the right of the individual in our example consists of his freedom to choose any one of several options. However, there are many rights that can hardly be expressed in this way. For example, the case of Mr A, who has a right not to be persecuted by the state because he is a Hindu, cannot be articulated through Mr A's freedom to make certain actual choices. Is it possible to articulate it through his hypothetical choices? The answer seems to be in the negative. Suppose, in order to achieve martyrdom, Mr A would very much like to be executed by the government for being a Hindu — indeed, if he could influence the government, he would see to it that this would be done. However, even in this case the government would violate Mr A's rights if, in deferense to Mr A's hypothetical choice, it decided to execute him.

Lastly, in our example, what actions are or are not permissible for the individual does not depend on the prior actions of other agents. (In fact, it does not depend on the prior fulfilment of any condition.) In contrast, there are many rights where what actions are permissible for the agent concerned depends crucially on the prior actions of other agents. In the next section we shall discuss rights of this type in some detail. Here we would just like to note that there are important rights which do not share this feature of our example.

Thus, the right discussed in this section has some special features which may not be shared by other rights. However, we do believe that many basic rights are structurally similar to this right. In the next section, we go beyond our counter-example with the purpose of assessing the applicability of S(1.1) in general, and argue that, in the context of most rights, formulation S(1.1) and stronger versions of it run into intuitive difficulties exactly similar to the ones we have discussed above.

# 3 Critique of Sen's Formulation: Generalization

How widespread is the difficulty that arose in our example? With the purpose of settling this question, let us try to identify systematically the type of rights that do indeed lend themselves to formulation S. Since our discussion of rights is mainly in the context of social choice theory, we shall concentrate on constitutional and legal rights.

To begin with, we note that (1.1) needs some modification. This is called for if it is to be applicable to the wide class of individual rights that are contingent on the fulfillment of certain conditions, as well as to the class of individual rights that are not contingent on any such condition.

Individual rights are indeed often contingent on the prior fulfilment of certain conditions: an individual $i$'s right to go to $j$'s party is contingent on the condition that $j$ has invited $i$ to come to $j$'s party; an individual's right to criticize the state may be contingent on "normal circumstances" prevailing and may be restricted when the country is engaged in a war; an individual's right to travel by a public bus may be subject to the availability of a vacant seat; and so on. On the other hand, some categories of rights, e.g. the right to believe or not to believe in the existence of God, may not be contingent on the prior fulfilment of any condition. The number of such non-contingent rights is perhaps rather small. To be able to take into account rights that are contingent on the prior fulfilment of a certain set of conditions, let $\Gamma = \{\gamma_1, \gamma_2, \ldots\}$ be the set of alternative contingencies under each of which a given right can be invoked. Then (1.1) should be modified as follows:

(3.1) If a certain contingency in $\Gamma$ occurs, then, for every individual $i$, there exist distinct alternatives, $x$ and $y$, such that $i$ is decisive over $\{x, y\}$.

Clearly, the non-contingent rights can be included in (3.1), where the set $\Gamma$ covers all conceivable contingencies.

To what extent is (3.1) consistent with our intuition about individual rights? In discussing this issue, it seems convenient to invoke the familiar distinction between *pas-*

*sive rights* and *active rights*.[8] A passive right $r_i$ of an individual $i$ just implies certain obligations of other agent(s) (individuals, groups, the society, the state, ...) to do or to refrain from doing something without providing $i$ with any power to do, to have, or to be anything specific. For example, the right of an individual $i$ not to be arrested without a proper warrant implies an obligation of the state not to arrest $i$ without a proper warrant; but it does not provide $i$ with any power to do, to have or to be anything specific. In contrast, an active right $r_i$ of an individual $i$ provides $i$ (or some other agent acting on behalf of $i$ under suitably specified conditions)[9] with a certain power to do, to have or to be something specific, which usually accompanies certain obligations of other agents to do or to refrain from doing something. For example, the right of an individual $i$ to criticize the state in normal circumstances is an active right. It provides $i$ with a power to do something that expresses his criticism, which is accompanied by an obligation of the state not to imprison, interfere with, persecute or execute $i$ for his doing that act. We shall argue that formulation S(3.1) is consistent neither with passive rights nor with active rights unless certain very restrictive conditions are fulfilled.

Consider passive rights first. By definition, a passive right $r_i$ of an individual $i$ simply imposes certain obligations on other agents without paying any particular attention to $i$'s preferences; neither does it provide $i$ with any power to choose anything in accordance with $i$'s preferences. It should be clear, therefore, that the category of passive rights cannot properly be captured by formulation S which postulates a link between the final social choice and the right-holder's preferences over certain pairs of social alternatives.

In order to avoid possible misunderstandings, let us examine once again the passive right against arrest without a proper warrant. It is true that this right might have been conferred originally with a certain presumption about people's "usual" preferences, which would go against arrest without a warrant. However, whatever may be the reason that lies behind the introduction of such a right in the first place, the formal structure of this right, once introduced, seems to be independent of the individuals involved. In other words, $i$'s right against arrest without a proper warrant prohibits a specific action of the state, which remains in force even if $i$ would like to be arrested without a warrant for reasons of his own. Likewise, the origin of passive rights such as the protection from being robbed, the provision of fire-fighting service in case one's house catches fire and so on may be traced back to the presumption that people usually prefer the state to provide these services. However, once these passive rights come into existence, their structure may often be independent of the preferences of specific individuals involved. In such cases, it is clear that the passive rights would not lend themselves to the preference-based formulation S(3.1).

How about active rights? To be specific, suppose that $r_i$ is an active right of an indi-

[8]See Feinberg [3; 4] for a very lucid discussion of this important distinction.

[9]Strictly speaking, whenever we talk about the power or freedom of an individual $i$ under an active right, we should refer to the power or freedom that he, *or some other agent acting on his behalf under suitably specified conditions*, enjoys. In what follows, we shall leave out the lengthy phrase 'or some other agent acting on his behalf under suitably specified conditions' for the sake of expositional convenience, but it should not be construed to imply that we subscribe to a narrow view of active rights, which visualizes these rights only in terms of direct choice made by the individual concerned.

vidual $i$, which becomes effective under any contingency in a subset $\Gamma(r_i)$ of $\Gamma$. (Needless to say, if $r_i$ is a non-contingent active right, then $\Gamma(r_i)$ covers all conceivable contingencies.) Since $r_i$ is an active right, $i$ is endowed with a power to choose (to do, to have, to be, ...) something from, say, a set $\Omega(\gamma) = \{\omega_1, \omega_2, \ldots\}$ of options when a contingency $\gamma \in \Gamma(r_i)$ obtains. Given the right $r_i$, and given $\gamma \in \Gamma(r_i)$, the choice of $\omega \in \Omega(\gamma)$ by $i$ may imply certain obligations $\sigma_q(\gamma, \omega)$ for another agent $q$ in a set $Q(\gamma, \omega)$.

It may help if we illustrate these notions in terms of an example. Consider an individual $i$'s active right $r_i$ of going or not going to $j$'s party, assuming that $j$ has invited $i$ in the first place. Here $\Gamma(r_i) = \{j$ invites $i$ to his party$\}$. Given that $j$ has invited $i$, $i$ is free to go or not to go. Thus, $\Omega(j$ invites $i$ to his party$)= \{$to go, not to go$\}$. If $j$ invites $i$ and $i$ chooses to go (resp. not to go), then every other individual $q$ has an obligation not to harass $i$ for going to $j$'s party (resp. not to harass $i$ for not going to $j$'s party). Hence, $Q(j$ invites $i$, to go) as well as $Q(j$ invites $i$, not to go) is the set of all individuals other than $i$. For all $q \in Q(j$ invites $i$, to go), $\sigma_q(j$ invites $i$, to go$) = \{$don't harass $i$ for going$\}$. Similarly, for all $q \in Q(j$ invites $i$, not to go), $\sigma_q(j$ invites $i$, not to go$) = \{$don't harass $i$ for not going$\}$.

We are now ready to argue that active rights, in general, cannot be properly articulated through formulation S(3.1).

Let $F = \{f_1, f_2, \ldots\}$ be the set of all feature indices involved in the formal description of the social alternatives. In relation to an active right $r_i$ of an individual $i$, suppose that $\gamma \in \Gamma(r_i)$ obtains, so that $r_i$ becomes effective, and $i$ knows that this has happened. This fact, in itself, will specify some features of the social alternatives. If $i$ exercises $r_i$ and chooses an option $\omega \in \Omega(\gamma)$, some other features of the social alternatives will be thereby specified. The agents $q \in Q(\gamma, \omega)$ are constrained by their obligations $\sigma_q(\gamma, \omega)$ under $r_i$. The fulfilment of these obligations will specify still further features of the social alternatives. If $i$ understands the nature of his endowed active right $r_i$, it is not unreasonable to presume that $i$ knows all these specifications of the features of the social alternatives which follow from the invocation of his active right $r_i$.

Let $F(\gamma, \omega)$ stand for the set of all feature indices which are thus specified by $\gamma \in \Gamma(r_i)$, $\omega \in \Omega(\gamma)$, and $\sigma_q(\gamma, \omega)$ ($q \in Q(\gamma, \omega)$), and known as such by $i$. The crucial question is whether or not $F(\gamma, \omega)$ exhausts all the feature alternatives.

Suppose, for some $\gamma \in \Gamma(r_i)$ and some $\omega \in \Omega(\gamma)$, that $F \setminus F(\gamma, \omega)$ is non-empty. Then, given $\gamma$, the option $\omega \in \Omega(\gamma)$ of $i$ will be associated not with one unique social outcome, but with several possible outcomes, any one of which may materialize depending on the exact fashion in which the residual features $f \in F \setminus F(\gamma, \omega)$ are determined. If these residual features matter at all for $i$'s preferences, we shall have exactly the same difficulties that we encountered in our previous example.

The general reasoning presented above may be illustrated by referring to the example of the party. To keep our reasoning transparent, let us assume that it is physically impossible for anyone to harass anyone else for going or not going to someone's party. While this assumption is not at all essential, it greatly simplifies our reasoning by enabling us to ignore the implied obligations of other agents.

Assume now that $j$ has decided to invite both $i$ and $k$, and this fact is known to both $i$ and $k$. This clearly determines a feature of the social alternatives as far as $j$'s invitation to $i$ and $k$ is concerned. Individual $i$ can now decide whether or not to go to $j$'s party. Suppose that he has to make his decision without knowing whether or not $k$ has decided to go to the party. If $i$ decides to go to $j$'s party, a further feature of the social alternatives will be determined, i.e. whether $i$ goes or does not go to $j$'s party. However, there remains a residual feature relating to $k$'s choice, and $i$ has decided whether or not to go to $j$'s party without knowing how this feature is going to be decided. Therefore, in $i$'s own mind, the option of his going to $j$'s party is associated with two possible social outcomes: ($i$ goes to $j$'s party, $k$ goes to $j$'s party) and ($i$ goes to $j$'s party, $k$ does not go to $j$'s party). Similarly for $i$'s option of not going. Given this leeway, it can now be easily shown that, irrespective of whether formulation S(3.1) gives to $i$ decisiveness over {($i$ goes to $j$'s party, $k$ goes to $j$'s party), ($i$ does not go to $j$'s party, $k$ goes to $j$'s party)} or over {($i$ goes to $j$'s party, $k$ does not go to $j$'s party), ($i$ does not go to $j$'s party, $k$ goes to $j$'s party)} or over {($i$ goes to $j$'s party, $k$ does not go to $j$'s party), ($i$ does not go to $j$'s party, $k$ does not go to $j$'s party)}, formulation S(3.1) contradicts our intuition just as in our previous example.

It should now be clear that the problem with formulation S(3.1) can be avoided only under a very restrictive condition, which reads as follows:

(3.2) For all $i$, all $\gamma \in \Gamma(r_i)$, and all $\omega \in \Omega(\gamma)$, the features in $F \setminus F(\gamma, \omega)$, if any, do not affect the relative desirability for $i$ of the options in $\Omega(\gamma)$.

It seems to us that (3.2) is a quite stringent condition which is likely to be violated in many important cases. If this is the price we have to pay in order to articulate active rights through formulation S(3.1), then it would seem necessary to search for a better alternative. Such an alternative formulation of rights will be discussed in the next section.

# 4 An Alternative Formulation of Individual Rights: Game Form Approach

In discussing individual rights, we have emphasized that every active right of $i$ implies the freedom of $i$ (or someone acting on $i$'s behalf) to choose from a certain set of options or actions; and $i$'s choice of one of these options, in its turn, implies obligations of certain other agents to do or not to do something. On the other hand, every passive right of $i$ just implies the obligation of certain other agents to do or not to do something. Thus, in general, a right of $i$ implies certain restrictions on the set of permissible actions, any one of which may be chosen by individuals including $i$ himself. This basic idea suggests a very natural formal framework for articulating rights, where the notion of game form plays the central role.[10]

Formally, a game form is a specification of:

[10]The concept of a game form is due originally to Gibbard [6]. The formulation of rights in terms of game forms originates in Gärdenfors [5] and Sugden [19].

(a) a set $N$ of $n$ players;
(b) a set $S_k$ of strategies for each player $k \in N$;
(c) a set $X$ of all feasible outcomes; and
(d) an outcome function which specifies exactly one outcome for each element of $\Pi_{k \in N} S_k$ (i.e. for each $|N|$-tuple of strategies, one strategy for each player).

Given a game form, when we specify the preferences of the players, we have a game.

The content of individual rights in this framework lies in a specification of the admissible strategies for each player $k \in N$, and the complete freedom of each player to choose any of the admissible strategies and/or the obligation of the agents not to choose a non-admissible strategy.

Let us illustrate this formulation in terms of the party example.[11] Individual $j$ has two admissible strategies:

$y_j$ : to invite $i$;
$y'_j$ : not to invite $i$;

whereas individual $i$ has two admissible strategies:

$y_i$ : if $j$ does not invite him, then he stays at home, and if $j$ invites him, then he declines and stays at home.
$y'_i$ : if $j$ does not invite him, then he stays at home, and if $j$ invites him, then he accepts and goes to the party.

Note that we have expressed the relevant game form in its normal form. Note also that $(y_i, y_j)$ leads to $j$'s inviting $i$ and $j$'s invitation being declined by $i$ who stays at home; $(y'_i, y_j)$ leads to $j$'s inviting $i$ and $j$'s invitation being accepted by $i$; and so on. It is clear that our intuitive notion of $j$'s right to invite or not to invite $i$ to his party and $i$'s right to accept or not accept $j$'s invitation is reflected in the specification of the admissible strategies for the two individuals. For example, it is not an admissible strategy for $i$ to extract an invitation at gun point if $j$ does not invite $i$ in the first instance, nor is it admissible for $i$ to go to $j$'s party without receiving an invitation from $j$ in the first place. It is, however, possible to specify the game form in such a way that the strategy,

$y''_i$: If $j$ does not invite $i$, then $i$ forces his way into the party with a gun, and if $j$ invites $i$ on his own, then $i$ goes to $j$'s party,

is a feasible strategy for $i$, but the outcome function is such that the combination $(y''_i, y_j)$ leads to $i$'s spending some time in prison. Thus, in general, the content of individual rights in this framework is reflected in the specification of the sets of admissible strategies, and the outcome function which reflects, so to speak, the 'rules of the game'.

This formulation of individual rights via a game form, game form formulation for short, accommodates most instances of what we would intuitively think of as individual

[11] Many other illustrations of this approach can be found in Suzumura [21].

rights, which cannot be so easily accommodated by formulation S. For example, readers can easily check that both the right to choose one's own shirt and the right to decide whether or not to go to the invited party, which we have shown to create so much difficulty for formulation S, can be articulated through the game form formulation without any difficulty whatsoever. In each case, an appropriate specification of the set of permissible strategies for each player can completely capture the intuitive content of the right concerned.

Several general remarks on the nature of the game form formulation seem to be in order.

First, it is obvious that the notion of a game form, by itself, has very little to do with rights. All that we are claiming is that rights are best modelled as game forms, with strategy sets being interpreted as the sets of legally or socially admissible strategies for each and every agent. Of course, one can think of numerous game forms in various other contexts (e.g. feuds between two mafia families, with murder, abduction and arson being the strategies), where the question of rights does not arise at all. It is the specific *interpretation* of the strategies and the outcome function that determines whether the game form is intended to capture some rights in the sense of game form formulation. Note that, in formulation S of individual rights as well, it is not the formal specification of decisiveness of an individual over a pair of social states as such, but the specific *interpretation* of the intuitive basis of such decisiveness, which brings rights into the picture.[12]

Second, how does the society decide which strategies should or should not be admissible for a specific player in a given context? In other words, how are rights granted in the first place in the framework of game form formulation of individual rights? This is an important question, but we ignore it here, since we are not addressing the problem of how or why rights come into existence. Our purpose is to discuss the relatively limited issue of the *formal structure* of individual rights and the implications thereof, assuming that the society, for some reason or other, has decided to grant certain rights to some agents. From the formal point of view, the important aspect is that the rights of an individual $i$ in game form formulation always take the form of $i$'s freedom to choose a strategy from the set of all *admissible* strategies for him, and/or a specification of the admissible strategies for some other agent(s). Depending on the situation, the intuitive interpretation of such specification can be in terms of either a prohibition of certain strategies for these other agents, or their obligation to adopt certain types of strategies.

Lastly, it is worth noting that our game form formulation is in terms of normal game forms, and to that extent it suffers from the usual drawbacks of normal game forms. Thus, it is well known that the intuition of certain extensive games cannot be adequately captured in terms of games in normal form. By relying on the normal game form, our

[12] Indeed, Sen [14, p.89] was careful enough to point out that the acceptability of formulation S will "depend on the nature of the alternatives that are offered for choice", and "if the choices are all non-personal, e.g. whether or not to outlaw untouchability, to declare war against another country", formulation S "should not have much appeal". It is only when two social alternatives are interpreted as differing solely with respect to someone's personal life that formulation S was intended to apply.

formulation also suffers from this problem.[13] However, despite this, it would seem to provide a far more flexible and adequate framework for analysing a wide range of rights than is provided by the classical formulation in Sen's terms.

# 5 Historical Background

In this chapter, we have concentrated on the problem of formulating, formally, the notion of individual rights, as distinct from the problems of compatibility of individual rights and other social values and possible ethical justifications for individual rights. No attempt will be made here to summarize our arguments. Instead, we shall give a brief historical account of the ways in which different writers have tried to capture the intuition about individual rights in an analytical framework.

Sen [13; 14] was, of course, the first writer to introduce the concept of individual rights into the formal theory of social choice and welfare economics, and to analyse the implications of such rights. Much of the literature on rights in social choice theory and welfare economics over the last two decades has evolved around Sen's formulation which we have discussed in detail in the preceding sections.

One of the earliest critiques of Sen's formulation of individual rights came from Nozick [11]. According to him,"[i]ndividual rights are co-possible; each person may exercise his rights as he chooses. *The exercise of these rights fixes some features of the world*. Within the constraints of these fixed features, a choice can be made by a social choice mechanism based on a social ordering, if there are any choices left to make!" (Nozick [11, p.166]: emphasis added). These lines have several nuances. However, what seems to be particularly important in the context of our analysis is that Nozick, unlike Sen, does not visualize individual rights in terms of restrictions on social choice which are linked to the individual's preference over pairs of social alternatives or complete descriptions of the society. Instead, Nozick visualizes the right of an individual in terms of the individual's freedom to choose from among several available options relating to some specific aspect of the social states, and the constraints on social choice are imposed when the individual, exercising his right, does choose one of the options. (Recall Nozick's [11, p.166] well-known example of his choosing to live in Massachusetts or New York.) Note that, under Nozick's conception, the individual's act of choice from among the alternative options fixes only some features of the social states; and this, rather than the individual's preferences over some pairs of social states, imposes the constraint on social choice. The similarity of all this to our discussion of the individual's right to choose his own shirt is obvious and does not need any further elaboration. It is of interest to note that Bernholz [1] seems to have taken a similar view of the formal structure of individual rights.

The explicit formulation of rights in terms of game forms was given in an important paper by Gärdenfors [5]. However, Gärdenfors chose to provide the formal representation of rights in terms of "effectivity functions" rather than in terms of game forms.[14] As far

[13]For a discussion of some of these issues, see Deb [2].

[14]For the notion of effectivity functions, see Peleg [12]. See Deb [2] for an illuminating discussion of various issues relating to the representation of rights in terms of effectivity functions.

as we are aware, the explicit use of normal game forms for representing rights was first suggested by Sugden [19].

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# Chapter 16
# Welfare, Rights, and Social Choice Procedure: A Perspective*

## 1 Introduction

It is slightly ironic that the *Bergson-Samuelson social welfare function* and the *Arrow social welfare function*, which have so much to contrast with each other in many important respects, have a basic feature in common.[1]. Despite the fact that it is the Arrow impossibility theorem and nothing else that poses a devastating criticism against the possibility of the *democratic* Bergson-Samuelson social welfare function, both concepts hinge on the informational basis which is *welfaristic* in nature.[2]

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[1]At the risk of reminding readers of what is obvious to them, note that the Bergson-Samuelson social welfare function is "a function of all the economic magnitudes of a system which is supposed to characterize some ethical belief .... Any possible opinion is admissible .... We only require that the belief be such as to admit of an unequivocal answer as to whether one configuration of the economic system is 'better' or 'worse' than any other or 'indifferent', and that these relationships are transitive .... The function need only be ordinally defined .... A more extreme assumption ... states that individuals' preferences are to 'count'. If any movement leaves an individual on the same indifference curve, then the social welfare function is unchanged, and similarly for an increase or decrease" (Samuelson [27, pp.221-228]. In contrast, "*[b]y [the Arrow] social welfare function will be meant a process or rule which*, *for each set of individual orderings* $R_1, \ldots, R_n$ *for alternative social states (one ordering for each individual)*, *states a corresponding social ordering of alternative social states*, $R$. ... [The Arrow] social welfare function ... is a method of choosing which social welfare function of the Bergson type will be applicable .... Since we are trying to describe social welfare and not some sort of illfare, we must assume that the social welfare function is such that the social ordering responds positively to alterations in individual values, or at least not negatively. Hence, if one alternative social state rises or remains still in the ordering of every individual without any other change in those orderings, we expect that it rises, or at least does not fall, in the social ordering" (Arrow [1, pp.23-25]).

[2]According to Sen [33, p.464], "[w]elfarism implies that any two states of affairs that are identical in terms of individual utility characteristics must be judged to be equally good no matter how different they are in nonutility respects, and also that any state that has more utility for someone and no less utility for anyone in comparison with another state is a better state than the other." The latter property, which is called the *Pareto Principle*, is also shared by the Bergson-Samuelson social welfare function and the Arrow social welfare function.

In the case of the Bergson-Samuelson social welfare function, this fact is quite explicit. For each profile $\boldsymbol{u} = (u_1, u_2, \ldots, u_n)$ of ordinal individual utilities, where $u_i$ $(i = 1, 2, \ldots, n)$ denotes person $i$'s ordinal utility and $n$ denotes the number of persons in the society, the Bergson-Samuelson social welfare function $f$ maps $\boldsymbol{u}$ into an ordinal index of social welfare: $\boldsymbol{u} = f(\boldsymbol{u})$.[3] Thus, social welfare judgements in accordance with the Bergson-Samuelson social welfare function depend on the information of individual utilities and nothing else. In the case of the Arrow social welfare function $F$, which maps each profile $\boldsymbol{R} = (R_1, R_2, \ldots, R_n)$ of individual preference orderings over the set $X$ of all conceivable social states, where $R_i$ $(i = 1, 2, \ldots, n)$ denotes person $i$'s individual preference ordering, into a social preference ordering $R = F(\boldsymbol{R})$, this fact is less conspicuous.[4] However, it is known that the Arrow social welfare function satisfies the following property with strong welfaristic flavour:[5]

> *Strong Neutrality:* For any pairs $\{x, y\}$ and $\{a, b\}$ of social states and for any profiles $\boldsymbol{R^1} = (R_1^1, R_2^1, \ldots, R_n^1)$ and $\boldsymbol{R^2} = (R_1^2, R_2^2, \ldots, R_n^2)$ of individual preference orderings, if $xR_i^1y$ holds if and only if $aR_i^2b$ holds for all $i = 1, 2, \ldots, n,$ then $xR^1y$ holds if and only if $aR^2b$ holds, where $R^1 = F(\boldsymbol{R^1})$ and $R^2 = F(\boldsymbol{R^2})$.

Thus, social welfare judgements in accordance with the Arrow social welfare function depend on the information of relative positions of social states in the individual preference orderings and all other characteristic features of social states are deemed completely irrelevant. It is against this common welfaristic feature that underlies traditional welfare economics and social choice theory that Sen's "Impossibility of a Paretian Liberal" was meant to cast a serious doubt. Whatever else we may say for or against Sen's impossibility theorem, it is in this arena that the value of his contribution should be tested in the final analysis. *Hic Rhodes, hic salta*.

The structure of this chapter is as follows. In Section 2, Sen's original formulation of the concept of individual liberty and his impossibility theorem are briefly recapitulated. Section 3 examines Sen's formulation of individual liberty in the light of several criticisms raised against it. In Section 4, we identify three crucial problems that should be squarely examined by any theoretical approach to the concept of individual liberty. Section 5 is devoted to the evaluation of Sen's criticism against the welfaristic foundation of normative economics. Section 6 concludes.

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[3] See Samuelson [27, p. 228].

[4] An *ordering* $R$ on a set $X$ is a binary relation defined over $X$ satisfying: (a) [*Completeness*]: For any $x, y$ in $X$, either $xRy$ or $yRz$ holds; and (b) [*Transitivity*]: For any $x, y$ and $z$ in $X$, $xRy$ and $yRz$ imply $xRz$. *A preference ordering* $R$ on $X$ is defined to mean that $xRy$ holds if and only $x$ is at least as preferable as $y$. When $x$ is strictly preferred to $y$, viz. when $xRy$ holds but $yRx$ does not hold, we write $xP(R)y$.

[5] See Sen [31; 32] for the formal proof of this important fact.

# 2 Sen's Concept of Individual Liberty and the Impossibility of a Paretian Liberal

Sen's concept of individual liberty is phrased in the context of social choice framework which is slightly more general than that of Arrow [1]. Let $X$ and $N = \{1, 2, \ldots, n\}$, where $n$ is a finite integer which is no less than 2, be the set of all conceivable social states and the set of all persons in the society, respectively. $\Sigma$ denotes a family of non-empty subsets of $X$. Each element of $\Sigma$ is meant to denote an *opportunity set*, which the society faces under suitably specified conditions. It is assumed that there exists no restriction on how the individual evaluates social states from his/her idiosyncratic point of view. Thus, each and every person can have whatever preference ordering over $X$ he/she cares to express. Given any profile $\boldsymbol{R} = (R_1, R_2, \ldots, R_n)$ of individual preference orderings, and given any opportunity set $S$ in $\Sigma$, the society must choose something from $S$, paying proper attention to the distribution of persons' wishes which is summarized by $\boldsymbol{R}$. Let $C(S, \boldsymbol{R})$ be the non-empty subset of $S$ consisting of all social states which the society chooses from $S$ when $\boldsymbol{R}$ summarizes peoples' wishes. $C(S, \boldsymbol{R})$ is to be called the *social choice set* for $(S, \boldsymbol{R})$. A function $C$ which is defined on the Cartesian product of $\Sigma$ and $\Omega$, where $\Omega$ stands for the set of all logically conceivable profiles, and maps each $(S, \boldsymbol{R})$ into $C(S, \boldsymbol{R})$ will be called the *collective choice rule*.

Let us say that a group $D$ of persons is *decisive* over a pair $\{x, y\}$ of social states if and only if $D$ can secure that $y$ (resp. $x$) does not belong to $C(S, \boldsymbol{R})$ as long as $x$ (resp. $y$) is available in $S$ by expressing unanimous preference within $D$ for $x$ (resp. $y$) against $y$ (resp. $x$). If it so happens that a singleton set $\{i\}$ is decisive over $\{x, y\}$ for some person $i$ in $N$, we say that the person $i$ is decisive over $\{x, y\}$. We are now ready to state the following:[6]

> *Sen's Minimal Liberty:* There are at least two persons such that for each of them there is at least one pair of social states over which he/she is decisive.

The intended meaning of this condition is illustrated by Sen as follows: "Given other things in the society, if you prefer to have pink walls rather than white, then [the] society should permit you to have this, even if a majority of the community would like to see your walls white. Similarly, whether you should sleep on your back or on your belly is a matter in which the society should permit you absolute freedom, even if a majority of the community is nosey enough to feel that you must sleep on your back (Sen [29, p. 152])."

Note that, to be concordant with this intuitive justification, the pair of social states which are mentioned in Sen's condition should be such that they differ only in the mentioned person's personal matter.

[6] There are many versions of Sen's condition of minimal liberty, depending on how we specify the social choice framework as well as on how we define *social preference*. See, among others, Sen [30; 33; 35; 36], and Pattanaik [25; 26]. The version used in the text is taken from Sen [29, p. 156, footnote 4]. Whichever version we may pick from among the many alternatives, the following points basically hold *mutatis mutandis*.

To make this crucial point explicit, let $X_0$ denote the set of all *impersonal* features of the society, and let $X_i$, where $i$ is any element in $N$, denote the set of all *personal* features of person $i$. Then, $X$ is the Cartesian product of $X_0, X_1, \ldots, X_n$, and each and every social state $x$ is represented by an $(n+1)$-tuple of feature alternatives: $x = (x_0, x_1, \ldots, x_n)$, where $x_0$ is taken from $X_0$ and $x_i$ for each $i$ in $N$ is taken from $X_i$. For convenience, let $x_{-i}$ for each $i$ in $N$ be defined by $x_{-i} = (x_0, x_1, \ldots, x_{i-1}, x_{i+1} \ldots, x_n)$ and let $x$ be denoted alternatively as $x = (x_i; x_{-i})$. Let $j$ and $k$ be two persons mentioned in Sen's condition, and let $\{x^i, y^i\}$ $(i = j, k)$ be the pair of social states over which person $i$ is decisive. In order for Sen's formulation to be consistent with his intuitive concept of individual liberty, we must have $x^i_{-i} = y^i_{-i}$ for $i = j, k$, so that $x^i$ and $y^i$ differ only in person $i$'s personal feature.

Turning to the other requirement on collective choice rule, let us now introduce a widely known condition which is welfaristic in nature:[7]

> *Pareto Principle:* If every person in the society prefers any social state $x$ to another social state $y$, then $y$ should never be socially chosen from any opportunity set $S$ which contains $x$.

Since the Pareto Principle has seldom been seriously challenged as a reasonable requirement on social welfare judgements, there is no wonder that Sen's impossibility theorem to the effect that *there exists no collective choice rule satisfying Sen's minimal liberty as well as the Pareto principle* caused a stir. As Sen [29, p.157] put it in his first paper on the *impossibility of a Paretian liberal*, "the moral [of this impossibility theorem] is that in a very basic sense liberal values conflict with the Pareto principle. ... [I]f someone does have certain liberal values, then he may have to eschew his adherence to Pareto optimality." A truly devastating criticism against the welfaristic basis of normative economics indeed.

Before proceeding to the critical examination of Sen's condition of minimal liberty, it may be worth examining Gibbard's [14] extension of this condition. The gist of his extension is that if a person, say $i$, is warranted by the society's collective choice rule to be decisive over $\{x^i, y^i\}$, where $x_i = (x^i_i; x^i_{-i})$ and $y^i = (y^i_i; x^i_{-i})$, it does not make much intuitive sense to deny $i$'s decisiveness over $\{z^i, w^i\}$, where $z^i = (x^i_i; z_{-i})$ and $w^i = (y^i_i; z_{-i})$. After all, if Ian is empowered to paint his bedroom walls pink rather than white when all other persons paint theirs yellow, why should we not empower him to use pink rather than white when all other persons are using blue instead? Likewise, why should we not empower John, Kevin and Liz to choose the colour of their bedroom walls freely when we empower Ian in this way? Presumably, it was these considerations that led Gibbard [14] to formulate the following natural extension of Sen's condition:

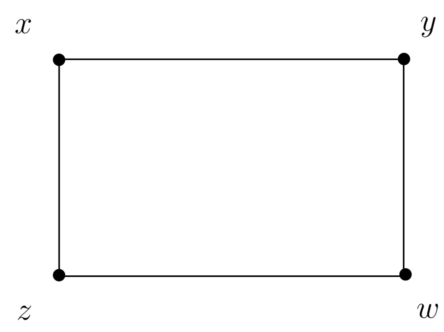
> *Gibbard's Libertarianism*: Each person $i$ in $N$ is decisive over the pair of social states $\{x^i, y^i\}$, where $x^i = (x_i; x_{-i})$ and $y^i = (y_i; x_{-i})$, whatever may be the specification of $x_i, y_i$, and $x_{-i}$.

[7] This version of the Pareto Principle is also taken from Sen [29, p.156].

Despite the common intuitive root of Sen's minimal liberty and Gibbard's libertarianism, the logical consequence of Gibbard's libertarianism is even more disturbing than Sen's impossibility of a Paretian liberal. Indeed, it is shown by Gibbard [14] that *there exists no collective choice rule satisfying Gibbard's libertarianism*. The gist of this result can be illustrated by the situation where $N = \{1, 2\}$, $X_0 = \{x_0\}$ and $X_i = \{a, b\}$ for $i = 1, 2$. Let $x = (x_0, a, a)$, $y = (x_0, a, b)$, $z = (x_0, b, a)$ and $w = (x_0, b, b)$. Suppose that the two persons have the following preference orderings: $xP(R_1)z$, $zP(R_1)w$, and $wP(R_1)y$ for person 1, and $zP(R_2)w$, $wP(R_2)y$ and $yP(R_2)x$ for person 2. Given this profile $\boldsymbol{R} = (R_1, R_2)$ and an opportunity set $S = \{x, y, z, w\}$, Gibbard's libertarianism dictates that the social choice set $C(S, \boldsymbol{R})$ cannot but be empty, given the decisiveness of person 1 (resp. person 2) over $\{x, z\}$ and $\{y, w\}$ (resp. $\{x, y\}$ and $\{z, w\}$). Whichever state in $S$ the society chooses, it cannot but violate either person 1's or person 2's decisiveness. The moral is that Sen's concept of liberty in the form generalized by Gibbard generates a system of individual claim rights to collective choice rule, which is self-contradictory.

The Gibbard impossibility theorem leads us to an interesting further question. Under what conditions can we assure the existence of a collective choice rule which materializes a system of individual claim rights generated by Sen's requirement of individual liberty? A complete answer to this question may be found in Suzumura ([42]; [43]; [46, Chpater 7]), but the essence of the answer is simple, which can be intuitively illustrated in terms of Figure 1. Note that person 1 is decisive over $\{x, z\}$ and $\{y, w\}$ and person 2 is decisive over $\{x, y\}$ and $\{z, w\}$. Thus, we can start from any state in $S$, say $x$, and follow a path along the edges of the rectangle in Figure 1, say from $x$ to $y$ to $w$ to $z$, and come back to $x$ again. Along this loop, each and every edge consists of a pair of social states over which either person 1 or person 2 has decisiveness. It is the existence of such a critical loop that underlies the Gibbard impossibility theorem. Excluding the occurence of such a critical loop is necessary and sufficient for the existence of a collective choice rule which materializes a system of individual claim rights generated by Sen's requirement of individual liberty.

*Figure 1: Gibbard's Impossibility Theorem*




*Note:* person 1 has decisiveness over $\{x, z\}$ and $\{y, w\}$, while person 2 has decisiveness over $\{z, w\}$ and $\{x, y\}$.

# 3 Sen's Formulation of Individual Liberty: Critical Examination

Given the basic nature of Sen's criticism, it is all too natural that the impossibility of a Paretian liberal has been under careful scrutiny along several lines.[8] Given our present purpose, we have only to focus on the way in which Sen crystallized his intuition on individual liberty in terms of the analytical framework of social choice theory.

The first misgivings, which are frequently expressed in the literature, criticize Sen's formulation of individual liberty as having failed to consider "a strong libertarian tradition of free contract", according to which "a person's rights are for his to use or to bargain away as he finds fit (Gibbard [14, p.397])." This viewpoint was most conspicuously formulated by Harel and Nitzan [19]. lt is through the careful examination of their proposal that we can pinpoint the crucial problem underlying this escape route from Sen's impossibility theorem.[9]

The gist of the Harel-Nitzan proposal can be crystallized in terms of a simple example due to Sen [29]. There is a single copy of *Lady Chatterley's Lover*. Everything else being the same, there are three social states: Mr. $P$ (the prude) reading it ($r_P$), Mr. $L$ (the lascivious) reading it ($r_L$), and no one reading it ($r_0$). Mr. $P$ ranks them in the descending order of $r_0, r_P, r_L$, whereas Mr. $L$ ranks them in the descending order of $r_P, r_L, r_0$. Since to read a book or not is ordinarily construed as a person's private matter and no other person's business, Sen endows Mr. $P$ (resp. Mr. $L$) with decisiveness over $\{r_P, r_0\}$ (resp. $\{r_L, r_0\}$).[10] Given this system of claim rights based on the decisiveness of persons, and given the profile $\boldsymbol{R} = (R_P, R_L)$ of individual preference orderings we have specified, the social choice set $C(\{r_P, r_L, r_0\}, \boldsymbol{R})$ cannot but be empty, vindicating Sen's impossibility theorem. In this situation, Harel and Nitzan call our attention to the fact that Mr. $P$ (resp. Mr. $L$) has *ordinally stronger preference* for $r_0$ against $r_L$ than that for $r_0$ against $r_P$ (resp. for $r_P$ against $r_0$ than that for $r_L$ against $r_0$).[11] Thus, so the Harel-Nitzan argument goes, Mr. $P$ has incentive to exchange his claim right based on his decisiveness over $\{r_0, r_P\}$ with the claim right of Mr. $L$ based on his decisiveness over $\{r_L, r_0\}$. Mr. $L$ is similarly motivated. If this mutually beneficial exchange of claim rights are in fact realized between Mr. $P$ and Mr. $L$, bringing Mr. $P$ (resp. Mr. $L$) to be decisive over $\{r_L, r_0\}$ (resp. $\{r_0, r_P\}$), then the impossibility result identified by Sen evaporates. Indeed, the social choice after the realization of voluntary exchange of claim rights will be $\{r_P\}$.

[8] For surveys of some of these works, see Sen [30; 35; 36], Suzumura [46, Chapter 7], and Wriglesworth [50].

[9] The following analysis is based on Suzumura [48]. See also Breyer [6].

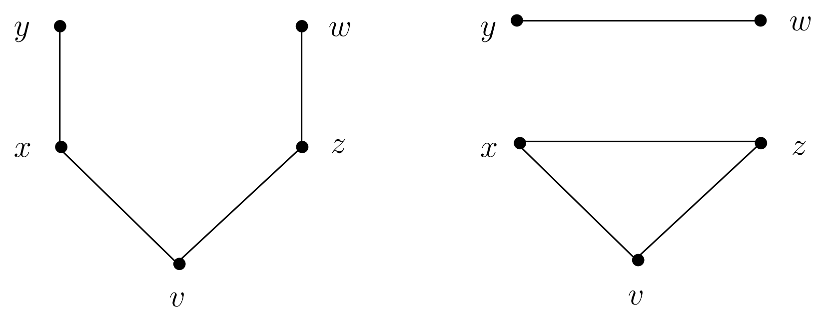
[10] Note that there exists no critical loop in the distribution of decisiveness in this example, so that the Gibbard impossibility theorem does not have any bite in this context.

[11] When a preference ordering $R$ is such that $xP(R)y$, $yP(R)z$ and $xP(R)z$ hold, we say that the preference for $x$ against $z$ is ordinally stronger than that for $x$ against $y$. Likewise, the preference for $x$ against $z$ is ordinally stronger than that for $y$ against $z$. It was Blau [5] who introduced this concept into social choice theory, but the origin of the concept goes back at least as far as to Luce and Raiffa [21].

Note, however, that this "resolution" of the impossibility of a Paretian liberal has very little to commend itself to a person with liberal belief in the ordinary sense of the word. Indeed, to enable Mr. $A$ to choose whether Mr. $B$ should or should not read a book, not in view of Mr. $B$'s own preferences but in view of Mr. $A$'s preferences, is not liberalism but paternalism, and a liberal may well regard paternalism as the worst form of despotism imaginable.

The problem with the Harel-Nitzan scheme does not end there. Consider the situation where $N = \{1, 2\}$, $X = \{v, w, x, y, z\}$ and person 1 (resp. person 2) is decisive over $\{x, y\}$ and $\{v, z\}$ (resp. $\{v, x\}$ and $\{w, z\}$). As is clear from the LHS of Figure 2, there is no critical loop in the system of claim rights generated by this distribution of decisive power. Suppose that the profile of individual preference orderings are as follows: $xP(R_1)v$, $vP(R_1)w$, $wP(R_1)y$, $yP(R_1)z$, $vP(R_2)y$, $yP(R_2)z$, $zP(R_2)x$, and $xP(R_2)w$. It is clear that person 1 has ordinally stronger preference for $x$ against $z$ than that for $x$ against $y$. Likewise, person 2 has ordinally stronger preference for $y$ against $w$ than that for $z$ against $w$. In the situation like this, Harel and Nitzan allow the two persons to realize the mutually beneficial exchange of *social states* $y$ and $z$ to *create* a new pair $\{x, z\}$ for 1 and $\{w, y\}$ for 2, over which they are decisive. This is obviously bizarre. To exchange a pair of social states, over which a person has a claim right, with another pair of social states, over which the exchange partner has a claim right, has a clear meaning, but to exchange social states between persons so as to concoct new decisive pairs of social states does not make any sense at all. Worse still, if this bizarre exchange is somehow enforced, the resulting assignment of claim rights has a critical loop, even though such a loop did not exist before the exchange. See the RHS of Figure 2.

*Figure 2: Logical Difficulty of Harel-Nitzan Libertarian Right*




*Note:* In LHS, 1 (resp. 2) has decisiveness over $\{x, y\}$ and $\{v, z\}$ (resp. $\{v, x\}$ and $\{w, z\}$), while in RHS, 1 (resp. 2) has decisiveness over $\{x, z\}$ and $\{v, z\}$ (resp. $\{w, y\}$ and $\{v, x\}$).

We cannot but conclude that the criticism against Sen's formulation of individual liberty along this line has serious problems of its own, and does not succeed in presenting a meaningful alternative concept of individual liberty, let alone a 'resolution' of the impossibility of a Paretian liberal.

There is another string of critics who also find Sen's articulation of individual liberty in terms of decisiveness rather at odds with what an ordinary liberal would claim. To bring his point home, recollect Sen's motivation for his minimal liberty condition to the following effect: "If you prefer to have pink walls rather than white, then [the] society should permit you to have this" and also that "whether you should sleep on your back or on your belly is a matter in which the society should permit you absolute freedom". Not many people with liberal belief would have anything to say against Sen's intuitive motivation. However, the actual formulation of this intuition in terms of the relevant person's decisiveness in social choice may make such a person raise an eyebrow. He/she may well ask: Why don't we simply leave the matter of choosing the colour of one's bedroom walls, or choosing one's sleeping posture, to the relevant person's warranted individual choice, rather than articulating such a right through his/her decisiveness in *social choice*?

It was in this vein that Nozick made the following famous remark on Sen's impossibility of a Paretian liberal:

> "A more appropriate view of individual rights is as follows. Individual rights are co-possible; each person may exercise his rights as he chooses. The exercise of these rights fixes some features of the world. ... If l have a right to choose to live in New York or in Massachusetts, and I choose Massachusetts, then alternatives involving my living in New York are not appropriate objects to be entered in a social ordering (Nozick [22 p.166])."

Capitalizing on, and generalizing Nozick's observation, Sugden [38; 39; 40] and Gaertner, Pattanaik and Suzumura [12] have developed an alternative approach to individual libertarian rights, which came to be known as the *game form approach* to individual rights.[12] This approach articulates individual libertarian rights as (i) the complete freedom of each player to choose any admissible strategy, and (ii) the obligation of each player not to choose an inadmissible strategy for himself/herself, and not to prevent anyone from choosing an admissible strategy.

In the case of Nozick's counterargument against Sen, for example, the game form that captures Nozick's right to choose to live in New York or Massachusetts can be formulated neatly as follows: Nozick's set of admissible strategies, say $S_{Nozick}$, should contain "to live in New York" and "to live in Massachusetts", and the set of admissible strategies for all other persons should not contain such strategies as "to harass Nozick if he chooses to live in Massachusetts", "to force Nozick to live in New York at gunpoint", and so forth,

[12] A *game form* is a specification of a set $N$ of *players*, a set $S_i$ of *admissible strategies* for each player $i$ in $N$, a set $A$ of *feasible outcomes*, and an *outcome function* $g$ which maps each strategy profile $\boldsymbol{s} = (s_1, s_2, \ldots, s_n)$, where $s_i$ is in $S_i$ for each $i$ in $N$, into a social outcome $g(\boldsymbol{s})$ in $A$. Given a game form $G = (N, \{S_i\}, g)$, if a profile $\boldsymbol{R} = (R_1, R_2, \ldots, R_n)$ of preference orderings of the players is specified, we have a *game* $(G, \boldsymbol{R})$. Gärdenfors [13] developed a related but distinct game theoretic approach to individual liberty. See also Bernholz [4], Deb [7], Gibbard [15], Hammond [17; 18], Pattanaik [23; 24], Sen [34; 36], and Suzumura [47] for more detailed account of the alternative approaches to individual liberty. The following exposition of the game form approach is based on Pattanaik and Suzumura [25; 26].

and the outcome function $g$ should be such that $g(\boldsymbol{s})$ is a social state in which Nozick lives in Massachusetts (resp. he lives in New York) if $s_{Nozick}$ (viz. Nozick's component of $\boldsymbol{s}$) is "to live in Massachusetts" (resp. "to live in New York").[13]

Two remarks are in order at this juncture. First, unlike the first alternative approach based on the voluntary exchange of libertarian rights, which not only accused Sen's approach of being out of line with traditional liberal values, but also asserted that the impossibility of a Paretian liberal could be resolved by appropriately reformulating what a liberal should claim, the game form approach does not claim to be a resolvent of Sen's impossibility theorem. Quite to the contrary, it was conjectured that the Sen impossibility problem "persists under virtually every plausible concept of individual right... (Gaertner, Pattanaik, and Suzumura [12, p. 161])." We will have more to say on this point in the next section.

Second, the game form articulation of individual libertarian rights based on the intuitive concept of freedom of choice is not just an alternative approach to Sen's classical articulation of individual liberty. It is also meant to cast serious doubt on Sen's approach. To bring this point home, let us examine a modified version of the *Lady Chatterley's Lover* case. Suppose that both Mr. $P$ and Mr. $L$ own a copy of this book. Everything else remaining the same, there exist four social states: $(r, r), (r, n), (n, r)$ and $(n, n)$, where $r$ (resp. $n$) stands for "to read it" (resp. "not to read it"). Suppose further that their preference orderings over $\{(r, r), (r, n), (n, r), (n, n)\}$ are described as follows:[14]

$R_P : (n, n), (r, r), (n, r), (r, n)$
$R_L : (n, r), (r, n), (r, r), (n, n)$.

Following the game form approach and the intuitive concept of freedom of choice, let us entrust each and every person to choose either to read this book or not to read it in accordance with his individual preference. However, this is not a straightforward problem of preference optimization. The effect on a person of his choice from the set of options $\{r, n\}$ hinges squarely on what the other person chooses from the same set of options, and no one is within his right to know the other's choice beforehand. In this sense, the problem of choice faced by Mr. $P$ and Mr. $L$ is that of choice under uncertainty. If they follow the maximin principle of choice under uncertainty, the maximin choice of Mr. $P$ (resp. Mr. $L$) is $n$ (resp. $r$), thereby generating a social state $(n, r)$ through unhindered exercise of their respective freedom of choice. However, since $(r, r)$ and $(n, r)$ differ only in Mr. $P$'s reading or not reading this book, and Mr. $P$ prefers $(r, r)$ to $(n, r)$, the realization of $(n, r)$ cannot but be regarded that Mr. $P$'s liberty is violated if we subscribe to Sen's articulation of individual liberty, even though nobody's freedom of choice is violated in this case.

[13]Lest we should be misunderstood that the game form approach hinges on the supposition that each and every person is empowered to control some aspects of socia1 states *directly*, let us emphasize that no such unwarranted restriction is needed for the workability of this approach. Those who are interested are referred to Gaertner, Pattanaik and Suzumura [12], Pattanaik [24], and Suzumura [47], where many examples are worked out in order to illustrate and substantiate this claim.

[14]Preference orderings are represented horizontally, with the less preferred alternative to the right of the more preferred alternative.

We have thus shown that the game form articulation of individual liberty is a viable alternative to Sen's formulation, and it poses a serious doubt on the compatibility of Sen's approach with our intuition about freedom of choice. In the next section, we will identify three crucial problems in the theory of individual rights within the conceptual framework of game form approach.

# 4 Articulation, Realization, and Initial Conferment of Rights

In discussing individual libertarian rights, three distinct issues should be addressed. The first issue is the *formal structure* of rights. The second issue is the *realization* of conferred rights. The third issue is the *initial conferment* of rights. In the previous section, we have contrasted Sen's articulation of the formal structure of individual libertarian rights and the game form articulation of rights. In Sen's approach, the issue of the realization of conferred rights boils down to the existence of a collective choice rule which realizes the conferred individual decisiveness in social choice, whereas Sen never addressed himself to the issue of the initial conferment of rights.[15] The rest of this section is devoted to explaining how the game form approach treats the second and third issues.[16]

The issue of the realization of conferred rights is treated by the game form approach as follows. Let $A$ be the set of feasible social states. Given a game form $G_A = (N, \{S_i\}, g_A)$ which articulates the conferred individual rights when $A$ prevails, and given a profile $\boldsymbol{R} = (R_1, R_2, \ldots, R_n)$ of individual preference orderings over the set of social states, we have a game $(G_A, \boldsymbol{R})$. Let $T(G_A, \boldsymbol{R})$ be the set of all social states which the society predicts to appear when the game $(G_A, \boldsymbol{R})$ is played.[17] It is clear that the conferred rights $G_A$ will be realized through the play of the game $(G_A, \boldsymbol{R})$ and a social state in $T(G_A, \boldsymbol{R})$ will materialize as a result of the play of this game.

The issue of the initial conferment of rights requires us to expand our conceptual framework rather substantially. To ask and answer how and why a game form representing individual rights come to be conferred in the first place, it is not enough that we are informed of the individual preference orderings over the set of social outcomes. To bring this point home, consider the following simple problem.

A father is to divide a cake among three children fairly. Method I is that the father divides this cake into three equal pieces, and tells them to take a piece each or leave it.

[15] Since Sen's interest was focussed on the basic conflict between non-welfaristic claim of rights and welfaristic claim of Pareto optimality, it was unnecessary for him to provide a full characterization of rights, neither was it necessary for him to develop a theory of the initial conferment of rights.

[16] The following analysis is essentially based on Pattanaik and Suzumura [25; 26]. Those who are interested in some technical details are referred to these original sources.

[17] If the prevailing concept of equilibrium is given by $E$ and the set $B_E(G_A, \boldsymbol{R})$ of pure strategy equilibria is non-empty, then it is natural to assume that: $T(G_A, \boldsymbol{R}) = \{x^* \in A | x^* = g_A(\boldsymbol{s})$ for some $\boldsymbol{s} \in B_E(G_A, \boldsymbol{R})\}$. The case where there exists no pure strategy equilibrium, but a mixed strategy equilibrium does exist, and the case where there exists no equilibrium are discussed in Pattanaik and Suzumura [26].

Method II is that the children are given the opportunity to discuss how this enticing cake should be divided fairly among them, and cut it into three pieces in accordance with the conclusion they arrive at. If they happen to conclude that equal division should be the outcome, and if we are informed only of the outcomes, we cannot but conclude that these two methods are the same. It is clear, however, that this is certainly inappropriate. In the case of method I, three children are not provided with any right to participate in the process through which their dividend is determined, whereas in the case of method II, they are endowed with such a right. To capture this important difference, we must enlarge the description of social state in such a way that, not only the social *outcomes*, but also the *process* or *mechanism* through which such outcomes are brought about, are included.[18]

This conceptual expansion can be attained as follows. Let $x$ and $y$ be two (conventionally defined) social states, and let $\theta$ and $\eta$ be two decision-making mechanisms. The ordered pair $(x, \theta)$ [resp. $(y, \eta)$] denotes an extended social state in which the outcome $x$ (resp. $y$) is entailed through the decision-making mechanism $\theta$ (resp. $\eta$). It is assumed that people are prepared to make judgements of the following type: It is better to obtain an outcome $x$ through a mechanism $\theta$ than to obtain an outcome $y$ through a mechanism $\eta$. In what follows, we focus on the situation where the decision-making mechanism is specified by the *rights-system* $\boldsymbol{G}$ which specifies a game form $G_A$ for each set of feasible outcomes $A$. Let $\boldsymbol{Q} = (Q_1, Q_2, \ldots, Q_n)$ be the profile of extended individual preference orderings over the pairs $(x, \boldsymbol{G}^1), (y, \boldsymbol{G}^2)$, etc. Note in passing that, for any fixed rights-system $\boldsymbol{G}$, the profile $\boldsymbol{Q}$ induces a profile $\boldsymbol{Q_G} = (Q_{1\boldsymbol{G}}, Q_{2\boldsymbol{G}}, \ldots, Q_{n\boldsymbol{G}})$ over the set of conventionally defined social states by $xQ_{i\boldsymbol{G}}y$ if and only if $(x, \boldsymbol{G})Q_i(y, \boldsymbol{G})$ for all $x, y$ in $X$ and all $i$ in $N$.

Suppose that a feasible set of outcomes $A$, rights-system $\boldsymbol{G}$, and a profile of extended individual preference orderings $\boldsymbol{Q}$ are given. We then obtain a game $(G_A, \boldsymbol{Q_G})$, the play of which will determine a set $T(G_A, \boldsymbol{Q_G})$ of realizable social states. For the sake of simplicity in exposition, it is assumed in what follows that $T(G_A, \boldsymbol{Q_G})$ consists only of a single element, say $\tau(G_A, \boldsymbol{Q_G})$.[19] In this case, a feasible extended social state is given by $(\tau(G_A, \boldsymbol{Q_G}), \boldsymbol{G})$.

We are now ready to explain how this framework treats the issue of the initial conferment of rights. Let $\Psi$ be the *the extended social welfare function* which maps each profile $\boldsymbol{Q} = (Q_1, Q_2, \ldots, Q_n)$ of extended individual preference orderings into an extended social welfare ordering: $Q = \Psi(\boldsymbol{Q})$. Given a set $A$ of feasible social states, the socially optimal conferment of rights is nothing other than the rights-system $\boldsymbol{G}^*$ such that $(\tau(G^*_A, \boldsymbol{Q}_{\boldsymbol{G}^*}), \boldsymbol{G}^*)\Psi(\boldsymbol{Q})(\tau(G_A, \boldsymbol{Q_G}), \boldsymbol{G})$ holds for any feasible rights-system $\boldsymbol{G}$.

Before closing this summary account of the game form approach to individual libertarian rights, two remarks are due. First, unlike Sen's classical articulation of rights, the game form articulation of rights does not assign any role whatsoever to individual preferences. However, in the realization of rights articulated by the rights-system, as well as in the initial conferment of rights, this theory does assign crucial role to the profile of

[18]See Arrow [1, Chapter 7, Section 6], and Sen [37] for further forceful endorsement of this viewpoint.
[19]See Pattanaik and Suzumura [26] for a fuller exposition without this simplifying assumption.

extended individual preference orderings. In the former case, it is the induced preference profile $\boldsymbol{Q_G}$, together with the set $A$ of feasible social states, which determines the game $(G_A, \boldsymbol{Q_G})$ to be played as well as the outcome of the play $\tau(G_A, \boldsymbol{Q_G})$. In the latter case, it is the extended social welfare ordering $Q = \Psi(\boldsymbol{Q})$ which determines the rights-system to be conferred. Thus, in the full theory of game form approach to rights, there are important niches for individual preference orderings. To recapitulate, although the formal contents of the conferred game form rights are independent of individual preferences, the extended individual preferences play a crucial role in deciding the rights-system to be conferred, as well as in socially realizing the individual freedom of choice thus conferred.

Second, unlike in the context of the Sen-Gibbard rights, the game form approach to rights does not have any counterpart of the Gibbard impossibility theorem. In other words, the problem of internal inconsistency of rights never surfaces in the game form approach. To the extent that the initial conferment of rights is performed in accordance with the scenario of the game form approach, the conferred rights will be realized through the actual play of the game, thereby excluding any possibility of internal inconsistency of rights.

# 5 Sen's Criticism Against Welfarism: An Evaluation

Back, then, to the central focus of this chapter. What does the game form approach clarify about the impossibility of a Paretian liberal? Does it fortify, or qualify, or even nullify Sen's criticism against welfarism which is based on the basic conflict between the welfaristic Pareto principle and the non-welfaristic claim of individual liberty? In what follows, we will contend that the main thrust of Sen's criticism against welfarism remains intact even if Sen's articulation of individual liberty is rejected and replaced by the game form articulation.

To begin with, consider yet another variant of Sen's *Lady Chatterley's Lover* case. As in the first variant used in Section 3 to crystallize a conceptual difficulty of Sen's approach, suppose that both Mr. $P$ and Mr. $L$ have a copy of *Lady Chatterley's Lover*, and their preference orderings over the set of feasible social states $\{(r, r), (r, n), (n, r), (n, n)\}$ are given by:

$R_P : (n, n), (r, n), (n, r), (r, r)$
$R_L : (r, r), (r, n), (n, r), (n, n).$

As in the first variant, the issue of individual liberty contained in this situation may be captured neatly by the game form $G = (N, \{n, r\}, \{n, r\}, g)$, where $N = \{P, L\}$ and the outcome function $g$ is such that $g(s_P, s_L) = (s_P, s_L)$, where $s_P$ and $s_L$ are taken from $\{n, r\}$. Unlike in the first variant, however, the preference profile $\boldsymbol{R} = (R_P, R_L)$ that defines a game $(G, \boldsymbol{R})$ has a *dominant strategy equilibrium* $(n, r)$, which is Pareto dominated by $(r, n)$. Thus, the voluntary exercise of freedom of choice yields a social state which is Pareto dominated by another feasible social state. This is the first instance in which Sen's impossibility of a Paretian liberal recurs in the context of realizing conferred game form rights.

As a matter of fact, Sen's impossibility recurs also in the context of initial conferment of game form rights. To show this possibility unambiguously, consider a situation where $N = \{C, D\}$ ($C$ = "consequentialist"; $D$ = "deontologist"). There are two issues to be decided on. The first issue is the religion, and there are two options: $b$ ="Buddhism" and $c$ = "Christianity". The second issue is whether or not a book is to be read, and there are two options: $r$ = "to read it" and $n$ = "not to read it". Thus, the set $A$ of physically possible social states consists of 16 alternatives. A typical element of $A$ is denoted by $(c, n; b, r)$, which is a state where Mr. $C$ believes in Christianity and does not read the book, and Mr. $D$ believes in Buddhism and reads the book. There are two feasible rights-systems: $\boldsymbol{G^1} = \{G^1\}$ and $\boldsymbol{G^2} = \{G^2\}$.[20] The game form $G^1 = (N, \{S_i^1\}, g^1)$, where $S_i^1$ is the Cartesian product of $\{b, c\}$ and $\{r, n\}$ for $i = C, D$ and $g^1(\boldsymbol{s}) = \boldsymbol{s}$ for all $\boldsymbol{s} = (s_1, s_2)$ such that $s_i$ is in $S_i^1$ for $i = C, D$, is the one where the two persons are empowered to choose their religion as well as reading or not reading the book freely. In contrast, the game form $G^2 = (N, \{S_i^2\}, g^2)$, where $S_i^2 = \{r, n\}$ for $i = C, D$ and $g^2(\boldsymbol{s}) = \boldsymbol{s}$ for all $\boldsymbol{s} = (s_1, s_2)$ such that $s_i$ is in $S_i^2$ for $i = C, D$ is the one where the two persons are only allowed to choose reading or not reading the book freely, the matter of choosing common religion being decided by the society. If the social choice of common religion is $t$ in $\{b, c\}$ and the strategy pair $\boldsymbol{s}$ is chosen, then the social state will be given by $(t, s_1; t, s_2)$.

Let $\boldsymbol{Q} = (Q_C, Q_D)$ be the profile of extended individual preference orderings. Mr. $C$ is a die hard consequentialist who cares only about the outcomes of social interactions and nothing else. Thus, for all social state $x$ in $A$, $(x, \boldsymbol{G^1})I(Q_C)(x, \boldsymbol{G^2})$ holds true, where $I(Q_C)$ is the indifference relation generated by $Q_C$. For each pair $(u, v)$, where $u$ (resp. $v$) refers to Mr. $C$'s (resp. Mr. $D$'s) religion, and for each $\boldsymbol{G} = \boldsymbol{G^1}$ and $\boldsymbol{G^2}$, let $Q_{C\boldsymbol{G}}(u, v)$ be defined by:

$$Q_{C\boldsymbol{G}}(u, v) : (u, r; v, r), (u, n; v, r), (u, r; v, n), (u, n; v, n),$$

which, in turn, is used to define $Q_{C\boldsymbol{G}}$ by:

$$Q_{C\boldsymbol{G}} : Q_{C\boldsymbol{G}}(b, c), Q_{C\boldsymbol{G}}(b, b), Q_{C\boldsymbol{G}}(c, b), Q_{C\boldsymbol{G}}(c, c).$$

Mr. $D$ is a deontologist whose belief in the procedural justice in allowing people to choose their religion has such predominant importance that, for all $x, y$ in $A$, he holds that $(x, \boldsymbol{G^1})P(Q_D)(y, \boldsymbol{G^2})$. For each pair $(u, v)$ of religions of Mr. $C$ and Mr. $D$ and for each $\boldsymbol{G} = \boldsymbol{G^1}$ and $\boldsymbol{G^2}$, we define $Q_{D\boldsymbol{G}}(u, v)$ by:

$$Q_{D\boldsymbol{G}}(u, v) : (u, n; v, n), (u, n; v, r), (u, r; v, n), (u, r; v, r),$$

which, in turn, is used to define $Q_{D\boldsymbol{G}}$ by:

$$Q_{D\boldsymbol{G}} : Q_{D\boldsymbol{G}}(c, c), Q_{D\boldsymbol{G}}(b, c), Q_{D\boldsymbol{G}}(c, b), Q_{D\boldsymbol{G}}(b, b).$$

Let us examine the game $(G^1, \boldsymbol{Q}_{\boldsymbol{G^1}})$. It is easy, if tedious, to check that $(b, r)$ is the dominant strategy for Mr. $C$, and $(c, n)$ is the dominant strategy for Mr. $D$. Thus,

[20] Throughout this example, the feasible set $A$ is fixed, which is why $\boldsymbol{G^1}$ as well as $\boldsymbol{G^2}$ consists of only one game form each.

$(b, r; c, n)$ in $A$ is the dominant strategy equilibrium in the game $(G^1, \boldsymbol{Q_{G^1}})$. In the situation where there exists a dominant strategy equilibrium, it is very natural to assume that $\tau(G^1, \boldsymbol{Q_{G^1}}) = ((b, r; c, n), \boldsymbol{G^1})$. Turning to the game $(G^2, \boldsymbol{Q_{G^2}})$, it is again easy to confirm that $r$ (resp. $n$) is the dominant strategy for Mr. $C$ (resp. Mr. $D$) irrespective of whether the social choice of religion turns out to be $b$ or $c$. Thus, $\tau(G^2, \boldsymbol{Q_{G^2}}) = ((b, r; b, n), \boldsymbol{G^2})$ or $((c, r; c, n), \boldsymbol{G^2})$ depending on the social choice of $b$ or $c$. Recollect that Mr. $D$ holds a lexicographic preference for $(x, \boldsymbol{G^1})$ against $(y, \boldsymbol{G^2})$ whatever may be $x$ and $y$. Thus, he must surely prefer $\tau(G^1, \boldsymbol{Q_{G^1}})$ to $\tau(G^2, \boldsymbol{Q_{G^2}})$. Mr. $C$ being a consequentialist, he is indifferent between $\tau(G^1, \boldsymbol{Q_{G^1}}) = ((b, r; c, n), \boldsymbol{G^1})$ and $((b, r; c, n), \boldsymbol{G^2})$ and he prefers $((b, r; c, n), \boldsymbol{G^2})$ to $((b, r; b, n), \boldsymbol{G^2})$ as well as to $((c, r; c, n), \boldsymbol{G^2})$. By transitivity of $Q_C$, Mr. $C$ must then prefer $\tau(G^1, \boldsymbol{Q_{G^1}})$ to $\tau(G^2, \boldsymbol{Q_{G^2}})$. Thus, as long as the extended social welfare function $\Psi$ satisfies the Pareto principle, $\boldsymbol{G^1}$ must be the rights-system to be conferred. However, if $\boldsymbol{G^1}$ is conferred and the game $(G^1, \boldsymbol{Q_{G^1}})$ is played, $(b, r; c, n)$ will be the social outcome, which is Pareto dominated by another feasible social state $(b, n; c, r)$.

We have thus shown that Sen's Pareto libertarian paradox recurs not only in the context of realizing game form rights, but also in the context of initial conferment of game form rights. It is in this sense that we contend that Sen's criticism against welfarism survives without losing an iota of its importance even if his articulation of libertarian rights has to be replaced by the allegedly more proper game form articulation. We close this section by re-stating our conviction that the Sen impossibility problem "persists under virtually every plausible concept of individual rights".

# 6 Concluding Remarks

To argue for the *logical* relevance of Sen's criticism against welfarism is one thing, and to argue for its *empirical* relevance in the actual context where welfare economics is set in motion is quite another. In this chapter, we have confirmed that Sen's impossibility of a Paretian liberal does not lose its logical relevance even in the light of the many criticisms recently raised against Sen's method of articulating individual liberty in terms of a person's decisive power in social choice. How about the empirical relevance? Is Sen's impossibility just a theoretical curiosity which is amusing as a logical exercise in the classroom yet can be safely neglected once we turn our attention to the pressing economic problems where the real bite of welfare economics is seriously tested? Quite to the contrary, it seems to us that there are many real situations where serious conflict occurs between the claim of individual rights and the desire for social efficiency.

Suffice it to visualize a local city where small and traditional retailers are engaging in hand-to-mouth business. From the viewpoint of improving social efficiency in retailing service in this city, it makes sense to allow a few large-scale organized retailers to enter this city. If we do so, however, those small retailers who have been doing business in this city over many years will almost surely be unable to cope with the large-scale retailers and expelled from retailing business. Should we pursue the improvement in social efficiency at the cost of depriving small retailers of their "rights" of doing business? Or, should we respect these “rights” at the cost of missing an opportunity to improve social efficiency in retailing business? This is a typical and realistic situation where policy makers are confronted with the conflict between right and efficiency.

To the extent that welfare economics claims to serve as the theoretical foundations of economic policy, there is no way of avoiding such conflict between two basic values — the welfaristic value of social efficiency, on the one hand, and the non-welfaristic claim of individual rights, on the other. Although Sen [29] posed this serious problem in terms of a deceptively simple parable, the problem he thereby posed is neither simple nor unrealistic.

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