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Comment on "Actuarial Neutrality across Generations Applied to Public Pensions under Population Ageing: Effects on government finances and National Saving"

by

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I believe Dr.Oksanen wrote a very useful paper for everyone in public finance, from empirical economists interested in intergenerational equity questions, to theoreticians working on three-generation overlapping models. What this paper provides is a very simple notion of actuarial neutrality across generations in public pensions, expressed in a very general social accounting framework. Unlike most overlapping generation models, the result is not limited to steady-states and is derived from very general partially funded public pension models.

1. I will follow his basic notation, so I will skip basic presentation of his model and jump right to the heart of his model. Denoting the budget balance of the pension system at period t in per-capita form, it is given by

$$c_t w_t L_t + \rho_t A_{t-1} = s_t w_t R_t + A_t$$

where c_t t is the contribution rate, ρ_t is the interest factor on pension fund A_{t-1}, s_t is the replacement ratio of pension, and R_t is the number of pensioners per worker. The left-hand side denotes the gross revenue and the right-hand side denotes the gross expenditure, including the transactions with respect to funds.

The author's ingenuity is in the translation of the interest factor (which is one plus the rate of interest) ρ_t into the following form;

$$\rho_t = \frac{w_t L_t}{w_{t-1} L_{t-1}}(1+d_t)\,.$$

where d_t is the premium over the growth rate of the economy. This translation allows him to rewrite the budget equation in the following reduced form; namely,

$$c_t = (1-z_{t-1})\frac{s_t}{f_{t-1}} + z_t\,\frac{s_{t+1}}{f_t(1+d_t)}$$

where z_t is the funded proportion of the pension and f_t is the fertility factor (one plus the rate of increase in population).

In order to define neutrality, actually we need only two simple definitions; the relative size of pension benefit to wage bill is defined as theta t, or

$$\theta_t \equiv \frac{IPD_t}{w_t L_t} = \frac{s_{t+1}}{f_t(1+d_t)}$$

while the relative size of pension asset to wage bill is defined as qt, or

$$q_t \equiv A_t /(w_t L_t)$$

Then the actuarial neutrality across generations is defined as the state in which total public debt minus financial asset of the system relative to the wage bill remains constant; namely, the difference between theta and q remains unchanged across generations, or

$$\theta_t - q_t = \theta_{t-1} - q_{t-1}.$$

In other words, if generation q leaves exactly what it has inherited from the previous generation and the increase in the pension debts (theta), then it is observing actuarial neutrality across generations; or

$$q_t = q_{t-1} + [\theta_t - \theta_{t-1}].$$

2. In order to draw behavioral implications from this criterion, it is best to express c_t in terms of θ_t, q_t and d_t; namely,

$$c_t = (\theta_{t-1} - q_{t-1})(1+d_t) + q_t$$

We can then solve for actuarially neutral pension contribution rate;

$$\begin{aligned} c_t &= (\theta_{t-1} - q_{t-1})(1+d_t) + q_{t-1} + [\theta_t - \theta_{t-1}] \\ &= \theta_t + d_t[\theta_{t-1} - q_{t-1}] \end{aligned}$$

Given the data on theta's, q and d, we can solve for c_t. I believe this is the main result of this paper. Notice that we did not have to assume anything regarding the behavior of our economy, not even steady-states.

3. Suppose the pension has been run as a pure pay-as-you-go system; in the first period the first generation administrator sets the system parameters as

$$c_1 = \theta_1$$
$$q_1 = 0.$$

For instance, they collect exactly what they expect to receive in retirement.

Now, however, prior to the start of the second period, there was a sudden one-time increase in future values of theta due to a sudden drop in the fertility rate. Then under the actuarially neutral pension system, the system parameters are given by

$$c_2 = \theta_2 + d_2\theta_1$$
$$q_2 = \theta_2 - \theta_1$$

The second term of the right-hand-side of the first equation shows that the second generation's contribution is raised above what they expect to receive in retirement, reflecting the deficiency in the number of future contributors when they receive their pensions. The pension thus moves from a pure pay-as-you-go to partially funded system. It is perhaps instructive to write what happens to the third generation,

$$c_3 = \theta_3 + d_3[\theta_2 - q_2] = \theta_3 + d_3\theta_1$$
$$q_3 = q_2 + [\theta_3 - \theta_2] = q_2$$

As long as the change is a one-shot event, and no change in d takes place, the system follows the previous parameters.

4. We should add that Oksanen's analysis is very general: actually, it is more general than he shows, requiring no fixed rate of returns on interest factors. The connection between the usual pay-as-you-go pension analysis and his can be easily seen by rewriting

$$\rho_t = \frac{w_t L_t}{w_{t-1} L_{t-1}} (1 + d_t)$$
$$= (1 + \omega_t) f_t (1 + d_t)$$
$$\cong (1 + \omega_t + n_t + d_t)$$

where ω_t is the increase in wage rate, and n_t is the rate of increase in labor force. Thus the premium can be given by

$$d_t = \rho_t - (1 + \omega_t + n_t) .$$

In other words, d_t is the difference between the rate of interest and the sum of the rate of growth in wage rate ω_t and the rate of increase in labor force n_t. Thus, if the latter exceeds the interest rate, the criterion allows the pension to collect less than what they expect to receive even under his criterion.

I congratulate Dr.Oksanen for writing this excellent paper, providing a very pragmatic, clear-cut solution for a very complex problem.

Reference

H.Oksanen, "Actuarial Neutrality across Generations Applied to Public Pensions under Population Ageing: Effects on government finances and National Saving", January 2006