Envy-Free Configurations in the Market Economy*

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First Version: February 2001
This Version: July 2001

* We are grateful to Shasikanta Nandeibam and William Thomson for useful discussions. Tadenuma acknowledges the Scientific Research Grant for Priority Areas (B) Number 603 from the Ministry of Education, Science and Culture, Japan.

Abstract

Using an extended framework in which an agent is endowed with three types of preference orders: an allocation preference order, an opportunity preference order, and an overall preference order, this paper introduces several notions related to efficiency and equity-as-no-envy and examines the performance of competitive market mechanisms. We also axiomatically characterize the equal income Walras rule for pure exchange economies.

JEL Classification Numbers: D63, D71

1 Introduction

Recently Suzumura and Xu (1999, 2000) have developed an extended framework in which an agent is modelled to have *extended preferences* such as follows: it is better for the agent to have a consequence x from an opportunity set A than to have a consequence y from an opportunity set B. Following Suzumura and Xu (1999, 2000), this paper is to develop a similar extended framework to re-examine some classical problems of efficiency and equity-as-no-envy of competitive market mechanisms. Our fundamental standpoint is that the performance of economic mechanisms should be evaluated on the bases of not only final consumption bundles of agents but also opportunity sets from which consumption bundles are chosen. This approach is in line with the recent contributions in the literature of non-welfaristic approach to welfare economics and social choice (see, among others, Pattanaik and Xu (1990), and Sen (1988, 1997)).

Given an economy, we define a *configuration* as a pair of an allocation and a distribution of opportunity sets. Thus, a configuration specifies a final consumption bundle and an opportunity set for each agent in the economy. A pair of a consumption bundle and an associated opportunity set is called an *individual state*. Each agent is assumed to be endowed with three preference orders: an allocation preference order that ranks consumption bundles, an opportunity preference order that ranks opportunity sets, and an overall preference order that ranks individual states. Based on each of the three types of preference orders, we introduce the notions of Pareto-optimality and no-envy. In the literature, Pareto-optimality and no-envy with respect only to allocation preference orders have been considered. *Allocation-Pareto-optimality* corresponds to the usual efficiency notion in economics. *Allocation-No-Envy* was introduced by Kolm (1971) and Foley (1967), and has been discussed extensively in the literature. (See, for example, the survey of Thomson and Varian (1985).) It requires that no agent prefers the consumption bundle of any other agent to his own.

The notions of no-envy and Pareto-optimality based on opportunity preference orders and overall preference orders are new. We say that a configuration is *opportunity-envy-free* if no agent prefers any other agent's opportunity set to his opportunity set. In the literature, there have been some discussions of *equality of opportunities* that are related to our notion of opportunity-no-envy. On one hand, authors like Archibald and Donaldson (1979), Klappholz (1972), and Thomson (1994) start with equality of opportunities as *identical* opportunity sets for the agents in the economy.[1] On the other hand, there is an increasing literature that discusses various issues relating to equality in the distribution of opportunity sets. (See Peragine (1999) for a survey of this literature.) We say that a configuration is *overall-envy-free* if no agent prefers the individual state of any other agent to his own.

We say that a feasible configuration is *opportunity-Pareto-optimal* if there exists no other feasible configuration at which every agent's opportunity set is at least as good as the original one, and some agent's opportunity set is strictly better than the original one. Similarly, a feasible configuration is *overall-Pareto-optimal* if there exists no other feasible configuration at which the individual state of every agent is at least as good as the original one, and the individual state of some agent is strictly better than the original one. These notions are natural extensions of the standard notions of allocation-no-envy and allocation Pareto-optimality in our framework.

Equipped with these concepts, we examine the performance of competitive market mechanisms. We also axiomatically characterize the equal income Walras rule.

The remainder of the paper is organized as follows. In Section 2, we lay down the basic notation and definitions. Section 3 introduces the concepts of Pareto-optimality and no-envy in our framework, and then examines the logical relationships among those concepts. In Section 4, we introduce several normative properties to be imposed on social choice rules. Section 5 provides characterizations of the equal income Walras rule for pure exchange economies. We conclude in Section 6.

2 Allocations, Opportunities and Configurations

There are n agents and k goods. Let $N = \{1, \ldots, n\}$ be the set of agents. An *allocation* is a vector $x = (x_1, \ldots, x_n) \in \mathbb{R}^{nk}_+$ where for each $i \in N$, $x_i = (x_{i1}, \ldots, x_{ik}) \in \mathbb{R}^k_+$ is a *consumption bundle* of agent i.[2] There exist some fixed amounts of social endowments of goods, which are represented by the vector $\Omega \in \mathbb{R}^k_{++}$. An allocation $x \in \mathbb{R}^{nk}_+$ is *feasible* if $\sum_{i=1}^n x_i \leq \Omega$.[3] Let

[1]Thomson (1994) also discusses other notions of equality of opportunities.

[2]As usual, $\mathbb{R}_+$ is the set of all non-negative real numbers, and $\mathbb{R}_{++}$ is the set of all positive real numbers.

[3]Vector inequalities are as usual: $\geq, >$ and $>>$.

Z be the set of all feasible allocations.

For each $i \in N$, an *opportunity set* for agent i is a set in $\mathbb{R}^k_+$. We consider a particular class of opportunity sets, namely the class of budget sets. For each $i \in N$, a *budget set* for agent i at a price vector $p \in \mathbb{R}^k_+$ and a consumption bundle $x_i \in \mathbb{R}^k_+$ is defined by

$$B(p, x_i) = \{y_i \in \mathbb{R}^k_+ \mid p \cdot y_i \leq p \cdot x_i\}$$

Let $\mathcal{B} = \{B(p, x_i) \mid p \in \mathbb{R}^k_+, x_i \in \mathbb{R}^k_+\}$. A *distribution of budget sets* is an n-tuple $B = (B_1, \ldots, B_n) \in \mathcal{B}^n$. A *configuration* is a pair $(x, B) \in \mathbb{R}^{nk}_+ \times \mathcal{B}^n$ such that $x_i \in B_i$ for all $i \in N$. For each $(x, B) \in \mathbb{R}^{nk}_+ \times \mathcal{B}^n$ and each $i \in N$, the *individual state of agent i at* (x, B) is the pair (x_i, B_i).

A configuration $(x, B) \in \mathbb{R}^{nk}_+$ is *feasible in the market economy* if $x \in Z$ and there exist $p \in \mathbb{R}^k_+$ and $\omega = (\omega_1, \ldots, \omega_n) \in Z$ such that $B_i = B(p, \omega_i)$ for all $i \in N$. Let $\mathbb{Z}$ be the set of all configurations that are feasible in the market economy.

Note that the feasibility condition above requires that the distribution of budget sets is generated from an initial feasible allocation and a price vector, and that for each agent, the specified consumption bundle is in his budget set. However, whether an agent actually chooses the consumption bundle depends on his preferences. (See the notion of "decentralizability" which will be introduced later.)

Given a set X, a *preference order* on X is a reflexive, transitive and complete binary relation on X. Each agent $i \in N$ is endowed with the following three preference orders.[4]

1. An *allocation preference order* on $\mathbb{R}^k_+$, denoted R^A_i, which is continuous, monotonic and convex.

2. An *opportunity preference order* on $\mathcal{B}$, denoted R^O_i, which is *monotonic* in the following sense:

(i) $\forall B_1, B_2 \in \mathcal{B}, B_1 \subseteq B_2 \Rightarrow B_2 \; R^O_i \; B_1$

[4] One could argue that the overall preference order is closely related to the allocation preference order and the opportunity preference order, and perhaps can be obtained by *aggregating* the allocation preference order and the opportunity preference order. This can be the case (see, for example, Bossert, Pattanaik and Xu (1994), and Suzumura and Xu (2000) for some discussions of the related issues). Since we focus on examining the performance of competitive market mechanisms in the extended framework, we do not wish to discuss the aggregation problem, if there is any, in detail. Instead, we simply assume that the overall preference order is monotonically linked with the allocation preference order and the opportunity preference order.

(ii) $\forall B_1, B_2 \in \mathcal{B}, B_1 \subset \text{int}B_2 \Rightarrow B_2 \; P_i^O \; B_1$, where $\text{int}B_2$ is the relative interior of B_2 in $\mathbb{R}_+^k$.[5]
3. An *overall preference order* on $\mathbb{R}_+^k \times \mathcal{B}$, denoted $\bar{R}_i$, which is related with R_i^A and R_i^O as follows:
$\forall (x_i, B_i), (y_i, C_i) \in \mathbb{R}_+^k \times \mathcal{B}$
(i) $x_i \; R_i^A \; y_i$ and $B_i \; R_i^O \; C_i \Rightarrow (x_i, B_i) \; \bar{R}_i \; (y_i, C_i)$
(ii) $x_i \; P_i^A \; y_i$ and $B_i \; P_i^O \; C_i \Rightarrow (x_i, B_i) \; \bar{P}_i \; (y_i, C_i)$

Let $\mathcal{R}^A, \mathcal{R}^O$, and $\bar{\mathcal{R}}$ denote the classes of allocation preference orders, opportunity preference orders, and overall preference orders, respectively. Let $\mathcal{R} = \mathcal{R}^A \times \mathcal{R}^O \times \bar{\mathcal{R}}$. A *preference profile* is a list $R = (R_1, \ldots, R_n) \in \mathcal{R}^n$ where $R_i = (R_i^A, R_i^O, \bar{R}_i)$ for all $i \in N$.

Let $R_i = (R_i^A, R_i^O, \bar{R}_i) \in \mathcal{R}$ be given. We say that agent $i \in N$ is a *consequentialist at* R if $\forall (x_i, B_i), (y_i, C_i) \in \mathbb{R}_+^k \times \mathcal{B} : (x_i, B_i) \bar{R}_i (y_i, C_i) \Leftrightarrow x_i \; R_i^A \; y_i$. We say that agent $i \in N$ is a *non-consequentialist at* R if $\forall (x_i, B_i), (y_i, C_i) \in \mathbb{R}_+^k \times \mathcal{B} : (x_i, B_i) \bar{R}_i (y_i, C_i) \Leftrightarrow B_i \; R_i^O \; C_i$.[6]

A *social choice rule* is a mapping φ that associates with each $R \in \mathcal{R}^n$ a non-empty set $\varphi(R) \subseteq \mathbb{Z}$. Given an $R \in \mathcal{R}^n$ and given a social choice rule φ, let $\varphi_Z(R) = \{x \in Z \mid \exists B \in \mathcal{B}^n, \; (x, B) \in \varphi(R)\}$.

3 Pareto-Optimality and No-Envy

In this paper, we focus on "decentralizable" configurations, at which the specified consumption bundle for each agent is actually the best one in his opportunity set by his allocation preference order. If a configuration (x, B) were not decentralizable, then there had to be some restriction on the freedom of choice for some agents. Hence, decentralizability is a fundamental requirement of autonomous decisions. (See discussions on concepts of freedom in Sen (1993).)

Let $R = (R_1, \ldots, R_n) \in \mathcal{R}^n$ be given, where $R_i = (R_i^A, R_i^O, \bar{R}_i)$ for all $i \in N$. We say that a configuration $(x, B) \in \mathbb{Z}$ is *decentralizable* for R if for all $i \in N$ and all $y_i \in B_i$, $x_i \; R_i^A \; y_i$. Let $D(R)$ denote the set of decentralizable configurations for R.

[5] As usual, the strict preference order associated to R_i^O is denoted P_i^O. Similar notation is used for other preference orders.

[6] Our notions of a consequentialist and a non-consequentialist correspond, respectively, to an *extreme consequentialist* and an *extreme non-consequentialist* proposed in Suzumura and Xu (1999, 2000)

A configuration $(y, C) \in \mathbb{Z}$ *allocation-Pareto-dominates* $(x, B) \in \mathbb{Z}$ for R if $y_i \; R_i^A \; x_i$ for all $i \in N$ and $y_i \; P_i^A \; x_i$ for some $i \in N$. A configuration $(x, B) \in \mathbb{Z}$ is *allocation-Pareto-optimal* for R if no feasible configuration allocation-Pareto-dominates it. Let $E^A(R)$ be the set of allocation-Pareto-optimal configurations for R. We define *opportunity-Pareto-domination* and *opportunity-Pareto-optimality* by replacing allocation preference orders with opportunity preference orders in the above definitions. We also define analogously *overall-Pareto-domination* and *overall-Pareto-optimality* based on overall preference orders.

Given our assumptions on R_i^A for all $i \in N$, the following proposition is immediate.

Proposition 1 *For every $R \in \mathcal{R}^n$, if a feasible configuration is decentralizable for R, then it is allocation-Pareto-optimal for R.*

Sen (1993) considers the following conditions of opportunity preference orders (See Sen's Axiom O on p.530):
(i) for all $B_i, C_i \in \mathcal{B}$, $B_i \; R_i^O \; C_i$ *only if* there exists $x_i \in B_i$ such that $x_i \; R_i^A \; y_i$ for all $y_i \in C_i$, and
(ii) for all $B_i, C_i \in \mathcal{B}$, $B_i \; P_i^O \; C_i$ *only if* there exists $x_i \in B_i$ such that $x_i \; P_i^A \; y_i$ for all $y_i \in C_i$.
Let $\hat{\mathcal{R}}^O$ be the class of opportunity preference orders satisfying the above conditions.

Although Sen (1993) introduces these conditions as *necessary conditions* for opportunity preferences, it turns out that under completeness of opportunity preference orders, these are also *sufficient conditions*. Then, the next result immediately follows from Proposition 1.

Proposition 2 [Sen, 1993, p.534] *Let $R = (R_1, \ldots, R_n) \in \mathcal{R}^n$ be such that for all $i \in N$, $R_i^O \in \hat{\mathcal{R}}^O$. Then, no decentralizable configuration for R is opportunity-Pareto-dominated by another decentralizable configurations for R.*

We can extend the above result to overall-Pareto-optimality.

Proposition 3 *Let $R = (R_1, \ldots, R_n) \in \mathcal{R}^n$ be such that for all $i \in N$, $R_i^O \in \hat{\mathcal{R}}^O$. Then, no decentralizable configuration for R is overall-Pareto-dominated by another decentralizable configuration for R.*

Proof: Let $(x, B) \in \mathbb{Z}$ be a decentralizable configuration for R. By Proposition 1, (x, B) is allocation-Pareto-optimal for R. Suppose to the contrary that there exists $(y, C) \in \mathbb{Z}$ that is decentralizable for R, and that overall-Pareto-dominates (x, B). Since (y, C) overall-Pareto-dominates (x, B), we have that
(i) for all $i \in N, (y_i, C_i)\bar{R}_i(x_i, B_i)$ and
(ii) for some $j \in N, (y_j, C_j)\bar{P}_j(x_j, B_j)$.
From (ii) and by the definition of the overall preference order $\bar{R}_j$, it follows that (ii.1) $y_j \; P_j^A \; x_j$ or (ii.2) $C_j \; P_j^O \; B_j$. Suppose that (ii.1) holds. Since (x, B) is allocation-Pareto-optimal for R, there exists $h \in N$ such that $x_h \; P_h^A \; y_h$. From the definition of $\bar{R}_h$ and (i) requiring $(y_h, C_h)\bar{R}_h(x_h, B_h)$, we must have $C_h \; R_h^O \; B_h$. Because $R_h^O \in \hat{\mathcal{R}}^O$, there exists $z_h \in C_h$ such that $z_h \; R_h^A \; w_h$ for all $w_h \in B_h$. Since (y, C) is decentralizable, it follows that $y_h \; R_h^A \; z_h$, and by the transitivity of R_h^A, $y_h \; R_h^A \; w_h$ for all $w_h \in B_h$. Letting $w_h = x_h$, we have a contradiction with $x_h \; P_h^A \; y_h$.

Suppose that (ii.2) holds. Then, from $R_j^O \in \hat{\mathcal{R}}^O$, there exists $z_j \in C_j$ such that $z_j \; P_j^A \; w_j$ for all $w_j \in B_j$. Since (y, C) is decentralizable, we have $y_j \; R_j^A \; z_j$, and $y_j \; P_j^A \; w_j$ for all $w_j \in B_j$. Hence, $y_j \; P_j^A \; x_j$. Starting with this and from (ii.1), we can derive a contradiction. In summary, therefore, (x, B) cannot be overall-Pareto-dominated by another decentralizable configuration. ■

In the literature on opportunity set rankings, many authors have considered and axiomatically characterized opportunity preference orders that do not belong to $\hat{\mathcal{R}}^O$ (See, for example, Pattanaik and Xu (1990).) For such preference orders, a decentralizable configuration may be opportunity-Pareto-dominated by another decentralizable configuration. Consider the following example.

Example 1. There are two agents $N = \{1, 2\}$ and two goods. Define $R_1^O \in \mathcal{R}^O$ as follows: for all $B_1, C_1 \in \mathcal{B}$, $B_1 \; R_1^O \; C_1$ if and only if the area of B_1 is greater than or equal to that of C_1. Define $R_2^O \in \mathcal{R}^O$ as follows: for all $B_2, C_2 \in \mathcal{B}$, $B_2 \; R_2^O \; C_2$ if and only if there exists $x_2 \in B_2$ such that $x_2 \; R_2^A \; y_2$ for all $y_2 \in C_2$. Note that $R_2^O \in \hat{\mathcal{R}}^O$, but $R_1^O \notin \hat{\mathcal{R}}^O$. Let $R_1^A, R_2^A \in \mathcal{R}^A$ be such that the indifference curves are drawn as in Figure 1. In the Edgeworth box, (x, B) and (y, C) are two decentralizable configurations such that $x_2 \; P_2^A \; y_2$ and the area of B_1 is greater than that of C_1. Hence, for all $i \in N$, $B_i \; P_i^O \; C_i$. Thus, (y, C) is opportunity-Pareto-dominated by (x, B).

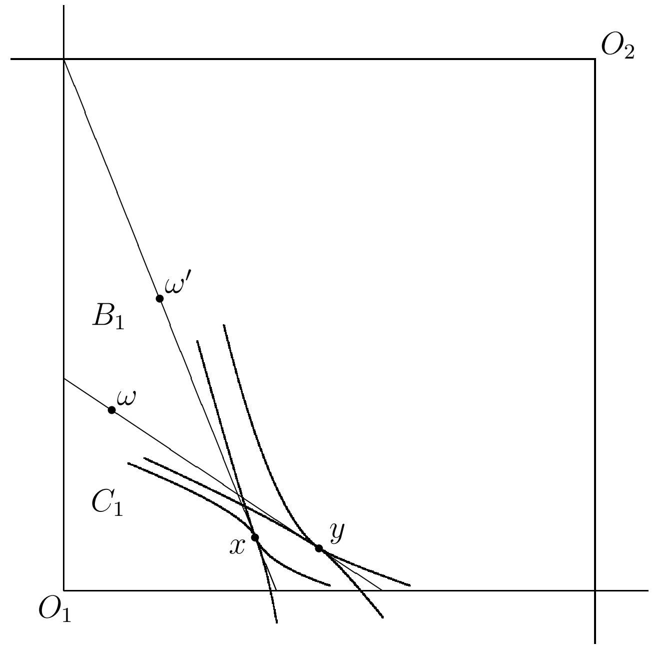

Figure 1: A decentalizable configuration may be opportunity-Pareto-dominated by another decentralizable configuration (Example 1).

Note that in the economy of Example 1, the "transfer paradox" occurs:[7] when the initial allocation is changed from ω to ω', namely, when some amounts of both goods are transferred from agent 2 to agent 1, agent 1 is nevertheless worse off at his equilibrium consumption bundle according to his allocation preference order. Despite this fact, agent 1 may prefer having the opportunity set B_1 generated by the initial bundle ω'_1 rather than C_1 corresponding to ω_1 according to his opportunity preference order.

The above example shows the vulnerability of Sen's result claiming the opportunity-Pareto-optimality of a decentralizable configuration. The opportunity preference order that Sen considers is very restrictive, and if we drop the restriction, we can no longer guarantee opportunity-Pareto-optimality of a decentralizable configuration.

The example can also be extended to overall-Pareto-optimality: for some class of opportunity preference orders, a decentralizable configuration may

[7]The "transfer paradox" was first pointed out by Leontief (1936).

be overall-Pareto-dominated by another decentralizable configuration. This is the case when, for instance, agent 1 is a non-consequentialist and agent 2 a consequentialist in the above example.

Next, we introduce the concepts of equity-as-no-envy. A configuration $(x, B) \in \mathbb{Z}$ is *allocation-envy-free* for R if for all $i, j \in N$, $x_i \; R_i^A \; x_j$. Let $F^A(R)$ be the set of allocation-envy-free configurations for R. A configuration $(x, B) \in \mathbb{Z}$ is *opportunity-envy-free* for R if for all $i, j \in N$, $B_i \; R_i^O \; B_j$. Let $F^O(R)$ be the set of opportunity-envy-free configurations for R. A configuration $(x, B) \in \mathbb{Z}$ is *overall-envy-free* for R if for all $i, j \in N$, $(x_i, B_i)\bar{R}_i(x_j, B_j)$. Let $\bar{F}(R)$ be the set of overall-envy-free configurations for R.

The next result follows from the monotonic link between the allocation preference order, the opportunity preference order, and the overall preference order.

Proposition 4 *For every $R \in \mathcal{R}^n$, if a configuration is allocation-envy-free* and *opportunity-envy-free for R, then it is overall-envy-free for R.*

Proof. Suppose that $(x, B) \in \mathbb{Z}$ is both allocation-envy-free and opportunity-envy-free for R. Since (x, B) is allocation-envy-free, we have $x_i \; R_i^A \; x_j$ for all $i, j \in N$. Similarly, $B_i \; R_i^O \; B_j$ for all $i, j \in N$ follows from (x, B) being opportunity-envy-free. Then, from the definition of $\bar{R}_i$ for each $i \in N$, we have $(x_i, B_i)\bar{R}_i(x_j, B_j)$ for all $i, j \in N$. Therefore, (x, B) is overall-envy-free. ■

With decentralizability, we have the following logical relations:[8] opportunity-no-envy implies both allocation-no-envy and overall-no-envy; overall-no-envy implies allocation-no-envy but dose not imply opportunity-no-envy; allocation-no-envy implies neither opportunity-no-envy nor overall-no-envy.

Proposition 5
(i) *There exist $R \in \mathcal{R}^n$ and $(x, B) \in \mathbb{Z}$ such that (x, B) is decentralizable and allocation-envy-free but is neither opportunity-envy-free nor overall-envy-free for R.*
(ii) *For every $R \in \mathcal{R}^n$, if $(x, B) \in \mathbb{Z}$ is decentralizable and opportunity-envy-free for R, then it is allocation-envy-free and overall-envy-free for R.*

[8]Without decentralizability, there is no logical relation between any two of the three conditions of no-envy.

(iii) *For every* $R \in \mathcal{R}^n$, *if* $(x, B) \in \mathbb{Z}$ *is decentralizable and overall-envy-free for* R, *then it is allocation-envy-free for* R. *However, there exist* $R \in \mathcal{R}^n$ *and* $(x, B) \in \mathbb{Z}$ *such that* (x, B) *is decentralizable and overall-envy-free but is not opportunity-envy-free for* R.

Proof. (i) For simplicity, we consider a two-agent and two-good economy, and assume that $\Omega = (10, 10)$. Suppose that R_1^A and R_2^A are represented by the following utility functions, respectively:

$$\begin{aligned} U_1(x_{11}, x_{12}) &= x_{11}x_{12}^2 \\ U_2(x_{21}, x_{22}) &= x_{21}^2 x_{12} \end{aligned}$$

Let $x_1 = (3, \frac{120}{19})$ and $x_2 = (7, \frac{70}{19})$, and let $x = (x_1, x_2)$. Define $p = (20, 19)$. Let $B_1 = B(p, x_1), B_2 = B(p, x_2)$ and $B = (B_1, B_2)$. Then, (x, B) is a decentralizable configuration. Because $U_1(x_1) > U_1(x_2)$ and $U_2(x_2) > U_2(x_1)$, (x, B) is allocation-envy-free. However, since $B_1 \subset \text{int}B_2$, we have $B_2 \; P_1^O \; B_1$. Hence, (x, B) is not opportunity-envy-free. Let $\bar{R}_1 \in \bar{\mathcal{R}}$ be such that $(x_2, B_2)\bar{P}_1(x_1, B_1)$.[9] For such $\bar{R}$, (x, B) is not overall-envy-free.

(ii) Let $R \in \mathcal{R}^n$ be given, and let $(x, B) \in \mathbb{Z}$ be a decentralizable and opportunity-envy-free configuration for R. Since (x, B) is decentralizable, there exists a price vector $p \in \mathbb{R}_+^k$ such that for all $i \in N$, $B_i = B(p, x_i)$. If for some $i, j \in N$, $B_i \neq B_j$, then either $B_i \subset \text{int}B_j$ or $B_j \subset \text{int}B_i$. By monotonicity of R_i^O and R_j^O, this implies that either $B_j \; P_i^O \; B_i$ or $B_i \; P_j^O \; B_j$, which is in contradiction with the assumption that (x, B) is opportunity-envy-free. Hence, it must be true that for all $i, j \in N, B_i = B_j$. Because (x, B) is decentralizable, for all $i, j \in N$ and all $y_j \in B_j = B_i$, $x_i \; R_i^A \; y_j$. In particular, $x_i \; R_i^A \; x_j$. Thus, (x, B) is allocation-envy-free. From Proposition 4, (x, B) is also overall-envy-free.

(iii) Let $R \in \mathcal{R}^n$ be given and let $(x, B) \in \mathbb{Z}$ be a decentralizable configuration for R where $B_i = B(p^*, x_i)$ for all $i \in N$. Suppose that (x, B) is not allocation-envy-free for R. Without loss of generality, suppose that $x_2 \; P_1^A \; x_1$. Since (x, B) is decentralizable, $x_1 \; R^A \; y_1$ for all $y_1 \in B_1$. Hence, we must have that $x_2 \notin B_1$. Because $B_1 = B(p^*, x_1), B_2 = B(p^*, x_2)$ and $x_2 \notin B_1$, we have $B_1 \subset \text{int}B_2$. It follows from strict monotonicity of R_1^O that $B_2 \; P_1^O \; B_1$. Together with $x_2 \; P_1^A \; x_1$, we have $(x_2, B_2)\bar{P}_1(x_1, B_1)$. Therefore, (x, B) is not overall-envy-free for R. Thus, if (x, B) is decentralizable and overall-envy-free, then it must be allocation-envy-free.

[9] This relation holds if, for example, agent 1 is a non-consequentialist.

Next, consider the same economy as (i) above, the same allocation preference orders and opportunity preference orders, and the configuration (x, B). However, let $\bar{R}_1 \in \bar{\mathcal{R}}$ be such that $(x_1, B_1)\bar{R}_1(x_2, B_2)$. Then, (x, B) is decentralizable and overall-envy-free, but is not opportunity-envy-free. ■

4 Properties of Social Choice Rules

In this section, we propose several normative properties that a social choice rule should satisfy. Let φ denote a social choice rule throughout this section.

The first property, Decentralizability, requires a rule to guarantee autonomous choice of every individual from the opportunity set assigned to him.

Decentralizability: $\forall R \in \mathcal{R}^n, \varphi(R) \subseteq D(R)$.

The next property, Allocation-Pareto-Optimality, requires that the final allocations of resources should be Pareto-efficient (in the standard sense).

Allocation-Pareto-Optimality: $\forall R \in \mathcal{R}^n, \varphi(R) \subseteq E^A(R)$

By Proposition 1, Decentralizability implies Allocation-Pareto-Optimality. That is, for all $R \in \mathcal{R}^n$, $D(R) \subseteq E^A(R)$.

The next three properties, Allocation-No-Envy, Opportunity-No-Envy and Overall-No-Envy, are concerned about the distributional equity of configurations chosen by a social choice rule. Allocation-No-Envy, for example, requires that any configurations selected by the rule should be allocation-envy-free. The meanings of Opportunity-No-Envy and Overall-No-Envy are analogous.

Allocation-No-Envy: $\forall R \in \mathcal{R}^n, \varphi(R) \subseteq F^A(R)$

Opportunity-No-Envy: $\forall R \in \mathcal{R}^n, \varphi(R) \subseteq F^O(R)$

Overall-No-Envy: $\forall R \in \mathcal{R}^n, \varphi(R) \subseteq \bar{F}(R)$

Our final properties, Allocation Independence and Independence, are about informational requirements of a social choice rule. Allocation Independence requires that the selection of allocations depends on agents' allocation preference orders only: if two preference profiles R and R' are such that $R_i^A = {R'}_i^A$ for all $i \in N$, then the allocations figured in configurations selected by a social choice rule for R are identical to those chosen for R'.[10] Independence stipulates that the selection of configurations depends on agents' allocation preference orders and opportunity preference orders: if two preference profiles R and R' are such that $R_i^A = {R'}_i^A$ for all $i \in N$ and $R_i^O = {R'}_i^O$ for all $i \in N$, then the configurations selected by a social choice rule for R are identical to those chosen for R'. Formally, they may be stated as follows.

Allocation Independence: $\forall R, R' \in \mathcal{R}^n$, if $R_i^A = R_i'^A$ for all $i \in N$, then $\varphi_Z(R) = \varphi_Z(R')$.

Independence: $\forall R, R' \in \mathcal{R}^n$, if $R_i^A = R_i'^A$ and $R_i^O = R_i'^O$ for all $i \in N$, then $\varphi(R) = \varphi(R')$.

5 Characterizations of the Equal Income Walras Rule

The *Equal Income Walras Rule*, denoted W^e, is defined as follows. For every $R \in \mathcal{R}^n$: $(x, B) \in W^e(R)$ if and only if there exists $p \in \mathbb{R}_+^k$ such that for each $i \in N$, $B_i = B(p, \frac{\Omega}{n}), x_i \in B_i$, and $x_i \; R_i^A \; y_i$ for all $y_i \in B_i$.

Note that for all $R \in \mathcal{R}^n$,

$$W^e(R) \subseteq D(R) \cap F^A(R) \cap F^O(R) \cap \bar{F}(R)$$

and that W^e satisfies Independence as well as Allocation Independence.

From the proof of Proposition 5(ii), it is clear that for every $R \in \mathcal{R}^n$, if (x, B) is decentralizable and opportunity-envy-free for R, then x is an equal income Walras allocation. The next proposition immediately follows from this observation.

Proposition 6 *A social choice rule φ satisfies Decentralizability and Opportunity-No-Envy if and only if $\varphi(R) \subseteq W^e(R)$ for all $R \in \mathcal{R}^n$.*

It should be noted that, from Propositions 5, if a configuration is decentralizable and is opportunity-envy-free, then it is both allocation-envy-free

[10] As a "dual" to Allocation Independence, we may define Opportunity Independence requiring that the selection of distributions of opportunity sets depends on agents' opportunity preference orders only. However, it can be shown that there exists no social choice rule satisfying Opportunity Independence and Decentralizability together.

and overall-envy-free. However, when a configuration is decentralizable and overall-envy-free, it is allocation-envy-free but may not be opportunity-envy-free. In other words, in the presence of Decentralizability, Opportunity-No-Envy is more demanding than Overall-No-Envy, which is in turn stronger than Allocation-No-Envy.

The following proposition "partially" characterizes the equal income Walras rule by using the notion of Overall-No-Envy.

Proposition 7
(i) *The equal income Walras rule W^e satisfies Allocation Independence, Decentralizability and Overall-No-Envy.*
(ii) *If a social choice rule φ satisfies Allocation Independence, Decentralizability and Overall-No-Envy, then $\varphi_Z(R) \subseteq W^e_Z(R)$ for all $R \in \mathcal{R}^n$.*

Proof. Clearly, W^e satisfies Allocation Independence, Decentralizability and Overall-No-Envy.

Let φ be a social choice rule satisfying Allocation Independence, Decentralizability and Overall-No-Envy. Suppose that there exist $R \in \mathcal{R}^n$ and $x \in \varphi_Z(R)$ such that $x \notin W^e_Z(R)$. Let $R' \in \mathcal{R}^n$ be such that for all $i \in N$, ${R'}^A_i = R^A_i$, and all i are non-consequentialists at R'. By Allocation Independence, $x \in \varphi_Z(R')$. Let $B \in \mathcal{B}^n$ be such that $(x, B) \in \varphi(R')$. Since φ satisfies Decentralizability, (1) there exists $p \in \mathbb{R}^k_+$ such that for all $i \in N$, $B_i = B(p, x_i)$ and for all $y_i \in B_i$, $x_i \ {R'}^A_i \ y_i$. If $(x, B) \in W^e(R')$, then $x \in W^e_Z(R')$, and since W^e satisfies Allocation Independence, we have $x \in W^e_Z(R)$, which is a contradiction. Thus, $(x, B) \notin W^e(R')$. From (1), $(x, B) \notin W^e(R')$ holds true only if there exist $h, j \in N$ such that $B_h \subset \text{int}B_j$. But then, because agent h is a non-consequentialist at R', $(x_j, B_j)\bar{P}'_h(x_h, B_h)$, which means that $(x, B) \notin \bar{F}(R')$. Since φ satisfies Overall-No-Envy, we must have $(x, B) \notin \varphi(R')$, a contradiction with $(x, B) \in \varphi(R')$. Thus, for all $R \in \mathcal{R}^n$, $\varphi_Z(R) \subseteq W^e_Z(R)$. ■

Two remarks are in order. First, the conclusion in part (ii) of Proposition 7 cannot be strengthened to $\varphi(R) \subseteq W^e(R)$ for all $R \in \mathcal{R}^n$ as the next counterexample shows. Define the social choice rule ψ as follows: if every $i \in N$ is a consequentialist at $R \in \mathcal{R}^n$, then

$$\psi(R) = \{(x, B) \mid x \in W^e_Z(R) \text{ and } (x, B) \in D(R)\}$$

and otherwise

$$\psi(R) = W^e(R)$$

Then, ψ satisfies Decentralizability, Allocation Independence and Overall-No-Envy. However, there exists $R^* \in \mathcal{R}^n$ such that $\psi(R^*) \nsubseteq W^e(R^*)$. Indeed, let $N = \{1,2\}, k = 2$, and $\Omega = (10,10)$. Suppose that both agents 1 and 2 are consequentialists at $R^* \in \mathcal{R}^2$, and that ${R^*}_1^A$ and ${R^*}_2^A$ are represented by the following utility functions:

$$\begin{aligned} U_1(x_{11}, x_{12}) &= x_{11} + 2x_{12} \\ U_2(x_{21}, x_{22}) &= 2x_{21} + x_{22} \end{aligned}$$

Then, $(x, B) \in \psi(R^*)$ if and only if $x = ((0,10),(10,0))$ and for some $(p_1, p_2) \in \mathbb{R}^2_+$ with $\frac{1}{2} \leq \frac{p_1}{p_2} \leq 2$, $B_i = B(p, x_i)$ for all $i \in N$. If we take (p_1, p_2) such that $\frac{p_1}{p_2} \neq 1$, then $B_1 \neq B_2$, and hence $(x, B) \notin W^e(R^*)$. Thus, $\psi(R^*) \nsubseteq W^e(R^*)$.

Secondly, in the hypothesis in part (ii) of Proposition 7, Overall-No-Envy cannot be weakened (in the presence of Decentralizability) to Allocation-No-Envy: there exists a social choice rule that satisfies Allocation Independence, Decentralizability and Allocation-No-Envy, and that $\varphi_Z(R) \nsubseteq W^e_Z(R)$ for some $R \in \mathcal{R}^n$. For example, define the social choice rule Ψ by $\Psi(R) = D(R) \cap F^A(R)$ for all $R \in \mathcal{R}^n$. Then, Ψ satisfies the above three properties, and yet $\Psi_Z(R) \supset W^e_Z(R), \Psi_Z(R) \neq W^e_Z(R)$ for some $R \in \mathcal{R}^n$.

To completely characterize the equal income Walras rule using the notion of Overall-No-Envy, we turn to the following proposition which uses Independence.

Proposition 8
(i) *The equal income Walras rule W^e satisfies Independence, Decentralizability and Overall-No-Envy.*
(ii) *If a social choice rule φ satisfies Independence, Decentralizability and Overall-No-Envy, then $\varphi(R) \subseteq W^e(R)$ for all $R \in \mathcal{R}^n$.*

Proof. It is clear that the social choice rule W^e satisfies Independence, Decentralizability and Overall-No-Envy.

Let φ be a social choice rule satisfying Independence, Decentralizability and Overall-No-Envy. Suppose that there exist $R \in \mathcal{R}^n$ and $(x, B) \in \varphi(R)$ such that $(x, B) \notin W^e(R)$. Following the same argument as in the proof of Proposition 7, there exist $h, j \in N$ such that $B_h \subset \text{int}B_j$. Let $R' \in \mathcal{R}^n$ be such that for all $i \in N$, ${R'}_i^A = R_i^A$ and ${R'}_i^O = R_i^O$, and every i is a non-consequentialist at R'. By Independence, $(x, B) \in \varphi(R')$. Agent h being a non-consequentialist at R', it follows that $(x_j, B_j)\bar{P}'_h(x_h, B_h)$, and hence $(x, B) \notin \bar{F}(R')$. However, since φ satisfies Overall-No-Envy and $(x, B) \in \varphi(R')$, we must have $(x, B) \in \bar{F}(R')$. This is a contradiction. Therefore, for all $R \in \mathcal{R}^n$, $\varphi(R) \subseteq W^e(R)$. ■

It may be remarked that there exists a social choice rule φ such that $\varphi(R) \subseteq W^e(R)$ for all $R \in \mathcal{R}^n$, and that violates Independence and Allocation Independence. The next example shows this fact. Define ψ as follows: if agent 1 is a non-consequentialist at $R \in \mathcal{R}^n$, then

$$\psi(R) = \{(x, B) \mid (x, B) \in W^e(R) \text{ and } \forall (y, C) \in W^e(R), x_1 \; R_1^A \; y_1\}$$

and otherwise

$$\psi(R) = W^e(R)$$

Note that there exist $R, R' \in \mathcal{R}^n$ such that
(i) $R_i^A = R'^A_i, R_i^O = R'^O_i$ for all $i \in N$
(ii) there exist $x, y \in W_Z^e(R) = W_Z^e(R'), x \neq y$ such that $x_1 \; P_1^A \; y_1$
(iii) agent 1 is a non-consequentialist at R but is not a non-consequentialist at R'.
For such R, R', $\psi(R) \neq \psi(R')$ and $\psi_Z(R) \neq \psi_Z(R')$ even though $R_i^A = R'^A_i, R_i^O = R'^O_i$ for all $i \in N$. Hence, ψ violates Independence and Allocation Independence.

Our final remark is that there are social choice rules that satisfy Independence, Decentralizability and Allocation-No-Envy, and such that $\varphi(R) \not\subseteq W^e(R)$ for some $R \in \mathcal{R}^n$. An example is the correpondence Ψ defined in the second remark to Proposition 7.

6 Concluding Remarks

We have developed an extended framework in which an agent is endowed with three preference orders: an allocation preference order, an opportunity preference order and an over-all preference order. The extended framework reflects the idea that an agent cares not only about his final consumption bundle but also about the opportunity set from which his consumption bundle is chosen. Using this framework, we have examined some classical problems such as Pareto-optimality and equity-as-no-envy of competitive market mechanisms.

In this paper, we have accomplished the following. First, we have demonstrated that the classical notions like Pareto-optimality and equity-as-no-envy can be naturally extended to examine desirability of not only allocations but distributions of opportunities or pairs of allocations and opportunities as well. Secondly, we have shown that, under certain conditions, competitive market mechanisms can achieve allocation-Pareto-optimality, opportunity-Pareto-optimality and overall-Pareto-optimality simultaneously. However, the conditions for these claims to be true are very restrictive. For the purpose of comparison, we have constructed examples to show the difficulty for competitive market mechanisms to achieve both allocation-Pareto-optimality and opportunity-Pareto-optimality as well as both allocation-Pareto-optimality and overall-Pareto-optimality. Finally, we have established some axiomatic characterizations of the equal income Walras rule that figures prominently in the theory of fair allocation for exchange economies.

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