Fiscal Policy Switching:
Evidence from Japan, US, and UK

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Abstract

This paper estimates fiscal policy feedback rules in Japan, the United States, and the United Kingdom, allowing for stochastic regime changes. Using Markov-switching regression methods, we find that the Japanese data clearly reject the view that fiscal policy regime is fixed; i.e., the Japanese government has been adopting either of Ricardian or Non-Ricardian policy *at all times*. Instead, our results indicate that fiscal policy regimes evolve over time in a stochastic manner. This is in a sharp contrast with the U.S. and U.K. results in which the government's fiscal behavior is consistently characterized by Ricardian policy.

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1 Introduction

Recent studies about the conduct of monetary policy argue that fiscal policy regime has important implications for the choice of desirable monetary policy rules, in particular, monetary policy rules in the form of inflation targeting (Sims (2005), Benigno and Woodford (2006)). Needless to say, we can safely believe that fiscal regime during the peace time is characterized as "Ricardian" in the terminology of Woodford (1995), or "passive" in the terminology of Leeper (1991). In such a case, we are allowed to design an optimal monetary policy rule without paying any attention to fiscal regimes. However, if the economy is unstable in terms of fiscal situations, it would be dangerous to choose a monetary policy rule independently of fiscal policy regimes. For example, some researchers argue that rapid accumulation of public debt in Japan is an evidence for the lack of fiscal discipline of the Japanese government.[1] If this is the case, it would be possible that participants in the government bond market will come to have doubts about the government's intention to repay public debt. Given this environment, it would not be desirable to design a monetary policy rule without paying any attention to the future evolution of fiscal policy regime. The purpose of this paper is to estimate fiscal policy feedback rules in Japan, the United States, and the United Kingdom for more than a century, so as to acquire a deeper understanding about the evolution of fiscal policy regime.

One of the most important features of the recent studies on fiscal policy rules is the recognition that fiscal policy regime is *not* fixed over time, but it evolves in a stochastic way.[2] For example, Favero and Monacelli (2005) and Davig and Leeper (2005) estimate fiscal policy rules for the United States during the postwar period, under the assumption that there are two alternative fiscal regimes, i.e. "passive" and "active", and that stochastic fluctuations between the two regimes may be characterized by a Markov process. They find that fiscal regime switching occurred fairly frequently: Davig and Leeper (2005) reports that there were twelve fiscal regime changes during the period of 1948-2004; Favero and Monacelli (2005) find that fiscal policy was even more unstable than monetary policy.[3]

However, these pioneering works still have some shortcomings. First, they do not make an empirical distinction between *locally* and *globally* Ricardian policy rules. For example, Favero and Monacelli

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[1] See, for example, @@@. Iwamura et al. (2005) provides an empirical evidence against that view.

[2] Afonso (2005) provides a comprehensive list of the recent empirical studies on fiscal policy rules.

[3] These studies are in a sharp contrast with researches on fiscal sustainability initiated by Hamilton and Flavin (1986) about two decades ago, which typically investigate whether fiscal variables such as the debt-GDP ratio are characterized by a stationary or a nonstationary process without any break (Trehan and Walsh (1988, 1991), Wilcox (1989), Ahmed and Rogers (1995)). Given recent experiences in Japan, US, and EU, the assumption of no change in fiscal policy regime seems to be unrealistic.

(2005) specifies a locally Ricardian rule, and asks whether the US government follows this rule or deviates from it. However, as pointed out by Bohn (1998) and Canzoneri et al. (2001), the transversality condition may be satisfied even if the debt-GDP ratio is not a stationary process, or equivalently, even of a government deviates from a locally Ricardian policy rule. Second, they do not pay much attention to a government's tax smoothing behavior. As pointed out by Barro (1986) and Bohn (1998), the tax-smoothing behavior could create a negative correlation between public debt and primary surplus. Without properly controlling for it when estimating a government's reaction function, researchers are easy to get a biased estimate for fiscal policy reaction to an increase in public debt. Third, their empirical method based on maximum likelihood estimation ignores the possibility that the debt-GDP ratio is nonstationary in the long run. Specifically, maximum likelihood estimators fail to follow a standard limiting distribution if there are nonstationary regimes that are visited too often or for too long time, so that the debt-GDP ratio does not satisfy stationarity even asymptotically.[4]

The main findings of the paper are as follows. First, the Japanese data clearly reject the view that fiscal policy regime is fixed; i.e., the Japanese government has been adopting either of Ricardian or Non-Ricardian policy *at all times*. Instead, our results indicate that fiscal policy regimes evolve over time in a stochastic manner. This is in a sharp contrast with the U.S. and U.K. results in which the government's fiscal behavior is consistently characterized by at least globally Ricardian policy. Second, the Japanese government had a strong fiscal discipline before the 1920s, which is consistent with the fact that the government had been forced to maintain balanced budget under the gold standard system until its termination in 1917. Third, the Japanese government lost fiscal discipline during the WWII. The estimated date of restoring discipline after the war is consistent with the fact that fiscal restructuring led by the allied powers started in late 1948. Fourth, we find that the Japanese government has been deviating even from a globally Ricardian rule over the last thirty years. Moreover, some of our results indicate that the debt-GDP ratio is nonstationary not only within a regime, but also in the long run.

The rest of this paper is organized as follows. Sections 2 and 3 explain our empirical approach,

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[4]The asymptotic properties of MLE have been addressed only recently. To our knowledge, the only existing result of asymptotic normality of MLE is given by Douc et al. (2004). A critical condition of such a result is that the observed process is stable in the long run ("Harris recurrent"), in a sense that the impact of a shock vanishes in the long run. It is indeed possible to switch between two AR (p) processes, one stable and one unstable. In that case, however, the unstable regime must not be visited too often or for too long time. This in turn imposes conditions on the transition probabilities of the Markov chain (Francq and Zakoian (2001)). Given that our ultimate concern in the present paper is to know whether or not fiscal variables such as the debt-GDP ratio are indeed stable or not in the long run, it would not be appropriate to use MLE.

and Section 4 explains our data set. Section 5 presents regression results. Section 6 concludes the paper. Appendix A provides detailed explanation about our data set.

2 Ricardian fiscal policy

2.1 Government's budget constraint

We start by looking at the government's budget constraint. Let us denote the nominal amount of public debt and base money at the end of period t by B_t and M_t. Also, we denote the one-period nominal interest rate starting in period $t-1$ by i_{t-1}, the nominal government expenditure (excluding interest payments) and the nominal tax revenue in period t by G_t and T_t. Then, the consolidated body of the government and the central bank is subject to a flow budget constraint of the form:

$$M_t + B_t = (1 + i_{t-1})B_{t-1} + M_{t-1} + (G_t - T_t).$$

Dividing both sides of this equation by the nominal GDP, Y_t, we obtain:

$$m_t + b_t = \frac{1 + i_{t-1}}{1 + n_t}b_{t-1} + \frac{1}{1 + n_t}m_{t-1} - s_t,$$

where m_t, b_t, s_t, and n_t are defined by

$$m_t \equiv \frac{M_t}{Y_t}; \ b_t \equiv \frac{B_t}{Y_t}; \ s_t \equiv \frac{T_t - G_t}{Y_t}; \ n_t \equiv \frac{Y_t - Y_{t-1}}{Y_{t-1}}.$$

Denoting the total liabilities for the consolidated body by $w_t(\equiv m_t + b_t)$, the transition equation for w_t can be expressed as:

$$w_t - w_{t-1} = \frac{i_{t-1}}{1 + n_t}w_{t-1} - \frac{n_t}{1 + n_t}w_{t-1} - \left[\frac{i_{t-1}}{1 + n_t}m_{t-1} + s_t\right]. \tag{1}$$

Note that $\frac{i_{t-1}}{1+n_t}m_{t-1}$ represents seignorage, and that an increase in primary surplus s_t or seignorage reduces the total liabilities. Also, note that an increase in the nominal growth rate n_t contributes to lowering the total liabilities through the second term on the right-hand side, $-\frac{n_t}{1+n_t}w_{t-1}$, which is sometimes called "growth dividend" (Bohn (2005)).

Equation (1) can be rewritten as:

$$w_t = q_t[w_{t+1} + s_{t+1}] + \frac{i_t}{1 + i_t}m_t, \tag{2}$$

where q_t represents a discount factor that is defined by

$$q_t \equiv \frac{1 + n_{t+1}}{1 + i_t}.$$

Integrating equation (2) forward from the current period and taking expectations conditional on information available in period t, we obtain a present-value expression of the budget constraint:

$$w_t = s_t + E_t \sum_{j=1}^{T} \left(\prod_{k=0}^{j-1} q_{t+k} \right) s_{t+j} + E_t \sum_{j=0}^{T-1} \left(\prod_{k=0}^{j-1} q_{t+k} \right) \left(\frac{i_{t+j}}{1+i_{t+j}} \right) m_{t+j} + E_t \left(\prod_{k=0}^{T-1} q_{t+k} \right) w_{t+T}.$$

This implies that the transversality condition is given by

$$\lim_{T \to \infty} E_t \left(\prod_{k=0}^{T-1} q_{t+k} \right) w_{t+T} = 0. \tag{3}$$

2.2   Locally Ricardian

Woodford (1995) proposes to call a fiscal policy commitment "Ricardian" if it implies that the transversality condition, equation (3), necessarily holds for all possible paths of endogenous variables (in particular, prices). More specifically, Woodford (1995, 1998) propose two types of Ricardian fiscal policy rule.

The first type, which is referred to as "locally Ricardian", can be expressed as:

$$s_t + \frac{i_{t-1}}{1+n_t} m_{t-1} = \left[\lambda_t + \frac{i_{t-1}}{1+n_t} \right] w_{t-1} + \nu_t, \tag{4}$$

where λ_t is a time-varying but positive parameter that represents the government's responsiveness to changes in total liabilities, and ν_t is an exogenous stationary variable. Note that the left hand side of equation (4) represents the sum of primary surplus and seignorage. Equation (4) requires the government to create a surplus in period t more than enough to cover its interest payment in that period, $\frac{i_{t-1}}{1+n_t} w_{t-1}$.

By substituting (4) into (1), we can fully characterize the dynamics of w_t:

$$w_t = \left[1 - \lambda_t - \frac{n_t}{1+n_t} \right] w_{t-1} - \nu_t. \tag{5}$$

Under the assumption that n_t is an exogenous process (i.e., the government treats its value as exogenously given when it makes a fiscal decision in period t),[5] this equation implies that w_t would be a stationary process and thus satisfies the transversality condition if the sum of λ_t and $\frac{n_t}{1+n_t}$ is less than unity.[6]

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[5]It is possible that n_t could be an endogenous variable in the sense that government's fiscal behavior could have non-negligible consequences on the path of n_t. For example, as argued by Woodford (2001) among others, it might be possible that if the government does not react at all to changes in total liabilities (namely, $\lambda_t = 0$), then inflation endogenously emerges ($n_t > 0$), and consequently the coefficient on w_{t-1} in equation (5) becomes less than unity.

[6]To be precise, w_t is a stationary process if $-\frac{n_t}{1+n_t} < \lambda_t < 2 - \frac{n_t}{1+n_t}$.

An alternative specification to equation (4) would be of the form:

$$s_t + \frac{i_{t-1}}{1+n_t}m_{t-1} + \frac{n_t}{1+n_t}w_{t-1} = \left[\lambda_t + \frac{i_{t-1}}{1+n_t}\right]w_{t-1} + \nu_t. \tag{6}$$

Now the government seeks to adjust the sum of primary surplus, seignorage, and growth dividend in response to changes in total liabilities. An important difference from equation (4) is that the government reduces primary surplus when growth dividend is positive, for example, due to high inflation, and increases it when it is negative; on the other hand, equation (4) requires the government to create primary surplus independently of the level of growth dividend. It is straightforward to see that the transition equation corresponding to (5) is now given by

$$w_t = [1 - \lambda_t]w_{t-1} - \nu_t, \tag{7}$$

and that w_t is a stationary process if λ_t satisfies the condition that $0 < \lambda < 1$.

Favero and Monacelli (2005) adopts a policy reaction function very close to equation (6). According to their definition, the government with fiscal discipline seeks to make primary deficit lower than "debt-stabilizing deficit", which is given by

$$-\left[\frac{i_{t-1}}{1+n_t} - \frac{n_t}{1+n_t}\right]w_{t-1}.$$

Given this definition, debt-stabilizing deficit would become positive if n_t takes a sufficiently large positive value, then the government is allowed to run a deficit.

2.3 Globally Ricardian

The idea that the government should create surplus at least to cover its interest payments seems to be a useful one from the practical point of view,[7] but the transversality condition does not necessarily require it. Specifically, as shown by Bohn (1998) and Canzoneri et al. (2001), the transversality condition could be satisfied even if the government reacts to an increase in total liabilities by less than the amount needed to cover its interest payments. This is the second type of Ricardian, which is referred to as "globally Ricardian".

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[7]If we rewrite equation (4) as $s_t - \frac{i_{t-1}}{1+n_t}b_{t-1} = \lambda_t w_{t-1} + \nu_t$, we see that the rule requires that not primary surplus but traditional fiscal surplus (i.e., primary surplus less interest payment) should be adjusted in response to a change in total liabilities, which is in the same spirit of the Maastricht Treaty and the Stability and Growth Pact. See Woodford (2001) for more on this.

Globally Ricardian policy can be expressed as:

$$s_t + \frac{i_{t-1}}{1+n_t} m_{t-1} = \gamma_t w_{t-1} + \nu_t, \tag{8}$$

where γ_t is a time-varying and positive parameter. Note that equations (4) and (6) require the government to create primary surplus enough to cover its interest payments in each period, while the government is now allowed to issue additional public debt to pay interest on the existing debt at the beginning of that period. Under this policy rule, the dynamics of w_t is now given by:

$$w_t = \left[1 - \gamma_t - \frac{n_t}{1+n_t} + \frac{i_{t-1}}{1+n_t} \right] w_{t-1} - \nu_t \tag{9}$$

or

$$w_t = \left[\frac{1}{q_{t-1}} - \gamma_t \right] w_{t-1} - \nu_t, \tag{10}$$

which implies that the transversality condition (equation (3)) is satisfied if $0 < \gamma_t < 1$. Note that this condition does not necessarily guarantee that w_t is a stationary process; in fact, it allows w_t to grow forever, but at a rate lower than an interest rate in each period. In that sense, a globally Ricardian rule imposes a weaker requirement on government behavior as compared with a locally Ricardian rule.

Bohn (1998, 2005) adopts a policy reaction function very close to equation (8), and look at the US data to know whether γ_t is positive or not.[8] Equation (8) is an appropriate estimating equation when the government adopts globally Ricardian policy, or when it actually adopts locally Ricardian policy but interest rates do not fluctuate so much during the sample periods. In the latter case, we would be able to empirically distinguish between locally and globally Ricardian just by looking at whether or not the estimated coefficient on w_{t-1} is greater than the sample average of nominal interest rates. However, if the government adopts locally Ricardian policy and fluctuations in interest rats are not so small, then Bohn's specification might not be appropriate. For example, the estimated coefficient on w_{t-1} would be biased towards zero if fluctuations in interest rates are quite large during the sample period, while those in public debt are negligibly small.

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[8]However, Bohn (1998, 2005) do not consider the possibility that fiscal regime evolves over time in a stochastic way.

3 Estimation method

3.1 Estimating equations

Given each of the above definitions of Ricardian fiscal policy, we estimate an equation of the form:

$$b_t = \begin{cases} \mu_0 + (\alpha_0 + \eta_t)b_{t-1} + u_{0t}, & \text{if} \quad S_t = 0 \\ \mu_1 + (\alpha_1 + \eta_t)b_{t-1} + u_{1t}, & \text{if} \quad S_t = 1 \end{cases} \tag{11}$$

where $u_{it} = \varepsilon_{it} - \nu_t$ with $\varepsilon_{it} \sim i.i.d.N(0, \sigma_i^2)$. $\{S_t \in (0,1)\}$ is a two-state Markov chain with transition probabilities $p_{ij} = \Pr(S_t = j \mid S_{t-1} = i)$. Note that, given that the current regime is i, the expected average duration of staying in the same regime is $(1 - p_{ii})^{-1}$. Also, note that we use public debts issued by the government b_t as a dependent variable, rather than the total liabilities w_t, assuming that the base money m_t is small relative to the public debt, and that fluctuations in seignorage play much less important role compared with those in primary surplus.

Depending on the definition of observable variables, namely η_t and ν_t, we specify four different estimating equations:

Specification 1 $\eta_t = \nu_t = 0$: This is a benchmark case in which no exogenous variables are included. Hence, b_t follows a simple Markov-switching AR(1) process.

Specification 2 $\eta_t = 0$, and $\nu_t = -g_t^m$: This is a case in which the government's tax smoothing behavior is incorporated through g_t^m (military expenditures relative to GDP). As pointed out by Barro (1986) and Bohn (1998), the government's tax-smoothing behavior could create a negative correlation between public debt and primary surplus. To illustrate this, consider the situation in which the government increases its expenditures, but only temporarily (such as in the case of a war). The government could increase taxes simultaneously by the same amount in accordance with the increase in expenditures, but it is costly to change marginal tax rates over time, since it increases the excess burden of taxation. Recognizing this, an optimizing government seeks to smooth marginal tax rates over time. This implies that a temporary increase in government expenditures would lead to a decrease in primary surplus and an increase in public debt. Bohn (1998) argues that such a negative correlation between primary surplus and public debt should be properly controlled for when estimating

a government's reaction function; otherwise researchers are easy to get an imprecise estimate for fiscal policy reaction to an increase in public debt. Bohn (1998, 2005) shows that empirical results for the U.S. sharply differ depending on whether or not temporary government expenditures are included as an independent variable, while Iwamura et al. (2005) results a similar finding for Japan during the post-war period.

Specification 3 $\eta_t = -\frac{n_t}{1+n_t}$, and $\nu_t = -g_t^m$: This specification corresponds to equation (5) with $\nu_t = -g_t^m$ and $\alpha_i = 1 - \lambda_i$. Note that when n_t is very close to 0, specification 3 reduces to specification 2. This condition might hold in a very stable economy without any experience of high inflation, but unfortunately, this is not the case in many countries including Japan, which experienced three-digit inflation just after the end of the WWII. Of course, Japan is not an exception, and one can easily find other examples in which the accumulation of public debt leads to uncontrollably high inflation. Specifications 3 and 4 are not identical for those countries.

Note that locally Ricardian policy requires that α_i should be smaller than unity, while stationarity of the debt-GDP ratio requires that the coefficient on b_{t-1}, $\alpha_i + \eta_t$, is less than unity. These two conditions are closely related but not identical except for the case of $n_t = 0$.

Specification 4 $\eta_t = -\frac{n_t}{1+n_t} + \frac{i_{t-1}}{1+n_t}$, and $\nu_t = -g_t^m$: This corresponds to equation (9) with $\nu_t = -g_t^m$ and $\alpha_i = 1 - \gamma_i$. This differs from specification 3 in that interest payments, $\frac{i_{t-1}}{1+n_t}$, is included in η_t, reflecting tha fact that the government is not required to create surplus to cover its interest payments. Note that globally Ricardian policy requires that $\alpha_i - \frac{n_t}{1+n_t}$ should be less than unity, implying that, when n_t is always equal to zero, b_t continues to grow forever, but at a rate lower than the borrowing cost in each period.

3.2 Estimation

We estimate equation (11) by employing a Bayesian approach via the Gibbs sampler, instead of a classical approach based on maximum likelihood (ML) estimation. The Bayesian approach has the following advantages. First, the ML estimation has a potential disadvantage in that inference on S_t is conditional on the estimates of the unknown parameters. We estimate the parameters of the model, then making inference on S_t conditional on the estimates of the parameters, as if we certainly know the true values of the parameters. In contrast, the Bayesian approach allows both the unknown

parameters and S_t to be random variables. Therefore, inference on S_t is based on the joint distribution of the parameters and S_t (See Albert and Chib (1993)).

Second, ML estimators follow a non-standard limiting distribution when the process is nonstationary in the long run (or globally nonstationary). To our knowledge, such limiting distributions have not been derived in Markov-switching models. On the other hand, the Bayesian method can approximate the joint and marginal distributions of the parameters and S_t via the Gibbs sampler. The method is valid even when the observed process exhibits non-stationarity (or explosive) behavior in the long run. To illustrate this point, suppose there are two fiscal policy regimes: one is a stable regime in which the debt-GDP ratio is characterized by a stationary process, and the other one is an unstable regime in which the debt-GDP ratio is characterized by a nonstationary process. Note that the mere existence of an unstable regime does not necessarily imply global unstability: the system could still be globally stable if the unstable regime is not visited too often or for too long time. In this sense, the transition probabilities of the Markov chain are important determinants of global stability or unstability. On the other hand, as shown by Francq and Zakoian (2001), it is possible that the system is globally unstable even when both of the two regimes are stable. An important thing to be emphasized here is that it would not be appropriate to employ MLE if researchers have only limited knowledge about global stability of the system.[9]

Third, for the Markov switching models, the likelihood is often not uni-modal but multi-modal. Therefore, numerical algorithms such as EM and Newton-Rapson sometimes converge to a local maximum on the likelihood surface. This is a typical problem facing with data in practice, regardless of which optimization algorithms are used. Maddala and Kim (1998) refer to the fragility of the ML estimation method because multiple local maxima are often found.

3.3 Gibbs Sampling

Albert and Chib (1993) is the first to present a Bayesian analysis of the Markov-switching models using the Gibbs sampler. The Gibbs sampler is to approximate the joint and marginal distributions of the parameters of interest from conditional distributions of subsets of parameters given the others (See Kim and Nelson (1999) for an introduction of Gibbs sampling). It is useful in this case because

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[9]An alternative empirical framework to study fiscal regime shifts would be to use a methodology proposed by Bai and Perron (1998), in which a multiple linear regression with m breaks (or m+1 regimes) are investigated within the classical framework. However, it requires the process to be weakly stationary in each regime. Therefore, their method cannot be applied in our context.

the joint distributions are hard to obtain.

Following Kim and Nelson (1999), we here describe a procedure to estimate the following models:

$$
b_t^* = \begin{cases} \mu_0 + \alpha_0 b_{t-1} + \varepsilon_{0t}, & \text{if} \quad S_t = 0 \\[2mm] \mu_1 + \alpha_1 b_{t-1} + \varepsilon_{1t}, & \text{if} \quad S_t = 1 \end{cases}
$$

where $b_t^* = b_t - \eta_t b_{t-1} + \nu_t$ and $\varepsilon_{it} \sim i.i.d.N(0, \sigma_i^2)$ for $i = 0, 1$ with $\sigma_{S_t}^2 = \sigma_0^2(1 + h_1 S_t)$ and $h_1 > 0$. $\{S_t \in (0, 1)\}$ is a two-state Markov chain with transition probabilities $p_{ij} = \Pr(S_t = j \mid S_{t-1} = i)$.

3.3.1 Prior Distributions

We describe the choice of priors for the unknown parameters. Let $\widetilde{h}_1 = 1 + h_1$ with $h_1 > 0$. Then, the priors are the following:

$$
\begin{aligned}
\mu_i &\sim N(\psi, \omega^{-1}), \ \ \alpha_i \sim N(\phi, c^{-1}), \\
\sigma_0^2 &\sim IG(\frac{\upsilon}{2}, \frac{\delta}{2}), \ \ \widetilde{h}_1 \sim IG(\frac{\upsilon}{2}, \frac{\delta}{2})_{1(\widetilde{h}_1 > 1)}, \\
p_{11} &\sim beta(u_{11}, u_{10}), \ \ p_{00} \sim beta(u_{00}, u_{01})
\end{aligned}
$$

The parameters used are $\psi = 0$, $\omega = 25$, $\phi = 0$, $c = 1$, $(\upsilon, \delta) = (0, 0)$, $u_{00} = u_{11} = 8$, and $u_{10} = u_{01} = 2$. Hence the prior of σ_i^2 is non-informative. Other parameters are chosen so that priors are informative but relatively diffused. The means and standard deviations of the prior distributions are presented in the following table.

Priors for the parameters

	Distribution	Mean	Std
μ_i	Normal	0.00	0.20
α_i	Normal	0.00	1.00
p_{ii}	Beta	0.80	0.12
σ_0^2	Inverted Gamma	—	—
$\widetilde{h}_1$	Inverted Gamma	—	—

3.3.2 Computational Algorithm

The needed posterior conditional distributions for implementing Gibbs sampling are easily obtained from the priors and the assumptions of the data generating process. The following steps 1 through 5 are iterated to obtain the joint and marginal distributions of the parameters of interest.

Step 1: Generate p_{11} and p_{00} conditional on $\widetilde{S}_T = (S_1, ..., S_T)$. Let n_{ij} refers to the total number of transition from state i to j, which can be counted from $\widetilde{S}_T$. Then,

$$p_{11} \mid \widetilde{S}_T \quad \sim \quad beta(u_{11} + n_{11}, u_{10} + n_{10})$$

$$p_{00} \mid \widetilde{S}_T \quad \sim \quad beta(u_{00} + n_{00}, u_{01} + n_{01})$$

Step 2: Generate μ_i conditional on $\widetilde{S}_T$, σ_i^2, and ϕ_i: We have the regression $y_t = \mu_i + \varepsilon_{it}$ where $y_t = b_t^* - \alpha_i b_{t-1}$ for $t \in \{t : S_t = i\}$. Hence, the posterior distribution is $\mu_i \sim N(\psi_*, \omega_*^{-1})$ where

$$\omega_* = \sum_{t \in \{t:S_t=i\}} 1/\sigma_i^2 + \omega, \; \psi_* = \omega_*^{-1} \left[\sum_{t \in \{t:S_t=i\}} y_t/\sigma_i^2 + \omega\psi \right]$$

Step 3: Generate α_i conditional on $\widetilde{S}_T$, σ_i^2, and μ_i: Let $d_t^* = b_t^* - \mu_i$, then we have the regression $d_t^* = \alpha_i b_{t-1} + \varepsilon_{it}$ for $t \in \{t : S_t = i\}$. Hence, the posterior distribution is $\phi_i \sim N(\phi_{i*}, c_{i*}^{-1})$ where

$$c_{i*} = \sum_{t \in \{t:S_t=i\}} b_{t-1}^2/\sigma_i^2 + c, \; \phi_{i*} = c_{i*}^{-1} \left[\sum_{t \in \{t:S_t=i\}} b_{t-1}d_t^*/\sigma_i^2 + c\phi \right]$$

Step 4: Generate σ_0^2 and σ_1^2 conditional on $\widetilde{S}_T$, μ_i, and α_i: We first generate σ_0^2 conditional on h_1 and then generate $\widetilde{h}_1 = 1 + h_1$ to indirectly generate σ_1^2. Conditional on h_1, the posterior distribution of σ_0^2 is as follows:

$$\sigma_0^2 \sim IG\left(\frac{v_{0*}}{2}, \frac{\delta_{0*}}{2}\right)$$

where

$$v_{0*} = v + T$$
$$\delta_{0*} = \delta + RSS_0 + RSS_1/(1 + h_1)$$

with $RSS_i = \sum_{t \in \{t:S_t=i\}} (b_t^* - \mu_i - \alpha_i b_{t-1})^2$. Conditional on σ_0^2, the posterior distribution of $\widetilde{h}_1 = 1 + h_1$ is as follows:

$$\widetilde{h}_1 \sim IG\left(\frac{v_{1*}}{2}, \frac{\delta_{1*}}{2}\right)_{1(\widetilde{h}_1 > 1)}$$

where

$$v_{1*} = v + T_1$$
$$\delta_{1*} = \delta + RSS_1/\sigma_0^2$$

with $T_1 = \sum_{t=1}^{T} S_t$. Once $\widetilde{h}_1$ is obtained, we can calculate σ_1^2.

Step 5: Generate $\widetilde{S}_T = (S_1, ..., S_T)$ conditional on other parameters. It is based on multi-move Gibbs-sampling, introduced by Carter and Kohn (1994) in a state-space model. Here the procedure for generating $\widetilde{S}_T$ using the multi-move Gibbs-sampling is same as Kim and Nelson (1999).

We iterate step 1 through 5 $M + N$ times. Discard the realizations of the first M iterations but keep the last N iterations to form a random sample of size N on which statistical inference can be made. M must be sufficiently large that the Gibbs sampler has converged. N is also chosen to large enough to obtain the precise empirical distributions. Under these considerations, we set $M = 5000$ and $N = 10000$.

4   Data

We construct a data set covering 1886-2004 for Japan, 1840-2005 for the United States, and 1830-2003 for the United Kingdom.[10]

4.1   Japan

Nominal GDP and government expenditures   A single data set covering the entire sample period is not available, so that we collect data from various sources and link them in a consistent way. As for the period after 1929, we use a data set produced by the Japanese government (various versions of SNA). As for the period before 1930, we basically use "Estimates of Long-Term Economic Statistics of Japan since 1868" (LTES) produced by K. Ohkawa, M. Shinohara, and M. Umemura.[11]

We need to make various adjustments to link the different data sets in a consistent way. First, the LTES reports figures only for the combined body of the general government and various public enterprises for the period of 1885-1969, so that we need to construct a data set only for the general government. To do so, we collect information about profits and capital formations for public enterprises from their annual reports and so on, and subtract them from the LTES figures. Second, the data is completely missing during the final stage of the WWII (FY1944 and 1945). To fill this, we construct a series of real GDP by using a production index as a proxy,[12] and a series of GDP deflator by

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[10]See Appendix A for details.

[11]We use various definitions of the general government; the definition by the Economic Counsel Board for 1885-1954, OLD SNA for 1955-1969, 68SNA for 1970-1979, and 93SNA for 1980-2004. Note that these definitions are slightly different from each other, because special accounts held by the central government and business accounts held by local governments are sometimes classified as a part of the general government, and sometimes not.

[12]A similar methodology was employed by the Japanese central bank in its various publications on financial and economic activities around the end of the WWII. (Bank of Japan (1950))

using an aggregate price index produced by the Economic Planning Agency. With regard to general government expenditures, we use "Net Total of General Government Expenditures," which is reported in Emi and Shionoya (1966).

Public debt Outstanding amount of public debt at the end of each fiscal year has been made available by the Ministry of Finance since the 1880s, but these are figures reported in budget documents, which are slightly different from the SNA definition. We make adjustments by excluding the amount of public debt accumulated in the colonial special accounts, which is held by the central government and enterprise special accounts.

4.2 US and UK

For the United States, our main data sources are "Historical Statistics of the United States" (Carter et al. (2006)) and "Historical Tables, Budget of the United States Government". For the United Kingdom, they are "British Historical Statistics" (Mitchell (1988)), Public Sector Finances Databank by HM Treasury and "Annual Abstract of Statistics" published by Office for National Statistics.

5 Empirical results

5.1 Unit root tests

Table 1 conducts the standard ADF tests for the debt-GDP ratio in Japan, the United States, and the United Kingdom. Specifically, we run a AR (p) regression of the form:

$$b_t = \mu + \alpha b_{t-1} + \sum_{j=1}^{p-1} \phi_j \triangle b_{t-j-1} + u_t \tag{12}$$

for the entire sample period with no break. We repeat a similar regression with various lag lengths ($p = 1$ to 10). According to the table, the estimates of α are very close to unity in all of the three countries, and the null hypothesis $H_0 : \alpha = 1$ cannot be rejected for each of the three countries.

Does this necessarily imply that the debt-GDP ratio follows a unit root process throughout the entire sample period, or its time series property evolves over time? To see this, Figure 2 conducts rolling regressions of equation (12) with the window of 40 years. For example, the estimated value for the year of 1925 is from a regression conducted over the period of 1885 to 1925. The lag length is set at $p = 2$. The result for Japan, shown in Panel A, reveals the following. First, the estimate of α shows

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a very large fluctuation, ranging from 0.6 to 1.1. This indicates that the time series property of the debt-GDP ratio changes significantly over time. Second, the estimate of α shows a sharp rise during the WWII, suggesting that the Japanese government lost fiscal discipline during this period. On the other hand, it shows a sharp decline during the period just after the war, probably reflecting the fact that very high inflation during that period quickly eroded the real value of public debt (namely, partial default). Third, the value of α stays very close to (or slightly above) unity since the beginning of the 1990s, suggesting that the debt-GDP ratio follows a unit root process or even an explosive process during this recent period.

The estimates of α for U.S. and U.K., presented in Panels B and C, differ from the Japanese one in that they do not fluctuate so much. These estimates basically stay below unity, except that the U.S. estimate shows a sharp rise during the WWII. Also, they show a sharp decline during high inflation periods (1920s in the U.S. and U.K., and 1950s in the U.K.), again probably reflecting the fact that the real value of public debt quickly decreased due to high inflation.

5.2   Empirical results for Japan

Table 2 presents regression results for Japan using a two state Markov switching model. Panel A of the table is a benchmark regression in which no exogenous variables are included (specification 1). The estimate of α in regime 0 is 0.517, indicating that the debt-GDP ratio is characterized by a stationary process that converges to its mean quite (and probably, unrealistically) quickly. On the other hand, the estimate of α in regime 1 is 1.116, with its upper and lower bounds being 1.164 and 1.067, respectively. Since the 95 percent confidence interval exceeds unity, we can say that the debt-GDP ratio follows an explosive process. Panel A of Figure 3 presents the estimated probability of regime 1 in each year of the sample period, showing that the years except 1945-1970 belong to regime 1. The lower chart of Panel A shows the estimated coefficient on b_{t-1}, which is calculated as a weighted average of the coefficients in regimes 0 and 1, with the estimated probabilities of each regime being used as a weight. The shaded area represents the 95 percent confidence interval. Again, this chart indicates that the coefficient exceeds unity except the period of 1945-1970.

Panel B of Table 2 conducts a similar regression, but we now put military expenditures as an exogenous variable (specification 2). Regime 0 is again characterized by a stationary process, but the estimate of α in regime 1 is 1.082 with the lower bound of 1.042, again implying that the debt-

GDP ratio is characterized by an explosive process. The estimated probability of regime 1, which is presented in Panel B of Figure 3, is similar to Panel A, except that the probability is now lower in 1890-1905 and 1915-1920, reflecting the fact that growth dividend was higher during these years due to the emergence of high inflation.

Panel C of Table 2 reports regression results for the case in which military expenditures and $-\frac{n_t}{1+n_t} b_{t-1}$ are included as exogenous variables (specification 3). Again, regime 0 is characterized by a stationary process, and regime 1 by an explosive process. But a notable difference is that the estimate of α is much closer to unity, indicating that convergence to its mean is much slower than before. We can see this difference more clearly in the estimated coefficient on b_{t-1}, reported in Panel C of Figure 3, which now shows fluctuations only around unity. These differences can be interpreted as reflecting that the effect of growth dividend is now properly controlled for. Panel C of Figure 3 shows that the probability of regime 1 is close to unity in 1930-1950 and 1970-2004, while the probability of regime 0 is high in 1885-1925 and 1950-1970. These results suggest that the former periods are characterized by the lack of fiscal discipline, while the latter periods are characterized by a locally Ricardian rule. Also, the estimated transition probability is very close to unity ($p_{11} = 0.945$ and $p_{00} = 0.938$), indicating that the average duration of staying in a regime is about 17 years.

Panel D reports results for the case in which military expenditures and $\left(\frac{i_{t-1}}{1+n_t} - \frac{n_t}{1+n_t} \right) b_{t-1}$ are included as exogenous variables (specification 4). The results are basically the same as Panel C, but the estimate of α in regime 1 is now 1.053 with the lower bound being slightly closer to unity.

We may summarize findings from Table 2 and Figure 3 as follows. First, the Japanese government has made several large changes with respect to its fiscal behavior over the last 120 years. Second, Japanese fiscal policy is characterized by a locally Ricardian rule in 1885-1925 and 1950-1970. The former corresponds to the period in which the gold standard system was adopted: the Japanese government was forced to keep balanced budget in order to maintain the gold standard system until 1917, when the Japanese government decided to terminate it following the same decisions made by the core countries of the system.[13] The latter period is consistent with the fact that fiscal restructuring after the WWII started in December 1948, when the Supreme Commander for Allied Powers (SCAP) directed the Japanese government to implement balanced budget in order to stop runaway inflation.[14]

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[13]See Shizume (2001) for more on the Japanese government's fiscal behavior during the gold standard period.
[14]See, for example, Cohen (1950) and Yamamura (1967) for more details on the "Dodge Line".

5.3 Sensitivity analysis

AR (2) model In our baseline regressions reported in Table 2, we assume that the government adjusts primary surplus in period t in response to a change in public debt at the beginning of period t. Given that we use annual data, this seems to be a good approximation to actual policy making. However, as often pointed by researchers and practitioners, it usually takes more than one year before fiscal decision is finally made. If this is the case, our baseline specification might not be an appropriate one. To cope with such a potential problem, we change the lag structure of our estimating equation (equation (11)) into

$$
b_t = \begin{cases} \mu_0 + \alpha_0 \sum_{k=1}^{K} \omega_k b_{t-k} + \eta_t b_{t-1} + u_{0t}, & \text{if} \quad S_t = 0 \\ \\ \mu_1 + \alpha_1 \sum_{k=1}^{K} \omega_k b_{t-k} + \eta_t b_{t-1} + u_{1t}, & \text{if} \quad S_t = 1 \end{cases}
$$

where ω_k is a parameter representing the lag structure of fiscal decision making.[15] We conducted a lag search to end up with $K = 2$. Regression results reported in Table 3 and Figure 4 are almost the same as reported before, and the coefficient on $\triangle b_{t-1}$ ($\equiv b_{t-1} - b_{t-2}$), denoted by θ in Table 3, is very close to zero in each regime.

Alternative definition of military expenditures Regression results in Table 2 indicate that military expenditures play an important role as an independent variable. Table 4 uses an alternative definition of military expenditures to see the robustness of earlier findings. In our baseline regressions reported in Table 2, we use military expenditures spent only by the forces at home. However, a non-negligible portion of expenditures spent by the forces at overseas was financed by the general government through the issue of public debt at the final stage of the WWII, so that this portion must be included in our regression analysis.

Emi and Shionoya (1966) reports expenditures for the forces at overseas for the years of 1937-1945. Their figures mainly consist of (1) expenditures that were financed by transfers from colonial special accounts and enterprise special accounts, neither of which belong to the general government, and (2) those financed by the issue of public debt plus public borrowings. We construct an alternative military

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[15]Note that this specification differs from a partial adjustment model, such as the one adopted by Favero and Monacelli (2005), in that the coefficient on lagged values of b depends only on the current regime (not on past regimes).

expenditure series by estimating the first part and subtracting it from the total.[16] Figure 5 compares a new and old series for 1937-1945. We see almost no difference for 1937-1941, but the difference becomes non-negligible after that, reaching about 1.8 times in 1944.

Table 4 and Figure 6 report regression results using a new expenditure series. A notable difference from the earlier results is that the estimate of α in regime 0 is now very close to unity and its upper bound exceeds unity, so that we fail to reject the null of a unit root. These results indicate that it is very important to properly controlling for military expenditures in our regression analysis.

No restriction on the coefficient of interest payments   As we can see from equations (4) and (8), a sole difference between locally and globally Ricardian rules is what kind of restriction we impose on the coefficient of interest payments $\frac{i_{t-1}}{1+n_t}w_{t-1}$. Locally Ricardian rules impose a restriction that the coefficient should be equal to zero, while globally Ricardian rules impose a restriction that it should be unity. The former corresponds to specification 3, and the latter corresponds to specification 4. An important implication of these restrictions, whether it might be zero or it might be unity, is that these specifications allow a switching only between locally Ricardian rules and the other rules (i.e., rules that do not belong to locally Ricardian) in the case of specification 3, and a switching only between globally Ricardian rules and the other rules in the case of specification 4. These specifications would be inappropriate, for example, if policy switching occurs between locally and globally Ricardian rules.

To cope with this potential problem, Table 5 conducts a similar regression as before, but we now do not impose a priori restriction on the coefficient of interest payments. Specifically, we add a new dependent variable $\frac{i_{t-1}}{1+n_t}w_{t-1}$ to equation (11) with

$$\eta_t = -\frac{n_t}{1+n_t}; \ \nu_t = -g_t^m.$$

The coefficient on the new dependent variable should be close to zero if a true rule is well approximated by a locally Ricardian rule, and it should be unity in the case of globally Ricardian. The estimated coefficients presented in Table 5 indicate that it is 0.628 in regime 0 (stationary regime) and 0.506 in regime 1 (nonstationary regime). More importantly, the lower bound in regime 0 is 0.235, rejecting the null of zero, while the upper bound in regime 0 is slightly lower than unity (0.990), again rejecting the null of unity. This means that a true rule is not well approximated by the two extreme ones (namely, locally and globally Ricardian rules), but it is located between the two. The same results can be seen

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[16]See Appendix A for details about this procedure.

for regime 1.[17] However, the estimated values of α in Table 5 tend to fall between those obtained in specifications 3 and 4 of Table 2, confirming that the main results about fiscal policy behavior in Table 2 holds without any substantial modifications.

Three state model Table 6 checks the robustness of the findings in Table 3 by extending analysis to a three state model. Panel A, which reports regression results for specification 3, shows that regime 0 is characterized by a stationary process ($\alpha = 0.926$); regime 1 by an explosive process ($\alpha = 1.081$); and regime 2 by another highly explosive process ($\alpha = 1.313$). Figure 8 shows that the periods belonging to regime 1 in Figure 3 are again classified as regime 1,[18] suggesting that the number of regimes allowed in Table 2 (namely, two regimes) is not an inappropriate description of a true model. These results, together with results for specification 4, almost confirm the earlier findings: (1) the periods of 1885-1920 and 1950-1970 belong to regime 0 (regime with fiscal discipline); (2) the periods of 1920-1950 belong to regime 1 (regime without discipline).

5.4 Global stationarity or nonstationarity

Regression analysis in this section has sought to know whether the debt-GDP ratio is a stationary process within a regime. However, as we discussed in section 3.2, even if the ratio is stationary within a regime, it does not necessarily imply that it is stationary in the long run. This is simply because regime change occurs stochastically in accordance with transition probabilities. Thus what we have to know is where the debt-GDP ratio goes to in the long run, given policy shocks and transition probabilities, or put differently, whether its distribution converges over time to somewhere or not. The process is said to be globally stationary if the distribution converges to somewhere over time, while stationarity within a regime is called local stationarity. Global stationarity implies that the effect of policy shocks on the debt-GDP ratio becomes smaller and smaller over time and finally disappears in the long run. Turning to actual economic activities, investors in government bonds markets are interested in whether this global stationarity is satisfied or not, and policymakers, in particular central banks, are interested in this property when designing monetary policy rules.

Francq and Zakoian (2001) shows a somewhat interesting result about the relation between local

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[17]These results suggest that neither empirical researches focusing only on locally Ricardian rules nor those focusing only on globally Ricardian rules might be employing an appropriate estimating equation.

[18]The exceptions are 1944 and 1970-1980 during which the debt GDP ratio recorded an extremely high growth rate, so that they are classified as regime 2.

and global statinarity; namely, local stationarity is neither necessary nor sufficient condition for global stationarity. For example, suppose there are two regimes, and one satisfies local stationary and the other does not. Even in this combination, the process could be globally stationary. On the other hand, even if each of the two regimes satisfies local stationarity, it does not necessarily imply global stationarity.

As we saw in Table 2, regression results using two-state model show that one regime satisfies stationarity, and the other one does not. Also, as we saw in Table 6, regression results using three-state model indicate that one regime satisfies stationarity, but the other two do not. Given these results, one may wonder if they imply global stationarity or nonstationarity. To address this, we conduct a simulation analysis in the following way. First, we generate a time series of b_t using

$$
b_t = \begin{cases} \hat{\mu}_0 + \left(\hat{\alpha}_0 - \frac{n_t}{1+n_t} \right) b_{t-1} + u_{0t}, & \text{if} \quad S_t = 0 \\[2ex] \hat{\mu}_1 + \left(\hat{\alpha}_1 - \frac{n_t}{1+n_t} \right) b_{t-1} + u_{1t}, & \text{if} \quad S_t = 1 \end{cases}
$$

for a two-state model, and the corresponding equation for a three-state model. Here parameters with hat represent the estimated ones in the earlier regressions. More specifically, we randomly draw policy shocks and policy regimes using the parameters and transition probabilities obtained from regressions of specification 3, and generate a replication for the time series of debt-GDP ratio over 1000 years, for various paths of the nominal growth rate (n_t) that are exogenously determined. We repeat this 5000 times to obtain a distribution of the debt-GDP ratio in every year of the 1000 years. We can say that the debt-GDP process is globally stationary if this distribution is stable over time, and otherwise globally nonstationary.

Table 7 reports the first, second, and third quantiles of the simulated distribution in $T = 500$ (500 years later) and $T = 1000$ for two-state and three-state models. Panel A assumes that the initial regime is a stationary one (namely, regime 0, $S_0 = 0$), and that the debt-GDP ratio at period 0 is zero. On the other hand, Panel B assumes that the initial regime is a nonstationary one (regime 1, $S_0 = 1$), and that the initial debt-GDP ratio is unity (100 percent). The simulation results from two-state model show that the distribution is stable over time, irrespective of initial conditions and the assumed values of nominal growth rates (n), clearly indicating that the process satisfies global stationarity. On the other hand, the results from three-state model show that distributions in $T = 500$

and $T = 1000$ differ significantly for the case of $n = 0$, 0.03, and 0.06, implying that the process is globally nonstationary. However, when n goes up to 0.10, distributions in $T = 500$ and $T = 1000$ become identical, suggesting that sufficiently high nominal growth could make the process globally stationary.[19]

Figure 9 conducts a similar simulation in order to forecast the future path of the debt-GDP ratio over the next 100 years. To make the initial condition as close as the current situation in Japan, we assume that the initial regime is $S_0 = 1$, and that the debt-GDP ratio in period 0 is 1.7, which is the actual figure at the end of 2005. According to the result from two-state model with 3 percent nominal growth, the "third quantile" line goes up until it reaches 3 in $T = 20$, indicating that further increase in the debt-GDP ratio is quite likely to occur over the next 20 years. After that, however, it turns to a declining trend due to switching to a stationary regime, and converges to a quite narrow (and probably tolerable) band within 100 years. On the other hand, the result from three-state model with 3 percent nominal growth shows that the median of the distribution increases quite quickly to reach an unrealistic and intolerable level within 50 years, and that its variance increases over time, clearly indicating global nonstationarity.

5.5 Empirical results for U.S.

Table 8 presents regression results for the United States using a two state model. Results for specification 3, presented in Panel A, indicate that each of regimes 0 and 1 is characterized by a stationary process. This implies that the U.S. government's fiscal behavior during the entire sample period is well described by switching between locally Ricardian policy rules. If we turn to the results for specification 4, presented in Panel B, it again indicates that each of regimes 0 and 1 satisfies stationarity, implying that U.S. fiscal policy is characterized by a switching between globally Ricardian rules.

These results suggest that the U.S. government fiscal behavior has been consistently very close to locally Ricardian policy throughout the entire sample period. In fact, the estimated coefficient on b_{t-1}, presented in Figure 10, has been consistently and statistically significantly below unity. If we compare these results with those reported in the previous studies about US fiscal policy, we find some similarities. For example, Bohn (2005) regresses US primary surplus on public debt in the sample period of 1793 to 2003, and reports that the OLS estimate of the coefficient on public debt is

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[19]The threshold for nominal growth rates is about 8 percent, which is lower than the sample average (13.7 percent).

positive and significantly different from zero when he properly controls for tax smoothing effects. Bohn interprets this result as an evidence for a globally Ricardian rule; but since the estimated coefficient is typically greater than the average level of interest rates, this could be interpreted as suggesting even a locally Ricardain rule. Bohn (1998) conducts a similar exercise using the sample of 1916-1995 to find that the coefficient on public debt is significantly positive not only for the entire sample period, but also for five subsamples including the postwar period. These results reported by Bohn (1998, 2005) are consistent with ours.

Favero and Manacelli (2005) estimate an equation that is very close to our specification 1 (equation (6)) using maximum likelihood method, and reports that US government behavior has been deviating from Ricardian policy for most of the entire sample period (1961-2002), except that it was close to a locally Ricardian rule during the period of 1995-2001. It might not be easy to compare their results with ours, because their empirical methodology differs from ours in several respects. But we still make some comparison by adjusting our sample period to theirs; Panels C and D of Table 8 conduct a similar regression as Panels A and B, but now using the postwar period data. Regression results indicate that the estimate of α in regime 0 is less than unity, suggesting that it a stationary regime as before, but that the upper bound of α in regime 1 is slightly exceeding unity, failing to reject the null of a unit root. Fluctuations in the estimated coefficient on b_{t-1}, presented in Panels C and D of Figure 10, show that it has been slightly higher than unity since the year of 1975, implying the possibility that the US government started to deviate from Ricardian policy around 1975. However, the figure clearly shows that the estimated coefficient on b_{t-1} is consistently less than unity during the period before 1975, and that there is no evidence for returning to Ricardian policy around the year of 1995, as detected by Favero and Monacelli (2005). Given these results, there still exist some inconsistency with their results.[20]

Given that US fiscal policy is characterized by switching between stationary regimes, we may apply a multiple linear regression model with multiple breaks, proposed by Bai and Perron (1998) to the US data. This model does not require researchers to assume that policy regime switching is a recurrent one, and that it has a Markov property. This is an important advantage, but on the other hand, it requires that the debt process to be weakly stationary in each regime, so that we are not allowed to apply it to the Japanese data. Regression results reported in Table 9 show that regime changes occur

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[20]We also estimated specification 1, which is very close to an estimating equation employed by Favero and Monacelli (2005), for the entire sample period as well as for the postwar period, only to find that both regimes are stationary ones.

four times (namely, there are five different regimes) both for specifications 3 and 4. According to the result for specification 4, the estimates of α is slightly higher than unity during the wartime (regime 3, 1917-1943), but it is significantly smaller than unity in the other four regimes. These results may be interpreted as confirming our earlier results obtained from Markov switching regression.[21]

5.6 Empirical results for U.K.

Table 10 presents regression results for the United Kingdom using a two state model. Results for specification 3 indicate that regime 0 is characterized by a stationary process, while regime 1 is characterized by a unit root process (the upper bound of α slightly exceeds unity). On the other hand, results for specification 4 indicate that each of regime 0 and regime 1 is characterized by a stationary process, implying that the U.K. government's fiscal behavior is characterized by switching between globally Ricardian rules.

6 Conclusion

This paper has estimated fiscal policy feedback rules in Japan, the United States, and the United Kingdom, allowing for stochastic regime changes. Using Markov-switching regression methods, we find the following. First, the Japanese data clearly reject the view that fiscal policy regime is fixed; i.e., the Japanese government has been adopting either of Ricardian or Non-Ricardian policy *at all times*. Instead, our results indicate that fiscal policy regimes evolve over time in a stochastic manner. This is in a sharp contrast with the U.S. and U.K. results in which the government's fiscal behavior is consistently characterized by Ricardian policy. Second, the Japanese government had a strong fiscal discipline before the 1920s, which is consistent with the fact that the government had been forced to maintain balanced budget under the gold standard system until its termination in 1917. Third, the Japanese government lost fiscal discipline during the WWII. The estimated date of restoring discipline after the war is consistent with the fact that fiscal restructuring led by the allied powers started in late 1948. Fourth, we find that the Japanese government has been deviating even from a globally Ricardian rule over the last thirty years. Moreover, some of our results indicate that the debt-GDP ratio is nonstationary not only within a regime, but also in the long run.

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[21]However, results for specification 3 are not so informative since α exceeds unity in three regimes out of the five. This result might be interpreted as evidence against using the Bai-Perron method even to the US data.

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Table 1: Unit Root Tests

p	Japan		U.S.		U.K.	
	α	t-stat	α	t-stat	α	t-stat
1	0.9990	-0.02	0.9868	-1.04	0.9864	-1.17
2	0.9607	-1.31	0.9784	-2.09	0.9798	-2.21
3	0.9702	-0.94	0.9818	-1.74	0.9800	-2.15
4	0.9729	-0.80	0.9804	-1.86	0.9817	-1.94
5	0.9756	-0.68	0.9808	-1.79	0.9825	-1.84
6	0.9691	-0.82	0.9806	-1.78	0.9813	-1.94
7	0.9720	-0.71	0.9815	-1.67	0.9853	-1.54
8	0.9824	-0.43	0.9817	-1.63	0.9819	-1.91
9	0.9716	-0.67	0.9839	-1.42	0.9826	-1.82
10	0.9740	-0.58	0.9797	-1.81	0.9827	-1.78

Notes: We conduct the standard ADF tests, $b_t = \mu + \alpha b_{t-1} + \sum_{j=0}^{p-1} \phi_j \Delta b_{t-j-1} + u_t$, with various lag length. The null hypothesis is $\alpha = 1$ and the 10% critical value is -2.57 when the sample size is 100 (see Hamilton (1994)). Sample periods for Japan, U.S. and U.K. are 1885-2004, 1840-2005 and 1830-2003, respectively.

Table 2: Two State Model for Japan

Panel A: Specification 1

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	0.0355	0.0512	0.0669	-0.0549	-0.0267	0.0023
α	0.4783	0.5177	0.5529	1.0674	1.1168	1.1649
σ^2	0.0006	0.0010	0.0018	0.0037	0.0050	0.0068
p_{11}	0.9193	0.9658	0.9941			
p_{00}	0.7804	0.9078	0.9811			

Panel B: Specification 2

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	0.0380	0.0535	0.0688	-0.0863	-0.0591	-0.0321
α	0.3785	0.4134	0.4476	1.0424	1.0821	1.1233
σ^2	0.0005	0.0009	0.0015	0.0024	0.0033	0.0044
p_{11}	0.8978	0.9564	0.9900			
p_{00}	0.7706	0.8984	0.9792			

Panel C: Specification 3

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0133	0.0036	0.0164	-0.0300	0.0073	0.0476
α	0.8681	0.9178	0.9762	1.0167	1.0641	1.1110
σ^2	0.0003	0.0005	0.0007	0.0022	0.0033	0.0049
p_{11}	0.8552	0.9378	0.9867			
p_{00}	0.8778	0.9448	0.9864			

Panel D: Specification 4

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0067	0.0056	0.0169	-0.0524	-0.0150	0.0193
α	0.8126	0.8550	0.8998	1.0103	1.0536	1.1003
σ^2	0.0003	0.0004	0.0006	0.0022	0.0033	0.0050
p_{11}	0.8631	0.9440	0.9876			
p_{00}	0.8921	0.9469	0.9831			

Notes: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower- and upper-bounds of the 95% confidence interval, and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 3: AR(2) Model

Panel A: Specification 3

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0088	0.0069	0.0185	-0.0298	0.0075	0.0383
α	0.8572	0.9004	0.9578	1.0151	1.0558	1.1028
θ	0.0218	0.1227	0.3150	-0.0099	0.1009	0.1999
σ^2	0.0003	0.0004	0.0006	0.0020	0.0030	0.0045
p_{11}	0.8782	0.9454	0.9883			
p_{00}	0.8752	0.9412	0.9827			

Panel B: Specification 4

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0058	0.0079	0.0189	-0.0489	-0.0125	0.0248
α	0.8007	0.8429	0.8933	0.9979	1.0449	1.0871
θ	0.0317	0.1213	0.2190	-0.0024	0.0854	0.1745
σ^2	0.0002	0.0004	0.0005	0.0019	0.0030	0.0045
p_{11}	0.8689	0.9467	0.9855			
p_{00}	0.8868	0.9439	0.9876			

Notes: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower- and upper- bounds of the 95% confidence interval, and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 4: Alternative Definition of Military Expenditures

Panel A: Specification 3

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0307	-0.0076	0.0064	0.0018	0.0261	0.0476
α	0.9116	0.9620	1.0137	1.0091	1.0362	1.0628
σ^2	0.0004	0.0005	0.0007	0.0006	0.0009	0.0014
p_{11}	0.8489	0.9399	0.9881			
p_{00}	0.8852	0.9472	0.9843			

Panel B: Specification 4

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0504	-0.0268	0.0029	-0.0129	-0.0006	0.0118
α	0.8636	0.9513	1.0126	1.0127	1.0313	1.0478
σ^2	0.0003	0.0005	0.0007	0.0005	0.0008	0.0012
p_{11}	0.8966	0.9608	0.9954			
p_{00}	0.8764	0.9470	0.9881			

Notes: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower- and upper-bounds of the 95% confidence interval, and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 5: No Restriction on the Coefficient of Interest Payments

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
Estimated coefficient on interest payments	0.2351	0.6283	0.9907	0.1046	0.5064	0.8778
μ	-0.0088	0.0052	0.0174	-0.0431	-0.0062	0.0336
α	0.8228	0.8772	0.9367	1.0155	1.0617	1.1075
σ^2	0.0003	0.0004	0.0006	0.0022	0.0033	0.0048
p_{11}	0.8668	0.9420	0.9884			
p_{00}	0.8826	0.9454	0.9850			

Notes: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower- and upper- bounds of the 95% confidence interval, and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 6: Three State Model

Panel A: Specification 3

	Regime 0			Regime 1			Regime 2		
	LB	Mean	UB	LB	Mean	UB	LB	Mean	UB
μ	-0.0448	-0.0018	0.0232	-0.0533	-0.0241	0.0193	-0.3370	-0.0425	0.0438
α	0.8518	0.9261	1.0657	1.0428	1.0819	1.1122	1.1850	1.3136	1.5440
σ^2	0.0001	0.0003	0.0006	0.0005	0.0007	0.0011	0.0006	0.0081	0.0154
p_{00}	0.7937	0.9111	0.9783						
p_{01}	0.0053	0.0560	0.1523						
p_{10}	0.0018	0.0388	0.1067						
p_{11}	0.8357	0.9235	0.9796						
p_{20}	0.0015	0.0666	0.2667						
p_{21}	0.0149	0.1353	0.3272						

Panel B: Specification 4

	Regime 0			Regime 1			Regime 2		
	LB	Mean	UB	LB	Mean	UB	LB	Mean	UB
μ	-0.0002	0.0153	0.0262	-0.0731	-0.0602	-0.0469	-0.0624	-0.0266	0.0051
α	0.7958	0.8283	0.8673	1.0625	1.0840	1.1013	1.1971	1.2783	1.3412
σ^2	0.0001	0.0003	0.0004	0.0004	0.0006	0.0008	0.0005	0.0014	0.0032
p_{00}	0.7839	0.8994	0.9698						
p_{01}	0.0103	0.0607	0.1548						
p_{10}	0.0062	0.0457	0.1155						
p_{11}	0.8498	0.9261	0.9739						
p_{20}	0.0015	0.0556	0.1903						
p_{21}	0.0317	0.1393	0.3257						

Notes: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower- and upper- bounds of the 95% confidence interval, and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 7: Globally Stationary or Nonstationary?

Panel A: $S_0 = 0$ and $b_0 = 0$

Quantile	Two State Model			Three State Model		
	First	Median	Third	First	Median	Third
0% Growth						
T=500	-0.1471	0.0006	0.1525	-1.1E+10	-3.1117	1.4E+10
T=1000	-0.1593	-0.0007	0.1497	-2.5E+21	-2.7E+07	2.6E+21
3% Growth						
T=500	-0.0751	0.0005	0.0772	-3.6E+04	0.0414	4.9E+04
T=1000	-0.0790	-0.0011	0.0786	-5.9E+09	0.6536	1.1E+10
6% Growth						
T=500	-0.0569	0.0003	0.0574	-4.2741	-0.0016	4.4939
T=1000	-0.0591	-0.0012	0.0584	-16.665	0.0085	20.506
10% Growth						
T=500	-0.0476	0.0001	0.0463	-0.1160	0.0008	0.1205
T=1000	-0.0478	-0.0006	0.0466	-0.1175	0.0010	0.1192
13.7% Growth						
T=500	-0.0423	-0.0003	0.0413	-0.0579	0.0013	0.0615
T=1000	-0.0428	-0.0004	0.0403	-0.0598	-0.0001	0.0592

Panel B: $S_0 = 1$ and $b_0 = 1$

Quantile	Two State Model			Three State Model		
	First	Median	Third	First	Median	Third
0% Growth						
T=500	-0.1476	0.0017	0.1623	6.1E+08	2.9E+11	8.9E+13
T=1000	-0.1524	-0.0028	0.1515	1.0E+19	5.2E+22	1.8E+26
3% Growth						
T=500	-0.0800	-0.0007	0.0780	131.32	1.7E+05	7.2E+07
T=1000	-0.0780	-0.0007	0.0751	4.2E+05	1.3E+10	9.2E+13
6% Growth						
T=500	-0.0573	-0.0003	0.0582	-0.2212	0.4199	121.10
T=1000	-0.0575	0.0006	0.0585	-1.8212	0.2312	359.11
10% Growth						
T=500	-0.0481	-0.0007	0.0467	-0.1082	0.0011	0.1195
T=1000	-0.0458	0.0007	0.0467	-0.1131	0.0006	0.1172
13.7% Growth						
T=500	-0.0427	-0.0009	0.0416	-0.0603	-0.0009	0.0574
T=1000	-0.0400	0.0008	0.0413	-0.0573	0.0013	0.0600

Notes: We randomly draw policy shocks and policy regimes using the parame-
ters obtained from regressions of specification 3, and generate 5000 replications
for the time series of debt-GDP ratio (1000 years), for various paths of the
nominal growth rate (n_t), which are exogenously determined. The figures in
the table represent the first, second, and third quantiles of the simulated distri-
bution in T=500 (namely, 500 years later) and T=1000. The average growth
rate over the entire sample was 13.7 percent.

Table 8: Two State Model for U.S.

Panel A: Specification 3, 1840-2005

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0039	-0.0013	0.0014	0.0055	0.0242	0.0458
α	0.8734	0.8805	0.8885	0.9025	0.9393	0.9760
σ^2	0.00003	0.00007	0.0001	0.0004	0.0007	0.0010
p_{11}	0.8448	0.9161	0.9702			
p_{00}	0.8635	0.9287	0.9704			

Panel B: Specification 4, 1840-2005

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0031	-0.0012	0.0008	0.0062	0.0277	0.0518
α	0.8448	0.8526	0.8594	0.8432	0.8811	0.9165
σ^2	0.00003	0.00005	0.00007	0.0007	0.0010	0.0014
p_{11}	0.8101	0.8922	0.9530			
p_{00}	0.8637	0.9243	0.9644			

Panel C: Specification 3, 1948-2004

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0426	-0.0111	0.0114	-0.0182	0.0085	0.0332
α	0.8478	0.8887	0.9441	0.9261	0.9699	1.0189
σ^2	0.00005	0.0001	0.0002	0.0002	0.0004	0.0006
p_{11}	0.8250	0.9273	0.9849			
p_{00}	0.7771	0.9068	0.9777			

Panel D: Specification 4, 1948-2004

|	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0522	-0.0300	-0.0073	-0.0443	-0.0245	-0.0057
α	0.8492	0.8907	0.9285	0.9330	0.9653	1.0004
σ^2	0.00005	0.0001	0.0002	0.0001	0.0002	0.0004
p_{11}	0.8379	0.9292	0.9833			
p_{00}	0.7636	0.9024	0.9794			

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower- and upper- bounds of the 95% confidence interval, and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 9: Multiple Break Tests for U.S.

	Panel A: Specification 3			Panel B: Specification 4	
	μ	α		μ	α
Regime 1	-0.0093	0.9555	Regime 1	-0.0120	0.9167
1840-1872	(0.0026)	(0.0240)	1840-1872	(0.0028)	(0.0256)
Regime 2		1.0009	Regime 2		0.9781
1873-1916		(0.0265)	1873-1916		(0.0283)
Regime 3		1.0469	Regime 3		1.0264
1917-1943		(0.0125)	1917-1943		(0.0133)
Regime 4		0.9071	Regime 4		0.8868
1944-1972		(0.0057)	1944-1972		(0.0061)
Regime 5		1.0135	Regime 5		0.9450
1973-2004		(0.0079)	1973-2004		(0.0084)

Note: The constant term is imposed to be identical across regimes. The maximum number of breaks is 5 with ϵ=0.15. Figures in parentheses denote standard errors.

Table 10: Two State Model for U.K.

Panel A: Specification 3

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	0.0837	0.1026	0.1220	-0.0341	-0.0264	-0.0180
α	0.8043	0.8213	0.8369	0.9884	0.9955	1.0019
σ^2	0.0003	0.0004	0.0005	0.0004	0.0005	0.0006
p_{11}	0.9128	0.9544	0.9817			
p_{00}	0.6603	0.8175	0.9248			

Panel B: Specification 4

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0033	0.0692	0.0952	-0.0411	-0.0336	-0.0239
α	0.7986	0.8202	0.8686	0.9640	0.9709	0.9773
σ^2	0.0003	0.0004	0.0005	0.0003	0.0004	0.0006
p_{11}	0.9027	0.9474	0.9816			
p_{00}	0.6323	0.7900	0.9199			

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower- and upper-bounds of the 95% confidence interval, and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Figure 1: Public Debt (Relative to GDP)

Panel A: Japan
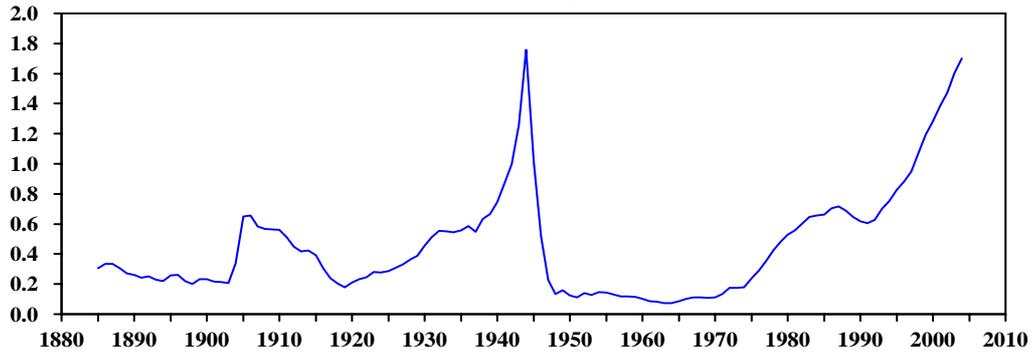
Panel B: U.S.
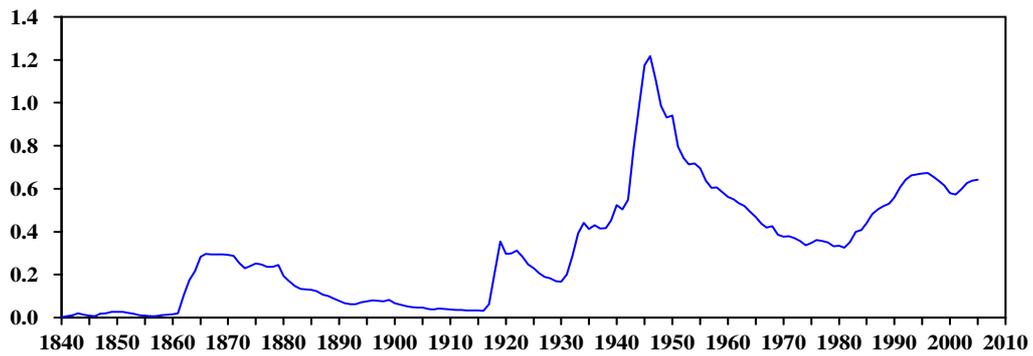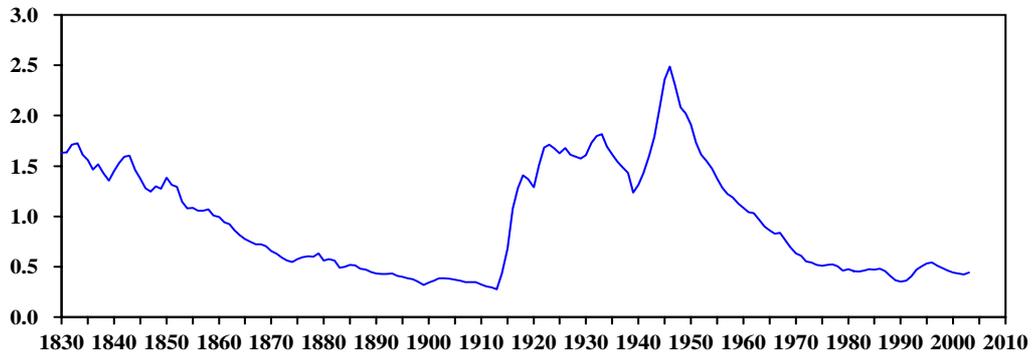
Panel C: U.K.

Figure 2: Estimated Coefficient on b_{t-1} from Rolling Regressions
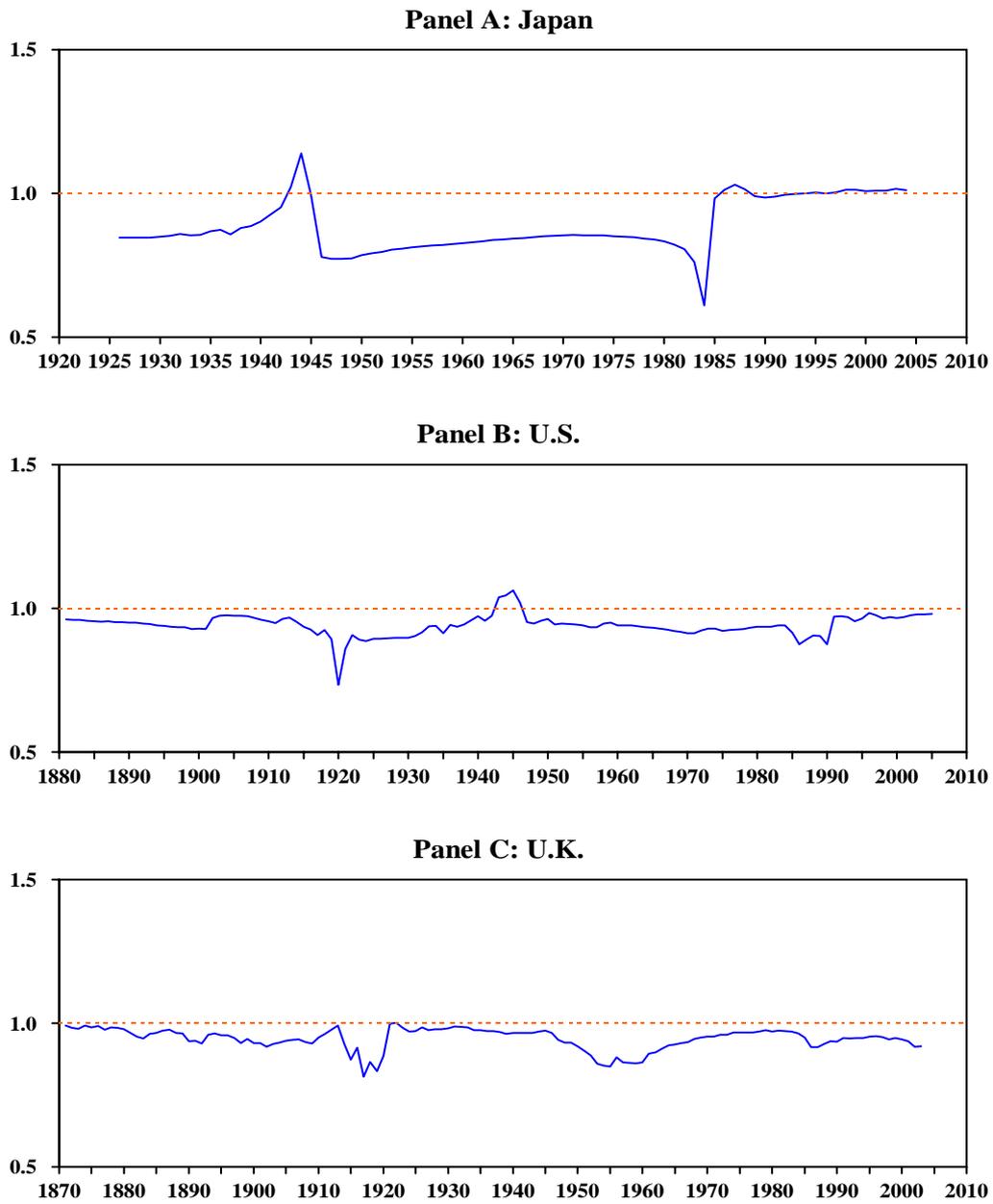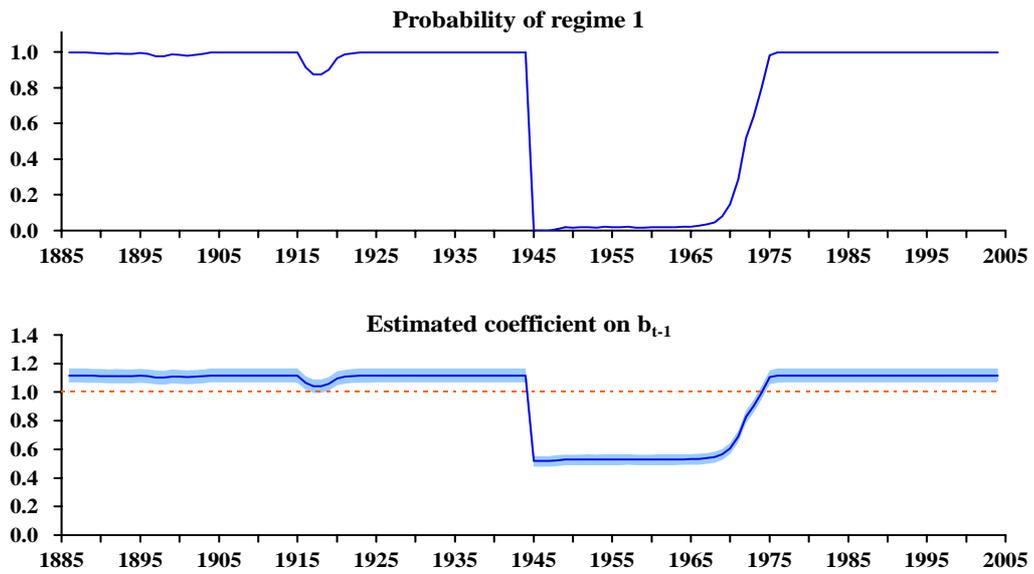
Panel A: Japan

Panel B: U.S.

Panel C: U.K.

Figure 3: Two State Model for Japan

Panel A: Specification 1

Probability of regime 1

Estimated coefficient on b_{t-1}

Panel B: Specification 2

Probability of regime 1

Estimated coefficient on b_{t-1}

Figure 3: Two State Model for Japan, Continued

Panel C: Specification 3

Probability of regime 1

Estimated coefficient on b_{t-1}

Panel D: Specification 4

Probability of regime 1

Estimated coefficient on b_{t-1}
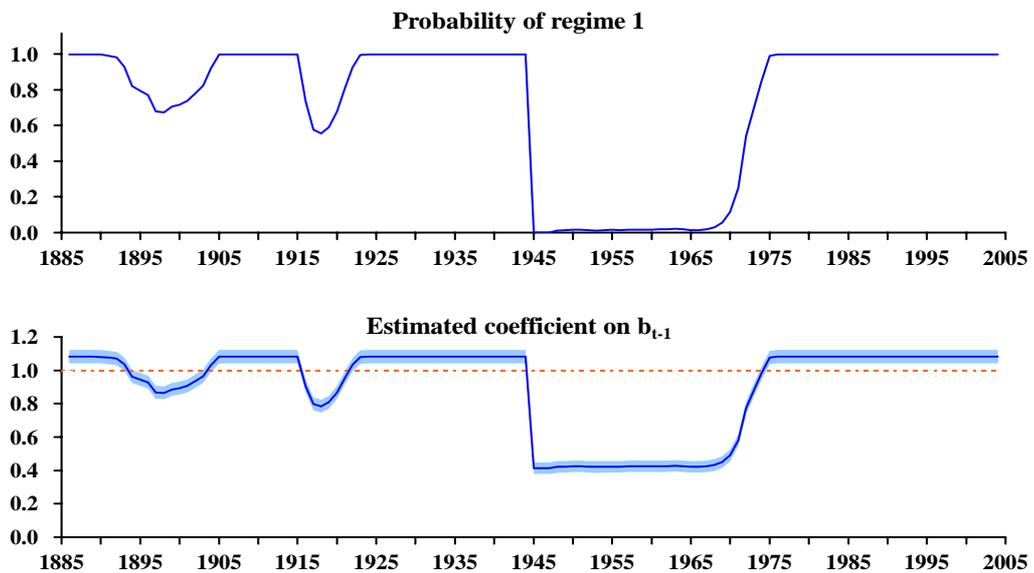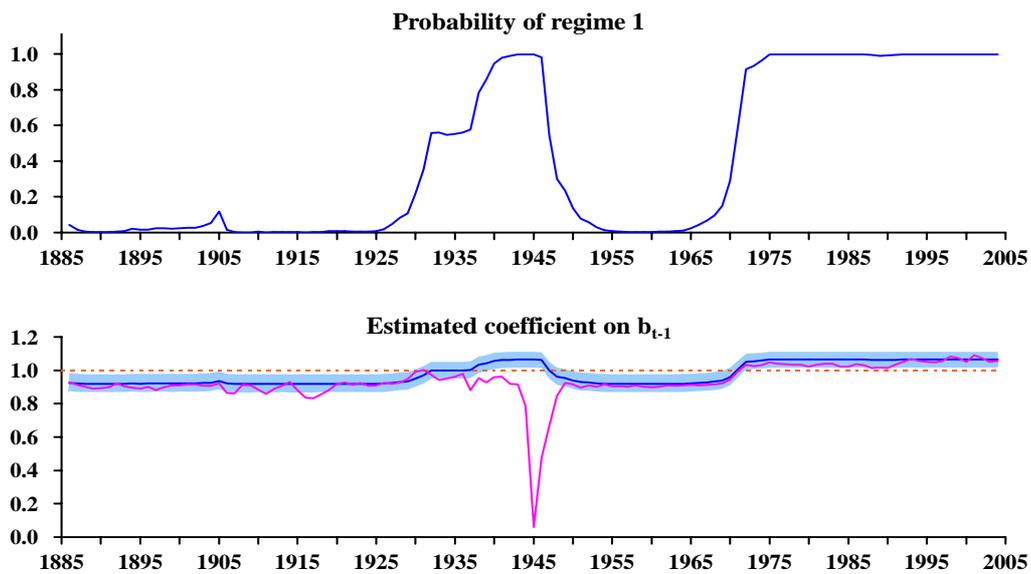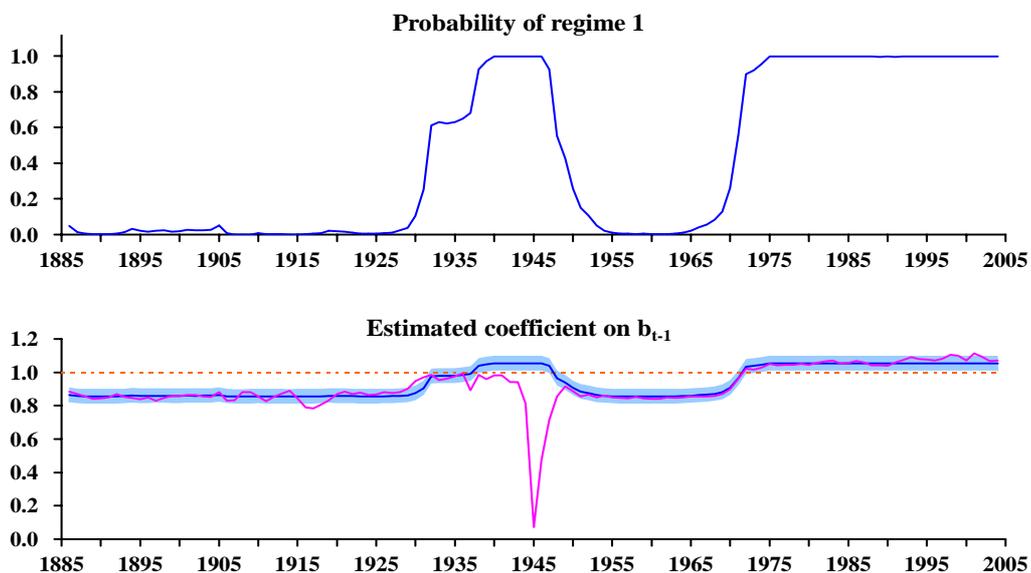
Figure 4: AR(2) Model

Panel A: Specification 3

Probability of regime 1

Estimated coefficient on b_{t-1}

Panel B: Specification 4

Probability of regime 1

Estimated coefficient on b_{t-1}
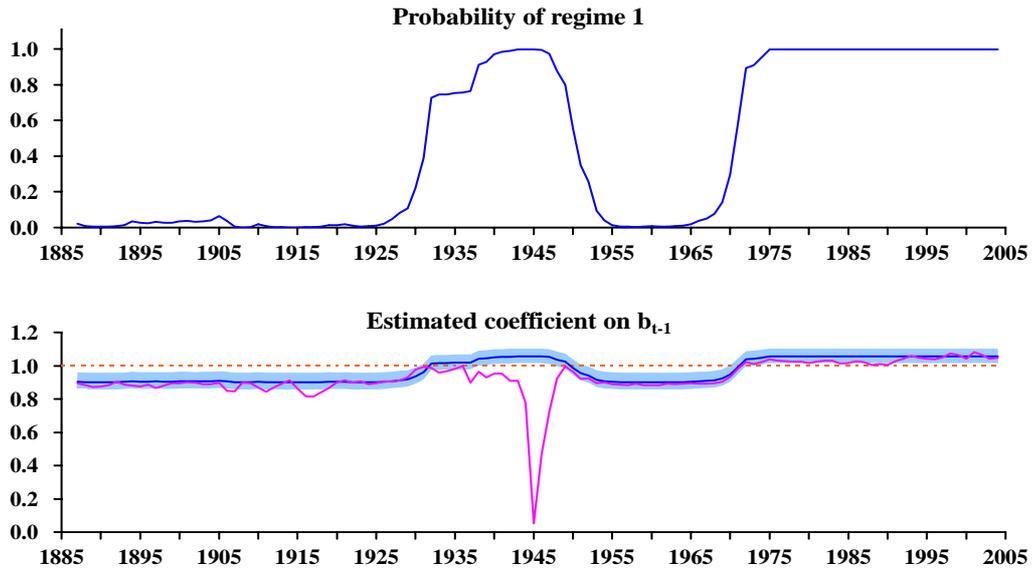
Figure 5: Military Expenditures for 1937-1945 in Japan

(Hundred million yen)
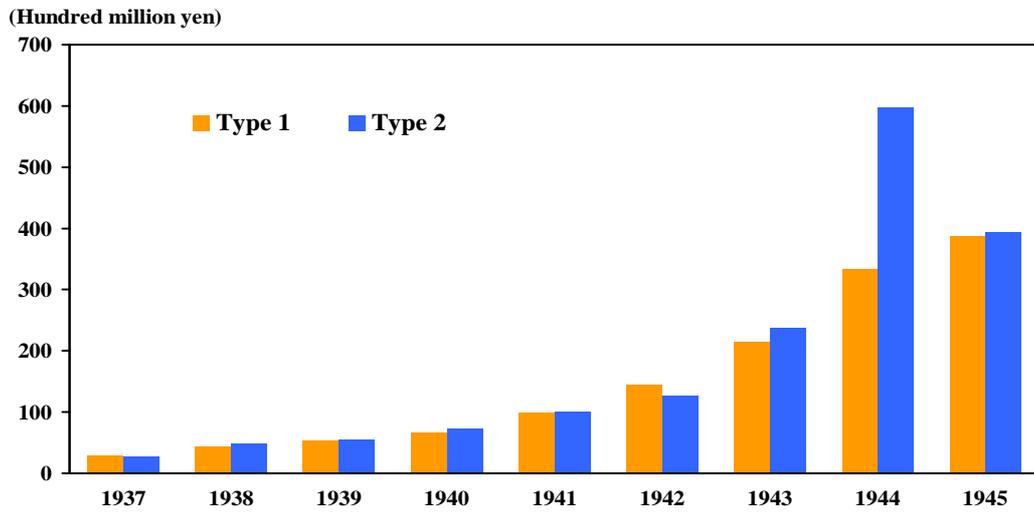
Figure 6: Alternative Definition of Military Expenditures

Panel A: Specification 3

Probability of regime 1

Estimated coefficient on b_{t-1}

Panel B: Specification 4

Probability of regime 1

Estimated coefficient on b_{t-1}
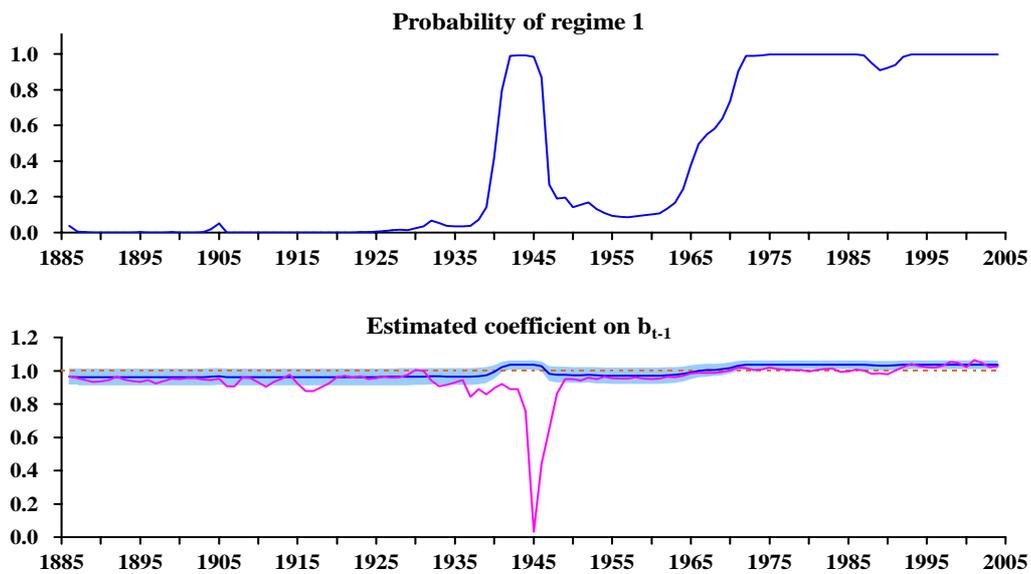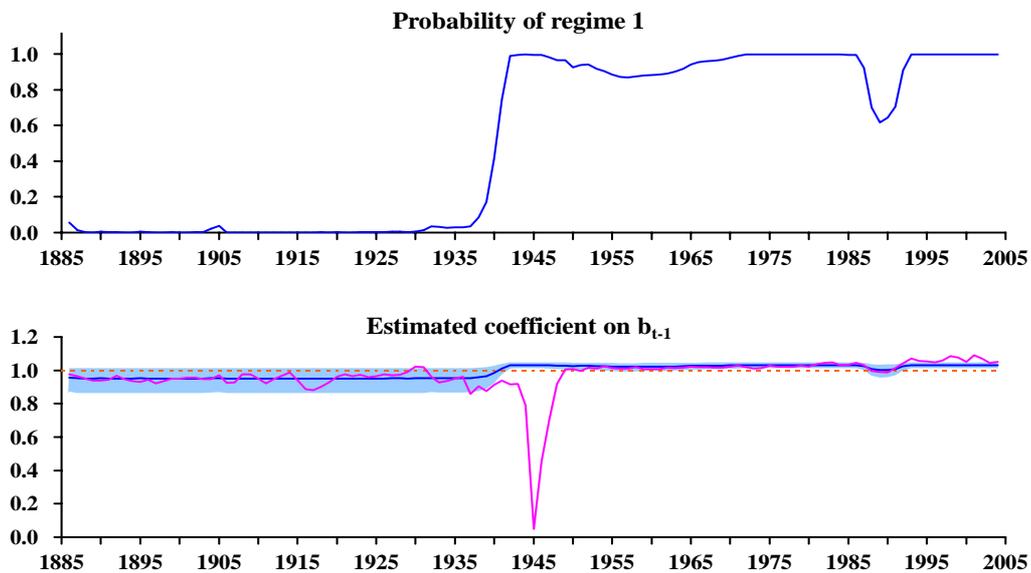
Figure 7: No Restriction on the Coefficient of Interest Payments
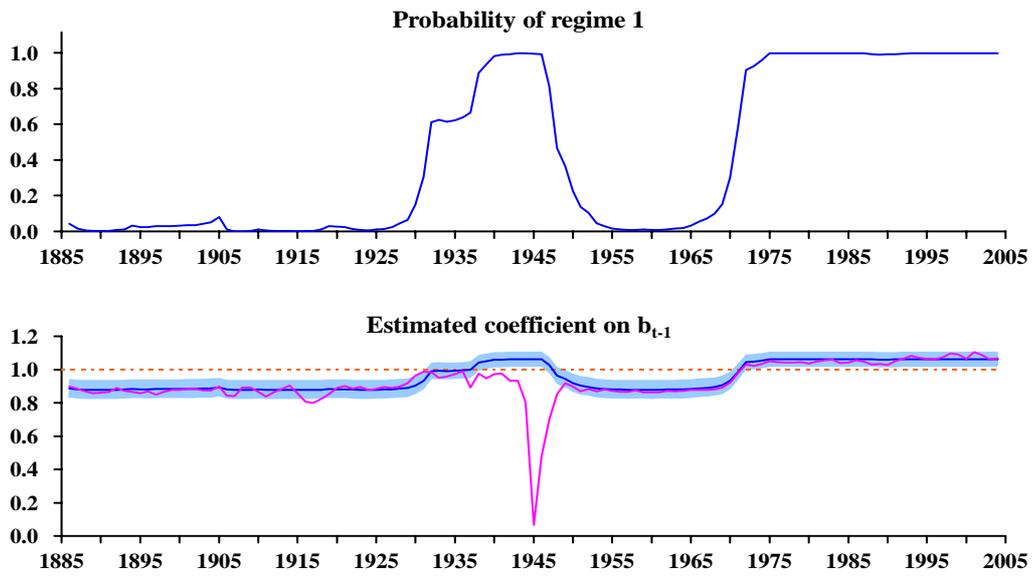
44

Figure 8: Three State Model for Japan

Panel A: Specification 3

Probability of regime 0

Probability of regime 1

Probability of regime 2

Estimated coefficient on b$_{t-1}$
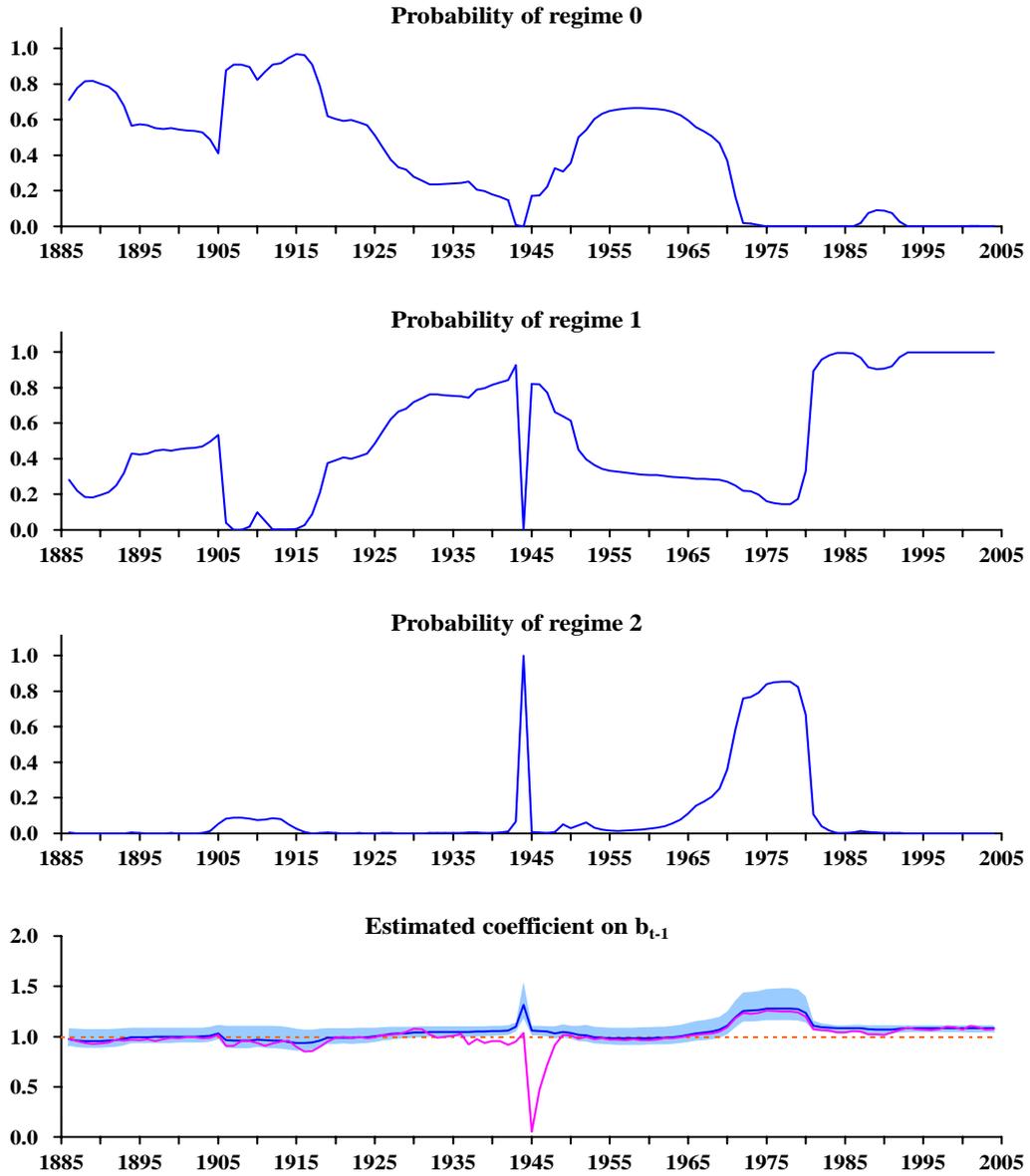
Figure 8: Three State Model for Japan, Continued

Panel B: Specification 4

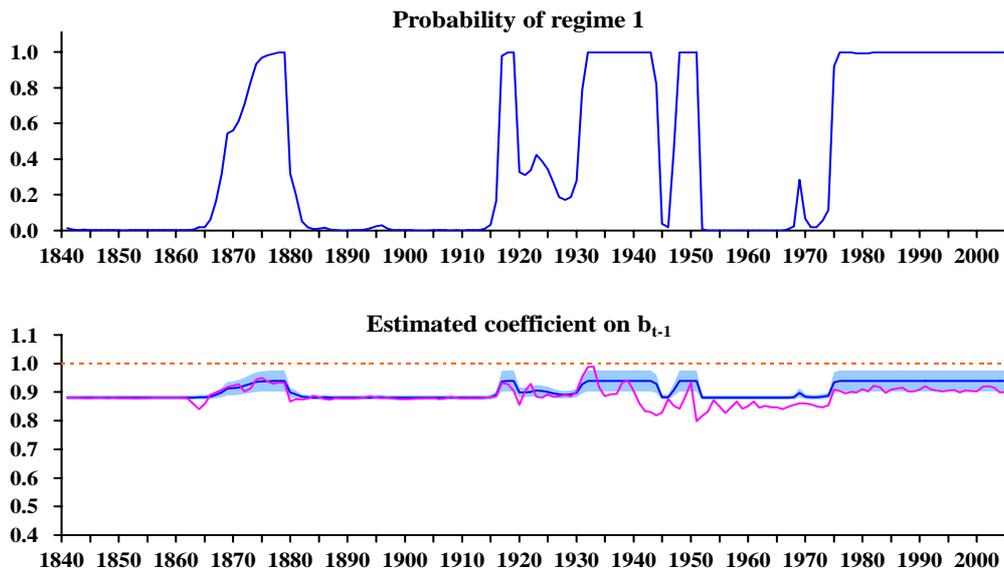**Probability of regime 0**

Probability of regime 1

Probability of regime 2

Estimated coefficient on b_{t-1}
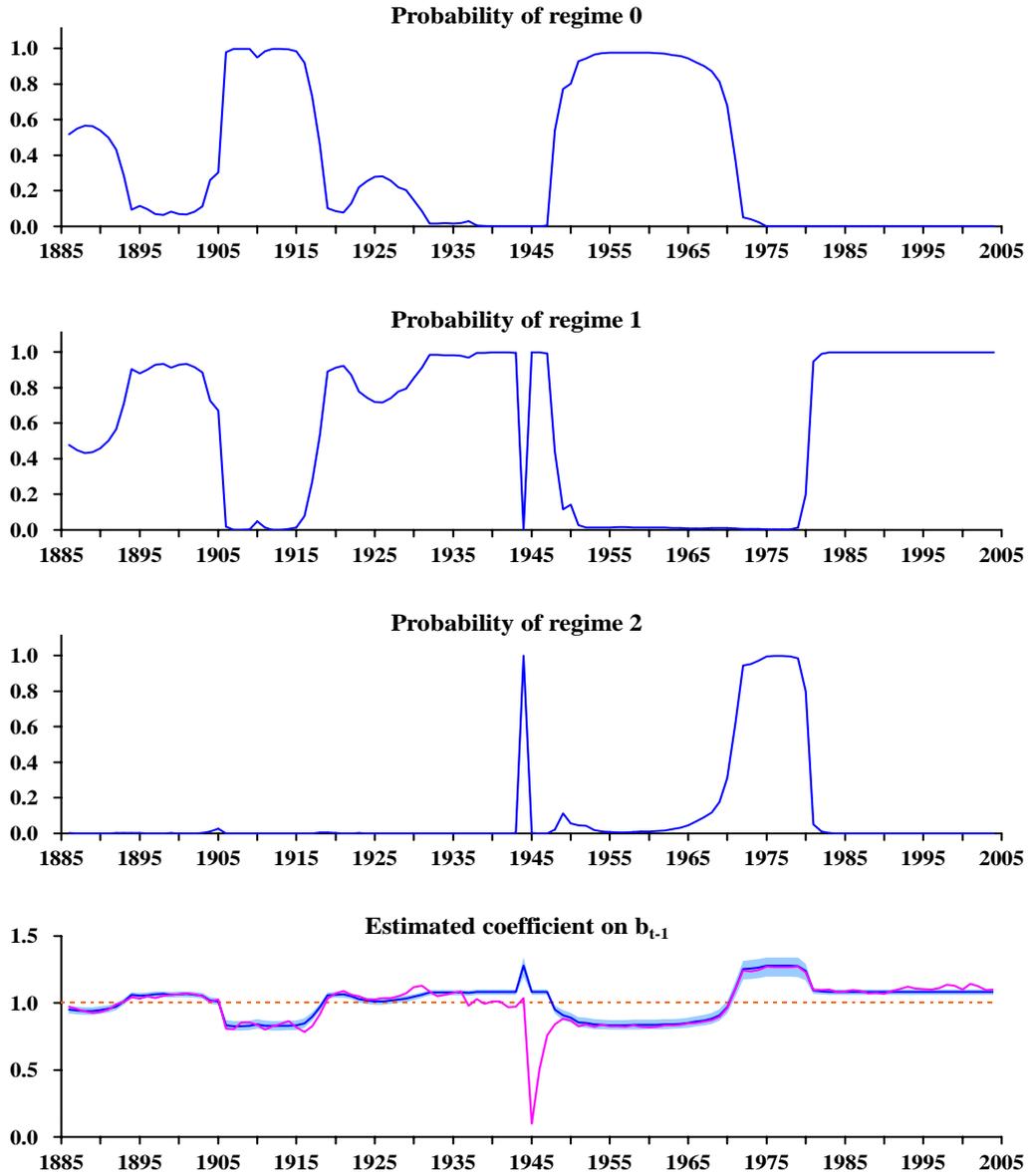
Figure 9: Globally Stationary or Nonstationary?

Panel A

Simulated data with 3% growth (Specification 3 of two state model)

Panel B

Simulated data with 13.7% growth (Specification 3 of two state model)

Panel C

Simulated data with 3% growth (Specification 3 of three state model)

Panel D

Simulated data with 13.7% growth (Specification 3 of three state model)
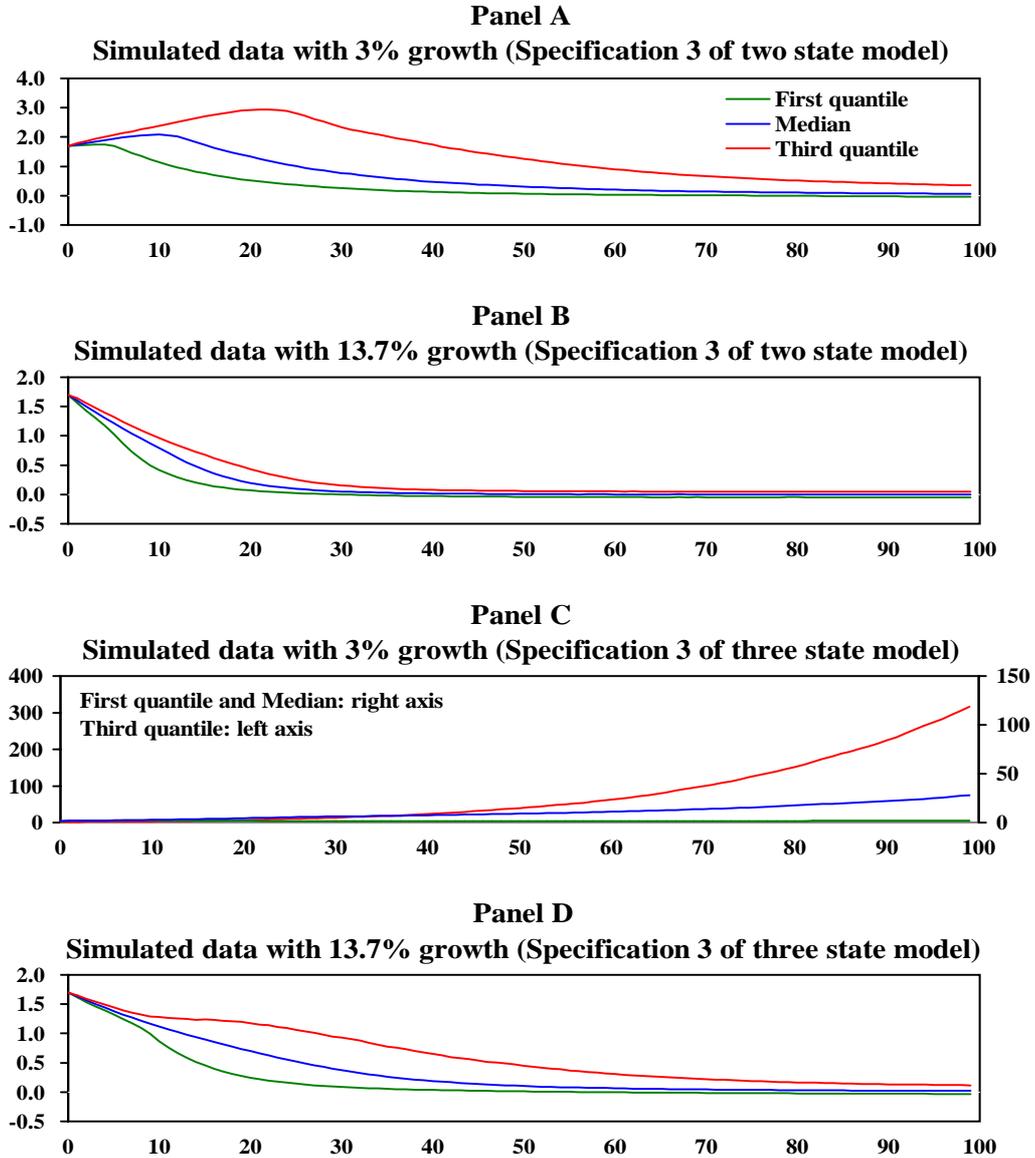
Notes: The data of size 120 is generated from Specification 3 using estimated values with b_0=1.7 and S_0=1. In all cases, we replicate it 5000 times to compute the first, second, and third quantiles.

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Figure 10: Two State Model for U.S.

Panel A: Specification 3, 1840-2005

Probability of regime 1

Estimated coefficient on b_{t-1}

Panel B: Specification 4, 1840-2005

Probability of regime 1

Estimated coefficient on b_{t-1}
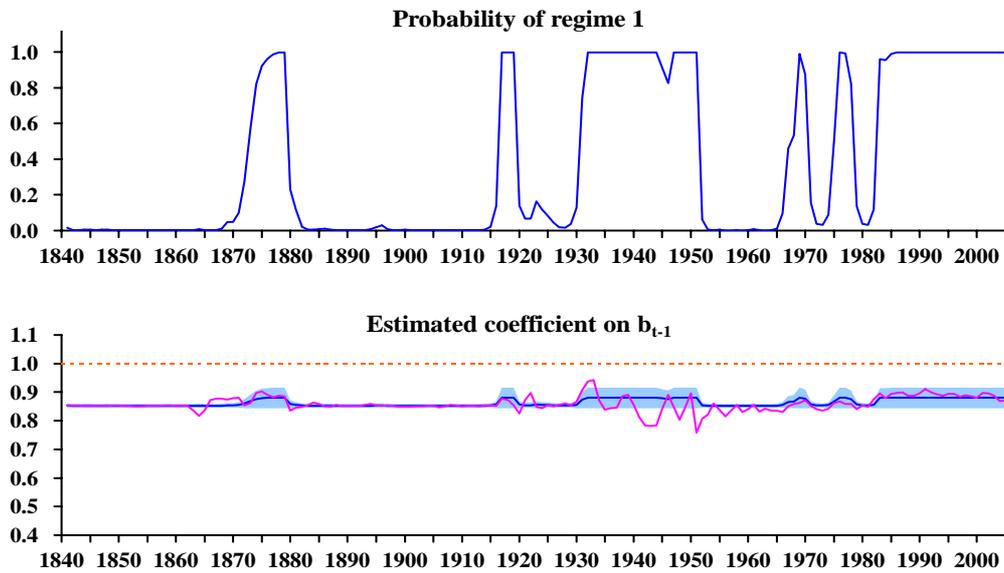
Figure 10: Two State Model for U.S., Continued

Panel C: Specification 3, 1948-2004

Probability of regime 1

Estimated coefficient on b_{t-1}

Panel D: Specification 4, 1948-2004

Probability of regime 1

Estimated coefficient on b_{t-1}

Figure 11: Two State Model for U.K.

Panel A: Specification 3

Probability of regime 1

Estimated coefficient on b_{t-1}

Panel B: Specification 4

Probability of regime 1

Estimated coefficient on b_{t-1}