J.M. Keynes versus F.H. Knight：
How to Deal with Risk, Probability and Uncertainty

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Abstract

The purpose of this paper is to discuss and compare two giants in the history of economic thought, J.M. Keynes and F.H. Knight, with special reference to risk, probability, and uncertainty.

It is in 1921 that both of them published apparently published similar books on the economics of risk and uncertainty. While Knight's contribution on risk and uncertainty is now well recognized, Keynes's accomplishments on probability and uncertainty have been rather ignored in the shadow of his most famous book *The General Theory of Employment, Interest and Money* (1936). This paper aims to focus on his earlier yet equally important book *A Treatise on Probability* (1921), and shed a new light on his outstanding ideas and everlasting influences on his later work including *The General Theory*. It is really interesting to see that Keynes's concept of probability and uncertainty can be well compared to Knight's distinction between a measurable risk and a non-measurable uncertainty.

Key words: Keynes, Knight, risk, probability, uncertainty

1. 1921 as a Miracle Year: An Introduction

There is no doubt that J.M. Keynes and F.H. Knight are two towering figures in the history of the economics of risk and uncertainty. Knight's contribution on risk and uncertainty is well known to many economists: In fact, his book *Risk, Uncertainty and Profit* (1921) is now regarded as a classic in the economics profession. In start contrast to this, Keynes' accomplishments on risk and uncertainty have been rather underestimated in the dark shadow of his most famous book *The General Theory of Employment, Interest and Money (1936).* The main purpose of this paper is to mend such an unfortunate tendency by focusing on his earlier yet equally important book *A Treatise on Probability* (1921). It is interesting to see that the two economic giants published apparently similar books on risk, probability and uncertainty in the same year . Therefore, 1921 may be referred to as a sort of miracle year in the long history of economic thought.

The relation between Keynes and Knight, two giants in the history of economic thought, is so delicate and complicated that cannot be described by a simple passage. It is true that they were contemporaries and lived through two world wars, the First World War (1914-18) and the Second World War (1939-45). It is noted, however, that they were poles apart in origins. Keynes (1883-46) was born with a silver spoon in the United Kingdom: He spent his young days at a rich Victorian house, in a peaceful Cambridge district. During all his proactive career in the fields of academics and practical affairs, he could return to this house, full of nice memories, and to his beloved parents. His roots were deep in Harvey road, which maintained the traditional values of the British society. He spent a very colorful life until his untimely death in 1946, first as a university instructor, then as a high government officer, and sometimes as a art collector. His academic accomplishments in monetary and macroeconomic theories were so novel and revolutionary that he was regarded by most people as the greatest economist in the 20th century.

By contrast, Knight (1885-72) was born with a wooden spoon in the United States: His career began from rather unpromising roots in Maclean County, Illinois. He was the eldest of eleven children , being raised by parents who strongly believed in Christian fundamentalism. It is said that he received general education in small colleges in rural Tennessee and proceeded to the University of Tennessee. From there, he moved on to the Graduate School of Economics, Cornell University. His concentration and hard work resulted in a Ph.D. thesis entitled "A Theory of Business Profit", written in 1925-15, and later published with some revisions in 1921 under the revised title *Risk,*

Uncertainty and Profit. It was in 1927 that he finally appointed Professor of Economics at the University of Chicago, and continued to live there as the "Grandpa of Chicago until he finished his long life in 1972.

In short, Keynes and Knight have very different backgrounds in many ways. Comparison of these two men might be characterized as a silver spoon versus a wooden spoon, or as an elite in the Old World versus a man in the street in the New World, or perhaps as a colorful "practical man" with many talents versus a plain and colorless professor living at the ivory tower. Considering such personal and educational differences between the two, it might be understandable to see that very few books and papers on Keynes versus Knight have ever been published in academic circles around the world. In this paper, however, I intend to break such an unfortunate tendency by carefully comparing the two great economists from an angle of probability and uncertainty.

The contents of this paper are as follows. Session 2 will focus on the way how Keynes has dealt with the concepts of probability and uncertainty. The meaning of probability and Keynes' strange chart of probability will carefully be discussed. Session 3 will attempt to compare Keynes and Knight in terms of risk, probability and uncertainty. It will be shown that the figures of multiple rings are quite helpful for such a comparison. Session 4 will turn attention to J.R. Hicks, still another great economist in the 20th century, who has regarded economics on the verges of both sciences and history. It will be seen that Hicks' approach is more or less influenced by Keynes and Knight. Some final remarks will be made in the final session.

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2. Keynes on Probability and Uncertainty

2.1 The Meaning of Probability

The concept of probability is ambiguous and unclear. It may change with persons, places, and the times. Besides, there are several words that are more or less similar to probability: Risk, uncertainty, likelihood, ambiguity, complexity, and so on.

According to *Oxford Dictionary of English*, we can see the standard usage of probability as follows.

PROBABILITY

(1) [noun] the quality or state of being probable: the extent to which something is likely to happen or be the case.

(2) [mathematics] the extent to which an event is likely to occur, measured by the ratio of the favorable cases to the whole number of cases possible.

Probability has double meanings, (1) and (2). The first meaning is commonly used in everyday life. For instance, a man in the street may say that it is probable that it will rain tomorrow. The second meaning is more technical than the first, and attached to a special meaning by a professional person such as a statistician or mathematician. For example, let us consider the game of rolling the dice. Then the probability of having one is 1/6 since there is only one favorable case (namely, one) and the whole number of cases possible is six (namely, 1, 2, 3, 4, 5, 6). The first meaning is subjective and personal whereas the second one is objective and scientific.

Keynes's position is delicate and complicated. It is not based on the second meaning of probability at all, but seems to be rather akin to the first one. On the one hand, his concept of probability is more general and comprehensive than it is adopted in the mathematical theory of probability. On the other hand, it is more academically and scientifically used than in everyday life.

He devoted more than ten years of hard work to establish his own doctrine, which took a middle position between the subjective and objective theory of probability. The following poet written at the very end of his probability book (1921) is likely to show his unique opinion:

" O False and treacherous Probability,
Enemy of truth, and friend to wickednesse:
With whose bleare eyes Opinion leans to see,
Truth's feeble party here, and barennesse."
(Keynes,1921, p.466)

This is an old, romantic Byron-like rhyming verse with classical wordings. According to the young Keynes, probability is double-edged, having dual meanings. For one thing, it may be a false and treacherous concept, even possibly becomes the enemy of truth. For another, it is a guide to truth in the sense that it plays the role of a feeble party on the truth-finding journey. In a sense, the concept of probability may be barren or fruitful. It depends: No one really knows in advance which one is applicable. [1)]

In order to understand Keynes' position on probability and uncertainty, we have found it very convenient to start with his charming chart of probability. A detailed discussion on the chart will be our aim of the next subsection.

2.2 Keynes's Strange Chart of Probability: An Attractive Partial-Order Network

In a long history of economic science, there are two different kinds of great books, nicely-written and badly-written books. In our opinion, Keynes's first masterpiece *A Treatise on Probability* (1921) is of the first kind whereas his second one *The General Theory* (1936) is of the second kind.

Keynes' earlier book is a well-organized, nicely-written book, yet not easy to read. It looks like a high peak which waits its turn to be conquered by a well-trained climber. No matter how difficult to climb, there must be a guiding route for the climber to follow. In our opinion, such a route surely exists in its bulky and seemingly difficult book. As is seen in Fig. 1, it is featured by a very charming chart of partial network.

Fig. 1 around here

Interestingly enough, Fig. 1 was depicted as a rather small figure in Keynes (1921), page 39. In order to make the partial-order property of his chart much clearer, we dare to add many arrows (→—) to the original diagram. For example, "*V*→—*A*" means that *A* is more probable than *V*. Fortunately, detailed explanations by Keynes himself are available, and will be reproduced here:

> "[The] properties are illustrated in the annexed diagram. *O* represents impossibility, *I* certainty, and *A* a numerically measurable probability intermediate between *O* and *I*; *U*, *V*, *W*, *X*, *Y*, *Z* are on-numerical probabilities, of which, however, *V* is less than the numerical probability *A*, and is also less than *W*, *X*, and *Y*. *X* and *Y* are both greater than *W*, and greater than *V*, but are not comparable with one another, or with *A*. *V* and *Z* are both less than *W*, *X*, and *Y*, but are not comparable with one another; *U* is not quantitatively comparable with any of the probabilities *V*, *W*, *X*, *Y*, *Z*. Probabilities which are numerically comparable will all belong to one series, and the path of this series, which we may call the numerical path or strand, will be represented by *OAI*." (Keynes, 1921, p.39)

As many people may remember, Frankenstein's dreadful monster in the SF movie was a creation of Dr. Frankenstein, a notorious German scientist. Likewise, Keynes's charming chart of probability was a creation of Mr. Keynes, a famous English economist.

No sooner did I see this chart than I had a flash of inspiration. This must be the very essence of his concept of probability! It was so regrettable, however, that he failed to provide any concrete illustration for the chart. Besides, it seems to me that almost all economists have been content themselves with its mere reproduction without going further. So I have made up my mind to have the courage of escaping a blind alley toward a new open area.

In the following, I will attempt to provide the reader with one possible concrete illustration:

Point O = 0 %, or impossibility.
Point I = 100%, or certainty.
Point A = a numerically measurable probability between O and I, say 60%.
Point U = a non-numerical probability, or any unspecified point between O and I.
Point V = a non-numerical probability, which may be represented by an interval valued probability, say 30%~40%.
Point W = 45%~65%.
Point X = 50%~80%.
Point Y = 45%~100%.
Point Z = 0%~65%.

As the reader may notice, I have partially employed the concept of interval valued probability $[\alpha, \beta]$, with the lower limit α and the upper limit β, which may be regarded as a natural extension of traditional, single-valued probability, α or β. In fact, any point on the path *OVA*, *OVW*, *OZW*, *WYI* and *WXI* may correspond to a certain interval valued probability. It is noted that Keynes himself developed such an upper-lower probabilistic interval approach to probability. In fact, he once remarked:

> "The sphere of inexact numerical comparison is not quite so limited, Many probabilities, which are incapable of numerical measurement, can be placed nevertheless *between* numerical limits. And taking particular non-numerical probabilities as standards a great number of comparisons or approximate measurements become possible. If we can place a probability in order of magnitude with some standard probability, we can obtain its approximate measure by comparison". (Keynes, 1921,p.150)

How exactly we can understand this paragraph remains a mystery. Keynes himself considered and compared numerical and non-numerical probabilities, and did exact and approximate measures. It is my belief that interval valued probabilities of the form $[\alpha, \beta]$ can represent well Keynes's intention of inexact or approximate measures. [2)]

Now. let us get back to Fig. 1. On the one hand, every point on the bottom line *OAI* is more certain than all other points in the chart, thereby it can be associated with a specific numerical probability. On the other hand, *U* on the uppermost half circle represents the most uncertain point: It is not quantitatively comparable with any other probability. In other words, it is not possible to represent *U* neither by an interval valued probability nor by a single-valued probability.

The main results which have been derived so far are very important, and will be summarized below.

(1) Keynes's charming chart of probability is a collection of probability networks represented by partial ordering. For instance, the expression '*O* →— *A*', which is my own creation, means that '*A* is more probable than *O*'. It is easy to see that such ordering is not a total ordering, but merely a partial ordering.

(2) The area in which the traditional theory of probability is applicable is limited to the bottom line *OAI*. Every point on this path can be associated with a single-valued probability: For instance, *O* stands for 0 %, *A* for 60 %, and *I* for 100 %. Therefore, any two points on the path are easily comparable with each other. In contrast, all the points off *OAI* are no longer quantifiable by a single value.

(3) The path *OVWXI* indicates a path on which any two points are comparable by means of interval valued probability. Note that *V* = [30%, 40%], *W* = [45%, 65%], and *X* = [50%, 80 %]. In order to give a nice interpretation, let us suppose that a young girl who wishes to become a professional singer must enter a good school of music for very hard training. There are three schools available —— School *V*, School *W*, and School *X*. The probabilities for admission to *V*, *W* and *X* are respectively are described by the intervals [30%, 40%], [45%,, 65%], and [50%, 80%]. Apparently, getting into *X* is more probable than getting into *W*, which in turn more probable than getting into *V*.

(4) While there exists a path from *V* to *A*, there are no paths available from *W* to *A*, and from *X* to *A*. Surely, 60% is more probable than [30%, 40%]. It is not correct to say, however, that 60% is more probable or less probable than [45%, 65%], or that 60% is numerically comparable to [50%, 80%].

(5) The path *OZWYI* shows another possible ordering. It is fair to say that *Y* = [45%, 100%] is more probable than *W* = [45%, 65%], which in turn more probable than *Z*

= [0%, 65%]. However, comparison between W and A = 60% is not possible. Nor can Z or X be compared to A.

The job of comparing the three points Y, W and Z may be a bit harder than before, requiring our wiser judgment based on common sense. Let me assume that the possibility of admission to School Y is greater than 45%, the one to W is somewhere between 45% and 65%, and the one to Z is less than 65%. Then it would be fairly reasonable to say that getting into School Y is more probable than W, which is in turn more probable than Z.

⑹ The most outer path OUI represents the most vague probability among any other path. Although it presumably lies somewhere between the impossibility point O and the certainty point I, its comparison with any other point in Keynes's chart constitutes a sort of 'mission impossible.'

In conclusion, Keynes's charming chart of probability looks like the unconquered, highest peak in the great land of Keynes. What I have attempted to do so far is nothing but the discovery of one possible route of mountain-climbing. Probably, there would be alternative routes available, which requires further investigation.

2.3 Keynes's Theory of Induction: Reappraisal of Several Case Studies

This subsection will discuss Keynes's theory of induction. It is my strong belief that reappraisal of several case studies gives us important clues for understanding the very unique features of the Keynes theory.

In the following, let us discuss and shed a new light on a series of case studies, which were originally introduced by Keynes in *A Treatise on Probability* (1921) and *the General Theory* (1936).

[Case Study 1] Deciding for or against an umbrella

England is a country where the rain falls frequently and unexpectedly. Suppose that Mr. Smith is going to start out for a walk when the barometer is high but the clouds are black. The problem of interest is whether he should carry with him an umbrella in the expectation of rain. The possibility of rain may be small if the barometer is high , whereas it may not so if the clouds are black. Therefore, if the barometer is high but the clouds are black, it is not always rational that one possibility should prevail over the other in his mind. It could be nonsense to numerically measure the probability of rain, or even compare the likeliness of rain against non-rain It is really an arbitrary matter for Mr. Smith to decide for or against the umbrella. In case the weather report is available, it is likely to affect the probability of rain and his umbrella decision.

According to Keynes, this simple case shows the essence and difficulty of probability.

Watching the weather forecast in the TV set, the weather man often says that probably it will rain tomorrow afternoon. The forecast is not perfect, and may change whenever a new information becomes available. Keynes was a refined gentleman with many hobbies. In fact, he was not only interested in weather forecast, but also in horseracing and beauty contest.

[Case Study 2] Breeding racehorses : Recovering the damages by breach of a contract

The contract at issue was that Cyllene, a racehorse owned by the defendant, should in the season of the year 1909 serve one of the plaintiff's mares. In the summer of 1908, however, the defendant sold Cyllene for £30,000 to go to South America without the consent of the plaintiff. The plaintiff claimed a sum equal to the average profit he had made through having a mare served by Cyllene during the past four years. During those four years he had had four colts which had sold at £3300. Upon that basis his loss was estimated at £700.

Mr. Justice Jeff said that he was desirous to find some mode of legally making the defendant compensate the plaintiff; but the question of damages presented insurmountable difficulties. The estimate could only be based on a succession of contingencies. First of all, it was assumed that Cyllene would be alive and well; that the mare would be well bred and not barren; that the foal would be born alive and healthy. In such a case he could only rely on the weighing of chances. Therefore, the damages must be a matter of speculation, but there may be some reasons for compensation.

This second case teaches us that the concept of probability in social sciences is fundamentally different from the one in natural sciences since a series of human factors are working behind the scene. Keynes is interested in another similar case, which arose out an offer of a Beauty Prize by the *Daily Express*. The prize was to be a theatrical engagement to a handsome prince and, according to the press, the probability of subsequent marriage.

[Case Study 3] Assessing the damages in the court: Beauty contest number one

Corresponding to the offer, 6000 photographs were submitted, and a certain number were to be selected and published in the newspaper in the following fashion. The United Kingdom was to be divided into districts and the photographs of the selected candidates living in each district were to be submitted to the readers of the paper in the district. Those readers were to select by their votes the ladies whom they considered

the most beautiful, and interestingly enough, Mr. Hicks was then to make an appointment with the 50 ladies obtaining the greatest number of votes and himself select 12 of them.

The plaintiff, who came out head of the districts, submitted that she had not been given a reasonable opportunity of keeping an appointment, that she had thereby lost of her chance of one of the 12 prizes and claimed damages accordingly. The jury found that the defendant had not taken reasonable means to give the plaintiff an opportunity of presenting herself for selection, and assessed the damages at £100. Two questions arose: Relative to what evidence ought the probability to be calculated, and was it numerically measurable? The attitude of Lord Justice is clear, The plaintiff had evidently suffered damage, and justice required that she should be somehow compensated. It was equally evident, however, relative to the information available and taking account of the arbiter's taste, it was not possible to estimate the probability of the amount of damages with numerical precision. The result was that rough justice should be done: the plaintiff's loss was estimated at 12/50 of the value of the prize.

According to Keynes, these three cases have not received serious consideration so far. By writing a book on probability, he intended to mend such negligence, putting the things on the right track.

2.4 Keynes on uncertainty : *The General Theory*

As was seen in case study 3 above, Keynes was deeply excited by the beauty contest, and continued to consider its economic implications throughout his life. In his later book *The General Theory* (1936), he picked up again a new version of the beauty contest in order to shed a new light on professional investment .

[Case Study 4] Thinking of the investment bubble: Beauty contest number two

According to Keynes, professional investment may be likened to those newspaper competitions in which the competitors are asked to pick out the six prettiest faces out of a hundred photographs. The prize is no longer getting a chance to date a handsome prince, but rather a handsome amount of money given to a beauty queen. Remarkably, such a prize is awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole. As a result, each competitor has to pick up, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are likely to look at the problem from the same viewpoint. Keynes remarked:

"It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligence to anticipating what average pinion expects the average opinion to be. And there are some, I [Keynes] believe, who practice the fourth, fifth and higher degrees".
(Keynes, 1921, p.156)

In the 1920s and 1930s when both *A Treatise on Probability* and *the General Theory* were published, the British Empire constituted the largest empire on the earth: It was often called 'the empire on which the sun never sets." One of the biggest event which excited the British people was the expedition to the South Pole, which then remained a virgin soil to the Empire.

[Case Study 5] Going on the South Pole expedition : The role of animal spirits

There was a strong sense of rivalry between the British team led by R.F. Scott and the Norwegian team led by R. Amundsen: Both teams wanted to be the first to reach the South Pole. When Scott reached the pole on 17 January 1021, he found that Amundsen had preceded him by mere 34 days, and unfortunately died on the return journey from the pole.

Whether and to what extent Scott's expedition would be successful might be controversial even to him. What motivated him to engage in such an adventure could not be explained by a mathematical calculations, but rather depended on spontaneous optimism or 'animal spirits.' In this connection, Keynes remarked with great eloquence:

"Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large portion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits — of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities". (Keynes, 1936, p.161)

To the best of our knowledge, Keynes was the first economists who employed the

word 'animal spirits' and discussed its economic consequences. As Keynes pointed out above, it is so important to make a distinction between the instability due to speculation and the instability due to human nature , or the nature of spontaneous optimism. While the first instability is related to mathematical expectation, or numerical probability, the second instability is connected with a sort of true uncertainty that is nor measurable nor comparable. We have to do something positive even under the second instability. According to Keynes, it is animal spirits that plays a critical role under such circumstances. Individual initiative is only adequate when reasonable calculation is supplemented and supported by animal spirits, so that the thought of possible failure may effectively be put aside. According to Keynes, the roll of animal spirits in the working and performance of the market economy cannot be overemphasized.

3. Keynes versus Knight: With special reference to risk, probability, and uncertainty

3.1 Keynes on probability and uncertainty

We are in a position to compare Keynes and Knight from the viewpoint of risk, probability, and uncertainty. Their analyses look similar but are more or less different. In the following, we will show pictorially the similarities and differences between them.

The Keynes world of probability and uncertainty is well illustrated in Fig. 2. It constitutes a quadruple ring structure. The outer two rings are drawn by solid lines, but the inner rings by dotted lines. Whether the boundary lines are solid or dotted correspond to the degree of connectedness between the two separated areas.

Fig. 2 around here

The key concept of Keynes's *The General Theory* (1936) is uncertainty, which is indicated by the most outer ring in Fig. 2. This is the field in which probabilities are neither numerical nor comparable. In the famous case of beauty contest mentioned above, the factor which may play a critical role is not what each competitor thinks himself, but rather what he thinks likeliest to catch the fancy and rumors of the other competitors. The resulting situation would be a sort of bubble followed by another bubble. In the case of the competition of expedition to the South Pole between rival countries, the decision may depend on aggressive character of an explorer rather than

his mathematical expectation. In the dark shadow of uncertainty and fear, what makes a man go ahead without hesitation is no more than his animal spirits.

The area lying the inside of the uncertainty ring represents the vast field of probability, which is thought of as the key concept in Keynes' *A Treatise on Probability* (1921). Probability may small or large, depending on the degree of belief and/or evidence. If probability is small enough, it may be non-comparable. While interval-valued estimation may be feasible at this stage, single-valued estimation is not so. As probability gets larger and larger, it will become comparable, and gradually numerical. The core of the quadruple ring structure indicates the area where numerical representation is available: It is in this small area that the traditional probability analysis is effectively applicable.

Now let us look at the matter from an opposite viewpoint. In other words, let start from the center core, and gradually move away toward the outer rings. As mentioned above, the core indicates the numerical area where risk in a narrow sense is present in the sense that such convenient concepts such as a probability density curve and a cumulative distribution curve can be employed. If we dare to weaken the concept of 'risk proper' by introducing the more realistic factors of 'distortion' and 'weight,' then the 'probability' ring in Fig. 2 will largely expand outward. It is interesting to note that such 'distortion and weight factors' play a central role in modern theory of prospect which has been promoted by Kahneman and Tversky (1979).

Even if we get into a wider area of the rings, the concept of probability in a broad sense , whether it is based on point-valued or interval-valued, remains to be important. According to Keynes, the theory of probability should be logical because it is concerned with the degree of belief. More exactly, it is a sort of 'common belief' which it is rational to have within a group in a given conditions, and it does not necessarily correspond to the actual beliefs of particular individuals, which may or may not be rational as can be seen in the famous contest. The degree of belief thus defined cannot be fixed: It may change by acquisition of new information. In fact, it is rather common that an opinion is probable on the evidence at first hand, but on further information becomes untenable.

Keynes thinks that uncertainty is radically different from probability. In sharp contrast to the latter, the former has nothing to with numerical representation. A person faces uncertainty when he walk around with no light at a dark night. Since the walking path is new and unknown to him, his experience does not help him much. What motivates him to walk ahead is his animal spirits or spontaneous optimism that he can achieve something that he never seen before.

A year later than *The General Theory* (1936), Keynes published a follow-up paper (1937) to clarify the concept of uncertainty:

> "By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain. Even the weather is moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years later hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know". (Keynes, 1937, pp.113-114)

Keynes holds an unshakable belief that uncertainty is categorically different from probability: It is something more than that. The game of roulette is subject to the mathematical theory of probability, thereby does not belong to the world of uncertainty. So is a Victory bond because the prospect of returns remains within the anticipated range with an upper limit and a lower limit. The expectation of human life may change country to country, and time to time. Given a country and a year, however, a man's average life expectancy may easily be calculated by a life insurance company. A weatherman says that the probability of rain tomorrow is around 20%, so that the weather does not belong to true uncertainty, but rather to a wider range of probability. By contrast, once a war begins, a lot of uncertain things may happen. We do not know in advance how the war will proceed, nor are we in a position to know its economic consequences. The price of copper and the rate of interest may go up during the war, but the price of copper and the rate of interest 20 years later are beyond imagination. In respect to those and many other events, we have no scientific basis to calculate exact probability.

In recent years, the important role of animal spirits in the working of the market economy has drawn much attention in the economic circle. George Akerlof and Robert Shiller are influential American economists, and have written their joint book *Animal Spirits* (2009). They remarked:

> "To understand the economy then is to comprehend how it is driven by the animal spirits. Just as Adam Smith's invisible hand is the keynote of classical economics,

Keynes's animal spirits are the keynote to a different view of the economy —— a view that explains the underlying instability of capitalism."

(Akerlof & Shiller, 2009, Preface, p. ix)

According to Akerlof and Schiller, Keynes's animal spirits are keynotes to understanding the modern economy. They are really intermingled with uncertainty and imperfect knowledge. Its importance may be well compared to Adam Smith's individual hand as the keynote of the classical economy.

In short, we live in the world of probability and uncertainty. Without inquiry into these subjects, our economic science would not be complete. We will turn our attention to Knight, who was one of Keynes's contemporaries and did a great work on the field of risk and uncertainty.

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3.2 Knight on risk and uncertainty

The question of much interest is how Knight discussed the concepts of risk and uncertainty. A related question is whether and to what extent his work differed from the work of Keynes. [3)]

If we want to illustrate the world of Knight by help of a figure, we may have a triple ring structure in Fig. 3. This new figure may be well compared to the quadruple ring structure of Keynes. While there were four rings for Keynes, there are now only three rings for Knight. Could such decrease in the number of rings by one really make the systems of Keynes and Knight entirely different? Or could we regard these two as essentially the same ?

Fig. 3 around here

As is seen in Fig.3, the most important ring of uncertainty is located in the most outer position. In the long history of economics, *Risk, Uncertainty and Profit* (1921) is presumably the first book whose title contains the word 'uncertainty.' It is so remarkable to see that the two terms, risk and uncertainty, are intentionally separated. In fact, Knight made the following remark:

"Uncertainty must be taken in a sense radically distinct from the familiar notion of risk, from which it has never been properly separated. The term 'risk,' as

loosely used in everyday speech and in economic discussion, really covers two things which, functionally at least, in their causal relation to the phenomena of economic organization, are categorically different." (Knight, 1921, p.19)

Uncertainty must be radically distinct from risk. The two terms 'risk' and 'uncertainty' should be categorically different. Here Knight himself is using extremely strong expressions such as "radically distinct" and "categorically different." The question which would naturally arise is how these two are really different. In this connection, Knight remarked:

"A *measurable* uncertainty, or "risk" proper, as we shall use the term, is so far different from an *unmeasurable* one that it is not in effect an uncertainty at all. We shall accordingly restrict the term "uncertainty" to cases of non-quantitative type. It is this "true" uncertainty, and not risk, as has been argued, which forms the basis of a valid theory of profit and accounts for the divergence between actual and theoretical competition. (Knight, 1921,p.20)

According to Knight, uncertainty serves as the key concept for understanding the effectiveness and limitations of the market economy. There are two different kinds of uncertainties——*measurable* uncertainty and *non-measurable* (or *unmeasurable*) uncertainty. The problem of measurability is so important that he deliberately employs italics for the words "measurable" and "non-measurable." Measurable uncertainty is not in effect uncertainty at all, but rather should be called "risk" proper. By contrast, non-measurable uncertainty is what he may call "true" uncertainty, thus being the main theme of his famous book *Risk, Profit and Uncertainty* (1921).

Knight finds that there are three different types of probability situation. The first type is *a priori* probability, and occupies the center core in the triple ring structure in Fig. 3. For instance, let us toss a coin. Then the probability that we get "one" is 1/6, a mathematically determined fraction. The second type is statistical probability, which correspond to a small ring or donut which encircles the core. A typical example of this type is provided by the proposition that the average life expectancy of a 50 years old man in Japan now is 30 years. These two types, *a priori* or statistical, belong to the same field of measurable risk. A more important concept is non-measurable uncertainty, which is shown by the outer ring encircling the inner ring of risk. Knight called this third type of probabilistic situation "estimates" or "judgments," and paid a special attention to it:

"The distinction here is that there is *no valid basis of any kind* for classifying instances. This form of probability is involved in the greatest logical difficulties of all, and no very satisfactory discussion of it can be given, but its distinction from the other types must be emphasized and some of its complicated relations indicated." (Knight, 1921, p.225)

Knight finds that estimates or judgment should not be perfect, and are liable to err. Its distinction from *a priori* or statistical probability must be emphasized. Indeed, it is meaningless to speak of either calculating probability *a priori* or of determining it empirically by studying a large number of instances. The essential fact is that the "instance" is so unique that there are not a sufficient number to form a scientific basis for any real probability. We simply do not know enough. We have to do our decisions on the basis of imperfect estimates or judgment, however. We are now subject to non-measurable uncertainty.

With the introduction of true uncertainty into the real economy, Knight continues to say, its character is completely changed. Here we see the emergence of a new economic functionary, the entrepreneur, whose function is to do a new venture on the basis of a limited amount of knowledge. Such a venture itself may be of the nature of a gamble, involving a large proportion of unpredictable factors. The entrepreneur is categorically different from the conventional type of manager who is supposed to do mere routine functions with no courage to do something new.

To sum up, the two great economists, Knight and Keynes, newly introduced non-measurable uncertainty into economic science, and energetically discussed how it differs from measurable risk. So far so good. We have to recognize, however, that Knight is Knight whereas Keynes is Keynes: There must be some differences between them. Those differences may look small in some situations, but might be very large in other situations.

Let us take a close look again at Fig. 1 and Fig. 2. Then a comparison of these two multiple ring structures may teach us that Knight and Keynes are different with respect to their views of probability and uncertainty in a delicate way. It is really remarkable to see that the 'intermediate belt' of probability which plays a very important role in Keynes's world is not present in the world of Knight. Knight really intended to do a direct comparison between risk and uncertainty without help of ' any middle passage'. As a result is that unlike Keynes, Knight found little interest in 'distortions and weights' of probability , interval valued probability and the like.

In the real world, the area in which uncertainty is measurable, and the one where uncertainty is not so cannot be so clearly divided. There should be some gaps and/or overlaps. It is in such a chaotic world that Keynes as a practical man preferred to live. although he was born with a silver spoon, his 'academic spoon' became very flexible as his age advanced, and could sometimes be bent rather easily.

By contrast, Knight as an academic man used to live in an ivory tower throughout his entire life, whence he decided a clear-cut line between the measurable area and the non-measurable area. Whereas he was born with a wooden spoon, his 'academic spoon' was strong and became even stronger as he got older.

In short, the relation between the 'silver spoon' and the 'wooden spoon' was too delicate to describe by one word. These 'academic spoons' formed between them the complicated history of separation, approach, decoupling, and reunion.

4. Hicks on decision and probability: How he was influenced by Keynes and Knight

4.1 Economics on the edges of both sciences and history

In one of his later works *Causality in Economics* (1979), J. R. Hicks concentrated his attention on the difference between natural sciences and economics. In his opinion, economic knowledge is extremely imperfect. There are very few economic laws which we know with precision: Indeed, they are subject to errors and ambiguities, which would be regarded as intolerable by many natural scientists. Hicks remarked:

> "Economics is a leading example of uncertain knowledge; it is knowledge, yet it is evidently uncertain". (Hicks, 1979, p. 2)

So he thinks that economics does not belong to the core of natural sciences, but rather is located at the edge of the sciences. Causality in economics is not as simple as one in natural sciences. The relation between cause and effect may not be one-to-one: In fact, one cause may have many effects whereas one effect may have many causes. It would be the case that a certain degree of reciprocity or mutual causality exists between cause and effect. For instance, when a person makes a sales decision in the market, his decision may not be completely independent but is likely to be influenced by the pressure of the market.

The economist is concerned with the future as well as the past. Whereas the past provides him with his facts, he uses and processes those facts for future prediction. In this connection, the following sentences are quite interesting:

"Economics, if it is on the edge of sciences, is also on the edge of history; facing both ways, it is a key position. So a consideration of economics ... may throw light in both directions. (Hicks, 1979, p.4)

In my opinion, this is a very important point when we consider the place of economics in relation to sciences and to history. Hicks proceeds to think that the same key position is occupied by other social sciences, and presumably by statistics. They are all on the edge of sciences and on the edge of history. What we are saying may be more readily be understood if it is illustrated pictorially.

Fig. 4 around here

As is seen in Fig. 4, there are two overlapping rings, with a intersection area between them The left ring and the right ring respectively indicate the field of sciences and the field of history. The intersection of the two fields shows the area in which both scientific and historical approaches are applicable. It is noted that such a pluralistic view was already pioneered by the two economists, Keynes and Knight. In particular, in the following, we will discuss more exactly why we must look at probability and statistics in both directions.

4.2 Decision and probability

It should be remembered that Hicks was once a president of Manchester Statistical Society, which was founded in 1833 as one of the earliest of such societies. As he noted, statistics used to mean social and economic statistics rather than mathematical statistics. Even Adolphe Quetelet (1796-1874), who began his career as a Belgian astronomer and mathematician, became very famous in his later years by introducing statistical methods to many social sciences including demography, sociology, and criminology.

Generally speaking , probability is regarded as a basis on which the field of statistics should be built. When we speak of statistics, it has a multiple meanings: Social scientists attach one meaning to it whereas natural scientists, another meaning. Such diversity of statistics may be a natural consequence of the wide range of views for probability.

It is conventional to say that there are two theories of probabilities. They are (1) the frequency theory that starts with many data and is induction oriented, and (2) the axiomatic theory that begins with many axioms and is deduction oriented. Most modern works on statistical mathematics tends to take the first frequency theory as their starting point.

There exists a third approach which lies somewhere between the frequency and axiomatic theories, however. According to Hicks, one of the leading proponents of this alternative approach has been Keynes in his *Treatise on Probability* (1921). This means that Hicks, who has praised Keynes as the greatest economist in the 20th century, has also thought of Keynes as a great proponent of the alternative theory of probability. In fact, Hicks remarked:

> "I have myself come to the view that the frequency theory, though it is thoroughly at home of the natural sciences, it is not wide enough for economics. Indeed. on those points where Keynes and Jeffreys appear to differ, I generally find myself on the side of Keynes". (Hicks, 1979, p.105)

Regarding the theory of probability, Hicks generally find himself on the side of Keynes. Hicks considered an example of that the European war would come to an end within a year. Surely, this was a probability which, in the particular year of 1944, most people would have assessed to be a high one. It is quite clear, however, that it does not fall within the frequency definition since it is not a matter of trials which could be repeated. Hicks was a very eloquent speaker on this point. Like Keynes (1921), Hicks was so eager to pick up this third kind of probability in economics.

> "Investments are made, securities are bought and sold, on a judgment of probabilities. This is true of economic behavior; it is also true of economic theory The probabilities of 'states of the world' that are formally used by economists, as for instance in portfolio theory, cannot be interpreted in terms of random experiments. Probability, in economics, must mean something wider.
> (Hicks, 1979, p.107)

The view that probability in economics must mean something wider clearly shows the fundamental position of Hicks on the relation between economics and probability theory. This may more readily understood if it is pictorially illustrated.

Fig. 5 around here

As is seen in Fig. 5, the Hicks world of probability and judgment consists of four rings. The inner three rings are related to a variety of probabilities, with the most outer ring being associated with uncertainty. More specifically, the smallest ring , namely the center core, indicates the area in which probabilities are unquestionably numerical: They may be represented by specific distribution functions such as binomial and normal distribution functions. The larger ring covering the numerical core shows the field where probabilities are orderable (or comparable according to Keynes), but not expressible as numbers. The even bigger ring covering the orderable ring is the one in which they are neither orderable nor comparable.

Hicks maintains that in even the non-orderable ring, probability judgments can sometimes be made, and that they can be rational since they could be improved by additional information. To sum up, the grand field of probability contains those three rings, namely numerical, orderable and non-orderable ones. By contrast, the most outer ring in the figure is associated with the area in which even probabilities are nonexistent and may be replaced by ambiguities or chaos.

Hicks has devoted his full energy into many types of probabilities, so that he seemed to pay much less attention to uncertainty than both Keynes and Knight did. The young Hicks energetically investigated the modern utility theory, one of the cores of microeconomic theory, and discussed how and to what extent the ordinal utility differs from the cardinal utility. The ordinal utility is concerned with only orders, not with absolute levels, so that it must be a more general concept than the cardinal utility. It seems to me that there is a nice correspondence between probabilities and utilities: In fact, numerical and orderable probabilities respectively correspond to cardinal and ordinal utilities. Some preference orderings such lexicographic ones cannot be represented by any utility function. Likewise, there exist some probabilities which cannot be even ordered in a simple fashion. It is rather unfortunate that Hicks has found much less interest in the concept of true uncertainty or total ambiguities. [4)]

5. Beyond Keynes and Knight: Concluding Remarks

In the above, we have intensively discussed Keynes' theory of probability versus

Knight's theory of uncertainty. In particular, we have focused on 1921 as a sort of miracle year, in which the two great books, *A Treatise on Probability* by Keynes and *Risk, Uncertainty and Profit* by Knight, were both published. We believe that this was no accident, but rather marked the start of the coming age of uncertainty in an appropriate manner.

We are now in another age of uncertainty and complexity. In fact, in June 2009, the Lionel Robbins Lectures took place at the London School of Economics. Paul Krugman, a famed Nobel laureate, was an invited speaker and remarked:

" Most macroeconomics of the past 30 years was spectacularly useless at best, and positively harmful at worst". (Krugman, 2009)

I am in general agreement with Krugman. And I would like to even say that most economic theories of the past 30 years, whether they are micro-oriented or macro-oriented, was spectacularly useless at best, and positively harmful at worst.

According to John K. Galbraith (1977), The 20th century may be characterized as the Age of Uncertainty. We would contrast the great certainties in economic thought in the 19th century with the great uncertainty in the 20th century. In the 19th century capitalists were quite certain of the success of capitalism, and socialists of the success of socialism. Little of this certainty survives in the 20th century, however. [5)]

We live now in the new 21st century. Although the Cold War was ended with the fall of the socialist Soviet Union in 1991, the so-called Riemann Shock in 2008 gave a crushing blow to the U.S. and all other capitalist countries. Besides, a series of nuclear power plant accidents such as the Three Mile disaster (1979), the Chernobyl disaster (1986) and the Fukushima disaster (2011) recalled all the people of the existence of dreadful risks that were far beyond human capacities. Surely, we are still facing a great number of human and natural risks and uncertainties.

As Joseph Stiglitz, another Nobel winning economist, once remarked that a new economic paradigm would be strongly needed.[6)] There have existed so many old economic paradigms. It seems that almost all of them are either useless or even harmful, with possible exceptions. What we have to do now is no less than the construction of a new economic paradigm. In order to carry such a task, we have to learn new lessons from old teaching of Keynes and Knight. We believe that the Keynes spirits are still alive, and so are the Knight spirits. We need to have a new kind of animal spirits, so that we may go beyond Keynes and Knight.

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Footnotes

1) It is quite probable that Keynes's romantic poem aforementioned was influenced by Hume's view over human sentiments. See Hume (1739-40). The point I want to stress here is that the Keynes poem has almost completely been neglected until today, which may constitute a puzzle in the long history of the economic thought of risk and uncertainty.

2) The role of interval valued probability in Keynes's approach was intensively investigated by Brady (2004).

3) For a detailed discussion on economic thought of Knight, see Sakai (2010)

4) It is noted that Hicks (1979) has introduced the new idea of "gray zone," meaning that outside of which probabilities are clear, but within which they are not clear at all. It remains an open question to see to what extent the gray zone is related to the concept of interval valued probability.

5) See Galbraith (1977), Preface.

6) See Stiglitz (2010) . It is noted that a new approach based on socio-economic physics is emerging in the academic circle. For a detailed discussion on this point, see Aoki, Aoyama, Aruka and Yoshikawa (2011).

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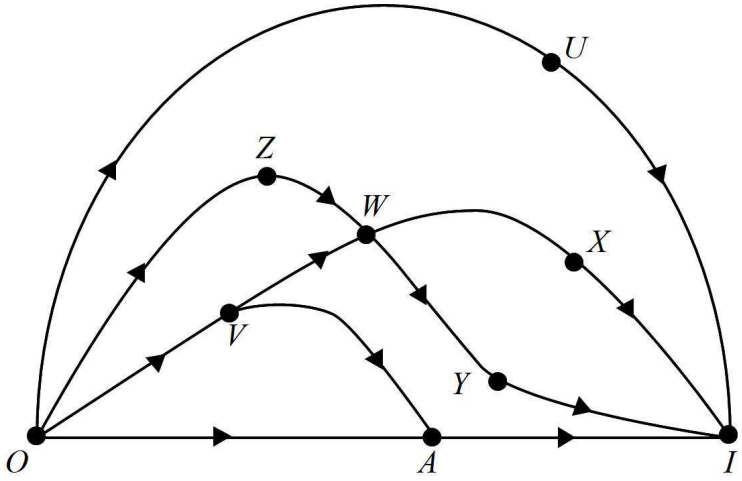

Remark: "$O \rightarrowtail A$" means that A is more probable than O. These arrows were newly introduced by the author on the original chart of Keynes.

Fig. 1. Keynes's Charming Chart of Probability: A Partial Order Network

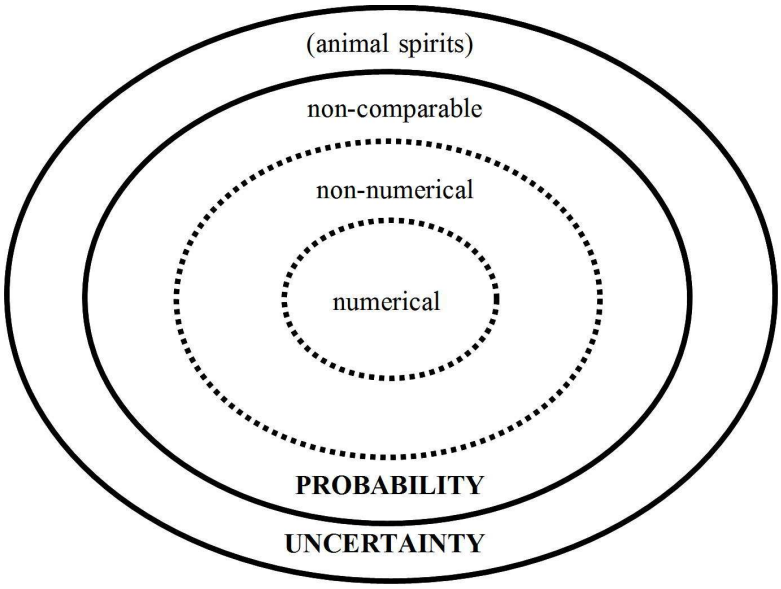

Fig. 2. Keynes on Probability and Uncertainty

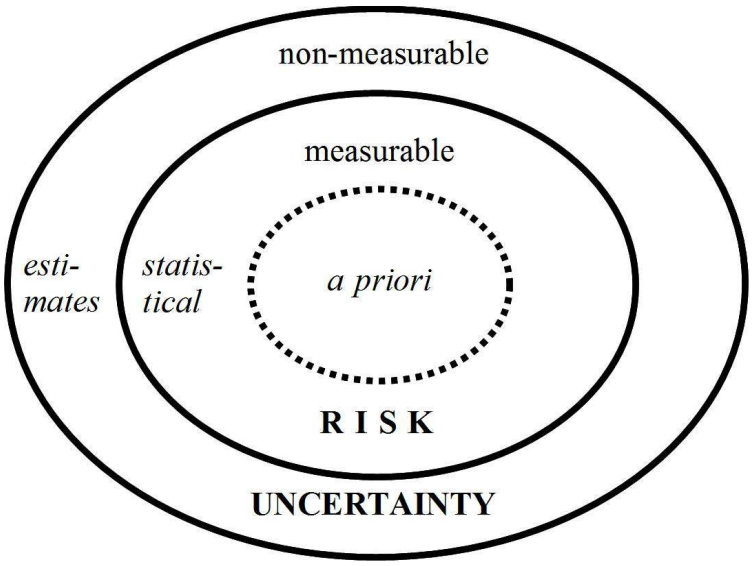

Fig. 3. Knight on Risk and Uncertainty

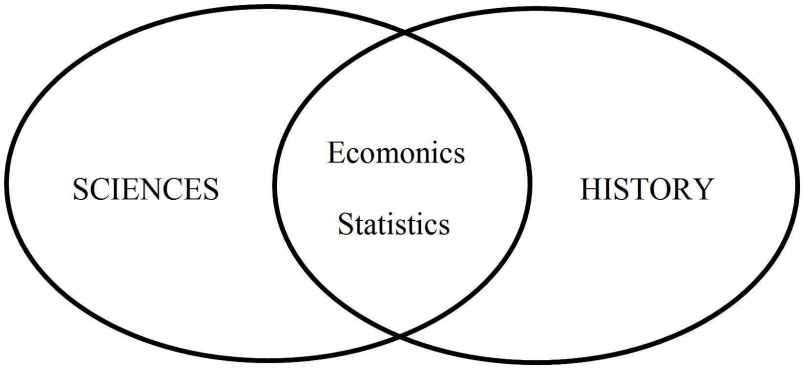

Fig. 4. Hicks' Dual Visions on Economics and Statistics

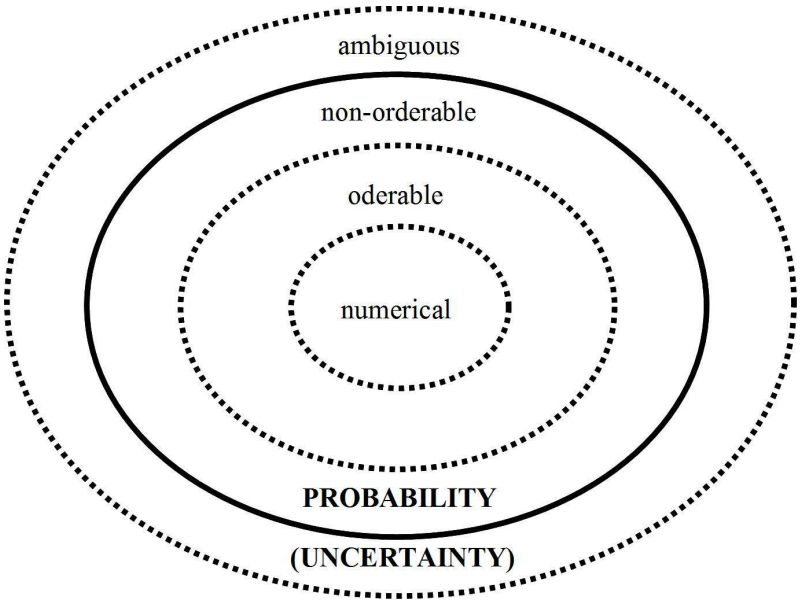

Fig. 5. Hicks on Probability and Judgment