Downward Nominal Wage Rigidity and Inflation Dynamics during and after the Great Recession*

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Abstract

I develop a New Keynesian model which embeds downward nominal wage rigidity (DNWR) for heterogeneous workers in order to address two inflation puzzles: the missing deflation during the Great Recession and the subsequent excessive disinflation. I demonstrate that DNWR makes the observed Phillips curve flatter during recessions by creating a time-varying wedge between the output gap and marginal cost. The endogenous evolution of the cross-sectional wage distribution generates non-linear dynamics such as the sign-, size-, and state-dependence of aggregate shocks. These non-linearities are reinforced in the presence of the zero lower bound of the nominal interest rate. I calibrate the model to match the wage distribution in the U.S. data and find that the model quantitatively resolves the inflation puzzles.

JEL classification: E12, E24, E52.

Keywords: Downward nominal wage rigidity, wage distribution, Phillips curve, Great Recession, zero lower bound.

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1 Introduction

During the 2008–2009 financial crisis and its aftermath, the U.S. economy experienced only a slight decline of inflation despite suffering from a severe economic downturn, a situation called the missing deflation puzzle.[1] In addition, several years later, the recovery of inflation was excessively slow although economic activity sluggishly but steadily improved. Inflation falling short from the 2 percent target despite the absence of many adverse factors was called as a mystery by the Chair of the Federal Reserve in Yellen (2017), whereas Constâncio (2015) has labeled the phenomenon as excessive disinflation.[2]

These observations cast doubt on one of the fundamental theories in modern monetary economics: the Phillips curve relationship between inflation and level of economic activity. To this end, understanding the causes behind the aforementioned inflation puzzles is important not only for improvement of macro economic models to allow them to better capture the real world but also for formulation of desirable policies because the Phillips curve governs the output-inflation trade-off in the economy.

In this paper, I argue that downward nominal wage rigidity (DNWR) resolves the missing deflation puzzle during the Great Recession as well as the excessive disinflation puzzle in subsequent years. Among several hypotheses to potentially address these inflation puzzles, I have at least two reasons for focusing on DNWR. First, recent empirical studies using micro data have uncovered severe DNWR during the Great Recession and its aftermath (e.g., Daly and Hobijn (2015), Fallick et al. (2016)). I incorporate these micro evidences into a general equilibrium model to study the aggregate implications in terms of inflation dynamics. Second, numerous studies point out that the Phillips curve relationship between inflation and a measure of the output

[1] For example, Hall (2011), in his Presidential Address to the American Economic Association, mentions that the little response of inflation to the long-lasting slack after the Great Recession is inconsistent with most of economic theories.

[2] Constâncio (2015) points out that excessive disinflation has been a common feature of inflation dynamics in advanced economics in recent years.

gap became ambiguous after the Great Recession (e.g., Stock and Watson (2010), Ball and Mazumder (2011)). However, I find that the marginal cost representation of the Phillips curve, which is directly derived from firms' price setting behavior, remained stable according to the data. The fact implies that a puzzle lies in the relationship between the output gap and marginal cost rather than in the Phillips curve itself. In this regard, DNWR accounts for the time-varying relationship between the output gap and marginal cost as I explain the mechanism shortly.

To investigate the consequences of DNWR on inflation dynamics, I introduce DNWR for individual workers into a New Keynesian dynamic stochastic general equilibrium (DSGE) model. Building upon a rich literature on DNWR, this paper is the first attempt to build a DSGE model including both nominal price rigidity and an explicit constraint on downward nominal wage adjustment for individual workers. The existing studies abstract some of these model ingredients although some economists and central bankers have mentioned DNWR as a hypothesis to account for the inflation puzzles. The abstraction potentially misleads quantitative conclusions given the highly non-linear nature of DNWR and the substantial heterogeneity of individual wages that is observed in data. The setting of this paper, on the other hand, enables me to evaluate quantitative implications of DNWR in an empirically plausible environment.

The key mechanism in my model is as follows. DNWR creates a time-varying wedge between real wages and the marginal rate of substitution (MRS) of consumption for labor by impeding wage adjustments. Real wages are tightly linked to the marginal cost of producing one unit of output, whereas the MRS is related to the output gap in a general equilibrium. Therefore, the DNWR constraint prevents declines in marginal cost during recessions even if the output gap contracts. The dampened responses of marginal cost lead to scant declines of inflation because inflation is determined by the current and future marginal costs under the New Keynesian Phillips Curve (NKPC). Consequently, DNWR accounts for the flattening of the observed

Phillips curve during recessions even though the marginal cost representation of the NKPC remains unchanged.[3]

My model embeds heterogeneous workers who have different level of desired wages and decide their wages subject to the DNWR constraint. By doing so, I can utilize the micro evidence on DNWR to calibrate my model. More importantly, this heterogeneous agent setting implies that the model's aggregate dynamics depend on the cross-sectional wage distribution. I indeed find that the endogenous evolution of the cross-sectional distribution generates non-linear dynamics in the several important dimensions. First, the model's responses are asymmetric depending on the *sign* of an exogenous shock. A contractionary shock makes the DNWR constraint binding for a larger share of workers, which leads to a stronger downward wage and price stickiness than upward at the aggregate level. Second, the aggregate dynamics are also affected by the *size* of an exogenous shock because a larger shock creates more drastic changes in the share of workers with or without being constrained. Hence, the mechanism behind missing deflation becomes particularly apparent for a large and contractionary shock, and the Great Recession is the most obvious example. Third, the *state* of the economy before hitting an exogenous shock is another determinant of the aggregate dynamics. In particular, I find that the state-dependence is the key to addressing excessive disinflation. Intuitively, once workers' desired wages fall short of their actual ones during recessions, wage increases are postponed in the recovery phase as long as the DNWR constraint remains binding.[4,5] This channel is particularly strong in the presence of heterogeneity because a substantial fraction of workers

[3]To be precise, I focus on the price Phillips curve to study inflation dynamics, whereas several studies in the literature investigate the effects of DNWR on the wage Phillips curve (e.g., Akerlof et al. (1996), Benigno and Ricci (2011), Daly and Hobijn (2014)).

[4]In addition to this backward-looking channel of DNWR, my model accommodates the forward-looking channel to compress wage increases due to precautionary motives for future downturns, which previous studies have emphasized (e.g., Elsby (2009)). However, I find that the backward-looking channel is particularly significant after severe recessions.

[5]Several studies have indeed found a pent-up wage deflation mechanism in the data after the Great Recession (e.g., Daly and Hobijn (2015)). I demonstrate this mechanism's link to the excessive disinflation and investigate quantitative results in a DSGE model.

can be left with being constrained when the aggregate economy starts to improve.

It is noteworthy that these effects of DNWR are reinforced by the presence of the ZLB of the nominal interest rate. As the existing literature has demonstrated, the impacts of an exogenous shock are amplified under the ZLB due to the lack of offsetting monetary policy responses (e.g., Christiano et al. (2011)). However, since scope for downward adjustments of wages and prices are limited due to DNWR in my model, the ZLB's amplification effects upon a contractionary shock are exclusively absorbed by further contractions of real quantities. Moreover, even when the economy is not at the ZLB, agents expect that future recessions would become severe because the economy might be constrained by the ZLB. Therefore, they compress wage increases in preparation for a future downturn, which leads to low wage growth and price inflation. Overall, the presence of the ZLB help the model reconcile the sluggish fluctuations of inflation and large variations of real quantities during and after the Great Recession.[6]

To quantify these implications, I calibrate the model to match various moments of the wage distribution in the U.S. data. The calibrated model predicts that inflation declines by only 3.1 percentage points in the year-on-year rate in a severe recession that replicates the Great Recession. Moreover, the model suggests that, starting from such severe recession, the recovery of inflation is four times slower than recovery from the median state. These quantitative results are in stark contrast to those of a stylized New Keynesian model without DNWR, which predicts a massive deflation and a quick recovery upon a severe recession shock. They are indeed consistent with the data in which inflation declined sluggishly with an initial drop of around 2 percentage points.

This paper is related to several bodies of research. First, it falls into the growing body of literature on the inflation puzzles in recent years. However, the existing studies focus on the formation of inflation expectations (e.g., Coibion and Gorodnichenko (2015), Bianchi and Melosi (2017)), financial frictions (e.g., Del Negro et al. (2015),

[6]This finding is distinct from previous studies such as Gust et al. (2017), who find the responses of both price and real variables are enlarged at the ZLB.

Christiano et al. (2015), Gilchrist et al. (2017)), or real rigidities (e.g., Lindé and Trabandt (2018)) to explain the missing deflation puzzle. The wage channel proposed in this paper is complementary to but distinctive from these studies. Moreover, I find the quantitative significance of the wage channel. This finding is all the more important because the existing studies find it challenging to quantitatively match the slight decline in inflation found in the data.[7] Regarding the excessive disinflation puzzle, Constâncio (2015) mentions several hypotheses such as anchored expectations and increased international competition, although a formal analysis has not been provided in the literature to my knowledge. In this regard, I show that my model resolves the excessive disinflation as well as the missing deflation coherently.

Second, this paper contributes to the literature on DNWR. In particular, this paper is related to the studies that have explored the aggregate implications of DNWR (e.g., Akerlof et al. (1996), Elsby (2009), Benigno and Ricci (2011)) and its consequences after the Great Recession (e.g., Daly and Hobijn (2014), Schmitt-Grohé and Uribe (2016), Schmitt-Grohé and Uribe (2017)). However, in a crucial difference of my model from these studies, I introduce DNWR into a New Keynesian setting with nominal price rigidity. I show that the interaction between nominal price rigidity and DNWR accounts for the joint dynamics of marginal cost, inflation, and output found in the data.[8] In addition, compared with the studies that approximate DNWR by a smooth asymmetric adjustment cost function (e.g., Kim and Ruge-Murcia (2009), Aruoba et al. (2018)), my model's explicit DNWR constraint generates substantial non-linear dynamics that enable the model to match the data under parameterization in accordance with the micro evidence.

Third, this paper's model is classified as a heterogeneous agent model with aggre-

[7]For example, Del Negro et al. (2015) and Kara and Pirzada (2016) estimate a medium-scale DSGE model and identify much higher degree of nominal price rigidity than implied by individual price changes in the micro data.

[8]For example, under the flexible price setting that is used by existing studies, marginal cost is constant over time contrary to the data. I also find that adding nominal price rigidity strengthens the persistence of inflation because inflation depends not only on the current marginal cost but also on the future ones.

gate uncertainty, which was initiated by Krusell and Smith (1998). Among others, it is closely related to Gornemann et al. (2016) and Blanco (2018), who apply the equilibrium concept to the New Keynesian model, but remains distinct from them in that I examine the heterogeneity of individual wages.

The remainder of the paper is organized as follows. Section 2 provides the empirical evidence that motivates my model. Section 3 develops my baseline model with DNWR, and Section 4 describes the computation method as well as calibration procedure. Section 5 presents numerical results, which is followed by an investigation of the Great Recession in Section 6. Section 7 offers conclusion.

2 Motivating evidence

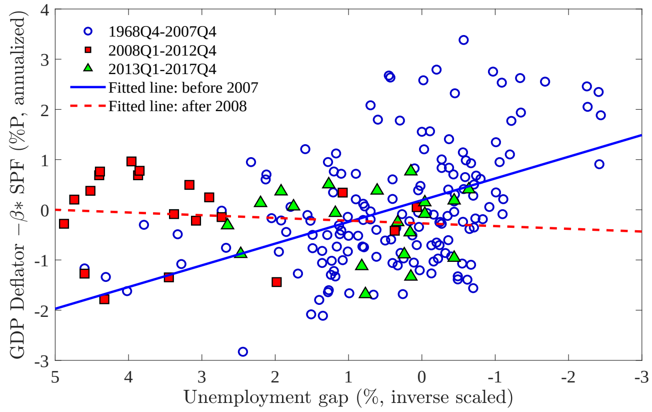
This section discusses empirical evidence that motivates the model analysis presented in the subsequent sections. Specifically, I estimate two representations of the NKPC: the output gap and marginal cost representations. It should be noted that they are interchangeable only when the output gap and marginal cost have a one-to-one relationship.[9,10] Indeed, I document that, whereas the farmer became flatter after the Great Recession in line with the findings of previous studies, the latter remained stable in the data.

Before proceeding to the regression analysis, Figure 1 depicts the relationship between inflation and the output gap in the U.S. data. Following Coibion and Gorodnichenko (2015), I use the unemployment gap as a proxy variable for the output gap. The inflation rate on the vertical axis is adjusted using inflation expectations so that the slope of the fitted lines represents that of the NKPC. The figure suggests that the

[9] A linear relationship between the output gap and marginal cost is often derived as a consequence of a frictionless labor market. I discuss this point using a three-equation New Keynesian model in Online Appendix F.

[10] In the empirical literature, early studies such as Gali and Gertler (1999) obtain significant estimates for the slope of the NKPC when using marginal cost instead of the output gap as a regressor. On the other hand, many recent studies including Ball and Mazumder (2011) and Coibion and Gorodnichenko (2015) report flattening of the NKPC after the Great Recession in terms of a measure of the output gap.

Figure 1: Phillips curve for U.S. data



Notes: The horizontal axis is the unemployment gap as a proxy for the output gap (x_t). The vertical axis represents the inflation rate minus the inflation expectation term ($\pi_t - \beta\pi_t^e$) where the discount factor β is calibrated to be 0.995. The inflation measure is the GDP implicit price deflator, whereas the inflation expectations are the median forecast of the SPF. The slope of the fitted lines correspond to the slope of the NKPC in each sub-sample.

slope of the output gap NKPC is flatter in the sample after 2008. This implies that inflation showed only a slight decline during the Great Recession, and its recovery was slow afterward compared with predictions of the NKPC.

I estimate the NKPC as follows:

$$\begin{aligned}\pi_t &= \beta_0\pi_t^e + \gamma_0\pi_{t-1} + \kappa_0 x_t \\ &\quad + \beta_1 postGR_t\pi_t^e + \gamma_1 postGR_t\pi_{t-1} + \kappa_1 postGR_t x_t + e_t. \end{aligned} \tag{1}$$

Several points are noteworthy in the specification. First, the expectation term $\mathbb{E}_t[\pi_{t+1}]$ is replaced with survey-based expectation π_t^e. This follows findings of the literature that survey-based expectation measures improve a regression model's fit (e.g., Adam and Padula (2011), Coibion and Gorodnichenko (2015), Furhrer (2017)). Second, following conventions in the literature, the lagged inflation term π_{t-1} is added to incorporate persistence of inflation in the data. Third, the error term e_t captures exogenous innovations to current inflation such as markup shocks. Under this assumption, the ordinary least square (OLS) delivers an unbiased estimator.[11] Lastly, to capture potential changes in coefficients after the Great Recession, I include interaction terms with a dummy variable for the post-Great Recession periods $postGR_t$.

Regarding the data series, I use the GDP implicit price deflator as a measure of inflation.[12] The inflation expectations are the median forecast of the GDP deflator in the Survey of Professional Forecast (SPF) provided by the Federal Reserve Bank of Philadelphia.[13] I consider two cases for the choice of variable x_t: the output gap and marginal cost. The unemployment gap is a measure of the output gap. To estimate marginal cost, I follow the insights of Hall (1986) by resorting to firms' optimization behavior, assuming the Cobb-Douglas production function with overhead labor adjustment (CDOH) in a baseline case. The construction of marginal cost is described in Online Appendix A.

Table 1 presents the regression result of the NKPC (1). The coefficients without interaction are highly significant with the anticipated sign. Regarding the changes in coefficients after the Great Recession, the coefficient for the output gap in the full sample in columns (2) is less than half of that before the Great Recession in columns (1). The interaction term with the post-Great Recession dummy variable is significantly negative in column (3), which implies a flattening of the NKPC in terms of the output gap. In contrast, the coefficient on marginal cost in columns (4) and (5) is quite stable over time with one standard error of each coefficient covering the other. The interaction term is consistently insignificant in column (6).

In Online Appendix A, I assess robustness for various alternative cases including other measures for the output gap and marginal cost, a different specification of the NKPC, and alternative assumptions on the inflation expectations. These cases

[11] More discussion is provided by Adam and Padula (2011).

[12] This follows Gali and Gertler (1999). The GDP deflator is considered preferable over the Consumer Price Index (CPI) or the Personal Consumption Expenditures (PCE) inflation for this analysis because the inflation rate in the NKPC should reflect the pricing behavior of domestic firms.

[13] Several studies use the inflation expectations in the Michigan survey. However, the survey does not provide the forecast of the GDP deflator.

Table 1: OLS estimation of NKPC

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x_t	Unemployment gap			Marginal cost (CDOH)		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	0.610***	0.570***	0.621***	0.420***	0.469***	0.445***
	(0.082)	(0.078)	(0.080)	(0.080)	(0.074)	(0.079)
π_{t-1}	0.423***	0.448***	0.410***	0.573***	0.524***	0.548***
	(0.077)	(0.075)	(0.076)	(0.081)	(0.075)	(0.080)
x_t	0.332***	0.158***	0.311***	0.231***	0.172***	0.191***
	(0.059)	(0.046)	(0.056)	(0.076)	(0.061)	(0.072)
$postGR_t \times \pi_t^e$			0.0544			0.368
			(0.218)			(0.222)
$postGR_t \times \pi_{t-1}$			–0.164			–0.365
			(0.222)			(0.234)
$postGR_t \times x_t$			–0.334***			–0.0508
			(0.091)			(0.129)
Adjusted R^2	0.956	0.948	0.950	0.953	0.947	0.947
N	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity-corrected standard errors are reported in parentheses. The sign of the coefficient for the unemployment gap is flipped for comparison purposes. $postGR_t$ is a dummy variable that takes a value of 1 from 2008Q1 to 2012Q4. The time-series spans quarterly from 1968Q4 to 2007Q4 for column (1) and (4), and from 1968Q4 to 2016Q4 for the other columns. The start point of the time-series corresponds to when the SPF became available.

confirm the baseline results in that the coefficient on marginal cost is stable over time whereas the coefficient on the output gap drops after the Great Recession or is insignificant even before in some specifications.[14]

Given the various issues regarding estimating the NKPC,[15] I take the findings in this section as suggestive only. However, they cast doubt on the tight connection between the output gap and marginal cost, which is often assumed in a stylized New Keynesian model. In the next section, therefore, I develop a model in which labor market friction arising from DNWR creates a wedge between these two elements. The model is capable of accounting for the flattening of the observed Phillips curve relationship between inflation and the output gap following the Great Recession while keeping the marginal cost representation of the NKPC unchanged.

[14]For instance, the coefficient of the output gap is insignificant under the rational expectation assumption regardless of sample periods. The result is indeed consistent with the existing literature (e.g., Gali and Gertler (1999), Adam and Padula (2011)).

[15]Mavroeidis et al. (2014) provide a comprehensive survey.

3 Model

This section develops a DSGE model that embeds the DNWR constraint for individual workers. Workers are heterogeneous in that they are subject to idiosyncratic shocks and therefore have different level of desired wages. Other aspects of the economy share many features with a standard New Keynesian model (e.g., Erceg et al. (2000), Ireland (2004)). The economy consists of monopolistically competitive firms that set prices and face quadratic price adjustment costs, households who make saving-consumption decisions and supply differentiated labor services, and a central bank that follows an interest rate feedback rule.

3.1 Households

There is a continuum of households indexed by j on the unit interval, each of whom supplies a differentiated labor service to the production sector. The aggregate labor supply takes the Dixit-Stiglitz form:

$$H_t \equiv \left(\int_0^1 h_t(j)^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}} \tag{2}$$

where θ_w represents the labor demand elasticity. The user of these labor services minimizes the cost of using a certain amount of composite labor inputs, taking each labor service provider's wage as given. The FOC for the cost minimization problem leads to the following individual labor demand function:

$$h_t(j) = \left(\frac{w_t(j)}{W_t} \right)^{-\theta_w} H_t \tag{3}$$

where the aggregate wage index W_t is defined as

$$W_t \equiv \left(\int_0^1 w_t(j)^{1-\theta_w} dj \right)^{\frac{1}{1-\theta_w}}. \tag{4}$$

The utility function of each household j is assumed to be additively separable in utility from consumption $c_t(j)$ with risk aversion parameter σ and in disutility from hours worked $h_t(j)$ with the Frisch labor supply elasticity $1/\eta$.[16] The disutility from labor is subject to an idiosyncratic shock $\chi_t(j)$, which follows an *i.i.d.* log-normal distribution.[17] The time-varying discount factor β_t is exogenous and common for each household. Expected lifetime utility is given by

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty} D_{t,t+s}\left(\frac{1}{1-\sigma}c_{t+s}(j)^{1-\sigma} - \frac{1}{1+\eta}\chi_{t+s}(j)h_{t+s}(j)^{1+\eta}\right)\right] \tag{5}$$

$$where \ \ log(\chi_t(j)) \sim i.i.d.N(0,\sigma_\chi^2)$$

$$D_{t,t+s} = \beta_{t+s}D_{t,t+s-1} \ \ for \ s \geq 1, \ and \ \ D_{t,t} = 1.$$

The discount factor β_t follows an AR(1) process as follows:

$$log(\beta_t) = (1-\rho_d)log(\bar{\beta}) + \rho_d log(\beta_{t-1}) + \epsilon_{d,t} \ , \ \epsilon_{d,t} \sim i.i.d.N(0,\sigma_d^2) \tag{6}$$

where $log(\bar{\beta})$ represents the unconditional mean of $log(\beta_t)$. A positive innovation $\epsilon_{d,t}$ causes households to lose their desire to consume in the current period, which can be interpreted as a contractionary demand shock.

Household j's budget constraint in period t is given by

$$c_t(j) + \frac{a_t(j)}{P_t} \leq (1+\tau_w)\frac{w_t(j)}{P_t}h_t(j) + R_{t-1}\frac{a_{t-1}(j)}{P_t} + \frac{\tau_t(j)}{P_t} + \Phi_t(j) \tag{7}$$

[16]This specification is extensively used in the literature. On the other hand, several studies investigate the role of non-separable preferences (e.g., King et al. (1988), Christiano et al. (2011)). In this regard, Guerrón-Quintana (2008) shows that non-separability makes consumption more responsive to real wages due to the complementarity of consumption and labor. This implies that non-separable preferences would reinforce my quantitative results to match large variations in real quantities and small fluctuations in price variables found in the data.

[17]An idiosyncratic labor disutility shock can act as a trigger for the cross-sectional dispersion of wages. Though a labor productivity shock can play a similar role, a labor disutility shock has an advantage of keeping the aggregation of the budget constraint concise, which allows one to derive the aggregate consumption Euler equation without idiosyncratic components. Moreover, wages have a natural interpretation that they represent the amount paid for a unit of hours worked. Similar specifications are used by Benigno and Ricci (2011) and Daly and Hobijn (2014).

where $a_t(j)$ is the amount of asset holding, $\tau_t(j)$ is the lump-sum tax, and $\Phi_t(j)$ is the share of producer's real profits distributed to household j. The model assumes that households do not internalize the changes, if any, of $\tau_t(j)$ or $\Phi_t(j)$. P_t, R_{t-1}, and τ_w denote the aggregate price index, gross nominal interest rate, and the labor subsidy, respectively.

Households' nominal wages might be subject to the DNWR constraint.[18] The specification largely follows Daly and Hobijn (2014) by assuming that $1-\alpha$ fraction of households is not allowed to reduce their nominal wages in each period with $0 < \alpha < 1$, whereas the remaining fraction α is free to change their wages without constraint:

$$w_t(j) \geq w_{t-1}(j) \quad \textit{with prob. } 1-\alpha. \tag{8}$$

This assumption reflects a well known empirical fact that nominal wage reduction is rare to occur. For instance, Baratteiri et al. (2014) study the frequency of individual nominal wage changes in the Survey of Income and Program Participation (SIPP) during 1996–1999 and report that nominal wage reductions only amount to 12.3 percent of all non-zero nominal wage changes after correcting for measurement errors.

In what follows, I impose two additional assumptions, both of which are widely used in the literature. First, I assume that each household has access to complete contingent claim markets which guarantees that consumption is identical across households, i.e., $c_t(j) = C_t$, though they are still subject to an uninsurable idiosyncratic labor disutility shock. Second, I assume that the labor subsidy is set so as to remove the steady-state wage markup that arises from households' monopolistic power over their labor service. That is, $1+\tau_w = \bar{\mu}_w$, with $\bar{\mu}_w \equiv \theta_w/(\theta_w - 1)$ being the steady-state wage markup.

[18] Wages function as a workers' choice variable because of workers' monopolistic power over their labor service. One can take the situation that wages are posted by firms along with the corresponding labor demand, and workers make a decision among available choices. To this end, the literature provides ample evidence on firms' reluctance to offer nominal wage cuts (e.g., Bewley (1999), Chemin and Kurmann (2014)).

Each household j maximizes expected lifetime utility (5) by choosing consumption $c_t(j)$, asset holdings $a_t(j)$, and nominal wages $w_t(j)$ subject to the budget constraint (7), individual labor demand (3), and the DNWR constraint (8) if applicable. The FOCs for the problem take the following form:

$$\mathbb{E}_t\left[\beta_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{R_t}{\Pi^p_{t+1}}\right]=1 \tag{9}$$

$$\psi_t(j)=\left(\frac{w_t(j)}{P_t}-mrs_t(j)\right)\left(C_t^{-\sigma}\frac{\theta_w h_t(j)}{w_t(j)}\right)+\beta_t\mathbb{E}_t[\psi_{t+1}(j)] \tag{10}$$

$$where \quad mrs_t(j)\equiv\frac{\chi_t(j)h_t(j)^\eta}{C_t^{-\sigma}}$$

$\Pi^p_t \equiv P_t/P_{t-1}$ is the gross price inflation rate and $mrs_t(j)$ is the marginal rate of substitution (MRS) of household j. $\psi_t(j)$ denotes the Lagrange multiplier for the DNWR constraint, which represents the shadow value of easing the DNWR constraint by one unit. The complementary slackness conditions for the DNWR constraint (8) are given by

$$\psi_t(j)\geq 0 \tag{11}$$

$$\psi_t(j)(w_t(j)-w_{t-1}(j))=0. \tag{12}$$

The optimality conditions for individual wage setting are characterized by (11), (12), and (10). If not for the DNWR constraint, then, $\psi_t(j)=0$ for all j and t. In that case, the optimality conditions are reduced to

$$\frac{w^f_t(j)}{P^f_t}=mrs^f_t(j) \tag{13}$$

where x^f_t denotes the variable x_t under flexible prices and wages. Equation (13) suggests that the real wage is equalized to the MRS in a frictionless labor market. On the other hand, in the presence of the DNWR constraint, the complementary slackness conditions imply that either of the following must hold true:

$$w_t(j) = w_{t-1}(j) \tag{14}$$

$$or$$

$$\psi_t(j) = 0. \tag{15}$$

The former condition corresponds to the case where the DNWR constraint binds in the current period, whereas the latter indicates the case where it does not and is rearranged using (10):

$$\frac{w_t(j)}{P_t} = mrs_t(j) - \beta_t \mathbb{E}_t[\psi_{t+1}(j)] \left(C_t^{-\sigma} \frac{\theta_w h_t(j)}{w_t(j)} \right)^{-1}. \tag{16}$$

The first term in the RHS of (16) coincides with the optimal wage under flexible wages, whereas the second term represents the reserved wage hike due to DNWR. It is worth pointing out that the optimal wages implied by (16) are weakly lower than the MRS because $\psi_t(j)$ is non-negative. Intuitively, a household internalizes the possibilities that the DNWR constraint might bind in the future period and therefore desires to hold a buffer to prevent the future constraint from binding even if it does not bind in the current period. In this way, DNWR endogenously generates upward wage stickiness. This property is in line with the findings of previous studies (e.g., Elsby (2009)).

To summarize the aforementioned conditions, the optimal wage of household j subject to the DNWR constraint follows the rule:

$$w_t(j) = max \left\{ w_t^d(j), w_{t-1}(j) \right\} \tag{17}$$

where the desired wage $w_t^d(j)$ satisfies (16). A household not subject to the constraint determines the wage according to (16) only.

3.2 Firms

There is a continuum of monopolistically competitive firms indexed by i on the unit interval, each of which produces a differentiated good. Firm i uses composite labor inputs $h_t(i)$ with a linear production technology[19]:

$$y_t(i) = Z_t h_t(i) \tag{18}$$

$$where \;\; h_t(i) = \left(\int_0^1 h_t(i,j)^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}} \tag{19}$$

with $h(i,j)$ denoting the labor service supplied by household j and used in firm i. Technology Z_t is exogenous and common for each firm.

The FOC of the cost minimization problem to determine the labor inputs $h_t(i)$ takes the following form:

$$mc_t(i) = \frac{W_t}{Z_t P_t} \equiv MC_t \tag{20}$$

where MC_t is the real marginal cost of producing one unit of output. Marginal cost is identical across firms because each firm has identical labor demand elasticity and takes wages as given.

I next formulate the firms' profit maximization problem. Firms face the quadratic price adjustment costs of Rotemberg (1982) with parameter ϕ_p governing the degree of price stickiness. A firm chooses its price to maximize expected profits (21) subject to the individual good demand function (22), which is analogous to the individual

[19]Although my model analysis uses a simple linear production function, I allow for richer specifications to accommodate various properties of the data when constructing marginal cost series for the empirical analysis in the previous section.

labor demand function:

$$\max_{p_t(i)} \ : \ \mathbb{E}_t \left[\sum_{s=0}^{\infty} Q_{t,t+s} \Phi_{t+s}(i) \right] \tag{21}$$

$$\begin{aligned} where \quad & Q_{t,t+s} = D_{t,t+s} \left(\frac{C_{t+s}}{C_t} \right)^{-\sigma} \\ & \Phi_t(i) = (1+\tau_p) \frac{p_t(i)}{P_t} y_t(i) - MC_t y_t(i) - \frac{\phi_p}{2} \left(\Pi_t^p(i) - \Pi^* \right)^2 C_t \\ s.t. \quad & y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta_p} Y_t. \end{aligned} \tag{22}$$

where $\Pi_t^p(i) \equiv P_t(i)/P_{t-1}(i)$. $\Phi_t(i)$ is the real profits of firm i, and $Q_{t,t+s}$ is the stochastic discount factor between t and $t+s$. τ_p denotes the production subsidy. As in the households' problem, I assume that the production subsidy removes the steady-state price markup that arises from firms' monopolistic power. That is, $1+\tau_p = \bar{\mu}_p$ with $\bar{\mu}_p \equiv \theta_p/(\theta_p - 1)$.

I focus on the symmetric equilibrium where each firm sets an identical price. The FOC for the profit maximization problem yields the NKPC:

$$(\Pi_t^p - \Pi^*)\Pi_t^p = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) (\Pi_{t+1}^p - \Pi^*)\Pi_{t+1}^p \right] + \kappa \left(MC_t - 1 \right) \tag{23}$$

where MC_t is defined in (20). $\kappa \equiv \theta_p/\phi_p$ represents the slope of the NKPC. The aggregate production function is given by

$$Y_t = Z_t H_t. \tag{24}$$

3.3 Central bank

The baseline model assumes a Taylor (1993)-type monetary policy rule where a central bank sets the gross nominal interest rate R_t to stabilize the gross inflation rate Π_t

and output Y_t around their target level Π^* and Y^*[20]:

$$R_t = R^* \left(\frac{\Pi_t^p}{\Pi^*}\right)^{\delta_\pi} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} \tag{25}$$

where $R^* = \Pi^*/\beta$. The ZLB is introduced in Section 6.

In the following, I investigate the recursive competitive equilibrium. Definitions of the equilibrium as well as the government budget constraint and market clearing conditions are provided in Online Appendix B.

3.4 Analytical example

This subsection briefly discusses the key mechanism of the model by imposing several additional assumptions to analytically characterize the equilibrium. Specifically, I consider log-utility from consumption, linear-disutility from labor, and no idiosyncratic disutility shocks, i.e., $\sigma = 1$, $\eta = 0$, and $\chi_t(j) = 1$, in this subsection.

Under flexible prices and wages, the labor market equilibrium condition equalizes the marginal product of labor (MPL) and the MRS through real wages:

$$MPL_t^f = \frac{W_t^f}{P_t^f} = MRS_t^f \tag{26}$$

where $MPL_t^f = Z_t$ and $MRS_t^f = Y_t^f$. Note that the notation j for each household is dropped in (26) because the MRS does not have idiosyncratic components in this example.

In contrast, in the presence of DNWR and nominal price rigidity, the MPL and MRS are no longer equalized. The labor market equilibrium condition leads to:

$$MC_t MPL_t = \frac{W_t}{P_t} = \mu_{w,t} MRS_t \tag{27}$$

[20] My numerical analysis focuses on the consequences of a discount factor shock. Because the shock does not change the allocation under flexible prices and wages, (25) is identical to the rule that responds to the output gap.

where $MPL_t = Z_t$ and $MRS_t = Y_t$. The wage markup $\mu_{w,t}$ summarizes the discrepancy between real wages and the MRS due to DNWR, whereas $MC_t(\equiv 1/\mu_{p,t})$ captures the time variations in marginal cost that result from imperfect price adjustments.

By combining (26) and (27) and taking the logarithm of both sides, the relationship between the output gap and marginal cost is given by

$$\widehat{MC}_t = \hat{Y}_t^{gap} + \hat{\mu}_{w,t} \tag{28}$$

where $\hat{Y}_t^{gap} \equiv \hat{Y}_t - \hat{Y}_t^f$ and $\hat{X}_t \equiv log(X_t)$. By substituting (28) into the linearized version of (23), one can immediately see the wage markup appears in the output gap representation of the NKPC as follows:

$$\begin{aligned} \pi_t^p &= \beta \mathbb{E}_t \left[\pi_{t+1}^p\right] + \kappa \widehat{MC}_t \\ &= \beta \mathbb{E}_t \left[\pi_{t+1}^p\right] + \kappa \hat{Y}_t^{gap} + \kappa \hat{\mu}_{w,t} \end{aligned} \tag{29}$$

where $\pi_t^p \equiv log(\Pi_t^p)$. Equations (28) and (29) suggest that DNWR creates a time-varying wedge between the output gap and marginal cost; this wedge appears as a shifting parameter in the output gap representation of the NKPC. Intuitively, during recessions when the wedge increases due to the binding DNWR constraint, the output gap representation of the NKPC shifts up, which makes the observed relationship between inflation and the output gap become flatter. To quantify these implications, I solve the model numerically in the next section.

4 Numerical Method and Calibration

This section presents my equilibrium computation method and calibration procedure.

4.1 Modified Krusell-Smith algorithm

I solve my model taking heterogeneity of individual workers' wages into account. In this regard, unlike a time-dependent constraint such as the staggered contract of Calvo (1983), whether the DNWR constraint binds or not crucially depends on the previous period's wages. Therefore, in order to characterize equilibrium, one needs to keep track of the cross-sectional wage distribution. Moreover, I choose to solve my model in a dynamic setting that incorporates aggregate uncertainty because my main focus is on inflation dynamics at business cycle frequencies.

To deal with heterogeneity and aggregate uncertainty, I adopt the insights of Krusell and Smith (1998), who propose a computation method that summarizes the information in cross-sectional distribution into a small number of moments of that distribution.

Although their original algorithm requires aggregate jump variables to have a closed form solution in terms of aggregate state variables, that condition does not hold in my New Keynesian setting.[21] Therefore, I propose a modified algorithm containing the following steps. First, I formulate a guess for the aggregate law of motion (ALM) of the state variable, that is, aggregate real wages $\tilde{W}_t$.[22] Second, given the guess for the ALM, I solve for aggregate jump variables as an equilibrium outcome of the aggregate part of the economy. Notice that it is a New Keynesian system that includes the consumption Euler equation (9), the NKPC (23), and the Taylor rule (25). Importantly, this step is feasible because the aggregate part is independent of individual workers' behavior once conditional on aggregate real wages. Third, given all the aggregate variables, I derive an individual worker's real wage $\tilde{w}_t(j)$ as a solution to the individual wage setting problem. Finally, I numerically integrate individual wagees to recover the aggregate wage and update the guess for the ALM. For solving each of the aggregate and individual parts of the model, I use a global

[21] More precisely, aggregate jump variables should be solved interdependently with the future expectations in a wide class of the New Keynesian models including mine.

[22] As in a standard New Keynesian model, level of nominal prices and wages is undetermined.

method to address the occasionally binding nature of DNWR and the ZLB. The details of computation are provided in Online Appendix C.

4.2 Calibration

The model's time frequency is quarterly. The externally fixed parameters are listed in Panel (A) of Table 2. The choices of the discount factor β and target inflation rate Π^* correspond to an annual real interest rate of 2 percent and an annual inflation rate of 2 percent, respectively. Relative risk aversion σ is set at 2.0 and the Frisch labor supply elasticity η^{-1} is at 4.0 in line with the macro literature. The values of θ_w and θ_p imply the steady-state markup is 12.5 percent. I follow Fernández-Villaverde et al. (2015) and choose $\delta_\pi = 1.50$ and $\delta_y = 0.25$. The degree of price stickiness ϕ_p is calibrated according to the micro evidence reported by Nakamura and Steinsson (2008).[23] The implied slope of the NKPC is considerably steeper, implying that inflation is more volatile, than a typical estimate of a medium-scale DSGE model.[24] In other words, I assess whether my model is able to match the inflation dynamics in the data under parameterization that is consistent with the micro evidence.

I use the moment matching method to calibrate the parameters for cross-sectional wage distribution. The target moments are the share of non-zero wage-changes and their standard deviations in the U.S. data reported in the literature. I choose the model parameters to minimize the distance between the target moments and the corresponding model moments. Details are provided in Online Appendix D. The results are listed in Panel (B) of Table 2. The value for the fraction of workers without the DNWR constraint, α, implies that around 5 percent of workers can cut their nominal wages in each quarter. This number is broadly consistent with other empirical studies that use different data source. For example, Grigsby et al.

[23] They find that the median frequency of individual price changes excluding temporary sales is 11–13 percent per month. The value implies a slope of the NKPC of around 0.15. The corresponding parameter value is $\phi_p = 60.0$ in my model.

[24] E.g., Smets and Wouters (2007) and Del Negro et al. (2015).

Table 2: Calibration

Panel (A): Fixed parameters

Parameter	Symbol	Value	Target/Source
Average discount factor	β	0.995	Annual real interest rate = 2.0%
Relative risk aversion	σ	2.00	Externally fixed
Frisch labor supply elasticity	η^{-1}	4.00	King and Rebelo (1999)
Labor demand elasticity	θ_w	9.00	S.S. wage markup = 12.5%
Goods demand elasticity	θ_p	9.00	S.S. price markup = 12.5%
Price adjustment cost	ϕ_p	45.0	Slope of NKPC = 0.20
(Corr. Calvo parameter)	-	(0.64)	Nakamura and Steinsson (2008)
Responsiveness to inflation	δ_π	1.50	Fernández-Villaverde et al. (2015)
Responsiveness to output	δ_y	0.25	Same as above
Target inflation rate	Π^*	1.005	Annual inflation rate = 2.0%
Target output	Y^*	1.000	Externally fixed

Panel (B): Parameters for cross-sectional wage distribution

Parameter	Symbol	Value	Target/Source
Fraction of workers without the DNWR constraint	α	0.0503	Workers with wage changes=0.266; Barattieri et al. (2014)
S.D. of idiosyncratic labor disutility shock	σ_χ	0.1811	S.D. of wage changes=0.108; (annual) Fallick et al. (2016)

Notes: Barattieri et al. (2014) report the unconditional share of wage-changes for job-stayers and job-changers at a quarterly frequency. They identify the share to be between 0.211 and 0.266 depending on the assumptions used in their estimation. I use the most conservative value of 0.266. Fallick et al. (2016) report the standard deviation of wage changes, which is not available in Barattieri et al. (2014).

Panel (C): Parameters for aggregate exogenous processes

Parameter	Symbol	Value	Target/Source
AR(1) coefficient of discount factor	ρ_β	0.863	First-order autocorr. of output=0.85
S.D. of innovations to discount factor	σ_β	0.0057	S.D. of output=1.55%

Notes: Targets are the HP-filtered real GDP from 1955Q1 to 2007Q4. The end point of the sample is determined to exclude the ZLB periods.

(2018), who use administrative payroll data, find nominal wage cuts are quite rare especially for job stayers. In addition, because the model has positive trend inflation, workers can decrease their real wages up to inflation by freezing nominal wage changes. In what follows, I focus on the consequences of a discount factor shock β_t, while

Table 3: Untargeted moments

Panel (A): Cross-sectional moments

Description	Model	Data	Source
Fraction of wage cuts out of non-zero wage changes	0.120	0.123	Barattieri et al. (2014)
		0.22, 0.32	Author's calculation based on Elsby et al. (2016)
Mean wage changes (annual)	0.020	0.033	Fallick et al. (2016)
Median wage changes (annual)	0.0021	0.028	Fallick et al. (2016)

Notes: The data source and the sample period of each paper is as follows; Barattieri et al. (2014): SIPP, 1996-1999. Fallick et al. (2016): Employment Cost Index (ECI), 1982-2014. Elsby et al. (2016): Current Population Survey (CPS), 1980-2012. Elsby et al. (2016)'s result of 0.22 is for hourly paid workers, and they found 0.32 for non-hourly paid workers.

Panel (B): Time-series moments

	σ (%)		ρ		Sk	
	Model	Data	Model	Data	Model	Data
Y	(1.55)	1.55	(0.85)	0.85	–0.29	–0.49
H	1.55	1.82	0.85	0.90	–0.29	–0.28
π^p	0.60	0.58	0.90	0.86	1.10	1.25
π^w	0.66	0.74	0.82	0.40	1.41	0.23
W/P	0.47	0.84	0.95	0.78	2.08	0.53
i	1.27	0.82	0.86	0.95	0.73	1.17

Notes: The standard deviation σ, first-order autocorrelation ρ, and skewness Sk are reported. Targeted moments are shown in parentheses. In the data series, Y is the real GDP, H is the total hours in the non-farm business sector, π^p is the GDP implicit price deflator, π^w is the compensation per hour in the non-farm business sector, and i is the effective federal funds rate. Y and H are taken as log and detrended by the HP-filter. π^p, π^w, and i are the quarterly rate. The sample period is from 1955Q1 to 2007Q4. The end point of the sample is determined to exclude the ZLB periods.For computing the moments in the models, I simulate the economy for 51,000 periods and discard the initial 1,000 observations.

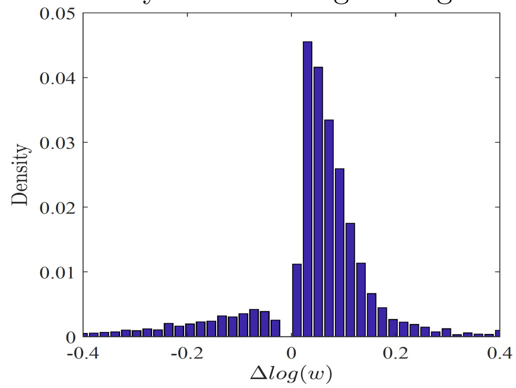
keeping technology Z_t constant.[25,26] In this regard, abundant evidence suggests that a demand-side shock, particularly a wedge in intertemporal substitution, is a key element that accounts for the Great Recession.[27] As Justiniano et al. (2011) point out, the time-varying discount factor is a parsimonious way to capture this shock. The AR(1) process is calibrated to match the persistence and variance of the real GDP in the post-war U.S. data, which are listed in Panel (C) of Table 2.

[25] Although the DSGE literature often models various shocks to account for the data, I focus on a single source of shock due to the computation burden of my heterogeneous agent model. A similar strategy has been used in the literature of heterogeneous agent models (e.g., Krusell and Smith (1998), Nakamura and Steinsson (2010)).

[26] I investigate the effects of a technology shock in Online Appendix E.

[27] Previous studies reach that conclusion by a reduced form regression analysis (Hall (2011)) or a structural model analysis (e.g., Christiano et al. (2014), Gust et al. (2017)).

Figure 2: Stationary non-zero wage-change distribution



Notes: The spike at zero wage-change is excluded.

To check the validity of the calibration, Table 3 reports various untargeted moments. Panel (A) displays cross-sectional moments. The model delivers much less wage cuts than increases and a lower median than mean, consistently with the data.[28,29] Figure 2 graphically shows the stationary distribution of non-zero wage-changes.[30] In Panel (B) of Table 3, the model matches the time-series moments in the data fairly well. In particular, it captures the lower variance of price inflation and wage growth than that of output as well as their positive skewness.[31]

5 Numerical Results

This section presents numerical results for the calibrated model with a particular focus on aggregate dynamics. I focus on the case without the ZLB to isolate the

[28]Elsby et al. (2016) find larger fraction of wage cuts, but the CPS data they use is subject to reporting errors. In contrast, Barattieri et al. (2014) estimate the fraction after correcting for measurement errors.

[29]Mean wage changes are slightly lower than the data because I do not consider trend technology growth, which is in turn because my model is inconsistent with real trend growth. However, if I take such growth into account, the model would identify a higher degree of DNWR to match zero-wage-changes in the data under higher wage growth.

[30]The definition of the stationary equilibrium is provided in Appendix B.

[31]The model generates considerably higher skewness of wage growth and real wages than that found in the data. However, I find that the skewness decreases once technology shocks are added to the model because they can make the DNWR constraint binding less often by affecting the marginal product of labor. Details are presented in Online Appendix E.

non-linear effects of DNWR from those of the ZLB.[32]

5.1 Non-linear dynamics

Figure 3 presents the generalized impulse responses (GIR) to a 2 S.D. discount factor shock. GIR are defined as the conditional expectations of each variable on the initial shock. Details are presented in Online Appendix C. A discount factor shock generates comovements of output and inflation. More importantly, responses of wage growth, marginal cost, and price inflation display substantial asymmetry to contractionary and expansionary shocks; they are much more sluggish downward than upward. Intuitively, the DNWR constraint binds for a larger share of workers upon a contractionary shock. This prevents real wages and therefore marginal cost from declining as long as nominal prices are also sticky. The dampened responses of marginal cost, in turn, lead to a small drop in inflation through the NKPC. On the other hand, the responses of output and hours worked are larger to a contractionary shock because the quantity of labor, instead of wages, should be adjusted in the labor market equilibrium.[33] Further discussion on the causes behind the non-linearity is provided in Section 5.3.

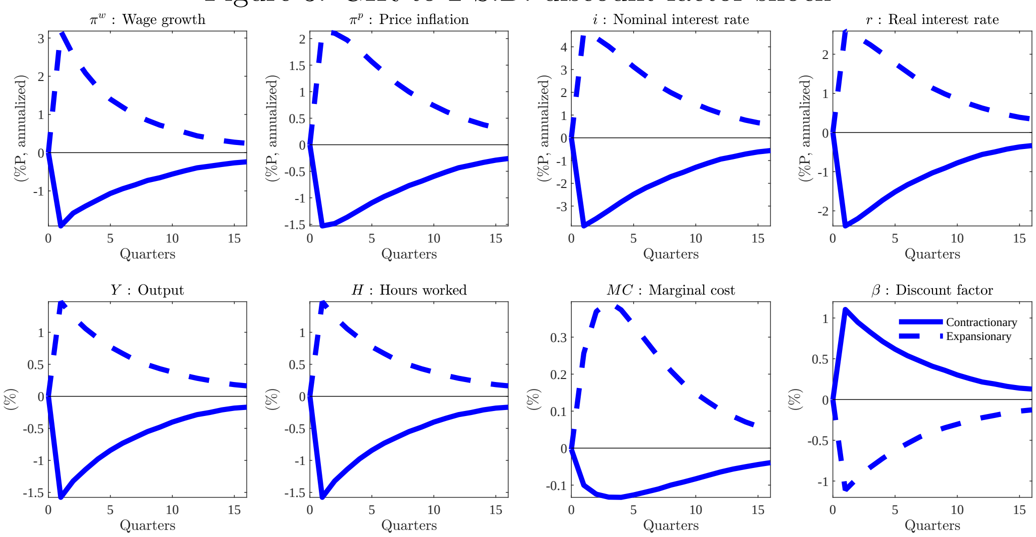
Table 4 reports the degree of asymmetry to different size of shocks. It measures the relative size of GIR upon a contractionary shock compared with those upon an expansionary one.[34] It can be seen that the asymmetry is increasing with the size of a shock. For example, the inflation responses to a contractionary shock are 15 percents smaller than those to an expansionary one when the magnitude is 1 S.D., whereas the gap increases to 40 percents when 3 S.D. shocks hit. These properties arise because a larger shocks changes the share of the workers whose DNWR constraint is binding more drastically. In other words, the effects of DNWR are particularly strong upon

[32] I investigate the role of the ZLB in Section 6.

[33] The degree of asymmetry of output responses is relatively smaller than that of inflation because the concavity of the utility function makes households resist a decline of consumption more strongly than they appreciate an increase, and this effect partly offsets the asymmetry arising from DNWR.

[34] Details are presented in Online Appendix E.

Figure 3: GIR to 2 S.D. discount factor shock



Notes: The horizontal-axes represent the time horizon after the initial shock. The vertical-axes are in the deviation from the stochastic mean.

Table 4: Degree of asymmetry to different size of shocks

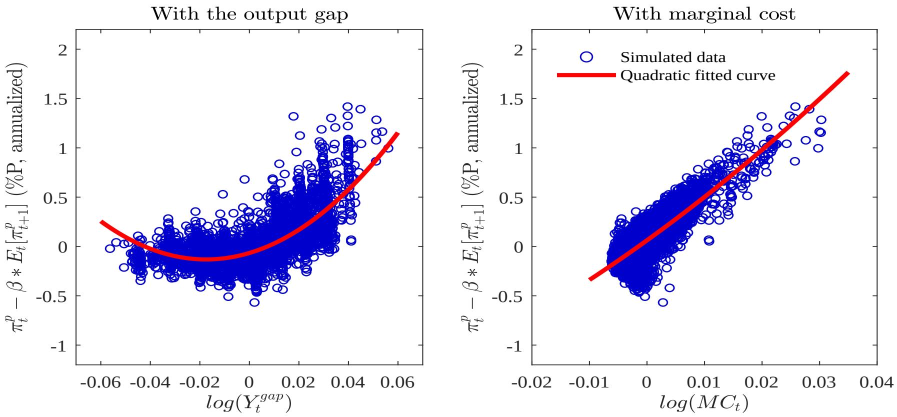
ϵ_1	π^w	π^p	i	r	Y	H	MC	β
1.0 S.D.	0.82	0.85	0.90	0.95	1.04	1.04	0.64	1.00
2.0 S.D.	0.65	0.70	0.81	0.90	1.08	1.08	0.34	1.00
3.0 S.D.	0.54	0.60	0.74	0.86	1.12	1.12	0.21	1.00
4.0 S.D.	0.47	0.54	0.69	0.82	1.15	1.15	0.14	1.00

Notes: The table reports the relative size of GIR upon a contractionary shock to that upon an expansionary one in the initial four-quarters. The formal definition is provided in Online Apendix E. Values smaller (larger) than 1 indicate that the GIR is smaller (larger) upon a contractionary shock.

a large and contractionary shock such as the Great Recession.

In Online Appendix F, I compare GIR in various models with different specifications of wage adjustments.[35] I verify that my model with DNWR generates a smaller drop of inflation on impact and more persistent responses upon a contractionary shock compared with other models.

Figure 4: Simulated Phillips curve



Notes: Each panel displays scatter plots of the simulated data along with the corresponding quadratic fitted curves. Because I discretize exogenous state variables with the relatively small number of grid points, I use interpolation between observations to generate a smooth transition path. Specifically, I simulate the model economy for 5,000 periods on the discretized state space and interpolated at the middle point of exogenous states by using spline interpolation. The total number of observations is 9,999 in each panel.

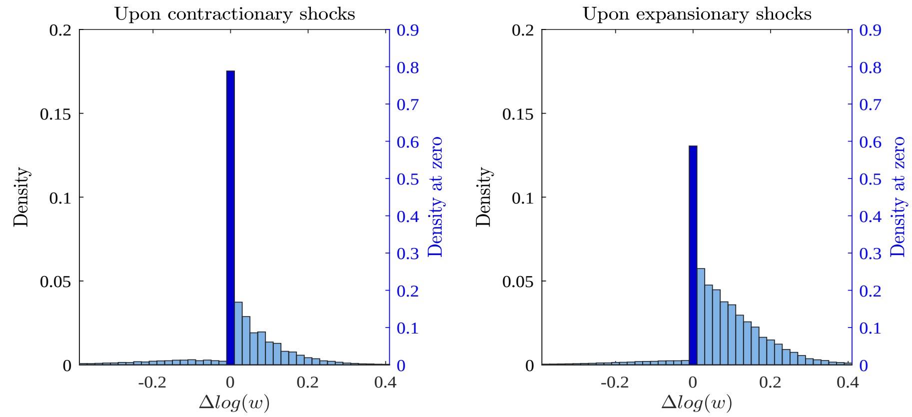
5.2 Simulated Phillips curve

Figure 4 plots two representations of the Phillips curve for the simulated model economy. The output gap representation of the NKPC, shown in the left panel, becomes flatter when the output gap takes negative values, and the quadratic fitted curve exhibits strong convexity. This occurs because the wage markup arising from the binding DNWR constraint shifts the NKPC upward in recessions, as demonstrated in (29). Because the effect is increasing with the size of shocks, the Phillips curve looks almost flat on its left end. Furthermore, it can be seen that inflation does not accelerate substantially even on its right end. This is because workers compress their wage increases due to precautionary motives for another downturn. In contrast, the marginal cost representation of the NKPC stays almost linear in the right panel. This reflects the fact that the firms' price-setting behavior given the level of marginal cost does not change over the business cycle in my model.[36]

[35] I analyze a flexible wage model, a quadratic wage adjustment cost model, and an asymmetric wage adjustment cost model.

[36] Precisely speaking, deviations can emerge from the time variations in the stochastic discount factor. However, they are quantitatively small because they appear only in the second and higher

Figure 5: Cross-sectional distribution upon aggregate shocks



Notes: The left panel computes cross-sectional density upon a 2 S.D. contractionary discount factor shock, and the right panel shows the case for an expansionary one. The spike at zero-wage-changes is shown in the right axis, whereas other histograms are in the left axis.

5.3 Discussion and relation to existing literature

Shifts of cross-sectional wage distribution. The key feature to understanding the non-linear dynamics of my model is the endogenous shifts of the cross-sectional distribution. Figure 5 shows the wage-change distribution in response to contractionary and expansionary discount factor shocks.[37] The size of the spike at zero-wage-changes indicates that a larger share of workers is constrained at DNWR to freeze their wages upon a contractionary shock. That leads to stronger downward sluggishness of the aggregate wage than upward.[38]

Another important feature is the role of idiosyncratic shocks. My calibration suggests that the standard deviation of idiosyncratic shocks is much larger than that of aggregate shocks. Consequently, a non-trivial fraction of workers experiences wage increases even after a contractionary aggregate shock. Since downward wage adjust-

orders.

[37]Daly and Hobijn (2014) show their simulated wage distribution in the stationary equilibrium and along a deterministic path of exogenous process. On the other hand, I solve the model in a dynamic setting. Therefore, agents internalize future aggregate uncertainty, which leads to precautionary motives to compress wage hikes.

[38]This observation is consistent with the micro evidence of Daly and Hobijn (2015) and Fallick et al. (2016), who document that the fraction of workers with zero wage changes substantially increased after the Great Recession.

ment is truncated at zero as long as workers are subject to the DNWR constraint, the average wage change becomes higher than in the case without idiosyncratic shocks. This further impedes adjustment of the aggregate wage upon a contractionary shock.

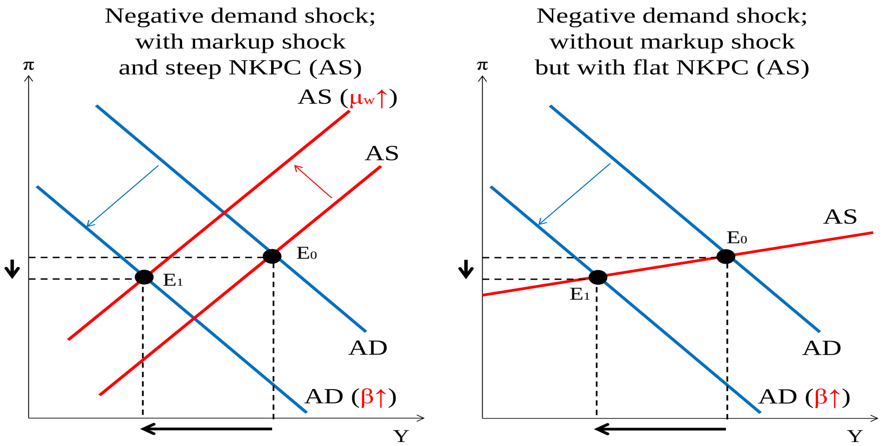
Time-varying wage markup. The wage markup in my model is closely related to but distinct from that in the model with a Calvo (1983)-type staggered wage contract, which is widely used in the literature (e.g., Erceg et al. (2000), Christiano et al. (2005)). First of all, fluctuations in wage markup in my model are asymmetric during booms and recessions. Intuitively, a contractionary shock generates a negative income effect to reduce the MRS. However, because the DNWR constraint prevents nominal wages from declining, the wage markup should increase so that the labor supply curve meets labor demand. In contrast, the wage markup does not respond to an expansionary shock unless the DNWR constraint binds.

Moreover, the fluctuations depend on the size of the exogenous shock. In this regard, it is important to note that an exogenous shock has two effects on the wage markup. First, an exogenous shock affects the individual wage markups for workers whose DNWR constraint is already binding; this can be considered a direct effect. In addition, an exogenous shock changes the portions of workers with and without the binding constraint by changing their desired wages. Consequently, the total effect is increasing with the size of shocks.[39]

Markup shock to the NKPC. King and Watson (2012) point out that a standard medium-scale DSGE model requires sizable and frequent exogenous markup shocks to the NKPC in order to account for actual inflation. In line with their finding, Del Negro et al. (2015) argue that if one adopts a relatively steep NKPC, a large positive markup shock should be accompanied by a negative demand shock to solve

[39] In contrast, the staggered contract of Calvo (1983), in which a constant fraction of workers face the constraint in each period, lacks the second effect. Therefore, the model does not deliver significant non-linearity.

Figure 6: Demand and markup shocks with different slope of the NKPC



Notes: This figure corresponds to Figure 5 of Del Negro et al. (2015).

the missing deflation puzzle. Instead, they propose that a sufficiently flat Phillips curve, as in the right panel of Figure 6, can resolve the puzzle. To this end, my model generates an endogenous rise in the wage markup upon a contractionary demand shock. The wage markup shifts the NKPC (aggregate supply curve) upward, which causes the decline of inflation to be moderate despite a large shift in the aggregate demand curve as shown in the left panel of Figure 6.

Stable inflation expectations. Several studies emphasize, to address the missing deflation puzzle, the fact that the inflation expectations were stable during and after the Great Recession. Some of them attribute it to the departure from a full information and rational expectation (FIRE) model (Coibion and Gorodnichenko (2015)) or discrete regime changes of the economy (Bianchi and Melosi (2017)). On the other hand, I argue that the stable inflation expectations are consistent with my model although I stick to a FIRE model without regime switching. To see this point, iterating the linearized version of the NKPC forward yields

$$\mathbb{E}_t[\pi_{t+1}] = \kappa \mathbb{E}_t \left[\sum_{s=0}^{\infty} D_{t+1,t+s+1} \widehat{MC}_{t+s+1} \right]. \tag{30}$$

Equation (30) indicates that the inflation expectations $\mathbb{E}_t[\pi_{t+1}]$ are the infinite sum of the discounted values of future marginal costs. Therefore, the model can address the stable inflation expectations as long as the future marginal costs are sufficiently stabilized. In Section 6, I show that the model indeed predicts a moderate decline of the inflation expectations, which is broadly consistent with the survey based inflation expectations in the data after the Great Recession.

Choice of computation method. Whereas I use a modified version of the Krusell and Smith (1998) algorithm to solve my model, recent studies propose various new computation methods to deal with a heterogeneous agent model that typically gain computation efficiency. For instance, Reiter (2009) approximates cross-sectional distribution with finite dimensional histograms, whereas Winberry (2016) parameterizes the distribution by using a family of polynomial functions. Moreover, Ahn et al. (2017) and Kaplan et al. (2018) build a continuous time model where the evolution of the distribution is formulated in the Kolmogorov forward equation and its boundary conditions. The key innovation of these studies to achieve efficient computation is to apply the first order approximation around the stationary distribution in terms of aggregate dynamics. However, as is discussed in Ahn et al. (2017), this class of solution methods cannot capture the sign- and size-dependence of aggregate shocks because the aggregate dynamics are approximated in the first order. In contrast, I find that such non-linear dynamics are crucial to accounting for the inflation puzzles in my model. In addition, the presence of the ZLB is another reason to resort to a global solution method because the first order approximation cannot be applied due to the kink in a monetary policy rule.

6 Modeling the Great Recession

This section investigates the model's validity to account for the inflation dynamics during and after the Great Recession.

6.1 ZLB

A number of studies have pointed out that the ZLB is one of crucial elements to understand the Great Recession (e.g., Aruoba et al. (2018), Basu and Bundick (2017)). Therefore, in the following analysis, I extend my model to introduce the ZLB into the Taylor (1993) rule[40]:

$$R_t^d = R^* \left(\frac{\Pi_t^p}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{31}$$

$$R_t = max\{\ R_t^d\ ,\ 1\ \}. \tag{32}$$

In Online Appendix E, I present various properties of the ZLB including time-series moments and GIR. I also investigate the effects of forward guidance by modeling the history-dependent rule of Reifschneider and Williams (2000).

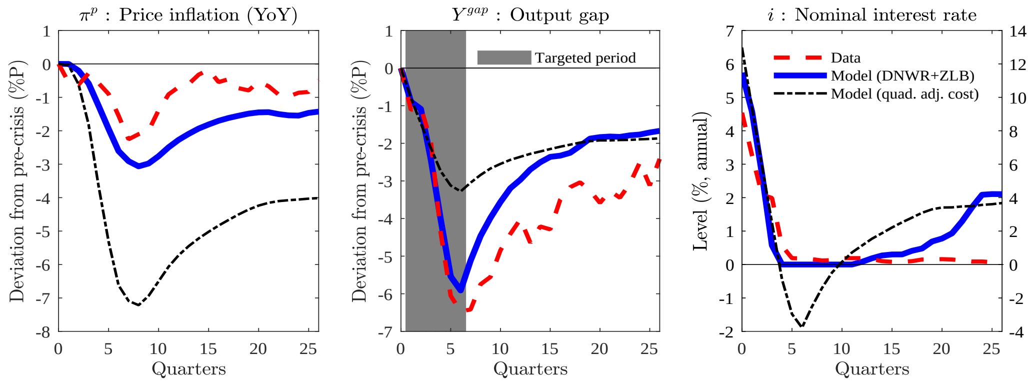
6.2 Missing deflation

Figure 7 and Figure 8 display the model's median responses in severe recession episodes. I define a severe recession as the top 0.5 percent (once in 50 years) episodes in which the output gap fluctuations are closest to those in the data for the Great Recession (2008Q1–2009Q2) in the simulated economy. For comparison purposes, the figure also depicts the responses of a quadratic wage adjustment cost model with a representative agent, which is often used in the DSGE literature.[41]

[40] In this regard, a number of studies argue that the ZLB is an essential element to understand the Great Recession (e.g., Christiano et al. (2015), Basu and Bundick (2017), Gust et al. (2017)).

[41] Details are described in Online Appendix F.

Figure 7: Severe recession episodes (1)



Notes: Red dashed lines show the data, whereas blue real lines are the benchmark model with DNWR and the ZLB. Black dash-dotted lines are a quadratic wage adjustment cost model without the ZLB, whose details are described in Online Appendix F. Each model series shows the median responses of the severe recession episodes. I define a severe recession as the top 0.5 percent (once in 50 years) episodes in which the output gap fluctuations are closest to those in the data for the Great Recession (2008Q1–2009Q2) in the simulated economy. To remove jolting movements due to the discretized state space, I smooth the series by taking the three-quarter moving average after the targeted periods. The nominal interest rates of the quadratic wage adjustment cost model are shown in the right axis. For the data series, the inflation rate π^p is the year-on-year growth rate of the GDP implicit price deflator, the output gap Y^{gap} is the one estimated by the Congressional Budget Office, and the nominal interest rate $i$is the effective federal funds rate. Each data series except for the nominal interest rate is in the deviation from the business cycle peak before the Great Recession, which is defined by the NBER (2007Q4).

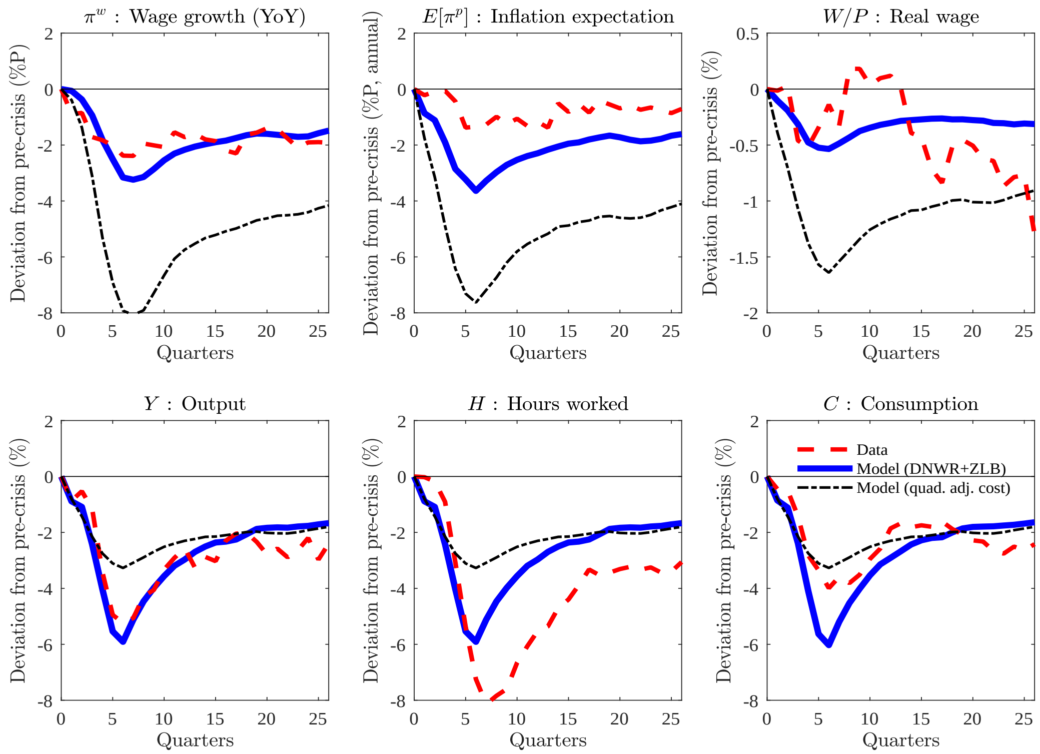
In Figure 7, the severe recession episodes lead to a 3.1 percentage points decline in year-on-year inflation rate despite a sharp drop in the output gap in the benchmark model with DNWR and the ZLB.[42] As is investigated in Section 5, the DNWR constraint binds for a substantial share of workers upon a large contractionary shock, which leads to little responses of marginal cost and therefore inflation even though the output gap declines sharply. Moreover, the lack of offsetting monetary policy responses at the ZLB further enlarge the non-linearity.[43] This quantitative result is consistent with the data, in which the inflation rate in the GDP deflator declined by 2.3 percentage points around the Great Recession.[44] The model also replicates

[42]The decline of the CBO-estimated output gap during the Great Recession roughly corresponds to 3 standard deviations of the prior series.

[43]Comparison of the models with and without the ZLB is presented in Online Appendix E, whereas the relation to the existing literature is discussed in Online Appendix G.

[44]The peak before the Great Recession is 2.5 percent as of 2007Q4, whereas the bottom is 0.2 percent as of 2009Q3.

Figure 8: Severe recession episodes (2)



Notes: The definition of each series of model follows that of Figure 7. For the data series, the inflation expectation $E[\pi^p]$ is the median forecast for one quarter ahead GDP deflator in the SPF, while consumption C is the HP filtered real personal consumption expenditures. The definition of the other series follows that of Figure 3. Each series is in the deviation from the business cycle peak before the Great Recession defined by the NBER (2007Q4).

the sluggish recovery of inflation after the initial drop in the data, which is further investigated in the next subsection.

In contrast, the quadratic wage adjustment cost model predicts a much larger decline of inflation (-7.2 percentage points), even though the model fails to match the sharp drop of the output gap in the data.[45] It should be noted that the wage adjustment cost is calibrated to match the frequency of wage changes in data, which is also used as a calibration target of the benchmark model. In other words, the distinct outcomes of two models comes not from calibration strategy but from model specifications, such as asymmetry of wage adjustment and heterogeneity.

[45]In Online Appendix F, I show that once wage rigidity is omitted at all the model generates much larger deflation.

The benchmark model generates approximately 2 years of ZLB episodes, followed by subsequent low interest rate periods. Although still shorter than in the data, my model addresses longer spells of the ZLB episodes compared with previous studies.[46] To interpret this result, the interaction between DNWR and nominal price rigidity generates substantial persistence of inflation, and this results in sluggish movements of the desired interest rate implied by the Taylor rule.[47]

In Figure 8, the benchmark model matches the sluggish declines of wage growth and real wages in the data, consistent with the implications of DNWR. In addition, the model is broadly consistent with stable inflation expectations found in the data because the inflation expectations reflect the future marginal costs under the NKPC.[48,49]

Essentially, my model does not have a number of features that previous studies argue are important to account for the inflation puzzles.[50] Nevertheless, the model suggest that DNWR and its interaction with the ZLB can replicate the key features of inflation dynamics observed in the data.

6.3 Excessive disinflation

This subsection investigates my model's implications for the excessive disinflation after the Great Recession. Figure 9 shows GIR to a 1 S.D. expansionary shock arising from different initial states and the divergent responses across these initial states. In particular, when starting from a 3 S.D. recession state, which corresponds

[46] See Fernández-Villaverde et al. (2015) for this issue.

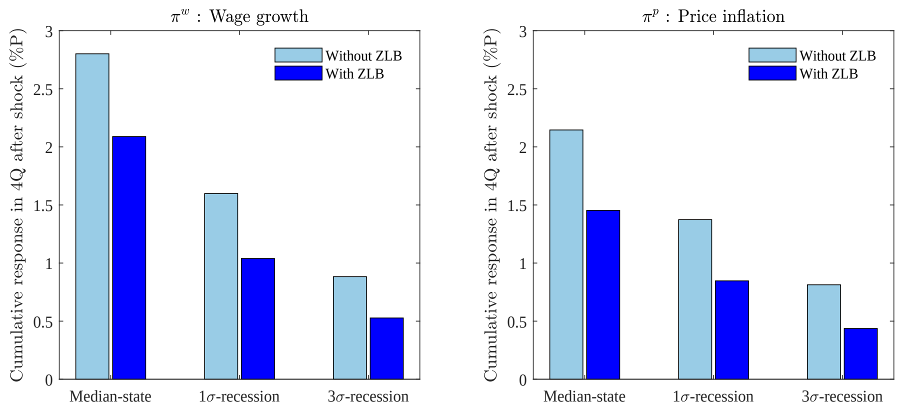
[47] Another potential explanation to address long spells of the ZLB episodes is forward guidance. I study this point by modeling the history-dependent rule in Online Appendix E.

[48] The discrepancy between the model and the data can reflect deviations from the rational expectations. More discussion is provided in Online Appendix G.

[49] The model is also consistent with sharp drops of real quantities. Though it cannot perfectly match the relatively larger drop of hours worked and the smaller decline of consumption in the data, they are presumably because I abstract away capital investment, which is not the main focus of this paper.

[50] E.g., high degree of nominal price rigidity, exogenous shocks to the inflation expectations, financial frictions, etc.

Figure 9: GIR to 1 S.D. expansionary shock from different initial states



Notes: Each bar shows the cumulative responses in four-quarters after a 1 S.D. expansionary discount factor shock from different initial states. The magnitude is normalized by the responses of output. To draw each initial state, I simulate the model economy in each state for six-quarters, which correspond to the length of the NBER recession around 2008 (2008Q1–2009Q2).

to a severe recession similar to the Great Recession, the responses of wage growth and price inflation are approximately four times smaller than those from the median state. This is due to the state-dependent nature of DNWR. During a severe recession, most workers' desired wages fall short of their actual ones due to the binding DNWR constraint. In a recovery phase, even when their desired wages start to rise as the economy improves, workers do not see raises in actual wages as long as the constraint remains binding. In other words, their wage increases are postponed until their desired wages exceed the actual ones. The delayed responses of wages lead to a slow recovery of inflation through the formation of marginal cost.

Interestingly, the recovery of wage growth and price inflation becomes slower in the presence of the ZLB. This is the case even if the ZLB does not bind. I find that this is attributed to the forward-looking channel of DNWR. Workers internalize the fact that recessions are more severe in the presence of the ZLB. Consequently, they have stronger motives, compared to the case without the ZLB, to compress their wage increases in a recovery phase in preparation for future downturns.[51]

[51] I verify that inflation becomes less skewed in the presence of the ZLB. I also find that the variance of inflation becomes slightly smaller even though that of output is substantially expanded. The details are presented in Online Appendix E.

7 Conclusion

In this paper, I introduce DNWR for heterogeneous workers into an otherwise standard New Keynesian DSGE model. DNWR accounts for the flattening of the observed Phillips curve relationship between inflation and the output gap during recessions. The endogenous evolution of the cross-sectional wage distribution generates non-linear dynamics such as the sign-, size-, and state-dependence of aggregate shocks. Moreover, the interaction between DNWR and the ZLB reinforces these non-linearities. The calibrated model successfully matches the key moments of the inflation dynamics during and after the Great Recession, which are often referred to as the missing deflation and excessive disinflation puzzles.

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For Online Publication:
Appendix for "Downward Nominal Wage Rigidity and Inflation Dynamics during and after the Great Recession"

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This Appendix is structured as follows:

- Appendix A provides details of empirical evidence on the New Keynesian Phillips Curve (NKPC). It contains construction of marginal cost measures and robustness check for the baseline estimation of the NKPC. It also assesses empirical relationship between the output gap and marginal cost.
- Appendix B provides supplemental details of the model.
- Appendix C provides details of computation methods. It explains the modified Krusell-Smith algorithm and other elements for the equilibrium computation, as well as construction of the generalized impulse responses (GIR).
- Appendix D provides details of calibration procedure.
- Appendix E provides additional numerical results. It studies role of the zero lower bound (ZLB) and technology shocks, as well as details of the GIR analysis.
- Appendix F provides model comparison. It investigates a flexible wage model, a quadratic wage adjustment cost model, and an asymmetric wage and price adjustment cost model.
- Appendix G discusses miscellaneous topics regarding my model's connections to the existing literature.

A Empirical Evidence

A.1 Construction of marginal cost

Firm's cost minimization problem. Since it is infeasible to directly observe firms' marginal cost, I follow the insights of Hall (1986) to resort to firms' optimizing behavior to estimate marginal cost. I consider a cost minimization problem:

$$\min_{\{I_t^j\}_{j=1}^J} : \sum_{j=1}^{J} P_t^j I_t^j \tag{A.1}$$

$$s.t. \quad Y_t = F(I_t^1, ..., I_t^J, Z_t^1, ..., Z_t^J) \tag{A.2}$$

where Y_t denotes output, and I_t^j is factor input j with P_t^j being the corresponding factor price. Production technology is given by F with Z_t^j being factor j augmenting technology. Firms are assumed to be price taker in factor markets. The first order condition (FOC) for the problem takes the following form:

$$P_t^j = \lambda_t \frac{\partial F_t}{\partial I_t^j} \quad for\ j = 1, ...J \tag{A.3}$$

where λ_t denotes the Lagrangian multiplier that represents the nominal marginal cost of producing one unit of output. The FOC can be rearranged to:

$$MC_t \equiv \frac{\lambda_t}{P_t} = \underbrace{\left(\frac{\partial log(F_t)}{\partial log(I_t^j)}\right)^{-1}}_{(\theta_t^j)^{-1}} \underbrace{\frac{P_t^j I_t^j}{P_t Y_t}}_{s_t^j}. \tag{A.4}$$

θ_t^j and s_t^j are the output elasticity with respect to and the expenditure share of input j, respectively. Equation (A.4) allows one to construct the real marginal cost MC_t with s_t^j and θ_t^j.

In my baseline estimation, I use the labor share of income for the non-farm business sector to represent the expenditure share s_t^J. To estimate the elasticity θ_t^J, I impose assumptions on the functional form of the production technology F. Following Basu (1996), Gagnon and Khan (2005), and Nekarda and Ramey (2013), I consider labor and capital inputs with labor augmenting technology and assume the Cobb-Douglas production function with overhead labor (CDOH) adjustment for a baseline case. Alternative specifications of the Cobb-Douglas production function (CD) and a production function with constant elasticity of substitution (CES) are also investigated.

Specification of production function. For a baseline case, I assume the Cobb-Douglas production function with overhead labor (CDOH):

$$F(H_t, K_t, Z_t) = \{Z_t(H_t - \bar{H})\}^{\alpha} K_t^{1-\alpha} \tag{A.5}$$

where Y_t denotes output, Z_t labor augmenting technology, H_t labor inputs, K_t capital inputs. $\bar{H}$ is the overhead component of labor inputs that is not directly linked to value added production. The FOC for labor inputs implies

$$\theta_t^H \equiv \frac{\partial log(F_t)}{\partial log(H_t)} = \alpha \frac{H_t}{H_t - \bar{H}} \tag{A.6}$$

Using (A.4), the FOC is rearranged to the specification of marginal cost:

$$MC_t^{CDOH} = \frac{1}{\alpha}\left(1 - \frac{\bar{H}}{H_t}\right) s_t^H \tag{A.7}$$

with $s_t^H = \frac{W_t H_t}{P_t Y_t}$ being the labor share. The specification leads to

$$\widehat{MC}_t^{CDOH} = \frac{\bar{H}/H^{ss}}{1 - \bar{H}/H^{ss}} \hat{H}_t + \hat{s}_t^H \tag{A.8}$$

where $\hat{x}$ denotes the log-deviation from the steady-state. For parameterization, I borrow the estimated value of Basu (1996) to calibrate $\bar{H}/H^{ss} = 0.288$. The value is broadly in line with other estimates in the literature such as 0.20 of Ramey (1991) and 0.14 of Bartelsman et al. (2013). Bartelsman et al. (2013) point out as a reference that in the U.S. manufacturing industries non-production workers compose of roughly 30 percent of total employment and managers of 10 percent.

For robustness check, I consider alternative specifications: the Cobb-Douglas production function (CD) and a production function with constant elasticity of substitution (CES). The Cobb-Douglas production function is given by

$$F(H_t, K_t, Z_t) = (Z_t H_t)^{\alpha} K_t^{1-\alpha}. \tag{A.9}$$

The FOC for labor inputs formulates marginal cost to be proportional to the labor share:

$$MC_t^{CD} = \frac{1}{\alpha} s_t^H \tag{A.10}$$

and

$$\widehat{MC}_t^{CD} = \hat{s}_t^H. \tag{A.11}$$

On the other hand, under a CES production function,

$$F(H_t, K_t, Z_t) = \left\{ \alpha \left(Z_t H_t \right)^{\frac{\nu-1}{\nu}} + (1-\alpha) K_t^{\frac{\nu-1}{\nu}} \right\}^{\frac{\nu}{\nu-1}} \tag{A.12}$$

marginal cost is given by

$$MC_t^{CES} = \frac{1}{\alpha} \left(\frac{Y_t}{Z_t H_t} \right)^{\frac{\nu-1}{\nu}} s_t^H \tag{A.13}$$

and

$$\widehat{MC}_t^{CES} = \frac{\nu-1}{\nu} (\hat{Y}_t - \hat{Z}_t - \hat{H}_t) + \hat{s}_t^H \tag{A.14}$$

where ν represents the elasticity of substitution between labor and capital inputs. I follow Gali et al. (2007) to calibrate $\nu = 0.5$. For the series of Z_t, I use the utilization-adjusted quarterly-TFP for the U.S. business sector constructed based on Fernald (2014).

Detrending. An important issue when using the series of the labor share in the U.S. data is detrending, because the data displays a low frequent downward trend. Though there is substantial debate regarding the causes behind the trend, various studies attribute it to structural changes of the economy such as offshoring of manufacturing industries and declining relative price of investment goods due to advances in information technology, while others emphasize mismeasurement of the data (e.g., Elsby et al. (2013), Karabarbounis and Neiman (2013)). Since the main focus of this paper is on business cycle fluctuations that are related to inflation dynamics, I use a filtering method to extract cyclical components of the labor share. Similar methods are employed by Mavroeidis et al. (2014) when they estimate the NKPC. Specifically, each series of marginal cost is detrended by the Hamilton filter. Hamilton (2017) argues that the Hamilton filter has desirable time-series properties compared to the HP-filter, which is widely used in business cycle analysis. In particular, the one-sided method of the Hamilton filter potentially addresses the so called end-of-sample problem of the HP-filter.

A.2 Robustness checks for estimation of NKPC

This subsection assesses robustness for the baseline estimation of the NKPC in the main article. Specifically, I investigate alternative measures for the output gap and marginal cost in aggregate data, marginal cost in industry level data, the purely forward-looking NKPC, and the rational expectation assumption.

A.2.1 Alternative measures for marginal cost

I estimate the baseline specification of the NKPC (A.15) with alternative measures of marginal cost:

$$\begin{aligned}\pi_t = {} & \beta_0 \pi_t^e + \gamma_0 \pi_{t-1} + \kappa_0 x_t \\ & + \beta_1 postGR_t \pi_t^e + \gamma_1 postGR_t \pi_{t-1} + \kappa_1 postGR_t x_t + e_t. \end{aligned} \tag{A.15}$$

More precisely, I use the marginal cost based on the Cobb-Douglas production function (CD) and a production function with constant elasticity of substitution (CES).

Table A.1 reports the results. The estimation results display quite similar patterns to the baseline specification of the Cobb-Douglas production function with overhead labor (CDOH). The coefficients of marginal cost remain stable after the Great Recession, and the interaction terms with the post-Great Recession dummy are not statistically significant.

Table A.1: OLS estimation with alternative measures for marginal cost

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Marginal cost (CD)			Marginal cost (CES)		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	0.446***	0.485***	0.466***	0.419***	0.469***	0.444***
	(0.079)	(0.074)	(0.078)	(0.080)	(0.074)	(0.079)
π_{t-1}	0.548***	0.509***	0.528***	0.574***	0.525***	0.548***
	(0.080)	(0.075)	(0.079)	(0.082)	(0.075)	(0.081)
x_t	0.205**	0.151**	0.170**	0.168***	0.128***	0.136***
	(0.082)	(0.067)	(0.076)	(0.056)	(0.045)	(0.052)
$postGR_t \times \pi_t^e$			0.289			0.435*
			(0.219)			(0.224)
$postGR_t \times \pi_{t-1}$			–0.312			–0.406*
			(0.234)			(0.233)
$postGR_t \times x_t$			–0.0891			0.00287
			(0.154)			(0.098)
Adjusted R^2	0.952	0.946	0.946	0.953	0.947	0.947
N of obs.	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. $postGR_t$ is a dummy variable that takes 1 from 2008Q1 to 2012Q4. Sample period is from 1968Q4 to 2007Q4 for *Before GR* and from 1968Q4 to 2016Q4 for *Full sample*, respectively. The starting period corresponds to the period when the SPF became available.

A.2.2 Alternative measures for output gap

I next explore alternative measures for the output gap. Following the literature, I use the detrended output[1], and the employment rate, which is defined as the ratio of employment out of the population older than 16 years. Notice that the employment rate potentially addresses issues regarding the changes in the labor force participation after the Great Recession.

Table A.2 reports the regression results. In line with my baseline case, the interaction terms of the output gap measures with the post-Great Recession dummy variable are significantly negative, though the evidence is a little weaker than the baseline case. The results indicate that the flattening of the output gap representation of the NKPC also holds with alternative measures.

[1] I construct the detrended output by removing a quadratic trend from the log output. Though many of previous studies only take a linear trend into account (e.g., Gali and Gertler (1999), Adam and Padula (2011)), I find the quadratic term is highly significant especially in the sample that covers after the Great Recession.

Table A.2: OLS estimation with alternative measures for output gap

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Detrended output			Employment rate		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	0.555***	0.581***	0.578***	0.462***	0.495***	0.479***
	(0.085)	(0.078)	(0.083)	(0.080)	(0.075)	(0.079)
π_{t-1}	0.447***	0.423***	0.426***	0.532***	0.498***	0.513***
	(0.084)	(0.077)	(0.082)	(0.080)	(0.075)	(0.079)
x_t	0.0948***	0.0870***	0.0931***	0.317**	0.233*	0.309**
	(0.021)	(0.019)	(0.020)	(0.138)	(0.119)	(0.138)
$postGR_t \times \pi_t^e$			0.134			0.218
			(0.210)			(0.215)
$postGR_t \times \pi_{t-1}$			–0.187			-0.256
			(0.230)			(0.221)
$postGR_t \times x_t$			–0.106*			–0.419*
			(0.056)			(0.223)
Adjusted R^2	0.954	0.949	0.948	0.951	0.945	0.945
N of obs.	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. The detrended output is constructed by removing a quadratic trend from the log output, whereas the employment rate is detrended by the HP filter. $postGR_t$ is a dummy variable that takes 1 from 2008Q1 to 2012Q4. Sample period is from 1968Q4 to 2007Q4 for $Before\ GR$ and from 1968Q4 to 2016Q4 for *Full sample*, respectively. The starting period corresponds to the period when the SPF became available.

A.2.3 Purely forward looking NKPC

I estimate the purely forward looking NKPC (A.16) to assess the robustness in terms of the specification of the NKPC:

$$\pi_t = \beta\pi_t^e + \kappa x_t + \beta_1 postGR_t\pi_t^e + \kappa_1 postGR_t x_t + e_t. \tag{A.16}$$

As in the baseline case, I use the unemployment gap and marginal cost based on the Cobb-Douglas production function with overhead labor as measures of the output gap and marginal cost, respectively.

Table A.3 reports the estimation results. Similar to the hybrid NKPC, the coefficient of the marginal cost is stable after the Great Recession, and the interaction term of the marginal cost and the dummy variable for the post-Great Recession period is insignificant.

Table A.3: OLS estimation of the purely forward looking NKPC

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Unemployment gap			Marginal cost (CDOH)		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	1.048***	1.029***	1.046***	0.998***	0.996***	0.996***
	(0.023)	(0.024)	(0.023)	(0.024)	(0.023)	(0.023)
x_t	0.458***	0.229**	0.427***	0.157*	0.128**	0.121
	(0.063)	(0.052)	(0.060)	(0.082)	(0.066)	(0.078)
$postGR_t \times \pi_t^e$			-0.126			–0.00331
			(0.137)			(0.096)
$postGR_t \times x_t$			–0.445***			0.0358
			(0.099)			(0.126)
Adjusted R^2	0.947	0.936	0.941	0.934	0.930	0.929
N of obs.	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. The sign of the coefficient of the unemployment gap is flipped for comparison purposes. $postGR_t$ is a dummy variable that takes 1 from 2008Q1 to 2012Q4. Sample period is from 1968Q4 to 2007Q4 for *Before GR* and from 1968Q4 to 2016Q4 for *Full sample*, respectively. The starting period corresponds to the period when the SPF became available.

A.2.4 Rational expectation assumption

I investigate the rational expectation for seeing robustness in terms of expectation formations. Following Gali and Gertler (1999), the inflation expectations can be replaced with the realized inflation and the rational expectation error:

$$\mathbb{E}_t[\pi_{t+1}] = \pi_{t+1} + \tilde{e}_{t+1} \tag{A.17}$$

where $\tilde{e}_{t+1}$ is the rational expectation error that satisfies $\mathbb{E}_t[\tilde{e}_{t+1}] = 0$. The NKPC is rearranged to:

$$\pi_t = \beta\pi_{t+1} + \gamma\pi_{t-1} + \kappa x_t + e_{t+1} \tag{A.18}$$

with $e_{t+1} \equiv \beta\tilde{e}_{t+1}$. Notice that the OLS estimator of (A.18) is biased because e_{t+1} might be correlated with π_{t+1}. Therefore, I use instrument variables for π_{t+1} to derive the GMM estimator. It should be noted that any variables at and before period t are valid instruments, because the rational expectation error e_{t+1} is orthogonal to any variable in the information set at period t. I follow Gali and Gertler (1999) to use the lagged variables that are potentially correlated with π_{t+1} as instruments.

Table A.4 reports the estimation results. The coefficient of the unemployment gap is insignificant in column (1) and (2), and weakly significant with a negative sign to in column (3). Although the result is inconsistent with the theory of the NKPC, it is in line with the findings of previous empirical studies. For example, Gali and Gertler (1999) and Sbordone (2002) obtain insignificant estimates for the coefficient of the output gap under the rational expectation assumption. More recently, Adam and Padula (2011) find that the coefficient of the output gap is significant only when they use a survey based expectation measure instead of the rational expectation assumption. On the other hand, I confirm the baseline result regarding the marginal cost representation of the NKPC. In other words, the coefficient of marginal cost is significantly positive over time and does not show a significant decline after the Great Recession.

Table A.4: GMM estimation under rational expectation assumption

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Unemployment gap			Marginal cost (CDOH)		
	Before GR	Full sample		Before GR	Full sample	
π_{t+1}	0.730***	0.707***	0.713***	0.706***	0.698***	0.702***
	(0.051)	(0.039)	(0.058)	(0.048)	(0.050)	(0.054)
π_{t-1}	0.260***	0.281***	0.274***	0.281***	0.290***	0.284***
	(0.051)	(0.039)	(0.058)	(0.049)	(0.050)	(0.054)
x_t	-0.0273	-0.0166	-0.0352*	0.129***	0.0817***	0.111***
	(0.021)	(0.011)	(0.021)	(0.039)	(0.030)	(0.040)
$postGR_t \times \pi_{t+1}$			0.174			–0.00192
			(0.285)			(0.165)
$postGR_t \times \pi_{t-1}$			0.182			0.0270
			(0.154)			(0.163)
$postGR_t \times x_t$			0.199**			–0.206
			(0.088)			(0.153)
N of obs.	208	244	244	208	244	244

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The two step GMM with an HAC weight matrix is employed. Instruments are the first to forth lagged GDP implicit price deflator, marginal cost, labor share, output gap, wage growth rate, commodity price inflation, short-long term interest rate spread, post-Great Recession dummy. HAC corrected standard errors are reported in parentheses. $postGR_t$ is a dummy variable that takes 1 from 2008Q1 to 2012Q4. Sample period is from 1955Q1 to 2007Q4 for *Before GR* and from 1955Q1 to 2016Q4 for *Full sample*, respectively.

A.2.5 Marginal cost in industry level data

One concern regarding the estimation of the NKPC using aggregate series is that the estimated marginal cost might include considerable measurement errors. In this subsection, therefore, I investigate the intermediate share in industry level data as an alternative measure of marginal cost. To this end, it is notable that the firm's cost minimization condition should hold for any factor inputs. Moreover, various studies suggest that intermediate inputs are promising in many dimensions (e.g., Basu (1995), Nekarda and Ramey (2013), Bils et al. (2018)). For example, adjustment costs for intermediates are considered to be low relative to those for capital or labor. The assumption of no overhead component seems more defensible for intermediates. In addition, using industry level data removes composition bias among industries.

I use the KLEMS 2017 dataset to construct the intermediate share. The KLEMS 2017 dataset is annual from 1947 to 2014, covering 65 industries. I focus on 60 industries in the non-farm business sector, including 18 manufacturing and 42 non-manufacturing.

Since a measure of inflation expectations is not available for each industry, I rely on the rational expectation assumption to estimate the industry level NKPC:

$$\pi_{i,t} = \alpha_i + \beta\pi_{i,t+1} + \gamma\pi_{i,t-1} + \kappa x_{i,t} + e_{i,t+1} \tag{A.19}$$

$$where\ \mathbb{E}_t[\pi_{i,t+1}] = \pi_{i,t+1} + \tilde{e}_{i,t+1}. \tag{A.20}$$

I define $e_{i,t+1} = \beta\tilde{e}_{i,t+1}$ with $\tilde{e}_{i,t+1}$ being the rational expectation error. α_i is an unobserved industry fixed effect. For variable $x_{i,t}$, I use the intermediate share as a measure of marginal cost, and the detrended output as the output gap. In the estimation, I employ the two step GMM model procedure to correct industry fixed effects, which is called the Arellano-Bond estimator. Since I take the first difference of (A.19) to remove industry fixed effects, valid instruments for moment conditions are one-period more lagged than those in the standard GMM estimator. More discussion is provided by Arellano and Bond (1991).

Table A.5 presents the estimation results. The coefficient of the detrended output is insignificant in each specification, which is in line with our GMM estimation with aggregate data. On the other hand, the coefficient of the intermediate share is significantly positive and the interaction term is not significant in each case. The purely forward looking NKPC yields similar results to the hybrid NKPC. These results confirm the observations of the baseline estimation that the decline of the slope of the NKPC is not supported in terms of marginal cost.

Table A.5: Dynamic panel GMM estimation of NKPC in industry level data

	Dependent : Output price inflation π					
	(1)	(2)	(3)	(4)	(5)	(6)
	Hybrid NKPC			Purely forward looking NKPC		
	before GR	Full sample		Before GR	Full sample	
π_{t+1}	0.358^{+++}	0.239^{+++}	0.240^{+++}	0.403^{+++}	0.263^{***}	0.264^{***}
	(0.045)	(0.069)	(0.070)	(0.075)	(0.093)	(0.096)
π_{t-1}	0.321^{+++}	0.227^{+++}	0.235^{+++}			
	(0.033)	(0.061)	(0.059)			
$Output_t$	0.0139	0.00224	0.0117	-0.0214	-0.0130	-0.0120
	(0.024)	(0.024)	(0.027)	(0.029)	(0.027)	(0.031)
$postGR_t \times Output_t$			-0.0690			-0.00946
			(0.065)			(0.076)
AR(p) test	1	1	1	1	1	1
N of ind.	60	60	60	60	60	60
N of total obs.	3,180	3,540	3,540	3,180	3,540	3,540

	Dependent : Output price inflation π_t					
	(1)	(2)	(3)	(4)	(5)	(6)
	Hybrid NKPC			Purely forward looking NKPC		
	Before GR	Full sample		Before GR	Full sample	
π_{t+1}	0.359^{+++}	0.244^{+++}	0.244^{+++}	0.393^{+++}	0.261^{***}	0.254^{***}
	(0.045)	(0.072)	(0.073)	(0.073)	(0.093)	(0.095)
π_{t-1}	0.316^{+++}	0.222^{+++}	0.225^{+++}			
	(0.032)	(0.060)	(0.060)			
$IntShare_t$	0.0164^{**}	0.0337^{*}	0.0347^{*}	0.0259^{**}	0.0375^{*}	0.0413^{**}
	(0.008)	(0.018)	(0.020)	(0.013)	(0.019)	(0.021)
$postGR_t \times IntShare_t$			0.00270			0.00799
			(0.005)			(0.005)
AR(p) test	1	1	1	1	1	1
N of ind.	60	60	60	60	60	60
N of total obs.	3,180	3,540	3,540	3,180	3,540	3,540

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, $^{+++}$ $p < 0.001$

Notes: The two step GMM is employed. Output is taken log and detrended by the Hamilton filter. Output price inflation and intermediate price inflation are in log difference while the intermediate share is in log level. Instruments are the second to forth lagged output price inflation rate, the intermediate share, intermediate price inflation, the detrended output. Windmeijer corrected standard errors are reported in parentheses. *AR(p) test* indicates the order of AR process implied by the Arellano-Bond AR(p) test. Industry fixed effects are included. $postGR_t$ is a dummy variable that takes 1 from 2008 to 2012. A dummy variable for year 2009 is included as an independent variable to control for volatile price inflation of the year, which is not reported in table. Sample period is from 1955 to 2014 for $beforeGR$ and from 1955 to 2014 for $fullsample$, respectively.

A.3 Relationship between marginal cost and output gap

This subsection examines empirical relationship between marginal cost and the output gap using the industry level data of the KLEMS 2017. As is discussed in the previous subsection, the intermediate share has desirable properties to measure marginal cost.[2]

I run the following panel regression of marginal cost on measures of the output gap

$$IntShare_{i,t} = \beta_0 + \beta_1 x_{i,t} + \beta_2 D_{i,t} x_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t} \tag{A.21}$$

or

$$IntShare_{i,t} = \beta_0 + \beta_1 x_t + \beta_2 D_t x_t + \alpha_i + \epsilon_{i,t} \tag{A.22}$$

where α_i and γ_t are the fixed effects for industry i and time t, respectively. $IntShare_{i,t}$ denotes the intermediate share, which is a measure of marginal cost. For measures of the output gap x, I use the industry level detrended output $Output_{i,t}$ and the aggregate unemployment gap $UnempGap_t$. In order to capture potential non-linearity, I include interaction terms with dummy variables D, which indicate the post-Great Recession periods $postGR$, or the higher- and lower-quantiles of each series of x, HQT and LQT.

Table A.6 presents the regression results. The coefficient of the detrended output is significantly positive in column (1)-(4). The finding supports the procyclicality of marginal cost (counter-cyclical markup) in line with previous studies (e.g., Basu (1995), Gali et al. (2007), and Bils et al. (2018)). Moreover, the data identifies convex relationship between marginal cost and the detrended output. To be precise, the interaction term with the post-Great Recession dummy is significantly negative in column (2), implying that marginal cost did not decline as much as the detrended output did after the Great Recession. Interestingly, the interaction term with the lower quartile in column (3) is negative, whereas that with the higher quartile in column (4) is positive. These observations imply that the convex relationship is present over business cycles, and it significantly appeared after the Great Recession. Similar results are found with respect to the aggregate unemployment gap in

[2]Despite the desirable properties, I use the labor share in aggregate data to estimate the NKPC in the main article, because the survey based inflation expectations are not available for each industry.

column (5)-(8).

Table A.6: Panel regression of marginal cost on the output gap measures

	Dependent variable : $IntShare_{i,t}$							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Linear	post-GR	LQT	HQT	Linear	post-GR	LQT	HQT
$Output_{i,t}$	0.316^{+++}	0.371^{+++}	0.395^{+++}	0.258^{+++}				
	(0.031)	(0.035)	(0.048)	(0.043)				
$postGR_t \times Output_{i,t}$		-0.245***						
		(0.076)						
$LQT_{i,t} \times Output_{i,t}$			-0.164**					
			(0.078)					
$HQT_{i,t} \times Output_{i,t}$				0.129*				
				(0.066)				
$UnempGap_t$					0.676^{+++}	1.159***	1.118***	0.242
					(0.167)	(0.362)	(0.410)	(0.223)
$postGR_t \times UnempGap_t$						-1.004**		
						(0.393)		
$LQT_t \times UnempGap_t$							-0.902**	
							(0.438)	
$HQT_t \times UnempGap_t$								1.074
								(1.064)
Industry FE	YES	YES	YES	YES	YES	YES	YES	YES
Time FE	YES	YES	YES	YES	NO	NO	NO	NO
N of ind.	60	60	60	60	60	60	60	60
N of total obs.	1,800	1,800	1,800	1,800	1,800	1,800	1,800	1,800

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, $^{+++}$ $p < 0.001$

Notes: Standard errors are clustered within each industry and reported in parentheses. The intermediate share and the detrended output are taken log and detrended by the Hamilton filter. The sign of the coefficient of the unemployment gap is flipped for a comparison purpose. $postGR_t$ is a dummy variable that takes 1 from 2008 to 2012. $LQT_{i,t}$ and $HQT_{i,t}$ is a dummy variable that takes 1 if the observation is in the lower and higher 25 percentile of the sample, respectively. Sample is annual data from 1985 to 2014 for 60 industries in the non-farm business sector, including 18 manufacturing and 42 non-manufacturing. In column (5)-(8), dummy variables for year 2008 and 2009 are included to control for the large variations during the financial crisis.

B Model

This section provides supplemental details of the model.

B.1 Government and market clearing

In the economy, the government is passive. It levies lump-sum tax on households and distributes it as subsidies to remove the monopolistic distortion of firms and households with a balanced budget:

$$\int_0^1 \tau_t(j)dj = \tau_w \int_0^1 w_t(j)h_t(j)dj + \tau_p \int_0^1 p_t(i)y_t(i)di \tag{B.23}$$

where $\tau_t(j)$, τ_w, τ_p denote the lump-sum tax on households, the labor subsidy to workers, and the production subsidy to firms, respectively.

Market clearing conditions are given as follows.

$$A_t \equiv \int_0^1 a_t(j)dj = 0 \tag{B.24}$$

$$Y_t = C_t + \frac{\phi_p}{2}(\Pi_t - \Pi^*)^2 C_t \tag{B.25}$$

$$H_t = \int_0^1 h_t(i)di. \tag{B.26}$$

The asset market clearing (B.24) is trivial because consumption is identical across households and it is therefore not necessary to keep track of individual asset holdings to characterize equilibrium. The goods market clearing (B.25) should hold at the aggregate level since I restrict the attention to the symmetric equilibrium. It should be noticed that the labor market clearing (B.26) is automatically satisfied in the symmetric equilibrium because each firm i uses the identical level of labor input.

B.2 Definition of recursive competitive equilibrium

Notation. In the following definition, x denotes the variable in the current period, while x' and x_{-1} represent the one period ahead and lagged variables, respectively. The definitions of each variable follow those in the main article. That is, C: consumption, Y: output, H: hours worked, Π: gross price inflation rate, R: gross nominal interest rate, β: discount factor, Z: technology, W: nominal wage, $\tilde{W}$: real wage, χ: labor disutility. Capital letters denote aggregate variables.

Definition. *A recursive competitive equilibrium is a household's policy function for individual real wages $\tilde{w} = h(\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z)$, a policy function for a set of aggregate jump variables*

$X \equiv \{C, Y, H, \Pi, R\} = f(g_{-1}, \beta, Z)$, and a law of motion Γ for cross-sectional density of individual real wages g, such that

(i) a household's policy function h solve the recursive wage setting problem,[3]

$$\begin{aligned}
V^{dnwr}\Big(\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z\Big) = \max_{\tilde{w}} : \; & -\frac{1}{1+\eta}\chi h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) \\
& + \beta\mathbb{E}\Big[V\big(\tilde{w}, \chi'; g, \beta', Z'\big)|\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z\Big] \qquad \text{(B.27)} \\
s.t. \quad & h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H \\
& \tilde{w} \geq \tilde{w}_{-1}/\Pi
\end{aligned}$$

$$\begin{aligned}
V^{no}\Big(\chi; g_{-1}, \beta, Z\Big) = \max_{\tilde{w}} : \; & -\frac{1}{1+\eta}\chi h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) \\
& + \beta\mathbb{E}\Big[V\big(\tilde{w}, \chi'; g, \beta', Z'\big)|\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z\Big] \qquad \text{(B.28)} \\
s.t. \quad & h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H
\end{aligned}$$

$$\begin{aligned}
\textit{where} \quad & \mathbb{E}\Big[V\big(\tilde{w}, \chi'; g, \beta', Z'\big)|\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z\Big] \\
& = (1-\alpha)\mathbb{E}\Big[V^{dnwr}\big(\tilde{w}, \chi'; g, \beta', Z'\big)|\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z\Big] \\
& \quad + \alpha\mathbb{E}\Big[V^{no}\big(\chi'; g, \beta', Z'\big)|\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z\Big] \qquad \text{(B.29)}
\end{aligned}$$

$$\textit{and} \qquad \tilde{W} \equiv \left(\int \tilde{w}^{1-\theta_w} dg\right)^{\frac{1}{1-\theta_w}}. \qquad \text{(B.30)}$$

(ii) an aggregate policy function f solves the Euler equation (B.31), the NKPC (B.32), the monetary policy rule (B.33), as well as the production function (B.34) and the market clearing conditions (B.35),

[3] The recursive problem is equivalent to satisfying the FOCs in the main article.

$$\mathbb{E}\left[\beta\left(\frac{C'}{C}\right)^{-\sigma}\frac{R}{\Pi'}\mid g_{-1},\beta,Z\right]=1 \tag{B.31}$$

$$(\Pi-\Pi^*)\Pi=\beta\mathbb{E}\left[\left(\frac{C'}{C}\right)^{-\sigma}\left(\frac{Y'}{Y}\right)(\Pi'-\Pi^*)\Pi'\mid g_{-1},\beta,Z\right]+\kappa\left(MC-1\right) \tag{B.32}$$

$$R=R^*\left(\frac{\Pi}{\Pi^*}\right)^{\delta_\pi}\left(\frac{Y}{Y^*}\right)^{\delta_y} \tag{B.33}$$

$$Y=ZH \tag{B.34}$$

$$Y=C+\frac{\phi_p}{2}(\Pi-\Pi^*)^2C. \tag{B.35}$$

(iii) a law of motion Γ *is generated by* g*, that is, the cross-sectional density* g *satisfies*

$$g=\Gamma(g_{-1},\beta,Z). \tag{B.36}$$

B.3 Definition of stationary equilibrium

Definition. *A stationary equilibrium is a household's policy function for individual real wages* $\tilde{w}=h(\tilde{w}_{-1},\chi)$*, aggregate variables* $X=\{\tilde{W},C,Y,H,\Pi,R\}$*, and a probability distribution* $p(\tilde{w},\chi)$*, such that* *(i) a household's policy function h solves a recursive wage setting problem,*

$$V^{dnwr}\Big(\tilde{w}_{-1},\chi\Big) = \max_{\tilde{w}}: \; -\frac{1}{1+\eta}\chi h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) + \beta\mathbb{E}\Big[V\big(\tilde{w},\chi'\big)|\tilde{w}_{-1},\chi\Big] \tag{B.37}$$

$$s.t. \quad h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H$$

$$\tilde{w} \geq \tilde{w}_{-1}/\Pi$$

$$V^{no}\Big(\chi\Big) = \max_{\tilde{w}}: \; -\frac{1}{1+\eta}\chi h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) + \beta\mathbb{E}\Big[V\big(\tilde{w},\chi'\big)|\tilde{w}_{-1},\chi\Big] \tag{B.38}$$

$$s.t. \quad h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H$$

$$\textit{where} \quad \mathbb{E}\Big[V\big(\tilde{w},\chi'\big)|\tilde{w}_{-1},\chi\Big] = (1-\alpha)\mathbb{E}\Big[V^{dnwr}\big(\tilde{w},\chi'\big)|\tilde{w}_{-1},\chi\Big] + \alpha\mathbb{E}\Big[V^{no}\big(\chi'\big)|\tilde{w}_{-1},\chi\Big] \tag{B.39}$$

(ii) aggregate jump variables X solve the labor demand condition, the production function, the goods market clearing condition, the constant inflation condition, and the Fischer equation,

$$\tilde{W} = Z \; , \quad Y = ZH \; , \quad Y = C \; , \quad \Pi = \Pi^* \; , \quad R = \Pi^*/\bar{\beta} \tag{B.40}$$

(iii) a probability distribution p is a stationary distribution, that is,

$$p(\tilde{w}',\chi') = \int_{\chi}\int_{\tilde{w}:\tilde{w}'=h(\tilde{w},\chi')} p(\tilde{w},\chi)P(\chi'|\chi)d\tilde{w}d\chi \tag{B.41}$$

(iv) the aggregate hours H satisfies the market clearing condition,

$$H = \left(\int_{\chi}\int_{\tilde{w}} \Big\{(\tilde{w}/\tilde{W})^{-\theta_w} Hp(\tilde{w},\chi)\Big\}^{\frac{\theta_w-1}{\theta_w}} d\tilde{w}d\chi\right)^{\frac{\theta_w}{\theta_w-1}} \tag{B.42}$$

C Computation

C.1 Details of equilibrium computation algorithm

C.1.1 Modified Krusell-Smith algorithm

Approximated equilibrium. To deal with the infinite dimensionality of the cross-sectional distribution, Krusell and Smith (1998) propose an approximated equilibrium where each agent perceives the evolution of aggregate state variables as being a function of a small number of moments of the cross-sectional distribution. Adopting their insights, I assume that the aggregate endogenous sate variable, aggregate real wages $\tilde{W}_t$, follows an aggregate law of motion (ALM):

$$\tilde{W}_t = \Gamma(\tilde{W}_{t-1}, \beta_t, Z_t). \tag{C.43}$$

Specification of the ALM. To parameterize the ALM Γ, I conjecture that it is approximated by a log-linear form:

$$log(\tilde{W}^s) = \ B_0^s + B_1^s log(\tilde{W}_{-1}) \ , \quad for \quad (\beta_t, Z_t) = (\beta^s, Z^s) \tag{C.44}$$

where the superscript s denotes the exogenous state of the economy. It should be noted that, even though the ALM takes a simple functional form in terms of $\tilde{W}_{-1}$, it can capture rich non-linear dynamics because of the semi-parametric specification of the coefficients B^s. In other words, it accommodates different impacts of the past state variables across exogenous states s.

The log-linear specification of own lagged values is widely used in the literature to approximate the ALM (e.g., Krusell and Smith (1998), Nakamura and Steinsson (2010)). For robustness check, I include the higher order moments and the higher order polynomials in the ALM. I also introduce a jump variable such as output and interest rate as a regressor, which is a strategy to increase the forecasting power of the ALM proposed in the literature (e.g., Vavra (2013)). However, I find that the resulting equilibrium dynamics are numerically close to the baseline specification.

Algorithm. I sketch the outline of the equilibrium computation. The algorithm takes the following steps in each iteration $m = 1, 2, 3...$

1. (Initial guess) Each agent uses the ALM with the coefficients $B^{(m)} = \{B_0^{s(m)}, B_1^{s(m)}, ..., , B_k^{s(m)}\}_{s=1}^{S}$ to forecast the current period aggregate state variable $\tilde{W}$.

2. (Aggregate problem) Given the aggregate state variables $\{\beta, Z, \tilde{W}\}$, the policy function for aggregate jump variables $f^{(m)}$ is obtained by solving a New Keynesian system, i.e., the Euler equation, the NKPC, and the Taylor rule, along with the production function and the market clearing condition. Notice that, given $\tilde{W}$, the aggregate part of the economy does not depend on the cross-sectional distribution.

3. (Individual problem) Given all the aggregate variables, i.e., the aggregate state variables $\{\beta, Z, \tilde{W}\}$ and the aggregate jump variables implied by $f^{(m)}$, individual households solve their wage setting problem to derive their policy function $h^{(m)}$.

4. (Stochastic aggregation) Given the aggregate policy function $f^{(m)}$ and the individual policy function $h^{(m)}$, the model economy is simulated with N households for T periods. The initial distribution is set at the one in the stationary equilibrium, and the initial T_0 periods are discarded. The simulation delivers the series of aggregate variables $\{X_t^{(m)}\}_{t=T_0+1}^{T}$. I set $N = 10,000$, $T = 51,000$, and $T_0 = 1,000$.[4]

5. (Update) Using the simulated variables $\{X_t^{(m)}\}_{t=T_0+1}^{T}$, the suggested coefficients $\hat{B}$ are obtained by running the OLS of the ALM. Then, the coefficients $B^{(m+1)}$ are updated according to the rule:

$$B^{(m+1)} = \lambda \hat{B} + (1 - \lambda) B^{(m)} \tag{C.45}$$

   where λ is the weight for updating. I set λ at 0.1.

6. Repeat from step 1 to step 5 until convergence criterion for the coefficients B are attained.

[4]I confirm that the computation results do not change even if I further increase N or T.

Convergence criterion of the ALM. I use two convergence criterion for the ALM coefficients B. First, I repeat iterations until the maximum quadratic distance between the original and updated coefficients becomes smaller than 10^{-4}. This is a standard criteria used in the literature.

In addition, to guarantee the accuracy of the ALM, I use the Den Haan (2010) statistics as an additional convergence criteria. The statistics measures the maximum distance between the aggregate state variables computed according to the ALM, $\tilde{W}_t^{alm}$, and those derived from equilibrium conditions in the simulation, $\tilde{W}_t^{sim}$:

$$DH(B) = sup_{t\in[T_0+1,T]}\ |log(\tilde{W}_t^{sim}) - log(\tilde{W}_t^{alm})|. \tag{C.46}$$

Den Haan (2010) proposes to use the statistics rather than R^2 to check the accuracy the ALM, because R^2 only measures the average error in the one-period ahead forecast. I repeat iterations until $DH(B)$ becomes smaller that $0.5 * 10^{-2}$. The critical value indicates that the cumulative error of agents' prediction of the aggregate real wage is smaller than 0.5% over 50,000 periods. The accuracy is in the same order as the one used in the literature.[5]

C.1.2 Global solution methods

Global solution methods. I use a global solution method for solving each of the aggregate and individual parts of the model. In this regard, a local solution method, such as the perturbation method, cannot be applied due to non-linearity of my model. More precisely, since the DNWR constraint makes the individual policy function kinked, the function is no longer differentiable. The presence of the ZLB is another source of non-linearity.

I use the policy function iteration method of Coleman (1990) for the aggregate problem, whereas I employ the value function iteration method for the individual one. In both methods, I discretize the state spaces and numerically search for the functions which satisfy the equilibrium conditions. To evaluate expectations, I use the linear interpolation and the numerical integration over the discretized grid points.

[5]In this regard, Den Haan (2010) compares several computation algorithms to solve a heterogenous agent model, and find that the Krusell-Smith algorithm, which is the most accurate one, gives around 0.2% as the Den Haan (2010) statistics over 10,000 periods.

Discretization of exogenous variables. I discretize the AR(1) process of exogenous variables by using the Tauchen (1986) method. I truncate the state space of discount factor at $[-5.0 * \sigma_\beta, 5.0 * \sigma_\beta]$.[6] The state space of idiosyncratic labor disutility shocks is truncated at $[-2.33 * \sigma_\chi, 2.33 * \sigma_\chi]$ to remove the upper and the lower 1 percentile of households. I use 11 grid points for each exogenous process.

C.2 Construction of generalized impulse responses

Definition. Following Koop et al. (1996), I define the generalized impulse responses (GIR) as follows:

$$
\begin{aligned}
GIR(y, t, \epsilon_1) &= \mathbb{E}[y_t|\epsilon_1] - \mathbb{E}[y_t] && \text{(C.47)}\\
&= \mathbb{E}\Big[\mathbb{E}[y_t|x_0 = \tilde{x}_0, x_1 = \tilde{x}_1 e^{\epsilon_1}, \cdots, x_t = \tilde{\tilde{x}}_t, \omega_0 = \tilde{\omega}_0]|\epsilon_0\Big]\\
&\quad - \mathbb{E}\Big[\mathbb{E}[y_t|x_0 = \tilde{x}_0, \beta_1 = \tilde{x}_1, \cdots, x_t = \tilde{x}_t, \omega_0 = \tilde{\omega}_0]\Big] && \text{(C.48)}
\end{aligned}
$$

where y_t is the target variable of GIR, while x_t and ω_t denote the exogenous and endogenous state variables, respectively. $t \geq 1$ is the length of periods after the initial shock. $\{\tilde{x}_s\}_{s=1}^t$ and $\{\tilde{\tilde{x}}_s\}_{s=1}^t$ are generated with and without the exogenous shocks ϵ_1. Notice that the initial state $\tilde{x}_0$ and ω_0 is integrated out so that GIR capture the impacts of an exogenous shock without being dependent on the initial conditions.

Computation. Since GIR do not have a closed form solution, a simulation based method is employed to compute them. The construction of GIR takes the following steps:

1. Draw an initial state $\tilde{\omega}_0$ and $\tilde{x}_0$ randomly.

2. Draw a series of exogenous shocks $\{\tilde{x}_s\}_{s=1}^t$ and $\{\tilde{\tilde{x}}_s\}_{s=1}^t$ with or without the initial shock ϵ_0, given the initial state.

3. Simulate the model economy along with the path of exogenous variables.

[6] The range corresponds to $\pm 5.0 * \sqrt{1 - \rho_\beta^2} \approx \pm 2.5$ of the unconditional standard deviation of discount factor.

4. Repeat the procedure 1-3 for 10,000 times and take the mean to compute expectations.

D Calibration

D.1 Moment matching method

Definition. I define the moment matching estimator for the list of model parameters Θ:

$$\hat{\Theta} = argmin_{\Theta} : \ \mathbf{g}(\boldsymbol{\Theta})\mathbf{W}\mathbf{g}(\boldsymbol{\Theta})' \tag{D.49}$$

$$where = \mathbf{g}(\boldsymbol{\Theta}) \equiv \mathbf{d} - \mathbf{m}(\boldsymbol{\Theta}). \tag{D.50}$$

$\mathbf{d}$ is a vector that collects target moments in data, whereas $\mathbf{m}(\boldsymbol{\Theta})$ is a vector that contains the corresponding moments of the model. $\mathbf{W}$ is a weighting matrix.

Baseline calibration. In my baseline model, I calibrate the parameters for the cross-sectional wage distribution by using the moment matching method. That is, $\Theta = \{\alpha, \sigma_\chi\}$, where each symbol is presented in the main article. I use an identity matrix for W. Therefore, the estimator $\hat{\Theta}$ is obtained by minimizing sum of quadratic distance between the target moments and corresponding moments in the model. Since the number of targets coincides with that of model parameters to calibrate, they are just identified.

To generate the model moments, I solve the stationary equilibrium of the model due to the computation burden of deriving the recursive competitive equilibrium. However, I verify that the resulting cross-sectional moments in the recursive competitive equilibrium under the calibrated parameter values are fairly close to those in the stationary equilibrium.

E Additional Numerical Results

E.1 Details of GIR analysis

E.1.1 Degree of asymmetry

To quantify the degree of asymmetry of GIR, I introduce the following measure:

$$Asym(y, k, \epsilon_0) = \sum_{t=1}^{k} |GIR(y, t, \epsilon_0)| / \sum_{t=1}^{k} |GIR(y, t, -\epsilon_0)| \tag{E.51}$$

where y is a target variable and ϵ_0 is the initial exogenous shock with $\epsilon_0 > 0$. k is time-horizon, which is set at 4. The measure compares relative size of GIR to contractionary and expansionary shocks. Table E.7 shows that the degree of asymmetry is increasing in the size of shock. For example, inflation responses to a contractionary shock are 15% smaller than those to an expansionary one if the magnitude is 1 S.D., whereas the difference is enlarged to 40% if 3 S.D. shocks hit.

Table E.7: Degree of asymmetry to different size of shocks

ϵ_1	π^w	π^p	i	r	Y	H	MC	β
1.0 S.D.	0.82	0.85	0.90	0.95	1.04	1.04	0.64	1.00
2.0 S.D.	0.65	0.70	0.81	0.90	1.08	1.08	0.34	1.00
3.0 S.D.	0.54	0.60	0.74	0.86	1.12	1.12	0.21	1.00
4.0 S.D.	0.47	0.54	0.69	0.82	1.15	1.15	0.14	1.00

Notes: The table reports the asymmetry measure defined in (E.51). Smaller (larger) values than 1 indicate that the GIR is smaller (larger) upon a contractionary shock.

E.1.2 Half-lives

The half-lives of an exogenous shock are defined as follows:

$$HL(y, \epsilon_0) = \arg\min_t |GIR(y, t, \epsilon_0)| / |GIR(y, 1, \epsilon_0)| < 0.5 \tag{E.52}$$

where y is a target variable and ϵ_0 is the initial exogenous shock. The half-lives are a measure of persistence of an exogenous shock. Table E.7 shows the half-lives of different sign and size of shocks. The half-lives of output and hours worked are the same as those of exogenous shocks because my model does not have endogenous mechanisms to generate persistence of real quantities such as habit formation. On the other hand, the half-lives of marginal cost are considerably longer than those of exogenous shocks reflecting sluggish adjustment of real wages due to DNWR, while it does not guarantee strong persistence of wage growth. The half-lives of price inflation are longer that those of wage growth. This is because forward-looking firms responds not just the current marginal costs but also the future ones under nominal price rigidity. Regarding asymmetry to the sign of a shock, wage growth and price inflation display stronger persistence upon a large shock.

Table E.8: Half-lives of different sign and size of shocks

				Contractionary shock				
ϵ_1	π^w	π^p	i	r	Y	H	MC	β
1.0 S.D.	5	8	7	7	6	6	13	6
2.0 S.D.	6	8	7	7	6	6	16	6
3.0 S.D.	7	9	7	7	6	6	16	6
4.0 S.D.	7	9	7	7	6	6	16	6
				Expansionary shock				
ϵ_1	π^w	π^p	i	r	Y	H	MC	β
1.0 S.D.	5	8	8	8	6	6	12	6
2.0 S.D.	5	8	7	8	6	6	11	6
3.0 S.D.	4	8	7	8	6	6	10	6
4.0 S.D.	4	8	7	8	6	6	10	6

Notes: The table reports the asymmetry measure defined in (E.52).

E.2 Details of ZLB analysis

E.2.1 Monetary policy rules with ZLB

Taylor rule with ZLB. A number of studies argue that the ZLB is an essential element to understand the Great Recession (e.g., Christiano et al. (2015), Basu and Bundick (2017), Gust et al. (2017)). Based on these observations, I introduce the ZLB into the Taylor (1993) rule:

$$R_t^d = R^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\delta_\pi} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} \tag{E.53}$$

$$R_t = max\{\ R_t^d\ ,\ 1\ \}. \tag{E.54}$$

History-dependent rule with ZLB. Although many of the previous studies that investigate the role of the ZLB after the Great Recession assume the standard Taylor rule as in (E.53),[7] the Federal Reserve announced a commitment to keep low interest rate policy in the future when they faced the ZLB. To capture the effects of such forward guidance, I consider the history-dependent rule proposed by Reifschneider and Williams (2000):

$$R_t^d = R^* \left(\frac{R_{t-1}^d}{R_{t-1}}\right) \left(\frac{\Pi_t}{\Pi^*}\right)^{\delta_\pi} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} \tag{E.55}$$

$$R_t = max\{\ R_t^d\ ,\ 1\ \}. \tag{E.56}$$

Under this rule, a central bank keeps track of the gap between the desired and actual interest rates R_{t-1}^d/R_{t-1}. Once the economy is constrained at the ZLB, a central bank continues low interest rate policy until the gap is cleared, even if the standard Taylor rule implies interest rate hikes. Although the existing literature proposes several ways to implement forward guidance in a general equilibrium model (e.g., Eggertsson and Woodford (2003), Del Negro et al. (2012), McKay et al. (2016)), the history-dependent rule is distinguished from others in that it is fully embedded in the rational expectation equilibrium as a monetary policy rule. In other words, the central bank's commitment to low interest rate policy is expected by agents ex-ante.[8]

Computation with ZLB. Under the Taylor rule with the ZLB, I use the same log-linear specification for the ALM (C.44) as the baseline model without the ZLB. However, I find that numerical errors tend to be larger, which makes it difficult to achieve the convergence criterion in Appendix C. On the other hand, even if I increase the number of iterations, the model does not display a tendency to diverge, and the model's moments are quite stable over iterations. Therefore, I use looser criterion for convergence. To be precise, I stop iterations when the maximum quadratic distance between the updated and original coefficients becomes smaller than $0.5 * 10^{-3}$ and the Den Haan (2010) statistics smaller than 10^{-2}.

[7] Few exceptions include Basu and Bundick (2015) and Katagiri (2016).

[8] In addition, Basu and Bundick (2015) argue that the history-dependent rule has desirable properties to remove the contractionary bias of the ZLB, that is, the bias that a central bank charges higher interest rate than the desired one on average over business cycles in the presence of the ZLB.

Under the history-dependent rule with the ZLB, the previous period's interest rate gap becomes another state variable. Therefore, I modify the ALM in the following way:

$$log(\tilde{W}^s) = \ B_0^s + B_1^s log(\tilde{W}_{-1}) + B_2^s log(R_{t-1}^d / R_{t-1}) \ , \quad for \quad (\beta_t, Z_t) = (\beta^s, Z^s) \tag{E.57}$$

where R_{t-1}^d and R_{t-1} are the desired and actual interest rate in the previous period, respectively. I use the same convergence criterion as the baseline model without the ZLB.

E.2.2 Time-series moments with ZLB

Table E.9 reports time-series moments under different monetary policy rules with and without the ZLB. The table also reports the unconditional probability and the average duration of ZLB episodes.

Several points are noteworthy in the table. First, the standard deviation of output and hours worked is enlarged in the presence of the ZLB. This result is in line with previous studies, which point out that the impacts of an exogenous shocks are amplified at the ZLB due to the lack of offsetting monetary policy responses (e.g., Christiano et al. (2011)). Moreover, since the amplification effects works strongly during recessions, output and hours worked are more negative skewed. Second, interestingly, the standard deviation of price inflation and wage growth is slightly smaller under the Taylor rule with the ZLB than the case without the ZLB. Intuitively, although the fluctuations of real quantities are amplified at the ZLB, downward adjustment of price variables are restricted by DNWR. Moreover, since agents internalize the severity of recessions with the ZLB, they compress wage increases in booms, more strongly than the case without the ZLB, in preparation for downturns. These effects result in smaller variations of price inflation and wage growth. Consistently, price variables are less positive skewed in the presence of the ZLB. Third, regarding the length of the ZLB episodes, my model generates relatively longer spells than previous studies (e.g., Fernández-Villaverde et al. (2015)). This is because DNWR along with nominal price rigidity generates substantial persistence of price inflation, which leads to sluggish movements of the desired interest rate implied by the Taylor rule. Fourth, the history-dependent rule partly offsets the adverse effects of the ZLB by committing to future low interest rate policy at the ZLB.

Consistently, the duration of the ZLB episodes are longer than the Taylor rule with the ZLB. At the same time, it is also notable that the history-dependent rule cannot fully restore the economy without the ZLB, because the model takes into account aggregate uncertainty, and therefore the future monetary easing is discounted in the current values.

Table E.9: Time-series moments with ZLB

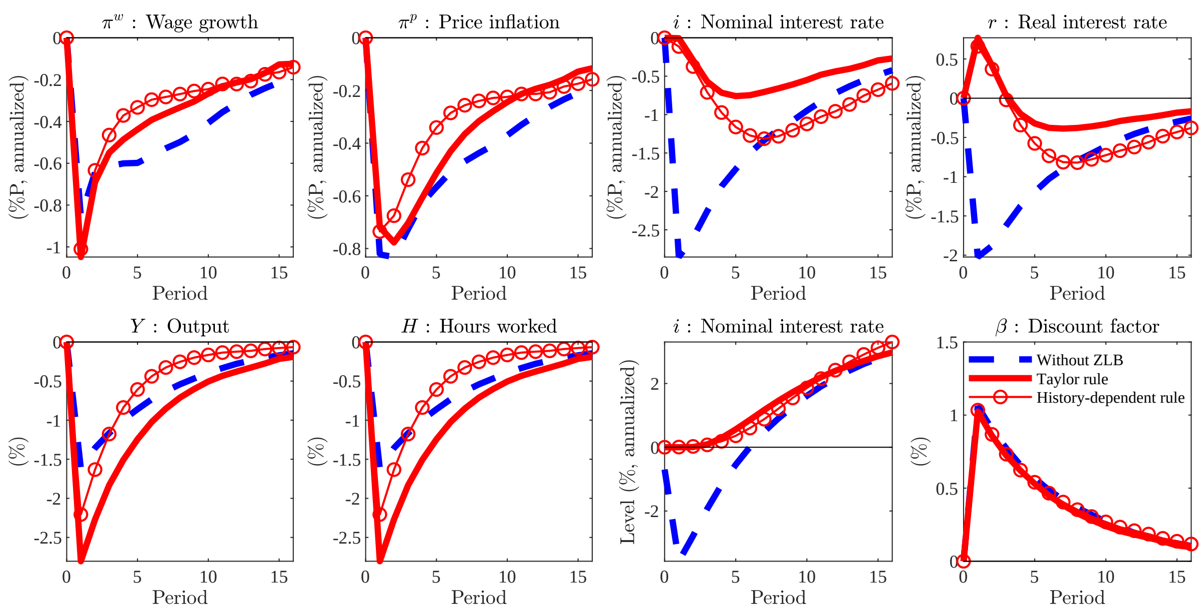
	Data	Model		
		Taylor rule without ZLB	Taylor rule with ZLB	History-dependent rule with ZLB
$\sigma(Y)$	1.55	1.55	2.25	1.71
$\sigma(\Delta Y)$	0.89	0.85	1.23	1.04
$\sigma(H)$	1.82	1.55	2.25	1.71
$\sigma(\Delta H)$	0.82	0.85	1.23	1.04
$\sigma(\pi^p)$	0.58	0.60	0.53	0.57
$\sigma(\pi^w)$	0.74	0.66	0.61	0.65
$\sigma(W/P)$	0.84	0.46	0.57	0.53
$\sigma(i)$	0.82	1.26	1.06	1.20
$\rho(Y)$	0.85	0.85	0.85	0.81
$\rho(\Delta Y)$	0.29	-0.10	-0.10	-0.11
$\rho(H)$	0.90	0.85	0.85	0.81
$\rho(\Delta H)$	0.58	-0.10	-0.10	-0.11
$\rho(\pi^p)$	0.86	0.90	0.92	0.90
$\rho(\pi^w)$	0.40	0.81	0.80	0.81
$\rho(W/P)$	0.78	0.94	0.95	0.95
$\rho(i)$	0.95	0.89	0.89	0.89
$Sk_1(Y)$	-0.49	-0.38	-0.71	-0.39
$Sk_1(\Delta Y)$	-0.37	-0.03	-0.00	0.08
$Sk_1(H)$	-0.28	-0.38	-0.71	-0.39
$Sk_1(\Delta H)$	-0.68	-0.03	-0.00	0.08
$Sk_1(\pi^p)$	1.25	1.50	1.00	1.12
$Sk_1(\pi^w)$	0.23	1.75	1.30	1.35
$Sk_1(W/P)$	0.53	2.26	1.80	1.77
$Sk_1(i)$	1.17	1.01	1.12	1.01
$Sk_2(Y)$	-0.05	-0.11	-0.17	-0.05
$Sk_2(\Delta Y)$	-0.00	-0.00	0.00	-0.01
$Sk_2(H)$	-0.06	-0.11	-0.17	-0.05
$Sk_2(\Delta H)$	-0.16	-0.00	0.00	-0.01
$Sk_2(\pi^p)$	0.30	0.36	0.27	0.31
$Sk_2(\pi^w)$	0.04	0.38	0.34	0.36
$Sk_2(W/P)$	0.04	-0.11	0.39	0.41
$Sk_2(i)$	0.15	0.23	0.33	0.25
$Prob.(zlb)$	-	0.00	0.27	0.30
$Dura.(zlb)$	-	0.0	4.6	6.9

Notes: The standard deviation σ, the first order autocorrelation ρ, and the skewness Sk, as well as the unconditional probability of the ZLB episodes $Prob.(zlb)$ and the average duration of them $Dura.(zlb)$ are reported. The duration is in terms of quarters. For the skewness, in addition to the standard definition Sk_1, an alternative skewness measure Sk_2 is reported. Sk_2 is defined as $Sk_2 = (\mu - Q)/\sigma$ with the mean μ and the median Q, and bounded between -1 and 1. Kim and White (2003) argue that Sk_2 is robust to outliers. Regarding the moments of data, Y is the real GDP, H is the total hours in the non-farm business sector, π^p is the GDP implicit price deflator, π^w is the compensation per hour in the non-farm business sector, and i is the effective federal funds rate. Y and H are taken log and detrended by the HP-filter, while ΔY and ΔH are the quarterly growth rate. π^p and π^w are the quarter-on-quarter growth rate. i is the annual rate divided by 4 (quarterly rate). Sample period is from 1955Q1 to 2007Q4. For computing the moments of the models, I simulate the economy for 51,000 periods and discard the initial 1,000 observations. Each model is solved under the same parameter values as the baseline model.

E.2.3 GIR with ZLB

Figure E.1 displays GIR under different monetary policy rules with and without the ZLB. In line with the findings in the time-series moment analysis above, the responses of output and hours are amplified under the Taylor rule with the ZLB, whereas those of wage growth and price inflation are not. This helps one reconcile the small decline of inflation and the large drop of output observed in the data during the Great Recession. Under the history-dependent rule, due to the imperfect substitutability of the current and the future monetary easing, the initial responses are close to those under the Taylor rule with the ZLB, though they quickly recover because of the extended low interest rate policy.

Figure E.1: GIR with and without the ZLB



Notes: To compute GIR, I generate two time paths: (i) a path starting from a recession state in which the economy is constrained by the ZLB, (ii) a path starting from the same recession state but being hit by another 2 S.D. contractionary shock. The difference between the two paths gives the GIR at the ZLB. In the case without the ZLB, the initial exogenous states are drawn from the higher 25 percentile of discount factor. The number of simulations is set at 50,000. The horizontal axes represent the time horizon after the initial shock. The vertical axes are in the differences between two paths except for the nominal interest rate in the bottom row, which is shown in the level.

E.3 Effects of technology shock

E.4 Specification of technology shock

This subsection investigates the effects of a technology shock in my model. For this exercise, I assume that the aggregate technology Z_t follows an AR(1) process:

$$log(Z_t) = \rho_z log(Z_{t-1}) + \epsilon_{z,t} \ , \ \epsilon_{z,t} \sim N(0, \sigma_z^2). \tag{E.58}$$

For parameterization, I follow Fernández-Villaverde et al. (2015) to set $\rho_z = 0.900$ and $\sigma_z = 0.0025$.

E.4.1 Time-series moments with technology shock

Table E.10 reports the time-series moments in the model with technology shocks. Compared to the baseline model only with discount factor shocks, the model with technology shocks account for moderate positive skew of wage growth and real wages. This is because a positive technology shock make the DNWR constraint binding for less workers by raising the marginal product of labor. The model also generates larger standard deviation of hours worked than that of output, consistently with the data.

Table E.10: Time-series moments with technology shock

	Data	Model		
		β shock only	Z shock only	Both β and Z
$\sigma(Y)$	1.55	1.55	0.15	1.55
$\sigma(\Delta Y)$	0.89	0.85	0.04	0.86
$\sigma(H)$	1.82	1.55	0.44	1.62
$\sigma(\Delta H)$	0.82	0.85	0.06	0.87
$\sigma(\pi^p)$	0.58	0.60	0.20	0.53
$\sigma(\pi^w)$	0.74	0.66	0.16	0.59
$\sigma(W/P)$	0.84	0.46	0.39	0.68
$\sigma(i)$	0.82	1.26	0.26	1.14
$\rho(Y)$	0.85	0.85	0.80	0.85
$\rho(\Delta Y)$	0.29	-0.10	-0.16	-0.09
$\rho(H)$	0.90	0.85	0.93	0.86
$\rho(\Delta H)$	0.58	-0.10	0.03	-0.08
$\rho(\pi^p)$	0.86	0.90	0.80	0.91
$\rho(\pi^w)$	0.40	0.81	0.91	0.82
$\rho(W/P)$	0.78	0.94	0.97	0.96
$\rho(i)$	0.95	0.89	0.80	0.90
$Sk_1(Y)$	-0.49	-0.38	0.07	-0.08
$Sk_1(\Delta Y)$	-0.37	-0.03	0.04	-0.01
$Sk_1(H)$	-0.28	-0.38	-0.03	-0.10
$Sk_1(\Delta H)$	-0.68	-0.03	0.06	-0.02
$Sk_1(\pi^p)$	1.25	1.50	0.11	0.74
$Sk_1(\pi^w)$	0.23	1.75	0.27	0.98
$Sk_1(W/P)$	0.53	2.26	0.21	0.33
$Sk_1(i)$	1.17	1.01	0.14	0.47
$Sk_2(Y)$	-0.05	-0.11	-0.00	-0.03
$Sk_2(\Delta Y)$	-0.00	-0.00	0.00	-0.00
$Sk_2(H)$	-0.06	-0.11	-0.01	-0.04
$Sk_2(\Delta H)$	-0.16	-0.00	0.00	-0.00
$Sk_2(\pi^p)$	0.30	0.36	0.04	0.23
$Sk_2(\pi^w)$	0.04	0.38	0.11	0.31
$Sk_2(W/P)$	0.04	-0.11	0.06	0.05
$Sk_2(i)$	0.15	0.23	0.05	0.15

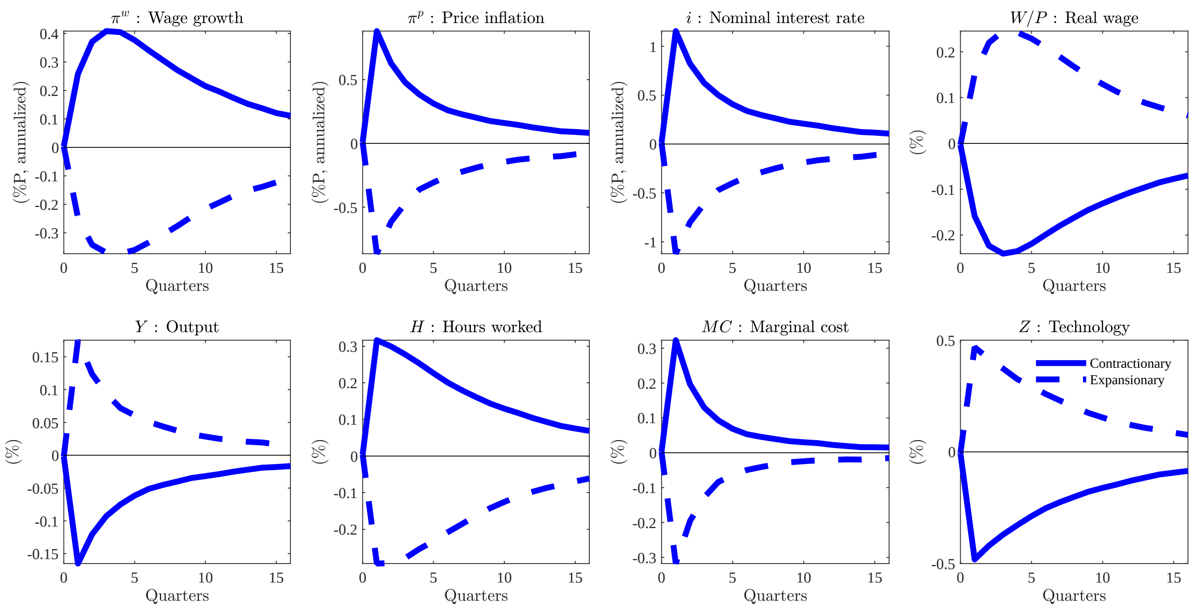
Notes: The definition of the data series is presented in Table E.9. Each model is solved under the same parameter values except for those of exogenous shocks. For the model only with technology shocks, I truncate the state space of technology at $\pm 5.0\sigma_z$, and discretize it with 11 grid points. For the model with both of discount factor shocks and technology shocks, I truncate each state space at $\pm 3.5\sigma$ and use 5 grids to discretize it.

E.4.2 GIR to technology shock

Figure E.2 shows GIR to a 2 S.D. technology shock. A technology shock does not generate significant asymmetry in GIR. In addition, the relative responses of price inflation to those of output are much larger than that upon a discount factor shock in the main article. This is because a technology shock directly affects marginal cost, which results in almost symmetric effects on price inflation through the NKPC. These effects are particular strong because real wages display considerable stickiness in the presence of DNWR and nominal price rigidity.

In the existing literature, several studies suggest the presence of negative technology shocks after the Great Recession from both exogenous and endogenous sources (e.g., Christiano et al. (2015), Guerrón-Quintana and Jinnai (ming)). The GIR analysis in this subsection implies that negative technology shocks, if any, reinforce the conclusion of this paper by offsetting declines of inflation that result from contractionary demand shocks in the Great Recession.

Figure E.2: GIR to 2 S.D. technology shock



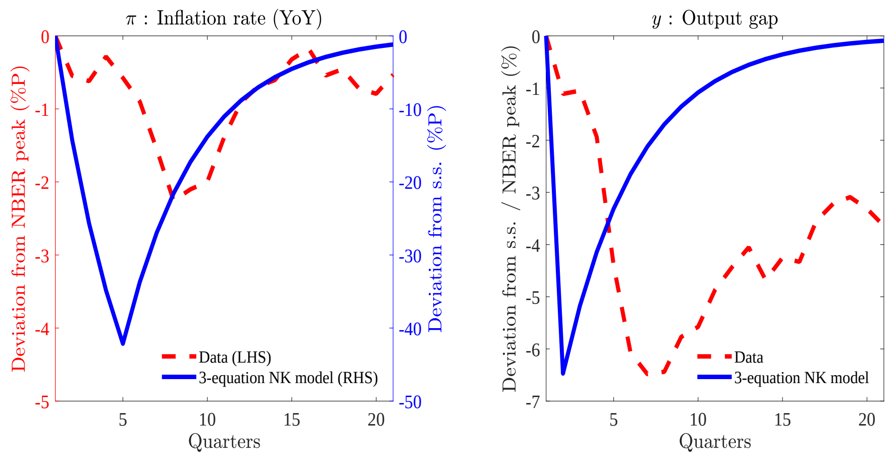
Notes: GIR are computed in the model only with technology shocks. The horizontal axes represent the time horizon after the initial shock. The vertical axes are in terms of the deviation from the stochastic mean.

F Model Comparison

F.1 Example in three-equation New Keynesian model

This subsection takes a look at the core of the problem of a stylized New Keynesian model in resolving the inflation puzzles. For this purpose, I consider a linearized three-equation

Figure F.3: IR in three-equation New Keynesian model and data



Notes: IR of the model are those to an exogenous innovation $e_{\epsilon,t}$. Size of shock is calibrated to match the drop of the output gap in data. For the series of data, the inflation rate is the year-on-year growth rate of the GDP implicit price deflator. The output gap is the one estimated by the Congressional Budget Office (CBO). Each data series is displayed as the deviation from the business cycle peak before the Great Recession that is defined by the NBER (2007Q4).

New Keynesian system that consists of the Euler equation (F.59), the NKPC (F.60), and the Taylor rule (F.61):

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}]) + \epsilon_t \tag{F.59}$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \tilde{\kappa} y_t \tag{F.60}$$

$$i_t = \delta_\pi \pi_t + \delta_y y_t \tag{F.61}$$

$$where \quad \epsilon_t = \rho_\epsilon \epsilon_{t-1} + e_{\epsilon,t}, \quad e_{\epsilon,t} \sim i.i.d. N(0, \sigma_\epsilon^2).$$

y_t, π_t, and i_t denote the output gap, the inflation rate, and the nominal interest rate, respectively. Each variable is in the log-deviation from the zero-inflation steady state. In this model, the Euler equation (F.59) along with the Taylor rule (F.61) governs the demand side of the economy, whereas the NKPC (F.60) the supply side. For simplicity, I suppose that the error term in the Euler equation ϵ_t, which follows an AR(1) process, is the only exogenous component of the economy. For the numerical exercise below, I choose $\rho_\epsilon = 0.80$, and the choice of the other parameter values is provided in the main article.

Figure F.3 compares the impulse responses (IR) of the three-equation New Keynesian model and the data after the Great Recession. It is immediate to see that the model predicts massive deflation (-42 percentage points), whereas the actual inflation rate declined moderately (-2.3 percentage points). It is also notable that the inflation rate in the model quickly reverts toward its steady-state value, whereas the recovery of the actual inflation was sluggish with a sizable gap from the pre-crisis rate even 20 quarters after the business cycle peak.

Among others, one of the key features to bring about these undesirable results in the stylized New Keynesian model is the relationship between the output gap and marginal cost. To see this point, it is convenient to decompose the output gap representation of the NKPC (F.60) into the marginal cost representation of the NKPC, which is formulated from a firms' profit maximization problem under nominal price rigidity,

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa mc_t \tag{F.62}$$

and the relationship between marginal cost and the output gap

$$mc_t = (\sigma + \eta) y_t \tag{F.63}$$

with mc_t denoting the marginal cost of producing one unit of output. The derivation of the equations follows Walsh (2010). Notice that Equation (F.63) is an outcome of the labor market equilibrium. To be precise, the households' marginal rate of substitution between consumption and hours worked should be equalized to real wages in a frictionless labor market. Real wages are proportional to firms' marginal cost given the marginal product of labor, whereas the marginal rate of substitution is related to households' consumption and hours worked, and it is therefore tied with the output gap in general equilibrium. Since inflation is pined down by current and future marginal costs through the NKPC (F.62), it is quite difficult to obtain a small decline of inflation and a large drop of the output gap simultaneously as long as the linear relationship between the output gap and marginal cost (F.63) is taken as given.

F.2 Comparison with different specifications of wage adjustment

This subsection compares aggregate dynamics across models with different specifications of wage adjustment that are often used in the literature. Specifically, I solve and calibrate a flexible wage model and a quadratic wage adjustment cost model, and compare them with my baseline model with DNWR.[9]

F.2.1 Flexible wage model

Wage setting. The friction less labor market equilibrium implies that real wages are equalized to the marginal rate of substitution.

$$\frac{W_t}{P_t} = \frac{H_t^{\eta}}{C_t^{-\sigma}} \tag{F.64}$$

Other parts of the model. The Euler equation, the NKPC, the Taylor rule, and the resource constraint.

$$1 = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}^p} \right] \tag{F.65}$$

$$(\Pi_t^p - \Pi^*)\Pi_t^p = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) (\Pi_{t+1}^p - \Pi^*)\Pi_{t+1}^p \right] + \frac{\theta_p}{\phi_p} \left(\frac{W_t}{Z_t P_t} - 1 \right) \tag{F.66}$$

$$R_t = R^* \left(\frac{\Pi_t^p}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{F.67}$$

$$Y_t = Z_t H_t = C_t + \frac{\phi_p}{2} \left(\Pi_t^p - \Pi^* \right)^2 C_t \tag{F.68}$$

Computation. I discretize the state space and use the policy function iteration method to solve the model.

Calibration. The same parameter values are used as the baseline model in the main article.

[9] Notice that the quadratic wage adjustment cost model coincides with the Calvo-type staggered contract model in the first order, which is widely used in the existing New Keynesian literature. (e.g., Erceg et al. (2000), Christiano et al. (2005))

F.2.2 Quadratic wage adjustment cost model

Wage setting. Households are subject to the following budget constraint with quadratic wage adjustment cost:

$$C_t + \frac{A_t}{P_t} \leq (1+\tau_w)\frac{W_t}{P_t}H_t - \frac{\phi_w}{2}\left(\Pi_t^w - \Pi^*\right)^2 H_t + R_{t-1}\frac{A_{t-1}}{P_t} + \frac{\Phi_t}{P_t} \tag{F.69}$$

where the notation follows the benchmark model. The FOC yields the wage Phillips curve:

$$(\Pi_t^w - \Pi^*)\Pi_t^w = \beta_t \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{H_{t+1}}{H_t}\right)(\Pi_{t+1}^w - \Pi^*)\Pi_{t+1}^w\right] + \frac{\theta_w}{\phi_w}\left(\frac{H_t^\eta}{C_t^{-\sigma}} - \frac{W_t}{P_t}\right) \tag{F.70}$$

Other parts of the model. The Euler equation, the NKPC, the Taylor rule, and the resource constraint.

$$1 = \beta_t \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{R_t}{\Pi_{t+1}^p}\right] \tag{F.71}$$

$$(\Pi_t^p - \Pi^*)\Pi_t = \beta_t \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{Y_{t+1}}{Y_t}\right)(\Pi_{t+1}^p - \Pi^*)\Pi_{t+1}\right] + \frac{\theta_p}{\phi_p}\left(\frac{W_t}{Z_t P_t} - 1\right) \tag{F.72}$$

$$R_t = R^*\left(\frac{\Pi_t^p}{\Pi^*}\right)^{\delta_\pi}\left(\frac{Y_t}{Y^*}\right)^{\delta_y} \tag{F.73}$$

$$Y_t = Z_t H_t = C_t + \frac{\phi_p}{2}\left(\Pi_t^p - \Pi^*\right)^2 C_t + \frac{\phi_w}{2}\left(\Pi_t^w - \Pi^*\right)^2 C_t \tag{F.74}$$

Computation. The same solution method is used as the flexible wage model.

Calibration. I calibrate the parameter value for the degree of wage stickiness ϕ_w according to the micro evidence reported by Barattieri et al. (2014). I use their most conservative estimate, which indicates that the frequency of individual wage changes is 26.6% per quarter.[10] The value implies the slope of the wage Phillips curve is 0.0977 and the corresponding parameter $\phi_w = 92.1$.[11] Other parameters are set at the same values as the baseline model

[10] Note that the value is also used as a calibration target of the baseline model.

[11] The slope of the wage Phillips curve in the Calvo model is given by $(1-\xi)(1-\beta\xi)/\xi$ where ξ is the probability of not being able to change wages. The calibrated values of $\beta = 0.995$ and $\xi = 1 - 0.266$ lead to the slope to be $(1-0.734)(1-0.995*0.734)/0.734 = 0.0977$. It implies $\phi_w = \theta_w/0.0977 = 92.1$ in the model here.

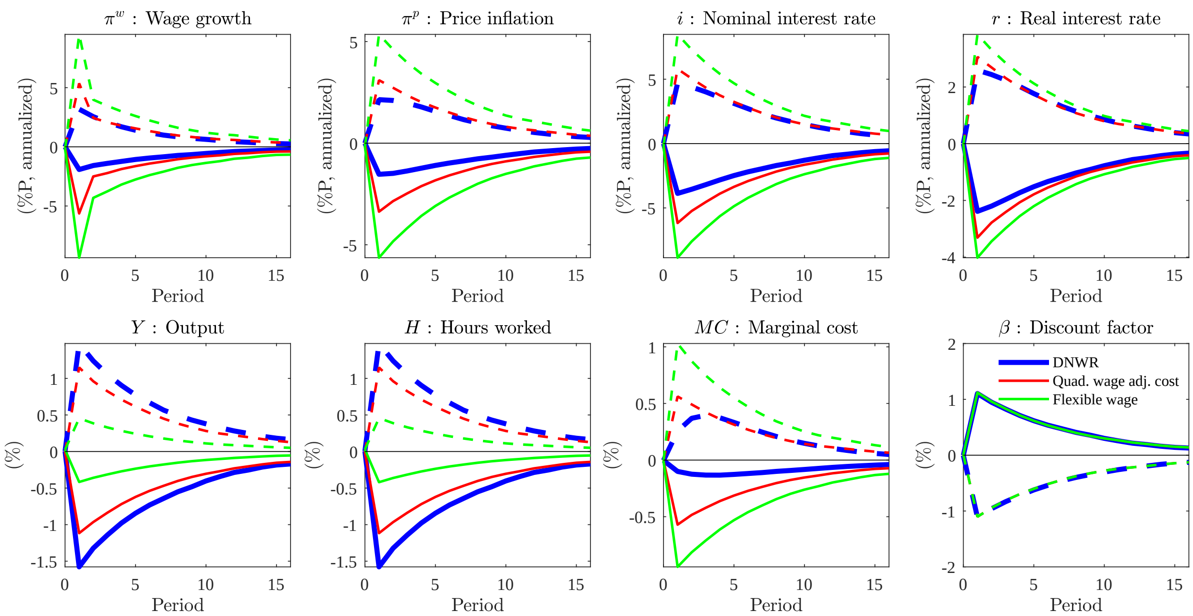
in the main article

F.2.3 Comparison of GIR

Figure F.4 compares GIR to a 2 S.D. discount factor shock in different models. In the flexible wage model, wage growth, marginal cost, and price inflation respond strongly to a discount factor shock. Since the effects of an exogenous shock are absorbed by adjustments of price variables, real quantities such as output and consumption do not react a lot. On the other hand, the quadratic wage adjustment model generates moderate responses in price variables and sizable responses of quantity variables. However, there are several important differences from the model with DNWR. First, the quadratic adjustment cost model does not bring about significant asymmetry since the wage adjustment cost is symmetric by construction.[12] As a result, the inflation responses to a 2 S.D. contractionary discount factor shock is almost twice as large as those in the model with DNWR, while the responses to an expansionary shock with the same magnitude is only slightly larger. Second, the propagation of an exogenous shock is not as stringent as the model with DNWR. The half-lives of wage growth and price inflation to a 2 S.D. contractionary shock are 2 and 6 quarters in the quadratic wage adjustment cost model, whereas they are 6 and 8 quarters in the model with DNWR. The difference reflects the state-dependency of DNWR. To be precise, after a contractionary shock, workers does not react to improvements of the state of the economy at all as long as the DNWR constraint binds, whereas the quadratic adjustment cost model allows for gradual responses in each period.

[12]Since the model is solved by a global method, non-linearity can arise from other parts of the model than the wage adjustment such as the curvature of the utility function. However, I find that such non-linearity is quantitatively small.

Figure F.4: GIR to discount factor shock in different models



F.3 Comparison with asymmetric wage and price adjustment cost model

This subsection compares the baseline model in the main article with a model that embeds asymmetric adjustment costs by focusing on non-linearity in each model. In this regard, Kim and Ruge-Murcia (2009) and Fahr and Smets (2010) propose to use an asymmetric wage adjustment cost function to approximate DNWR. More recently, Aruoba et al. (2017) build an asymmetric wage and price adjustment cost model to find that the model can capture the non-linearity in the data well. Here, I calibrate an asymmetric wage and price adjustment cost model based on the estimated parameters of Aruoba et al. (2017).

Model equations. The model consists of the Euler equation, the price Phillips curve, the wage Phillips curve, the Taylor rule, and the resource constraint. Derivations follow Aruoba et al. (2017).

$$1 = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi^p_{t+1}} \right] \tag{F.75}$$

$$\Phi'_p(\Pi^p_t)\Pi^p_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) \Phi'_p(\Pi^p_{t+1})\Pi^p_{t+1} \right] + \theta_p \left(\frac{W_t}{Z_t P_t} - 1 \right) \tag{F.76}$$

$$where\ \Phi_p \equiv \phi_p \left(\frac{exp(-\psi_p(\Pi^p_t - \Pi^*)) + \psi_p(\Pi^p_t - \Pi^*) - 1}{\psi_p^2} \right)$$

$$\Phi'_w(\Pi^w_t)\Pi^w_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{H_{t+1}}{H_t} \right) \Phi'_w(\Pi^w_{t+1})\Pi^w_{t+1} \right] + \theta_w \left(\frac{H^\eta_t}{C_t^{-\sigma}} - \frac{W_t}{P_t} \right) \tag{F.77}$$

$$where\ \Phi_w \equiv \phi_w \left(\frac{exp(-\psi_w(\Pi^w_t - \Pi^*)) + \psi_w(\Pi^w_t - \Pi^*) - 1}{\psi_w^2} \right)$$

$$R_t = R^* \left(\frac{\Pi^p_t}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{F.78}$$

$$Y_t = Z_t H_t = C_t + \Phi_p(\Pi^p_t)C_t + \Phi_w(\Pi^w_t)C_t \tag{F.79}$$

where ϕ and ψ govern the slope and the curvature of the Phillips curve, respectively.

Computation. Following Aruoba et al. (2017), I use the second order perturbation method around the steady state to solve the model.

Calibration. The parameter values for adjustment cost functions are set according to the posterior mean of Aruoba et al. (2017) for the sample of 1960Q1-2007Q4. They are listed in Table F.11. Other parameters are identical to those in the baseline model.

Table F.11: Parameter values in an asymmetric wage and price adjustment cost model

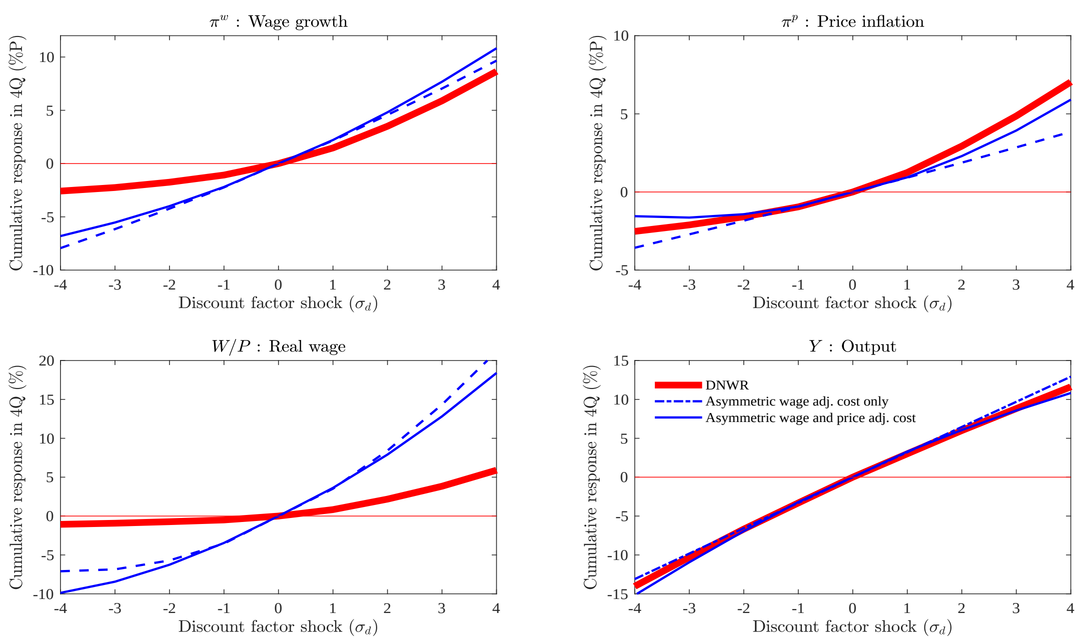
ϕ_p (ξ_p)	ψ_p	ϕ_w (ξ_w)	ψ_w
450 (0.87)	150	*28.1 (0.57)*	67.4

Notes: ξ is the Calvo parameter corresponding to ϕ with $\theta/\phi = (1-\xi)(1-\beta\xi)/\xi$. Italic values are the author's calculation based on Aruoba et al. (2017). For example, the posterior mean of Aruoba et al. (2017) for the slope of price Phillips curve is 0.02. In our model, $0.02 = \theta_p/\phi_p$ and $\theta_p = 9$ implies $\phi_p = 450$.

F.3.1 Comparison of non-linearity

Figure F.5 compares the cumulative responses of key variables in the initial 4 quarters after different sign and size of discount factor shocks. Interestingly, the non-linearity of price inflation and output are quite similar in two models. However, it is worth pointing out that the baseline model with DNWR is calibrated consistently with the frequency of individual price changes in micro data, whereas Aruoba et al. (2017) identify a much higher parameter value for the degree of price stickiness (flatter Phillips curve). Moreover, the responses of wage growth and real wages display much stronger non-linearity in the model with DNWR. This is because the non-linearity of the model with DNWR stems from asymmetric individual wage adjustments, whereas Aruoba et al. (2017)'s estimates indicate strong asymmetry in price adjustments rather than wage adjustments. Inthis regard, I find that the baseline model with DNWR matches the moderate declines of wage growth and real wages after the Great Recession fairly well.[13]

Figure F.5: Comparison of non-linearity



Notes: Each panel displays the cumulative responses in the initial 4 quarters after discount factor shocks. For the model with asymmetric wage adjustment cost only, the parameter for the asymmetry of price adjustment cost ψ_p is set at zero.

[13]The analysis is reported in Section 6 of the main article.

G Discussion and relation to existing literature

This section provides miscellaneous topics regarding the my model's connections to the existing literature.

G.1 Implication to inflation targeting

Figure G.6 displays the distribution of price inflation in time-series simulation of the model and the data. I estimate the kernel density to smooth jagged observations. In the model, the distribution of inflation is positive skewed because of the asymmetric effect of DNWR, which is consistent with the data. As a consequence, the median inflation rate in the model is substantially lower than the 2 percent of the calibrated target rate in the Taylor rule, whereas the mean inflation rate is almost around the target. In other words, lower inflation rates than the target level are more likely to realize in each period even if the target rate is achieved as the mean inflation.

This finding might be counterintuitive, but is indeed consistent with a wide class of Taylor-type monetary policy rules. To see this point, taking the the unconditional expectation of a Taylor rule leads to:

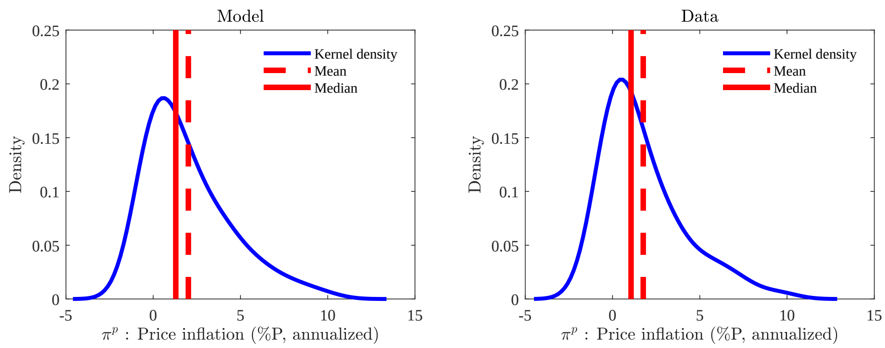
$$log(\mathbb{E}[R_t]) - log(R^*) \cong \delta_\pi(log(\mathbb{E}[\Pi_t]) - log(\Pi^*)) + \delta_y(log(\mathbb{E}[Y_t]) - log(Y^*)) \qquad \text{(G.80)}$$

Notice that the equation does not strictly hold because I ignore the Jensen's inequality terms. Equation (G.80) implies that inflation is stabilized in the mean under the Taylor rule, although the equation does not guarantee the stabilization of the median inflation to the target rate.

G.2 Connection to literature on ZLB

My model contributes to the literature on the ZLB by investigating the interaction of the ZLB and DNWR. As the existing literature has demonstrated, the impacts of an exogenous shock are amplified under the ZLB due to the lack of offsetting monetary policy responses (e.g., Christiano et al. (2011)). However, because scope for downward adjustments of price

Figure G.6: Distribution of inflation



Notes: The kernel density estimator is computed from the model and the data. The size of bandwidth is set at 1%. In the model, I simulate the model economy for 51,000 periods and discard the initial 1,000 observations. The data series is the GDP implicit price deflator. Sample period is from 1955Q1 to 2007Q4. The end point of the sample period is determined to exclude the ZLB periods.

variables is limited due to DNWR in my model, the ZLB's amplification effects upon a contractionary shock are exclusively absorbed by further contractions of real quantities. In other words, the ZLB significantly amplifies the fluctuations of real quantities without generating large inflation movements in the presence of DNWR. This finding is consistent with but distinct from previous studies such as Gust et al. (2017), who find the responses of both price and real variables are enlarged at the ZLB.

In contrast to my model's results, a recent paper by Amano and Gnocchi (2017) argue that DNWR mitigates the severity of recessions in the model with the ZLB. They claim that this is because declines in real interest rates upon a contractionary demand shock are mitigated due to the smaller declines in inflation expectations in the presence of DNWR. An important difference is that they use a piecewise-linear perturbation method developed by Guerrieri and Iacoviello (2015) to deal with the occasionally binding constraints, whereas I derive a fully non-linear solution using a global method. As Guerrieri and Iacoviello (2015) carefully discuss, the piecewise-linear perturbation method ignores precautionary behavior arising from uncertainty because the approximation remains linear conditional on the probabilities of switching across regimes. Therefore, the method understates the severity of recessions at the ZLB and DNWR. Indeed, Guerrieri and Iacoviello (2015) find that the piecewise-linear perturbation method delivers a smaller drop of consumption upon a contractionary demand shock at the ZLB than that of the non-linear solution.[14] In addition, non-linearity of other equations of the model, including price adjustment costs and disutility of labor, can enlarge a drop of output upon a contractionary shock. The non-linear effects are considered particularly significant upon a large shock such as the Great Recession.

As for implications of DNWR and the ZLB in booms, I find that, since agents take into account the severity of recessions when the economy is constrained at the ZLB, they compress wage increases in booms for precautionary purposes for future downturns. This interaction of DNWR and the ZLB has not been studied in the existing studies, although several studies mentions the forward-looking effect of DNWR in the economy without the ZLB (e.g., Elsby (2009)). This finding is important because it helps the model reconcile the sluggish fluctuations of inflation and large variations of real quantities after the Great Recession.

G.3 Connection to literature on micro evidence for DNWR

A crucial assumption of my model is the DNWR constraint for individual wage settings. Here, I quickly review the literature on the micro evidence for DNWR to assess the validity of the assumption. More comprehensive literature review is found in Basu and House (2016). McLaughlin (1994) is one of the earliest studies to test for the presence of DNWR using individual wage data in the U.S. Though his result was not favorable for DNWR, subsequent studies found evidence of it. For instance, Card and Hyslop (1997), using the individual wage data in the CPS, find a large spike at zero in wage change distribution. Lebow et al. (1995) report asymmetry of wage change distribution arising from DNWR, and Kahn (1997) propose a formal test to verify the asymmetry of distribution. To elaborate their findings, Gottschalk (2005) corrects for a measurement error problem in self-reported data by applying an econometric strategy that detects structural breaks. Barattieri et al. (2014) employ his technique to find that nominal wage reductions correspond only to 12.3 percent of non-zero nominal wage changes in the SIPP during 1996-1999 after correcting for measurement errors.

[14] The importance of incorporating uncertainty at the ZLB is also emphasized by Basu and Bundick (2017) and Nakata (2017), and non-linearity of the economy around the ZLB is studied by Fernández-Villaverde et al. (2015).

One might be concerned about the possibility that benefits such as bonus, pensions, and other supplementary payments are used to adjust the total compensation of workers even if wages are downward rigid. However, evidence is mixed in the literature. Kurmann and McEntarfer (2017) report that, using administrative worker-firm linked data in Washington state, the spike at zero of the changes in the average hourly compensation (sum of wages and benefits) is much smaller than that of wages only, and declines of compensation are not rare in individual workers' data. They claim that their results are less affected by measurement errors than the studies using self-reported data because they use administrative data. On the other hand, Lebow et al. (2003), computing changes of wages and benefits separately in the individual data of the ECI, document that, although benefits change more frequently than wages, changes of benefits are not systematically related to wage changes. Based on these observations, they conclude that the hypothesis that benefits are used to offset nominal wage rigidities is not supported in the data.[15] Moreover, Gu and Prasad (2018) find that benefits come to more rigid over time, due to the increases of quasi-fixed components such as health insurance and defined contributions (IRAs, 401k, etc.). They report that the increased rigidity of benefits made the total compensation countercyclical especially after the Great Recession.

Another interesting finding in the empirical literature is international differences in the degree of DNWR, though this paper exclusively focuses on the inflation dynamics in the U.S.[16] For instance, Smith (2000) investigates the weekly average compensation in the U.K. during 1991-1996 to report that only 1 percent of workers are constrained by DNWR after correcting measurement errors and long-term contracts. Elsby et al. (2016) report that the degree of DNWR in the U.K. is weaker than the U.S. using a longer time-series of data and argue that the downward flexibility of nominal wages in the U.K. resulted in the relatively rapid adjustment of real wage after the Great Recession. On the other hand, a sequence of studies by Kuroda and Yamamoto (2003) and Kuroda and Yamamoto (2014) documents that, after the financial crisis of the late 1990s in Japan, DNWR disappeared from the

[15] It should be noted that the unit of observation of the ECI is jobs instead of workers. How much their empirical results are affected by the difference in the unit of observations would be a subject of future research.

[16] In terms of wage rigidities in general, Dickens et al. (2007) find, in the International Wage Flexibility Project, the degree of nominal and real wage rigidity significantly varies across countries.

individual wage data in Japan although it was present until the mid 1990s. The consequence of these cross-country variations in the degree of DNWR for the inflation dynamics of each country would be an interesting research question, although it is beyond the scope of this paper.

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