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Time-varying local projections with stochastic volatility

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Abstract

This study discusses a general approach to dynamic modeling using the local projection (LP) method. Previous studies have proposed time-varying (TV) parameters in LPs; however, they did not address possible variations in error variances. Overlooking this could introduce significant bias in the estimate of the TV parameter, and consequently, the estimated impulse response. We develop an estimation strategy for LPs with stochastic volatility (SV) and illustrate the importance of SV inclusion using simulated data. Application to a topical macroeconomic time-series analysis illustrates the benefits of the proposed approach in terms of improved predictions.

JEL Classification: C15, C22, C53

Keywords: Local projections; Time-varying parameters; Stochastic volatility.

1 Introduction

Local projection (LP) method, proposed by Jordà (2005), has become a standard framework for estimating impulse response (IR) functions. This method regresses a variable of interest at a future time point onto a structural shock and covariates to capture the effects of the shock on the target variable. Unlike the traditional vector autoregression (VAR) framework, which can produce IRs susceptible to model misspecification, the LP approach offers practical advantages,

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including robustness to misspecification and the feasibility of incorporating high-dimensional covariates and nonlinearity.

Significant progress has been made in recent years regarding the LP methodology. The practical validity and robustness of the LP have been extensively studied by Plagborg-Møller and Wolf (2021), Montiel Olea and Plagborg-Møller (2021), and Li et al. (2024). The methodology is closely linked to traditional econometric techniques such as instrumental variables (Ramey 2016; Ramey and Zubairy 2018), inverse propensity score (Jordà and Taylor 2016; Angrist et al. 2018), spline-based smoothed IRs (Barnichon and Brownlees 2019), and Bayesian inference (Tanaka 2020a,b; Ferreira et al. 2023; Brugnolini et al. 2024). The application of this methodology to empirical analysis has expanded across macroeconomic literature, particularly in assessing fiscal and monetary policy effectiveness (e.g., Jordà and Taylor 2016; Ramey 2016; Tenreyro and Thwaites 2016; Miranda-Agrippino and Ricco 2021) and other relevant topics (see Jordà 2023; Jordà and Taylor 2025; Inoue et al. 2025, for a comprehensive survey).

Several studies have extended the LP approach to address the time variation in IRs. Ruisi (2019) allows the coefficients associated with the explanatory variable to follow a random-walk process and estimates the model using a Bayesian methodology. Inoue et al. (2024) propose a more general form of instability in the IRs, using the path estimator developed by Müller and Petalas (2010), where the time variation can manifest as structural breaks, piecewise constant paths, and more complicated forms. Cloyne et al. (2023) employ the Kitagawa-Blinder-Oaxaca decomposition in the LP context to develop a new framework for the time-varying (TV) IRs.

However, few studies have considered time variation in error variance along with TV coefficients. In the context of the VAR and its TV framework (Cogley and Sargent 2001), Sims (2001) and Stock (2001) indicate that ignoring the possible variation in the variance could lead to bias in the drifting coefficients. They argue that coefficient estimates adjust to compensate for the misspecification of the variance, thereby exaggerating the time variation in the coefficients. By incorporating the TV variance, Cogley and Sargent (2005) and Primiceri (2005) develop the TV parameter (TVP) VAR model with stochastic volatility (SV), which has become a standard tool in dynamic modeling strategy for macroeconomic empirical analysis.

This study highlights the shortcomings of the misspecified TV-LP that neglects the TV variance and reveals the importance of including the TV variance. The analysis proposes a TV-LP model with SV, consistent with the discussion on TVP-VARs. It also develops an efficient estimation algorithm using a Bayesian method. A simulation study and its application to topical macroeconomic data provide crucial evidence that overlooking TV variance deteriorates the model's predictive ability across short to long horizons, preventing accurate estimation of the IR function. In a closely related study, Lusompa (2020) formulates and estimates the TV-LP with SV but does not include any data analysis using the developed strategy. By contrast, this study illustrates the application of the modeling and estimation framework of TV-LP with SV.

The remainder of the paper is organized as follows. Section 2 outlines the class of TV-LP, defines a new approach that includes SV, and discusses the computational strategy for model fitting. Using simulated data, Section 3 presents an extensive and detailed assessment of the advantages of TV variance inclusion in the context. Section 4 presents a real-data study of a topical macroeconomic issue on the relationship between the global economy and trade activity. Finally, Section 5 summarizes the study.

2 Methodology

2.1 General model form

Suppose $\{y_t, z_{it}; i = 1, \ldots, p; t = 1, 2, \ldots\}$ are time series, and we estimate the IRs of $\{y_{t+h}; h = 1, 2, \ldots\}$ in response to a unit change in a specific variable, assuming z_{1t}, without loss of generality. The standard LP method proposed by Jordà (2005) utilizes a simple regression:

$$y_{t+h} = \alpha_{1(h)} z_{1t} + \cdots + \alpha_{p(h)} z_{pt} + u_{(h)t+h},$$

where $\mathrm{Var}\left(u_{(h)t+h}\right) = \sigma^2_{(h)}$. The advantage of LP is that it requires estimating only the h-step-ahead-specific univariate regression using the least squares method to obtain the IRs by aligning the estimated coefficients, $\{\hat{\alpha}_{1(1)}, \hat{\alpha}_{1(2)}, \ldots\}$, which correspond to z_{1t}, the variable of interest. This approach avoids the estimation of multivariate regressions such as VARs and allows for the incorporation of many control variables $(z_{2t}, \ldots, z_{pt})$, which can be nonlinear if necessary.

Ruisi (2019) introduces TV coefficients into the LP framework. That is,

$$y_{t+h} = \alpha_{1(h)t} z_{1t} + \cdots + \alpha_{p(h)t} z_{pt} + u_{(h)t+h},$$

where the coefficients $\alpha_{i(h)t}$ $(i = 1, \ldots, p)$ now include the subscript t, assuming their dynamic property of the coefficient. This evolution accounts for the potential time variations in the model's structure and in the multipliers of y_{t+h}, thereby representing TV IRs $\{\alpha_{1(h)t}; h = 1, 2, \ldots; t = 1, 2, \ldots\}$.

A general form of the LP with TV coefficients includes variables that are not associated with the TV coefficients but are time invariant. This is expressed as follows:

$$y_{t+h} = \boldsymbol{\alpha}'_{(h)t}\boldsymbol{z}_t + \boldsymbol{\beta}'_{(h)}\boldsymbol{x}_t + u_{(h)t+h},$$

where $\boldsymbol{\alpha}_{(h)t} = (\alpha_{1(h)t}, \ldots, \alpha_{p(h)t})'$, $\boldsymbol{z}_t = (z_{1t}, \ldots, z_{pt})'$, $\boldsymbol{\beta}_{(h)} = (\beta_{1(h)}, \ldots, \alpha_{k(h)})'$, $\boldsymbol{x}_t = (x_{1t}, \ldots, x_{kt})'$. Note that $\boldsymbol{x}_t$ denotes the variables associated with time-invariant coefficients $\boldsymbol{\beta}_{(h)}$.

A critical limitation of the formulation by Ruisi (2019) is the assumption that the variance in the prediction error is time-invariant. As aforementioned, this assumption may introduce bias in the estimates of TV coefficients in the TVP-VAR context. Lusompa (2020) introduces a TV error variance in the TV-LP framework by employing a discount TV variance model for $\mathrm{Var}\left(u_{(h)t+h}\right)$. Conversely, this study utilizes a standard framework of SV, which has been extensively used in the macroeconomic literature, and the TVP-VAR framework (Shephard 2005; Primiceri 2005).

Therefore, the general form of a TV-LP with SV is expressed as follows:

$$y_{t+h} = \boldsymbol{\alpha}'_{(h)t}\boldsymbol{z}_t + \boldsymbol{\beta}'_{(h)}\boldsymbol{x}_t + u_{(h)t+h}, \quad u_{(h)t+h} \sim N(0, \sigma^2_{(h)t+h}), \tag{1}$$

where the error variance $\sigma^2_{(h)t+h}$ has subscript $t+h$. Following Ruisi (2019) and other studies, the TV coefficients are assumed to follow a first-order random walk process:

$$\boldsymbol{\alpha}_{(h)t+1} = \boldsymbol{\alpha}_{(h)t} + \boldsymbol{\xi}_{(h)t}, \quad \boldsymbol{\xi}_{(h)t} \sim N(\boldsymbol{0}, \boldsymbol{V}_{(h)}), \tag{2}$$

with the matrix $\boldsymbol{V}_{(h)} = \mathrm{diag}(v^2_{(h)1}, \ldots, v^2_{(h)p})$ diagonal. Define $\gamma_{(h)t} = \log(\sigma^2_{(h)t})$. The dynamics of the TV variances are specified as:

$$\gamma_{(h)t+1} = \gamma_{(h)t} + \eta_{(h)t}, \quad \eta_{(h)t} \sim N(0, w^2_{(h)}). \tag{3}$$

Thus, this formulation of log variances $\gamma_{(h)t}$ defines traditional SV models, which are originally used in financial econometrics (Aguilar and West 2000; Shephard 2005; Omori et al. 2007; Prado et al. 2021, chapter 7) and widely applied in macroeconomics context (Uhlig 1997; Cogley and Sargent 2005; Primiceri 2005). All disturbances $u_{(h)t}$, $\boldsymbol{\xi}_{(h)t} = (\xi_{(h)1t}, \ldots, \xi_{(h)pt})'$, and $\eta_{(h)t}$ are assumed to be mutually independent. We refer to the formulation of equations (1)–(3) as TV-LP-SV.

Notably, other forms of the underlying dynamics for TV coefficients and variances may be defined. As discussed by Jordà (2005) and others, the inclusion of nonlinearity and instrumental variables is straightforward in the TV-LP-SV context. Since the LPs are not generative models, we assume a parametric error distribution and simply treat the estimation equation as the linear regression model with the TV parameters. The estimation algorithm discussed below can easily be arranged for these additional/different components in the baseline formulation of equations (1)–(3).

2.2 Estimation method

To estimate under the proposed LP framework, we use the Markov chain Monte Carlo (MCMC) methods, employing traditional sampling schemes for a linear state space model or dynamic regression model (e.g., West and Harrison 1997; Chan et al. 2019; Ferreira et al. 2023). The standard LP framework and the assumption of independence in disturbances yield an estimation separable across horizon h; hence, we only develop a generic MCMC procedure to simulate the h-specific full joint posterior. For notational simplicity, we omit the subscript (h) in the following description. Based on observations $\boldsymbol{y}_{1:n} = \{\boldsymbol{y}_t\}_{t=1}^n$, the MCMC samplers for the TV coefficients, $\boldsymbol{\alpha} = \{\boldsymbol{\alpha}_t\}_{t=1}^n$, the SV, $\boldsymbol{\gamma} = \{\gamma_t\}_{t=1}^n$, and the parameters $\boldsymbol{\Theta} = (\boldsymbol{\beta}, \boldsymbol{v}, w)$, where $\boldsymbol{v} = (v_1, \ldots, v_p)$ are utilized to obtain samples from the joint posterior $\pi(\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\Theta}|\boldsymbol{y})$. We discuss the components of the MCMC methods in this section and provide details of the samplers in Appendix A.[1]

First, sampling the model parameters $\boldsymbol{\Theta}$ conditional on the state variables and other parameters is straightforward. Based on traditional priors, we implement sampling from standard distributions such as multivariate normal and inverse gamma distributions. Second, an efficient and promising sampling method for the TV coefficients $\boldsymbol{\alpha}$ is available: a Kalman filter and simulation smoother, or forward-filtering backward-sampling (FFBS) algorithm (e.g., Frühwirth-Schnatter 1994; de Jong and Shephard 1995; Ferreira et al. 2023), which generates the full conditional posterior of $\pi(\boldsymbol{\alpha}|\boldsymbol{y}, \gamma, \boldsymbol{\Theta})$. Third, the MCMC algorithm is completed by sampling the SV $\boldsymbol{\gamma}$: an efficient block sampling method for univariate SV models can be adopted (Shephard and Pitt 1997; Watanabe and Omori 2004).

[1] The code is available on the author's website. https://sites.google.com/site/jnakajimaweb/tvlp

As reported in existing studies employing Bayesian computations for similar models (e.g., Nakajima 2011), the performance of the MCMC algorithm summarized in this study is reasonable, with good mixing and low autocorrelation of the samples in the simulation and real data analyses.

Note that this discussion also applies to the TV-LP framework in a panel data setting by direct extension. The aforementioned estimation methodology can be easily extended to the case of panel-based TV-LP-SV, as expressed below:

$$y_{i,t+h} = \boldsymbol{\alpha}'_{(h)t}\boldsymbol{z}_{it} + \boldsymbol{\beta}'_{(h)}\boldsymbol{x}_{it} + \boldsymbol{\delta}'_{i(h)}\boldsymbol{g}_{it} + u_{i(h)t+h}, \quad u_{i(h)t+h} \sim N(0, \sigma^2_{i(h)t+h}), \tag{4}$$

where the subscript i denotes each individual, $i = 1, \ldots, I$. In this framework, the TV and time-invariant coefficients, $\boldsymbol{\alpha}_{(h)t}$ and $\boldsymbol{\beta}_{(h)}$, measure the common sensitivity of the dependent variable to the predictors, $\boldsymbol{z}_{it}$ and $\boldsymbol{x}_{it}$, respectively. The newly introduced, time-invariant coefficient $\boldsymbol{\delta}_{i(h)} = (\delta_{i1(h)}, \ldots, \delta_{iq(h)})'$ measures individual-specific sensitivity to the predictor $\boldsymbol{g}_{it} = (g_{i1t}, \ldots, g_{iqt})'$. When $g_{ijt} = 1$ for all i and t, the corresponding coefficient $\delta_{ij(h)}$ measures the individual fixed effects for each $i = 1, \ldots, I$.

Assume that the TV coefficients follow the process in equation (2) and SV follows the individual-specific process in equation (3) with $\gamma_{i(h)t} = \log(\sigma^2_{i(h)t})$; that is,

$$\gamma_{i(h)t+1} = \gamma_{i(h)t} + \eta_{i(h)t}, \quad \eta_{i(h)t} \sim N(0, w^2_{i(h)}). \tag{5}$$

Assuming the independence of error distributions across individuals, the MCMC algorithm is easily extended to the panel-based case.

3 Simulation example

This section illustrates the advantage of TV-LP-SV over the time-invariant variance, TV-LP-CV (constant variance), using the simulated data.

A sample size of $n = 100 + H$ is drawn from a one-period-ahead ($h = 1$) TV-LP-SV formulation of equations (1)–(3), with $p = 2$ and $k = 0$ (i.e., omitting the time-invariant coefficients, as they are irrelevant for this simulation study). The two predictors, $\boldsymbol{z}_t = (z_{1t}, z_{2t})'$ are generated from the following autoregressive process that imitates the dynamics of the business cycle:

$$\begin{aligned} z_{1t} &= 1.2z_{1,t-1} - 0.3z_{1,t-2} + \epsilon_{1t}, \\ z_{2t} &= 0.7z_{2,t-1} + 0.1z_{2,t-2} + \epsilon_{2t}, \end{aligned}$$

where $\epsilon_{jt} \sim N(0, 0.1^2)$ for $i = 1, 2$. We take the parameters $v_1 = v_2 = 0.1$ and generate the TV coefficients $\boldsymbol{\beta}_t$ based on equation (2). We also generate SV $\boldsymbol{\gamma}$ based on equation (3), where we examine two cases of variance for the SV process, $w = 0.1$ and 0.5.

The following prior distributions are used: $\boldsymbol{\beta} \sim N(\mathbf{0}, 10\boldsymbol{I})$, $v_i^2 \sim IG(20, 2)$, and $w^2 \sim IG(20, 2)$, where $\boldsymbol{I}$ denotes an identity matrix. We specify the distribution of the initial state process: $\boldsymbol{\alpha}_1 \sim N(\mathbf{0}, 10\boldsymbol{I})$ and $\gamma_1 \sim N(3, 2)$. For the TV-LP-CV, we assume $\sigma^2 \sim IG(10, 2)$. We run the MCMC sequences for 20,000 iterates after a burn-in of 2,000.

We focus on the predictive ability of the TV-LP formulation for horizons $h = 1, \ldots, H$, where we consider $H = 8$ in this analysis. This study uses a realistic recursive out-of-sample forecasting format, beginning with a dataset spanning the first $T_1 = 80$ observations. We estimate the TV-LP formulation and obtain forecasts for $(y_{T_1+1}, \ldots, y_{T_1+H})$. Subsequently, we update the dataset with one additional observation at $T_2 = T_1 + 1$ and refit the TV-LP to obtain forecasts up to $T_2 + H$. This analysis is repeated up to $T_J = 100$ until $J = 20$ sets of forecast bundles. The entire simulation is implemented for $N = 100$ different simulated datasets.

To assess the utility of the proposed framework, forecasts are expressed as a posterior predictive distribution. In the MCMC sampling steps, we generate future values of y_t, that is, $(y_{T_j+1}, \ldots, y_{T_j+H})$ on the dataset of $\boldsymbol{y}_{1:T_j}$ based on their conditional posterior predictive distribution. The results are summarized using the log predictive density ratio (LPDR) for forecasting h-period ahead of time T_j, as given by

$$\mathrm{LPDR}_{T_j}(h) = \log \frac{p_{\mathrm{SV}}(\boldsymbol{y}_{T_j+h}|\boldsymbol{y}_{1:T_j})}{p_{\mathrm{CV}}(\boldsymbol{y}_{T_j+h}|\boldsymbol{y}_{1:T_j})},$$

where $p_{\mathrm{M}}(\boldsymbol{y}_{T+h}|\boldsymbol{y}_{1:T})$ represents the predictive density under formulation M. The LPDR evaluates the relative forecasting accuracy of TV-LP-SV over the baseline specification, TV-LP-CV. Unlike the standard measure of root mean squared errors, which evaluates point forecasts from competing models, LPDR assesses the uncertainty in predictions between the models and point forecasts, as noted by Nakajima and West (2013) and McAlinn et al. (2020).

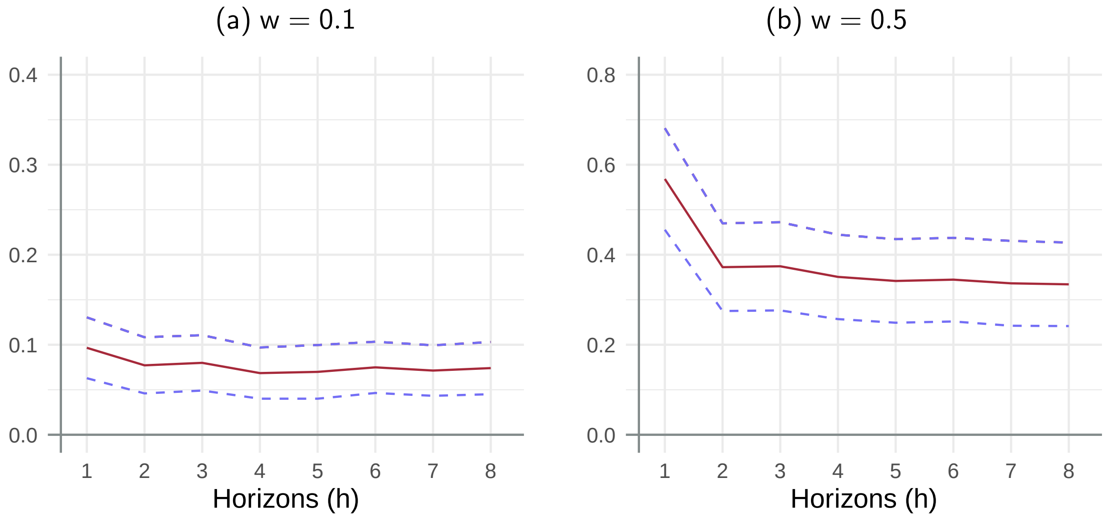


Figure 1: Simulation study: Log predictive density ratio (LPDR, solid) of the TV-LP-SV over TV-LP-CV (constant variance) for forecasting h-period ahead under the different degrees of the variance of SV, $w = 0.1$ (left) and 0.5 (right), with 95% confidence intervals (dashed).

Figure 1 shows the average of $\frac{1}{J}\sum_{j=1}^{J} \mathrm{LPDR}_{T_j}(h)$ across $N = 100$ simulated datasets. The LPDRs are all positive, indicating that TV-LP-SV dominates TV-LP-CV, with higher predictive performance for all h for both $w = 0.1$ and 0.5. While the density ratios are highest at $h = 1$ and decrease as h increases in this simulation settings, SV also plays a dominant role even in the longer-term horizons. The higher variation in SV, $w = 0.5$, yields a higher density ratio compared to $w = 0.1$. The shortcoming of ignoring time variation in the variance increases as the SV widens during the true process. A formal assessment of the LPDR is provided through a classical statistical test for $\frac{1}{J}\sum_{j=1}^{J} \mathrm{LPDR}_{T_j}(h) > 0$. The resulting test statistics indicate that the null hypothesis $\frac{1}{J}\sum_{j=1}^{J} \mathrm{LPDR}_{T_j}(h) = 0$ is rejected, with a significance level $p = 0.01$ for all the computed LPDR values in the figure.

4 Application: A study of Japan's international trade volumes

We use the TV-LP-SV framework to analyze Japan's international trade volumes, demonstrating that the SV component significantly improves multi-step, out-of-sample predictions. We find strong, significant evidence for time variation in the resulting IR of trade volumes to changes in the global economic growth. These results are particularly relevant for policy applications of the TV-LP framework.

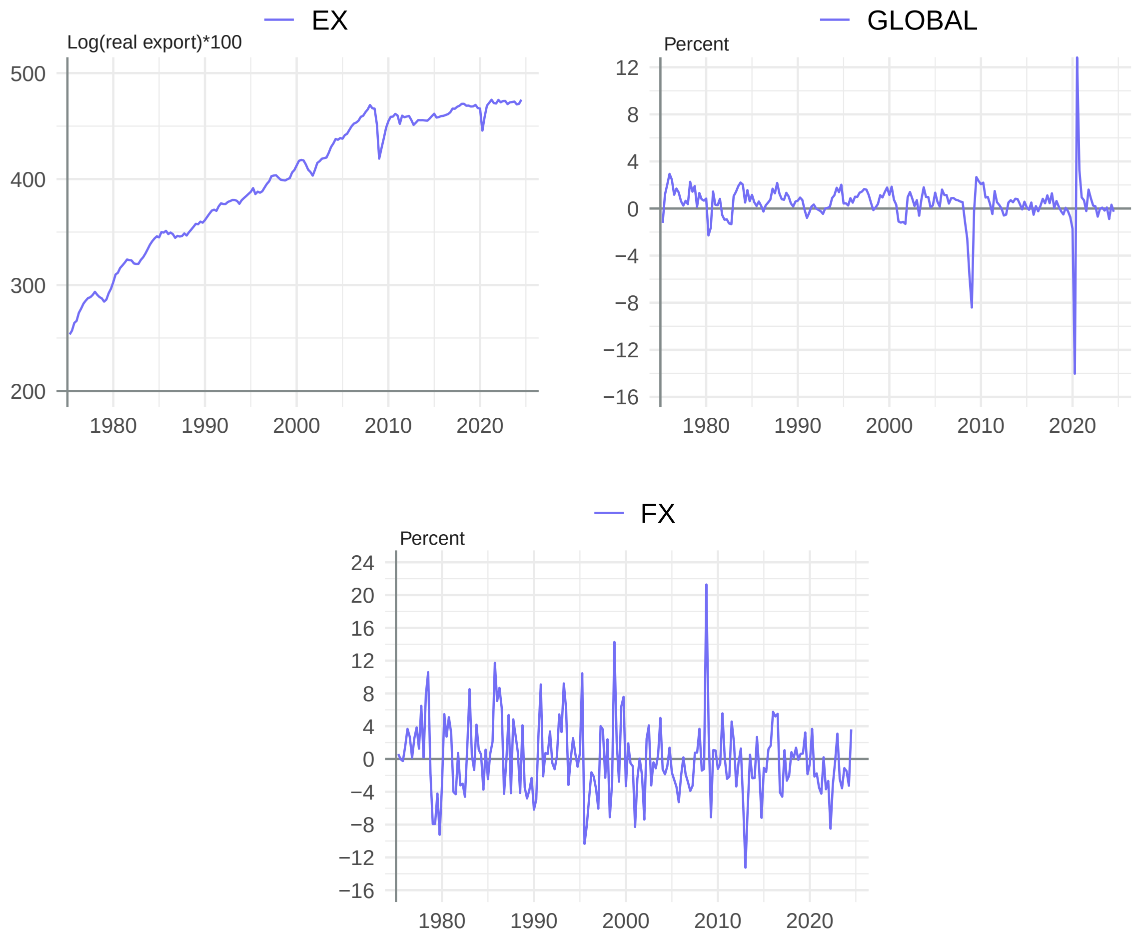


Figure 2: Data: Real export volumes (EX, logarithm of indices $\times 100$), the OECD's global economic business cycle index (GLOBAL, %), and real effective exchange rate of Japanese Yen (FX, %). GLOBAL and FX are quarter-on-quarter changes.

4.1 Data and settings

We analyze the quarterly time series of Japan's real trade volumes (EX) as a dependent variable and the OECD's global economic business cycle index (GLOBAL) and real effective exchange rate of Japanese Yen (FX) as predictors, in a context of topical interest (e.g., Bräuning and Sheremirov 2023). Appendix B provides additional details on the dataset. The dataset spans from 1975/Q2 to 2024/Q3 (Figure 2). Trade volumes deteriorated significantly during the global financial crisis (GFC) and the COVID-19 pandemic. We note a small decline owing to a specific domestic factor after the massive earthquake, the Great East Japan Earthquake in March 2011. We explicitly consider this event in our application study.

The analysis uses the following LP formulation:

$$\begin{aligned}
\mathrm{EX}_{t+h} &= \mathrm{EX}_t + \alpha_{1(h)t}\mathrm{GLOBAL}_{t-1} + \alpha_{2(h)t}\mathrm{FX}_{t-1} \\
&\quad + \alpha_{3(h)t}\Delta\mathrm{EX}_t + \alpha_{4(h)t} \\
&\quad + \beta_{1(h)}\mathrm{D}_{t+h} + \beta_{2(h)}\mathrm{D}_t + u_{(h)t+h},
\end{aligned}$$

where $\Delta\mathrm{EX}_t = \mathrm{EX}_t - \mathrm{EX}_{t-1}$. We take the lags of GLOBAL and FX as well as the lag of recent growth in EX by $\Delta\mathrm{EX}_t$. $\alpha_{4(h)t}$ is the TV intercept. D_t represents a dummy variable that equals one when the time t corresponds to the first quarter of 2011, when the massive earthquake occurred, and zero otherwise. Its coefficients $\beta_{i(h)}$ $(i = 1, 2)$, which are time-invariant, measure the changes in export volumes due to the destruction of domestic industrial production and other aspects after the earthquake.

Our empirical focus is on the sensitivity of trade volumes to the strength of the global economy and its time variations. In the above formulation, the sensitivity is measured by the TV coefficient $\alpha_{1(h)t}$, where we regress GLOBAL onto the change in EX over h quarters from the current period, $\mathrm{EX}_{t+h} - \mathrm{EX}_t$.

For all model analyses reported in this section, we use the same prior specifications and the simulation size as described in Section 3.

4.2 Out-of-sample forecasting performance

We summarize the out-of-sample predictive performances to compare the TV-LP-SV and TV-LP-CV frameworks. The same recursive forecasting format as in Section 3 is used to obtain $J = 40$ forecasts for each horizon, $h = 1, \ldots, H = 8$. The final forecast is based on the last observation (2024/Q3) in the dataset.

Figure 3 shows the log predictive density of those two models and $\frac{1}{J}\sum_{j=1}^{J} \mathrm{LPDR}_{T_j}(h)$, which evaluates the relative prediction performance of TV-LP-SV compared to TV-LP-CV. Evidently, the LPDRs are all positive, indicating that SV plays a crucial role in forecasting trade volumes over one to eight quarters.

Figure 4 shows the posterior means of the SV, $\sigma_{(h)t} = \exp(\gamma_{(h)t}/2)$ for $h = 2, 4, 6$, and 8, along with their 68% credible intervals. The figure shows several volatile periods in export volumes, particularly during the GFC. The deviation of the variance in TV-LP-SV from the time-

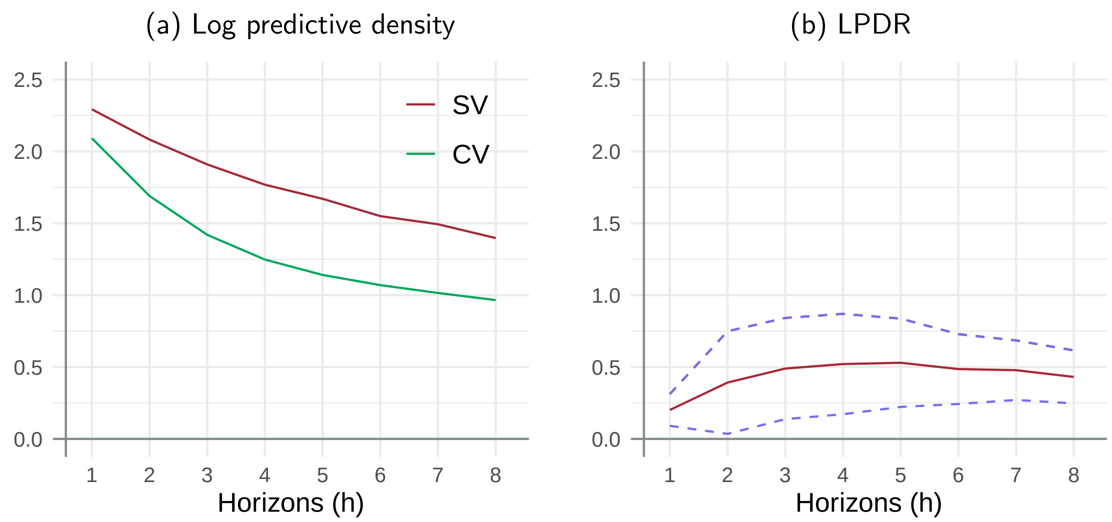


Figure 3: Performance of forecasting the trade volumes over h-quarter ahead. Left panel: Log predictive density of the TV-LP-SV and TV-LP-CV. Right panel: Log predictive density ratio (LPDR, solid) with 95% confidence intervals (dashed).

invariant variance in TV-LP-CV, depicted by the horizontal dashed line in the figure, influences the predictive performance of the frameworks, as observed in the forecasting analysis.

4.3 Time-varying impulse responses

Figure 5 shows the posterior means and credible intervals of the TV IR for the selected horizons, $h = 1, 2, 3, 4, 6$, and 8, estimated in the TV-LP-SV and TV-LP-CV frameworks. The TV-LP-SV framework shows clear evidence of time variations in the sensitivity of trade volumes to changes in global economic growth. The response generally increased from the 1980s and 1990s to the 2000s and 2010s and then declined in recent periods. Evidently, the shape of the IR changed over time, as observed in Figure 6. The responses typically lagged in the 1980s and 1990s but became hump-shaped in the 2000s and 2010s.

The figures reveal the dominant differences in IRs between the TV-LP-SV and TV-LP-CV frameworks. The IRs from TV-LP-CV show too much variation in the IR, as seen in Figure 5. In particular, the response from TV-LP-CV is larger than that from TV-LP-SV at horizons $h = 4$ and 6 in the late 2000s and early 2010s. A similar difference is observed in Figure 6(iv). This result underscores the practical relevance of SV in the TV-LP framework in connection with the enhanced predictive ability of TV-LP-SV over TV-LP-CV across short- to long-term horizons, as demonstrated in the forecasting analysis.

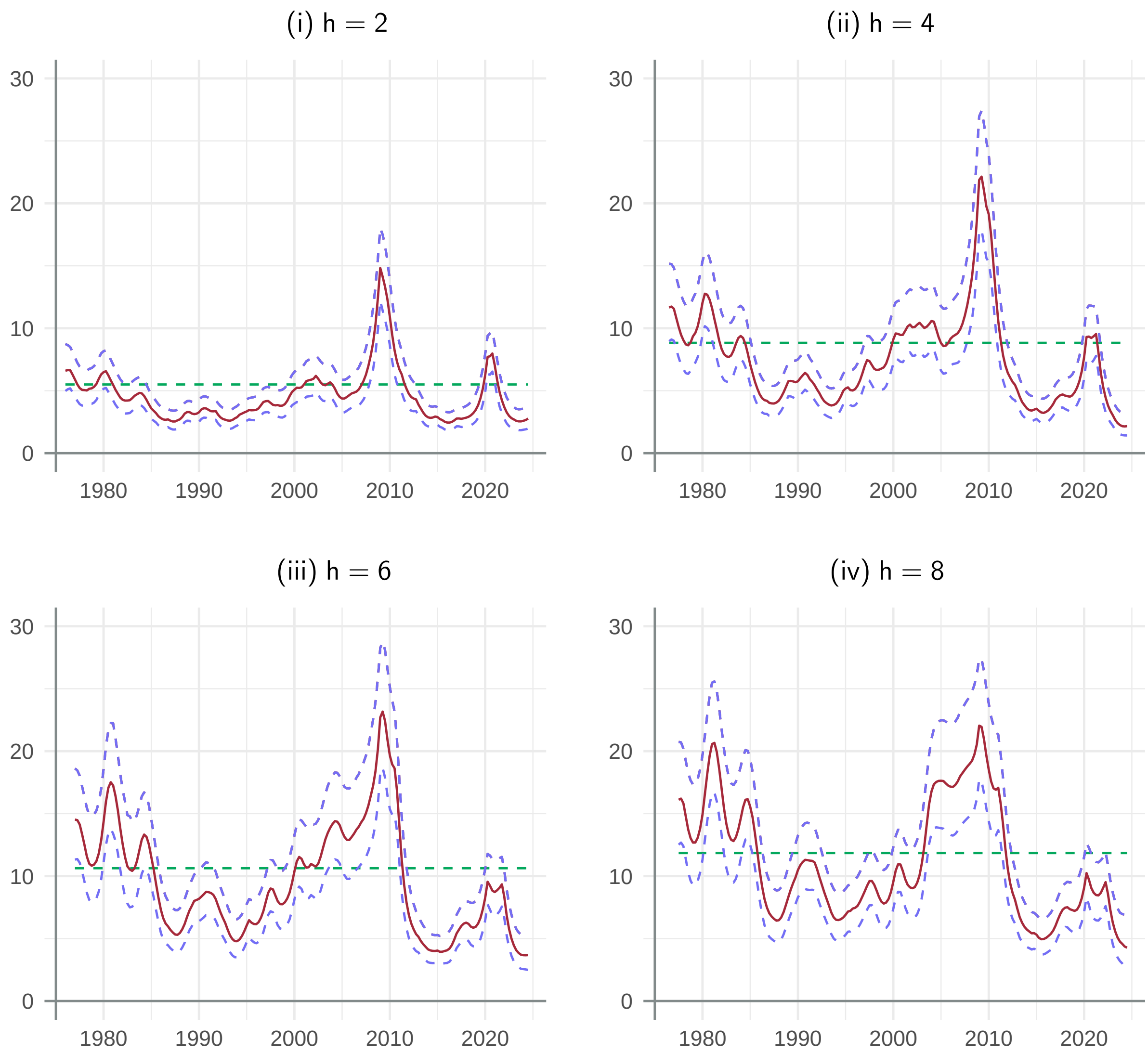


Figure 4: Posterior trajectories of $\sigma_{(h)t} = \exp(\gamma_{(h)t}/2)$ in the TV-LP-SV: posterior means (solid) and 68% credible intervals (dashed) for selected horizons. The horizontal line (dashed) graphs a posterior mean of $\sigma_{(h)}$ in the TV-LP-CV.

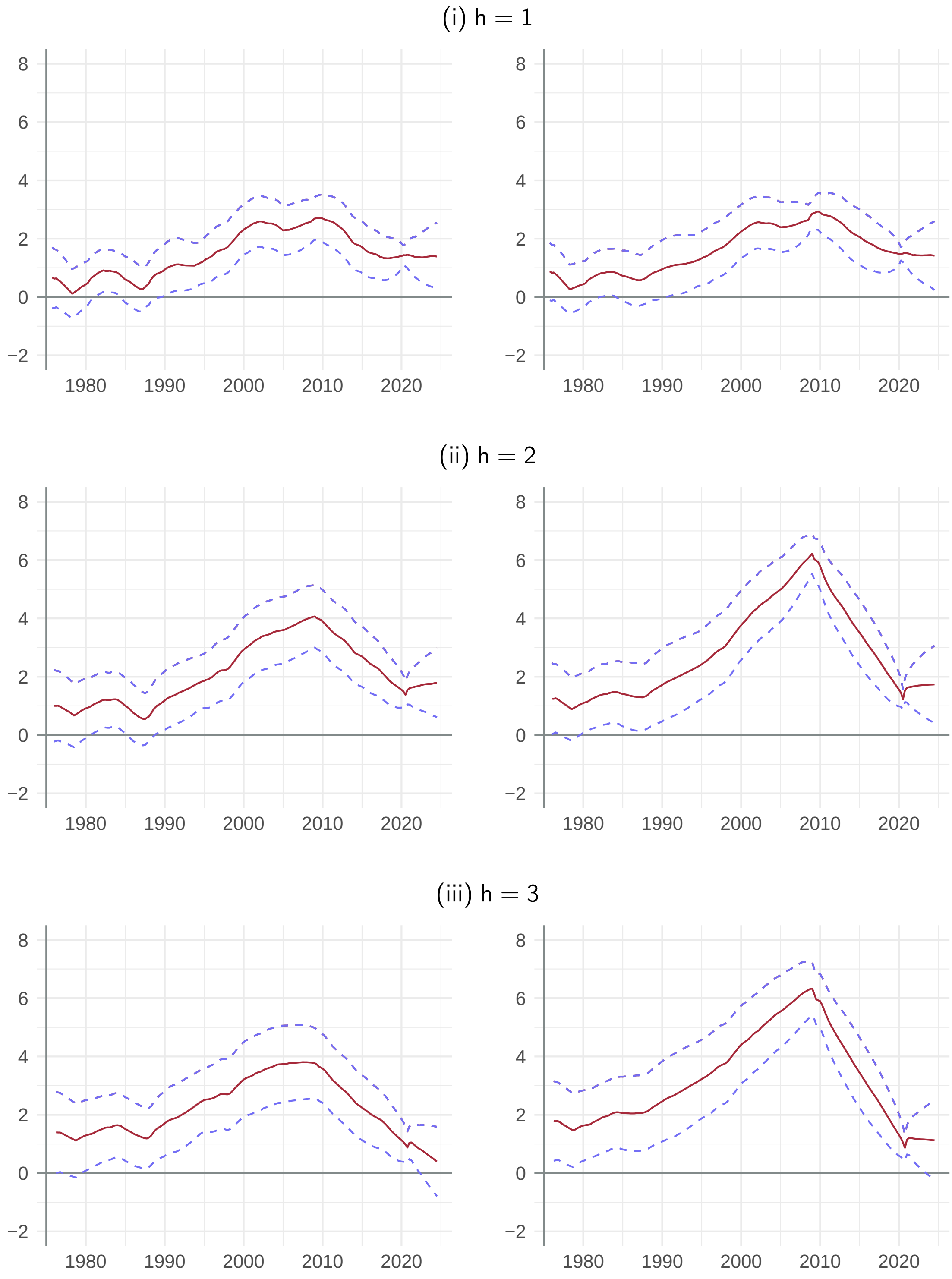
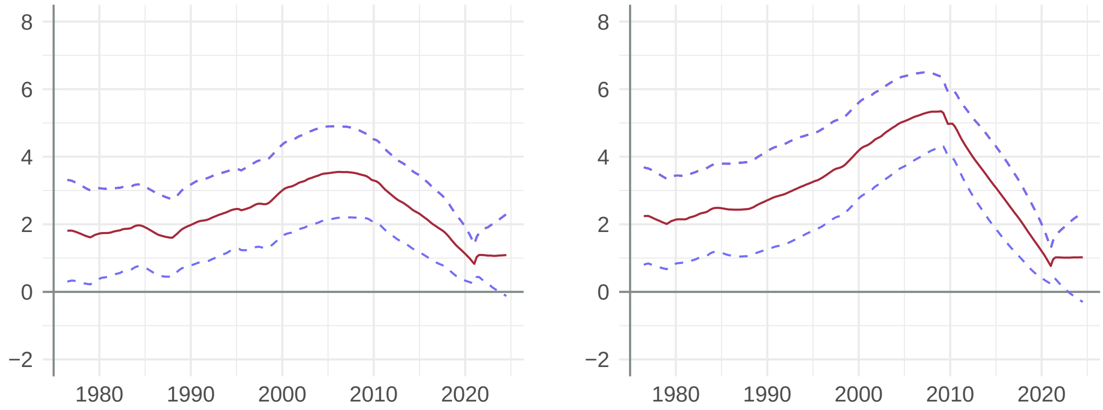
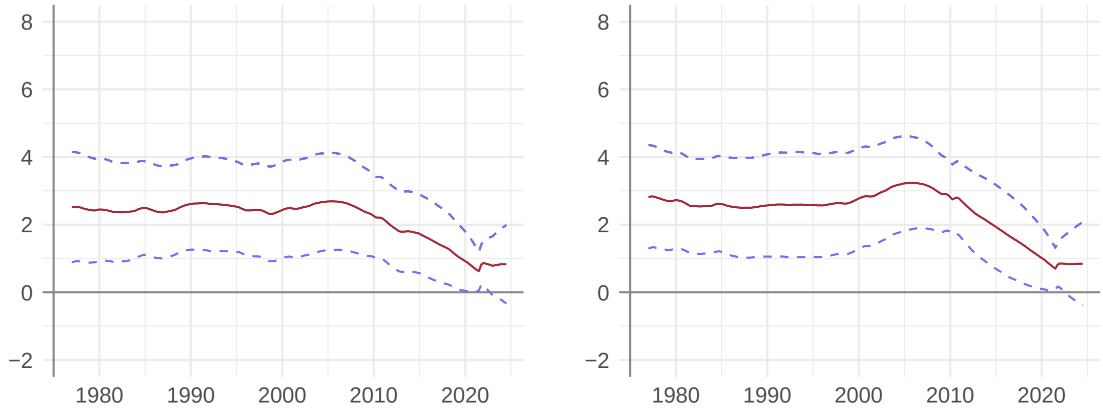


Figure 5: Time-varying impulse response trajectories from the TV-LP-SV (left) and TV-LP-CV (right) frameworks for $h = 1, 2, 3, 4, 6$, and 8: Posterior means (solid) and 68% credible intervals (dashed). The figure indicates the response of export volumes to a 1% change in the global economic growth index.

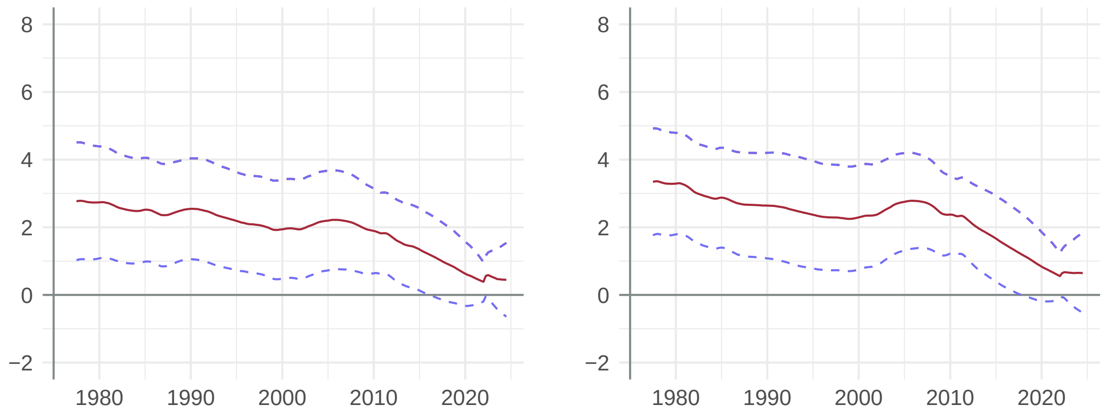
(iv) h = 4



(v) h = 6



(vi) h = 8



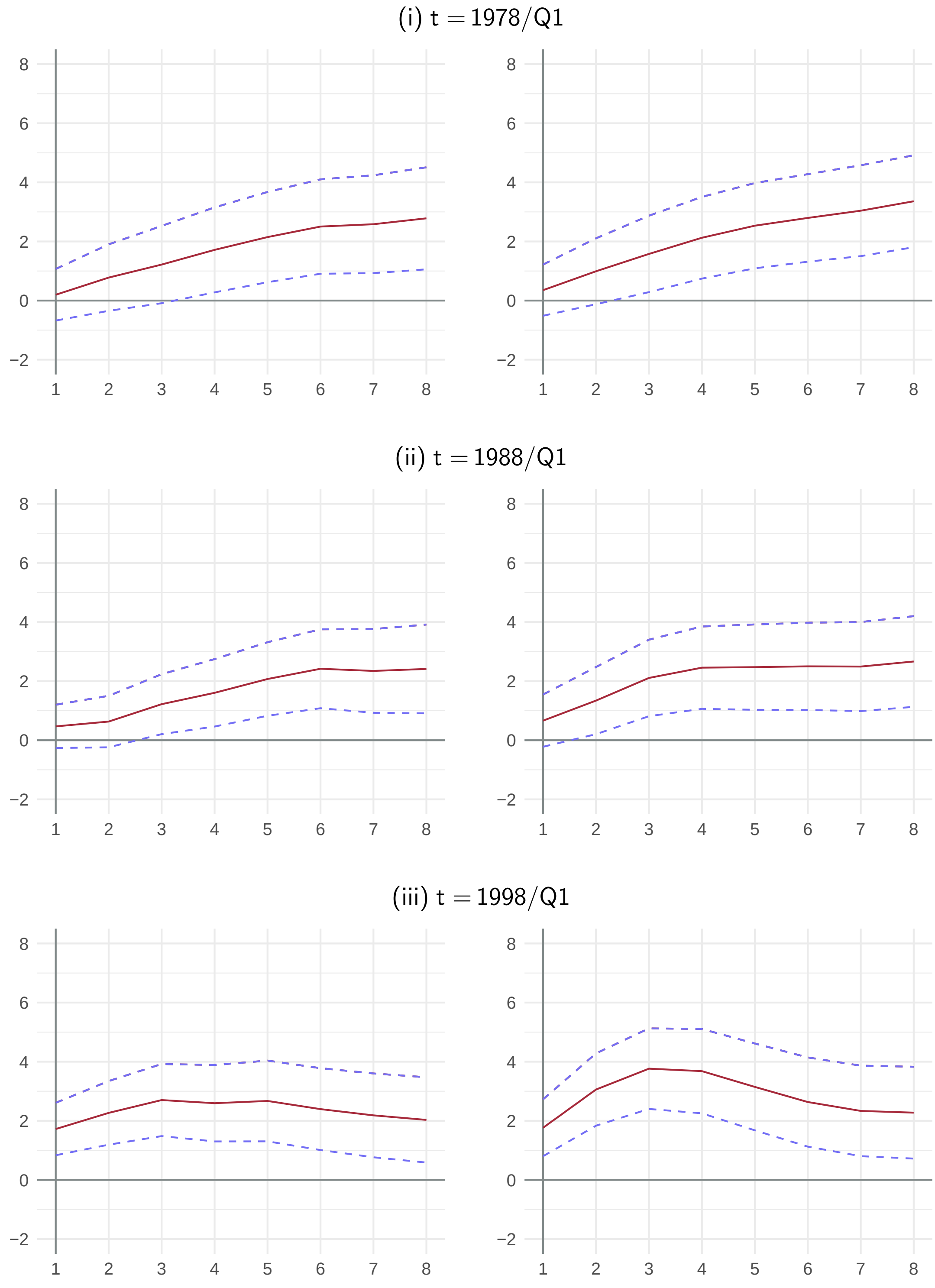
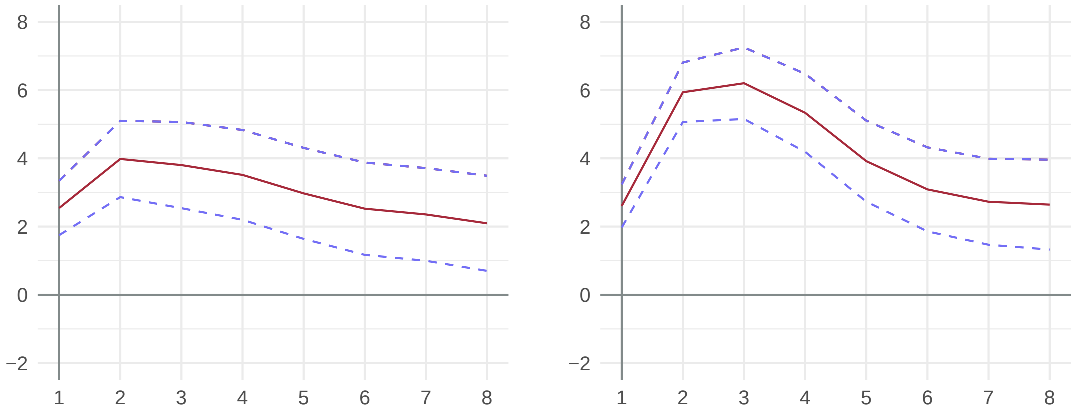
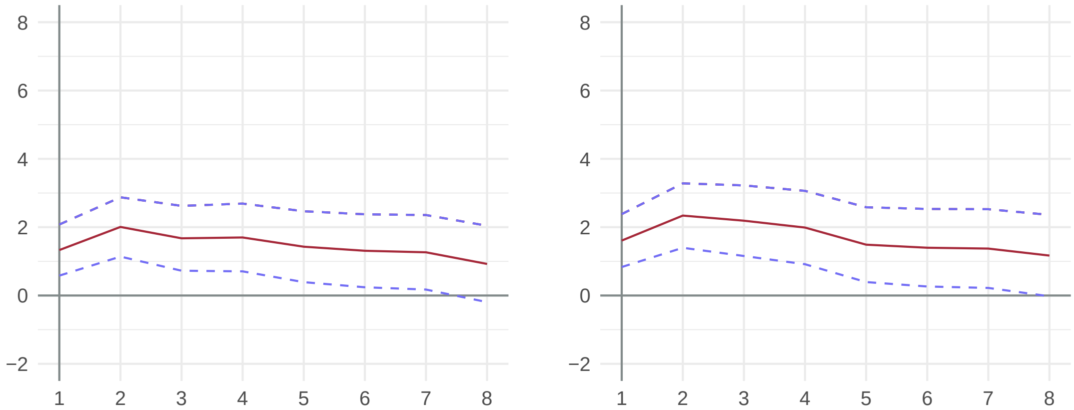


Figure 6: Impulse response trajectories from the TV-LP-SV (left) and TV-LP-CV (right) frameworks at the selected time points: Posterior means (solid) and 68% credible intervals (dashed). The figure indicates the response of export volumes to a 1% change in the global economic growth index.

(iv) $t = 2008/Q1$



(v) $t = 2018/Q1$



5 Conclusion

We introduce the time variation in error variances within the TV-LP framework and analyze its advantages over the time-invariant variance framework using simulated and real data analyses. The proposed approach provides a flexible structure for modeling the TV IR of a variable of interest, yielding higher predictive performance over short- to long-term horizons. Overlooking the time variation in variances can hinder the appropriate estimation of the TV IR in applications.

We highlight the potential of the proposed framework for application to broad real-world time-series analyses in the macroeconomic and financial market contexts. Nonlinearity and instrumental variables, which are standard refined approaches in the general LP framework (e.g., Jordà 2005), can be included in the TV-LP-SV strategy, which is a topic for future research.

Appendix A. MCMC algorithm

In the TV-LP-SV framework defined by equations (1)–(3), we document an MCMC algorithm for simulating the full joint posterior $\pi(\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\Theta}|\boldsymbol{y})$. We omit the subscript (h) in the following description for notational simplicity.

We assume prior forms of the following:

$$\begin{aligned} \pi(\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\Theta}) &= \pi(\boldsymbol{\alpha}) \times \pi(\boldsymbol{\gamma}) \times \pi(\boldsymbol{\Theta}) \\ &= f_N(\boldsymbol{\alpha}|\boldsymbol{\beta}_0, \boldsymbol{\Sigma}_0) \times \prod_{i=1}^{p} f_{IG}(v_i^2|n_{i0}/2, S_{i0}/2) \times f_{IG}(w^2|w_0/2, W_0/2), \end{aligned}$$

where $f_N(\cdot|\boldsymbol{m}, \boldsymbol{M})$ and $f_{IG}(\cdot|c, d)$ denote the probability density functions of the multivariate normal and inverse gamma distributions $N(\boldsymbol{m}, \boldsymbol{M})$ and $IG(c, d)$, respectively.

A.1 Sampling $\boldsymbol{\Theta}$

First, conditional on the state variables $(\boldsymbol{\alpha}, \boldsymbol{\gamma})$, sampling $\boldsymbol{\beta}$ is reduced to a generation from the conditional posterior of a standard linear regression. The conditional posterior distribution is $N(\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\Sigma}})$, where

$$\hat{\boldsymbol{\Sigma}} = \left(\boldsymbol{\Sigma}_0^{-1} + \sum_{t=1}^{n} \frac{\boldsymbol{x}_t \boldsymbol{x}_t'}{\sigma_t^2} \right)^{-1}, \qquad \hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\Sigma}} \left(\boldsymbol{\Sigma}_0^{-1} \boldsymbol{\beta}_0 + \sum_{t=1}^{n} \frac{\boldsymbol{x}_t \hat{y}_t}{\sigma_t^2} \right),$$

with $\hat{y}_t = y_t - \boldsymbol{\alpha}_t' \boldsymbol{z}_t$, $t = 1, \ldots, n$.

Second, conditional on state variable $\boldsymbol{\alpha}$, the posterior density of v_i^2 equals $f_{IG}(v_i^2 | \hat{n}_i/2, \hat{S}_i/2)$, where

$$\hat{n}_i = n_{i0} + n - 1, \qquad \hat{S}_i = S_{i0} + \sum_{t=1}^{n-1} (\alpha_{i,t+1} - \alpha_{it})^2,$$

for $i = 1, \ldots, p$. We generate v_i independent of $i = 1, \ldots, p$.

Third, conditional on the SV $\boldsymbol{\gamma}$, the posterior density of w^2 is $f_{IG}(w^2 | \hat{w}/2, \hat{W}/2)$, where

$$\hat{w} = w_0 + n - 1, \qquad \hat{W} = W_0 + \sum_{t=1}^{n-1} (\gamma_{t+1} - \gamma_t)^2.$$

A.2 Sampling $\boldsymbol{\alpha}$

Conditional on the SV and model parameters $(\boldsymbol{\beta}, \boldsymbol{v})$, equations (1)–(2) form a linear Gaussian state-space model or a dynamic linear model with known variances. We utilize the method of Kalman filter and simulation smoother, or FFBS algorithm (see e.g., Frühwirth-Schnatter 1994; de Jong and Shephard 1995; Chan et al. 2019; Ferreira et al. 2023).

A.3 Sampling $\boldsymbol{\gamma}$

Conditional on the TV coefficient and model parameters $(\boldsymbol{\beta}, w)$, equations (1) and (3) yield the following form of the univariate SV:

$$\begin{aligned} y_t^* &= \exp(\gamma_t/2) u_t, \\ \gamma_{t+1} &= \gamma_t + \eta_t, \\ (u_t, \eta_t)' &\sim N\left(\mathbf{0}, \mathrm{diag}(1, w^2)\right), \end{aligned}$$

where $y_t^* = y_t - \boldsymbol{\alpha}_t' \boldsymbol{z}_t + \boldsymbol{\beta}' \boldsymbol{x}_t$. We employ the efficient block sampling method for univariate SV models (Shephard and Pitt 1997; Watanabe and Omori 2004), which is adopted in this study.

Other efficient methods of mixture sampler (Kim et al. 1998; Omori et al. 2007) can be also used.

Appendix B. Data for the study of Japan's international trade volumes

This appendix documents the dataset used for the application described in Section 4. The series is quarterly and spans 1975/Q2 to 2024/Q3. The starting quarter of the sample period is determined by the availability of trade volume data. Although all the original series are monthly, we consider the average of the monthly figures to obtain the quarterly series.

Trade volume (EX)

We use the seasonally adjusted time series of Japan's real trade volumes provided by the Bank of Japan. We download the data from the bank's website. We define Y_t as the logarithm of the indices, multiplied by 100. As the dependent variable in the LP, we compute $y_{t+h} = Y_{t+h} - Y_t$, which is used for the estimation.

Global Economic business cycle index (GLOBAL)

We use the time series of industrial production indices aggregated within OECD countries, which is downloaded from the OECD's website. As an independent variable in the LP, we take a quarter-on-quarter change of the series.

Exchange rates (FX)

We use a time series of real effective exchange rates of the Japanese Yen (JPY) computed by the Bank for International Settlements (BIS). This series is downloaded from the BIS website.

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