Discussion Paper Series A No.753

On the estimation of the natural yield curve

Jouchi Nakajima
(Institute of Economic Research, Hitotsubashi University)
Nao Sudo
(Institute for Monetary and Economic Studies, Bank of Japan)
Yoshihiko Hogen
(Research and Statistics Department, Bank of Japan)
and
Yasutaka Takizuka
(Financial System and Bank Examination Department, Bank of Japan)

December 2023

Institute of Economic Research
Hitotsubashi University
Kunitachi, Tokyo, 186-8603 Japan

On the estimation of the natural yield curve*

Jouchi Nakajima† Nao Sudo‡ Yoshihiko Hogen§

Yasutaka Takizuka¶

December 2023

Abstract

This study discusses the estimation methodology of the natural yield curve, which is an extension of the natural rate of interest defined at a short-term interest rate to that defined for all maturities on a yield curve. To identify information about the latent factors forming the natural rate curve, the original estimation framework proposed by Imakubo et al. (2018), employs the pre-estimated potential growth rate, assuming that a change in the factors depends on a change in the potential growth rate. In contrast, this study examines an alternative approach that specifies that the levels of the factors depend on the potential growth rate. In an empirical analysis with recursive data updating, the differences in the resulting natural yield curve and its updated patterns between the two approaches are investigated.

JEL classification: C32, E43, E52, E58

Keywords: Natural yield curve; Natural rate of interest; Term structure

---

*The authors are grateful to Sohei Kaihatsu, Mitsuhiro Osada, Yu Sugioka, and Hiroki Yamamoto for their valuable comments and discussions. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Bank of Japan. Financial support from the Ministry of Education, Culture, Sports, Science, and Technology of the Japanese Government through a Grant-in-Aid for Scientific Research (No. 22K20157, 23H00050) and the Hitotsubashi Institute for Advanced Study is gratefully acknowledged.

†Institute of Economic Research, Hitotsubashi University (E-mail: nakajima-j@ier.hit-u.ac.jp)

‡Institute for Monetary and Economic Studies, Bank of Japan

§Research and Statistics Department, Bank of Japan

¶Financial System and Bank Examination Department, Bank of Japan

1 Introduction

The natural rate of interest is one of the most important variables in macroeconomic theory and has attracted attention from both theoretical and practical viewpoints of scholars as well as practitioners, such as central bankers and financial market participants. Because it cannot be observed directly, previous studies have attempted to develop a framework to estimate it by investigating how we can pin down information about it from a time series of macroeconomic variables. The main workhorse is the work of Laubach and Williams (2003); Holston et al. (2017, 2023), who employ a Kalman filtering approach to extract the natural rate of interest embedded in the IS and Phillips curves. Their estimation method has been discussed, and alternative identification strategies are proposed in several studies (e.g. Lewis and Vazquez-Grande, 2017; Wynne and Zhang, 2018; Kiley, 2020). Other studies estimate the natural interest rate based on vector autoregression models (e.g., Lubik and Matthes, 2015; Del Negro et al., 2017; Johannsen and Mertens, 2021). Approaches to the estimate of the natural rate using structural models, such as a dynamic stochastic general equilibrium model, are also proposed (e.g., Del Negro et al., 2017).

A seminal work by Brzoza-Brzezina and Kotłowski (2014) introduces the idea of the natural yield curve (NYC) as an extension of the natural interest rate defined at a short-term interest rate to one defined for all maturities on a yield curve. Imakubo et al. (2018) elaborate the idea and concept, develop an estimation framework of the NYC, and provide an empirical analysis using Japan's yield curve data. The natural yield curve is defined as the real yield curve in which the economy does not accelerate or decelerate. We assume that if the actual real yield curve matches the natural yield curve, the output gap will converge to zero; if the actual real yield curve lies above the natural yield curve, financial conditions are contractionary, leading to a contraction of a positive output gap (or an expansion of a negative output gap); by contrast, if the actual real yield curve lies below the natural yield curve, financial conditions are accommodative, leading to a contraction of a negative output gap (or an expansion of a positive output gap).

To estimate the NYC, we specify three latent factors describing each level and shape of the natural and actual yield curves. In line with the idea of the major model for the (short-term) natural rate of interest in Laubach and Williams (2003), the IS curve relates the output gap to the gaps in each factor between the natural and actual yield curves. The NYC factors are latent variables in this model and are assumed to follow an autoregressive process. Furthermore, in the model introduced by Imakubo et al. (2018), an innovation of the NYC factors is assumed to linearly depend on a change in the potential growth rate.

This paper discusses the specifications of the relationship between the factors and the potential growth rate. Contrary to the original specification, we focus on an alternative approach that specifies that the levels of the factors depend linearly on the potential growth rate. Between the two specifications, the concepts about a propagation of shock in the potential growth rate to the NYC factors are different. Consequently, the resulting estimated NYC may differ in the empirical analysis. Even for estimating the traditional natural rate of interest, several distinct specifications for linking the natural rate and the potential growth rate have been proposed among the studies (e.g., Laubach and Williams, 2003; Barsky et al., 2014; Del Negro et al., 2017; Lewis and Vazquez-Grande, 2017; Wynne and Zhang, 2018; Kiley, 2020), and there appears to be no consensus on which specification is better supported (see a discussion by Barsky et al., 2014). In this study, differences in the estimated NYC are investigated using Japan's yield curve data. In addition, a recursive estimation with data updated period by period is implemented, and the differences in the updated patterns of the estimated NYC are discussed.[1]

The remainder of this paper is organized as follows: Section 2 explains the original and alternative specifications of the NYC and discusses their differences. Section 3 provides an empirical analysis of Japan's data and discusses the differences in the estimated natural rate curves between the two specifications. Section 4 concludes the paper.

[1]Brand et al. (2021) and Dufrenot et al. (2022) extend the idea of the NYC presented by Brzoza-Brzezina and Kotłowski (2014) and Imakubo et al. (2018) to more general specifications.

2 The model

2.1 The basic idea

We begin with a basic idea of how to calculate the natural rate of interest, following Laubach and Williams (2003) and other studies that formulate the IS curve as follows:

$$y_t - y_t^* = \beta(r_t - r_t^*), \tag{1}$$

where y_t denotes the log of output, y_t^* the log of potential output, r_t the short-term real interest rate, and r_t^* the natural interest rate. The coefficient β measures the interest rate sensitivity of the economy, which is measured by the output gap on the left-hand side of the equation. While several other variables are included on the right-hand side of the equation in most previous studies, we consider only the interest rate gap, $r_t - r_t^*$, for simplicity, in this section.

Brzoza-Brzezina and Kotłowski (2014) and Imakubo et al. (2018) extend the idea of the natural rate of interest defined at a short-term maturity to the one defined for all maturities on a yield curve. We formulate the relationship between the output gap and interest rates as follows:

$$y_t - y_t^* = b \int_0^T \phi(\tau)(r_{\tau,t} - r_{\tau,t}^*)d\tau, \tag{2}$$

where $r_{\tau,t}$ and $r_{\tau,t}^*$ are the real and natural interest rates, respectively, for maturity τ at time t. We label $\{r_{\tau,t}^*\}_{\tau=0}^T$ as the NYC, an analog of the traditional natural rate of interest but extended to each point on the entire yield curve. $\phi(\tau)$ measures the difference in the sensitivity of the output gap to the interest rate gap at each maturity. We assume that $\phi(\tau) \geq 0$ for all τ and $\int_0^T \phi(\tau)d\tau = 1$.

A novel idea in the preceding studies is to approximate the real yield curve and NYC based on the three-factor Nelson and Siegel (1987) model. We define (L_t, S_t, C_t) as the level, slope, and curvature factors of the real yield curve and (L_t^*, S_t^*, C_t^*) as their analogs to the NYC. Specifically,

$$r_{\tau,t} = L_t + S_t \cdot \frac{1-e^{-\lambda\tau}}{\lambda\tau} + C_t \cdot \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \tag{3}$$

$$r^*_{\tau,t} = L^*_t + S^*_t \cdot \frac{1-e^{-\lambda\tau}}{\lambda\tau} + C^*_t \cdot \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \tag{4}$$

where λ is the decay rate parameter. We subtract equation (4) from equation (3) and substitute it into equation (2), which yields

$$y_t - y^*_t = b_L(L_t - L^*_t) + b_S(S_t - S^*_t) + b_C(C_t - C^*_t), \tag{5}$$

where (b_L, b_S, b_C) are the functions of (b, λ, τ). This model describes how each gap in the three factors affects the economy with $b_L, b_S, b_C > 0$. The NYC is defined as the real yield curve at which the economy neither accelerates nor decelerates, as we can see in the case where $L_t = L^*_t$, $S_t = S^*_t$, and $C_t = C^*_t$ in equation (5).

2.2 Estimation framework

Imakubo et al. (2018) propose the following specification to estimate the NYC model:

$$\begin{aligned}\begin{pmatrix} y_t \\ L_t \\ S_t \\ C_t \end{pmatrix} &= \begin{pmatrix} y^*_t \\ L^*_t \\ S^*_t \\ C^*_t \end{pmatrix} + \begin{pmatrix} a_y & b_L & b_S & b_C \\ 0 & a_L & 0 & 0 \\ 0 & 0 & a_S & 0 \\ 0 & 0 & 0 & a_C \end{pmatrix} \left[\begin{pmatrix} y_{t-1} \\ L_{t-1} \\ S_{t-1} \\ C_{t-1} \end{pmatrix} - \begin{pmatrix} y^*_t \\ L^*_t \\ S^*_t \\ C^*_t \end{pmatrix} \right] \\ &\quad + \begin{pmatrix} 1 & 0 & 0 & 0 \\ g_{yL} & 1 & 0 & 0 \\ g_{yS} & 0 & 1 & 0 \\ g_{yC} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon^y_t \\ \varepsilon^L_t \\ \varepsilon^S_t \\ \varepsilon^C_t \end{pmatrix}, \end{aligned} \tag{6}$$

$$\begin{pmatrix} \Delta y^*_t \\ L^*_t \\ S^*_t \\ C^*_t \end{pmatrix} = \begin{pmatrix} \Delta y^*_{t-1} \\ L^*_{t-1} \\ S^*_{t-1} \\ C^*_{t-1} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ h_{yL} & 1 & 0 & 0 \\ h_{yS} & h_{LS} & 1 & 0 \\ h_{yC} & h_{LC} & h_{SC} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon^{y^*}_t \\ \varepsilon^{L^*}_t \\ \varepsilon^{S^*}_t \\ \varepsilon^{C^*}_t \end{pmatrix}. \tag{7}$$

This model forms a state-space model with a change in the potential growth rate, and the three factors of the NYC included as state variables. Equation (6) specifies the IS curve of equation (5) with the lagged output gap and the factor gap dynamics as they follow a vector autoregressive (VAR) process. In equation (7), we assume that the NYC factors follow another VAR process with lagged coefficients set to one, namely, following the random-walk process, and that innovations of the change in the potential growth rate and of the NYC factors propagate as in the lower-triangular matrix of free parameters h.

Imakubo et al. (2018) treat the potential output as observed in estimating the NYC. We define $x_t = y_t^* - y_{t-1}^* (= \Delta y_t^*)$ and $\Delta x_t^* = \Delta y_t^* - \Delta y_{t-1}^* (= \varepsilon_t^{\Delta y_t^*})$ as the potential growth rate and the change from the previous period, respectively. Following the previous study, we rewrite equations (6) and (7) conditional on Δy_t^* using the definitions of x_t and Δx_t^* as

$$\begin{aligned}
\begin{pmatrix} y_t - y_t^* \\ L_t \\ S_t \\ C_t \end{pmatrix} &= \begin{pmatrix} a_y & b_L & b_S & b_C \\ 0 & a_L & 0 & 0 \\ 0 & 0 & a_S & 0 \\ 0 & 0 & 0 & a_C \end{pmatrix} \begin{pmatrix} y_{t-1} - y_t^* \\ L_{t-1} \\ S_{t-1} \\ C_{t-1} \end{pmatrix} \\
&+ \begin{pmatrix} b_L & b_S & b_C \\ 1 - a_L & 0 & 0 \\ 0 & 1 - a_S & 0 \\ 0 & 0 & 1 - a_C \end{pmatrix} \begin{pmatrix} L_t^* \\ S_t^* \\ C_t^* \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ g_{yL} & 1 & 0 & 0 \\ g_{yS} & 0 & 1 & 0 \\ g_{yC} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t^y \\ \varepsilon_t^L \\ \varepsilon_t^S \\ \varepsilon_t^C \end{pmatrix}, \\
\begin{pmatrix} L_t^* \\ S_t^* \\ C_t^* \end{pmatrix} &= \begin{pmatrix} L_{t-1}^* \\ S_{t-1}^* \\ C_{t-1}^* \end{pmatrix} + \begin{pmatrix} h_{yL} \\ h_{yS} \\ h_{yC} \end{pmatrix} \Delta x_t^* + \begin{pmatrix} 1 & 0 & 0 \\ h_{LS} & 1 & 0 \\ h_{LC} & h_{SC} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t^{L^*} \\ \varepsilon_t^{S^*} \\ \varepsilon_t^{C^*} \end{pmatrix}. \qquad (8)
\end{aligned}$$

Assuming that each of the disturbances ε_t^Z, where $Z \in (y, L, S, C, L^*, S^*, C^*)$, follows a normal distribution and is mutually independent, we estimate the parameters and the NYC factors based on these equations using the maximum likelihood method with the Kalman filter.

2.3 An alternative specification

The state equation (8) represents the dynamic aspect of the NYC factors as their changes from the previous period are driven by the change in the potential growth rate. In this original specification, we do not consider the possibility that the potential growth rate level is related to the levels of the NYC factors. Therefore, in the long run, the levels of the NYC factors does not depend on the potential growth rate, which is not necessarily consistent with the theoretical viewpoint that the natural rate of interest has a relevant relationship with the potential growth rate (e.g., Laubach and Williams, 2003; Wynne and Zhang, 2018). However, the specification is useful because the model is so flexible that it can capture a structural change in the NYC that deviates from a level consistent with the potential growth rate.

To address the points raised here further, we consider an alternative specification that relates the levels of NYC factors with the level of the potential growth rate. Equation (8) is modified as follows:

$$\begin{pmatrix} L_t^* \\ S_t^* \\ C_t^* \end{pmatrix} = \begin{pmatrix} \phi_L & 0 & 0 \\ 0 & \phi_S & 0 \\ 0 & 0 & \phi_C \end{pmatrix} \begin{pmatrix} L_{t-1}^* \\ S_{t-1}^* \\ C_{t-1}^* \end{pmatrix} + \begin{pmatrix} k_{yL} \\ k_{yS} \\ k_{yC} \end{pmatrix} x_t^* + \begin{pmatrix} 1 & 0 & 0 \\ k_{LS} & 1 & 0 \\ k_{LC} & k_{SC} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t^{L^*} \\ \varepsilon_t^{S^*} \\ \varepsilon_t^{C^*} \end{pmatrix}, \tag{9}$$

where $0 < \phi_X < 1$, for $X \in (L, S, C)$. In this alternative specification, the NYC factors follow a stationary VAR process conditional on the potential growth rate. Equation (9) characterizes the dynamics of the factors because their level depends on the potential growth rate. Conditional on information at time t, the expected level at which the factors converge without future shocks is $x_t^* k_{yX}/(1 - \phi_X)$ for $X \in (L, S, C)$. This feature is more consistent with economic theory than with the original specification. A caveat of this formulation is that the estimate of the NYC factors may heavily depend on the pre-estimated values of the potential growth rate, which could involve a larger variance than the original specification.

There appears to be no consensus in the literature on which specification out of these two approaches is better supported. Actually, even for the traditional natural rate of interest, it has been challenging to develop a generally suitable model with a wide consensus (see a discussion by Pescatori and Turunen, 2015). In the next section, we investigate their characteristics in estimating the NYC factors, implementing an empirical analysis in which the models are fitted to Japan's yield curve data to discuss the advantages and caveats of each approach.

3 Empirical analysis

3.1 Data and setup

Following Imakubo et al. (2018), we use the quarterly series of the output gap and potential growth rate estimated by the Bank of Japan and the series of real zero-coupon rates at 1-, 2-, 3-, 7-, 10, and 20 years maturities, which are computed by subtracting inflation expectations from the nominal zero-coupon rate. We compute the average of the daily figures of nominal rates within a quarter. For inflation expectations, we use the Consensus Forecasts reported by Consensus Economics, Inc. Because the series are available up to a 10-year horizon, we use the value of 10-year maturity (an average of six to ten years in the Consensus) for 20-year inflation expectations. As they are also available biannually for some periods, we interpolate the figures linearly to compute quarterly figures.

Additionally, we adjust for the effects of a consumption tax hike on inflation expectations. Imakubo et al. (2018) replace the figures that include the effects by linearly-interpolated ones based on the figures that are not affected. This study makes the adjustment more precise by extracting the effect sizes computed by a theoretical effect during the effective period beginning at the time of tax hike announcements from the original figures. Furthermore, the inflation expectations curve across the maturities is smoothed by a polynomial function of order six at each time point.

To obtain the three factors for the real yield curve, we estimate the dynamic Nelson-Siegel (DNS) model proposed by Diebold et al. (2006). The model consists of equations

(3) and

$$\begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} = \begin{pmatrix} \mu_L \\ \mu_S \\ \mu_C \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \left[\begin{pmatrix} L_{t-1} \\ S_{t-1} \\ C_{t-1} \end{pmatrix} - \begin{pmatrix} \mu_L \\ \mu_S \\ \mu_C \end{pmatrix} \right] + \begin{pmatrix} \xi_t^L \\ \xi_t^S \\ \xi_t^C \end{pmatrix},$$

where each of $\varepsilon_{\tau,t}$ and ξ_t^X for $X \in (L, S, C)$ follows a normal distribution, $\varepsilon_{\tau,t}$ are mutually independent and uncorrelated with ξ_t^X for any X, and ξ_t^X is mutually correlated. We estimate the DNS model using the maximum likelihood method with the Kalman filter.

We use the same sample period as in Imakubo et al. (2018), which spans from 1992/Q3 to 2014/Q4. While the dataset is basically the same, we use the latest figures of the output gap and potential growth, which have been updated from those used in the previous study, as well as the adjustment method of the consumption tax hike. Figure 1 shows the time series of the data used in this study.

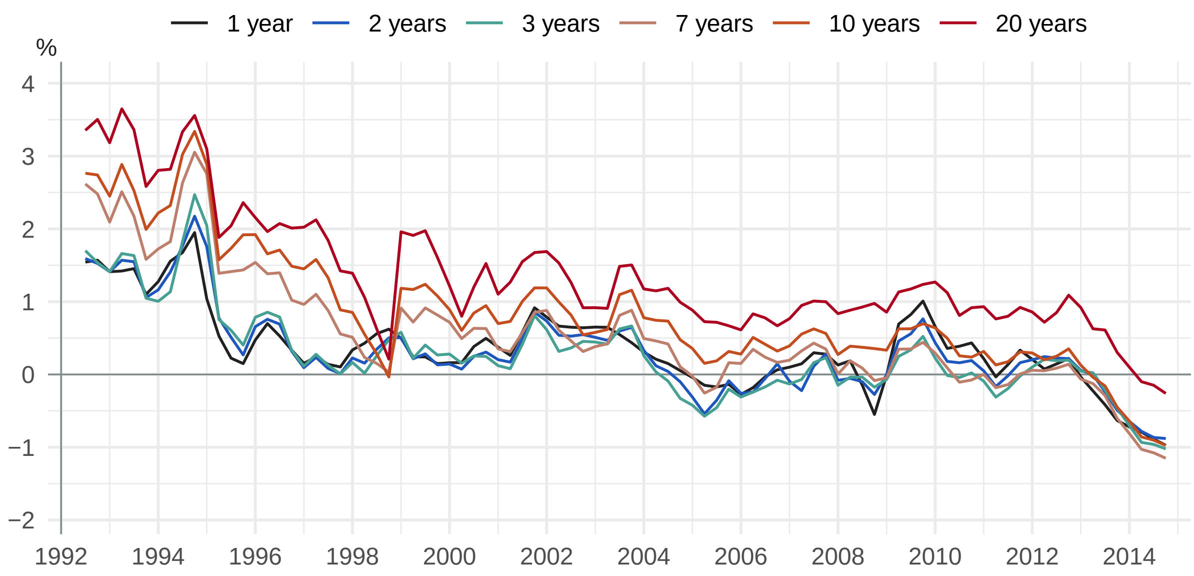
We implement a recursive estimation with quarterly updated data to examine how estimates of the NYC change in real-time when the data are updated. First, we estimate the model using data up to 2010/Q1. Next, we update the data by one quarter, up to 2010/Q2 and estimate the model. This sequential updating runs up to 2014/Q4, the end of the entire sample, which provides different real-time estimates for 20 quarters.

The forecasting performance of each specification is computed for comparing the specifications. Given the data up to time T, we compute a one-quarter ahead forecast of the output gap $y_{T+1} - y^*_{T+1}$, by assuming $x^*_{T+1} = x^*_T$, $y^*_{T+1} = y^*_T$, and ε^Z_{T+1}=0, where $Z \in (y, L, S, C, L^*, S^*, C^*)$. The forecasts are obtained sequentially for 20 quarters, and the root mean square error (RMSE) is computed for comparing the specifications.

3.2 Estimation results

Figure 2 plots the NYC factors estimated for the data spanning the entire sample period. The estimate of the original specification is similar to that in Imakubo et al. (2018) with a slight difference in the factors, owing to updating of the output gap and potential growth rate, and the adjustment method for the consumption tax hike. Between the estimates of the original and alternative specifications, the level factor

(i) Real interest rates



(ii) Output gap and potential growth rate

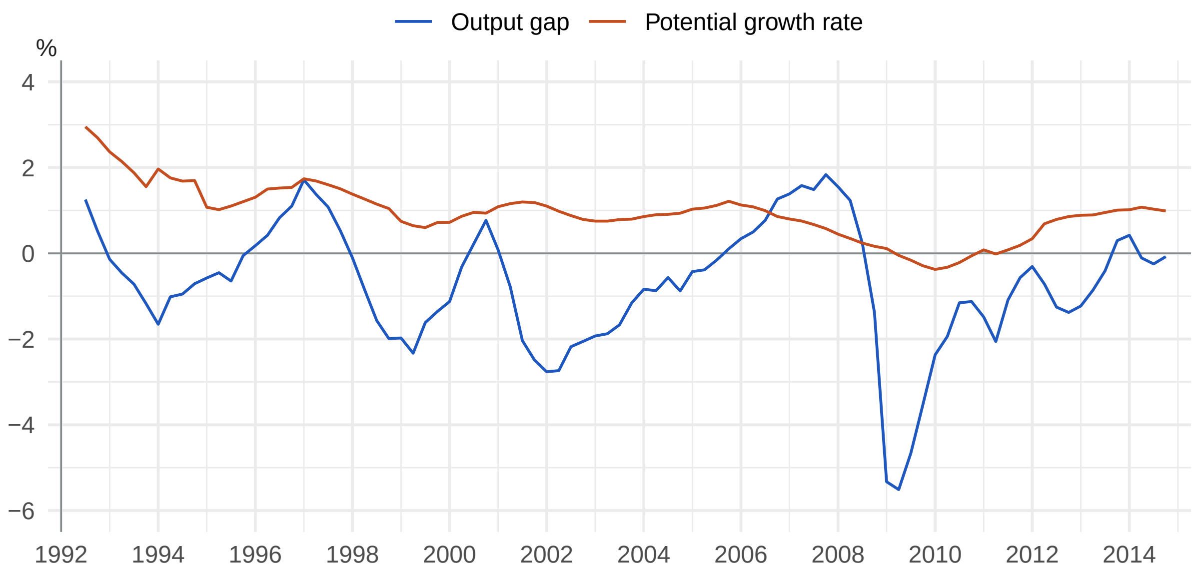


Figure 1: Japan's data. Real interest rates are nominal interest rates less the inflation expectations from the Consensus. Output gap and the potential growth rate are series provided by the Bank of Japan. All the series are quarterly.

(a) Level factor

(b) Slope factor

(c) Curvature factor

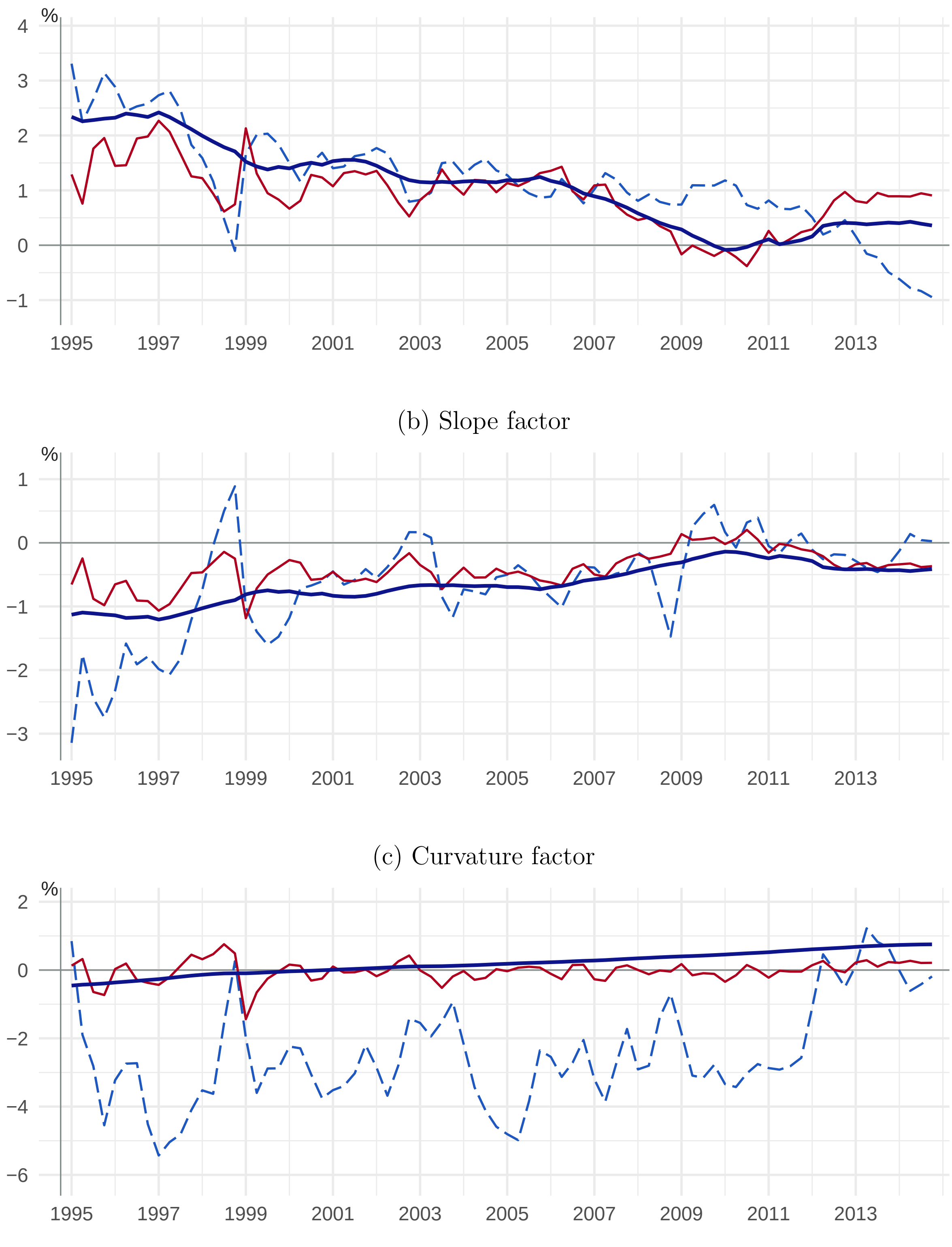


Figure 2: Factors of the real yield curve (dotted) and the NYC estimated by the original specification (bold) and the alternative (solid).

(a) 1-year maturity

Specification	2010/Q1	2011/Q1	2012/Q1	2013/Q1	2014/Q1
Original	0.112	0.131	0.133	0.111	0.075
Alternative	0.020	0.013	0.005	0.014	0.014

(b) 10-year maturity

Specification	2010/Q1	2011/Q1	2012/Q1	2013/Q1	2014/Q1
Original	0.162	0.184	0.184	0.157	0.099
Alternative	0.042	0.027	0.011	0.028	0.031

Table 1: Standard deviations of the estimate at the selected periods, computed across the sequentially updated datasets.

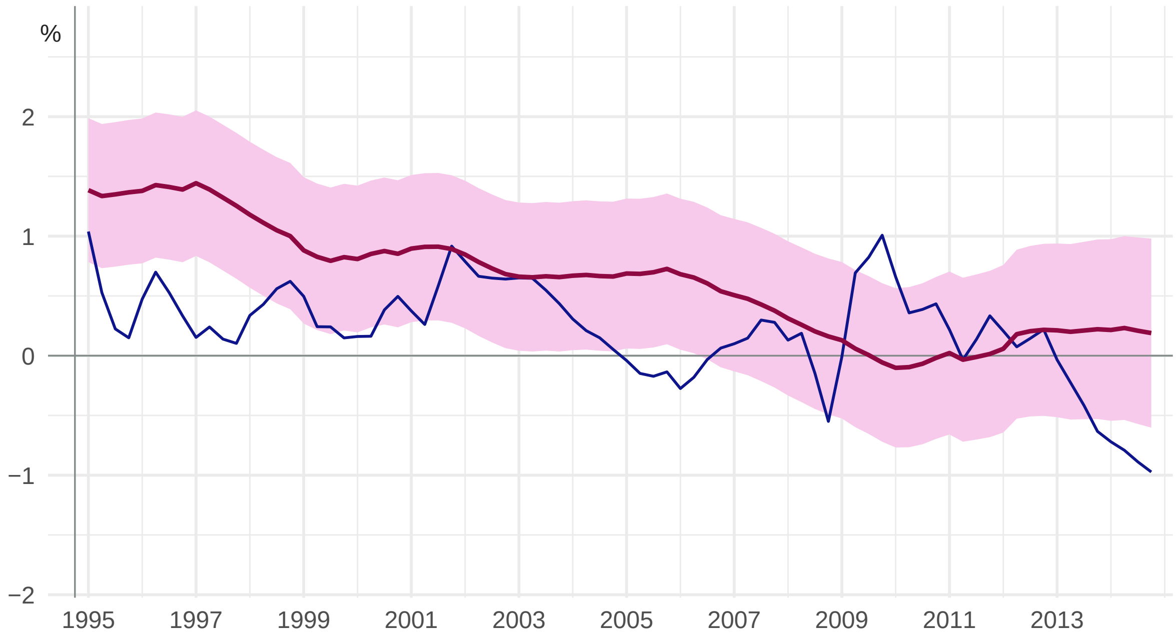
differs after the early 2010s when the potential growth rate gradually increases while the yield curve declines significantly.

Figures 3 and 4 plot the estimated NYC at the 1-year and 10-year maturities, respectively. The difference between the NYC factors leads to a deviation in the NYC at these maturities between the two specifications after the early 2010s. The 10-year natural rate of interest rate from the original specification is notably lower than that from the alternative specification. The estimate of the alternative has been partly affected by the information on the level of the potential growth rate since the early 2010s, which is not incorporated into the original. The 10-year natural rate of interest from the alternative specification rose in 2013, which could be partly supported by the level information and could be also driven by the dynamics of the stationary process returning to the stationary (i.e., historical) mean.

One characteristic in the estimated NYC from the alternative specification that it can fluctuate more than that from the original. In the 1990s and the early 2000s, the trajectory of the estimated NYC from the alternative had a larger variance than that of the original. This tendency could be a caveat in the alternative specification, which we must bear in mind when monitoring real-time estimates.

Table 1 reports the standard deviations of the estimates in selected periods computed across the sequentially updated datasets. For example, we obtain 20 estimates of the NYC in 2010/Q1 and 16 estimates in 2011/Q1. The results indicate that the

(a) Original specification



(b) Alternative specification

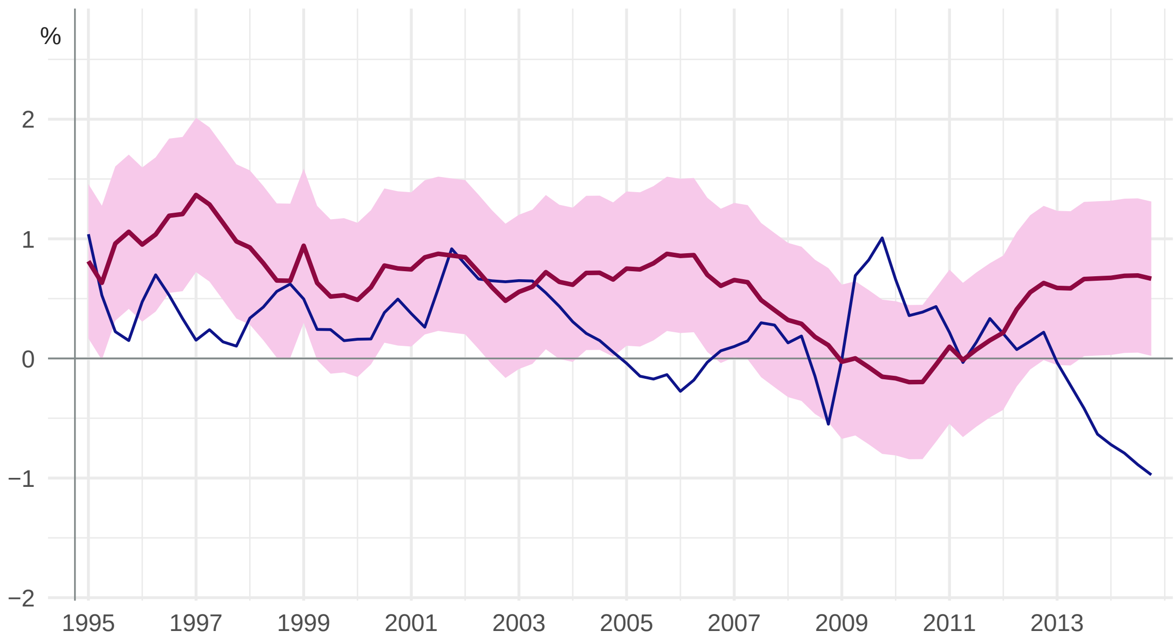
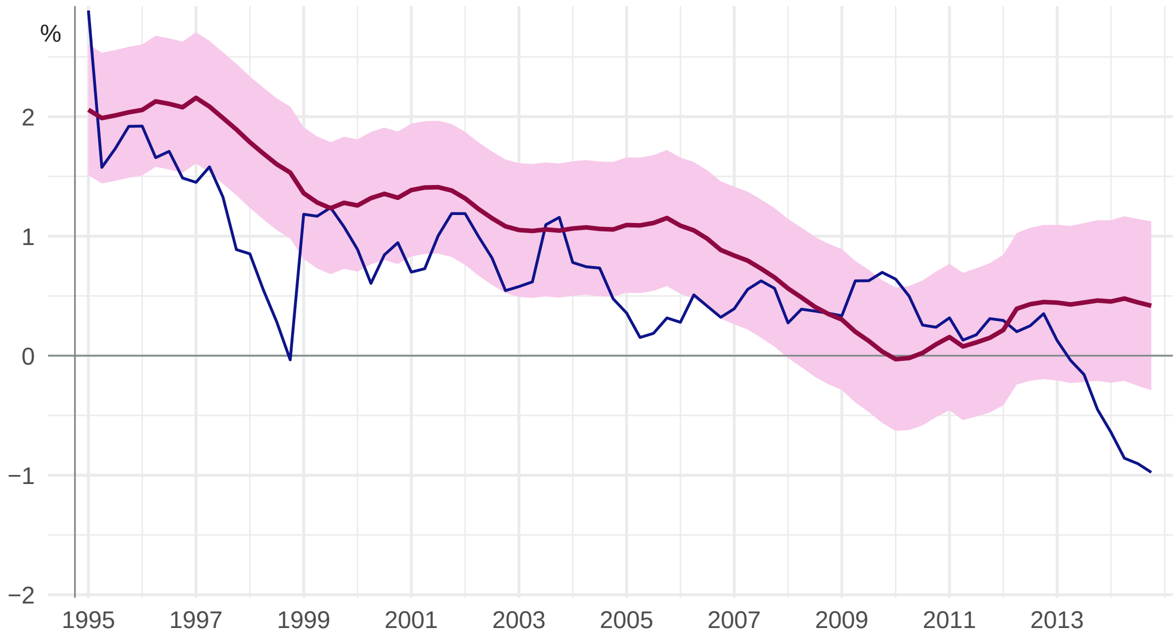


Figure 3: The estimated natural rate of interest at the 1-year maturity from (a) the original specification and (b) the alternative with the mean estimate (bold) and 95% confidence intervals (filled area) along with the real interest rates (solid).

(a) Original specification



(b) Alternative specification

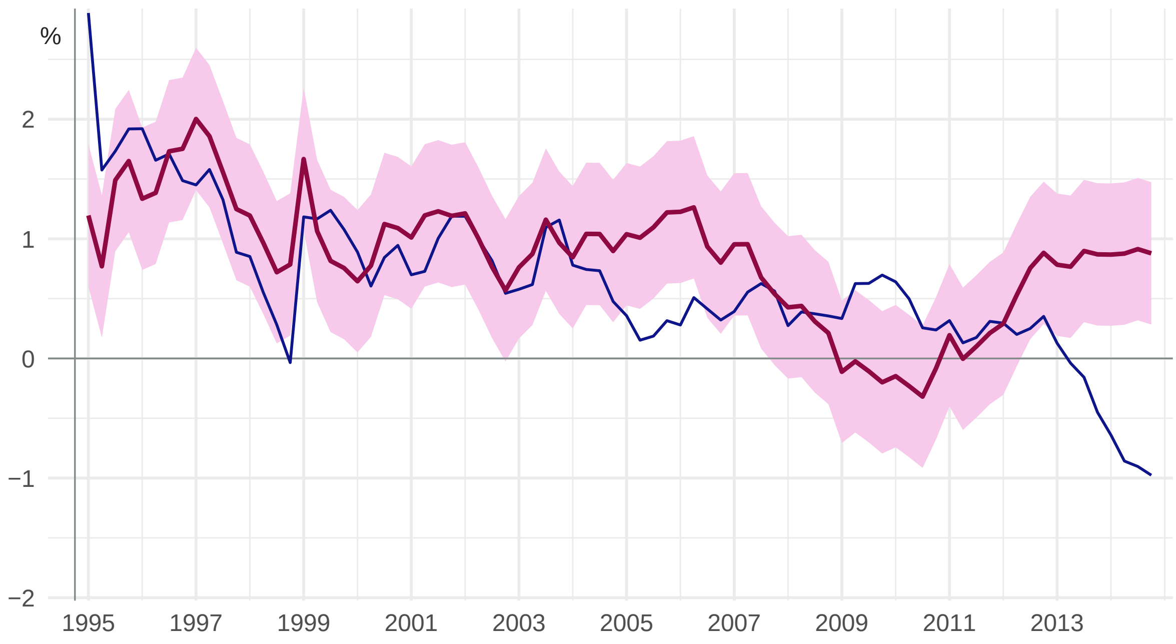


Figure 4: The estimated natural rate of interest at the 10-year maturity from (a) the original specification and (b) the alternative with the mean estimate (bold) and 95% confidence intervals (filled area) along with the real interest rates (solid).

standard deviations of the estimates from the alternative specification are clearly lower than those from the original specification. Therefore, in real time, the end-of-sample estimation uncertainty is smaller for the alternative specification than for the original specification. This result causes partly because in the alternative specification, the levels of the NYC factors are pinned down by the potential growth rate, which leads to more stable estimates than the original specification.

Table 2 reports the RMSEs for the forecasting and subsample periods. The RMSEs in the original specification is slightly lower than those in the alternative. While it is not obvious why the original specification dominates the alternative in terms of forecasting performance, the random-walk process of the NYC factors in the original specification could be more flexible than the alternative, capturing a temporal change that is relevant to the development of the output gap.

4 Conclusion

This study focuses on an alternative specification to estimate the natural yield curve by comparing its characteristics with the original specification in Imakubo et al. (2018). An empirical analysis of Japan's yield curve data provides three pieces of evidence. First, the estimated NYC from the alternative specification can fluctuate with a larger variance than the original. Second, the real-time estimation uncertainty is lower for the alternative specification. Third, the forecasting performance is higher in the original model. These results do not provide a concrete conclusion regarding which specification is better. These results are robust, even when we use a different series of inflation expectations to compute the real yield curve, as shown in the Appendix. A combination

Specification	Full sample	Sub-sample I	Sub-sample II
Original	0.434	0.483	0.451
Alternative	0.453	0.501	0.453

Table 2: RMSEs in forecasting the output gap. The full forecasting periods spans from 2010/Q1 to 2014/Q4, and the sub-sample periods span (I) from 2010/Q1 to 2011/Q4, and (II) from 2012/Q1 to 2013/Q4, respectively.

(a) 1-year maturity

Specification	2010/Q1	2011/Q1	2012/Q1	2013/Q1	2014/Q1
Original	0.134	0.161	0.179	0.169	0.137
Alternative	0.030	0.029	0.012	0.003	0.022

(b) 10-year maturity

Specification	2010/Q1	2011/Q1	2012/Q1	2013/Q1	2014/Q1
Original	0.183	0.221	0.214	0.189	0.151
Alternative	0.043	0.019	0.008	0.006	0.023

Table 3: Robustness check: Standard deviations of the estimate at the selected periods, computed across the sequentially updated datasets.

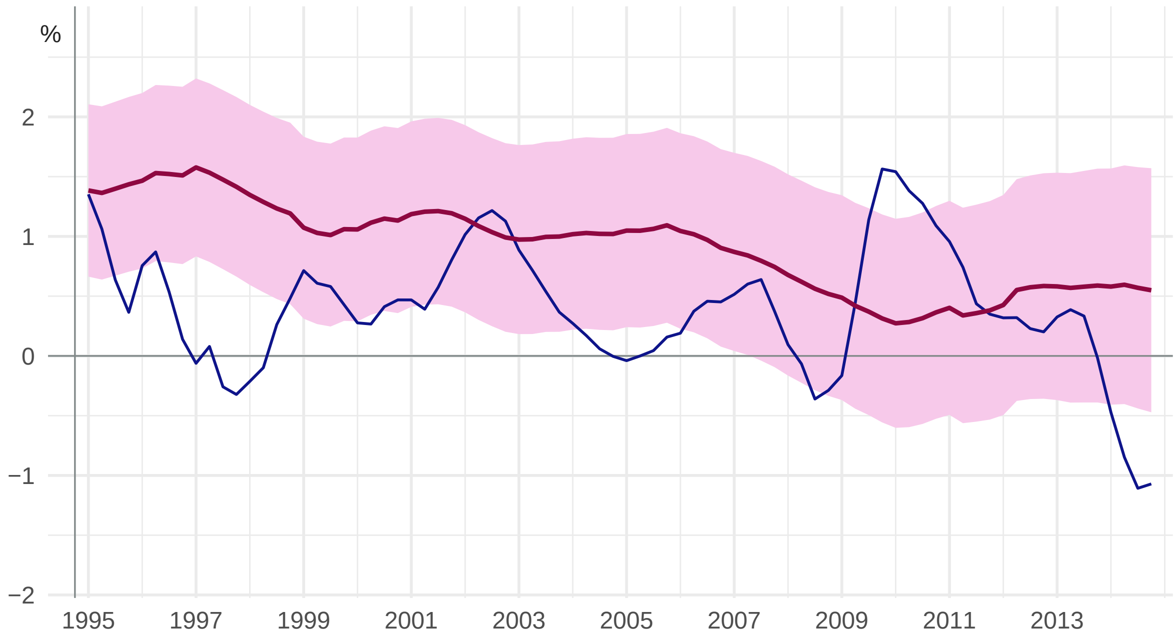
of two approaches or methods of model averaging would improve the estimates of the NYC, which is left for future research.

Appendix. Robustness check with different inflation expectations

This appendix examines the NYC estimation using a different series of inflation expectations to compute the real yield curve. Specifically, we employ real-time inflation forecasts obtained from the regime-switching trend inflation model developed by Kaihatsu and Nakajima (2018) and Nakajima (2023). The series are highly correlated with inflation expectations estimated from a term structure model of nominal and real yield curves developed by Imakubo and Nakajima (2015), which implies that inflation forecasts are compatible with the information on inflation expectations contained in the nominal yield curve.

We replace the series of inflation expectations in the data set with a four-quarter moving average series of inflation forecasts. As the trend inflation model uses CPI inflation rates, excluding the effects of the consumption tax hike, we do not need to adjust them in the NYC estimation. The same estimation procedure is implemented for the dataset.

(a) Original specification



(b) Alternative specification

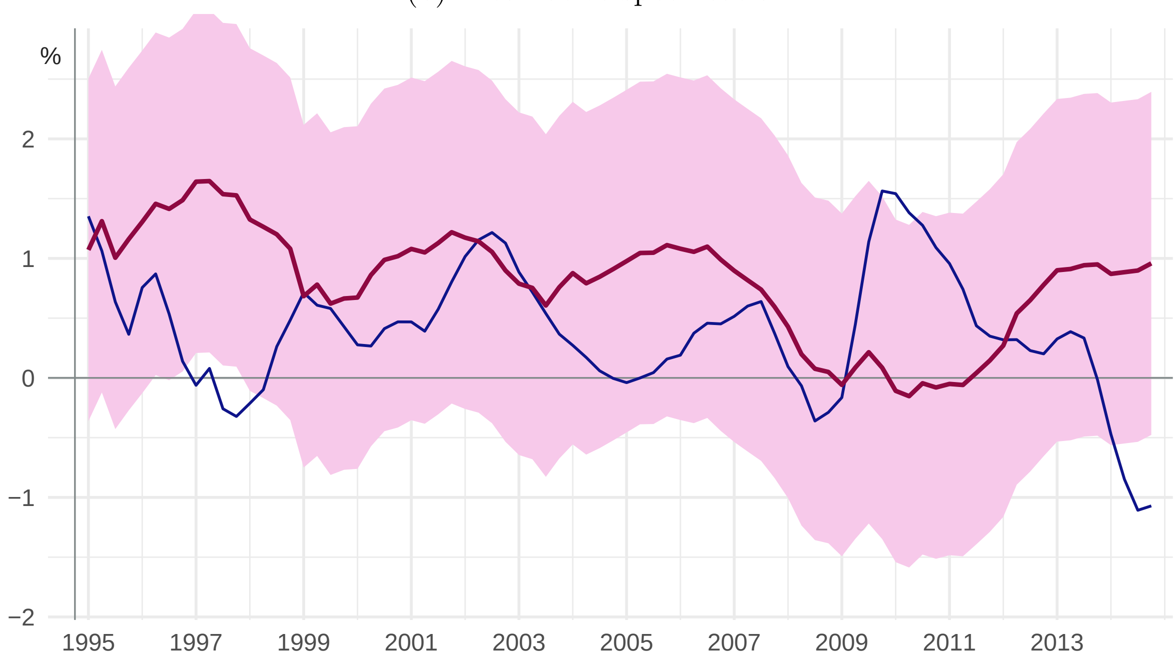
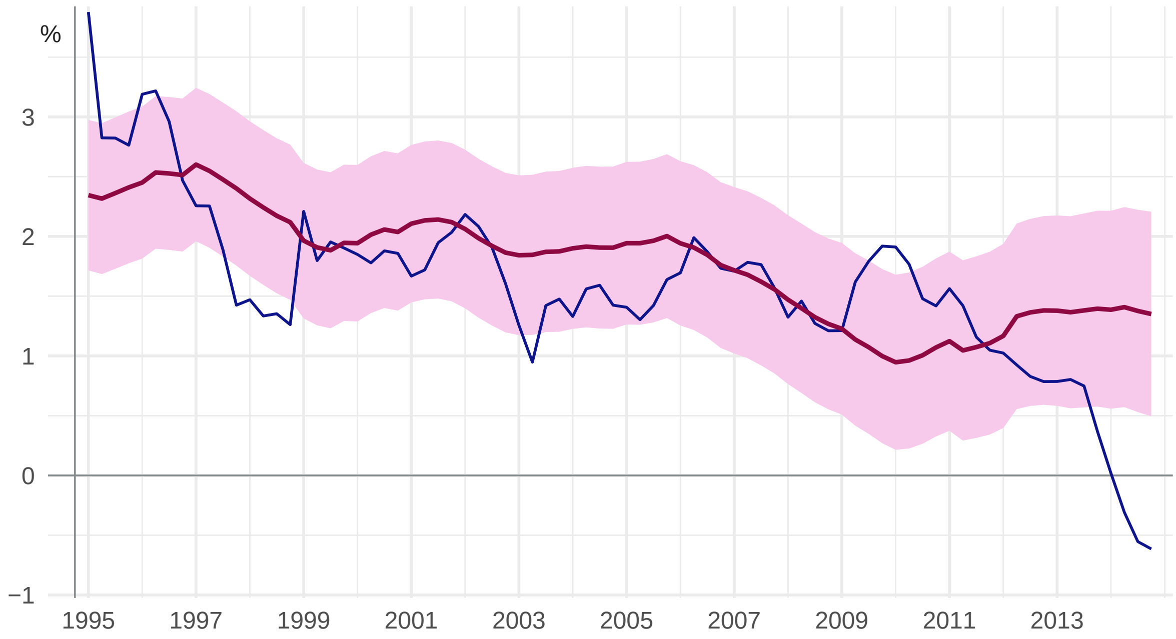


Figure 5: Robustness check: The estimated natural rate of interest at the 1-year maturity from (a) the original specification and (b) the alternative with the mean estimate (bold) and 95% confidence intervals (filled area) along with the real interest rates (solid).

(a) Original specification



(b) Alternative specification

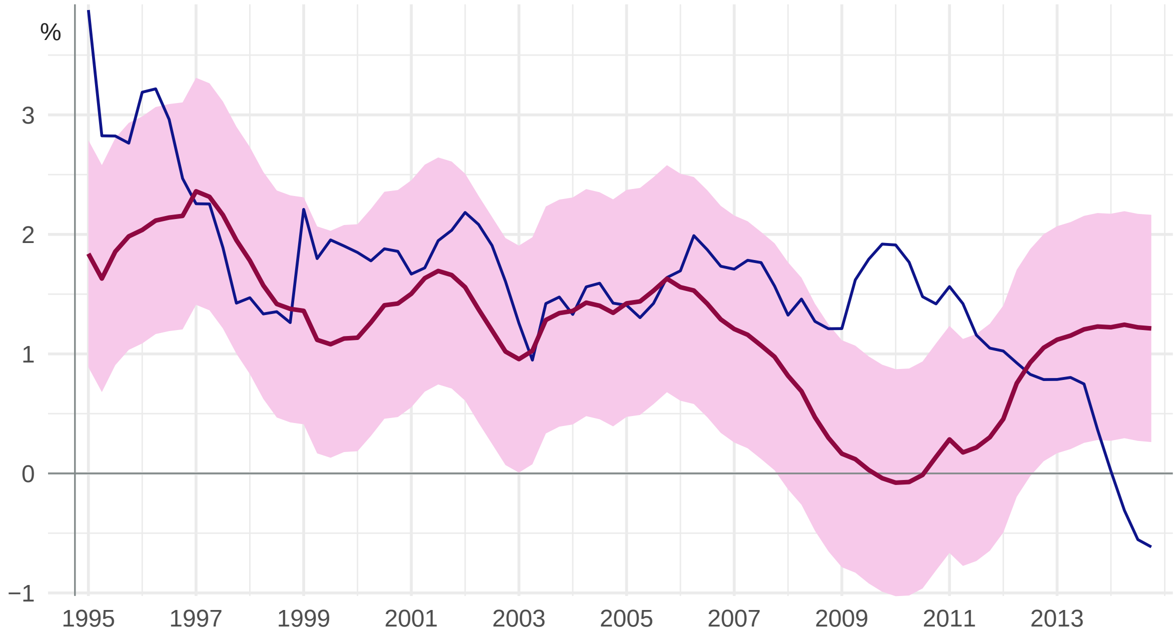


Figure 6: Robustness check: The estimated natural rate of interest at the 10-year maturity from (a) the original specification and (b) the alternative with the mean estimate (bold) and 95% confidence intervals (filled area) along with the real interest rates (solid).

Specification	Full sample	Sub-sample I	Sub-sample II
Original	0.465	0.547	0.466
Alternative	0.490	0.594	0.479

Table 4: Robustness check: RMSEs in forecasting the output gap. The full forecasting periods spans from 2010/Q1 to 2014/Q4, and the sub-sample periods span (I) from 2010/Q1 to 2011/Q4, and (II) from 2012/Q1 to 2013/Q4, respectively.

Figures 5 and 6 plot the estimated NYC at the 1-year and 10-year maturities, respectively. While we find some difference between the dataset in the main analysis and in this dataset with inflation forecasts, the overall trends of the gaps between the NYC and real yield curves are notably similar. Tables 3 and 4 report the standard deviations of the sequentially updated estimates and the RMSE in the forecasting exercise. Both results have the same implication as the main analysis, which indicates that the results are robust.

References

Barsky, R., A. Justiniano, and L. Melosi (2014). The natural rate of interest and its usefulness for monetary policy. *American Economic Review 104*(5), 37–43.

Brand, C., G. Goy, and W. Lemke (2021). Natural rate chimera and bond pricing reality. ECB Working Paper Series, No. 2612.

Brzoza-Brzezina, M. and J. Kotłowski (2014). Measuring the natural yield curve. *Applied Economics 46*(17), 2052–2065.

Del Negro, M., D. Giannone, M. P. Giannoni, and A. Tambalotti (2017). Safety, liquidity, and the natural rate of interest. *Brookings Papers on Economic Activity 2017*(1), 235–316.

Diebold, F. X., G. D. Rudebusch, and S. B. Aruoba (2006). The macroeconomy and the yield curve: A dynamic latent factor approach. *Journal of Econometrics 131*(1), 309–338.

Dufrenot, G., M. M. Rhouzlane, and E. Vaccaro-Grange (2022). Potential growth and natural yield curve in Japan. *Journal of International Money and Finance 124*, 102628.

Holston, K., T. Laubach, and J. C. Williams (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics 108*, S59–S75.

Holston, K., T. Laubach, and J. C. Williams (2023). Measuring the natural rate of interest after COVID-19. FRB of New York Staff Report, No. 1063.

Imakubo, K., H. Kojima, and J. Nakajima (2018). The natural yield curve: Its concept and measurement. *Empirical Economics 55*, 551–572.

Imakubo, K. and J. Nakajima (2015). Estimating inflation risk premia from nominal and real yield curves using a shadow-rate model. Working Paper Series No.15-E-1, Bank of Japan.

Johannsen, B. K. and E. Mertens (2021). A time-series model of interest rates with the effective lower bound. *Journal of Money, Credit and Banking 53*(5), 1005–1046.

Kaihatsu, S. and J. Nakajima (2018). Has trend inflation shifted?: An empirical analysis with an equally-spaced regime-switching model. *Economic Analysis and Policy 59*, 69–83.

Kiley, M. T. (2020). What can the data tell us about the equilibrium real interest rate? *International Journal of Central Banking 16*(3), 181–209.

Laubach, T. and J. C. Williams (2003). Measuring the natural rate of interest. *Review of Economics and Statistics 85*(4), 1063–1070.

Lewis, K. F. and F. Vazquez-Grande (2017). Measuring the natural rate of interest: Alternative specifications. Finance and Economics Discussion Series, 2017-059, Board of Governors of the Federal Reserve System.

Lubik, T. A. and C. Matthes (2015). Calculating the natural rate of interest: A comparison of two alternative approaches. Economic Brief, No. 15-10, Federal Reserve Bank of Richmond.

Nakajima, J. (2023). Estimating trend inflation in a regime-switching Phillips curve. IER Discussion Paper Series A, No. 750, Hitotsubashi University.

Nelson, C. R. and A. F. Siegel (1987). Parsimonious modeling of yield curves. *Journal of Business 60*(4), 473–489.

Pescatori, M. A. and M. J. Turunen (2015). Lower for longer: Neutral rates in the United States. IMF Working Paper, WP/15/135.

Wynne, M. A. and R. Zhang (2018). Estimating the natural rate of interest in an open economy. *Empirical Economics 55*, 1291–1318.