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# On the Relationship Between Interest Rate Policy & Debt Sustainability

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**Abstract**

A situation where interest rates are lower than the economic growth rate, $r < g$, has begun to emerge in many developed countries, as in Japan. Recent studies and policymakers have begun to consider the possibility that this $r < g$ economic phenomenon may make it unnecessary for governments to maintain fiscal discipline. We re-examine various monetary policies by a central bank including negative interest rate policy in the context of the fiscal sustainability. We identify a situation where the BOJ's bond purchasing policy can be justified: the policy lowers nominal interest rate and even make it possibly negative, and ease the fiscal burden. We also confirm that the conventional concern is applied: the BOJ is forced to help the fiscal authority, resulting in a huge inflation. We proceed by exploring these theoretical possibilities empirically. In particular, we thoroughly examine a recent study by Mian et al. (2022) which argues that Japan can significantly expand its fiscal stimulus without increasing taxes. We find that their results sensitively depend on their choices of various parameters. So, their policy recommendation needs to be taken with a grain of salt.

## 1 Introduction

Since 1990, the Japanese economy has been characterized by low growth, low interest rates, and low inflation, and has been the subject of much debate among researchers and policymakers. To cope with this situation, various monetary and fiscal policies have been implemented one after another. Unfortunately, the years passed without signs of recovery, and Japan was left with a large amount of government debt. This fact puts the fiscal sustainability of Japan in doubt, and Japan now needs to face these two difficult issues, the stagnation and large outstanding debt, simultaneously.

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Fortunately, there has been seemingly good news in the last decade. The interest rate has been lower than the growth rate of the economy so long, and this phenomenon has sparked a debate about fiscal policy that is different from what we have seen so far. If this phenomenon keeps continue, then it appears to be the case that any amount of deficit can be sustained: it is always possible for the government to issue more debt to finance the expenditure and pay back its debt. This free-lunch style policy is now vigorously debated about whether public finances can be sustained without fiscal reform (Blanchard (2019)). For example, Blanchard and Tashiro (2019) argue that fiscal stimulus without increasing taxes would solve the problem of insufficient demand in the Japanese economy. Economists start writing various papers based on a similar view. Modern monetary theory (MMT) can be interpreted as a different version of this free-lunch style policy. This is not a mere theoretical possibility. Alexandria Ocasio-Cortez, a house member in the US, said that she was open to using MMT.

In this paper, we re-examine monetary policies by a central bank in this context of the fiscal sustainability. In a typical monetary policy analysis, the question is whether the central bank design its monetary policy so that the economy does not produce too much (excess demand) or too little (shortage of demand). For example, a negative interest rate policy is suggested in a situation where the real interest rate is too high and the economy produces too little.[1] Unlike these studies, the focus of this paper is the interaction of the monetary policy and the fiscal sustainability. A narrower question we address is whether the negative interest rate policy can ease the fiscal burden and expand the fiscal space. If so, how, when, and why?

Before formulating our economic model which speaks to this issue, we begin by introducing macroeconomic facts about Japan. Since 1994, the fiscal deficit has been positive, and the ratio of the deficit to nominal GDP is on average about 4%. Reflecting this fact, the outstanding debt has been increasing over time. The Bank of Japan (BOJ) started to increase its debt holding aggressively since around 2012. This policy is a part of the larger unconventional monetary policies. We use these facts as inputs for our theoretical analysis.

Having introduced the basic facts about Japan, we develop an economic model which can be used for analyzing the fiscal sustainability and account for the facts jointly. We extend the model of Mian et al. (2022) by introducing money demand, and ignore the zero-lower bound constraint.[2]This simple generalization allows us to discuss fiscal and monetary policy simultaneously. We re-drive the main results of Mian et al. (2022) in our model, and analyze the narrower question we raised. Moreover, we identify the condition where the conventional concern holds, which is that the central

[1]Another example for studying the negative interest rate is Brunnermeier and Koby (2019). Brunnermeier and Koby (2019) study the effect of the negative interest rate on commercial banks' profitability. They show that the negative interest rate policy might be counterproductive since it shrinks profit by the banks.

[2]More precisely, we do not consider the case where the natural rate of interest becomes negative, and the economy cannot deliver such low real interest rate unless the nominal interest rate becomes negative.

bank might be forced to abandon the price stability, and support the fiscal authority. One limitation of our model is that we use a model where the real allocations are determined independently of fiscal and monetary policies. So, our model can only speak to monetary phenomena or the fiscal sustainability, but not welfare. Given this limitation, in this paper, we say a monetary policy is good if it helps the fiscal sustainability or maintain the price stability.

We move on to examine these theoretical possibilities empirically. In particular, we thoroughly examine the radical implications drawn by Mian et al. (2022). Mian et al. (2022) find that Japan has an ample fiscal space, and increases the fiscal expenditure drastically without having any tax increases. We discuss why Mian et al. (2022) draws this conclusion and identify the set of their parameter choices, delivering the radical recommendation for Japan. Under their parameter specification, the model has various implausible implications (and plausible implications too). So, we conclude that this radical implications needs to be taken with a grain of salt.

## 2 Basic Facts about Japan

We introduce various facts about the fiscal situation of Japan. The discussion will be developed around the budget flow constraints of the fiscal authority. Consider a simpler version of the flow budget constraint at a particular year $t$:

$$B_t + T_t = R_{t-1}B_{t-1} + G_t \tag{1}$$

where $B_t$ is the newly issued government debt, $R_t$ is the interest rate, $T_t$ is the total tax revenue, and $G_t$ is the government expenditure. The primary deficit denoted by $Z_t$ is defined as

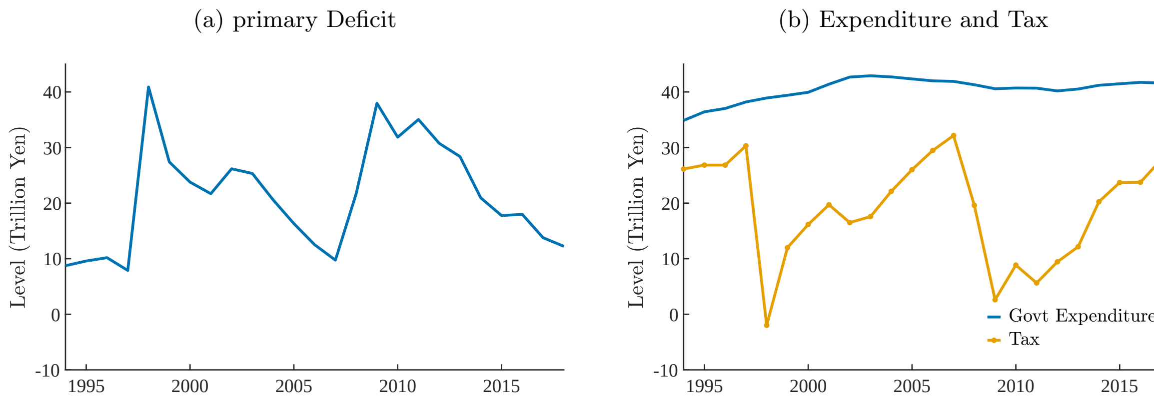
$$Z_t \equiv G_t - T_t. \tag{2}$$

If the primary deficit is equal to zero, $Z_t = 0$, all the expenditure is financed through the current tax. By using the flow budget constraint (1), the primary deficit is also written as follows:

$$Z_t = R_{t-1}B_{t-1} - B_t. \tag{3}$$

In Figure 1a and 1b we depict the logged levels of the government expenditure $G_t$, the total tax $T_t$, and the primary deficit $Z_t$. There are several points worth noting about these figures. First, Japan has persistently run budget deficits since 1994. Although the level of primary deficit varies from period to period, the average annual deficit is about 20 trillion yen. Second, variations in the primary deficit are almost entirely driven by variations in tax revenues. As Figure 1b shows,

Figure 1




fiscal spending moves quite smoothly relative to tax revenues. Tax revenues are strongly affected by business cycles and natural disasters, and fluctuate wildly.

Figure (1a) says that the primary deficit $Z_t$ has been positive. It follows from equation (3) that the fiscal authority has issued more debt to finance the expenditure and interest payments to the creditors.

$$B_t > R_{t-1}B_{t-1}. \tag{4}$$

The gross interest rate on the debt, $R_t$, is typically great than or equal to one. Combining this with equation (4), it is shown that the debt monotonically increases over time.

$$B_t > R_{t-1}B_{t-1} \geq B_{t-1}.$$

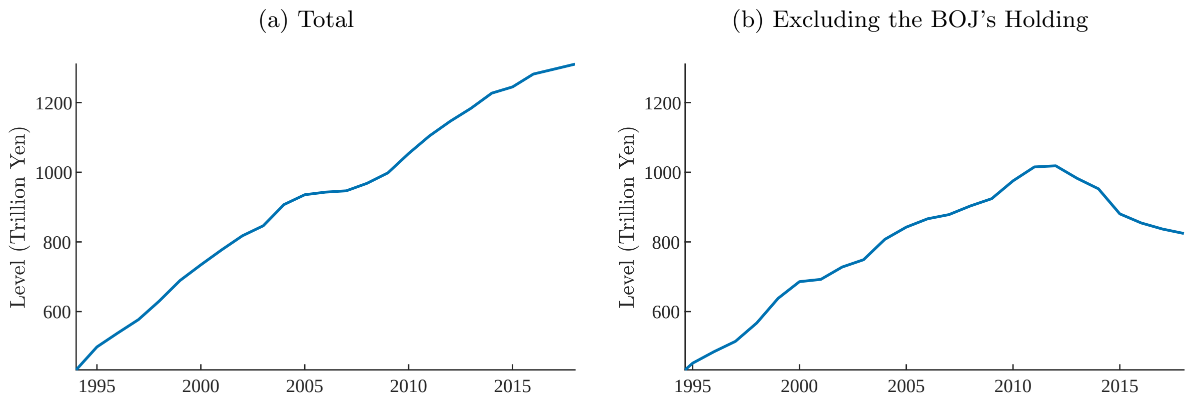
This observation that the debt increases over time is confirmed. Figure 2a plots the level of the outstanding Japanese government bond (JGB). The figure shows clearly that the total debt issued by the government has increased monotonically, and it exceeded 1200 trillion yen around 2015. These facts have sparked fears that the Japanese government bonds will eventually collapse and the collapse will have a devastating effect on the Japanese economy. It is not only the bureaucrats at the Ministry of Finance who have this concern, but it permeates the general public as well.

To consider whether it is possible to sustain the debt and the deficit, it is convenient to rewrite the flow budget constraint (1) as follows. First, we rewrite the budget constraint:

$$B_t = R_{t-1}B_{t-1} + Z_t.$$

Then we scale all the nominal variables by nominal GDP. Let $b_t$ denote the ratio of the total debt to nominal GDP, and $z_t$ the ratio of the fiscal deficit to nominal GDP. The above equation is written

Figure 2: Outstanding Debt




Notes. We measure of the total debt issued by Japan by the total liabilities of the general government. The data can be retrieved by the following URL, https://www.esri.cao.go.jp/en/sna/data/kakuhou/files/2018/tables/30si3_en.xlsx.

as

$$b_t = \frac{R_{t-1}}{G_t} b_{t-1} + z_t.$$

This version of the flow budget constraint is easier to work than the original flow budget constraint (1) since these scaled variables might have their steady state values.

This equation clarifies why some people like Blanchard (2019) argue that it is possible to sustain positive debt without balancing the budget. To see the argument, suppose that the economy reaches to a steady state where all the variables stay constant including the nominal interest rate and GDP growth rate. Then at the steady state, the budget constraint is

$$b = \frac{1}{1 - R/G} z. \tag{5}$$

If the nominal interest rate is smaller than the growth rate of nominal GDP, $R < G$, it is possible that both $b$ and $z$ are positive. That is, the government can run persistent deficit $z > 0$ without imposing any tax. To demonstrate how the fiscal authority supports the deficit level $z$ without taxes, consider the following scenario. The fiscal authority balances its budget $z = 0$ until $t = 0$, and decides to raise the deficit permanently from $t = 1$. The fiscal authority can issues $z$ unit of $b$ at date 1, $b_1 = z$, to finance the expenditure. At date 2, the government finances $z$ of date $t = 2$ by issuing debt and also repay the interest by issuing more debt. So, the fiscal authority needs to issue the following amount of the debt:

$$b_2 = z + \frac{R}{G} b_1 = z + \frac{R}{G} z.$$

The government can keep issuing more deficit to finance the expenditure. Then the total debt at date $t$ is

$$b_t = \left(1 + \frac{R}{G} + \left(\frac{R}{G}\right)^2 + \cdots + \left(\frac{R}{G}\right)^{t-1}\right) z,$$

which converges to (5). When the economy keeps growing, $G_t \to \infty$, the total debt issued by the fiscal authority, $B_t$, keeps increasing without bound, $B_t = bG_t \to \infty$.

Note that such policy becomes infeasible when $R > G$. If this is the case, the ratio $b_t$ diverges to infinity. Typically this is not an equilibrium since the households do not have such a large amount of the debt. So the current Japanese fiscal situation is only feasible when the nominal interest rate is less than the nominal GDP growth. Of course there is no guarantee that $(R, G)$ stays constant for large $b$. So, the question of the fiscal sustainability of Japan is rephrased as follows. How, why, and when is it possible that the nominal interest rate will be lower than the growth rate of nominal GDP forever $R < G$? We answer this question by using an economic model.

Before introducing our model, we quickly review the basic facts about these scaled variables, $b_t$ and $z_t$, and the nominal interest rate $R_t$. In Figure 3a, we plot the debt to the GDP ratio, and the ratio of the government debt excluding one held by the BOJ to GDP. Until 2012, the fraction of the government debt held by the BOJ had been almost constant. Since around 2012, the BOJ started to buying various types of the debt, lowering the long-term interest rate (See Figure 3b).

This point that the BOJ increased its bond purchase has a big implication for our economic model. In a typical economic model, the households have a motivation for holding the debt, and their collective behavior pins down the nominal interest rate. So, the relevant debt to GDP ratio is the one excluding the BOJ's bond purchase. However, there are many papers which ignores this point, and use the total debt to GDP ratio in their model. This might sound like a small and inconsequential mistake, but we demonstrate later that that is not the case. If we use a correct number, then a result obtained in a study does not hold any more.

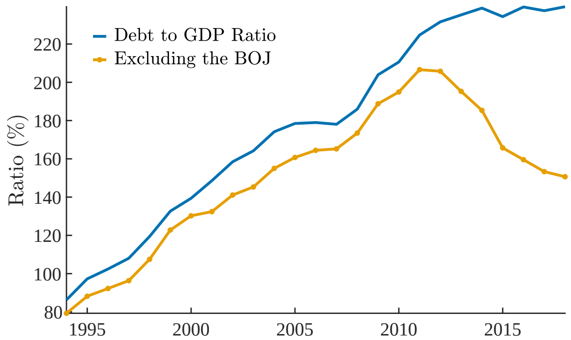
Finally the deficit to GDP ratio is displayed in Figure 4c. The ratio has fluctuated over the business cycle, but the ratio is almost always greater than 2%. The average level is around 4.1%. This number also plays a critical role in our model analysis.

## 3 Accounting Model

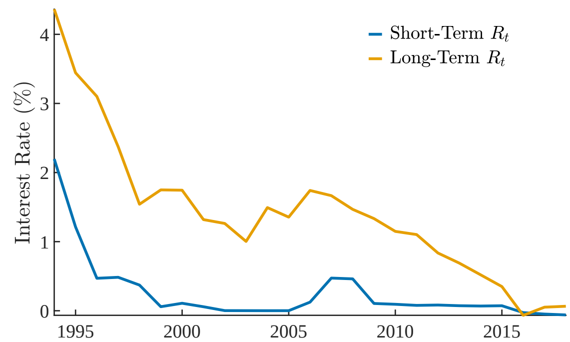
We extend the frictionless version of the model by Mian et al. (2022) by introducing another asset, called money. We use this model for our accounting purpose, and provide a positive analysis of Japan and its fiscal situation. The model presented here is a combination of the model by Mian et al. (2022) and the money-in-the-utility model. Since our accounting model has the typical dichotomy property, we cannot meaningfully discuss welfare. So, we evaluate various policies through the lens

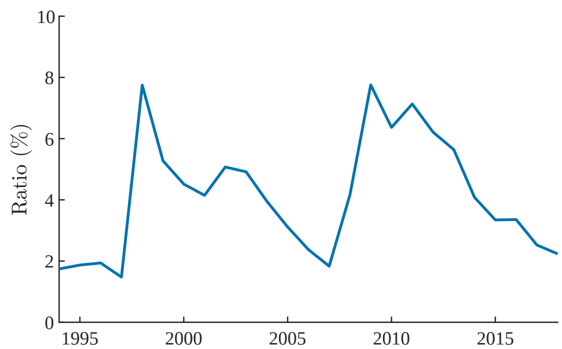
Figure 3

(a) Ratio of Debt to GDP




(b) The Long and Short Interest Rates, $R_t$




(c) Ratio of the Fiscal Deficit to GDP, $z_t$

of fiscal sustainability and price stability: a policy is good if its provides more fiscal space for Japan or stable inflation.

## 3.1 Model Setup

We consider an deterministic economy, which has infinite periods, $t = 0, 1, 2, \cdots$ . There are four agents in this economy: savers; firms; a central bank; and a fiscal authority.

**Savers** There is a continuum of savers uniformly distributed over $[0, 1]$. The savers provides one unit of labor inelastically to the firms at the wage rate, $W_t$. So, the total wage bill that the savers collectively get is $W_t$. They can save by purchasing bond issued by the fiscal authority or having money issued by the central bank. Let $B_t$ denote the stock of debt held by the households and, $M_t$ the money at the savers' hand at date $t$. The interest rate on the bond is denoted by $R_t$.

The maximization problem is formulated as follows:

$$\max_{\{C_t,B_t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty}\beta^t\left(\ln C_t + V\left(\frac{B_t}{P_tY_t},\frac{M_t}{P_tY_t}\right)\right) \tag{6}$$

$$s.t. \quad P_tC_t + B_t + M_t = W_t + T_t + R_{t-1}B_{t-1} + M_{t-1} \tag{7}$$

$$B_{-1} : \text{given},$$

where $C_t$ is consumption and $T$ is the lump-sum tax imposed by the fiscal authority. The savers directly obtain a util by having bond and money denoted by $v(\cdot,\cdot)$. There are many micro-foundations for this assumption, but for our purpose, it is sufficient to use this reduced-form specification. [3] To keep the stationary of the model, we assume that these util coming from bond and money are a function of the debt and money to GDP ratio, not the level of debt.

Let $\mathcal{L}$ denote the Lagrangian for the maximization problem (6), and $\beta^t\lambda_t$ denote the Lagrange multiplier for the budget constraint at date $t$.

$$\mathcal{L} = \sum_{t=0}^{\infty}\beta^t\left[\ln C_t + V\left(\frac{B_t}{P_tY_t},\frac{M_t}{P_tY_t}\right) + \lambda_t\left\{W_t + T_t + R_{t-1}B_{t-1} + M_{t-1} - (P_tC_t + B_t + M_t)\right\}\right].$$

The FONCs are

$$[C_t] \quad : \quad C_t^{-1} = \lambda_tP_t$$

$$[B_t] \quad : \quad \lambda_t = V_1\left(\frac{B_t}{P_tY_t},\frac{M_t}{P_tY_t}\right)\frac{1}{P_tY_t} + \beta\lambda_{t+1}R_t$$

$$[M_t] \quad : \quad \lambda_t = V_2\left(\frac{B_t}{P_tY_t},\frac{M_t}{P_tY_t}\right)\frac{1}{P_tY_t} + \beta\lambda_{t+1}$$

Combining these equations, we obtain the so-called the Euler equations.

$$\frac{1}{C_t} = V_1\left(\frac{B_t}{P_tY_t},\frac{M_t}{P_tY_t}\right)\frac{1}{Y_t} + \beta\frac{1}{C_{t+1}}\frac{R_t}{\pi_{t+1}} \tag{8}$$

$$\frac{1}{C_t} = V_2\left(\frac{B_t}{P_tY_t},\frac{M_t}{P_tY_t}\right)\frac{1}{Y_t} + \beta\frac{1}{C_{t+1}}\frac{1}{\pi_{t+1}} \tag{9}$$

The transversality condition is

$$\lim_{t\to\infty}\beta^t\left[-\frac{P_tY_t}{P_tC_t} + V_{1,t}\frac{B_t}{P_tY_t} + V_{2,t}\frac{M_t}{P_tY_t}\right] = 0, \tag{10}$$

[3] One foundation is that the money is needed for buying a good, which motivates the individuals to hold money. The literature gives a reason why the bond provides util. The function $v$ depends on the debt level since it captures the convenience yield by the government debt. The government debt typically pays lower interest rate than other similar safe assets. This discrepancy reflects the convenience that the government bond provides. See Krishnamurthy and Vissing-jorgensen (2012).

where $V_{i,t}$ is the derivative of function $V$ w.r.t. $i$-th element at date $t$. Equation (8), (9), (10), and (7) are the necessary and sufficient conditions for the optimality of the maximization problem (6).

**Firms** There is a representative firm which converts the labor service $N_t$ to the final good:

$$Y_t = \alpha_t L_t,$$

where $\alpha_t$ is the TFP and grows at constant rate $\mu$. All the markets are perfectly competitive. So the maximization problem for the firm is

$$\max_{Y_t} \qquad P_t Y_t - \frac{W_t}{\alpha_t} Y_t.$$

Then in an equilibrium,

$$P_t = \frac{W_t}{\alpha_t}.$$

**Central Bank** The central bank issue money and can buy the government bond. The amount of the government debt held by the central bank is denoted by $\tilde{B}_t \geq 0$. The "profit" that the central bank makes can be remitted to the fiscal authority. Let $\Omega_t$ denote this remittance. So, the flow budget constraint for the central bank is[4]

$$\Omega_t + \tilde{B}_t = R_{t-1}\tilde{B}_{t-1} + M_t - M_{t-1},$$

If the remittance term $\Omega_t$ is present, (as we will see) we can consolidate the budget constraints by the central bank and the fiscal authority to obtain a single budget constraint for the government. If not, namely $\Omega_t = 0$, then we need to study two budget constraints separately. By choosing a sequence of $\left\{\tilde{B}_t, M_t\right\}_{t=0}^{\infty}$, the central bank affects the economy.

**Fiscal Authority** The government imposes the tax on the savers and conduct fiscal stimulus, $X_t$, by buying the consumption good. Let $Z_t$ denote the (real) fiscal deficit. The government can finance its deficit by issuing the government debt, whose interest rate is $R_t$. The flow budget constraint is

$$\Omega_t + \bar{B}_t = R_t \bar{B}_{t-1} + P_t Z_t,$$

where $\bar{B}_t$ is the total government debt issued by the fiscal authority. We assume that the level of the government expenditure $G_t$ is $x$% of the total real GDP. Let $z_t$ denote the deficit to the nominal GDP ratio. We explore whether a constant (and positive) $z$ is feasible or not in an equilibrium.

[4]We use a slightly different version of the budget constraint than one used in Section 2 following Mian et al. (2022). The analysis below goes through even if we use the version of the flow budget constraints in Section 2.

**Market Clearing Condition and Competitive Equilibrium** The good market clearing condition is given by

$$C_t + X_t = \alpha_t L_t.$$

The labor market clearing condition is

$$L_t = 1.$$

The bond market clearing condition is

$$B_t + \tilde{B}_t = \bar{B}_t.$$

Since $X_t / A_t L_t = x$, the consumption level of the savers is

$$C_t = (1 - x) Y_t = (1 - x) \alpha_t. \tag{11}$$

A competitive equilibrium is defined as usual. The savers and the firms maximize their utility and profit, and all the market clears. Mostly we focus our analysis on the steady state (or balanced growth path).

**Equilibrium Characterization** Let $b_t$ and $m_t$ denote the debt to GDP ratio, and the money to GDP ratio respectively. The Euler equation (8) and (9) boil down to

$$\frac{R_t}{G_{t+1}} = \beta^{-1} \left[1 - (1 - x) V_{1,t}\right] \tag{12}$$

$$\frac{1}{G_{t+1}} = \beta^{-1} \left[1 - (1 - x) V_{2,t}\right] \tag{13}$$

where $G_t$ is the growth rate of nominal GDP, which is endogenously determined. Let $R_t^*$ denote the ratio of the nominal interest rate to the growth rate of nominal GDP. We scale the budget constraints of the fiscal authority and the central bank as follows. Let $\omega_t$ and $\tilde{b}_t$ denote the remittance to GDP ratio and the ratio of the bond held by the central bank to GDP. By using these notations, the flow budget constraint of the central bank can be written as

$$\omega_t + \tilde{b}_t = \frac{R_{t-1}}{G_t} \tilde{b}_{t-1} + m_t - \frac{m_{t-1}}{G_t}. \tag{14}$$

Similarly, we scale the budget constrain of the fiscal authority:

$$\omega_t + \bar{b}_t = \frac{R_{t-1}}{G_t} \bar{b}_{t-1} + z. \tag{15}$$

Finally the bond market clearing condition is written as follows:

$$b_t + \tilde{b}_t = \bar{b}_t. \tag{16}$$

Equation (12), (13), (14), (15), and (16) and the government monetary policy (which will be specified later ) characterize the dynamics of the economy.

For simplify our analysis, we begin our analysis by assuming that the function $V$ is additively separable.

$$V(b, m) = v(b) + w(m). \tag{17}$$

An analysis for non-separable $V$ function is briefly conduced in Appendix A.

### 3.2 Equilibrium Analysis

We can consolidate the budget constraints, (14) and (15), and work on the following single budget constraint:

$$R^* b + z = b + \left(1 - \frac{1}{G}\right) m.$$

There are two points worth noting about this steady-state budget constraint equation. First, note that the total government debt issued by the fiscal authority, $\bar{b}$, disappears from the flow budget constrains, implying that $\bar{b}$ does not affect equilibrium prices at all. What really matters is the amount of the debt held by the savers $b$, who are the marginal buyers of the government debt. So, when we take the model to data, it is important to use a correct measure for the debt to GDP ratio.

Second, the consolidated government can sustain any amount of fiscal deficit $z > 0$ by increasing money supply. The growth rate of nominal GDP $G$ is determined by the steady state version of equation (13).

$$\frac{1}{G} = \beta^{-1} \left[1 - (1 - x) w'(m)\right].$$

Since $w$ is a concave function, nominal GDP growth rate is an increasing function of the scaled money supply. It follows that the central bank can provide any level of the remittance, $(1 - 1/G) m$, to the fiscal authority. So, if the central bank gives up price stability and accept high inflation, then the government can support any level of the deficit $z$.

For a moment, assume that the central bank does not abandon its mandate, and tries to achieve its inflation target $\bar{\pi}^*$. By choosing the appropriate level of money supply $m^*$, the central bank can achieve the goal. More specifically, the central bank chooses $m^*$ so that the following equation holds:

$$\left(\frac{1}{G}=\right)\frac{1}{g_A\bar{\pi}^*}=\beta^{-1}\left[1-(1-x)\,w'(m)\right].$$

The central bank remit their profit to the fiscal authority. Then the fiscal authority can sustain the fiscal deficit $z$ if

$$z(b)=(1-R^*(b))\,b+\underbrace{\left(1-\frac{1}{G}\right)m^*}_{\text{Remittance from CB}}\quad. \tag{18}$$

Using equation (12), the debt Laffer curve (18) is written as

$$z(b)=\left(1-\beta^{-1}\left[1-(1-x)\,v'(b)\right]\right)b+\left(1-\frac{1}{G}\right)m^*. \tag{19}$$

If $z(\cdot)$ is single-peaked, then there are two steady states for $z<z^*$, where $z^*$ is the maximum deficit to GDP ratio. We summarize these findings in the following proposition.

**Proposition 1.** *Suppose that the central bank has a fiscal backing $\omega\neq 0$ , the debt Laffer curve (19) is single-peaked, and $z<\max_{b\geq 0} z(b)$. Then:*

1. *there are two steady state levels $b$ which is consistent with the ratio of the debt to GDP $z$;*

2. *the equilibrium prices and allocations are independent of $\bar{b}$.*

Proposition 1 is well-known, but has an implication for many recent studies about the fiscal pace. For example, Mian et al. (2022) calibrate their model for Japan, and assume that the debt to GDP ratio is around 230%. This debt is the total government debt issued by the government, not held by the savers. In the data, the debt held by the savers to GDP ratio is greatly smaller than this number. So, their calibration target is chosen wrongly, and this particular choice has a first order effect for their result. We explore this point in Section 4.

Proposition 1 characterizes the steady state of the economy. The model is so simple that we can fully characterize the entire dynamics. We have:

**Proposition 2.** *Suppose the debt Laffer curve (19) is single-peaked, and $z\in(0,\max_{b\geq 0} z(b))$. If the initial debt level is strictly smaller than $b_H$, $b_{-1}<b_H$, then*

$$b_t\to b_L.$$

*If the initial debt GDP ratio is $b_H$, then the debt level stays at that level. If $b_{-1}>b_H$, then there does not exist an equilibrium consistent with the constant debt, $z_t=z$.*

*Proof.* The dynamic equation is

$$b_t = \beta^{-1}\left[1-(1-x)\,v'(b_{t-1})\right]b_{t-1} + z - \left(1-\frac{1}{G}\right)m^*.$$

Then the debt increases at date $t$, $b_t > b_{t-1}$, if and only if

$$z > \left(1-\beta^{-1}\left[1-(1-x)\,v'(b_{t-1})\right]\right)b_{t-1} + \left(1-\frac{1}{G}\right)m^* = z(b_{t-1}).$$

Note that the debt Laffer curve is single-peaked. and $z < \max_{b\geq 0} z(b)$. So, there exists two steady state, $b_L < b_H$ such that $z(b_L) = z(b_H) = 0$. If $b_{t-1} > b_H$, then $z > z(b_{t-1})$. So, the debt increases over time, and eventually the interest rate becomes bigger than the nominal interest rate. Then the government cannot support constant $z > 0$. If $b_{t-1} < b_H$, then $b_t < b_{t-1} < b_H$. If $b_{t-1} < b_L$, then $b_{t-1} < b_t$. We have established Proposition (2). □

Finally we discuss the possibility of the free-lunch policy such that the government increases its fiscal spending without increasing tax. It turns out that such a policy is infeasible if the initial debt to GDP ratio is on the "wrong" side of the Laffer curve.

**Proposition 3.** *(Corollary 1 in Mian et al. (2022)) Suppose that the economy is at the steady state and the initial debt level is $b$. Then if $b \geq b^*$, then the fiscal authority cannot increase the fiscal deficit without increasing taxes. If not, then the fiscal authority can increase the fiscal deficit without increasing taxes.*

*Proof.* Suppose that the government increases its deficit to GDP ratio to $z' > z$. Then

$$z(b) < z'.$$

The initial debt level $b$ is greater than the new steady state debt $b'$ consistent with $z'$. So, Proposition (2) implies that there does not exists an equilibrium consistent with the new fiscal deficit $z'$. □

### 3.3 Desirability of Negative Interest Rate Rule

Now we consider additional policies taken by the central bank. In particular, we consider the following situation. Let $b_0$ denote the initial steady state debt and $R_0$ the associated initial interest rate. The economy is on the wrong side of the debt Laffer curve at the beginning, $b_0 = b_H$ for some $z < \max_{b\geq 0} z(b)$. To support the fiscal deficit $z$, the fiscal authority issues too much debt, and get trapped in a high debt with high interest rate steady state. Is it possible for the central bank to give a support for the fiscal authority? If so, how should the central bank provide help? We demonstrate how the central bank can provide such help for the fiscal authority. In particular, we demonstrate that the central bank can design its policy to move the economy to the right side of the Laffer curve.

Consider the following policy by the central bank. The central bank goes to the secondary market of the government debt, and buy the debt from the savers. By doing so, the nominal interest rate on the government debt goes down, encouraging the savers to sell their bond to the central bank. If the central bank buys $b_H - b_L$ units of the debt, then the savers owns $b_L$ units of the government debt and the nominal interest rate becomes lower and even negative. The central bank effectively undertake the debt, which is not profitable. The remittance declines, and even might become negative:

$$\omega = \underbrace{\left(R^* \left(b_L\right) - 1\right)}_{\leq 0} \underbrace{\left(b_H - b_L\right)}_{\geq 0} + \left(1 - \frac{1}{G}\right) m \geq \left(1 - \frac{1}{G}\right) m.$$

While the central bank might be insolvent superficially, the economy jump to the new steady state on the right side of the Laffer curve. Now, the fiscal authority can increase its deficit further without imposing higher taxes from Proposition 3. We summarize this finding in the following proposition.

**Proposition 4.** *Suppose that the initial steady state debt level is* $b_H$*. Then the central bank can buy the appropriate amount of the government debt so that the economy immediately jumps to* $b_H$*. The remittance from the central bank can be negative.*

This policy that the central bank buys the debt in a secondary market is a good policy since the policy provides more fiscal space for the fiscal authority. Arguably speaking, this is what the BOJ has done in the past. As Figure 3a indicates, the BOJ started to buying the various maturities of the JGB since around 2012. As predicted by the model, the long-term interest rate on the debt declined substantially, which helps the fiscal authority. From this perspective, the BOJ's unconventional policies successfully reduced the fiscal burden of the fiscal authority, and are good policies in this sense.

In the argument above, we assume that the economy at the beginning is characterized by the steady state. Namely, the deficit level is not too high, $z < \max_{b \geq 0} z\left(b\right)$. We now relax this condition, $z > \max_{b \geq 0} z\left(b\right)$. Then the debt increases over time since for any $b \geq 0$,

$$z > \left(1 - \beta^{-1} \left[1 - R^* \left(b_{t-1}\right)\right]\right) b_{t-1} + \left(1 - \frac{1}{G}\right) m^*.$$

Such an explosive path does not satisfy the transversality condition so that such an path is not an equilibrium. Even for this case, the central bank can sustain the deficit by abandoning its mandate, price stability. The central bank can increase its money supply so that the following equation holds:

$$z = \left(1 - \beta^{-1}\left[1 - R^*\left(b_{-1}\right)\right]\right) b_{-1} + \left(1 - \frac{1}{G\left(m^{**}\right)}\right) m^{**}.$$

Basically the central bank earns more seigniorage and remit to the fiscal authority. Note that in this case, the inflation rate increases, and possibly increases a lot. Let $\pi^{**}$ denote the (net) inflation rate associated with $m^{**}$. Then

$$\pi^{**} - \pi^* = \ln G\left(m^*\right) - \ln G\left(m^*\right) = \int_{m^*}^{m^{**}} \frac{G'\left(m\right)}{G\left(m\right)} dm.$$

So, the semi-elasticity, $G'\left(m\right)/G\left(m\right)$, plays a key role, which is known to be difficult to estimate. Also, note that we need to know the entire elasticity $\left(G'\left(m\right)/G\left(m\right)\right)_{m=m^*}^{m^{**}}$ to figure out the implication for $\pi^{**}$. This is even harder than estimating the current elasticity, and involves a difficult extrapolation problem. So, from the perspective of the price stability, this policy can be significantly bad.

## 4 Why Do Mian et al. (2022) Find Japan Has Ample Fiscal Space?

The model by Mian et al. (2022) is a special case of our model. In this model, there are no money (namely no central bank), and the bond only provides the convenience yield. So, the debt Laffer is simply

$$z\left(b\right) = \left(1 - \frac{R\left(b\right)}{G^*}\right) b. \tag{20}$$

They parameterize the ratio of the interest rate to the nominal GDP growth rate as follows:

$$\frac{R\left(b\right)}{G^*} = \frac{R_0}{G^*} + \varphi \frac{b - b_0}{b_0},$$

where $\varphi$ is the semi-elasticity, and they have an estimate for it. Substituting this expression into equation (20),

$$z\left(b\right) = \left(1 - \frac{R_0}{G^*} - \varphi \frac{b - b_0}{b_0}\right) b = \left(g^* - r_0 - \varphi \frac{b - b_0}{b_0}\right), \tag{21}$$

where $g^*$ and $r_0$ are the logged nominal GDP growth rate and the logged nominal interest rate. For the second equality, we use the first order approximation of $1 - R_0/G^*$. The debt Laffer curve (21) corresponds to their Laffer curve which is obtained in the continuous-time version of the model.

Mian et al. (2022) finds that Japan has an ample space for "free lunch" policies. That is, according to their model, Japan can increase the fiscal spending $x$ and simultaneously reduce the outstanding debt without increasing its tax $\tau$. We argue that this result is driven by the two implausibly strong assumptions. The first assumption is that the economy is characterized by the ZLB. This assumption is made in other contexts, but does not receive any empirical supports. Given the prolonged stagnation of the Japanese economy, it is not clear that the economy is not doing well because of the demand shortage problem as assumed by Mian et al. (2022). Indeed, the literature has a hard time to show that the Japanese poor economic performance is associated with the problems of the demand side. Wieland (2019) analyzes the Japanese economy before and after the 2011 earthquake and examines whether the implications of the NK model under the zero-lower bound constraint match the Japanese economy during this period. He concludes that it is difficult to characterize the Japanese economy with the NK model under the zero lower bound constraint. Moreover, Aruoba et al. (2018) use their structural model and examine whether the Japanese economy and the US economy are characterized by a deflationary equilibrium. Aruoba et al. (2018) find that the Japanese economy is characterized by a deflationary equilibrium, not the NK model with occasionally binding zero-lower bound constraint. So, the negative implications drawn from a NK model with occasionally binding zero-lower bound constraint are not applied to Japan. Ramey (2020) even argues that the low growth of the US economy is not associated with the demand shortage, but low productivity growth. Her argument is also applied for Japan: the unemployment rate for Japan has been low and stable, and the productivity growth rate has stagnated. The same argument is actually made by Hayashi and Prescott (2002). Giving these papers, it is far from clear that the Japanese economy lacks demand, and is well-characterized by Mian et al. (2022)'s model.

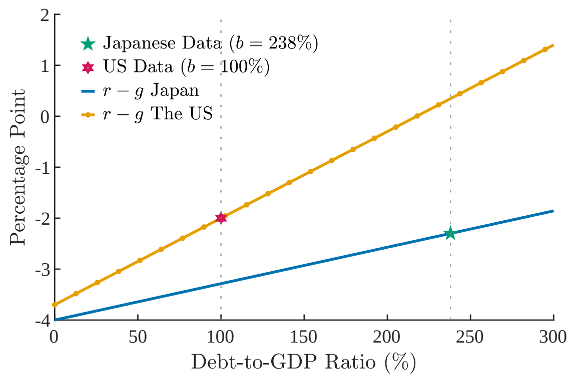
The second assumption is related to the calibration strategy used in Mian et al. (2022). The parameter choice by Mian et al. (2022) implies that Japan has ample fiscal space, but some of the parameter choices made by Mian et al. (2022) are implausible. We demonstrate that their result is greatly sensitive to their choice of the parameters.

To highlight the implausibility of the second assumption, recall the debt Laffer curve. The debt Laffer curve is

$$z = b\left[g^{*} - r_0 - \varphi\frac{b-b_0}{b_0}\right].$$

When $R$ is insensitive to the movement of $b$, the government can freely increase its spending without increasing the tax (since $r < g^{*}$). The limit case is that the interest rate is independent of $b$. Then any deficit level $z$ can be sustainable by issuing more debt. Such a free-lunch policy is limited when $R(b)$ increases quickly as the government issues more debt. In this case, the government needs to pay more interest rate payments. Moreover, when the nominal interest rate exceeds the growth rate, $g^{*}$, then the government is forced to increase its tax. So the fiscal space is determined by (1)

Figure 4: Spread




the initial difference between the nominal interest rate and the nominal interest rate (the initial condition) and (2) the semi elasticity of $r(b)$.

Now we show that the parameter choices by Mian et al. (2022) imply (1) and (2). The initial difference between the growth rate and the interest rate is 2% for the US, and 2.3% for Japan. Moreover, the convenience yield of the US drastically differs from one of Japan. Note that

$$r_{US}(b) - g^*_{US} = r_{US} - g^*_{US} + \varphi \frac{b - b_0^{US}}{b_0^{US}}$$
$$r_{JP}(b) - g^*_{JP} = r_{JP} - g^*_{JP} + \varphi \frac{b - b_0^{JP}}{b_0^{JP}}.$$

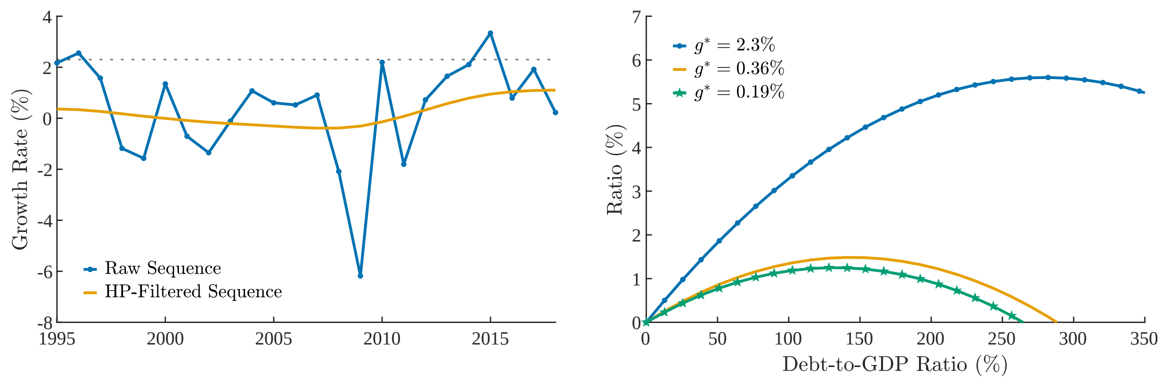
Figure 4 plots these two lines under the benchmark parameterization by Mian et al. (2022). Note that the spread between the nominal interest and the GDP growth rate for the US is higher than that of Japan for all the positive debt levels. This result is largely driven by the semi-elasticity term. The semi-elasticity of the debt with respect to the nominal interest rate for the US is $\varphi$ when the debt to GDP ratio is 100%. However, the semi-elasticity for Japan takes the same number $\varphi$ when the debt to GDP ratio is more than 200%. So, the convenience yields are fundamentally different. If the U.S. government debt-to-GDP ratio $b$ were at the same level as that of Japan 238%, the interest rate would exceed the growth rate, and the government cannot finance its fiscal expenditure by issuing its debt.

One economic interpretation of this calibration result is that the Japanese bond provides more convenience than the US Treasury debt, which is implausible. The US dollar is much more used and the US debt provides more collateral value than the JGB. Another interpretation is that individuals in Japan prefer to have the government debt for some reason. In this case, we need to strongly buy the linear functional form assumption, which is implausible. At some point, the Japanese individuals might stop liking to have the JGB when the debt to GDP ratio exceeds 300%. The data does not tell us whether this would not occur since the debt to the GDP ratio has never

Figure 5

(a) Nominal GDP Growth Rate

(b) Debt Laffer Curve




reached more than 250%. We conclude that the parameter choices for Japan by Mian et al. (2022) are implausible.

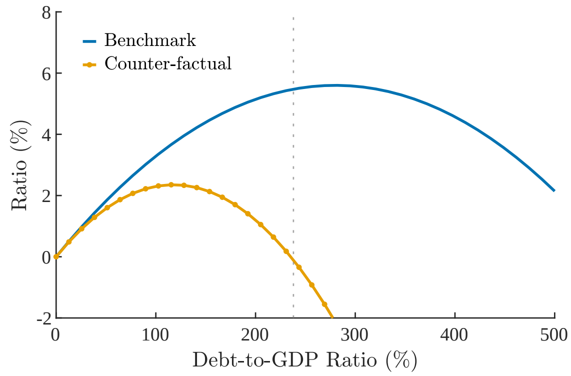
Another implausible parameter choice is that Mian et al. (2022) assume that the steady state growth rate of nominal GDP is 2.3%. In Figure 5a we plot the growth rate of nominal GDP and the growth rate of HP-filtered nominal GDP for Japan since 1994. It is clear from this figure that the growth rate of nominal GDP is far below 2.3%. The nominal GDP growth rate has exceeded 2.3% only twice since 19954. The average growth rate based on actual nominal GDP is 0.36%, and one based on the filtered nominal GDP is 0.19%. If we use these nominal growth rates, the shape of the debt Laffer curve significantly changes. In Figure 5b, we draw the debt Laffer curves for these values. Based on the parametrization of Mian et al. (2022), we find that the maximum deficit to GDP ratio is around 6%. When we assume that the nominal GDP growth rate is .36%, then the fiscal capacity of Japan shrinks significantly, and it is even infeasible to sustain 2% debt to GDP ratio. Given the fact that the ratio of the deficit to GDP is around 4%, our economic model implies that the fiscal authority needs to raise tax or the BOJ needs to abandon the mandate of price stability.

To further analyze the issue of their parametrization, suppose that the semi-elasticity for Japan is $\psi = 1.7\%$ when the debt to GDP ratio is 100%, $b = 100\%$. The spread between the nominal interest rate and nominal GDP growth is

$$r_{JP}(b) - g^*_{JP} = r_{JP} - g^*_{JP} + \varphi \frac{b - b_0^{US}}{b_0^{US}}. \tag{22}$$

The debt Laffer curve under this specification is depicted in Figure 6. If the spread is given by equation (22), then it is infeasible for the Japanese government to support the current fiscal spending without balancing the budget. The spread, $r_{JP}(b_{JP}) - g^*_{JP}$, is almost zero, and Japan

Figure 6: Debt Laffer Curve




does not have any fiscal space for free lunch policies. Moreover, the maximum deficit to GDP ratio consistent with the steady state is around 2%. As Figure XXX, the average ratio of the deficit to nominal GDP since 1994 is more than 4%, which exceeds the limit that the model implies. So to believe the implication of Mian et al. (2022), it is critically important to understand why the JGB provides more convenience than the US Treasury debt.

In sum, we lay out various problems of the model specification and calibration by Mian et al. (2022). We conclude that at this state, it is hard to strongly buy their argument, and we need to understand the Japanese fiscal condition more deeply before giving a drastic advice for Japanese policymakers.

## 5 Conclusion

This paper examines the monetary policy in the new economic environment where the nominal interest rate is lower then the growth rate of nominal GDP. We identify a situation where the central bank's bond purchase policy provides more fiscal space for the fiscal authority. Moreover, we confirm that the central bank is forced to support the fiscal sustainability by abandoning the price stability. While we keep the assumption that the new economic environment keeps continuing, Jorda et al. (2019) argue that this environment is rare from the historical viewpoint. Given this finding, we need to understand why the nominal interest rate is so low, and when this new environment turns into the old environment, where the nominal interest rate is higher than the growth rate of the economy. Such a research is left for the future.

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# A Analysis Without Separability

We consider the case where the convenience function is not additively separable. Assume that

$$V_i \geq 0$$
$$V_{ii}, V_{ij} \leq 0.$$

Suppose that the mandate of the central bank is to achieve the target inflation rate $\bar{\pi}^*$:

$$\frac{1}{g_A \bar{\pi}^*} = \beta^{-1} \left[1 - (1-x) V_2 (b, m)\right].$$

Since $V_{ij} \leq 0$, the implicit function $m(b)$ is declining.

$$m'(b) = \frac{\partial m}{\partial b} = -\frac{V_{21}(b,m)}{V_{22}(b,m)} < 0.$$

The consolidated budget constraint is

$$z(b) = (1 - R^*(b))\, b + \underbrace{\left(1 - \frac{1}{G}\right) m(b)}_{\text{Remittance from CB}},$$

where

$$R^*(b) = \beta^{-1} \left[1 - (1-x) V_1 (b, m(b))\right].$$

Note that

$$\frac{\partial R^*}{\partial b} = \frac{\partial R^*(b, m(b))}{\partial b} = \underbrace{R_1^*(b, m(b))}_{\geq 0} + \underbrace{R_2^*(b, m(b))}_{\geq 0} \times \underbrace{m'(b)}_{<0} \geq R_1^*(b, m(b)).$$

In this case, the debt level consistent with $1 = R^*(b)$ becomes larger than one in the separable case. The fiscal authority provides more liquidity into the market, so that the central bank does not issue a lot of money to deliver the target level of inflation. This is possible only if the debt can play a similar role as money (i.e. they are substitutable). However, the debt Laffer might shift downward since the remittance from the central bank to the fiscal authority declines.