Discussion Paper Series A No.728

Trial to Count the Discounted Envy ; Evaluation and Compensation

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December 2021

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December 7, 2021

Abstract

No envy or envy is the important object or problem the social choice researcher often study and many researcher try to decrease the envy existed in economy while keeping in mind the problem of social welfare. Feldman and Kirman(1974) try to count the number of envy of each subject or in economy directly . Their method is tractable in term of evaluating the degree of fairness and comparing the degree of fairness or envy among economies. But we consider that their method has serious problem. In short, they treat the envy of the richest subject equally with that of the poorest subject. So, we try to modify Feldman and Kirman (1974) from the point of justice and we introduce such a discounted envy that we discount envy each subject feel by the envy felt by the other subject, and we reconsider the problem of evaluation and compensation.

JEL H89; I30; D39; D63

Keywords envy(no-envy) fairness, extended utility function, distribution, evaluation

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1 Introduction

As standard for judging if social welfare attains , economists often select pareto efficient. where any subject could not expand his utility at the risk of other's utility.

But even if pareto efficient is satisfied in economy, we often find the situation that the vast unfairness of distribution is left ignored or that the walfare of the most misserable subject is left ignored. From the point of justice, we should not be ignore such a situation.

So, some economists began to try to introduce the problem of justice into economic model.

The Bergson-samuelsaon social welfare function (Bergson (1938) Samuelson(1938)) provide the basis for answer to distribution from point of view of justice. In short, they develop this problem on the space of vectors of utilities. However they are not in ordinal and they often rely on interpersonal comparison of utility which become object of issues associated with the measurement of each agent utility.

Arrow (1954) critisize the Bergson and samuelson and does not require utlity imformation and develop this problem in the formal social choice . But his model is too abstract to understand resource allocation ploblem

Modern economists often consider fainess as well as pareto-effient in economy and they try to descreasae envy under the constarained that parato efficient is satisfied and inquire the condition that the both conditions are satisfied simultaneoully

Folley(1967) formally first design envy or no-envy in ordinal case. He consider that subject feel envy when he compare the situation of his own with the situation of others , in short, he graps envy as a feeling that reflects negatively not only on the external circumstances in which subjects find themselves but also on the preference. Here, in his model, the external circumctances are defined by the consumption bundles the other subject receive.

So, He introduced analysis of envy into economics through interpersonal comparison of consumption bundles.

Archibald and Donaldson(1979), Chaudhuri(1985) more stricktly consider about envy. For example, they consider that, in order to say agent i as being envious against agent j , it is necessary to specify his preference as exhibiting consumption externalities that that fail to be monotonic with respect to agent$j$ Through such a consideration, Chaudhuri(1985) formalize such a notation and establish binary envy relation that Such a binary envy relation sometimes can be translated into relation using the extended utility shown as the below.(see Feldman and Kirman (1974)etc)

$$u_i(x_j)$$

It means that the utility subject i feel if the subject i is replaced by the position of subject j

By using entended utility function, Feldman and kirman (1974) consider that subject i feel envy against subject$j$ when the below relationship is satisfied.

$$u_i(x_j) - u_i(x_i) > 0$$

It means the utility of subject i if the subject i remains being put on his own position is smaller than the utility of rsubject i if he or she is replaced by the position of subject j

Moreover, by using this extended utility function and the above relationship, they define no envy or fairnes as situation that the above relationship is not seen in economy.

While they show thier unique way to measure the degree of env existed in econom. In short, they count directlly the case of the envy subject feel in the economy. as the below.

$$e_i(x) = \sum_{j=1}^{n} u_i(x_j) - u_i(x_i)$$

By using the number, they estimate the rnviness in economy and try to eliminate envy in economy.

Their method contribute to the envy ttheory in the two main points very well (see Fleurbaey (2007))

First by their method, we could measure degree of the envy exsisted in economy or society easilly and could compare the enviouness in economy of society.

Second, we get the way to redistribution for the compansate of enviness easilly.

But we consider that the trial of Feldman and Kirman(1974) has serious problem, In short, in their framework envy of each subject is treated equallity whether subject is the rich or the miserable. We should not treat the envy of the miserable with the envy of the rich equally.

So, we try to reconstruct the envy-count-modl and reconsider the evaluation and compensation.

2 The Model(Economy Space)

We consider n-trader m-good wconomy We assume that each of the m comodities is homogeneous and infinitely divisible.

In short, we consider An allovcation as the below.

$$x = (x_1, x_2, x_3, ..., x_n) \tag{1}$$

a point in $R^{mn}_{+,}$ whose i th component, x_{ij} is the bundle of goods assighned to trader i under x. We will let ω be a fixed initial allocation, we assume that o$\sum_{i=1}^{n} \omega_i > 0$ We also define $A(\omega)$ to be the set of allocations which are feasible in our economy. That is, $A(\omega) \equiv x : x \geqq 0, \sum_{i=1}^{n} \omega_i = \sum_{i=1}^{n} x_i$

We will assume that utility function of all traders are strictly quqsi concave If x_i and y_i are distinct nonnegative m vectors, $0 < \gamma < 1$ and $u_i(x_i) \geqq u_i(y_i)$ then $u_i(\gamma x_i + (1-\gamma)y_i) > u_i(y_i)$ We will also assume that all u_i are strictlly monotonic.

Based on the economiy space, Feldman and Kirman suggest the two bellow method to count or measure the envy of each subject and sum up each envy and existed in economy I Consider an allocation x. if a subset S of traders can redistribute its own resources in a way which makes all of its members at least as well off as$x$ makes them. and makes some of them better off Ş can block x

If there are bundles x_i for all i in S such that $\sum_{iinS} s_i = \sum_{i=1} \omega_i$ $u_i(s_i) \geqq u_i(x_i)$ for all i and $u_i(s_i) > u_i(x_i)$ for some j in s Then S block x.

An allocation is core is in the core if no group of traders cannot be block it. An allocation is Parato optimal is in the core if it cannot be blocked by the whole set of traders.

If p is a vector of prices and $\hat{x}$ is an allocation, and $\hat{x}_i$ maximizes $u_i(x_i)$ subject to $px_i \leqq p\omega_i$ for all i, then we say that p, $\hat{x}$) is a competitive allocation., and that $\hat{x}$ is a competitive equilibrium allocation.

We say that an allocation x is fair if every pair of traders, $u_i(x_i) \geqq u_i(x_j)$

3 The way for estimation of envy of subject or in economy 1 (ordinal case)

3.1 our revised against the count by Feldman and Kirman(1974) and our method

Based on the above economic space,,in order to evaluate the envy existed in economy, Feldman and Kirman (1974) define $C(x)$ ≡the number of pairs i, j for whom $u_i(x_i) < u_i(x_j)$

where subject i feel envy against dubject$j$.

In short, they count the number of the envies that are felt by each subject directlly and they estimate the envy againsst other subject in economny that is made of such a numberies

We consider that their method and the estimation for the envy existed in economy include serious problem. In short, they estimate the envy of the most misserable subject equally with the envy of the richest subject.

Their method or the estimation are not enough from view point of fairness or justice

We should not evaluate the envy of the rich subject who probably is felt envy by many subject equally with the envy of the poor who probably is felt by few subject.

So, We try to reconsider the method of Feldman and Kirman (1974)

We try to arrange the their method.

First, we introduce n_{ij} as the number of the object of such a subject i feel envy against (subject ij)

If we sum up n_{ij} over all the subjects in economy; from 1 to n and we could get the number of envies in economy as the below.

$$C(x) = \sum_{i=1}^{n} n_{ij} \tag{2}$$

This number is corespond to the number of Feldman and Kirman (1974), but in our setting, we could show the envy of subject clearlly. While, for our purpose, next we introduce also the number of the subject who feel against the subject i, (subject k), n_{ki}. And before suming up, we should divide n_{ij} by n_{ki} and could get the modeified number of envies of subject$i$ below number.

$$Cd_i = \frac{n_{ij}}{n_{ki}} \tag{3}$$

Next, we sum up the above number from 1 to n and get the below equation.

$$Cd(x) = \sum_{i=1}^{n} \frac{n_{ij}}{nk} \tag{4}$$

This is modified number of envies in economy. Furthermore, ,as the population of all the subjects in economy is n and the other subject than the object whom is felt envy by subject i and subject i feel envy against subject i., the below relationship is satisfied,.

$$n_{ki} = n - n_{ij} - n_i \tag{5}$$

So, we translate the population (5) into the above modeified envy number(4) and the below number.

$$Cdd((x) = \sum_{i=1}^{n} \frac{n_{ij}}{n - n_{ij} - n_i} \tag{6}$$

So, we get the modefied number of envy by which distinguish the envy of subject who is felt envy by many subjects and the envy of subject who felt envy by the few subject

3.2 Example

We will show the effectiveness of estimation by our method through example.(See illustlate A).

Now, we assume that there are three classes,; upper, middle, lower in economy. And we two cases; Case(A-1); There are nine subjects; four subjects in upper class, one subject in middle class and four subjects in lower class. Case(A-2); There are nine subjects; four subjects in upper class, one subject in middle class and one subject in lower class. Next, We notice the envy of the subject in the middle class. Because Feldman and Kirman (1974) count the number of the envies of subjects directly or mechanically by their method, we could conclude that the number of the envies of subject in the middle class has is 4. both in Case (A-1) and in Case(A-2.) While, by our method ,because the number of the envies of the subject in middle class by their conted number 4 is divided or is discounted by the number of the subjects in lower class 4, so we get the number 1 as the number of the subject in middle class. so we could estimate the envy of subject in middle class who is felt by the subject in the lower class more mode than Feldman and Kirman(1974).

Here, the estimation of the envy of the (particular)subject in the middle class has also influence on the estimation of envy of all the economy Now we consider the below two examples(distributions) against 9 subjects. Case (B-1) the same case the above example about the envy of subject in the middle class; There are four subjects in the upper class, one subject in the middle class and four subjects in the lower class Case(B-2) There are four subjects in the upper class, four subjects in the middle class and one subject in the lower class By Feldman and Kirman(1974) which count the number of envy directly or mechanically, we could count 24 envies in economy both in case(B- 1) and in case (B-2).

But we consider that we should estimate the envies existed in the case (B-2) more seriously than the envies existed in case (B-1,) because in case (B-2) there is only one subject in the lower class. While, by our method, as we do not count the number of envies of the subject in middle class mechanically but adjust those by discounting by the number of the envy of the lower class and we estimate the total number of envy in economy 24 in Case (B-2) and 17 in case (B-1). which is the result fit to our institution.

4 The way for Evaluation of envy of each subject or in economy 2(cardinal case)

ordinal case is certainlly tacvitable setting , but when we campare envy , it is important to measure how strong the subject feel envy

So, cardinal case is tacvitable setting use extensive utilityfunction

First, , Feldman and Kirman (1974) use the cardinal utility function dirrectly and define the envy of the each subject agansit the other subject

$$e_i(x) = \sum_{u_i(x_j)>u_i(x_i)} u_i(x_j) - u_i(x_i)$$

Additionally, they sum up the envy of the envy and suggest the envy existed in economy as the below.

$$e(x) = \sum_{i=1}^{n} e_i(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} u_i(x_j) - u_i(x_i)$$

But We consider also that there are serious plobrem in their method and estimation of the envy existed in economy by Feldman and Kirman(1974)

In short, they treat the envy of the most misserable subject equally with the envy of the richest subject

While, we consider that we should not treat the envy of the most misserable subject equally with the envy of the richest especially in argument about justice of fairness.,

We try to modefy the setting of Feldman and Kirman (1974) from such a problem consciousness.

4.1 our method

In order to do so, we introduce the cocept of the degree how the subject is felt by other subject as the below.

$$ee_i(x) \equiv e_i(x) = \sum_{u_j(x_i)>u_j(x_i)} u_j(x_i) - u_j(x_i)$$

it means the degree of the envy of subject i felt by the other subject

The larger B is, Bbecome, more the subject is felt by other subject. while, the smaller$B$ is, , less the subject is felt by other subject.

Next, We suggest to divide the envy of the subject e_i by B at first,

$$ed_i(x) = \frac{\sum_{j=1}^{n} u_i(x_j) - u_i(x_i)}{\sum_{j=1}^{n} u_i(x_j) - u_i(x_i)}$$

and sum up not sum up the envy of the subject

We sujes to tdivi the envy of each subject $e_i(x)$ by B. before suming up $e_i(x)$

By doing so, we can estimarte the envy of the misserable subject higher and the envy of the richest lower.

Our modyfiing setting setting effct the estimatimation the envy in economy.

See

$$ed(x)\sum_{i=1}^{n} e_i(x) =)\sum_{i=1}^{n} \frac{\sum_{j=1}^{n} u_i(x_j) - u_i(x_i)}{\sum_{j=1}^{n} u_i(x_j) - u_i(x_i)}$$

By the above setting, we could analyze envy or no envy between subjects including differentiation of utility of subjects

5 Method to eliminate envy in economy

Bsed on their above number, Feldman and Kirman (1974) try to minimize the envy existed in economy, in ordinal version or in cardinal version. required on the constraint that no trader be made worse off by a fairness increasing (Pareto-efficiant-constraint)

In short, at first, in ordinal version, they try to minimize $C(c)$ subject to $u_i(x_i) > u_i(\omega_i)$ for all i. But it cannot generally be solved by standard method. so they confine their analysis to a speciall case.(one composite good case)

In order to analyze, they will assume that every trader has the same strictly quasi-concave , monotonic function u. and u is homothetic : $\gamma \geqq 0, u(\gamma x) = \phi(\gamma)u(x)$ for Under these conditions it is possible to reduce the economy to one in which

(1)every trader has a bundle $\hat{x}_i$ which is proportionjal to$\sum_{i=1}^{n} \omega_i$.

(2) there is a social surplus bundle $L \equiv \sum_{i=1}^{n} \omega_i - \sum_{i=1}^{n} \hat{x}_i$. (3) $\hat{x}_i = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n$ is efficient in the economy with total resources $\sum_{i=1}^{n} \omega_i$ (4) $u(\hat{x}_i) = u(\omega_i)$ for all i.

where $\hat{x}_i = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n$ is efficient in the economy with total resources $\sum_{i=1}^{n} \omega_i$

And Feldman and Kirman (1974) prepare the arrangement for distribution. as the below (see illustlate B) and try to distribute the above surplus L into each subject, in order to eliminate envy in economy with keeping Pareto- effiecency constrained.

5.1 the arrangement for redistribution in order to eliminate envy

At first, they partition the traders $i$in the economy into $h \leqq n$ classes, $S_1, S_2, S_3, ..., S_h$ by putting traders with equal $\hat{x}_i$ into the same class,S_i They assume that the classes are numberered from richest to poorest $\hat{x}_{i1}. > \hat{x}_{i2} > ..... > \hat{x}_{in}$.

Moreover they define $\hat{\delta}_2 \equiv \hat{x}_{i1}. - \hat{x}_{i2}$ the difference between the wealth of menbers of classS_1 and the wealth of menbers of clas S_2

and define $n_h \equiv$the number of menbers of class S_h .

Moreover, based on such an above setting, they show solve the below minimize problem

$\min C(\tau(x) = \sum_{i<j} n_i n_j$

subject to $\sum_{\tau=2} \delta_\tau k_\tau < L$

5.2 the way of Kirman and Feldman(1974) and their limit

Since subject i with the same wealth is partioned into the same class, the only way to eliminate instances of envy without making anyone worse off is to make groups of trders move from lower classes to higher classes.

It is clear that any total migration upward can be represented as a vector of one-step upward moves therefore any movement upward can be represented by a vector$(k_1, k_2, ..., K_h)$, where $k_\tau \equiv$ the number of indivisuals who move from $S_\tau$to $S_{\tau-1}$

In short, the method of Feldman and Kirman (1974) to eliminate the envy is to decrease $C(x)$ along the above uproad step with distribution of the social surplus L as the below.

$\min C(\tau(x) = \sum_{i<j} n_i n_j$

subject to $\sum_{\tau=2} \delta_\tau k_\tau < L$

Here, when the social surplus L is enough large, such a redistribution can be operated and we could not have serious problem.

But, when the social surplus L is not enough large, we face serious problem, in short, we could not give answer to the way of distribution of L

While by our method, we can give solution

In order to see such a situation,

we develop $C(\tau(x) = \sum_{i<j} n_i n_j$ as the next.

$(n_2 - k_2 + k_3)(n_1 + k_2) + (n_3 - k_3 + k_4)(n_2 - k_2 + k_3 + n_1 + k_2) + ... = \sum_{i<j} n_i n_j + \sum_{\tau=2}(n_\tau - n_{\tau-1}) + \sum_{\tau=2} k_\tau (k_{\tau-1} - k_\tau)$

So, when the constrain is bind, we get the opyimal problem as the below.

$(n_2 - k_2 + k_3)(n_1 + k_2) + (n_3 - k_3 + k_4)(n_2 - k_2 + k_3 + n_1 + k_2) + ... = \sum_{i<j} n_i n_j + \sum_{\tau=2}(n_\tau - n_{\tau-1}) + \sum_{\tau=2} k_\tau (k_{\tau-1} - k_\tau) \theta (k_2 \delta_2 + k_3 \delta_3 + ... + \delta_\tau k_\tau)$ where θ is Lagrange multiplier.

Moreover, we differenciate the above equation by k_τ and We get the optimal condition $\delta_1, \delta_2, ...\delta_\tau$as the below.

$\theta\delta_\tau = 2k_\tau(n_\tau - n_{\tau-1}) - 2k_\tau + k_{\tau+1}$

But We could not get information about distribution from this caliculated result.

5.3 our method

While, in our model, from(6) we develop as the below. and get the optimal problem. $C(x) \equiv \frac{n_i n_{ij}}{n_{ki}} = \frac{n_i n_{ij}}{n-n_i-n_{ji}} =$

$$\frac{(n_2-k_2+k_3)(n_1+k_2)}{(n-n_1-k_2-k_3)} + \frac{(n_3-k_3+k_4)(n_1+n_2+k_3)}{(n-n_1-n_2-k_3-k_4)}. + ...$$

$$+ \frac{(n_{\tau 1} k_{\tau-1} {}_{-} k_\tau)(\sum_{i<\tau-1} n_i + k_{\tau-1})}{n-\sum_{i<\tau-1} n_i - n_{\tau-1} - k_\tau} + \frac{(n_\tau - k_\tau + k_{\tau+1})(\sum_{i<\tau} n_i + k_\tau)}{n-\sum_{i<\tau} n_i - n_\tau - k_{\tau+1}}.+.........$$

$$+\theta(k_2\delta_2 + k_3\delta_3 + ... + \delta_\tau k_\tau$$

Moreover, we differenciate this equation byk_τ and we could get the below.

$$\theta\delta_\tau =$$
$$\frac{\sum_{i<\tau} n_i - 2k_\tau + n_\tau - k_{\tau+1}}{n-\sum_{i<\tau-1} n_i - n_{\tau-1} - k_\tau} +$$
$$\frac{\sum_{i<\tau-1} n_i + k\tau - 1}{(n-\sum_{i<\tau-1} n_i + n_{tau} - n_{|tau-1} - k_\tau)^2} +$$
$$\frac{(n_{\tau+1} + k_{\tau-1} - 2k_\tau)(\sum_{i<\tau-1} n_i + k_{\tau-1})}{(n-\sum_{i<\tau-1} n_i + n_{tau} - n_{|tau-1} - k_\tau)^2}$$

MStill oreovere we diffirerantiarte this equation by $\sum_{i<\tau} n_i$and get the below.

$$\frac{\sum_{i<\tau} n_i - 2k_\tau + n_\tau - k_{\tau+1} + n - \sum_{i<\tau} n_i - n_{\tau-1} - k_\tau}{(n-\sum_{i<\tau} n_i - n_{\tau-1} - k_\tau)^2} +$$
$$2(n - \sum_{i<\tau} n_i + n_\tau - n_{\tau-1} - k_\tau)(\sum_{i<\tau} + k_{\tau-1})$$
$$\frac{(n-\sum_{i<\tau} n_i + n_\tau - n_{\tau-1} - k_\tau)^2 + 2(n-\sum_{i<\tau} n_i + n_\tau - n_{\tau-1} - k_\tau)(\sum_{i<\tau} + k_{\tau-1})}{(n-\sum_{i<\tau} n_i + n_\tau - n_{\tau-1} - k_\tau)^4} +$$
$$+ \frac{(n_{\tau+1} + k_{tau-1})(n-\sum_{i<\tau} n_i + n_\tau - k_{\tau-1} + 2k_\tau)^2 + 2(n-\sum_{i<\tau} n_i + n_\tau - n_{\tau-1} - k_{\tau-1})((n_{\tau+1} - k_{tau-1} + 2k_\tau)(\sum_{i<\tau} n_i + k_{\tau-1})}{(n-\sum_{i<\tau} n_i + n_\tau - n_{\tau-1} - k_\tau)^4} >$$
0

We could find that the sign of this caliculated result is plus. So, We could get information about distribution that the subjects who has more menber in upper class should be more distributed ratio from surplus L.

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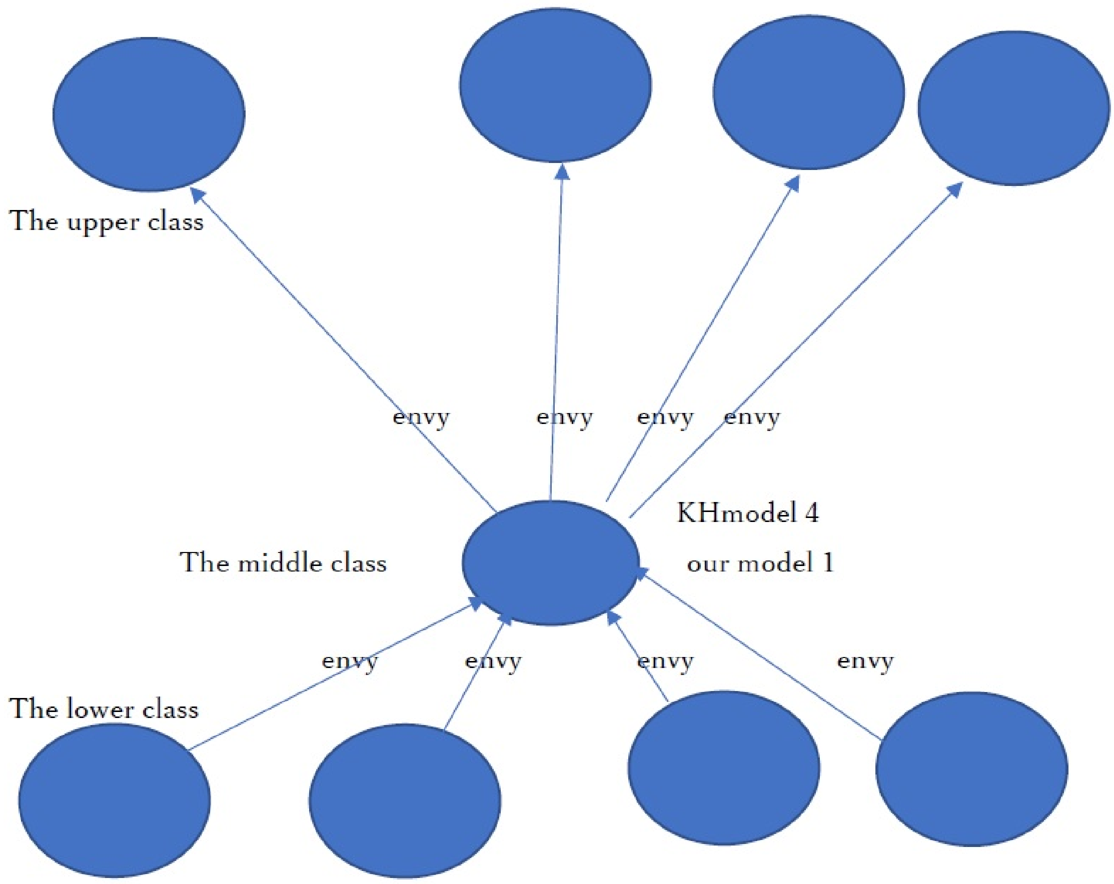

Figure 1: Case A1 Example about the estimation of the number of envy of subject in the middle class 1

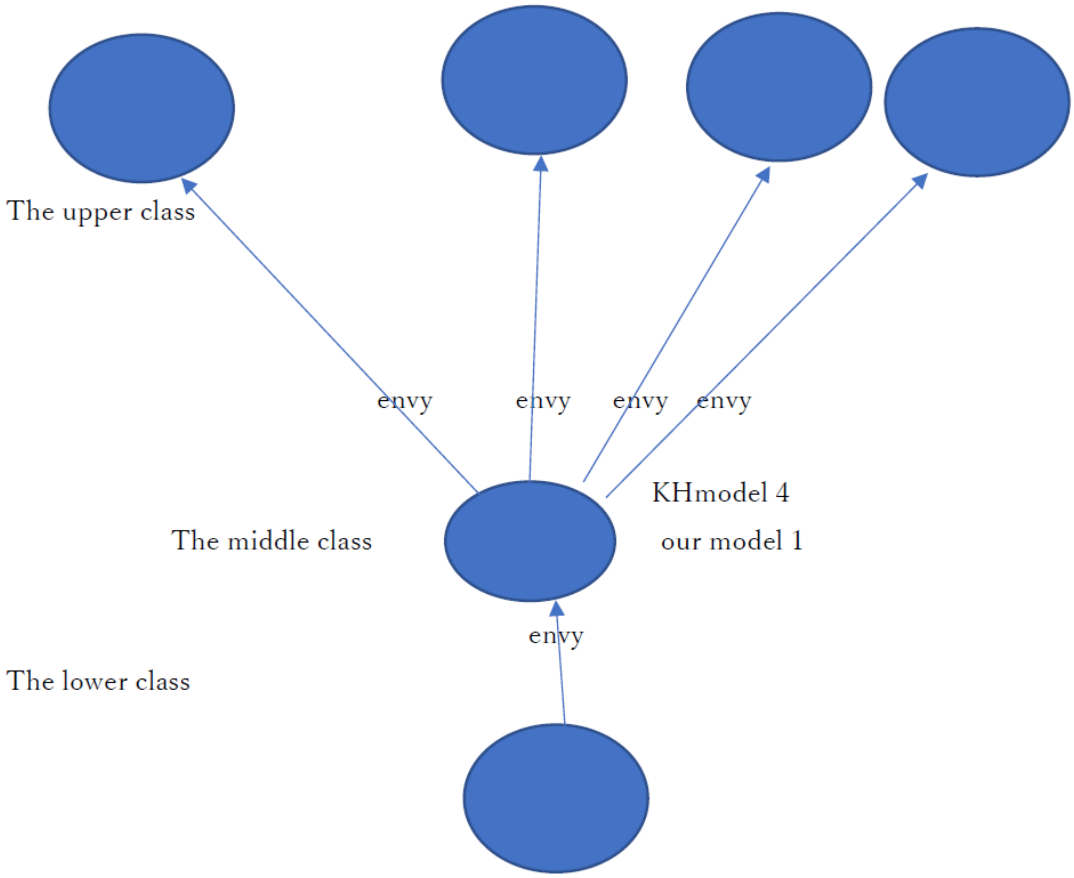

Figure 2: Case A2 Example about the estimation of the number of envy of subject in the middle class1

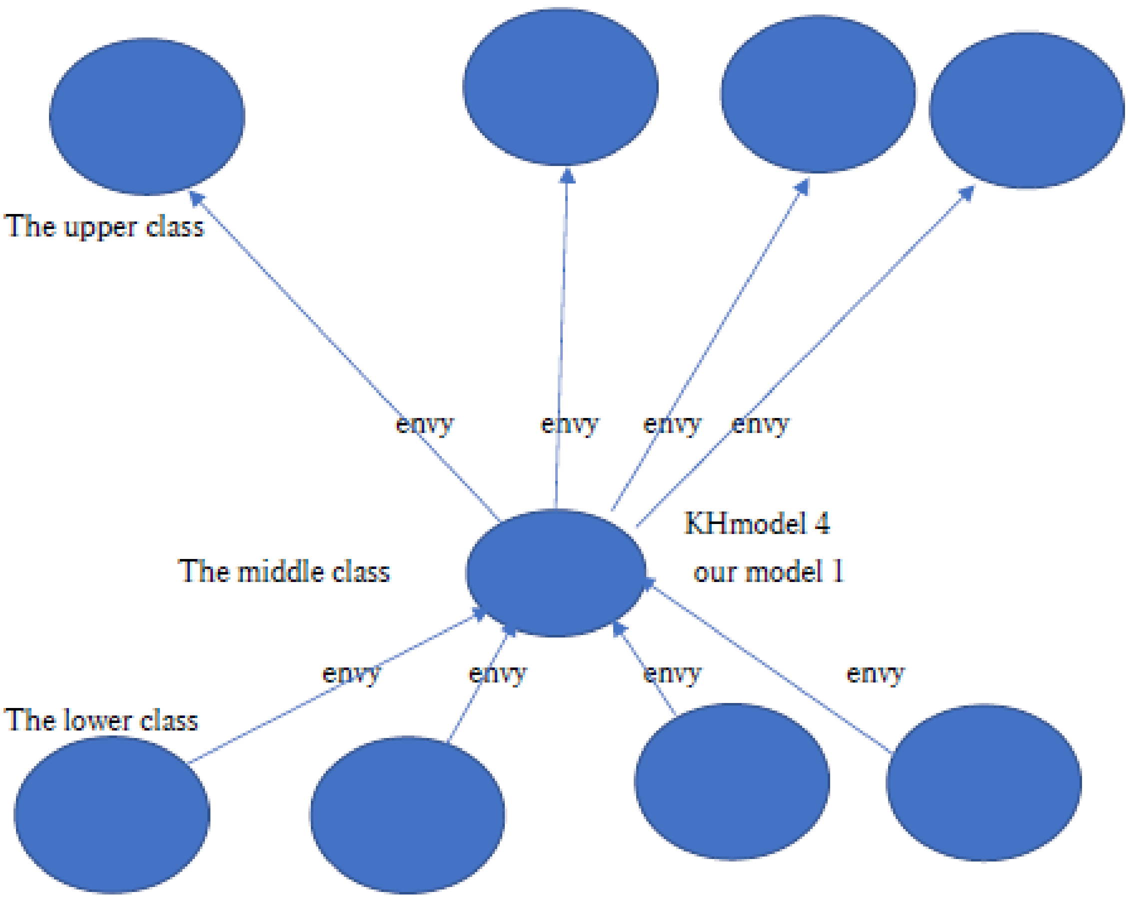

Figure 3: Case B1 Example about the estimation of the envy of subjects in the economy

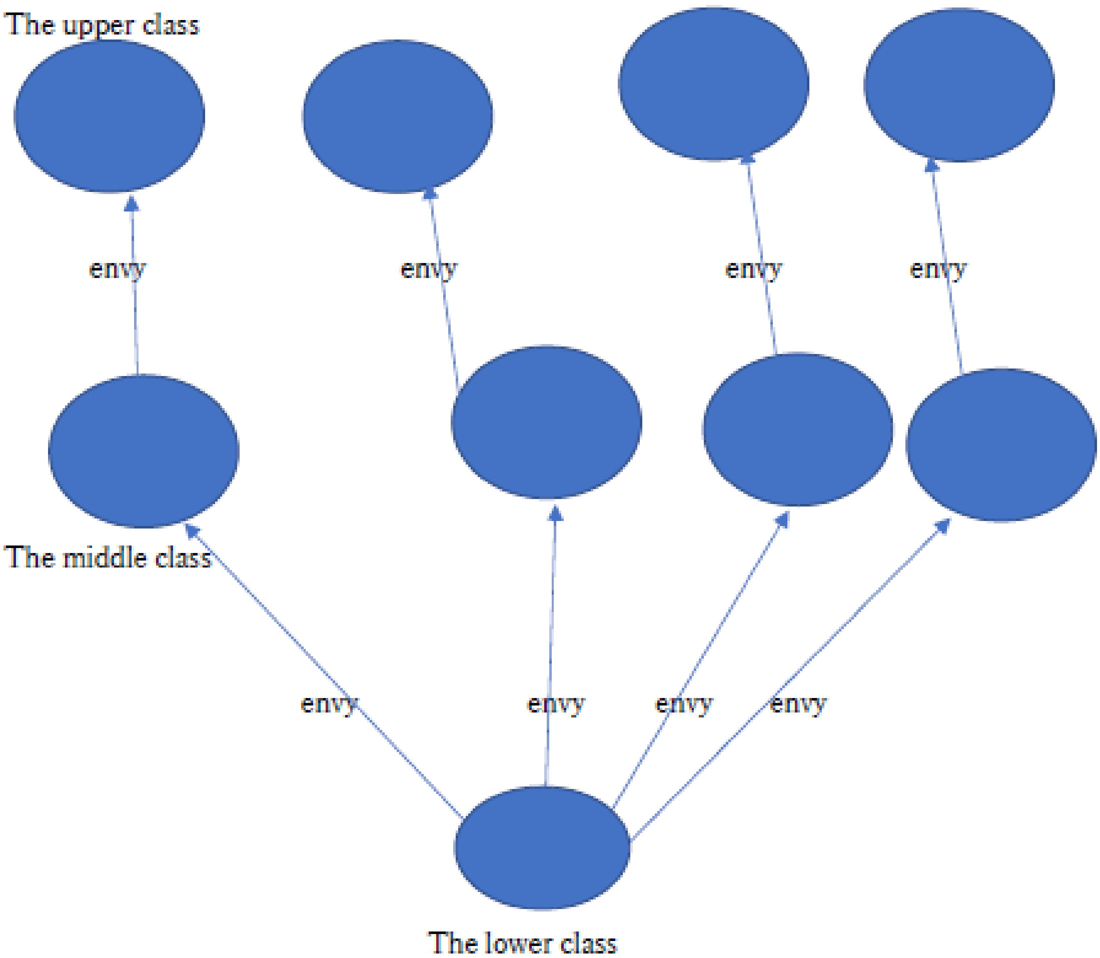

Figure 4: Case B2 Example about the estimation of the envy of subjects in the economy

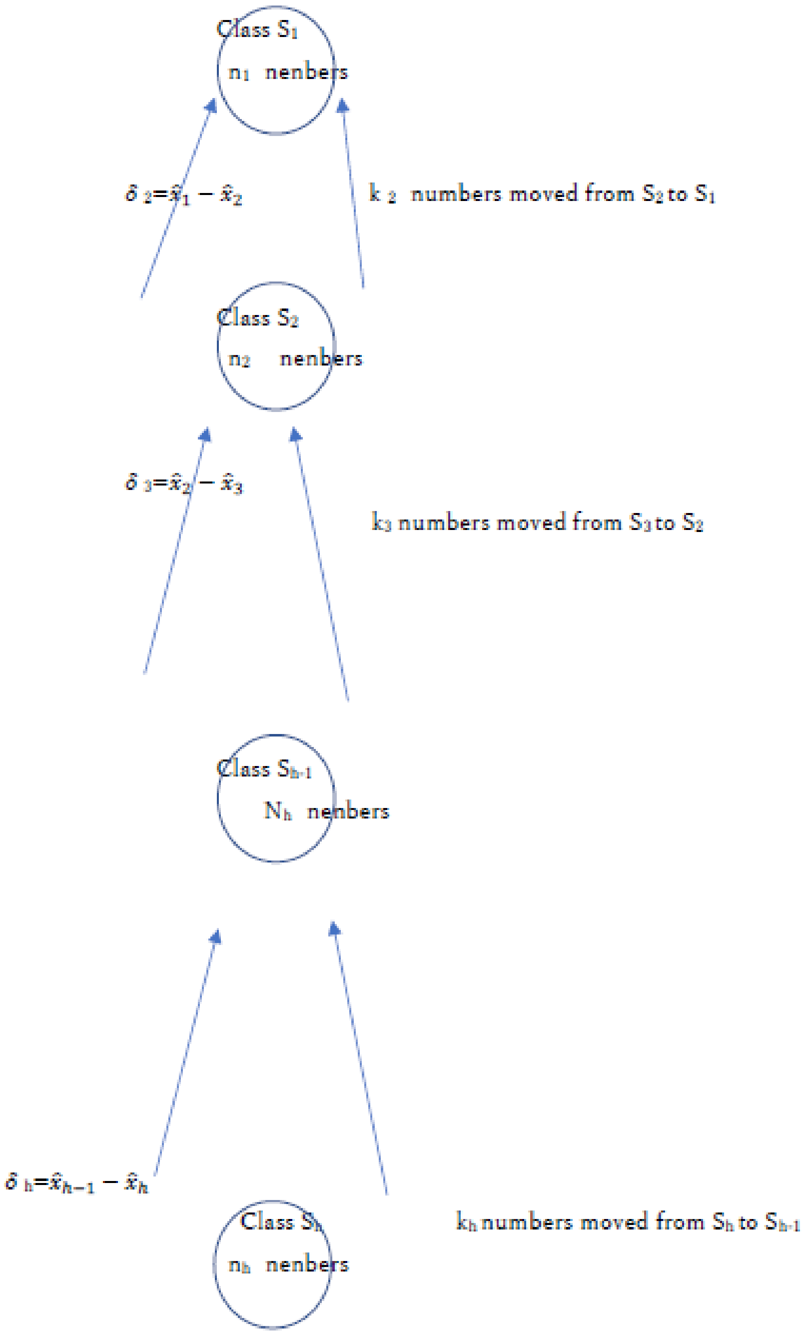

Figure 5: Illustration B elimination of the envy or distribution of surplus L