Discussion Paper Series A No.724

# On the Labor Theory of Value as the Basis for the Analysis of Economic Inequality in the Capitalist Economy

Naoki Yoshihara
(Department of Economics, University of Massachusetts Amherst,
Institute of Economic Research, Hitotsubashi University and
School of Management, Kochi University of Technology)

August 2021

Institute of Economic Research
Hitotsubashi University
Kunitachi, Tokyo, 186-8603 Japan

# On the Labor Theory of Value as the Basis for the Analysis of Economic Inequality in the Capitalist Economy*

Naoki Yoshihara†
University of Massachusetts at Amherst

July 8, 2021

**Abstract**

In this paper, we have reviewed the *labor theory of value* as the basis for the analysis of economic inequality in the capitalist economy. According to the standard Marxian view, the system of labor values of individual commodities can serve as the center of gravity for long-term price fluctuations in the precapitalist economy with simple commodity-production, where no exploitative social relation emerges, while in the modern capitalist economy, the labor value system is replaced by the prices of production associated with an equal positive rate of profits as the center of gravity, in which exploitative relation between the capitalist and the working classes is a generic and persistent feature of economic inequality. Some of the literature such as Morishima (1973, 1974) criticized this view by showing that the labor values of individual commodities are no longer well-defined if the capitalist economy has joint production.

Given these arguments, this paper firstly shows that the system of individual labor values can be still well-defined in the capitalist economy with joint production whenever the set of available production techniques is all-productive. Secondly, this paper shows that it is generally impossible to verify that the labor-value pricing serves as the center of gravity for price fluctuations in precapitalist economies characterized by the full development of simple commodity-production.

**JEL classification**: D63; D51.

**Keywords:** UE-Exploitation; The Labor Theory of Value; Prices of Production.

*I am grateful so much to the editor and the three anonymous referees for their suggestions toward the final revision of the paper. I am also thankful to Makoto Itoh and Takao Sasaki for the discussions with them on the earlier version of the paper.

†Department of Economics, University of Massachusetts Amherst, Crotty Hall, 412 North Pleasant Street, Amherst, MA 01002, U.S.; The Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-8603, Japan; and School of Management, Kochi University of Technology, 2-22, Eikokuji-Chou, Kochi, Kochi, 780-8515, Japan. (nyoshihara@econs.umass.edu)

# 1 Introduction

Recently, a vast literature has analyzed the persistent, and widening, inequalities in income and wealth observed in the vast majority of nations,[1] while some data show inequality in per-capita income between the richer developed countries and the poorer developing ones which has been expanding since 1820.[2] Thus, the issue of the long-run distributional feature of wealth and income in the capitalist economy should be at the heart of economic analysis, as Piketty (2014) emphasizes, and one of the central questions in economics should be to ask what the primary mechanism to generate such disparity persistently between the rich and the poor is.

To discuss such a mechanism in the modern capitalist economy, Karl Marx paid special attention to a particular form of inequality related to the systematic *underpayment* of labor in relation to their contribution to production, which is known as *exploitation as unequal exchange of labor* (UE-exploitation, hereafter). He then argued that an UE-exploitative relation between capitalists and workers is generic and persistent in the capitalist economy. Though the notion of UE-exploitation has been paid less attention in the mainstream economics, it should have been one of the prominent concepts relevant to capitalist economic systems, particularly in a number of debates and analyses of labor relations, especially focusing on the weakest segments of the labor force (see, e.g., ILO, 2005a,b).

Unlike the case of UE-exploitation in the feudal society, the application of this notion to the capitalist economy involves a fundamental difficulty, as the division of a worker's labor into working for him/herself and working for a capitalist is not observable. Moreover, the market contract between buyers and sellers of labor power is simply observed as an equal exchange of labor. Therefore, the existence of UE-exploitation in the capitalist economy should be measured through economic analysis. To promote such an analysis, one of the central issues in Marxian exploitation theory is to stipulate a suitable operational method to measure the difference between the labor expended and the labor received by an individual via his/her income, which is to answer the question of what a proper formal definition of UE-exploitation is. Indeed, there have been many proposals for such a definition, like Okishio (1963), Morishima (1974), Roemer (1982), Duménil (1980), and Foley (1982).[3]

Remember that Marx (1867, 1976) himself defined the notion of UE-exploitation on the basis of the labor theory of value (LTV, hereafter), and in particular, by means of the labor value of labor power. However, the recent literature on analytical Marxian exploitation theory, like the above mentioned works, suggests that the role of LTV in Marxian economics is quite limited. As Morishima (1973) and Roemer (1981, 1982) emphasized, the notion of labor values is rele-

[1] For example, see Piketty (2014).

[2] For instance, see Maddison (2001).

[3] The recent literature of axiomatic analysis of UE-exploitation, like Yoshihara (2010, 2017) and Veneziani and Yoshihara (2009, 2015, 2017a,b, 2018), addresses this fundamental question. These works introduce *axioms* to represent some basic properties that any appropriate definition of UE-exploitation should have, and then examine whether each of the well-known formal definitions of UE-exploitation satisfies them.

vant only in the formal definition of UE-exploitation, since it requires a formal definition of the amount of labor time that the workers can 'receive' via their wage revenue. This is indeed defined as the socially necessary labor time for the production of a real wage commodity bundle, that is the only part to which LTV is relevant. Moreover, as Steedman (1975) and Morishima (1973, 1974) argued, the additive model of labor valuation fails in the case of economies with joint production. Thus, in economies with joint production, it has been recognized that the formal definition of labor value of every individual commodity is not only unnecessary, but also generally impossible.

In this paper, we review such recognition of LTV in the literature. First, we will show that at least for some subclass of joint production economies, the standard claims of LTV and the basic theorems of UE-exploitation can be preserved. To do so, we introduce a specific notion of productiveness of production techniques, called *all-productiveness*, due to Kurz and Salvadori (1995, p. 239). Then, we model an economy with joint production, in which every available production technique is assumed to be all-productive. In such an economy, the labor value of every individual commodity is well-defined, thus a system of individual labor values can be identified for each production technique. Then, every alternative form of UE-exploitation can be represented by means of such a labor-value system, as in the case of economies with simple Leontief techniques[4].

Second, in this paper, we discuss the issue of the so-called *law of value* in LTV. The law of value implies that, in the case of the modern industrial capitalist economy, a system of *production prices* associated with *an equal positive rate of profits* can serve as the center of gravity for price fluctuations over the long term. Fortunately, such a claim of the law of value can be verified, which is one of implications from the main results in Duménil and Levy (1985) and Dana et. al (1989). Then, with reference to the literature of the so-called Fundamental Marxian Theorem, this consequence would also imply that the UE-exploitation is generic and persistent in the capitalist economy. Thus, we examine in this paper that whether a labor-value system can serve as the center of gravity for price fluctuations in the *precapitalist economy.* That is, the verification of the law of value in the precapitalist economy. By examining such an issue, while the UE-exploitation is generic and persistent in the capitalist economy, we discuss whether it is non-generic and non-persistent in the non-capitalistic market economies such as the simple commodity-production economy.

Before going into a more detail argument, let me briefly review the background arguments relevant to this issue. Although a typical interpretation of the LTV is that it asserts value/price proportionality, Marx's own view was that in a capitalist economy the system of labor values of individual commodities regulates the fluctuating market prices. It has been recognized that this view could be verified by showing firstly, the correlations between production prices and labor value magnitudes at the aggregate level, and secondly, that *prices of production* can serve as the center of gravity for price fluctuations over

[4] Note that an economy with a simple Leontief technique is often called a *single product system* in the Sraffian literature, like Kurz and Salvadori (1995).

the long term (that is, the law of value holds). In these two subjects, though the first one is relevant to the so-called *Transformation Problem*,[5] it is the second issue that this paper would like to focus.

As Itoh (2021) properly pointed out, in any type of society, "the total quantity of social labor time must somehow be allocated to the various kinds of concrete productive activity. The social division of labor in general represents not only the social organization of qualitatively different varieties of concrete labor, but also the quantitative division of abstract labor into the necessary branches. ... Marx makes clear that such a division of labor time into various necessary branches does exist under any form of production, ... Thus, it appears that abstract human labor, together with its concrete character, constitutes the common material basis for all societies. (Itoh, 2021, p. 97))" In a capitalist economy, such an allocation of labor time is exercised through market exchanges under free competition.[6] It implies that free exchanging activities in competitive markets are ultimately regulated on the basis of efficient quantitative-allocation of (abstract) social labor time among the various necessary branches, which is what the law of value shows.[7]

Note that both David Ricardo (1821, 1951) and Marx recognized that the full development of the law of value presupposes a freely competitive market economy with large-scale industrial production, in other words modern bourgeois society.[8] However, in contrast, it has been argued, since the era of Engels,[9] that the law of value holds generally for the period of precapitalist societies where the simple commodity-production is sufficiently developed. In market economies with *simple commodity-production*, unlike the case of the modern industrial capitalist economy, it has been recognized that the labor-value system serves as the center of gravity for the fluctuating market exchanges.

However, in this paper, we show that the system of labor values cannot be verified as the center of gravity for the long term price fluctuations, in that in any precapitalist economy with unequal private ownership of capital goods, an

[5]Regarding the recent literature on the Transformation Problem, Muhon and Veneziani (2017) provide a comprehensive survey.

[6]Thus, in the case of capitalist economies, a system of labor values of individual commodities is the objectified expression of an efficient quantitative-allocation of abstract human labor among the necessary branches, where each allocated abstract labor is objectified as the socially necessary labor time for the production of each commodity.

[7]The essentially same view about the law of value is also developed by Sasaki (2021, pp. 137-141).

[8]Indeed, Marx (1859, 1970) argued that: "operation of the law depends on definite historical pre-conditions. He [Ricardo] says that the determination of value by labour-time applies to "such commodities only as can be increased in quantity by the exertion of human industry, and on the production of which competition operates without restraint."

[9]For instance, see Engels, F., "*Supplement to Capital vol. III*" in Marx (1894, 1981), where Engels argues that "Marxian law of values hold general, as far as economic laws are valid at all, for the whole period of simple commodity-production, that is, up to the time when the latter suffers a modification through the appearance of the capitalist form of production. Up to that time prices gravitate towards the values fixed according to the Marxian law and oscillate around those values, so that the more fully simple commodity-production develops, the more the average prices over long periods uninterrupted by external violent disturbances coincide with values within a negligible margin." (Ibid., p. 1037)

infinite set of equilibrium prices including the labor-value pricing equilibrium is observed. It implies that the long-term equilibrium prices are generically indeterminate, and so none of the long term equilibrium prices, including the labor-value pricing equilibrium, can be verified as the center of gravity for price fluctuations.

# 2 Basic Models

Assume that there are $n$ types of physical commodities. There is a finite set, $\mathcal{P}$, of linear production techniques, or *techniques*, $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$, where $\boldsymbol{B}$ is the $n \times n$ nonnegative matrix of produced outputs, $\boldsymbol{A}$ is the $n \times n$ nonnegative matrix of produced inputs, and $\boldsymbol{L}$ is the $1 \times n$ positive vector of labor inputs. Let us call $\mathcal{P}$ a *production set*. Let $a_{ij}$ and $L_j$ denote, respectively, the amounts of physical input $i$ and labor used in the $j$-th production process of the production technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$, while $b_{ij}$ denotes the amount of good $i$ produced in production process $j$ of the technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$. Let $\boldsymbol{x}$ be the $n \times 1$ vector denoting the aggregate level of activating the various techniques. The vector of aggregate net output by activating the technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ with $\boldsymbol{x}$ activity level is $\boldsymbol{y} = (\boldsymbol{B} - \boldsymbol{A})\boldsymbol{x}$. If every process produces only one good (no joint production), then $\boldsymbol{B}$ is diagonal and activities can be normalized such that $\boldsymbol{B} = \boldsymbol{I}$, and can be simply denoted as $(\boldsymbol{A}, \boldsymbol{L})$. Let us call such $(\boldsymbol{A}, \boldsymbol{L})$ a *simple Leontief technique.*

## 2.1 All-Productiveness of Production Techniques

It is natural to assume that all of $n$ types of commodities can be produced as net outputs by means of the production set $\mathcal{P}$. This property would be formulated as follows: for any non-negative vector $\boldsymbol{y} \in \mathbb{R}^n_+$, there exist a technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L}) \in \mathcal{P}$ and a non-negative vector $\boldsymbol{x} \in \mathbb{R}^n_+$ such that $(\boldsymbol{B} - \boldsymbol{A})\boldsymbol{x} \geqq \boldsymbol{y}$.[10] If an economy is associated with a simple Leontief technique, so that $\mathcal{P} = \{(\boldsymbol{A}, \boldsymbol{L})\}$, then this producibility condition can be reduced to the standard notion of productiveness: a *technique* $(A, L)$ *is productive* if and only if there exists a positive vector $\boldsymbol{x} \in \mathbb{R}^n_{++}$ such that $\boldsymbol{x} > A\boldsymbol{x}$. Compared to this natural producibility condition, however, a much more stringent condition can be introduced as follows. A *technique* $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ *is all-productive* if and only if for any semi-positive vector $\boldsymbol{y} \in \mathbb{R}^n_+$, there exists a non-negative vector $\boldsymbol{x} \in \mathbb{R}^n_+$ such that $(\boldsymbol{B} - \boldsymbol{A})\boldsymbol{x} = \boldsymbol{y}$ (Kruz and Salvadori,1995, p. 239). A *production set is all-productive* if and only if every technique available in this set is all-productive. In the rest of the paper, we assume that the production set $\mathcal{P}$ is all-productive.

To see how much stringent the all-productiveness is, let us consider the simplest case: $n = 2$. Then, let

$$(\boldsymbol{B} - \boldsymbol{A}) = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}.$$

[10] Indeed, this condition is equivalent to the producibility of any nonnegative net outputs, formulated as **A5** in Roemer (1981, p. 36).

As argued by Kurz and Salvadori (1995, p. 239), if $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ is all-productive, then $(\boldsymbol{B} - \boldsymbol{A})$ is semi-positive invertible: $(\boldsymbol{B} - \boldsymbol{A})^{-1} \geq \mathbf{0}$. Since

$$(\boldsymbol{B} - \boldsymbol{A})^{-1} = \frac{1}{\alpha\delta - \beta\gamma} \begin{bmatrix} \delta & -\beta \\ -\gamma & \alpha \end{bmatrix},$$

all-productiveness of $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ implies that

$$\begin{aligned} (\boldsymbol{B} - \boldsymbol{A})^{-1}\mathbf{e}_1 &= \frac{1}{\alpha\delta - \beta\gamma} \begin{bmatrix} \delta & -\beta \\ -\gamma & \alpha \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \geqq \mathbf{0}; \\ \text{and } (\boldsymbol{B} - \boldsymbol{A})^{-1}\mathbf{e}_2 &= \frac{1}{\alpha\delta - \beta\gamma} \begin{bmatrix} \delta & -\beta \\ -\gamma & \alpha \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \geqq \mathbf{0}. \end{aligned}$$

From $(\boldsymbol{B} - \boldsymbol{A})^{-1}\mathbf{e}_1 \geqq \mathbf{0}$, it follows that $\delta \geqq 0$ and $\gamma \leqq 0$ whenever $\alpha\delta - \beta\gamma > 0$; and $\delta \leqq 0$ and $\gamma \geqq 0$ whenever $\alpha\delta - \beta\gamma < 0$. From $(\boldsymbol{B} - \boldsymbol{A})^{-1}\mathbf{e}_2 \geqq \mathbf{0}$, it follows that $\alpha \geqq 0$ and $\beta \leqq 0$ whenever $\alpha\delta - \beta\gamma > 0$; and $\alpha \leqq 0$ and $\beta \geqq 0$ whenever $\alpha\delta - \beta\gamma < 0$. Without loss of generality, let us consider the case of $\alpha\delta - \beta\gamma > 0$. Then, $\alpha > 0$, $\beta \leqq 0$, $\gamma \leqq 0$, and $\delta > 0$. This implies that $b_{11} - a_{11} > 0$, $b_{12} - a_{12} \leqq 0$, $b_{21} - a_{21} \leqq 0$, and $b_{22} - a_{22} > 0$. Thus, the matrix $(\boldsymbol{B} - \boldsymbol{A})$ satisfies the presumption of the Hawkins-Simon theorem, and so the technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ is essentially equivalent to simple Leontief, even though the non-diagonal elements of $\boldsymbol{B}$ could be positive. In other words, even if the technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ admits joint production, it is essentially a simple Leontief in terms of net output production, since any jointly produced commodity of any process cannot be a positive net output by the activation of this process alone.

There are plenty of cases that a process admits joint production as a positive net output of by-product in the class of economies with joint production. Such a rich class of joint production cases would be excluded from the consideration by the assumption of all-productive production set. However, such restriction allows us to define the system of labor values of individual commodities in economies with joint production, as will be discussed later.

# 3 Labor Values in Economies with All-productive Production Sets

If $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ is all-productive, a system of labor values of individual commodities for a technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ can be defined as the solution to the following system of equations:

$$\boldsymbol{vB} = \boldsymbol{vA} + \boldsymbol{L}. \quad (3.1)$$

Indeed the solution to (3.1) is a positive vector $\boldsymbol{v} > \mathbf{0}$ satisfying

$$\boldsymbol{v} = \boldsymbol{L}(\boldsymbol{B} - \boldsymbol{A})^{-1}. \quad (3.2)$$

As the production set $\mathcal{P}$ is all-productive, we can find such a solution for any system of equations defined from any technique available in $\mathcal{P}$. Therefore, if a technique $(\boldsymbol{B}^*, \boldsymbol{A}^*, \boldsymbol{L}^*)$ is selected according to the cost minimization principle, then a system of labor values associated with this cost minimizing technique is well-defined as $\boldsymbol{v}^* = \boldsymbol{L}^*(\boldsymbol{B}^* - \boldsymbol{A}^*)^{-1} > \mathbf{0}$. Correspondingly, a formal definition of UE-exploitation can be proposed by means of the labor-value system $\boldsymbol{v}^*$.

Let us examine whether the labor values defined as the solution to (3.1) is appropriate. It could be deemed appropriate if it can regulate market exchanges of commodities in order to allocate the total quantity of social labor time efficiently and appropriately to meet various social demands for commodities.[11]

To see it, let $\boldsymbol{c} > \mathbf{0}$ be an aggregate demand vector of all commodities. Let $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L}) \in \mathcal{P}$ be an optimal technique in terms of cost minimization, and $\boldsymbol{x}^c$ be the aggregate production activities to meet the social demand in the market economy which is defined as

$$\boldsymbol{x}^c = (\boldsymbol{B} - \boldsymbol{A})^{-1} \boldsymbol{c} > \mathbf{0}. \quad (3.3)$$

This implies that the *total living labor time* necessary to meet the demand $\boldsymbol{c}$ is $\boldsymbol{L}\boldsymbol{x}^c > 0$. Then, left multiplying both sides of the equations (3.3) by $\boldsymbol{L}$, and then taking (3.2) into account, we reach to the following equation:

$$\boldsymbol{L}\boldsymbol{x}^c = \boldsymbol{v}\boldsymbol{c}. \quad (3.4)$$

As in the case of the labor-value system in economies with simple Leontief techniques, the equation (3.4) implies that the total living labor time $\boldsymbol{L}\boldsymbol{x}^c$ is appropriately allocated to meet the social demand $\boldsymbol{c}$ through market exchange of commodities in accordance with the labor-value exchanging rates: $\boldsymbol{v}$.

Indeed, in the right hand side of the equation (3.4), $v_i c_i > 0$ for each commodity $i = 1, \ldots, n$ represents the *socially necessary labor time* for the production of this commodity to meet its social demand $c_i$. Therefore, the equation (3.4) implies that the total living labor time $\boldsymbol{L}\boldsymbol{x}^c$ is allocated to each commodity $i$'s production, and it is the social necessary labor time $v_i c_i$ that is allocated to each commodity $i$'s production in order to meet its social demand $c_i$. Note that the left hand side $\boldsymbol{L}\boldsymbol{x}^c$ of (3.4) is the aggregation of the labor time $L_i x_i^c$ expended in each production process $i = 1, \ldots, n$. As well recognized, each labor expenditure $L_i x_i^c$ is not necessarily identical to the socially necessary labor time to produce $c_i$.

[11] As argued in section 1, one of the essential claims of LTV is that, in any type of society, the total living labor time must be appropriately distributed to various spheres of production in order to meet the social needs for these products, and in the modern capitalist society, the allocation of the total labor is mediated through free competition in market exchanges, which results in the proportional distribution of the total labor time to the socially necessary labor time (labor value) of each commodity corresponding to its social needs. This claim is also explicitly represented by the following statements of Marx himself: “Every child knows, too, that the masses of products corresponding to the different needs required different and quantitatively determined masses of the total labor of society. That this necessity of the distribution of social labor in definite proportions cannot possibly be done away with by a particular form of social production but can only change the mode of its appearance, is self-evident. No natural laws can be done away with. What can change in historically different circumstances is only the form in which these laws assert themselves. And the form in which this proportional distribution of labor asserts itself, in the state of society where the interconnection of social labor is manifested in the private exchange of the individual products of labor, is precisely the exchange value of these products.” (Marx (1968))

To see this last point in more detail, assume that $n = 2$. Then, (3.3) is represented as follows:

$$\boldsymbol{x}^c = \begin{pmatrix} x_1^c \\ x_2^c \end{pmatrix} = \begin{pmatrix} \frac{b_{22}-a_{22}}{|\boldsymbol{B}-\boldsymbol{A}|}c_1 + \frac{-b_{12}+a_{12}}{|\boldsymbol{B}-\boldsymbol{A}|}c_2 \\ \frac{-b_{21}+a_{21}}{|\boldsymbol{B}-\boldsymbol{A}|}c_1 + \frac{b_{11}-a_{11}}{|\boldsymbol{B}-\boldsymbol{A}|}c_2 \end{pmatrix} > \mathbf{0}.$$

Therefore, we have:

$$L_1 x_1^c = L_1 \frac{b_{22}-a_{22}}{|\boldsymbol{B}-\boldsymbol{A}|}c_1 + L_1 \frac{-b_{12}+a_{12}}{|\boldsymbol{B}-\boldsymbol{A}|}c_2;\ L_2 x_2^c = L_2 \frac{-b_{21}+a_{21}}{|\boldsymbol{B}-\boldsymbol{A}|}c_1 + L_2 \frac{b_{11}-a_{11}}{|\boldsymbol{B}-\boldsymbol{A}|}c_2. \quad (3.5)$$

In contrast, it follows from (3.2) that

$$v_1 c_1 = L_1 \frac{b_{22}-a_{22}}{|\boldsymbol{B}-\boldsymbol{A}|}c_1 + L_2 \frac{-b_{21}+a_{21}}{|\boldsymbol{B}-\boldsymbol{A}|}c_1;\ v_2 c_2 = L_1 \frac{-b_{12}+a_{12}}{|\boldsymbol{B}-\boldsymbol{A}|}c_2 + L_2 \frac{b_{11}-a_{11}}{|\boldsymbol{B}-\boldsymbol{A}|}c_2. \quad (3.6)$$

By comparing (3.5) and (3.6), we can observe that to meet the social demand $c_1$ of commodity 1, $L_1 \frac{b_{22}-a_{22}}{|\boldsymbol{B}-\boldsymbol{A}|}c_1$ amount of living labor time is allocated from process 1 while $L_2 \frac{-b_{21}+a_{21}}{|\boldsymbol{B}-\boldsymbol{A}|}c_1$ amount of living labor time is allocated from process 2. Likewise, to meet the social demand $c_2$ of commodity 2, $L_1 \frac{-b_{12}+a_{12}}{|\boldsymbol{B}-\boldsymbol{A}|}c_2$ amount of living labor time is allocated from process 1 while $L_2 \frac{b_{11}-a_{11}}{|\boldsymbol{B}-\boldsymbol{A}|}c_2$ amount of living labor time is allocated from process 2. In other words, for the production of commodity 1 to meet the social demand $c_1$, $L_2 \frac{-b_{21}+a_{21}}{|\boldsymbol{B}-\boldsymbol{A}|}c_1$ portion of the total labor time $L_2 x_2^c$ expended in process 2 needs to be allocated, and for the production of commodity 2 to meet the social demand $c_2$, $L_1 \frac{-b_{12}+a_{12}}{|\boldsymbol{B}-\boldsymbol{A}|}c_2$ portion of the total labor time $L_1 x_1^c$ expended in process 1 needs to be allocated. In this way, (3.6) represents the state that the socially necessary labor time for each commodity's production is appropriately identified in order to meet its social demand, as the result of appropriate allocation of each process's total labor expenditure which is mediated by market exchange of commodities in accordance with the labor-value pricing.

In this way, the equation (3.4) represents the crucial condition that a proper formal definition of labor values should satisfy, and our proposal (3.1)-(3.2) is shown to pass this test. Thus, the well-known criticism against LTV developed by the Steedman-Morishima controversy is no longer applied to the class of all-productive joint production economies.

One remark is given for the above argument. It is true that all-productiveness of joint production techniques is mathematically quite stringent. However, this condition could be still reasonable from a view of economics. For instance, suppose that the joint production comes only from the existence of a non-negative square matrix $\boldsymbol{\Phi}$ of *fixed-capital input coefficients*, which is depreciated with a fixed ratio in each production period. This implies that $\boldsymbol{\Phi}$ appears at the beginning of one production period, while another non-negative square matrix $\boldsymbol{\Phi}'$ (with $\boldsymbol{\Phi}' \leq \boldsymbol{\Phi}$) of fixed-capital input coefficients appears at the end of this period. Here, $\boldsymbol{\Phi} - \boldsymbol{\Phi}'$ corresponds to a standard *depreciation matrix of fixed-capital goods.* Let $\boldsymbol{C}$ be the standard nonnegative square matrix of *circulating-capital input coefficients* which is productive, and define $\boldsymbol{B} \equiv \boldsymbol{I} + \boldsymbol{\Phi}'$ and $\boldsymbol{A} \equiv \boldsymbol{C} + \boldsymbol{\Phi}$. Then, such a technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ satisfies the presumption of the Hawkins-Simon theorem whenever the diagonal elements of $\boldsymbol{B} - \boldsymbol{A}$ are positive, which implies that it could be all-productive.

## 3.1 Remarks on Alternative LTV initiated by TSSI

By the way, there is some recent literature to propose an alternative formal definition of labor values, like *Temporal Single System Interpretation* (**TSSI**, hereafter) (Kliman and McGlone (1999)), which denies the value equations (3.1). According to **TSSI**, given that the technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L}) \in \mathcal{P}$ is used in period $t$, the labor values are determined at the end of period $t$ by:

$$\varepsilon_t \boldsymbol{v}_t \boldsymbol{B} = \boldsymbol{p}_{t-1} \boldsymbol{A} + \varepsilon_t \boldsymbol{L} \quad (3.7)$$

where $\boldsymbol{p}_{t-1} \geqq \boldsymbol{0}$ is the vector of market prices prevailed at the end of period $t-1$, and is assumed to be historically given at the end of period $t$. Moreover, $\varepsilon_t > 0$ is the *monetary expression of labor time* (**MELT**) at period $t$, which is to transform the unit of time to the unit of money. Therefore, the labor values are determined temporarily at each period, depending on the given market prices of the previous period, and so they may vary across different periods, even though no technical change from the present technique $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L})$ takes place.

In this short subsection, just one fundamental criticism is raised against the **TSSI** definition of labor values. That is, the **TSSI** system (3.7) of labor values cannot properly represent a profile of each commodity's socially necessary labor time as the allocation of the total living labor expenditure through competitive market exchanges of commodities, whenever such market exchanges are mediated in accordance with the **TSSI** labor-value pricing. This difficulty should not be regarded as the failure of competitive market exchanges, but rather be attributed to improperness of the **TSSI** system (3.7) of labor values, because it cannot meet the fundamental condition (3.4).

To see the last point, again let $\boldsymbol{c} > \boldsymbol{0}$ be an aggregate demand vector of all commodities, $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{L}) \in \mathcal{P}$ be an optimal technique in terms of cost minimization, and $\boldsymbol{x}^c$ be the aggregate production activities derived as the solution to (3.3). Then, from (3.7) it follows that

$$\begin{aligned} \varepsilon_t \boldsymbol{v}_t (\boldsymbol{B} - \boldsymbol{A}) &= \left(\boldsymbol{p}_{t-1} - \varepsilon_t \boldsymbol{v}_t\right) \boldsymbol{A} + \varepsilon_t \boldsymbol{L} \\ &\Leftrightarrow \quad \varepsilon_t \boldsymbol{v}_t = \left(\boldsymbol{p}_{t-1} - \varepsilon_t \boldsymbol{v}_t\right) \boldsymbol{A} (\boldsymbol{B} - \boldsymbol{A})^{-1} + \varepsilon_t \boldsymbol{L} (\boldsymbol{B} - \boldsymbol{A})^{-1} \\ &\Leftrightarrow \quad \boldsymbol{v}_t = \left(\frac{\boldsymbol{p}_{t-1}}{\varepsilon_t} - \boldsymbol{v}_t\right) \boldsymbol{A} (\boldsymbol{B} - \boldsymbol{A})^{-1} + \boldsymbol{v} \end{aligned}$$

where $\boldsymbol{v}$ is the labor-value system defined by (3.2), whereas $\boldsymbol{v}_t$ is the **TSSI** labor-value system given by (3.7). As $\frac{\boldsymbol{p}_{t-1}}{\varepsilon_t} \neq \boldsymbol{v}_t$, $\left(\frac{\boldsymbol{p}_{t-1}}{\varepsilon_t} - \boldsymbol{v}_t\right) \boldsymbol{A} (\boldsymbol{B} - \boldsymbol{A})^{-1} \neq \boldsymbol{0}$ holds.

Therefore, it follows from (3.4) that

$$\boldsymbol{v}_t \boldsymbol{c} = \left( \frac{\boldsymbol{p}_{t-1}}{\varepsilon_t} - \boldsymbol{v}_t \right) \boldsymbol{A} \left( \boldsymbol{B} - \boldsymbol{A} \right)^{-1} \boldsymbol{c} + \boldsymbol{v}\boldsymbol{c} = \left( \frac{\boldsymbol{p}_{t-1}}{\varepsilon_t} - \boldsymbol{v}_t \right) \boldsymbol{A} \left( \boldsymbol{B} - \boldsymbol{A} \right)^{-1} \boldsymbol{c} + \boldsymbol{L}\boldsymbol{x}^c,$$

and thus

$$\boldsymbol{v}_t \boldsymbol{c} \neq \boldsymbol{L}\boldsymbol{x}^c \Leftrightarrow \frac{\boldsymbol{p}_{t-1}}{\varepsilon_t} \neq \boldsymbol{v}_t.$$

In general, $\frac{\boldsymbol{p}_{t-1}}{\varepsilon_t} \neq \boldsymbol{v}_t$ holds, so that we conclude $\boldsymbol{v}_t \boldsymbol{c} \neq \boldsymbol{L}\boldsymbol{x}^c$. Thus, the market exchange rate determined by the **TSSI** labor-value system cannot appropriately stipulate a profile of each commodity's socially necessary labor time as an allocation of the total living labor to meet each commodity's social demand.

In this sense, the **TSSI** definition (3.7) of labor values is conceptually flaw.

# 4 On the (in)validity of the law of value in the precapitalist economy

## 4.1 Definition of the precapitalist economy with simple commodity-production

As argued in section 1 of this paper, it seems to be a common view within the Marxian camp that the labor-value system serves as the center of gravity for price fluctuations in the case of precapitalist economies, where the simple commodity-production is supposed to be sufficiently developed.[12] However, how should we characterize the basic features of the precapitalist economies with simple commodity-production, setting aside the more fundamental question of *whether the simple commodity-production economy had ever historically existed.*

In this respect, again we may refer to Marx (1894, 1981), where he asked what would happen if all commodities in the various spheres of production were sold at their labor values. In this context, he supposed that "the workers are themselves in possession of their respective means of production and exchange their commodities with one another."(Marx (1894, 1981, p. 276)) In this setting, Marx admitted *unequal private ownership of capital goods among private producers* by arguing that "If worker I has higher outlays, these are replaced by the greater portion of value of his commodities that replaces this 'constant' part, and he therefore again has a greater part of his product's total value to

[12] Indeed, when Marx (1867, 1976, p. 168) argued that "The production of commodities must be fully developed before the scientific conviction emerges, from experience itself, that all the different kinds of private labor (which are carried on independently of each other, and yet, as spontaneously developed branches of the social division of labor, are in a situation of all-round dependence on each other) are continually being reduced to the quantitative proportions in which society requires them. The reason for this reduction is that in the midst of the accidental and ever-fluctuating exchange relations between the products, the labor time socially necessary to produce them asserts itself as a regulative law of nature", it was under the supposition of fully developed commodity-production economy with the abstraction of capitalistic production.

transform back into the material elements of this constant part, while II, if he receives less for this, has also that much less to transform back." (Ibid., p. 277)

Under such a setting, Marx (1894, 1981) discussed the exchange of commodities at their labor values, and observed that "since I and II each receive the value of the product of one working day, they therefore receive equal values, after deducting the value of the 'constant' elements advanced" (Ibid., p. 277) and thus, "Profit rates would also be very different for I and II" (Ibid., p. 277) because of the different volumes of constant capital between them. However, "the difference in the profit rate would be a matter of indifference, ... just as in international trade the differences in profit rates between different nations are completely immaterial as far as the exchange of their commodities is concerned." (Ibid., p. 277)

Given these observations, Marx (1894, 1981) concluded that "The exchange of commodities at their values, or at approximately these values, thus corresponds to a much lower stage of development than the exchange at prices of production, for which a definite degree of capitalist development is needed." (Ibid., p. 277) Moreover, by arguing that "it is also quite apposite to view the values of commodities not only as theoretically prior to the prices of production, but also as historically prior to them" (Ibid., p. 277) Marx characterized the basic elements of precapitalist economies by (1) the "conditions in which the means of production belong to the worker" like "peasant proprietors and handicraftsmen who work for themselves", and (2) the features that "the means of production involved in each-branch of production can be transferred from one sphere to another only with difficulty, and the different spheres of production therefore relate to one another, within certain limits, like foreign countries or communistic communities." (Ibid., p. 277) These two basic properties imply that no labor nor capital is freely exchanged in markets. He also presumed the competitiveness of commodity markets by the three conditions specified in Marx (1894, 1981, pp. 278-279).[13] In this way, Marx argued that, under precapitalist economies characterized by the competitive commodity markets and the basic properties (1) and (2), the law of value governs prices and their movements by taking the system of labor values as the center of gravity.

Taking the above arguments into consideration, in the next subsection, we provide, as a thought experiment, a simple model of precapitalist economy,[14] in which unequal private ownership of capital goods among private producers; no labor market nor credit market is observed while commodity markets are

[13] "If the prices at which commodities exchange for one another are to correspond approximately to their values, nothing more is needed than (1) that the exchange of different commodities ceases to be purely accidental or merely occasional; (2) that, in so far as we are dealing with the direct exchange of commodities, these commodities are produced on both sides in relative quantities that approximately correspond to mutual need, something that is learned from the reciprocal experience of trading and which therefore arises precisely as a result of continuing exchange; and (3) that, as far as selling is concerned, no natural or artificial monopolies enable one of the contracting parties to sell above value, or force them to sell cheap, below value." (Ibid., pp. 278-279)

[14] Such a model has been discussed by Roemer (1982, Chapter 1) and Yoshihara and Kaneko (2016).

perfectly competitive; all agents have a common *leisure preference* in that they are primarily concerned with enjoyment of free hours (or leisure time), given that a common subsistence consumption bundle, necessary for their survival, is ensured. Note that leisure preference was ubiquitous in the pre-industrial society before the new time-discipline was imposed by the eighteenth century (see Thompson (1967), Cunningham (1980, 2014), and Kawakita (2010)).

With such a simple model, we examine whether the labor-value pricing equilibrium is established as the center of gravity for price fluctuations.

## 4.2 A simple model of precapitalist economy with a simple Leontief technique

Consider an economy with a simple Leontief technique,[15] so that assume $\mathcal{P} = \{(\boldsymbol{A}, \boldsymbol{L})\}$. Moreover, let $\boldsymbol{A}$ be productive and indecomposable. Let $\mathcal{N} \equiv \{N, S\}$ be the set of individuals, and the number of the types of commodities be $n = 2$. Let $\boldsymbol{b} \in \mathbb{R}^2_{++}$ be the *subsistence consumption bundle*, which every individual must consume for his survival in one period of production, regardless of whether supplying labor or not. For the sake of simplicity, the maximal amount of labor supply by every agent is equal to unity and there is no difference in labor skills among agents. Let $\overline{\boldsymbol{\omega}} \in \mathbb{R}^2_{++}$ be the social endowments of material capital goods at the beginning of the initial period of production. For the sake of simplicity, assume $\overline{\boldsymbol{\omega}} \equiv \boldsymbol{A}\left[\boldsymbol{I} - \boldsymbol{A}\right]^{-1}(2\boldsymbol{b})$. Every individual has the common consumption space $C \equiv \left\{\boldsymbol{c} \in \mathbb{R}^2_{+} \mid \boldsymbol{c} \geqq \boldsymbol{b}\right\} \times [0,1]$ with a generic element $(\boldsymbol{c}, l)$, where $\boldsymbol{c} \in \left\{\boldsymbol{c} \in \mathbb{R}^2_{+} \mid \boldsymbol{c} \geqq \boldsymbol{b}\right\}$ represents a consumption bundle and $l \in [0,1]$ represents an amount of labor expended. Moreover, every individual has the common *leisure preference* which is represented by a utility function $u : C \to \mathbb{R}$ defined as: for each $(\boldsymbol{c}, l) \in C$,

$$u(\boldsymbol{c}, l) = 1 - l.$$

That is, every individual does not concern about the increase of consumption goods beyond the subsistence level $\boldsymbol{b}$, but mainly concerns about the increase of free hours (leisure time), once the consumption of the subsistence bundle $\boldsymbol{b}$ is ensured.

An economy with a simple Leontief technique is specified by a profile $\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), \overline{\boldsymbol{\omega}} \rangle$, which we may call a *precapitalist economy*. Denote each individual's capital endowments at period $t$ by $\boldsymbol{\omega}^N_t = \left(\omega^N_{1t}, \omega^N_{2t}\right) > (0,0)$ and $\boldsymbol{\omega}^S_t = \left(\omega^S_{1t}, \omega^S_{2t}\right) > (0,0)$. Needless to say, assume that $\boldsymbol{\omega}^N_t + \boldsymbol{\omega}^S_t = \overline{\boldsymbol{\omega}}$ holds.

We explicitly take the time structure of production. Hence, the capital goods available at the present period of production cannot exceed the amount

[15] The following economic model is a simpler version of the model in Yoshihara and Kaneko (2016), though their interpretations are different. We will interpret the following model as a representation of the simple commodity production economy discussed in Marx (1894, 1981, pp. 275-279), while Yoshihara and Kaneko (2016) regard it as a model of pre-industrial world economy with no international labor nor credit market. Among the main theorems presented below, Theorems 1 and 3 are taken from Yoshihara and Kaneko (2016), but Theorems 2 and 4 are the new results in this paper.

of capital goods accumulated until the end of the preceding period of production. Moreover, the time structure of production is given as follows:

(1) Given market prices $\boldsymbol{p}_{t-1} = (p_{1t-1}, p_{2t-1}) \geq (0,0)$ at the beginning of the period $t$, each agent $\nu = N, S$ purchases, under the constraint of wealth endowment $\boldsymbol{p}_{t-1}\boldsymbol{\omega}_t^\nu$ , the capital goods $\boldsymbol{A}\boldsymbol{x}_t^\nu$ as inputs for the production at the present period, and the commodities $\boldsymbol{\delta}_t^\nu$ to sell, for a speculative purpose, at the end of the present period;

(2) Each agent is engaged in the production activity of period $t$ by inputting labor $\boldsymbol{L}\boldsymbol{x}_t^\nu$ and the purchased capital goods $\boldsymbol{A}\boldsymbol{x}_t^\nu$;

(3) The production activity is completed and $\boldsymbol{x}_t^\nu$ is produced as an output at the end of this period. Then, in goods markets with market prices $\boldsymbol{p}_t \geq (0,0)$, each agent earns the revenue $\boldsymbol{p}_t\boldsymbol{x}_t^\nu + \boldsymbol{p}_t\boldsymbol{\delta}_t^\nu$ derived from the output $\boldsymbol{x}_t^\nu$ and the speculative commodity bundle $\boldsymbol{\delta}_t^\nu$, with which he purchases the bundle $\boldsymbol{b}$ for the consumption at the end of this period and the capital stock $\boldsymbol{\omega}_{t+1}^\nu$ for the production of the next period. Therefore, the wealth endowment carried over to the next period $t+1$ is $\boldsymbol{p}_t\boldsymbol{\omega}_{t+1}^\nu$.

Given a price system $\left\{\boldsymbol{p}_{t-1}, \boldsymbol{p}_t\right\}$ prevailed at the beginning of period $t$, each agent $\nu\,(= N, S)$ solves the following optimization program $(MP_t^\nu)$:

$$\min_{x_t^\nu, \delta_t^\nu} l_t^\nu$$

$$\begin{aligned} \text{s.t.}\quad \boldsymbol{p}_t\boldsymbol{x}_t^\nu + \boldsymbol{p}_t\boldsymbol{\delta}_t^\nu &\geqq \boldsymbol{p}_t\boldsymbol{b} + \boldsymbol{p}_t\boldsymbol{\omega}_{t+1}^\nu; \\ l_t^\nu &= \boldsymbol{L}\boldsymbol{x}_t^\nu \leqq 1; \\ \boldsymbol{p}_{t-1}\boldsymbol{\delta}_t^\nu + \boldsymbol{p}_{t-1}\boldsymbol{A}\boldsymbol{x}_t^\nu &\leqq \boldsymbol{p}_{t-1}\boldsymbol{\omega}_t^\nu, \text{ where } \boldsymbol{\delta}_t^\nu \in \mathbb{R}_+^2\ ; \\ \boldsymbol{p}_t\boldsymbol{\omega}_{t+1}^\nu &\geqq \boldsymbol{p}_{t-1}\boldsymbol{\omega}_t^\nu. \end{aligned}$$

Denote the set of solutions for the optimization program of each agent $\nu$ at period $t$ by $\boldsymbol{O}_t^\nu\left(\left\{\boldsymbol{p}_{t-1}, \boldsymbol{p}_t\right\}\right)$.

For the sake of simplicity, let us focus on the case of stationary equilibrium prices (that is, $\boldsymbol{p}_t = \boldsymbol{p}_{t-1} = \boldsymbol{p}^*$). In this case, for any optimal solution $(\boldsymbol{x}_t^{*\nu}, \boldsymbol{\delta}_t^{*\nu}) \in \boldsymbol{O}_t^\nu(\boldsymbol{p}^*)$, it follows that $\boldsymbol{\delta}_t^{*\nu} = \boldsymbol{0}$ and $\boldsymbol{p}^*\boldsymbol{x}_t^{*\nu} - \boldsymbol{p}^*\boldsymbol{A}\boldsymbol{x}_t^{*\nu} = \boldsymbol{p}^*\boldsymbol{b}$.

Now, an equilibrium solution is ready to introduce.

**Definition 1:** For a precapitalist economy $\left\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), \left(\boldsymbol{\omega}_t^N, \boldsymbol{\omega}_t^S\right)\right\rangle$ at period $t$, where $\boldsymbol{\omega}_t^N + \boldsymbol{\omega}_t^S = \overline{\boldsymbol{\omega}}$, a *reproducible solution (RS) at this period* is a profile of a price system $\boldsymbol{p}^*$ and production activities $(\boldsymbol{x}_t^{*\nu})_{\nu\in\mathcal{N}}$ satisfying the following conditions:
(i) $(\boldsymbol{x}_t^{*\nu}, \boldsymbol{0}) \in \boldsymbol{O}_t^\nu(\boldsymbol{p}^*)$ $(\forall \nu \in \mathcal{N})$; (each agent's individual optimization)
(ii) $2\boldsymbol{b} \leqq [\boldsymbol{I} - \boldsymbol{A}]\left(\boldsymbol{x}_t^{*N} + \boldsymbol{x}_t^{*S}\right)$; (the demand-supply matching at the end of period $t$)
(iii) $\boldsymbol{A}\left(\boldsymbol{x}_t^{*N} + \boldsymbol{x}_t^{*S}\right) \leqq \boldsymbol{\omega}_t^N + \boldsymbol{\omega}_t^S$. (social feasibility of production at the beginning of period $t$)

In addition to the above definition, let us focus on the following subset of RS: A *reproducible solution is imperfectly specialized* if and only if $\boldsymbol{x}_t^{*\nu} > \mathbf{0}$ holds for each $\nu \in \mathcal{N}$.

By the property of imperfect specialization RS, it follows that $\boldsymbol{p}^* \in \mathbb{R}^2_{++}$ and $[\boldsymbol{I} - \boldsymbol{A}]\left(\boldsymbol{x}_t^{*N} + \boldsymbol{x}_t^{*S}\right) = 2\boldsymbol{b}$. The latter equation implies $\left(\boldsymbol{x}_t^{*N} + \boldsymbol{x}_t^{*S}\right) = [\boldsymbol{I} - \boldsymbol{A}]^{-1}(2\boldsymbol{b})$, therefore, $\boldsymbol{A}\left(\boldsymbol{x}_t^{*N} + \boldsymbol{x}_t^{*S}\right) = \boldsymbol{A}[\boldsymbol{I} - \boldsymbol{A}]^{-1}(2\boldsymbol{b}) = \overline{\boldsymbol{\omega}} = \boldsymbol{\omega}_t^N + \boldsymbol{\omega}_t^S$ holds.

Let $\mathbf{e}_i$ be the $i$-th unit vector (only the $i$-th component is unity, and any other is zero). Though there is no labor market in this economy, each agent $\nu \in \mathcal{N}$ should have his own personal view about the reward for his labor expenditure. That is, each agent $\nu \in \mathcal{N}$ should have a specific real number $w_t^\nu \geqq 0$, which represents $\nu$'s view that if the reward for the production in process $i$ by supplying one unit of labor is less than $w_t^\nu$, $\nu$ would not like to engage in such production activity. Formally speaking, $\frac{p_i - \boldsymbol{pA}\mathbf{e}_i}{L_i}$ represents the reward per unit of labor for the production activity in process $i$. Therefore, $\nu$ would not like to work in process $i$ if $\frac{p_i - \boldsymbol{pA}\mathbf{e}_i}{L_i} < w_t^\nu$. Moreover, $\nu$ would recognize that the production in process $i$ is *profitable* if $\frac{p_i - \boldsymbol{pA}\mathbf{e}_i}{L_i} \geqq w_t^\nu$, where the latter inequality is equivalent to $\frac{p_i - \boldsymbol{pA}\mathbf{e}_i - w_t^\nu L_i}{\boldsymbol{pA}\mathbf{e}_i} \geqq 0$. In this case, $\nu$ would also recognize that the production in process $i$ is more *profitable* than the production in process $j$ if and only if $\frac{p_i - \boldsymbol{pA}\mathbf{e}_i - w_t^\nu L_i}{\boldsymbol{pA}\mathbf{e}_i} > \frac{p_j - \boldsymbol{pA}\mathbf{e}_j - w_t^\nu L_j}{\boldsymbol{pA}\mathbf{e}_j}$. If so, then $\nu$ would like to be perfectly specialized in process $i$.

Given these arguments, we may say that, under the imperfectly specialized RS, where $\boldsymbol{x}_t^{*\nu} > \mathbf{0}$ holds for every $\nu \in \mathcal{N}$, there exists a suitable profile of personal views of labor rewards and return rates, $(w_t^{\nu *}, r_t^{\nu *})_{\nu \in \mathcal{N}}$, such that every agent is willing to work in both process 1 and process 2, in that $\frac{p_i - \boldsymbol{pA}\mathbf{e}_i}{L_i} \geqq w_t^\nu$ holds for every process $i = 1, 2$ and every agent $\nu \in \mathcal{N}$. Moreover, for every agent, both production processes are *equally profitable*: that is, for every $\nu \in \mathcal{N}$, $r_t^\nu = \frac{p_i - \boldsymbol{pA}\mathbf{e}_i - w_t^\nu L_i}{\boldsymbol{pA}\mathbf{e}_i}$ holds for every process $i = 1, 2$.

In summary, under the imperfectly specialized RS $\left\langle \boldsymbol{p}^*; (\boldsymbol{x}_t^{*\nu})_{\nu \in \mathcal{N}} \right\rangle$, every agent $\nu \in \mathcal{N}$ has a profile of his personal view of labor reward and return rate, $(w_t^{\nu *}, r_t^{\nu *})$, such that

$$\boldsymbol{p}^* = (1 + r_t^{\nu *})\, \boldsymbol{p}^* \boldsymbol{A} + w_t^{\nu *} \boldsymbol{L} \quad (4.1)$$

holds.

It is well-known that, in the so-called neoclassical Heckscher-Ohlin model of international trade, the factor price equalization theorem and the Heckscher-Ohlin theorem hold. Even in the model of precapitalist economy presented herein, where neither labor nor credit market exists, we can verify the following two theorems:

**Theorem 1 (Factor price equalization):** For any precapitalist economy $\left\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), \left(\boldsymbol{\omega}_t^N, \boldsymbol{\omega}_t^S\right) \right\rangle$ with $\boldsymbol{\omega}_t^N + \boldsymbol{\omega}_t^S = \overline{\boldsymbol{\omega}}$, let $\left\langle \boldsymbol{p}^*; (w_t^{\nu *}, r_t^{\nu *})_{\nu \in \mathcal{N}}, (\boldsymbol{x}_t^{*\nu})_{\nu \in \mathcal{N}} \right\rangle$ be an imperfect specialization RS at period $t$. Then, if $\frac{\boldsymbol{p}^* \boldsymbol{A}\mathbf{e}_1}{L_1} \neq \frac{\boldsymbol{p}^* \boldsymbol{A}\mathbf{e}_2}{L_2}$, then $\left(w_t^{N*}, r_t^{N*}\right) = \left(w_t^{S*}, r_t^{S*}\right)$ holds.

**Proof.** It follows from Yoshihara and Kaneko (2016; Theorem 1). ■

**Theorem 2 (Quasi-Heckscher-Ohlin theorem):** For any precapitalist economy $\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), (\boldsymbol{\omega}_t^N, \boldsymbol{\omega}_t^S) \rangle$ with $\boldsymbol{\omega}_t^N + \boldsymbol{\omega}_t^S = \overline{\boldsymbol{\omega}}$, let $\langle \boldsymbol{p}^*; (w_t^*, r_t^*), (\boldsymbol{x}_t^{*\nu})_{\nu \in \mathcal{N}} \rangle$ be an imperfect specialization RS at period $t$ with $\frac{\boldsymbol{p}^* \boldsymbol{A} \mathbf{e}_1}{L_1} > \frac{\boldsymbol{p}^* \boldsymbol{A} \mathbf{e}_2}{L_2}$. Then, if $\boldsymbol{p}^* \boldsymbol{\omega}_t^N > \boldsymbol{p}_t^* \boldsymbol{\omega}_t^S$, the wealthier agent, $N$, sells the more capital-intensive good, good 1, and purchases the more labor-intensive good, good 2. Correspondingly, the poorer agent, $S$, sells the more labor-intensive good 2 and purchases the more capital-intensive good 1.

Given an RS $\langle \boldsymbol{p}^*; (w_t^{\nu *}, r_t^{\nu *})_{\nu \in \mathcal{N}}, (\boldsymbol{x}_t^{*\nu})_{\nu \in \mathcal{N}} \rangle$ for an economy $\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), \overline{\boldsymbol{\omega}} \rangle$, the supply of labor hours to earn the revenue $\boldsymbol{p}^* \boldsymbol{b}$ for its own survival is $\boldsymbol{L} \boldsymbol{x}_t^{*\nu}$ for each agent $\nu = N, S$, while the amount of socially necessary labor for producing $\boldsymbol{b}$ as a net output is given by

$$\frac{1}{2} \boldsymbol{L} \left( \boldsymbol{x}_t^{*N} + \boldsymbol{x}_t^{*S} \right) = \boldsymbol{v} \boldsymbol{b} = \boldsymbol{L} \left[ \boldsymbol{I} - \boldsymbol{A} \right]^{-1} \boldsymbol{b}.$$

Then, the notion of UE-exploitation under precapitalist economies is formally defined as follows:

**Definition 2** [Yoshihara and Kaneko (2016)]**:** For a precapitalist economy $\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), \overline{\boldsymbol{\omega}} \rangle$, let $\langle \boldsymbol{p}^*; (w_t^{\nu *}, r_t^{\nu *})_{\nu \in \mathcal{N}}, (\boldsymbol{x}_t^{*\nu})_{\nu \in \mathcal{N}} \rangle$ be an RS at period $t$. Then:

$$\nu \text{ is an } \textit{exploiting} \text{ agent} \Longleftrightarrow \boldsymbol{L} \boldsymbol{x}_t^{*\nu} < \boldsymbol{v} \boldsymbol{b};$$
$$\nu \text{ is an } \textit{exploited} \text{ agent} \Longleftrightarrow \boldsymbol{L} \boldsymbol{x}_t^{*\nu} > \boldsymbol{v} \boldsymbol{b}.$$

Under the presumption of Definition 2, the following theorem indicates that if the quasi-Hecksher-Ohlin type of social division of labor is generated in the market-exchange relation, it is characterized as an exploitative relation:

**Theorem 3 (The generation of exploitative relations in precapitalist economies):** For any precapitalist economy $\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), (\boldsymbol{\omega}_t^N, \boldsymbol{\omega}_t^S) \rangle$ with $\boldsymbol{\omega}_t^N + \boldsymbol{\omega}_t^S = \overline{\boldsymbol{\omega}}$, let $\langle \boldsymbol{p}^*; (w_t^*, r_t^*), (\boldsymbol{x}_t^{*\nu})_{\nu \in \mathcal{N}} \rangle$ be an imperfect specialization RS at period $t$ with $\frac{\boldsymbol{p}^* \boldsymbol{A} \mathbf{e}_1}{L_1} > \frac{\boldsymbol{p}^* \boldsymbol{A} \mathbf{e}_2}{L_2}$. Then, if $r_t^* > 0$ and $\boldsymbol{p}_t^* \boldsymbol{\omega}_t^N > \boldsymbol{p}^* \boldsymbol{\omega}_t^S$, then the wealthier agent, $N$, is exploiting, and the poorer agent, $S$, is exploited, in terms of Definition 2. Conversely, if $r_t^* = 0$ or $\boldsymbol{p}^* \boldsymbol{\omega}_t^N = \boldsymbol{p}^* \boldsymbol{\omega}_t^S$ holds, then there is no exploitative relation.

**Proof.** It follows from Yoshihara and Kaneko (2016; Corollary 1). ■

In the above Theorem 3, the inequality $\boldsymbol{L} \boldsymbol{x}_t^{*N} < \boldsymbol{v} \boldsymbol{b} < \boldsymbol{L} \boldsymbol{x}_t^{*S}$ to represent *unequal exchange of labor* definitely implies the *generation of exploitative relation.* In an RS for a precapitalist economy, both of $N$ and $S$ earn the minimal income to purchase the subsistence bundle $\boldsymbol{b}$. However, there is a difference between the two agents in terms of their labor supply, and agent $N$ can enjoy more hours as freedom from the necessary labor for survival than agent $S$.

Given that $\boldsymbol{p}_t^* \boldsymbol{\omega}_t^N > \boldsymbol{p}^* \boldsymbol{\omega}_t^S$, Theorem 3 implies that an RS is UE-exploitative if and only if $r_t^* > 0$. It is well-know that the equilibrium prices $\boldsymbol{p}^*$ are labor-value pricing if and only if $r_t^* = 0$. Therefore, the existence of UE-exploitation in this precapitalist economy implies that the labor-value pricing could not be the unique equilibrium prices, and so it would not necessarily serve as the center of gravity for price fluctuations in the long term.

The last view is indeed verified by the following theorem:

**Theorem 4 (The existence of the continuum of RSs):** Consider a precapitalist economy $\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), \overline{\boldsymbol{\omega}} \rangle$, such that the Leontief technique $(\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b})$ is sufficiently productive: $\boldsymbol{vb} < 1$ holds. Moreover, for the unique Frobenius row vector $\boldsymbol{q} > \boldsymbol{0}$ of $\boldsymbol{A}$, let $\boldsymbol{L}$ and $\boldsymbol{q}$ be linearly independent. Then, there exists a unique positive profit rate $\overline{r} > 0$ such that $\boldsymbol{L}\left[\boldsymbol{I} - (1+\overline{r})\boldsymbol{A}\right]^{-1} \boldsymbol{b} = 1$ holds. Moreover, let us define a subset of the profit-wage curve as:

$$G_{\overline{r}} \equiv \left\{ (r,w) \in [0,\overline{r}] \times \left[1, \frac{1}{\boldsymbol{vb}}\right] \mid w = w(r) \equiv \frac{1}{\boldsymbol{L}\left[\boldsymbol{I} - (1+r)\boldsymbol{A}\right]^{-1} \boldsymbol{b}} \text{ for any } r \in [0,\overline{r}] \right\}.$$

Then, there exists $\theta^S \in \left(0, \frac{1}{2}\right)$ such that for $\left(\boldsymbol{\omega}_t^N, \boldsymbol{\omega}_t^S\right) \equiv \left(\left(1-\theta^S\right)\overline{\boldsymbol{\omega}}, \theta^S \overline{\boldsymbol{\omega}}\right)$ and for any $(r, w(r)) \in G_{\overline{r}}$, the corresponding price vector $\boldsymbol{p}(r) > \boldsymbol{0}$ is given by

$$\boldsymbol{p}(r) \equiv w(r)\,\boldsymbol{L}\left[\boldsymbol{I} - (1+r)\boldsymbol{A}\right]^{-1},$$

and $(\boldsymbol{p}(r), w(r), r)$ associated with a suitable profile of each agent's activities $\left(\boldsymbol{x}^N(r), \boldsymbol{x}^S(r)\right)$ constitutes an imperfect specialization RS at period $t$.

**Proof.** First of all, given that $\boldsymbol{L}$ and $\boldsymbol{q}$ being linearly independent, it can be ensured from Yoshihara and Kaneko (2016) that for any $r \in [0, R)$, where $\frac{1}{1+R} > 0$ is the Frobenius eigenvalue of $\boldsymbol{A}$, and for any $\boldsymbol{p}^r \equiv \boldsymbol{L}\left[\boldsymbol{I} - (1+r)\boldsymbol{A}\right]^{-1} > \boldsymbol{0}$, the vectors $\boldsymbol{p}^r[\boldsymbol{I} - \boldsymbol{A}]$ and $\boldsymbol{p}^r \boldsymbol{A}$ are linear independent. Then, since $\left[\boldsymbol{I} - (1+r)\boldsymbol{A}\right]^{-1}$ is continuous and strongly increasing with respect to $r$ and every component of $\left[\boldsymbol{I} - (1+r)\boldsymbol{A}\right]^{-1}$ is divergent to infinity as $r \to R$, we have $\lim_{r\to R} \boldsymbol{L}\left[\boldsymbol{I} - (1+r)\boldsymbol{A}\right]^{-1} \boldsymbol{b} = \infty$. Then, it follows from $\boldsymbol{L}\left[\boldsymbol{I} - \boldsymbol{A}\right]^{-1} \boldsymbol{b} = \boldsymbol{vb} < 1$ and the continuous and strong increasingness of $\boldsymbol{L}\left[\boldsymbol{I} - (1+r)\boldsymbol{A}\right]^{-1} \boldsymbol{b}$ with respect to $r$ that there exists $\overline{r} > 0$ such that $\overline{r} < R$ and $\boldsymbol{L}\left[\boldsymbol{I} - (1+\overline{r})\boldsymbol{A}\right]^{-1} \boldsymbol{b} = 1$ holds. Let the simplex of commodity prices be given by $\triangle \equiv \left\{\boldsymbol{p} \in \mathbb{R}_+^2 \mid \boldsymbol{pb} = 1\right\}$. Then, the wage rate $w(\overline{r}) = \frac{1}{\boldsymbol{L}[\boldsymbol{I}-(1+\overline{r})\boldsymbol{A}]^{-1}\boldsymbol{b}} = 1$ holds, so that any propertyless worker must supply one unit of labor to purchase the subsistence consumption vector whenever $(\boldsymbol{p}(\overline{r}), w(\overline{r}), \overline{r})$ prevails. This implies that for any $r > \overline{r}$ with $r < R$, $w(r) < 1$ holds, so that any propertyless worker cannot survive. Therefore, let us define the available class of profit rates as $[0, \overline{r}]$. Then, correspondingly, the class of available wage rates is given by $\left[1, \frac{1}{\boldsymbol{vb}}\right]$, where $w(\overline{r}) = 1$ and $w(0) = \frac{1}{\boldsymbol{vb}}$.

Define $G_{\overline{r}} \equiv \left\{ (r,w) \in [0,\overline{r}] \times \left[1, \frac{1}{\boldsymbol{vb}}\right] \mid w = w(r) \equiv \frac{1}{\boldsymbol{L}[\boldsymbol{I}-(1+r)\boldsymbol{A}]^{-1}\boldsymbol{b}} \text{ for any } r \in [0,\overline{r}] \right\}$, and $\boldsymbol{p}(r) \equiv w(r)\boldsymbol{L}[\boldsymbol{I}-(1+r)\boldsymbol{A}]^{-1}$. Then, consider the following problem: given any $(r, w(r)) \in G_{\overline{r}}$,

$$\min \theta \text{ s.t. } \boldsymbol{p}(r)(\boldsymbol{I}-\boldsymbol{A})\boldsymbol{x} = \boldsymbol{p}(r)\boldsymbol{b} \ \&\ \boldsymbol{p}(r)\boldsymbol{A}\boldsymbol{x} = \theta\boldsymbol{p}(r)\overline{\boldsymbol{\omega}} \ \&\ \boldsymbol{L}\boldsymbol{x} \leqq 1 \text{ for some } \boldsymbol{x} > \boldsymbol{0}.$$

Note that for $\boldsymbol{x}^{\boldsymbol{b}} \equiv [\boldsymbol{I}-\boldsymbol{A}]^{-1}\boldsymbol{b} > \boldsymbol{0}$, it follows that $\boldsymbol{p}(r)(\boldsymbol{I}-\boldsymbol{A})\boldsymbol{x}^{\boldsymbol{b}} = \boldsymbol{p}(r)\boldsymbol{b}$, $\boldsymbol{p}(r)\boldsymbol{A}\boldsymbol{x}^{\boldsymbol{b}} = \frac{1}{2}\boldsymbol{p}(r)\overline{\boldsymbol{\omega}}$, and $\boldsymbol{L}\boldsymbol{x}^{\boldsymbol{b}} < 1$. At a fixed $(r, w(r)) \in G_{\overline{r}}$, decreasing $\theta$ continuously from $\theta = \frac{1}{2}$ leads $\boldsymbol{x}(\theta; r) > \boldsymbol{0}$, which satisfies $\boldsymbol{p}(r)(\boldsymbol{I}-\boldsymbol{A})\boldsymbol{x}(\theta; r) = \boldsymbol{p}(r)\boldsymbol{b}$ and $\boldsymbol{p}(r)\boldsymbol{A}\boldsymbol{x}(\theta; r) = \theta\boldsymbol{p}(r)\overline{\boldsymbol{\omega}}$, to increase $\boldsymbol{L}\boldsymbol{x}(\theta; r)$ continuously. Therefore, the above problem is well-defined, and the solution $\theta(r) > 0$ should exist as $\theta(r) < \frac{1}{2}$ whenever $\boldsymbol{p}(r)(\boldsymbol{I}-\boldsymbol{A})$ and $\boldsymbol{p}(r)\boldsymbol{A}$ are linear independent, where the latter claim holds as confirmed above.

Next, consider

$$\max_{r\in[0,\overline{r}]} \theta(r)$$

and it has a unique solution, say $\theta^*$, as $\theta(r)$ is continuous over $[0,\overline{r}]$. Note that $\theta^* < \frac{1}{2}$, as $\theta(r) < \frac{1}{2}$ for any given $(r, w(r)) \in G_{\overline{r}}$. As argued above, at any fixed $(r, w(r)) \in G_{\overline{r}}$, decreasing the value $\theta$ from $\theta = \theta^*$ implies increasing the value $\boldsymbol{L}\boldsymbol{x}(\theta; r)$ from $\boldsymbol{L}\boldsymbol{x}(\theta^*; r) < 1$ by keeping $\boldsymbol{L}\boldsymbol{x}(\theta; r) < 1$, which implies that for any $(r, w(r)) \in G_{\overline{r}}$, $\theta(r) \leqq \theta^*$ and $\boldsymbol{L}\boldsymbol{x}(\theta^*; r) \leqq \boldsymbol{L}\boldsymbol{x}(\theta(r; r)) < 1$.

Now, let $\theta^S \equiv \theta^*$ and define $\left(\boldsymbol{\omega}_t^N, \boldsymbol{\omega}_t^S\right) \equiv ((1-\theta^*)\overline{\boldsymbol{\omega}}, \theta^*\overline{\boldsymbol{\omega}})$. As $\theta^* < \frac{1}{2}$, we have $\boldsymbol{\omega}_t^N > \boldsymbol{\omega}_t^S$. Then, for each $(r, w(r)) \in G_{\overline{r}}$, let $\boldsymbol{x}^S(r) \equiv \boldsymbol{x}(\theta^*; r) > \boldsymbol{0}$, and $\boldsymbol{x}^N(r) \equiv [\boldsymbol{I}-\boldsymbol{A}]^{-1} N\boldsymbol{b} - \boldsymbol{x}^S(r) > \boldsymbol{0}$. As $\boldsymbol{p}(r)(\boldsymbol{I}-\boldsymbol{A})\boldsymbol{x}^S(r) = \boldsymbol{p}(r)\boldsymbol{b}$ and $\boldsymbol{p}(r)\boldsymbol{A}\boldsymbol{x}^S(r) = \boldsymbol{p}(r)\boldsymbol{\omega}_t^S$ hold for $\boldsymbol{\omega}_t^S = \theta^*\overline{\boldsymbol{\omega}}$, we have $\boldsymbol{p}(r)(\boldsymbol{I}-\boldsymbol{A})\boldsymbol{x}^N(r) = \boldsymbol{p}(r)\boldsymbol{b}$ and $\boldsymbol{p}(r)\boldsymbol{A}\boldsymbol{x}^N(r) = \boldsymbol{p}(r)\boldsymbol{\omega}_t^N$ hold for $\boldsymbol{\omega}_t^N = (1-\theta^*)\overline{\boldsymbol{\omega}}$. Moreover, $\boldsymbol{L}\boldsymbol{x}^S(r) > \boldsymbol{L}\boldsymbol{x}^N(r) > 0$ holds for any $(r, w(r)) \in G_{\overline{r}}$, which implies $\boldsymbol{L}\boldsymbol{x}^N(r) < 1$ for any $(r, w(r)) \in G_{\overline{r}}$.

Finally, by construction of $\boldsymbol{x}^N(r)$, we have $\boldsymbol{L}\boldsymbol{x}^S(r) + \boldsymbol{L}\boldsymbol{x}^N(r) = \boldsymbol{vb}$, $\boldsymbol{A}\boldsymbol{x}^S(r) + \boldsymbol{A}\boldsymbol{x}^N(r) = \overline{\boldsymbol{\omega}}$, and $[\boldsymbol{I}-\boldsymbol{A}]\boldsymbol{x}^S(r) + [\boldsymbol{I}-\boldsymbol{A}]\boldsymbol{x}^N(r) = N\boldsymbol{b}$ for any $(r, w(r)) \in G_{\overline{r}}$. As $\boldsymbol{x}^\nu(r)$ is an optimal solution for $MP_t^\nu$ at the price system $(\boldsymbol{p}(r), w(r), r)$ for each $\nu = N, S$, we have $\boldsymbol{x}^\nu(r) \in \boldsymbol{O}_t^\nu(\boldsymbol{p}(r))$ for each $\nu = N, S$.

Thus, in summary, for each $(r, w(r)) \in G_{\overline{r}}$, a price system $(\boldsymbol{p}(r), w(r), r)$ associated with $\left(\boldsymbol{x}^N(r), \boldsymbol{x}^S(r)\right)$ constitutes an imperfect specialization RS for the economy $\left\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), \left(\boldsymbol{\omega}_t^N, \boldsymbol{\omega}_t^S\right)\right\rangle$ at period $t$. ■

Note that in Theorem 4, an imperfect specialization RS $(\boldsymbol{p}(r), w(r), r)$ is continuous at every $r \in [0,\overline{r}]$, and so we can specify a continuum set of imperfect specialization RSs as follows:

$$\left\{ (\boldsymbol{p}(r), w(r), r) \mid r \in [0,\overline{r}],\ w(r) = \frac{1}{\boldsymbol{L}[\boldsymbol{I}-(1+r)\boldsymbol{A}]^{-1}\boldsymbol{b}} \text{ and } \boldsymbol{p}(r) = w(r)\boldsymbol{L}[\boldsymbol{I}-(1+r)\boldsymbol{A}]^{-1} \right\},$$

which constitutes an one-dimensional equilibrium manifold. This equilibrium manifold also contains the *labor-value pricing equilibrium* $(\boldsymbol{p}(0), w(0), 0)$ where $w(0) = \frac{1}{\boldsymbol{vb}}$ and $\frac{\boldsymbol{p}(0)}{w(0)} = \boldsymbol{v}$.

From Theorem 4, we can see that for any precapitalist economy $\langle \mathcal{N}, (\boldsymbol{A}, \boldsymbol{L}, \boldsymbol{b}), \overline{\boldsymbol{\omega}} \rangle$, a continuum set of imperfect specialization RSs exists under some unequal initial distribution of capital goods $\overline{\boldsymbol{\omega}}$. This continuum set contains the labor-value pricing equilibrium, but there are infinitely many RSs with positive profit rates. By Theorem 3, such RSs are UE-exploitative.

In conclusion, unlike the standard Marxian view, it is impossible to verify that in a precapitalist economy with simple commodity-production, the labor-value pricing equilibrium serves as the center of gravity for the long terms price fluctuations. This is because, by Theorem 4, there are infinitely many candidates of equilibrium prices which could serve as the center of gravity, and no intrinsic mechanism in the precapitalist economy is found to select uniquely the labor-value pricing as the center of gravity.[16]

A final remark is given for what was wrong in Marx's (1894, 1981; Chapter 10) own arguments, from the viewpoint of main theorems in this section. Marx (1894, 1981; Chapter 10) presumed that even under the unequal private ownership of capital goods between workers I and II, they supply the same living labor time so that purchase and consume the same consumption bundle, where their individual return rates (profit rates) differ. His argument is insufficient in that it lacks the analysis of the social division of labor between the two producers which could generate through market competition.

In contrast, our analysis in this section shows that even if the two producers purchase and consume the same consumption bundle, the social division of labor between them emerges through market competition as Theorem 2 shows, and then it leads them to the inequality of the supply of the living labor time, due to the unequal initial endowments of capital goods, as Theorem 3 shows. In such a situation, profit rates are equalized among all processes under the imperfect specialization RS, as Theorem 1 shows. A possibility of these three features has been ignored by Marx (1894, 1981).

# 5 Conclusions

In this paper, we have reviewed LTV as the basis for the analysis of economic inequality in the capitalist economy. According to the standard Marxian view, the system of labor values of individual commodities can serve as the center of gravity for long-term price fluctuations in the precapitalist economy with simple commodity-production, where no UE-exploitative social relation emerges, while

[16] This criticism is particularly relevant if it is applied in the context of non-equilibrium price dynamics with no intertemporal structure. In contrast, if the issue of the center of gravity is considered in an intertemporal economy, it is Kaneko and Yoshihara (2019), among other works like Duménil and Levy (1985), Dana et. al (1989), and Veneziani (2007, 2013), which would provide us with the most relevant insights. In that paper where the intertemporal precapitalist economy with no labor nor credit market is considered, the path of intertemporal equilibrium labor allocations among nations converges to the egalitarian one, and so the UE-exploitation would tend to disappear in the infinite limit. However, *it does not necessarily imply that equilibrium prices converge to the labor-value pricing.* The egalitarian convergence of labor allocations takes place either because of the convergence of the prices to the labor values or because of the convergence of the wealth distributions to the egalitarian distribution.

in the modern capitalist economy, the labor value system is replaced by the prices of production associated with an equal positive rate of profits as the center of gravity, in which UE-exploitative relation between the capitalist and the working classes is a generic and persistent feature of economic inequality. Some of the literature such as Morishima (1973, 1974) criticized this view by showing that the labor values of individual commodities are no longer well-defined if the capitalist economy has joint production.

Given these arguments, this paper firstly shows that the system of individual labor values can be still well-defined in the capitalist economy with joint production whenever the set of available production techniques is all-productive. Under such a restricted class of joint production economies, the standard formal definition of individual labor values can be deemed appropriate, since the market exchanges of commodities in accordance with the labor-value pricing can allocate the total quantity of the living labor time in order to meet social demands for each and every commodity. With the same criterion, the paper also shows that an alternative LTV initiated by **TSSI** cannot be verified.

Secondly, this paper shows that it is generally impossible to verify that the labor-value pricing serves as the center of gravity for price fluctuations in pre-capitalist economies characterized by the full development of simple commodity-production. Though the labor-value pricing is just one candidate for the role of center of gravity, there are infinitely many other candidates, and no proper mechanism to uniquely select the labor-value pricing can be found within the simple commodity production economy.

# 6 References


Cunningham, H. (1980): *Lecture in the Industrial Revolution: c.1780-c.1880.* New York: St. Martin's Press.

Cunningham, H. (2014): *Time, Work and Leisure: Life changes in England since 1700.* Manchester, UK: Manchester University Press.

Dana, R-A., Florenzano, M., Le Van, C., and D. Levy (1989): "Production Prices and General Equilibrium Prices: A Long-Run Property of a Leontief Economy," *Journal of Mathematical Economics* 18, pp. 263-280.

Duménil, G. (1980): *De la Valeur aux Prix de Production*, Paris: Economica.

Duménil, G., and D. Levy (1985): "The Classicals and the Neoclassicals, A Rejoinder to Frank Hahn," *Cambride Journal of Economics* 9, pp. 327-345.

Foley, D. K. (1982): "The Value of Money, the Value of Labor Power, and the Marxian Transformation Problem," *Review of Radical Political Economics* 14, pp. 37-47.

International Labour Office. (2005a) *Human Trafficking and Forced Labour Exploitation*, International Labour Office, Geneva.

International Labour Office. (2005b) *Forced Labour: Labour Exploitation and Human Trafficking in Europe*, International Labour Office, Geneva.

Itoh, M. (2021) *Value and Crisis, Essays on Marxian Economics in Japan*, (2nd ed.), Monthly Review Press, New York.

Kaneko, S. and N. Yoshihara (2019): "On the General Impossibility of Persistent Unequal Exchange Free Trade Equilibria in the Pre-industrial World Economy," Working Paper 2019-05, UMass Amherst Economics Papers.

Kawakita, M. (2010): *Lectures on Modern British History*, (in Japanese) Gendai-Shinsho series 2070, Kodansha Ltd., Tokyo, Japan.

Kliman, A., and T. McGlone (1999): "A Temporal Single-system Interpretation of Marx's Value Theory," *Review of Political Economy* 11, pp. 33-59.

Kurz, H., and N. Salvadori (1995): *Theory of Production: A Long-Period Analysis*, Cambride University Press, Cambridge, UK.

Maddison, A. (2001). *The World Economy.* Paris: OECD.

Marx, K. (1859, 1970): *A Contribution to the Critique of Political Economy,* trans. by Ryazanskaya, S. W., Progress Publishers, Moscow, UK.

Marx, K. (1867, 1976): *Capital. A Critique of Political Economy, Vol.I.*, trans. by Fowkes, B., Penguin Books, Harmondsworth, UK.

Marx, K. (1868, 1988): "Marx to Ludwig Kugelmann in Hanover: london, 11 July 1868" in *Marx-Engels Collected Works*, vol. 43, p.68.

Marx, K. (1894, 1981): *Capital. A Critique of Political Economy, Vol.3*, trans. by Fernbach, D., Penguin Books, Harmondsworth, UK.

Morishima, M. (1973): *Marx's Economics*, Cambridge University Press, Cambridge.

Morishima, M. (1974): "Marx in the Light of Modern Economic Theory," *Econometrica* 42, pp. 611-632.

Muhon, S., and R. Veneziani (2017): "Value, Price, and Exploitation: The Logic of the Transformation Problem," *Journal of Economic Surveys*, 31, pp. 1387–1420.

Okishio, N. (1963): "A Mathematical Note on Marxian Theorems," *Weltwirtschaftliches Archiv* 91, pp. 287-299.

Piketty, T. (2014): *Capital in the Twenty-First Century*, Harvard University Press, Cambridge, MA.

Ricardo, D. (1951): *On the Principles of Political Economy and Taxation*, in Sraffa, P. (ed.), *The Works and Correspondence of David Ricardo, vol. I*, Cambridge. Cambridge University Press.

Roemer, J.E. (1981). *Analytical Foundations of Marxian Economic Theory.* Cambridge, MA: Harvard University Press.

Roemer, J. E. (1982): *A General Theory of Exploitation and Class*, Harvard University Press.

Sasaki, R. (2021): "Towards Understanding Marx's Theory of Equilibrium and Prices of Production," in *Marx-Engels-Jahrbuch 2019/20,* Edited by: Internationale Marx-Engels-Stiftung (IMES) Akademie Verlag | 2192-8207, pp. 135-157.

Steedman, I. (1975): "Positive Profits with Negative Surplus Values," *Economic Journal,* 85, pp. 114-123.

Thompson, E. P. (1967): "Time, Work-Discipline, and Industrial Capitalism," *Post & Present,* 38, pp. 56–97.

Veneziani, R. (2007): "Exploitation and Time," *Journal of Economic Theory* 132, pp. 189-207.

Veneziani, R. (2013): "Exploitation Inequality and Power," *Journal of Theoretical Politics* 25, pp. 526-545.

Veneziani, R., and N. Yoshihara (2015): "Exploitation in Economies with Heterogeneous Preferences, Skills and Assets: An Axiomatic Approach," *Journal of Theoretical Politics*, 27, pp. 8–33.

Veneziani, R., and N. Yoshihara (2017a): "One million miles to go: taking the axiomatic road to defining exploitation," *Cambridge Journal of Economics*, 41, pp. 1607–1626.

Veneziani, R., and N. Yoshihara (2017b): "Globalisation and Inequality in a Dynamic Economy: An Axiomatic Analysis of Unequal Exchange," *Social Choice and Welfare*, 49, pp. 445–468.

Yoshihara, N. (2010): "Class and Exploitation in General Convex Cone Economies," *Journal of Economic Behavior & Organization*, 75, pp. 281–296.

Yoshihara, N. (2017): "A Progress Report on Marxian Economic Theory: On the Controversies in Exploitation Theory since Okishio (1963)," *Journal of Economic Surveys*, 31, pp. 632–659.

Yoshihara, N., and S. Kaneko (2016): "On the Existence and Characterization of Unequal Exchange in the Free Trade Equilibrium," *Metroeconomica*, 67, pp. 210–241.

Yoshihara, N. and R. Veneziani (2009): "Exploitation as the Unequal Exchange of Labour: An Axiomatic Approach," IER Discussion Paper Series A. No.524, The Institute of Economic Research, Hitotsubashi University.

Yoshihara, N., and R. Veneziani (2018): "The theory of exploitation as the unequal exchange of labour," *Economics and Philosophy*, 34, pp. 381-409.