Discussion Paper Series A No.700

# Efficiency Invites Divide and Coercion in the Age of Increasing Returns to Scale

Yongsheng Xu
(Department of Economics, Andrew Young School of Policy Studies,
Georgia State University)
and
Naoki Yoshihara
(Department of Economics, University of Massachusetts Amherst,
Institute of Economic Research, Hitotsubashi University, and
School of Management, Kochi University of Technology)

October 2019

Institute of Economic Research
Hitotsubashi University
Kunitachi, Tokyo, 186-8603 Japan

# Efficiency invites Divide and Coercion in the Age of Increasing Returns to Scale

Yongsheng Xu*and Naoki Yoshihara†

October 10, 2019

**Abstract**

In the presence of (at least locally) increasing returns to scale technologies, the paper asks the question: does there exist an economic system which implements Pareto efficient allocations and respects the voluntary participation principle? To answer this question, the paper formulates an economic system as an allocation rule under economies with non-convex production possibility sets, and proposes a few weaker axioms to represent the voluntary participation principle. Then, the paper shows that any Pareto efficient allocation rule satisfies none of the axioms of the voluntary participation principle. The result suggests that pursuing Pareto efficiency in the presence of increasing returns to scale technologies leads to a dictatorial allocation rule, or forces someone to participate in the economic system without any guarantee of a minimal living standard.

Keywords: increasing returns to scale technologies, Pareto efficiency, allocation rule, individual rationality, minimal autonomy, voluntary participation
JEL Classification Numbers: D0, D2, D3, D5, D6, P0, P1, P2, P4

## 1 Introduction

Economists have been fascinated by technologies that exhibit increasing returns to scale. Increasing returns to scale technologies are regarded as an important factor behind economic growth (see, for example, Jones (2005)), and economists are paying more and more attention to such technologies. However, when production technologies exhibit increasing returns to scale, production possibility

*Department of Economics, Andrew Young School of Policy Studies, Georgia State University, Atlanta, GA 30303, U.S.A. Email: yxu3@gsu.edu

†Department of Economics, University of Massachusetts Amherst, Crotty Hall, Amherst, MA, 01002, USA; The Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-0004, Japan; and School of Management, Kochi University of Technology, Tosayamada, Kami-city, Kochi 782-8502, Japan. Email: nyoshihara@econs.umass.edu

sets are non-convex. Non-convexity of production possibility sets may bring certain difficulties for the market mechanism to allocate resources.

In the first place, it is well-known that non-convex production sets may create certain efficiency problems in a market economy (e.g., Guesnerie (1975) and Heal (1999)): market equilibrium outcomes may not be Pareto efficient. Pareto efficiency is a fundamental and highly desirable principle in economics. With rare exception, the principle of Pareto efficiency is brought in to evaluate an economic system that is used for allocating resources. In saying that market equilibrium outcomes may not be Pareto efficient, the market system is assessed along this efficiency principle by examining whether or not outcomes emerging from the market mechanism are efficient.

Secondly, production technologies exhibiting increasing returns to scale may cause uneven allocations of resources and result in uneven distributions of wealth in market economies. While increasing returns to scale create externality in social productivity by promoting cooperative works and the division of labor as many classical economists such as Smith (1776) and Marx (1867) emphasized, market exchanges between the industries with increasing returns to scale technologies and the other industries with non-increasing returns to scale technologies may lead to uneven development among the agents in the economy: agents specializing in the former industries (with technologies exhibiting increasing returns to scale) can get richer and agents specializing in the latter industries become poorer (see, for example, Krugman (1981)). It is therefore fundamentally important to know whether increasing returns to scale technologies are a driving force to generate uneven allocations of resources and wealth or it is the market competition that causes such divergence of resources and wealth.

It may be noted that, under the market mechanism, each individual can decide to participate in market exchanges: she would participate in market exchanges only if it is preferred to non-participating by her. This *voluntary participation* principle implicitly embedded in the market mechanism seems to suggest that, even if the market economic system can result in uneven allocations and distributions of resources and wealth mentioned earlier, the extent of such unevenness ought to be constrained.

With the above difficulties confronted by the market mechanism in the presence of increasing returns to scale technologies, we ask the following question: are the difficulties discussed above confined to the market mechanism only or are they widespread? More formally, we ask this question: in the presence of increasing returns to scale technologies, is there any economic system that is Pareto efficient and respects the principle of voluntary participation?

In this paper, we provide an answer to the above question. For this purpose, we first introduce and define the principle of voluntary participation. Admittedly, the principle of voluntary participation has been used, both implicitly and explicitly, by economists to evaluate economic systems. The principle can take various forms, but its basic idea is fairly simple and intuitive, and requires that individuals must not be *coerced* into participating in an economic system if they so desire. We therefore formulate the principle of voluntary participation in terms of individual rationality: individuals can do no worse than the "autarky" situation where they would be left alone and would be on their own individually. This version of the principle of voluntary participation seems to capture what Adam Smith (1776) had in mind when he explained how a market system would work.

Coupled with Pareto efficiency, we take this version of the principle of voluntary participation to examine the performance of an economic system. An economic system is viewed as an *allocation rule* that is used to select certain allocations for any given economy. We show that, *in the presence of increasing returns to scale technology, any efficient allocation rule violates the spirit of the principle of voluntary participation and is not individually rational.*

The remainder of the paper is organized as follows. In Section 2, we introduce the basic model for our discussion. Section 3 presents our basic results. Section 4 discusses alternative formulations of the principle of voluntary participation in different settings and examines their consequences together with the Pareto efficiency in those settings. We conclude in Section 5.

# 2 The basic set-up

There are $m \geq 2$ goods. Let $\mathbb{R}$ (resp. $\mathbb{R}_+$ and $\mathbb{R}_{++}$) denote the set of all real (resp. non-negative and positive) numbers. Let $\mathbb{R}^m$ (resp. $\mathbb{R}^m_+$ and $\mathbb{R}^m_{++}$) be the $m$-fold Cartesian product of $\mathbb{R}$ (resp. $\mathbb{R}_+$ and $\mathbb{R}_{++}$). For any $a, b \in \mathbb{R}^m$, $a > b$ denotes $[a_1 \geq b_1, \cdots, a_m \geq b_m$ and $a \neq b]$, and $a \gg b$ denotes $[a_1 > b_1, \cdots, a_m > b_m]$.

There is a fixed number $J$ of firms that are indexed by the set $\mathcal{J} = \{1, \cdots, J\}$. For each firm $j \in \mathcal{J}$, let $Y_j \subseteq \mathbb{R}^m$ be firm $j$'s production possibility set. Each $y_i = (y_{j1}, \cdots, y_{jm}) \in Y_j$ is called a production plan for firm $j$. We assume that each production possibility set $Y_j$ $(j \in \mathcal{J})$ is closed and $\mathbf{0} \in Y_j$. Note that we allow non-convex production possibility sets. Let $Y \equiv \sum_{j \in \mathcal{J}} Y_j$.

Let $\mathcal{N} = \{1, 2, \cdots, n\}$ be the set of individuals (consumers). Each individual $i \in \mathcal{N}$ is endowed with an initial endowment, $\omega_i = (\omega_{i1}, \cdots, \omega_{im}) \in \mathbb{R}^m_+$, and has a continuous, quasi-concave and monotonic[1] utility function over the consumption set $\mathbb{R}^m_+$. Let $\Omega = \sum_{i \in \mathcal{N}} \omega_i$. Assume $(Y + \{\Omega\}) \cap \mathbb{R}^m_{++} \neq \varnothing$.

An *economy*, to be denoted by $E$, is then defined as follows:

$$E \equiv \left\langle \mathcal{N}; (u_i, \omega_i)_{i \in \mathcal{N}}; (Y_j)_{j \in \mathcal{J}} \right\rangle .$$

An *allocation*, $(\boldsymbol{x}, \boldsymbol{y})$, in an economy $E$, specifies a consumption bundle $x_i$ for each individual $i \in \mathcal{N}$ and a production plan $y_j$ for each firm $j \in \mathcal{J}$. An allocation $(\boldsymbol{x}, \boldsymbol{y})$ is *feasible for the economy* $E$ if and only if $\sum_{i \in \mathcal{N}} x_i \leqq \Omega + \sum_{j \in \mathcal{J}} y_j$. Let the set of all feasible allocations for $E$ be denoted by $F(E)$.

Given an economy $\left\langle \mathcal{N}; (u_i, \omega_i)_{i \in \mathcal{N}}; (Y_j)_{j \in \mathcal{J}} \right\rangle$, a feasible allocation $(\boldsymbol{x}, \boldsymbol{y})$ is *Pareto efficient* for $E$ if and only if there exists no other feasible allocation

[1] $u_i$ is monotonic if, for all $a, b \in \mathbb{R}^m_+$, if $a > b$ then $u_i(a) \geq u_i(b)$ and if $a \gg b$ then $u_i(a) > u_i(b)$.

$(\boldsymbol{x}', \boldsymbol{y}')$ such that $[\forall i \in \mathcal{N} : u_i(x_i') \geq u_i(x_i)]$ and $[u_i(x_i') > u_i(x_i)$ for some $i \in \mathcal{N}]$.

Let $\mathcal{E}$ denote the set of all possible economies defined above. Given an economy $E$ in $\mathcal{E}$, there are various ways to select a feasible allocation to be the final outcome. In general, an *allocation rule*, $\varphi$, specifies a non-empty set of feasible allocations for each economy $E$ in $\mathcal{E}$: for any economy $E \in \mathcal{E}$, $\varnothing \neq \varphi(E) \subseteq F(E)$. The market mechanism is a prominent example of an allocation rule: for a given economy $E$, it selects competitive equilibrium allocations to be its final outcomes. We shall refer the allocation rule associated with the market mechanism as the *Walrasian rule*.

# 3 Any efficient allocation rule cannot be individually rational

In this section, we consider two properties to be imposed on an allocation rule and examine their consequences.

To begin with, we require that, for any economy, the allocations selected by an allocation rule be efficient:

**Pareto Efficiency**. For any economy $E \in \mathcal{E}$ and any allocation $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(E)$, $(\boldsymbol{x}, \boldsymbol{y})$ is Pareto efficient for $E$.

Our second property attempts to capture certain aspects of the principle of voluntary participation, and it requires that, for any economy, an allocation selected by an allocation rule should give each individual at least as much utility as the individual would get from an "autarky" situation:

**Individual Rationality**. For any economy $E = \left\langle \mathcal{N}; (u_i, \omega_i)_{i \in \mathcal{N}}; (Y_j)_{j \in \mathcal{J}} \right\rangle \in \mathcal{E}$ and any allocation $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(E)$, $u_i(x_i) \geq u_i(\omega_i)$ for all $i \in \mathcal{N}$.

In a production economy, there are perhaps alternative ways of defining an "autarky" situation. One simple and natural autarky situation is the one in which individuals are left with their initial endowments only. This seems a very reasonable reference point for individuals to compare if they would like to participate in an economic system: if an allocation rule selects an allocation that will make an individual worse off than this reference point, then there is good reason to believe that the individual would not like to participate in the economic system.

Our first result is presented in Theorem 1 below; its proof can be found in the Appendix.

**Theorem 1** *In the presence of non-convex production possibility sets and with $\omega_i \gg \mathbf{0}$ for every individual $i \in \mathcal{N}$, there exists no allocation rule that satisfies **Pareto Efficiency** and **Individual Rationality** simultaneously.*

Therefore, according to Theorem 1, in the presence of increasing returns to scale technologies, any efficient allocation rule cannot be individually rational. Stated differently, with increasing returns to scale technologies, if it is desirable to have an efficient allocation rule, then some individuals would have to be coerced into participating in the economic system.

In our formulation of **Individual Rationality**, it is required that no one be made worse off than the situation given by his/her initial endowment. Obviously, the allocation rule giving all the goods endowed and produced in an economy to a single individual is Pareto efficient and individually rational for this single individual (at the expense of all other individuals). But this is hardly reasonable. Therefore, we ask the following question: Can the property of **Individual Rationality** be weakened to allow only two individuals who are not to be coerced into participating in the economic system in order to salvage the impossibility? We proceed to answer this question below.

**Weak Individual Rationality**. For any economy $E = \left\langle \mathcal{N}; (u_i, \omega_i)_{i\in\mathcal{N}}; (Y_j)_{j\in\mathcal{J}} \right\rangle \in \mathcal{E}$ and any allocation $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(E)$, there exist at least two individuals $i, i' \in \mathcal{N}$ such that $\omega_i \gg \mathbf{0}$ and $\omega_{i'} \gg \mathbf{0}$, and $u_i(x_i) \geq u_i(\omega_i)$ and $u_{i'}(x_{i'}) \geq u_{i'}(\omega_{i'})$.

Theorem 2 below provides an answer to our question aboove; its proof is in the Appendix.

**Theorem 2** *In the presence of non-convex production possibility sets, there exists no allocation rule that satisfies* ***Pareto Efficiency*** *and* ***Weak Individual Rationality*** *simultaneously.*

Theorem 2 suggests that, in the presence of increasing returns to scale production technologies, an efficient allocation rule cannot be "individually rational" for just two individuals: the efficient allocation rule would have to "grab" lots of resources, endowed and produced, from one individual and give them to the other individual making the first individual worse-off than his initial endowment; the economic system associated with this kind of allocation rule would thus invite divide and coercion.

# 4 Alternative formulations of the principle of voluntary participation

In the formulation of **Individual Rationality** in the last section, we have used the initial endowment as a reference point to define an autarky situation. The formulation implies that each individual's well-being condition under the autarky situation is specified by the lives in which the individual uses his/her own initial endowments only. However, in production economies, the extent to which an individual can access to technologies is also quite relevant in determining a reference point to be used as the autarky situation. In other words, the information of ownership structures of production technologies matters for the specification of the reference point in production economies. We turn to two well-known ownership structures to discuss alternative formulations of the principle of voluntary participation.

*Private ownership economies.* To discuss this issue, let us first introduce a production economy with private ownership as discussed in the standard general equilibrium model. For each individual $i \in \mathcal{N}$ and each firm $j \in \mathcal{J}$, let $\theta_{ij}$ be individual $i$'s shares in firm $j$'s owned production possibility set: $0 \leq \theta_{ij} \leq 1$ for all $i \in \mathcal{N}$ and all $j \in \mathcal{J}$, and $\sum_{i \in \mathcal{N}} \theta_{ij} = 1$ for all $j \in \mathcal{J}$. For each $i \in \mathcal{N}$, let $\theta_i = (\theta_{i1}, \cdots, \theta_{iJ})$. A private ownership economy, to be denoted by $E^{PO}$, is defined as follows:

$$E^{PO} \equiv \left\langle \mathcal{N}; (u_i, \omega_i, \theta_i)_{i \in \mathcal{N}}; (Y_j)_{j \in \mathcal{J}} \right\rangle.$$

Let $\mathcal{E}^{PO}$ denote the set of all possible private ownership economies thus defined.

Given a private ownership economy, individual $i$ can freely access to resources within the set $\sum_{j=1,\ldots,J} \theta_{ij} Y_j + \{\omega_i\}$ regardless of whether she participates in the implementation of allocation rules. In such a case, the idea of voluntary participation can be formulated by the following condition:

**Individual Rationality under Private Ownership**. For any economy $E^{PO} \equiv \left\langle \mathcal{N}; (u_i, \omega_i, \theta_i)_{i \in \mathcal{N}}; (Y_j)_{j \in \mathcal{J}} \right\rangle \in \mathcal{E}^{PO}$ and any allocation $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(E^{PO})$, $u_i(x_i) \geq \max_{z_i \in \left(\sum_{j \in \mathcal{J}} \theta_{ij} Y_j + \{\omega_i\}\right)} u_i(z_i)$ for all $i \in \mathcal{N}$.

Obviously, this axiom is stronger than **Individual Rationality**. Therefore, as a corollary of Theorem 1, we have the following result:

**Corollary 3** *Suppose an allocation rule is defined over $\mathcal{E}^{PO}$. Then, in the presence of non-convex production possibility sets and with $\omega_i \gg \mathbf{0}$ for all $i \in \mathcal{N}$, there is no allocation rule that satisfies Pareto Efficiency and Individual Rationality under Private Ownership.*

*Public ownership economies.* In economies with public ownership, it begins with a presumption that production technologies are the *commons*, implying that every individual can freely access to those technologies under the autarky situation. Then, as Moulin and Roemer (1989) and Roemer (1996, ch. 6) suggest, the public ownership of technologies should guarantee at least the welfare level that each individual can enjoy under the free access to those technologies. A version of individual rationality in this context can be formulated as follows:

**Individual Rationality under Public Ownership.** For any economy $E = \left\langle \mathcal{N}; (u_i, \omega_i)_{i \in \mathcal{N}}; (Y_j)_{j \in \mathcal{J}} \right\rangle \in \mathcal{E}$, for any allocation $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(E)$, $u_i(x_i) \geq$

$\max_{z_i \in (Y + \{\omega_i\})} u_i(z_i)$ for all $i \in \mathcal{N}$.

Note that Individual Rationality under Public Ownership is stronger than Individual Rationality. The following result is then immediate:

**Corollary 4** *Suppose an allocation rule is defined over the set of all possible public ownership economies. Then, in the presence of non-convex production possibility sets, there is no allocation rule that satisfies Pareto Efficiency and Individual Rationality under Public Ownership.*

The above axiom of Individual Rationality under Public Ownership is sensible, in particular when the production technology is represented by an increasing returns to scale production function in economies with one-input and one-output, as Moulin (1987) discussed. Because in such economies, the public ownership regime brings positive externality by the operation of cooperative production.

However, in a broader class of economies, which contains a type of economies where the production technologies may not exhibit *purely* increasing returns to scale: they exhibit increasing returns to scale *locally*, but not *globally*.[2] Our assumption of non-convex production possibility sets allows such production technologies. For such economies, the public ownership regime may bring negative externalities. In those cases, Individual Rationality under Public Ownership seems too strong a requirement to be demanded for the principle of voluntary participation. This concern brings us to the following.

*Minimal living standards.* In view of the above discussion, we now consider a much weaker condition of voluntary participation. For this purpose, let $\epsilon > 0$ be 'sufficiently small'. Then, for each economy $E \in \mathcal{E}$, let $G(\epsilon E) = \epsilon(Y + \{\Omega\})$. $G(\epsilon E)$ specifies a minimal sphere of production technology and resources over which every individual can act autonomously. Then, we have the following weaker principle of voluntary participation requiring that an allocation rule should guarantee every individual at least the welfare level that s/he can enjoy from the chosen activities over this minimal sphere.

**Minimal Autonomy**. For any economy $E = \left\langle \mathcal{N}; (u_i, \omega_i)_{i \in \mathcal{N}}; (Y_j)_{j \in \mathcal{J}} \right\rangle \in \mathcal{E}$, for any allocation $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(E)$, $u_i(x_i) \geq \max_{z_i \in G(\epsilon E)} u_i(z_i)$ for all $i \in \mathcal{N}$.

This axiom can be motivated not only as a minimal condition of voluntary participation under public ownership regimes, but also as a condition to ensure all individuals the *minimal standard of living.* That is, we may say that there should be a proper $\epsilon > 0$ such that each individual's welfare ensured by the minimal standard of living should be at least as high as the welfare given by the resources feasibly provided from the production environment $G(\epsilon E)$. With this interpretation, Minimal Autonomy suggests that any individual can choose not

[2] Please see the production technology used in our proof of the Lemma in the Appendix.

to participate in the implementation of allocation rules unless the individual is guaranteed at least the minimal standard of living.

Now, we are ready to present our main result of this section. The proof of Theorem 5 is in the Appendix.

**Theorem 5** *In the presence of non-convex production sets, there exists no allocation rule that satisfies **Pareto Efficiency** and **Minimal Autonomy** simultaneously.*

The results discussed in this section send two messages. First, the main implication of Theorem 1 is still preserved even if the ownership structure of production technologies is seriously taken into account to define the reference point for voluntary participation. Secondly, Theorem 5 implies that in the presence of (at least locally) increasing returns to scale technologies, it is generally impossible to implement a moderate welfare state policy without sacrificing Pareto efficiency of resource allocations, taking for granted that any moderate welfare state policy should guarantee every individual at least a minimal standard of living.

In an interesting paper, Moulin (1987) shows the compatibility of Pareto efficiency and a stronger condition than our **Minimal Autonomy** for one-input and one-output economies with increasing returns to scale production functions. Our Theorem 5 is then a bit surprising. It is therefore of interest to know the main source for the contrast between our Theorem 5 and Moulin's (1987) possibility result.

One may initially conjecture that the impossibility result of Theorem 5 would be observed out of one-input and one-output economies. However, the domain of economies presumed in Theorem 5 contains the class of one-input and one-output economies, as we assume $m \geq 2$. Indeed, as discussed in the Appendix, the proof of Theorem 5 is given by considering an economy with $m = 2$.

The main source is that our domain of nonconvex production possibility sets is much broader than the domain of production possibility sets considered in Moulin (1987). In Moulin (1987), the production technology is purely increasing returns to scale, in that the average productivity is increasing with respect to labor inputs. Then, as argued by Moulin (1987), the cooperative game derived from such an economy is convex, and consequently, the existence of non-empty core is ensured there. In contrast, nonconvex production possibility sets considered in this paper do not necessarily exhibit purely increasing returns to scale. Under economies with such a more general type of production technologies, the corresponding cooperative game cannot be convex in general, and so the existence of non-empty core is no longer guaranteed. Indeed, the economy with a nonconvex production possibility set discussed in the proof of Theorem 5 not only has an empty core, but also has no interior efficient allocation, which implies the incompatibility between **Pareto Efficiency** and **Minimal Autonomy**.

# 5 Discussion and conclusion

In the presence of increasing returns to scale technologies, productive efficiency produces natural monopolies. However, this does not necessarily give us any definitive view on the distributional feature under the increasing returns to scale, since it would be generally possible to allocate the productive rewards in an equitable way even under the monopolized production process. In this respect, Krugman (1981), by focussing on a specific framework of international trade between a rich country and a poor country, gives us a case in which the increasing returns to scale technology in the manufacturing sector may cause the emergence of uneven development between these countries.

Our main results in this paper are to provide a comprehensive view, beyond perspective of the market economy, on economic systems used for allocating resources under increasing returns to scale. We have shown that, in the presence of (at least locally) increasing returns to scale technologies, any efficient allocation rule cannot be individually rational; in such an environment, to maintain efficiency of an allocation rule, some individuals would have to be coerced into participating in the economic system. As our Theorem 2 suggests further, in order to have an efficient allocation rule (and thus an economic system promoting efficiency), all but one individuals would have to be coerced into the participation of the economic system. Stated differently, the existence of increasing returns to scale technologies can only benefit one individual if efficiency is maintained as a goal for the economic system. In this sense, efficiency invites divide and coercion. Our results thus re-inforce Krugman's (1981) thesis.

If production possibility sets are all convex, and, given our assumptions on the utility functions of the individuals and on their initial endowments, the existence of a competitive market equilibrium is ensured, and therefore, the Walrasian rule is both Pareto efficient and individually rational in this environment. But once an economy has production technologies that exhibit (at least locally) increasing returns to scale, the incompatibility of Pareto efficiency and individual rationality occurs. Given the prevalence of increasing returns to scale in the modern economy of knowledge and ideas, our results present a real dilemma.

It seems that **Individual Rationality** and **Weak Individual Rationality** are fairly weak requirements on an allocation rule. Any further weakening of **Weak Individual Rationality** would call for a "dictator", but a 'dictatorial' allocation rule would be highly unacceptable. If this is a reasonable argument, then, to resolve the dilemma, Pareto efficiency would have to be sacrificed. Given the efficiency problems associated with increasing returns to scale in market economies, this route of non-insistence on Pareto efficiency might be a better way of resolving the dilemma.

**Individual Rationality** and **Weak Individual Rationality** can be interpreted differently from a different perspective. It can be argued that, a reasonable economic system should i) offer opportunities to individuals for them to freely choose whether or not to participate in the system, and ii) fulfill and respect individuals' decisions. Note that the decision "to participate" or "not to participate" in an economic system by an individual can be regarded as this individual's personal choice over his/her 'personal matters'. In this case, the participation of the individual to the economic system is due to his own free choice, and his choice is fulfilled and respected only if he prefers participation to "non-participation and living autarkically by himself'. Viewed this way, **Weak Individual Rationality** demands that there should be at least two individuals whose own choices over the issue of participation to the economic system being fulfilled and respected by the economic system. This interpretation of **Weak Individual Rationality** is closely related to and connected with Sen's (1970) Minimal Liberalism introduced for his now well-known liberal paradox: Sen's Minimal Liberalism requires the existence of two individuals and two pairs of social alternatives (concerning their respective personal matters), one for each individual, so that each individual's respective preferences over his pair of social alternatives be respected by a social decision rule. Sen (1970) then shows the incompatibility of Minimal Liberalism and the weak Pareto principle for an acyclic social decision rule. His result has become known as the Paretian liberal paradox. In a way and given our interpretation of **Weak Individual Rationality** above, our result of Theorem 2 demonstrates a similar paradox emerging in a very different environment.

Likewise, we have also introduced **Minimal Autonomy**, which is regarded as a weaker individual rationality condition suited in economies with public ownership of technologies, and it also can be motivated as a requirement of the minimal standard of living guaranteed for individuals in an economy. In a way, **Minimal Autonomy** specifies the minimal necessary condition that any welfare state policy should satisfy. Then, our Theorem 5 suggests the tension between Pareto efficiency and this minimal standard of living requirement in the presence of (at least locally) increasing returns to scale technologies. Thus, a trade-off between efficiency and **Minimal Autonomy** has to be made: whenever we take the minimal standard of living guaranteed as the first priority of the society in a welfare state, we may have to sacrifice economic efficiency.

# References


Roger Guesnerie, Pareto optimality in non-convex economies, *Econometrica* Vol. 43 (1975), pp. 1-29.

Geoffrey Heal, *The Economics of Increasing Returns*, Edward Elgar. 1999.

Charles I. Jones, *Growth and Ideas*, Chapter 16 in *Handbook of Economic Growth*, Volume 1B, edited by Philippe Aghion and Steven N. Nurlauf, Elsevier B.V. 2005.

Paul Krugman, Trade, Accumulation, and Uneven Development, *Journal of Development Economics* Vol. 8, (1981), pp. 149-161.

Karl Marx, *Capital*, Vol I. Lawrence & Wishart, London. 1954 [1867].

Hervé Moulin, A core selection for regulating a single-output monopoly. *The Rand Journal of Economics*, Vol. 18, (1987), pp. 397-407.

Moulin, H. and J. Roemer, Public ownership of the external world and private ownership of self. *Journal of Political Economy*, Vol. 97, (1989), 347-367.

John E. Roemer, *Theories of Distributive Justice*, Harvard Univ. Press. 1996.

Amartya Sen, The Impossibility of a Paretian Liberal, *Journal of Political Economy* Vol. 78, No. 1 (January and February 1970), pp. 152-157

Adam Smith, *An Inquiry into the Nature and Causes of the Wealth of Nations*, Wordsworth Editions. 2012 [1776].

# Appendix

Let $\varepsilon \in (0,1)$ be sufficiently small. For an economy $E = \left\langle \mathcal{N}; (u_i, \omega_i)_{i\in\mathcal{N}}; (Y_j)_{j\in\mathcal{J}} \right\rangle \in \mathcal{E}$, let $\varepsilon(Y + \{\Omega\}) = \{\varepsilon\mathbf{z} \in \mathbb{R}^m \mid \mathbf{z} \in Y + \{\Omega\}\}$.

Consider the following property:

**A1**. For any two individuals $i, i' \in \mathcal{N}$, any economy $E = \left\langle \mathcal{N}; (u_i, \omega_i)_{i\in\mathcal{N}}; (Y_j)_{j\in\mathcal{J}} \right\rangle \in \mathcal{E}$, for any allocation $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(E)$,

$$u_i(x_i) \geq \max_{z_i \in \varepsilon(Y+\{\Omega\})\cap\mathbb{R}^m_+} u_i(z_i) \text{ and } u_{i'}(x_{i'}) \geq \max_{z_{i'} \in \varepsilon(Y+\{\Omega\})\cap\mathbb{R}^m_+} u_{i'}(z_{i'}).$$

In this Appendix, we first provide the proof for the following lemma. Then, each of Theorems 1, 2 and 5 can be shown as a corollary to the Lemma.

**Lemma.** In the presence of non-convex production possibility sets, there exists no allocation rule that satisfies **Pareto Efficiency** and **A1** simultaneously.

**Proof.** We consider the case in which $m = 2$. Our proof method can be extended to the case involving more than 2 goods. Let $\varepsilon \in \left(0, \frac{1}{2}\right)$, and let $a \in \mathbb{R}_+$ be such that $a > \frac{1-\varepsilon}{\varepsilon}$. Note that $a > 1$. Let $1, 2 \in \mathcal{N}$ be two consumers whose utility functions are given below:[3]

$$u_1(x_{11}, x_{12}) \equiv \min\left\{\frac{x_{11}}{a}, x_{12}\right\} \text{ and } u_2(x_{21}, x_{22}) = \min\left\{x_{21}, \frac{x_{22}}{a}\right\}.$$

Suppose all other individuals have the same utility function as individual 1's utility function (it will become clear that this is not important).

[3]The specific utility functions are used here to provide a simplest economy to establish our result. We may note that, at the expense of increasing complexity, it is possible to consider a more standard type of utility functions, such as strongly monotonic utility functions rather than the Leontief utility functions, for the proof.

Consider the economy $E$ associated with $Y + \{\Omega\} \equiv comp\{(a,1),(1,a)\}$.[4] Thus, $\varepsilon(Y + \{\Omega\}) = comp\{\varepsilon(a,1), \varepsilon(1,a)\}$. Then, **A1** implies that, for any $(\boldsymbol{x}, y) \in \varphi(E)$, $u_1(x_1) \geqq \min\left\{\frac{\varepsilon a}{a}, \varepsilon\right\} = \varepsilon$ and $u_2(x_2) \geqq \min\left\{\varepsilon, \frac{\varepsilon a}{a}\right\} = \varepsilon$. The latter implies that $x_{21} \geqq \varepsilon$ and $x_{22} \geqq \varepsilon a$. Therefore, to be a Pareto efficient allocation, $x_{11} \leqq a - \varepsilon$ and $x_{12} \leqq 1 - \varepsilon a$. Therefore, $u_1(x_1) \leqq \min\left\{\frac{a-\varepsilon}{a}, 1 - \varepsilon a\right\} = 1 - \varepsilon a$. Thus,

$$\varepsilon \leqq u_1(x_1) \leqq 1 - \varepsilon a. \tag{1}$$

Likewise,

$$\varepsilon \leqq u_2(x_2) \leqq 1 - \varepsilon a. \tag{2}$$

However, note that

$$\begin{aligned} 1 - \varepsilon a - \varepsilon &= 1 - \varepsilon(a+1) \\ &< 1 - \varepsilon\left(\frac{1-\varepsilon}{\varepsilon} + 1\right) = 0, \end{aligned}$$

implying that

$$1 - \varepsilon a < \varepsilon.$$

Thus, there is no feasible allocation satisfying either (1) or (2). ■

**Proof of Theorem 2.** Let $a \in \mathbb{R}_+$ be such that $a > \frac{1-\varepsilon}{\varepsilon}$ is sufficiently larger than 1, and then consider the same economy as in the proof of Lemma. In particular, let us choose $\varepsilon \in (0, 1/2)$ sufficiently small so that $\varepsilon(Y + \{\Omega\}) \cap \mathbb{R}^m_+ \equiv comp\{\varepsilon(a,1), \varepsilon(1,a)\} \cap \mathbb{R}^m_+ \subseteq \{z \in \mathbb{R}^m_+ \mid z \leq \omega_i \wedge \omega_{i'}\}$ holds for the individuals with $\omega_i \gg \mathbf{0}$ and $\omega_{i'} \gg \mathbf{0}$.[5] Moreover, the profile of utility functions is defined as in the proof of Lemma by taking

$$u_i(x_{i1}, x_{i2}) \equiv \min\left\{\frac{x_{i1}}{a}, x_{i2}\right\} \text{ and } u_{i'}(x_{i'1}, x_{i'2}) = \min\left\{x_{i'1}, \frac{x_{i'2}}{a}\right\}.$$

Then, as $\varphi$ satisfies **Weak Individual Rationality** for the individuals $i, i' \in \mathcal{N}$, it also meets the property of **A1** for the specified economy with individuals $i, i' \in \mathcal{N}$. Thus, Lemma implies that there is no allocation rule that satisfies both **Pareto Efficiency** and **Weak Individual Rationality**. ■

**Proof of Theorem 1.** Note that if an allocation rule satisfies **Individual Rationality**, then it satisfies **Weak Individual Rationality**. Then, from Theorem 2, Theorem 1 follows easily. ■

**Proof of Theorem 5.** Let $a \in \mathbb{R}_+$ be such that $a > 1$, and then consider the same economy as in the proof of Lemma. Then, as $\varphi$ satisfies **Minimal Autonomy**, there exists a sufficiently small $\epsilon > 0$ such that $G(\epsilon E) \equiv comp\{\epsilon(a,1), \epsilon(1,a)\}$. By taking $a$ as sufficiently larger than 1, we can ensure

[4] For any two points, $(s_1, s_2), (t_1, t_2) \in \mathbb{R}^2$, $comp\{(s_1,s_2),(t_1,t_2)\}$ is defined as the set $\{(q_1,q_2) \in \mathbb{R}^2 \mid (s_1,s_2) \geq (q_1,q_2) \text{ or } (t_1,t_2) \geq (q_1,q_2)\}$.

[5] Note that for any $z, w \in \mathbb{R}^m_+$, $z \wedge w \equiv (\min\{z_k, w_k\})_{k=1,\dots,m}$.

that $a > \frac{1-\epsilon}{\epsilon}$ holds. Moreover, $\varphi$ satisfies **Minimal Autonomy**, it also satisfies **A1** for the individuals $1, 2 \in \mathcal{N}$. Then, Lemma implies that there is no allocation rule that satisfies both **Pareto Efficiency** and **Minimal Autonomy**. ■