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Does Dynamic Market Competition with Technological Innovation Leave No One Behind?

Yongsheng Xu
(Department of Economics, Andrew Young School of Policy Studies, Georgia State University)
and
Naoki Yoshihara
(Department of Economics, University of Massachusetts Amherst, Institute of Economic Research, Hitotsubashi University, and School of Management, Kochi University of Technology)

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Institute of Economic Research
Hitotsubashi University
Kunitachi, Tokyo, 186-8603 Japan

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Yongsheng Xu†and Naoki Yoshihara‡

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Abstract

In this paper, we examine the performance of the market mechanism by focusing on whether no one, in the 'long-run', can be left behind with technological innovation in the economy. We show that the market mechanism with technological innovation unavoidably leaves some individuals behind. We extend this negative result to a broader class of resource allocation mechanisms.

Keywords: dynamic market competition with technological innovation, Hicksian Optimism, the Walrasian allocation rule, Pareto efficiency, individual rationality

JEL Classification Numbers: D30, D51, D60, O33, P10, P20, P40

1 Introduction: Three Basic Functions of Market Mechanism

The market mechanism is the central subject in economics. Economics from the age of Adam Smith till the present day has viewed the market mechanism as having the following three basic functions:

i) it is a decentralized resource allocation mechanism that improves the welfare of consumers as a result of market exchange;

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†Department of Economics, Andrew Young School of Policy Studies, Georgia State University, Atlanta, GA 30303, U.S.A. Email: yxu3@gsu.edu

‡Department of Economics, University of Massachusetts Amherst, Crotty Hall, Amherst, MA, 01002, USA; The Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-0004, Japan; and School of Management, Kochi University of Technology, Tosayamada, Kami-city, Kochi 782-8502, Japan. Email: nyoshihara@econs.umass.edu

ii) it promotes the technological and social division of labor via specialization as motivated by the principle of comparative advantage, and consequently, it increases efficiency of production activities in the economy as a whole; and

iii) it promotes technical progress in order to enhance the productivity of the economy as a whole through a dynamic process of competition among the producers,[1] and consequently it improves individual welfare progressively.

Among the three functions of the market mechanism mentioned above, the first function is illustrated and explained in the Walrasian general equilibrium theory as the Fundamental Theorems of Welfare Economics, while the second function is captured and characterized in the Heckshar-Ohlin international trade theory as the Fundamental Theorems of International Trade (in particular, the Heckshar-Ohlin theorem).

In contrast, there are no basic results or theoretically rigorous analysis in the literature of contemporary economics that supports the argument for the third function of the market mechanism, despite some attempts to justify the market economy on this ground.[2] The third function of the market mechanism seems to contain two main points. First, the market mechanism can enhance the productivity of the economy as a whole through promoting technical progress in the dynamic process of competition among producers. Second, assuming that the dynamic competition in the market economy promotes technical innovation, the market mechanism can ensure the progressive improvement of individual welfare.

Regarding the first point, it can be argued that the market competition does not necessarily promote technological innovation, as the knowledge of a new technology can be regarded as a pure public good. Indeed, it is easily diffused once it is discovered (or invented) and publicized. As a consequence, a competitive producer may not have enough motive for R&D investment, as the rival producers can easily freeride on the successful result of an R&D before the competitive producer fully reaps the benefits of the R&D investment. See, for example, Schumpeter (1939, 1942), and the evidence presented in the study by Blundell, Griffith and van Reenen (1999) where they find, in the study of British manufacturing firms, that the monopolist invests more in its R&D. It is therefore further suggested that introducing the patent system would be a desirable incentive scheme to encourage R&D investment, though such a scheme regulates market competition by allowing the innovator to be a monopolist for some restricted period of time. On the other hand, we also have the argument that the market competition promotes technological innovation: more competition tends to diminish profits, which gives firms more incentive to innovate to capture profits (see, for example, Arrow (1962)). The recent literature on the topic has illustrated a complicated picture between competition and innovation (see, among others, Aghion et al. (2005), Igami and Uetake (forthcoming)). For

[1] Producers are motivated by capturing extra profits via the access to the most advanced technology.

[2] See, for example, Kornai (2013).

example, Aghion et al. (2005) have shown an inverted U-shaped relationship between innovation and competition, where some certain intermediate levels of competition give rise to the highest level of innovation.

However, even setting aside the incentive problem of R&D investment, the question that the market mechanism can ensure the progressive improvement of individual welfare remains. This is because the introduction of a new technology often involves a radical change of economic structure, which leads to the division of the population into the "winners" and the "losers".

The issue concerning the winners and losers seems to be easily dealt with by the Kaldor-Hicks compensation test (Hicks (1939), Kaldor (1939)). Indeed, if a technical progress due to innovation can be formulated as an expansion of the production possibility set available in the economy à la Schumpeter (1942), then a policy encouraging such innovations would always be socially desirable with respect to the (weaker) Kaldor-Hicks hypothetical compensation principle. This is because, a change from a competitive equilibrium under one economy to another competitive equilibrium under possibly another economy involves an *expansion* of the production possibility set in terms of the set inclusion, and consequently, the aggregate sum of all consumers' 'upper contour sets' definitely has a non-empty intersection with the new production possibility set. Therefore, even if some consumer becomes a "loser" with this change, this consumer can be (hypothetically) compensated via a suitable shift of an aggregate supply of commodity bundles, and a suitable redistribution of the bundle.

Such a compensation is, however, just hypothetical and the Kaldor-Hicks compensation principle provides no mechanism to implement such a compensation, and as a result, a loser would be left as a "loser" in reality. Therefore, judging such possible improvements of individual welfare by means of the Kaldor-Hicks compensation principle alone would not be much help if the hypothetical compensation cannot be actually implemented.

Nevertheless, neoclassical economics has attempted to justify the application of the (somewhat modified) hypothetical compensation principle to evaluate possible improvement of social welfare by appealing to the so-called *Hicksian Optimism*. As stated by Corden (1984, page 68), "if Pareto efficient policies are being pursued consistently over a long period, the *chances* are that eventually– though not at every particular step–everyone will be better off." He labels this principle as the *Hicksian Optimism* and attributes it to Hicks (1941) where Hicks writes:

> " ... then, although we could not say that all the inhabitants of that community would be necessarily better off than they would have been if the community had been organized on some different principle, nevertheless there would be a strong probability that almost all of them would be better off after the lapse of a sufficient length of time. Substantially, that is the creed of classical economics; if the 'improvements' are properly defined, it would appear to be a creed that is soundly based."

Therefore, if we follow through the above idea of Hicksian optimism, we may conjecture that the innovation and technological progress in the market place can eventually benefit every individual in the society, laying a foundation for the third function of the market mechanism, and for achieving some goals like "inclusive economic growth","shared prosperity", and "no one being left behind" envisioned and set forth in the 2030 Agenda for Sustainable Development as pledged by 193 United Nations Member States.[3]

The purpose of this paper is to examine this conjecture. While doing so, we propose a theoretical framework and introduce the notion of the Hicksian optimism in the context. We then examine whether the expansions of the society's production possibilities set due to innovation and technological progress can indeed benefit every member of the society *eventually* (after a long period of time).

The remainder of the paper is organized as follows. Section 2 presents our basic model. Section 3 examines whether the market mechanism can deliver the Hicksian optimism illustrated above. Section 4 extends our analysis to a broader class of resource allocation mechanisms and examines if the Hicksian optimism holds for those mechanisms. We conclude in Section 5.

2 A Basic Model

There are $m \geq 3$ goods. Let $\mathbb{R}$ (resp. $\mathbb{R}_+$ and $\mathbb{R}_{++}$) denote the set of all real (resp. non-negative and positive) numbers. Let $\mathbb{R}^m$ (resp. $\mathbb{R}^m_+$ and $\mathbb{R}^m_{++}$) be the m-fold Cartesian product of $\mathbb{R}$ (resp. $\mathbb{R}_+$ and $\mathbb{R}_{++}$). For any $a, b \in \mathbb{R}^m$, $a \geq b$ denotes $[a_1 \geq b_1, \cdots, a_m \geq b_m]$, $a > b$ denotes $[a \geq b$ and $a \neq b]$, and $a \gg b$ denotes $[a_1 > b_1, \cdots, a_m > b_m]$.

There is a fixed number J of firms that are indexed by the set $\mathcal{J} = \{1, \cdots, J\}$. For each firm $j \in \mathcal{J}$, let $Y_j \subseteq \mathbb{R}^m$ be firm j's production possibility set. Each $y_j = (y_{j1}, \cdots, y_{jm}) \in Y_j$ is called a production plan for firm j. We assume that each production possibility set Y_j $(j \in \mathcal{J})$ is closed, convex and $\{\mathbf{0}\} = Y_j \cap \mathbb{R}^m_+$.

Let $\mathcal{N} = \{1, 2, \cdots, n\}$ be the set of individuals (consumers). Each individual $i \in \mathcal{N}$ is endowed with an initial endowment, $\omega_i = (\omega_{i1}, \cdots, \omega_{im}) \in \mathbb{R}^m_+$, and has a continuous, quasi-concave and locally non-satiated utility function over the consumption set $\mathbb{R}^m_+$. Let $\Omega = \sum_{i \in \mathcal{N}} \omega_i$ be the vector of social endowments.

An *economy*, to be denoted by E, is then defined as follows:

$$E \equiv \left\langle \mathcal{N}; (u_i, \omega_i)_{i \in \mathcal{N}}; (Y_j)_{j \in \mathcal{J}} \right\rangle.$$

Let $\mathcal{E}$ denote the set of all possible economies defined above.

In the paper, we sometime consider a private ownership economy in which consumers own shares in firms and firm profits are distributed to share holders. In particular, for each $i \in \mathcal{N}$ and each firm $j \in \mathcal{J}$, let θ_{ij} be consumer i's shares in firm j's profits: $0 \leq \theta_{ij} \leq 1$ for all $i \in \mathcal{N}$ and all $j \in \mathcal{J}$, and $\sum_{i \in \mathcal{N}} \theta_{ij} = 1$ for all $j \in \mathcal{J}$. For each $i \in \mathcal{N}$, let $\theta_i = (\theta_{i1}, \cdots, \theta_{im})$. A private ownership

[3] See https://sustainabledevelopment.un.org/post2015/transformingourworld.

economy, to be denoted by E^{PO}, is defined as follows:

$$E^{PO} \equiv \left\langle \mathcal{N}; (u_i, \omega_i, \theta_i)_{i\in\mathcal{N}}; (Y_j)_{j\in\mathcal{J}} \right\rangle.$$

Let $\mathcal{E}^{PO}$ denote the set of all possible private ownership economies thus defined.

An *allocation*, $(\boldsymbol{x}, \boldsymbol{y})$, in an economy E, specifies a consumption bundle x_i for each individual $i \in \mathcal{N}$ and a production plan y_j for each firm $j \in \mathcal{J}$. An allocation $(\boldsymbol{x}, \boldsymbol{y})$ is *feasible for the economy* E if and only if $\sum_{i\in\mathcal{N}} x_i \leqq \Omega + \sum_{j=1,\ldots,J} y_j$. Let the set of all feasible allocations for E be denoted by $F(E)$. Given an economy $E = \left\langle \mathcal{N}; (u_i, \omega_i)_{i\in\mathcal{N}}; (Y_j)_{j\in\mathcal{J}} \right\rangle$, a feasible allocation $(\boldsymbol{x}, \boldsymbol{y})$ is *Pareto efficient* for E if and only if there exists no other feasible allocation $(\boldsymbol{x}', \boldsymbol{y}') \in F(E)$ such that $[u_i(x_i') \geq u_i(x_i)$ for all $i \in \mathcal{N}]$ and $[u_i(x_i') > u_i(x_i)$ for some $i \in \mathcal{N}]$.

Consider a simple 'dynamic' economy in which there are possibly an infinite number of periods. The periods are indexed by $\mathcal{T} = \{0, 1, 2, \cdots, \}$. An economy in period $t \in \mathcal{T}$, to be denoted by $E_t = \left\langle \mathcal{N}; (u_i, \omega_i^t)_{i\in\mathcal{N}}; \left(Y_j^t\right)_{j\in\mathcal{J}} \right\rangle$, is an element of $\mathcal{E}$. For each $t \in \mathcal{T}$ and each $E_t = \left\langle \mathcal{N}; (u_i, \omega_i^t)_{i\in\mathcal{N}}; \left(Y_j^t\right)_{j\in\mathcal{J}} \right\rangle \in \mathcal{E}$, let $\Omega^t \equiv \sum_{i\in\mathcal{N}} \omega_i^t$ be the total social endowments in period t, and $Y^t \equiv \sum_{j\in\mathcal{J}} Y_j^t$ be the aggregate production possibility set in period t. Thus, a *simple dynamic economy* can be defined formally as a collection of period economies: $(E_t = \left\langle \mathcal{N}; (u_i, \omega_i^t)_{i\in\mathcal{N}}; \left(Y_j^t\right)_{j\in\mathcal{J}} \right\rangle)_{t=0}^{\infty}$. Let $\mathcal{E}^\infty$ be the set of all simple dynamic economies.

Let φ be an *allocation rule* that maps each simple dynamic economy $E^\infty = (E_0, \cdots, E_t, \cdots) \in \mathcal{E}^\infty$ to a collection of sets of feasible allocations for the period economies: for each simple dynamic economy E^∞, $\varphi(E^\infty) = (\varphi(E_1), \cdots, \varphi(E_t), \cdots)$ such that, for each period $t \in \mathcal{T}$ and each period economy $E_t \in \mathcal{E}$, $\varnothing \neq \varphi(E_t) \subseteq F(E_t)$.

The market mechanism is a prominent example of an allocation rule: for a simple dynamic economy consisting of private ownership period economy $(E_t^{PO})_{t=0}^{\infty}$, it selects Walrasian competitive equilibrium allocations for each period economy E_t^{PO}. The Walrasian competitive equilibrium for a period economy E_t^{PO} is defined as follows: given a period private ownership economy,

$$E_t^{PO} \equiv \left\langle \mathcal{N}; \left(u_i, \omega_i^t, \theta_i^t\right)_{i\in\mathcal{N}}; \left(Y_j^t\right)_{j\in\mathcal{J}} \right\rangle,$$

a feasible allocation $(\boldsymbol{x}^{t*}, \boldsymbol{y}^{t*}) \in F\left(E_t^{PO}\right)$ and a price vector $\boldsymbol{p}_t^* \in \mathbb{R}_+^m$ constitute a *Walrasian competitive equilibrium for* E_t^{PO} if and only if the pair $\left(\left(\boldsymbol{x}^{t*}, \boldsymbol{y}^{t*}\right), \boldsymbol{p}_t^*\right)$ satisfies the following conditions:

(i) for each firm $j = 1, \ldots, J$, $\boldsymbol{p}_t^* \cdot y_j^{t*} \geq \boldsymbol{p}_t^* \cdot y_j^{t\prime}$ $(\forall y_j^{t\prime} \in Y_j^t)$;

(ii) for each individual $i \in \mathcal{N}$, $x_i^{t*} \in \arg\max_{x_i^t \in B\left(\boldsymbol{p}_t^*, \omega_i^t, \theta_i^t\right)} u_i\left(x_i^t\right)$, where

$$B\left(\boldsymbol{p}_t^*, \omega_i^t, \theta_i^t\right) \equiv \left\{ x_i^t \in \mathbb{R}_+^m \mid \boldsymbol{p}_t^* x_i^t \leq \boldsymbol{p}_t^* \omega_i^t + \sum_{j=1,\ldots,J} \theta_{ij}^t \boldsymbol{p}_t^* y_j^{t*} \right\};$$

(iii) $\sum_{i \in \mathcal{N}} x_i^{t*} = \Omega^t + \sum_{j=1,\ldots,J} y_j^{t*}$.

We shall refer the allocation rule associated with the market mechanism as the *Walrasian allocation rule.*

3 Hicksian Optimism and the Walrasian Allocation Rule

A movement from one Pareto efficient allocation to another Pareto efficient allocation is very likely to make someone worse off and yet someone else better off, and the Pareto principle itself cannot be used for making welfare judgments in such a movement. As discussed in Section 1, however, neoclassical economists believe that "if Pareto efficient policies are being pursued consistently over a long period, the *chances* are that eventually–though not at every particular step–everyone will be better off" (Corden, 1984, page 68), which is termed as the *Hicksian Optimism.*

Inspired by the above ideas, in what follows, we shall first introduce and define the 'improvements' that Hicks had in mind and then formulate the principle, the *Hicksian Optimism.* For this purpose, we consider a simple dynamic economy, $E^\infty = (E_t = \left\langle \mathcal{N}; (u_i, \omega_i^t)_{i\in\mathcal{N}}; \left(Y_j^t\right)_{j\in\mathcal{J}} \right\rangle)_{t=0}^\infty \in \mathcal{E}^\infty$, and an economy $E^* = \left\langle \mathcal{N}; (u_i, \omega_i^*)_{i\in\mathcal{N}}; \left(Y_j^*\right)_{j\in\mathcal{J}} \right\rangle \in \mathcal{E}$. We say that the simple dynamic economy $E^\infty = (E_t)_{t=0}^\infty$ is *expanding and converging* to the economy E^* if

(i) for all $t \in \mathcal{T}: Y^t + \{\Omega^t\} \subseteq Y^{t+1} + \left\{\Omega^{t+1}\right\}$,

(ii) for all $i \in N$ and all $t \in \mathcal{T}$: $\omega_i^t \leq \omega_i^{t+1}$, and $\omega_i^0 < \omega_i^* = \lim_{t\to\infty} \omega_i^t$,

(iii) $\lim_{t\to\infty} Y^t + \{\Omega^t\} \to Y^* + \{\Omega^*\} \supset Y^0 + \{\Omega^0\}$, and there is a sequence of allocations, $\{(\boldsymbol{x}^t, \boldsymbol{y}^t)\}_{t=0}^\infty$, such that $(\boldsymbol{x}^t, \boldsymbol{y}^t) \in \varphi(E_t)$ for all $t \in \mathcal{T}$, $\lim_{t\to\infty}(\boldsymbol{x}^t, \boldsymbol{y}^t) = (\boldsymbol{x}^*, \boldsymbol{y}^*) \in F(E^*)$.

Here, the expanding process from a period economy to the next period economy within a simple dynamic economy is characterized by condition (i), which represents the dynamic process of technical progress due to the success of technological innovation. The expansion is further re-enforced by condition (ii), where it requires that, for each individual, a period initial endowment is (weakly) expanding to its next period initial endowment, with the resulting initial endowment strictly larger than the period 0 initial endowment. Moreover, the dynamic transition of such expanding period-economies is characterized by condition (iii), which, in particular, requires that, after the expansion, the resulting economy is "bigger" than the initial economy.

In this formulation of the expanding and converging transition, we set aside the issue of how the R&D investment for technological innovation can be incentivized, as discussed in Section 1. Instead, we will focus on the issue whether such a dynamic transition of period-economies can improve individual and social welfare in the long run as captured by the idea of the Hicksian Optimism discussed in Section 1.

'The improvements' that Hicks had in mind are visualized as the improvements of the 'convergent economy' resulting from a sequence of expanding economies over the initial economy. Our notion of an expanding and converging economy requires more than what Hicks had in mind: the expansion occurs for any two periods in both aggregate resources (due to technical progress) and individual initial endowment, and is therefore much stronger than Hicks' requirement for the improvements of the economy. Now, comparing this eventual economy, the convergent economy of a sequence of expanding economies, to the initial economy, we can formulate the following principle requiring that no one be made worse off:

Hicksian Optimism (HO): For any economy $E^* = \left\langle \mathcal{N}; (u_i, \omega_i^*)_{i\in\mathcal{N}}; \left(Y_j^*\right)_{j\in\mathcal{J}} \right\rangle \in \mathcal{E}$, any simple dynamic economy $E^\infty = (E_t = \left\langle \mathcal{N}; (u_i, \omega_i^t)_{i\in\mathcal{N}}; \left(Y_j^t\right)_{j\in\mathcal{J}} \right\rangle)_{t=0}^{\infty} \in \mathcal{E}^\infty$, if E^∞ is *expanding and converging* to the economy E^*, then for the associated sequence of allocations $\{(\boldsymbol{x}^t, \boldsymbol{y}^t)\}_{t=0}^{\infty}$ with $(\boldsymbol{x}^t, \boldsymbol{y}^t) \in \varphi(E_t)$ for all $t \in \mathcal{T}$, we have $\lim_{t\to\infty} u_i\left(x_i^t\right) \geqq u_i\left(x_i^0\right)$ for all $i \in \mathcal{N}$.

According to our notion of the Hicksian Optimism, the improvements based on 'expansion policies' should leave no one behind–every individual will be made at least as well-off as before adopting those expansion policies.[4] Note that it is not required that everyone should (weakly) benefit at each period of the expansion policies. It merely requires that, eventually and this could be a long time requiring 'a great deal of human patience, more patience than is characteristic of the twentieth century, even of the economists of the twentieth century; more patience, perhaps, than we ought to ask' as observed by Hicks (1941) himself, no one is going to get hurt. Since our notion of an expanding and converging economy demands more than what Hicks requires for the expansion of the economy, our formulation of the Hicksian optimism is therefore much weaker than what Hicks had in mind, making our version of the Hicksian optimism even more appealing.

Our notion of Hicksian Optimism formally resembles various monotonicity properties proposed in the related literatures of axiomatic bargaining (Kalai (1977)), and of resource allocations (see, for example, Chambers and Hayashi (2017), Chun and Thompson (1988), Moulin and Roemer (1989), Moulin and Thompson (1988), and Roemer (1986)). Apart from being introduced in different frameworks and for different purposes, there is one more important difference: we require the expansion be such that, for every individual i, $\omega_i^* > \omega_i^0$ and $\omega_i^{t+1} \geq \omega_i^t$ for $t \in \mathcal{T}$; in other words, with the expansion of the economy, apart

[4]It may be of interest to note that our idea that 'no one is left behind' exposed in the Hicksian Optimism is similar to that advocated and used by the UNDP: "People get left behind when they lack the choices and opportunities to participate in and benefit from development progress." (UNDP, 2018)

from more aggregate resources becoming available, every one's initial endowment becoming (weakly) larger and larger and eventually strictly larger than his period 0 initial endowment.

The Hicksian Optimism seems a very mild requirement on an allocation rule. Before we examine a general allocation rule, we first ask the following question: how does the market mechanism (or the Walrasian allocation rule) fare with respect to the Hicksian Optimism? The answer is given in the following result whose proof can be found in the Appendix.

Theorem 1 *Suppose there are at least two individuals, i_1 and i_2, such that $\omega_{i_1}^0 \gg \mathbf{0}$ and $\omega_{i_2}^0 \gg \mathbf{0}$. Then, the Walrasian allocation rule does not satisfy Hicksian Optimism.*

Therefore, the market mechanism does not deliver the Hicksian Optimism: in a market system, economic expansions cannot avoid leaving some individuals behind–some individuals would be made worse off than the pre-expansion economy.

4 Any Efficient and Individually Rational Allocation Rule Is Not Hicksian Optimistic

In this section, we extend our analysis of the market mechanism to a broader class of mechanisms of allocating resources and examine the fate of the Hicksian optimism in these contexts. For this purpose, we introduce the following two familiar axioms for allocation rules that would be embedded in the class of resource allocation mechanisms that we have in mind.

Pareto Efficiency (PE): For each simple dynamic economy $E^\infty = \left\langle \mathcal{N}; (u_i, \omega_i^t)_{i\in\mathcal{N}}; \left(Y_j^t\right)_{j\in\mathcal{J}} \right\rangle_{t=0}^{\infty} \in \mathcal{E}^\infty$, for all $t \in \mathcal{T}$, and for every allocation $(\boldsymbol{x}^t, \boldsymbol{y}^t) \in \varphi(E_t)$ of E_t, $(\boldsymbol{x}^t, \boldsymbol{y}^t)$ is Pareto efficient for E_t.

Individual Rationality (IR): For each simple dynamic economy $E^\infty = \left\langle \mathcal{N}; (u_i, \omega_i^t)_{i\in\mathcal{N}}; \left(Y_j^t\right)_{j\in\mathcal{J}} \right\rangle_{t=0}^{\infty} \in \mathcal{E}^\infty$, for all $t \in \mathcal{T}$, and every allocation $(\boldsymbol{x}^t, \boldsymbol{y}^t) \in \varphi(E_t)$ of E_t, $[\forall i \in \mathcal{N} : u_i(x_i^t) \geq u_i(\omega_i^t)]$.

Pareto Efficiency requires that an allocation rule always pick up an efficient allocation in any given economy, and Individual Rationality requires that an allocation rule be such that anyone's welfare from the allocation chosen by the allocation rule is at least as great as his/her welfare derived from his/her initial endowment.[5] Note that both of the axioms, **PE** and **IR**, are satisfied by

[5]Note that we could have introduced a stronger individual rationalilty condition than this version of **IR** by incorporating information concerning individuals' ownerships of production technologies. Since our result is an impossibility result (see the theorem below), it becomes unnecessary to formulate a stronger individual rationality condition.

the Walrasian allocation rule. It is also of interest to note that, apart from the Walrasian allocation rule, there are other allocation rules satisfying **PE** and **IR**: the allocation rule that always selects 'core allocations' of any given economy is both Pareto efficient and individually rational.

The two axioms capture a broader class of allocation rules, and thus a broader class of resource allocation mechanisms. Can any such allocation rules deliver the Hicksian optimism? The answer is given in the following result and its proof can be found in the Appendix.

Theorem 2 *Suppose there are at least two individuals, i_1 and i_2, such that $\omega^0_{i_1} \gg \mathbf{0}$ and $\omega^0_{i_2} \gg \mathbf{0}$. Then, there exists no allocation rule satisfying Hicksian Optimism, Pareto Efficiency and Individual Rationality simultaneously.*

Therefore, any resource allocation mechanism characterized by an efficient and individually rational allocation rule is bound to leave some individual behind even if the economy is expanding due to technological innovation and even after a long period of time.

5 Conclusion

We have shown that the market mechanism characterized by the Walrasian allocation rule cannot deliver the Hicksian optimism–an ideal as envisioned by Hicks (1941) and, more recently, as advocated by the UNDP (2015) in its 2030 Agenda for Sustainable Development. Indeed, any resource allocation mechanism characterized by an efficient and individually rational allocation rule faces the same dilemma: with continued technological innovation and progress, some one is bound to be left behind even after a long period of time.[6]

Pareto efficiency is deeply rooted in economics and is highly appealing for resource allocation. Given the tremendous technological innovation made in recent human history and the great economic progress made in recent decades, the Hicksian optimism seems a very attractive property to be demanded for resource allocation mechanisms. If we follow these lines of reasoning, the property of individual rationality needs a careful and possibly critical examination.

To begin with, we note that, as our result in Section 3 shows, the unfettered market mechanism characterized by the Walrasian allocation rule cannot deliver Hicksian optimism. Part of the reasons is that the Walrasian allocation rule is both efficient and individually rational. Suppose we abandon individual rationality but still want to use the market mechanism to allocate resources. One possibility is to use the market mechanism coupled with a redistributive scheme

[6]It is of interest to note that, in a related paper by Chambers and Hayashi (2017), they show that, in a static pruduciton economy with constant returns to scale technologies, there is no allocation rule satisfying Pareto efficiency, technological monotonicity (introduced by Roemer (1986)), and a stronger version of individual rationality (what they call Free Access Lower Bound).

(perhaps similar to that required for the second welfare theorem) to ensure that, eventually, every individual is at least as well-off as period $t = 0$. Resource-wise, this is feasible since the economy is expanding and everyone's eventual initial endowment (ω_i^*) is bigger than his/her initial endowment, ω_i^0, in period 0. Of course, the Walrasian allocation rule coupled with a redistributive scheme may violate individual rationality introduced in Section 4, the reason being that, some individuals may be made worse off than their initial endowments in that period after redistributions.

If, on the other hand, one wants to go beyond the market mechanism, there are several studies on certain egalitarian type allocation rules for specific economies involving one input and one output, where it has been shown that the proposed allocation rules are Pareto efficient and satisfy some technological monotonicities. See the contributions by Moulin (1987, 1990), Moulin and Roemer (1989), and Roemer and Silvestre (1987). An important reason why they have obtained positive results is that their economies involve one input and one output and, in such specific economies, the property of individual rationality almost has no bite, while in our economy, individual rationality is no trivial and is rather stringent.

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Appendix

Proof of Theorem 2. Let $n = 2$ and $m = 3$, where the third commodity is used as a *primary factor*. Given $\varepsilon \in \left(0, \frac{1}{2}\right)$, let $a \in \mathbb{R}_+$ be such that $a > \frac{1-\varepsilon}{\varepsilon}$, which implies that $a > 1$. Let $\bar{\delta}$ and δ be sufficiently small with $\bar{\delta} > \delta > 0$. Let $(u_i)_{i \in \mathcal{N}}$ be defined by[7]

$$u_1(x_{11}, x_{12}, x_{13}) \equiv \min\left\{\frac{x_{11}}{a}(1+\delta), x_{12}\right\} \text{ and } u_2(x_{21}, x_{22}, x_{23}) \equiv u_1(x_{22}, x_{21}, x_{23})$$

For $t \in \mathcal{T}$, let $\omega_1^t = \omega_2^t = \left(\frac{t+1}{t+2a}\varepsilon(a+\bar{\delta}, 1+\bar{\delta}), 1\right)$.[8] Let the production possibility set Y^t at $t \in \mathcal{T}$, be a convex cone in $\mathbb{R}^2 \times (-\mathbb{R}_+)$ and the corresponding *output possibility set* of two commodities *when two units of good* 3 *are used* be represented by $Y_{1,2}^t \equiv \left\{(y_1, y_2) \in \mathbb{R}^2 \mid (y_1, y_2, -2) \in Y^t\right\}$.[9] Let $Y_{1,2}^0$ be given by the *comprehensive hull* of $(1,1) - \left(\Omega_1^0, \Omega_2^0\right)$ in $\mathbb{R}^2$, which is denoted by $Y_{1,2}^0 = comp\left\{(1,1) - \left(\Omega_1^0, \Omega_2^0\right)\right\}$. However, since the production possibility set is always a convex cone, the third good does not enter into any individual's utility function as defined above, and we will focus on any efficient allocation rule here, it is sufficient to have only the information of the output possibility set of two commodities, 1 and 2, when two units of good three are used at each $t \in \mathcal{T}$. Therefore, without loss of generality, we will use, in the following discussion, the notation Y^t as the representation of the output possibility set of two commodities 1 and 2 when two units of good three are employed. That is, as an abuse of notation, we will use the notation $Y^t + \{\Omega^t\}$ for $t \in \mathcal{T}$, such as $Y^0 + \left\{\Omega^0\right\} \equiv comp\left\{(1,1)\right\}$ for $t = 0$.

Let $\{Y^t + \{\Omega^t\}\}_{t=0}^{\infty}$ be a sequence of period economies such that for any $t \geqq 0$, $Y^t + \{\Omega^t\} \subseteq Y^{t+1} + \left\{\Omega^{t+1}\right\}$ and $\lim_{t\to\infty} Y^t + \{\Omega^t\} = Y^* + \{\Omega^*\}$ such that $Y^* + \{\Omega^*\} \equiv comp\left\{(a, 1+\delta)\right\}$. Note that $\lim_{t\to\infty} \omega_1^t = \lim_{t\to\infty} \omega_2^t = \left(\varepsilon(a+\bar{\delta}, 1+\bar{\delta}), 1\right) = \omega_1^* = \omega_2^*$. Since $\bar{\delta} > \delta$, it follows that $\varepsilon(a, 1+\delta) \ll (\omega_{11}^*, \omega_{12}^*)$ and $\varepsilon(a, 1+\delta) \ll (\omega_{21}^*, \omega_{22}^*)$.

Let an allocation rule φ satisfy **PE**, **IR**, and **HO**. Then, for any sequence $\{(\boldsymbol{x}^t, \boldsymbol{y}^t)\}_{t=0}^{\infty}$ such that $(\boldsymbol{x}^t, \boldsymbol{y}^t) \in \varphi(E_t)$, and any subsequence $\left\{\left(\boldsymbol{x}^{t^k}, \boldsymbol{y}^{t^k}\right)\right\}_{t^k=0}^{\infty}$ of $\{(\boldsymbol{x}^t, \boldsymbol{y}^t)\}_{t=0}^{\infty}$, let $\lim_{t^k\to\infty}\left(\boldsymbol{x}^{t^k}, \boldsymbol{y}^{t^k}\right) = (\boldsymbol{x}^*, \boldsymbol{y}^*) \in F(E^*)$ for $E^* = \left\langle \mathcal{N}; (X_i)_{i\in\mathcal{N}}; (u_i)_{i\in\mathcal{N}}; \Omega^*; Y^* \right\rangle$. Then, by **PE** and **IR**, every allocation $\left(\boldsymbol{x}^{t^k}, \boldsymbol{y}^{t^k}\right)$ at period t^k within the subsequence $\left\{\left(\boldsymbol{x}^{t^k}, \boldsymbol{y}^{t^k}\right)\right\}_{t^k=0}^{\infty}$ is Pareto efficient and

[7]The specific utility functions are used here to provide a simplest economy to establish our result. We may note that, at the expense of increasing complexity, it is possible to consider a more standard type of utility functions, where utility functions are strongly monotonic and are not the Leontief preferences of the first two goods, for the proof.

[8]The specification of the equal endowments for the two individuals is not essential, but simply for the sake of simplicity. Indeed, basically the same logic of proof can be worked out with the two individuals having different endowments.

[9]This production possibility set is the set of (net) outputs which can be produced by using all the individuals' endowments of good 3 as input.

individual rational for E_{t^k}, and so is $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ for E^*.[10] Therefore, $u_1(x_1^*) \geqq u_1(\varepsilon(a, 1+\delta), 0)$ and $u_2(x_2^*) \geqq u_2(\varepsilon(a, 1+\delta), 0)$. Thus, $u_1(x_1^*) \geqq \min\left\{\frac{\varepsilon a}{a}(1+\delta), \varepsilon(1+\delta)\right\} = \varepsilon(1+\delta)$ and $u_2(x_2^*) \geqq \min\left\{\varepsilon a, \frac{\varepsilon(1+\delta)^2}{a}\right\} = \frac{\varepsilon}{a}(1+\delta)^2$. Note that, as $\varepsilon a - \frac{\varepsilon}{a} > 0$, $\varepsilon a - \frac{\varepsilon(1+\delta)^2}{a} > 0$ holds for the sufficiently small $\delta > 0$. Then, $u_2(x_2^*) \leqq \frac{(1+\delta)^2(1-\varepsilon)}{a}$ holds. To see, suppose that $u_2(x_2^*) > \frac{(1+\delta)^2(1-\varepsilon)}{a}$. This implies that $x_{21}^* > \frac{(1+\delta)^2(1-\varepsilon)}{a}$, $x_{22}^* > (1+\delta)(1-\varepsilon)$. Then, $x_{11}^* < a - \frac{(1+\delta)^2(1-\varepsilon)}{a}$ and $x_{12}^* < \varepsilon(1+\delta)$. Therefore,

$$u_1(x_1^*) < \min\left\{(1+\delta)\left[1 - \frac{(1+\delta)^2(1-\varepsilon)}{a^2}\right], \varepsilon(1+\delta)\right\} = \varepsilon(1+\delta).$$

Note that $1 - \frac{(1+\delta)^2(1-\varepsilon)}{a^2} > \varepsilon$ holds, as $a^2 - (1+\delta)^2(1-\varepsilon) - a^2\varepsilon = a^2(1-\varepsilon) - (1+\delta)^2(1-\varepsilon) = \left(a^2 - (1+\delta)^2\right)(1-\varepsilon) = (a + (1+\delta))(a - (1+\delta))(1-\varepsilon) > 0$ by $a > 1$ and δ being sufficiently small. Then, since $u_1(x_1^*) < \varepsilon(1+\delta) \leqq u_1(x_1^*)$, we have a contradiction. Therefore, $u_2(x_2^*) \leqq \frac{(1+\delta)^2(1-\varepsilon)}{a}$ holds. By **HO**, for any $\left(\boldsymbol{x}^0, \boldsymbol{y}^0\right) \in \varphi(E_0)$, $u_2\left(x_2^0\right) \leqq \frac{(1+\delta)^2(1-\varepsilon)}{a}$ holds.

Let $\{Y'^t + \{\Omega'^t\}\}_{t=0}^{\infty}$ be a sequence such that, $Y'^0 + \{\Omega'^0\} = Y^0 + \{\Omega^0\}$, for any $t \geqq 0$, $Y'^t + \{\Omega'^t\} \subseteq Y'^{t+1} + \left\{\Omega'^{t+1}\right\}$ and $\lim_{t\to\infty} Y'^t + \{\Omega'^t\} = Y'^* + \{\Omega'^*\}$ such that $Y'^* + \{\Omega'^*\} \equiv \mathit{comp}\{(1+\delta, a)\}$. Then, by a symmetric argument, we obtain for any $\left(\boldsymbol{x}^0, \boldsymbol{y}^0\right) \in \varphi(E_0)$, $u_1\left(x_1^0\right) \leqq \frac{(1+\delta)^2(1-\varepsilon)}{a}$ holds.

Consider a feasible allocation $\left(\boldsymbol{z}, \boldsymbol{y}^0\right)$ with $\boldsymbol{z} = \left(\left(\frac{a}{a+1}, \frac{1}{a+1}, 0\right), \left(\frac{1}{a+1}, \frac{a}{a+1}, 0\right)\right)$ at E_0. Then, $u_1(z_1) = u_2(z_2) = \min\left\{\frac{a}{a+1}\frac{(1+\delta)}{a}, \frac{1}{a+1}\right\} = \frac{1}{a+1}$. However,

$$\frac{1}{a+1} - \frac{(1+\delta)^2(1-\varepsilon)}{a} > 0 \text{ for the sufficiently small } \delta > 0,$$

as $\frac{1}{a+1} - \frac{(1-\varepsilon)}{a} > 0 = \frac{a\varepsilon+\varepsilon-1}{a(a+1)} > 0$ by $a > \frac{1-\varepsilon}{\varepsilon}$. Thus, we have $u_i(z_i) > u_i\left(x_i^0\right)$ for all $i \in N$. This implies $\left(\boldsymbol{x}^0, \boldsymbol{y}^0\right)$ cannot be Pareto efficient, a contradiction.

■

Proof of Theorem 1. We note that the Walrasian rule satisfies Individual Rationality. Further, given our assumptions on each individual's preferences, by the first theorem of welfare economics, the Walrasian rule satisfies Pareto Efficiency. Then, noting that, for two individuals, i_1 and i_2, we have $\omega_{i_1}^0 \gg \mathbf{0}$ and $\omega_{i_2}^0 \gg \mathbf{0}$, Theorem 1 then follows easily from Theorem 2. ■

[10]The last property follows from the fact that the Pareto efficient and individually rational correspondence, that is, the allocation rule selecting all Pareto efficient and individually rational allocations at each and every economy, is upper hemi-continuous.