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The Dynamics of Exploitation and Inequality in Economies with Heterogeneous Agents

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Abstract

This paper analyses the relation between growth, inequalities, and exploitation as the unequal exchange of labour (UE exploitation). An economy with heterogeneous, intertemporally optimising agents is considered which generalises John Roemer's [52, 53] seminal models. First, a correspondence between profits and the existence (and intensity) of UE exploitation is proved in the dynamic context. This result is important, positively, because the profit rate is one of the key determinants of investment decisions, and, normatively, because it provides a link between UE exploitation and the functional distribution of income. Second, it is shown that asset inequalities are fundamental for the emergence of UE exploitation, but they are not sufficient for its persistence, both in equilibria with accumulation and growth, and, perhaps more surprisingly, in stationary intertemporal equilibrium paths. Labour-saving technical progress, however, may yield sustained growth with persistent UE exploitation by keeping labour abundant relative to capital. Persistent inequalities in income and labour exchanged arise from the interaction between labour market conditions and differential ownership of productive assets.

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1 Introduction

The relation between capital accumulation, growth, and inequality has long been at the centre of economic analysis. What is the effect of growth on inequalities? Is there a discernible pattern linking economic development and, for example, income inequalities? Conversely, do significant inequalities in wealth and/or political power hinder, or promote, capital accumulation and economic growth? This paper analyses the dynamics of growth and distribution, and the long-run relation between accumulation and inequalities, focusing on a specific measure of inequality, namely *exploitation as the unequal exchange of labour*.

The conventional view is that the concept of exploitation is ill-defined and metaphysical. It is ill-defined because it rests on a fundamentally flawed – if not logically inconsistent – approach to price theory. It is metaphysical because it rests on unobservable magnitudes that have no clear relevance for agents' behaviour in actual economies. At best, it may be seen as capturing the effects of non-competitive distortions and deviations from marginal productivity pricing in factor markets.

This view has been famously challenged by John Roemer, in a series of seminal contributions (Roemer [51, 53, 54, 57]). By applying the tools of contemporary economic theory, Roemer has derived many important conclusions, both substantive and methodological. Concerning methodology, he has argued that, contrary to the received view, exploitation theory can, and should be analysed by means of general equilibrium models with optimising agents.

Substantively, he has shown that a concept of exploitation as the unequal exchange of labour (henceforth, UE exploitation) can be identified which is logically consistent, well-defined, and with clear empirical foundations. The notion of UE exploitation captures the idea that while national income is the product of human labour, its distribution among agents is not proportional to the amount of labour – time, effort, and skills – contributed by them in productive activities. Significant wealth inequalities, and specifically differential ownership of productive assets, allow wealthy agents to appropriate a disproportionate share of national income.

In other words, the modern theory of UE exploitation directs our attention to the joint distribution of income and labour/leisure in the economy. As Fleurbaey [18] has argued, UE exploitation captures some inequalities in the distribution of material well-being and free hours that are – at least *prima facie* – of normative relevance. For instance, they are relevant for *inequalities of well-being freedom* (Rawls [50]; Sen [61]), because material well-being and free hours are key determinants of individual well-being freedom.[1] Further, a UE exploitation-free allocation coincides with the so-called *proportional solution*, a well-known fair allocation rule whereby every agent's income is proportional to her contribution to the economy (Roemer and Silvestre [60]; Moulin [42]). Proportionality is a widely held idea of equity (Tornblom [65]), whose philosophical foundations can be traced back to Aristotle (Maniquet [37]) and which can be justified in terms of the Kantian categorical imperative (Roemer [59]).

In this paper, we generalise Roemer's analysis in order to explore the growth/inequality nexus. First, we examine the relation between UE exploitation, the functional distribution of income, and capital accumulation, and the effect of inequalities in the allocation of income and labour/leisure bundles on the dynamics of a capitalist economy. Second, we investigate the dynamics of inequalities (in wealth holdings, in the functional distribution of income, *and* in the exchange of labour) in the equilibrium growth path of the economy, in order to identify the mechanisms guaranteeing the persistence of inequalities in capitalist economies (Piketty [46]; Piketty and Zucman [47]).

To be specific, we analyse the relation between wealth inequalities, profits, accumulation and UE exploitation in a dynamic general equilibrium model with heterogeneous optimising agents. Given

[1] The notion of well-being freedom emphasises an individual's ability to pursue the life she values. In the Rawls-Sen theory, inequalities in the distribution of well-being freedom are formulated as *inequalities of capabilities*, whereas they are formulated as *inequalities of (comprehensive) resources* in Dworkin's theory [16]. The resource allocation problem in terms of equality of capability is analysed by Gotoh and Yoshihara [23], whereas Roemer [56] analyse it in terms of equality of resources.

our focus on inequalities, we drop the standard representative agent assumption and assume that there exist two types of agents in the economy, capitalists and workers. Both types of agents have the same preferences over consumption bundles and solve an intertemporal optimisation programme. They also have the same skills and labour endowments. However, we allow for inequalities in their endowments of productive assets, which have some major behavioural implications.

Capitalists own wealth which allows them to hire workers and activate production processes by using a linear technology. They do not work and allocate their income optimally between consumption and savings. In contrast, workers own no wealth and their only source of income is their labour. Although they choose consumption bundles to maximise utility, like capitalists, we assume that they cannot accumulate. Empirically, this assumption can be seen as a stark way of capturing the extreme wealth inequalities,[2] and the relatively low degree of social mobility, that characterise capitalist economies, especially given that we abstract from skill heterogeneity and managerial labour.[3] It is also in line with recent findings by Krueger et al [33] who report higher consumption rates for agents with lower net worth.

Theoretically, it embodies Classical assumptions on behaviour: inequalities in asset ownership shape agents' feasible sets, and influence their attitudes, beliefs and choices. In general, the dynamics of capitalist economies is determined by the heterogeneous consumption and savings opportunities and choices related to different socio-economic status, as proxied by different positions in the wealth distribution (see Roemer [51], p.509).[4] However, similar assumptions have also been explored in recent analyses of the effect of wealth inequality on macroeconomic dynamics, and most notably in Kaplan and Violante [30] and Krueger et al [33].[5]

Perhaps more importantly, this assumption yields no significant loss of generality, either theoretically or formally. Theoretically, our aim is not to analyse social mobility, nor are we trying to explain *earning* inequalities arising from heterogeneous labour and skills. Formally, none of our key conclusions depends on ruling out workers' accumulation. On the one hand, the inclusion of workers' accumulation would *strengthen* our results on the transitory nature of inequalities in the basic economy without technical progress. On the other hand, as shown below, along any equilibrium growth paths, workers do not save, due to the downward pressure exercised by unemployment on the real wage.[6]

This kind of heterogeneity in savings behaviour is also consistent with two other key features of our model, namely the existence of wealth constraints and the incompleteness of credit markets,[7] whose importance has long been emphasised in the literature (see, for example, the classic papers by Loury [35]; Galor and Zeira [20]; Bénabou [10]; and Matsuyama [38]). In our economy, following Roemer [51, 53, 54], production takes time and entrepreneurs must lay out in advance the capital necessary to finance production. However, agents cannot access credit markets to finance either

[2]In the US, for example, agents in the bottom three quintiles of the wealth distribution own zero net worth (Allegretto [4]; Piketty [46]; Krueger et al [33]).

[3]Of course, the degree of social mobility in advanced economies is not zero. Nonetheless, recent research suggests that it is remarkably (and perhaps surprisingly) low and declining (Greenstone et al [24]). This is true even if one considers the very long run (Barone and Mocetti [7]) and without major differences between European countries and the US (Bénabou and Tirole [11], pp.702ff).

[4]Differential saving rates out of profit and wage income can also be motivated by noting that a sizeable proportion of savings and investment in advanced economies derives from firms' retained earnings, as famously argued by Kaldor [28].

[5]See, in particular, the model in section 6.2.1 of Krueger et al ([33]). In their general model, too, wealth-poor households make up a significant share of aggregate consumption and it is the lack of wealth at the bottom that is crucial for explaining aggregate consumption dynamics.

[6]This is reminiscent of a feature of the model by Banerjee and Newman ([5], p.291), whereby the abundance of labour drives the wage down which "makes it virtually impossible ... for workers to accumulate enough wealth".

[7]In economies with wealth constraints, the availability of production loans is a necessary condition for asset-poor agents to be able to undertake production activities and obtain nonzero and nonegligible wealth levels in equilibrium (Banerjee and Newman [6]).

production or accumulation.[8]

In this economy, we rigorously define the concept of UE exploitation (Definition 3): agents suffer from an unequal exchange of labour if the amount of labour they contribute to the economy is higher than the amount of labour they receive via their income. In other words, human labour is used as a numéraire to capture a relevant set of inequalities. This definition provides normatively relevant insights and the index measuring exploitative relations is based on empirically measurable variables (labour expended, technology, income, and consumption).

We first prove that at any general equilibrium, the profitability of capitalist production is synonymous with the existence of UE exploitation and a monotonic relation exists between the profit rate and the exploitation rate (Theorem 1). This result is important because the rate of profit is one of the key determinants of investment decisions, and of the long-run dynamics of capitalist economies. Thus, Theorem 1 can be interpreted as providing a link between inequality, exploitation, and growth. But the result is important also because it proves that, given private ownership of productive assets, profits are a counterpart of the transfer of social surplus and social labour from asset-poor agents to the wealthy. This provides a link between UE exploitation and the functional income distribution. This result confirms the normative and positive relevance of the concept of UE exploitation in examining the dynamics of capitalist economies, and it provides the foundations for the rest of the analysis.

We derive a number of results concerning the relation between inequality, UE exploitation, and growth. First, in the basic economy with constant labour supply and a given technology, there exists no equilibrium path with persistent growth and persistent UE exploitation even in the presence of significant asset inequalities (Proposition 1): absent any countervailing measures, accumulation leads capital to become abundant, and profits and exploitation to disappear. Second, and perhaps more surprising, asset inequalities and competitive markets alone do not guarantee the persistence of profits and exploitation, even in equilibrium paths in which capital does not become abundant. Neither at stable growth paths converging to a long-run steady state, nor at stationary states *without accumulation*, is UE exploitation a persistent feature of the economy, unless agents discount the future (Theorems 3 and 6).

These results have several implications, both normative and positive. From a normative viewpoint, they suggest that Roemer's claim that "differential distribution of property and competitive markets are sufficient institutions to generate an exploitation phenomenon, under the simplest possible assumptions" (Roemer [53], p.43) should at least be qualified. For asset inequalities are indeed fundamental for the emergence of UE exploitation, but they are not sufficient for its persistence, which raises some doubts on the idea that UE exploitation can be *reduced* to 'a kind of resource egalitarianism' (Roemer [58], p.2).

To be sure, one may argue that Theorems 3 and 6 suggest that, along equilibrium paths where capital remains scarce, a positive rate of time preference is all that is needed to guarantee the persistence of inequalities, and specifically of UE exploitation. Thus our own results provide the necessary qualification of Roemer's claim: differential distribution of (scarce) property and competitive markets are sufficient institutions to generate persistent inequalities and UE exploitation, *provided capitalists discount the future*.[9]

Although this is certainly a legitimate interpretation of Theorems 3 and 6, we do not think that it is the most insightful or normatively robust. On the one hand, in our view, one of the distinguishing features of exploitation theory is an emphasis on the structural, objective characteristics of capitalist economies, rather than on subjective and empirically contingent factors such as time preference.

[8] As in Patriarca and Vona [45] the economy displays intertemporal complementarities – whereby "production takes time and is carried on in vertical integrated firms with fixed proportions of capital and labor" (Patriarca and Vona [45], p.1643) – and markets open sequentially (produced inputs are bought before production takes place and must be financed out of beginning-of-period asset holdings).

[9] Despite the different formal and theoretical framework, the rate of time preference plays a conceptually similar role in the dynamics of asset inequalities as in Fischer [17].

On the other hand, we think that our results point to a more complex, structural role of asset inequalities in generating income inequalities and an unequal exchange of labour.

In particular, we take our results as suggesting that asset inequalities are a fundamental feature of capitalist economies, and a key determinant of its long run dynamics, but at the same time exploitation cannot be reduced to wealth inequalities *per se*. Differential ownership of (scarce) productive assets is causally necessary but normatively secondary in generating exploitation. The central role of asset inequalities is better understood in conjunction with labour market conditions and institutions, and the mechanisms that ensure the scarcity of capital.

Theorem 7 confirms this intuition: it proves that labour-saving technical progress may yield sustained growth with persistent exploitation and inequalities.[10] Innovations do not allow for persistent growth by affecting total factor productivity, or via their *direct* effect on output per worker. Also, they do not influence the functional distribution of income by altering marginal productivities. Rather, our analysis highlights the relevance of the interaction between technical change *and* labour market conditions in shaping distributive outcomes and the equilibrium growth paths of capitalist economies.[11] By raising labour productivity, labour-saving technical progress ensures the persistent abundance of labour, which in turn keeps wages low and guarantees the profitability conditions necessary for growth. Further, as the wage rate and employment remain relatively stagnant while the economy grows, the interaction between labour market conditions and technical progress leads the wage share in national income to fall steadily over time, as has been observed in most advanced countries in the last few decades (Karabarbounis and Neiman [31]; Piketty and Zucman [47]).

2 Related literature

This paper lies at the intersection of various strands of literature.

Roemer's [51, 53, 54] seminal contributions have sparked a vast debate. Several critiques have been expounded on his methodology and conclusions, mainly based on exegetical issues, but surprisingly little attention has been devoted to his models (Veneziani [66, 67]). In this paper, we take a different approach and critically evaluate Roemer's theory using a dynamic general equilibrium model. For, as Veneziani [66, 67] has argued, Roemer's [51, 53, 54] models are essentially static in that agents face no intertemporal trade-offs. As a result, they are not suitable to analyse exploitation and inequalities as *persistent* features of capitalist economies. Nor do they allow to analyse the relation between exploitation, inequality and accumulation.

This paper analyses the conditions both for the emergence and for the *persistence* of exploitation, and the relation between asset inequalities, exploitation, profits and growth, in a dynamic generalisation of Roemer's [51, 53, 54] economies with optimising agents. This allows us to assess the causal and moral relevance of asset inequalities in generating exploitation as a persistent feature of a capitalist economy where (a subset of) agents can save.

Our approach is reminiscent of earlier contributions by Devine and Dymski [14] and Hahnel [25], who have showed that if Roemer's [53] static model is allowed to run for many periods, capital accumulation eventually drives profits to zero. However, unlike in this paper, they focused on the T-fold iteration of the static model with myopic agents, without explicitly analysing intertemporal decisions. Moreover, they did not explicitly consider the possible determinants of the persistence of exploitation and inequalities. Some of our conclusions echo the results obtained in Veneziani [66, 67]. However, the latter contributions focus on simple subsistence economies with a given

[10] For a discussion of the role of technical change and a review of the literature, see Tavani and Zamparelli [64].

[11] This conclusion is similar in spirit to the main findings of Banerjee and Newman [5] where persistent inequalities derive from the interaction between labour market conditions and wealth constraints in a model of occupational choice.

technology, and therefore can only shed partial light on the determinants of exploitation and on the link between exploitation and growth.

The model incorporates a number of assumptions – most notably, concerning labour, technology, savings, and technical change – that can be traced back to the Classical economists, and that allow us "to focus on the effect of ongoing capital accumulation per se on the long run distribution of income between capital and labor" (Mookherjee and Ray [39], p.2). These assumptions have been formalised in some classic papers, such as Kaldor [27] and Pasinetti [44], which focused on the effect of income distribution on consumption and saving and sparked a vast debate (more recent work includes Bohm and Kaas [13] and Patriarca and Vona [45]). Unlike in these earlier contributions, however, we explicitly model micro behaviour and intertemporal trade-offs.

There is a vast literature analysing the effect of inequality on growth focusing on electoral competition and political economy (Bénabou [10]; Ray and Esteban [48]): in these contributions, inequality tends to have a detrimental effect on growth as it generates social conflict and insecure property rights, but "what really matters is not income inequality per se but inequality in the relative distribution of earning and political power" (Bénabou [10], p.18): a higher degree of pro-wealth bias in the political system tends to help growth. We abstract from political economy issues and in our model the role of capitalists in fostering accumulation implies that inequality always has beneficial effects on growth, consistent with standard insights in Classical economics.

Our results relate to a literature that explores the emergence and persistence of inequality, including human capital and nonconvexities (Galor and Zeira [20]; Galor and Tsiddon [21]), endogenous risk-taking or idiosyncratic and uninsurable unemployment risk (Becker, Murphy, and Werning [9]; Ray and Robson [49]; Krueger et al [33]), and aspirational preferences (Genicot and Ray [22]). Yet our analysis is conceptually closer to contributions taking a more structural perspective and emphasising the role of incomplete markets, credit constraints, and institutions (Loury [35]; Banerjee and Newman [5]; Matsuyama [38]), and the interaction between labour market conditions and technical change (Acemoglu [1, 2, 3]).

Compared with the literature just mentioned, our approach is distinctive given the focus on UE exploitation and on the relation between exploitation and the functional income distribution. In this respect, our paper is closer to the growing literature analysing the recent trend decline in the labor share both in the U.S. and globally, focusing on technical change and automation (Karabarbounis and Neiman [31]; Mookherjee and Ray [39]) or capital accumulation (Piketty [46]; Piketty and Zucman [47]). Although technical change and capital accumulation are central in our model, they play a different role in their interaction with labour market conditions. Thus, unlike in Karabarbounis and Neiman [31] and Mookherjee and Ray [39], innovations and accumulation do not affect distribution via their effect on the relative price of investment goods. Unlike in Piketty [46] and Piketty and Zucman [47], capital accumulation does not have a negative effect on the wage share and indeed when the conditions for long run accumulation are satisfied the growth rate of the economy is equal to the profit rate.

One important feature of the economy (less explored in the above literature) is the linearity of the production technology. Empirically, this assumption is in line with recent evidence casting doubts on the degree of substitutability between labour and capital, and showing some limitations of marginal productivity theory as an explanation of the remuneration of productive inputs (Foley and Michl [19]; Basu [8]). Theoretically, it allows us to examine some relatively unexplored mechanisms through which inequality affects growth, and vice versa. In economies with credit constraints and standard production functions with diminishing marginal productivity, for example, inequality tends to reduce growth because poorer agents cannot access production loans and therefore cannot invest in projects yielding higher returns (see Bénabou [10]; Matsuyama [38]; and the literature therein). Conversely, one of the main channels through which growth affects inequality is via the effect of accumulation on the dynamics of the marginal productivity of capital and labour. In our model, we analyse the relation between inequality and growth abstracting from any consideration concerning marginal productivity, while focusing on the role of labour market conditions and technical change.

Indeed, our model contributes to the literature on the generic indeterminacy of the functional income distribution in perfectly competitive economies with a linear technology (Mandler [36] and Yoshihara and Kwak [75]). By proving the existence of a monotonic relation between exploitation and profits, our paper suggests that distributive struggles between capitalists and workers may serve as a selection mechanism to identify an equilibrium factor income distribution, thus providing the foundations for an alternative to marginal productivity theory.

Finally, Theorem 1 proves that profits are synonymous with UE exploitation. The existence of a relation between exploitation and profits has a "prominent place in the modern formulation of Marxian economics" (Roemer [52], p.16), and therefore it has been dubbed the 'Fundamental Marxian Theorem' (FMT; Morishima [41]). The FMT has generated an extensive literature (see, e.g., the seminal contributions by Okishio [43] and Morishima [41]).[12] Nonetheless, the robustness and theoretical relevance of the FMT have been put into question: according to critics, the FMT holds only under very restrictive assumptions concerning preferences, endowments, and technology and, in any case, the relation between exploitation and profits is spurious, as the FMT simply captures the productivity of the economy (for a comprehensive discussion, see Yoshihara [72, 73]).

Yoshihara and Veneziani [68, 69] have recently shown that, contrary to the critics' claims, the main insights of the FMT continue to hold in static economies with general technologies and preferences. In this paper, we explicitly tackle the relation between exploitation, profits and growth, and provide a *dynamic* generalisation of the FMT in a model with maximising agents.

3 The Model

The economy consists of a sequence of non-overlapping generations. In each generation there is a set $\mathcal{N}_c = \{1, \ldots, N_c\}$ of capitalists with generic element ν, and a set $\mathcal{N}_w = \{1, \ldots, N_w\}$ of workers with generic element η. Agents live for T periods, where T can be finite or infinite, and are indexed by the date of birth kT, $k = 0, 1, 2, \ldots$ In every period t, they produce and exchange n commodities and labour. Let (p_t, w_t) denote the $1 \times (n+1)$ price vector in t, where w_t is the nominal wage.[13]

We analyse the consequences of wealth inequalities in economies with incomplete capital markets, and model differences in behaviour starkly. Let $\mathbf{0}$ be the null vector. In every t, each capitalist $\nu \in \mathcal{N}_c$ owns a $n \times 1$ vector of productive assets $\omega_t^\nu \geqq \mathbf{0}$,[14] where $\omega_{kT}^\nu \geq \mathbf{0}$ is the vector of endowments inherited, when born in kT. In every t, capitalists do not work but can hire workers in order to operate any activity of a standard Leontief technology (A, L), where A is a $n \times n$ non-negative, productive and indecomposable matrix of material input coefficients and $L > \mathbf{0}$ is a $1 \times n$ vector of direct labour coefficients.[15] For every ν, y_t^ν is the $n \times 1$ vector of activity levels that ν hires workers to operate at t. In every t each capitalist ν has to use her wealth, $p_t \omega_t^\nu$, to obtain the necessary material inputs. At the end of the production period, capitalists use their net income to pay workers and to finance consumption and accumulation. Thus, for each ν, in every t, s_t^ν is the $n \times 1$ vector of savings and $c_t^\nu \geqq \mathbf{0}$ is the $n \times 1$ consumption vector.

The choices available to workers are much limited. Each worker $\eta \in \mathcal{N}_w$ possesses no physical capital, $\omega_t^\eta = \mathbf{0}$ in every t, but is endowed with one unit of labour. Therefore workers obtain

[12] For a counterpoint on the normative relevance of the FMT and an alternative explanation of the origins of profits, see Hahnel [26].

[13] Throughout the paper, all variables and vectors are assumed to belong to a Euclidean space $\mathbb{R}^k$ of appropriate dimensionality k.

[14] For all vectors $x, y \in \mathbb{R}^p$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, p)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, p)$.

[15] In the basic model, technology remains unchanged over time. We introduce technical progress in section 6 below. The other assumptions on technology can also be relaxed, albeit at the cost of a significant increase in technicalities (see, for example, the general convex cone economies analysed in Veneziani and Yoshihara [70]).

income only by supplying labour and we assume that they use their income only to purchase consumption goods. To be precise, at all t, z_t^η is η's labour supply and c_t^η is η's $n \times 1$ consumption vector. Furthermore, the (work and consumption) choices available to workers are limited by the structural features of the economy and in particular by the presence of unemployment. Formally, for all $\eta \in \mathcal{N}_w$, in every t, there exists an upper bound $\widehat{z}_t^\eta$ to η's labour supply, which is determined by demand conditions.

We assume that, at all t, $c_t^\nu \geqq \mathbf{0}$ for all $\nu \in \mathcal{N}_c$, while there exists a reference consumption bundle $b > \mathbf{0}$, such that $c_t^\eta \geqq b$. Unlike capitalists, workers perform labour and need a minimum amount of consumption in return.[16] This incorporates the idea that capitalists are not essential in production and, together with the assumption that $\omega_t^\eta = \mathbf{0}$, for all $\eta \in \mathcal{N}_w$ and all t, it starkly outlines differences between workers and capitalists.[17]

Although some aspects of behaviour are determined by socio-economic status, we rule out heterogeneity in preferences over consumption. Formally, there is a continuous, strictly increasing, strictly quasi-concave, and homogeneous of degree one function $\phi : \mathbb{R}_+^n \to \mathbb{R}_+$, such that $\phi(c_t^h)$ describes agent h's welfare at t, where $h = \nu, \eta$, and we normalise ϕ by assuming that $\phi(c_t^h) = 0$ whenever $c_{it}^h = 0$ for some good i.

Credit markets are incomplete and there is no intertemporal trade between agents.[18] This is consistent with the lack of a pure accumulation motive – that is, the desire to maximise capital accumulation per se, which is often assumed in Marxist models (e.g., Morishima [40]; Roemer [52]). In our model, capitalists do not aim to maximise accumulation of capital per se, and production does not take place for production's own sake. However, Roemer's [51, 53, 54] static economies are generalised by allowing for intertemporal trade-offs during an agent's life.

Let $(p, w) = \{p_t, w_t\}_{t=kT,\dots,(k+1)T-1}$ denote the path of the price vector during the lifetime of a generation. Let $y^\nu = \{y_t^\nu\}_{t=kT,\dots,(k+1)T-1}$ denote ν's lifetime plan of activity levels and let a similar notation hold for c^ν, s^ν, ω^ν, z^η, and c^η. As a shorthand notation, let "all t" stand for "all t, $t = kT, \dots, (k+1)T - 1$." Let $0 < \beta \leqq 1$ be the discount factor. Capitalist ν is assumed to choose $\xi^\nu = (y^\nu, c^\nu, s^\nu)$ to maximise lifetime welfare subject to the constraint that (1) net revenues (profits) are sufficient for consumption and savings, all t; (2) wealth is sufficient for production plans, all t; (3) the dynamics of assets is determined by savings, all t; and (4) ν's descendants receive at least as many resources as she inherited. Formally, agent $\nu \in \mathcal{N}_c$ solves the following maximisation programme (MP^ν), whose value is denoted as $C(\omega_{kT}^\nu)$.

$$MP^\nu\colon\ C(\omega_{kT}^\nu) = \max_{\xi^\nu} \sum_{t=kT}^{(k+1)T-1} \beta^t \phi\left(c_t^\nu\right),$$

subject to

$$\begin{aligned}
\left[p_t\left(I - A\right) - w_t L\right] y_t^\nu &\geqq p_t c_t^\nu + p_t s_t^\nu, &(1)\\
p_t A y_t^\nu &\leqq p_t \omega_t^\nu, &(2)\\
\omega_{t+1}^\nu &= \omega_t^\nu + s_t^\nu, &(3)\\
\omega_{(k+1)T}^\nu &\geqq \omega_{kT}^\nu, &(4)\\
y_t^\nu &\geqq \mathbf{0}, \omega_t^\nu \geqq \mathbf{0}, c_t^\nu \geqq \mathbf{0}.
\end{aligned}$$

[16]The reference vector b does not identify a *physical* subsistence bundle. Rather, we interpret it as a socially-determined basic consumption standard which must be reached in order for workers to supply labour in the capitalist sector (Banerjee and Newman [5]). We assume b to be constant over time, but the model can be generalised to incorporate a time-varying b_t reflecting evolving social norms, culture, and so on.

[17]In a less schematic model, if profits fall below some threshold, capitalists would start to work.

[18]As already noted, credit markets are also incomplete in the sense that, within each period, agents cannot activate production with external finance. As Roemer [51, 53, 54] has shown, however, this assumption yields no significant loss of generality.

Similarly, worker $\eta \in \mathcal{N}_w$ chooses $\xi^\eta = (z^\eta, c^\eta)$ to maximise welfare subject to the constraint that at all t: (5) revenues are sufficient for η's consumption; and (6) subsistence is reached. Furthermore, at all t, (7) workers' labour supply is constrained both by their labour endowment *and* by labour market conditions, as captured by the (from the individual's viewpoint) exogenously given parameter $\widehat{z}_t^\eta$. Formally, agent $\eta \in \mathcal{N}_w$ solves the following maximisation programme (MP^η).

$$MP^\eta\colon \max_{\xi^\eta} \sum_{t=kT}^{(k+1)T-1} \beta^t \phi\left(c_t^\eta\right),$$

subject to

$$w_t z_t^\eta \geqq p_t c_t^\eta, \tag{5}$$

$$c_t^\eta \geqq b, \tag{6}$$

$$\min\left[1, \widehat{z}_t^\eta\right] \geqq z_t^\eta \geqq 0. \tag{7}$$

The optimisation programmes MP^ν and MP^η allow us to investigate the relation between wealth inequalities, growth, and exploitation in a dynamic context. Given the absence of capital markets and of any explicit bequest motive,[19] they are a natural generalisation of Roemer's [51, 53, 54] static profit or revenue maximisation programmes.

Let $\Omega_{kT} = \left(\omega_{kT}^1, \omega_{kT}^2, ..., \omega_{kT}^{N_c}\right)$. Let $E\left(\left(\mathcal{N}_c, \mathcal{N}_w\right), (A, L), \Omega_{kT}, (\beta, \phi)\right)$, or as a shorthand notation E_{kT}, denote the economy with population $(\mathcal{N}_c, \mathcal{N}_w)$, technology (A, L), endowments Ω_{kT}, discount factor β and welfare function ϕ. At all t, let $y_t = \sum_{\nu \in \mathcal{N}_c} y_t^\nu$, $c_t^c = \sum_{\nu \in \mathcal{N}_c} c_t^\nu$, $\omega_t = \sum_{\nu \in \mathcal{N}_c} \omega_t^\nu$, $s_t = \sum_{\nu \in \mathcal{N}_c} s_t^\nu$, $c_t^w = \sum_{\eta \in \mathcal{N}_w} c_t^\eta$, and $z_t = \sum_{\eta \in \mathcal{N}_w} z_t^\eta$. Following Roemer [51, 53, 54], the equilibrium concept can now be defined.[20]

Definition 1: A *reproducible solution* (RS) for E_{kT} is a price vector (p, w) and an associated set of actions $\left((\xi^\nu)_{\nu \in \mathcal{N}_c}, (\xi^\eta)_{\eta \in \mathcal{N}_w}\right)$ such that:

(i) ξ^ν solves MP^ν for all $\nu \in \mathcal{N}_c$;
(ii) ξ^η solves MP^η for all $\eta \in \mathcal{N}_w$;
(iii) $y_t \geqq Ay_t + c_t^c + c_t^w + s_t$, for all t;
(iv) $Ay_t \leqq \omega_t$, for all t;
(v) $Ly_t = z_t$, for all t;
(vi) $\omega_{(k+1)T} \geqq \omega_{kT}$.

Conditions (i) and (ii) require agents to optimise given the individual *and* the aggregate constraints limiting their choices; (iii) and (iv) require that there be enough resources for consumption and saving plans, and for production plans, respectively, at all t; (v) states that the amount of labour performed in the economy must be sufficient for production plans at all t; (vi) requires that resources not be depleted by any given generation.

Definition 1 is an intertemporal extension of the concept of RS first defined by Roemer [51] and it provides a general notion of Marxian equilibrium (Veneziani and Yoshihara [70]). It is important to note that the concept of RS does not impose market clearing, and allows for an aggregate excess supply of produced goods, and, crucially, labour.[21] Thus, Definition 1(v) is an ex post condition consistent with the existence of involuntary unemployment. For, although workers

[19] Though constraint 4 can also be interpreted as incorporating some sort of social norm on bequests.

[20] If technical change occurs as described in section 6 below, then the economy is more precisely described as $E\left(\left(\mathcal{N}_c, \mathcal{N}_w\right), (A, L_{kT}, \delta), \Omega_{kT}, (\beta, \phi)\right)$ but everything else remains unchanged.

[21] This is consistent with the Classical economists' emphasis on the conditions for the "reproducibility" of the economic system, rather than market clearing. As Roemer ([51], p.507) put it, "The concern is with whether the economic system can reproduce itself: whether it can produce enough output to replenish the inputs used, and to reproduce the workers for another period of work."

choose their labour supply optimally and aggregate labour supply equals labour demand ex post, labour market conditions act as a constraint on workers' choices ex ante in condition (7). In fact, by the monotonicity of ϕ, in our framework, the standard labour market clearing condition at t requires $Ly_t = N_w$, whereas involuntary unemployment occurs at t whenever $Ly_t = z_t < N_w$.

Therefore, we say that a RS is *unconstrained* if $Ly_t = z_t = N_w$, for all t, while a RS is *constrained* at t' if there exists some t' such that $N_w > z_{t'} = Ly_{t'}$ and $w_{t'} z_{t'} = N_w p_{t'} b$. Because workers are identical, we assume that at a constrained RS, all of them work an equal amount of time which allows them to reach subsistence. Given the absence of a subsistence sector and of the public sector, this seems an appropriate way of capturing unemployment in this model. Moreover, we also focus on the minimal wage rate in the case of involuntary unemployment.[22] Thus, formally, if a RS is constrained at t', then $\widehat{z}_{t'}^{\eta} = \frac{Ly_{t'}}{N_w}$ and $c_{t'}^{\eta} = b$, all $\eta \in \mathcal{N}_w$.

Given the focus on the persistence of exploitation and profits, the subset of RSs with stationary capital will be of particular interest. A *stationary reproducible solution* (SRS) for E_{kT} is a RS such that, at all t, $c_t^{\nu} = c^{\nu}$ and $s_t^{\nu} = \mathbf{0}$, all $\nu \in \mathcal{N}_c$, and $c_t^{\eta} = c^{\eta}$, all $\eta \in \mathcal{N}_w$.

Definition 2 captures the idea of capital scarcity as requiring that the total supply of productive assets is limited, relative to current demand (Skillman [63]).[23]

Definition 2: Let (p, w) be a RS for E_{kT}. The economy E_{kT} is said to exhibit *capital scarcity* at (p, w), in period t, if and only if $p_t A y_t^{\nu} = p_t \omega_t^{\nu}$, all $\nu \in \mathcal{N}_c$. If $p_t A y_t^{\nu} < p_t \omega_t^{\nu}$, some $\nu \in \mathcal{N}_c$, then capital is said to be *abundant* at (p, w), in period t.

To be sure, Definition 2 is not the only way of defining capital scarcity. For example, one may argue that capital scarcity should be defined as involving the existence of a positive profit rate.[24] We do not think that this is a particularly appropriate, or indeed promising, way of conceptualising capital scarcity in the context of our analysis. Rather than building the existence of positive profits into the notion of capital scarcity, Definition 2 separates the two concepts and allows us to examine the relation between asset inequalities, capital scarcity, exploitation and profits.

4 Exploitation and profits

We begin our analysis by deriving some preliminary results concerning the properties of RS's. Two properties immediately follow from the monotonicity of ϕ. First, because at the solution to MP^{ν}, $\omega_{(k+1)T}^{\nu} = \omega_{kT}^{\nu}$, all $\nu \in \mathcal{N}_c$, if (p, w) is a RS for E_{kT}, then it is also a RS for $E_{(k+1)T}$. Hence, we can interpret (p, w) as a steady state solution and focus on E_0 without loss of generality. Second, at any RS, it must be $p_t > \mathbf{0}$ and $w_t > 0$, all t.

Then, it is immediate to show that at a RS for E_0, in every period, constraints (1) and (2) are binding for all capitalists.

Lemma 1: *Let (p, w) be a RS for E_0. Then, for all t:*
(i) $[p_t (I - A) - w_t L] y_t^{\nu} = p_t c_t^{\nu} + p_t s_t^{\nu}$*, all* $\nu \in \mathcal{N}_c$*;*
(ii) if $p_t \geq p_t A + w_t L$*, then* $p_t A y_t^{\nu} = p_t \omega_t^{\nu}$*, all* $\nu \in \mathcal{N}_c$*.*

Let the profit rate of sector i at t be denoted as $\pi_{it} = \frac{[p_t(I-A) - w_t L]_i}{p_t A_i}$. The next Lemma proves that at a RS, in every period profits are non-negative and profit rates are equalised across sectors.

Lemma 2: *Let (p, w) be a RS for E_0. Then, at all t, $\pi_{it} \geqq 0$, for at least some i. Furthermore, if either $\pi_{it} > 0$, some i, or $c_t^c + c_t^w + s_t > \mathbf{0}$*, then $\pi_{it} = \pi_t$, all i.

[22]Note that at a RS, if $N_w > z_{t'} = Ly_{t'}$ for some t' then this does not necessarily imply $w_{t'} z_{t'} = N_w p_{t'} b$. If $w_{t'} z_{t'} > N_w p_{t'} b$, then $c_{t'}^{\eta} \geq b$ holds for all $\eta \in \mathcal{N}_w$.

[23]To be precise, a RS should be denoted as $\left((p, w), ((\xi^{\nu})_{\nu \in \mathcal{N}_c}, (\xi^{\eta})_{\eta \in \mathcal{N}_w})\right)$. In what follows, we write (p, w) for notational simplicity.

[24]We are grateful to an anonymous referee for this suggestion.

Proof. 1. Suppose that there is some t such that $p_{it} < p_t A_i + w_t L_i$, all i. Then $y^{\nu}_{it} = 0$, all i, $\nu \in \mathcal{N}_c$, for all ξ^{ν} that solve MP^{ν}, and thus $y_t = 0$. By Definition 1(v), this implies $z^{\eta}_t = 0$, all $\eta \in \mathcal{N}_w$, which violates Definition 1(ii).

2. Suppose that there is some t such that either $\pi_{it} > 0$, some i, or $c^c_t + c^w_t + s_t > \mathbf{0}$, but $\pi_{lt} < \pi_{jt}$, some j, l. Because wealth is used only to activate maximum profit rate processes, it follows that for all $\nu \in \mathcal{N}_c$, $y^{\nu}_{lt} = 0$, for all ξ^{ν} that solve MP^{ν}, and thus $y_{lt} = 0$. However, under the hypotheses stated, $A_l y_t + c^c_{lt} + c^w_{lt} + s_{lt} > \mathbf{0}$, which contradicts Definition 1(iii). ■

By Lemma 2, at a SRS, $\pi_{it} = \pi_t$, all i and all t. More generally, at any RS such that $\pi_{it} = \pi_t$, all i, we can consider price vectors such that $p_t = (1 + \pi_t) p_t A + w_t L$, all t. Furthermore, labour can be chosen as the numéraire, setting $w_t = 1$, all t, and in what follows we focus on RS's of the form $(p, \mathbf{1})$, where $\mathbf{1} = (1, ..., 1)'$.

Let $\lambda = L(I - A)^{-1}$ be the $1 \times n$ vector of Leontief employment multipliers, measuring the amount of labour (directly and indirectly) necessary to produce goods. By the assumptions on A, $\lambda > \mathbf{0}$ and throughout the paper we shall assume that the economy is sufficiently productive to guarantee the reproduction of workers. Formally, $\lambda b < 1$.

Let $y = \sum_{t=0}^{T-1} y_t$ and $c^w = \sum_{t=0}^{T-1} c^w_t$. Generalising Roemer [52, 53], the amount of labour received by workers at t is defined as the amount of labour embodied, or contained, in their consumption, λc^w_t. Similarly, the amount of labour received by workers during their whole life is defined as λc^w. Then, Roemer's definition of UE exploitation can be extended to the intertemporal context.[25]

Definition 3: The within-period (WP) exploitation rate at t is $e_t = \frac{(L y_t - \lambda c^w_t)}{\lambda c^w_t}$ and the whole-life (WL) exploitation rate is $e = \frac{(L y - \lambda c^w)}{\lambda c^w}$.

Both definitions convey normatively relevant information and provide important insights on the foundations of inequalities in capitalist economies. Nonetheless, below we shall focus on the WP definition because it is more appropriate in the analysis of the relation between inequality and growth, and of the evolution of exploitation over time.[26]

Theorem 1 proves the existence of a robust correspondence between UE exploitation and the functional distribution of income. To discuss it, we introduce a reasonable restriction on the welfare function ϕ.

Assumption 1 (A1): All commodities are normal goods, and are gross complement to each other.

Then, we can have the following characterization.[27]

Theorem 1 *Let $(p, \mathbf{1})$ be a RS for E_0 with $\pi_{it} = \pi_t$, all i and all t. Then (i) at all t, $e_t > 0$ if and only if $\pi_t > 0$. Furthermore, (ii) $e > 0$ if and only if $\pi_t > 0$, some t. Finally, (iii) if the economy E_0 satisfies A1, then at each t, there is a monotone-increasing functional relationship $\pi(e_t) = \pi_t$.*

Theorem 1 proves that a correspondence exists between positive profits and the existence of UE exploitation in general dynamic economies with intertemporally optimising agents. Because the rate of profit is one of the key determinants of investment decisions, and of the long-run dynamics of capitalist economies, Theorem 1 can be interpreted as providing a link between exploitation and growth. From a normative perspective, Theorem 1 is important because it establishes a link

[25] With technical change, the definition of the WL exploitation rate would need to be adjusted accordingly.

[26] As argued in Veneziani [66], the WP approach is also more suitable to capture the structural features of exploitative relations. It is worth reiterating, however, that a focus on the WP definition does *not* mean that WL exploitation is normatively irrelevant. In our economy, the existence of WP exploitation implies WL exploitation, and the disappearance of WP exploitation after a certain period does not make economic relations normatively unobjectionable. We are grateful to an anonymous referee for raising this issue.

[27] The proofs of all theorems are in Appendix A.

between wealth inequalities, the functional distribution of income, and exploitation: given private ownership of productive assets, profits are a counterpart of the transfer of social surplus and social labour from asset-poor agents to the wealthy.

As mentioned in section 2, in the linear production model with multiple reproducible capital goods and a homogeneous labour, the perfectly competitive mechanism *alone* would identify a continuum of RSs but cannot determinate an equilibrium functional income distribution, due to Mandler [36] and Yoshihara and Kwak [75]. In this respect, Theorem 1(iii) suggests that the class struggle over the rate of exploitation can function as an equilibrium selection mechanism to fix an equilibrium profit rate and thereby select an associated RS from the continuum of RSs, because $\pi(e_t)$ is a monotonically increasing function.

Theorem 1 also suggests that there is no RS with persistent accumulation and persistent exploitation. In fact, if $e_t > 0$, all t, then by Theorem 1 and Lemma 1(ii), and noting that $p_t > \mathbf{0}$, at a RS $Ly_t = LA^{-1}\omega_t$, all t. By Definition 1(ii) and (v), it must therefore be $LA^{-1}\omega_t \leqq N_w$, all t. Hence, if $\omega_{t+1} > \omega_t$, all t, $T-1 > t \geqq 0$, then $LA^{-1}\omega_t < N_w$, and the RS is constrained at all t, $T-1 > t \geqq 0$. Therefore $c_t^\eta = b$, all η, and $p_t b = \frac{LA^{-1}\omega_t}{N_w}$, all t, $T-1 > t \geqq 0$. By Lemma 1(i), and noting that $p_t > \mathbf{0}$, at a RS $(I-A)y_t = s_t + c_t^c + c_t^w$, all t, which implies $s_t = (I-A)A^{-1}\omega_t - c_t^c - c_t^w$, all t, or by the previous arguments, $\omega_{t+1} = A^{-1}\omega_t - c_t^c - N_w b$, all t, $T-1 > t \geqq 0$.

Given the linearity of MP^ν, there is at most one period in which, for any $\nu \in \mathcal{N}_c$, at the solution to MP^ν, both savings and consumption are positive at a constrained RS with accumulation.[28] Hence, given that capitalists are identical there is a period τ such that $c_t^c = \mathbf{0}$, all $t \geqq \tau$, and $\omega_{t+1} = A^{-1}\omega_t - N_w b$, all $t \geqq \tau$, which implies $\omega_t = \left(A^{-1}\right)^{t-\tau}[\omega_\tau - \omega_S] + \omega_S$, all $t \geqq \tau$, where $\omega_S = N_w A(I-A)^{-1}b$. Thus, by the productivity of A, given that workers' subsistence requires $\omega_t \geqq \omega_S$, all t, if T is sufficiently big, labour demand exceeds supply after a finite number of periods, driving π_t and e_t to zero. This can be summarised as follows.

Proposition 1: *For all $T > 0$, there is a $T' > T$ such that there is no RS with $\omega_{t+1} > \omega_t$, all t, $T'-1 > t \geqq 0$, and $e_t > 0$, all t.*

In other words, persistent accumulation and persistent exploitation and profits are inconsistent. At a broad conceptual level, this conclusion echoes Kalecki's [29] famous argument about capitalists' negative attitudes towards policies that promote growth and full employment. For Proposition 1 suggests that, absent any countervailing factors, capitalists will be *collectively* concerned with any long-run sustained accumulation that may significantly reduce capital scarcity, even though *individually* they may regard growth paths favourably.

The intuition behind Proposition 1 is simple. In our linear economy, *if* capitalists had a persistent incentive to accumulate, this would yield a continuous increase in the demand for labour which, provided the time horizon is sufficiently long, would eventually exceed the fixed labour supply making capital abundant and leading profits and exploitation to zero. It is important to note, however, that Proposition 1 does *not* prove that there are growth paths in which profits and exploitation disappear in equilibrium, because the anticipated fall in the profit rate might discourage rational agents from investing.

In the next sections, we explore optimal behaviour and the equilibrium dynamics of the economy.

5 Inequalities, Exploitation, and Time Preference

This section analyses the dynamic foundations of exploitative relations, focusing on stationary reproducible solutions. This is due to the theoretical relevance of SRS's, as discussed in Veneziani [66], but also because they represent a benchmark solution whereby the labour market may clear at all t. Lemma 3 provides a necessary condition for the existence of a SRS.

[28]This is proved rigorously below; see e.g. the analysis of MP^ν in the proof of Theorem 4.

Lemma 3: Let $(p, \mathbf{1})$ be a SRS for E_0 with $\pi_t > 0$, all t. Then $\beta(1+\pi_{t+1}) = 1$, all t.

Proof. 1. For all $\nu \in \mathcal{N}_c$, by Lemma 1, at any RS with $\pi_t > 0$, all t, it must be $p_t c_t^\nu = \pi_t p_t \omega_t^\nu - p_t s_t^\nu$, all t. At a SRS, the latter expression becomes $p_t c^\nu = \pi_t p_t \omega_0^\nu$, all t, ν, which implies $c^\nu \geq \mathbf{0}$.

2. Suppose, by way of contradiction, that $\beta(1+\pi_{t'+1}) > 1$, some $t' < T-1$. Take any capitalist $\nu \in \mathcal{N}_c$. Consider a one-period perturbation of ν's optimal choice such that $p_{t'} \mathrm{d}c_{t'}^\nu = -p_{t'} \mathrm{d}s_{t'}^\nu$, $p_{t'+1} \mathrm{d}c_{t'+1}^\nu = \pi_{t'+1} p_{t'+1} \mathrm{d}\omega_{t'+1}^\nu - p_{t'+1} \mathrm{d}s_{t'+1}^\nu$, $\mathrm{d}\omega_{t'+1}^\nu = \mathrm{d}s_{t'}^\nu = -\mathrm{d}s_{t'+1}^\nu$.

3. Since ϕ is homothetic, $c_t^\nu = c^\nu$ implies that at a SRS, at all t it must be $p_{t+1} = k_t p_t$ for some $k_t > 0$. Therefore consider $\mathrm{d}c_{t'}^\nu = h_{t'} c^\nu$ and $\mathrm{d}c_{t'+1}^\nu = h_{t'+1} c^\nu$ for some $h_{t'}, h_{t'+1} \geq 0$, and the one period perturbation can be written as $h_{t'} p_{t'} c^\nu = -p_{t'} \mathrm{d}s_{t'}^\nu$ and $h_{t'+1} p_{t'} c^\nu = \pi_{t'+1} p_{t'} \mathrm{d}s_{t'}^\nu + p_{t'} \mathrm{d}s_{t'}^\nu$.

4. By the homogeneity of ϕ it follows that $\phi\left(c^\nu + \mathrm{d}c_{t'}^\nu\right) + \beta\phi\left(c^\nu + \mathrm{d}c_{t'+1}^\nu\right) = (1+h_{t'})\,\phi\left(c^\nu\right) + (1+h_{t'+1})\,\beta\phi\left(c^\nu\right) > \phi\left(c^\nu\right) + \beta\phi\left(c^\nu\right)$ if and only if $h_{t'} + h_{t'+1}\beta = \left[-1 + \beta\left(1+\pi_{t'+1}\right)\right] \frac{p_{t'}}{p_{t'} c^\nu} \mathrm{d}s_{t'}^\nu > 0$. Therefore, if $\beta(1+\pi_{t'+1}) > 1$, there is a sufficiently small $\mathrm{d}s_{t'}^\nu$ with $p_{t'} \mathrm{d}s_{t'}^\nu > 0$ such that $h_{t'} + h_{t'+1}\beta > 0$, a contradiction. A similar argument holds if $\beta(1+\pi_{t'+1}) < 1$, some $t' < T-1$. ∎

Intuitively, if $\beta(1+\pi_{t'+1}) > 1$, some t', then the cost (in terms of overall welfare) of reducing consumption at t' is lower than the benefit of saving, producing and consuming in $t'+1$, and vice versa if $\beta(1+\pi_{t'+1}) < 1$. Only if $\beta(1+\pi_{t'+1}) = 1$ are costs and benefits equal.

Let $\frac{1}{1+\widetilde{\pi}}$ be the Frobenius eigenvalue of A: by the assumptions on A, $\widetilde{\pi} > 0$. Let $\pi_\beta \equiv \frac{1-\beta}{\beta}$ and let p_β denote the solution of $p = (1+\pi_\beta)pA + L$: for all $\pi_\beta \in [0, \widetilde{\pi})$, p_β is well defined and strictly positive. By the homotheticity of ϕ, let c_β denote a vector identifying the optimal proportions of the different consumption goods corresponding to p_β.[29] Theorem 2 analyses MP^ν.[30]

Theorem 2 *(i) Let $1 > \beta > \frac{1}{1+\widetilde{\pi}}$. If $\pi_t = \pi_\beta$, all t, then for all $\nu \in \mathcal{N}_c$ there is an optimal ξ^ν such that $s_t^\nu = \mathbf{0}$, all t. Moreover, if T is finite, $C(\omega_0^\nu) = \phi(c_\beta)(1-\beta^T)\frac{p_\beta \omega_0^\nu}{\beta p_\beta c_\beta}$, while if $T \to \infty$, $C(\omega_0^\nu) = \phi(c_\beta)\frac{p_\beta \omega_0^\nu}{\beta p_\beta c_\beta}$. (ii) Let $\beta \leqq 1$. If $\pi_t = 0$, all t, then for all $\nu \in \mathcal{N}_c$ there is an optimal ξ^ν such that $s_t^\nu = \mathbf{0}$, all t, and $C(\omega_0^\nu) = 0$.*

Consider a subset of the set of conceivable aggregate endowments of productive assets, namely vectors of the form $\omega_0 = \gamma_0 N_w A(I-A)^{-1}b$, where γ_0 is a positive real number capturing the abundance of aggregate capital relative to the subsistence requirements of workers. If $\gamma_0 = 1$, then aggregate endowments are barely sufficient to guarantee the subsistence of all workers and it is not difficult to show that the only RS for E_0 requires $\pi_t = 0$ and $s_t = \mathbf{0}$, all t.

Let π' be defined by $\gamma_0 \lambda b = L[I-(1+\pi')A]^{-1}b$: π' is the profit rate such that a worker supplying an amount of labour $\gamma_0 \lambda b$ can purchase the subsistence bundle b at prices $p' = L[I-(1+\pi')A]^{-1}$.

Given Theorem 2, the next result proves the existence of a SRS.[31]

Theorem 3 *Let $\omega_0 = \gamma_0 N_w A(I-A)^{-1}b$, $\gamma_0 > 1$. Let π' be defined by $\gamma_0 \lambda b = L[I-(1+\pi')A]^{-1}b$.*

(i) Let $\gamma_0 \lambda b < 1$. If $\beta(1+\pi') = 1$ and $c_\beta = kb$ for some $k > 0$, there is a SRS for E_0 with $\pi_t = \pi'$, all t;

(ii) Let $\gamma_0 \lambda b = 1$. Let $\beta \in [\frac{1}{(1+\pi')}, 1)$ be such that $c_\beta = kb$ for some $k > 0$. Then there is a SRS for E_0 with $\pi_t = \pi_\beta$, all t;

(iii) Let $\gamma_0 \lambda b \leqq 1$. If $\beta = 1$, there is a SRS for E_0 such that at all t, $\pi_t = 0$ and $p_t A y_t^\nu = p_t \omega_t^\nu$, all $\nu \in \mathcal{N}_c$. Further, there is no SRS with $\pi_t > 0$, some t.

[29] The vector c_β is determined up to a scalar transformation. If ϕ_i' denotes the partial derivative of ϕ with respect to the $i-$th entry, then $\frac{\phi_i'(c_\beta)}{\phi_j'(c_\beta)} = \frac{p_{i\beta}}{p_{j\beta}}$, for all i, j.

[30] In the case with $\pi_t = 0$, all t, Theorem 2 does not rule out the possibility that for some $\nu \in \mathcal{N}_c$, $s_t^\nu \neq \mathbf{0}$, for some t, at the solution to MP^ν. However, for all $\nu \in \mathcal{N}_c$ at any ξ^ν that solves MP^ν, it must be $\lambda s_t^\nu = 0$, all t.

[31] The restriction $\omega_0 = \gamma_0 N_w A(I-A)^{-1}b$ is necessary given the linearity of MP^ν and MP^η. No theoretical conclusion depends on this restriction, which in any case encompasses a rather large set of economies.

Remark 1 *By Lemma 3, Theorem 3(i)-(ii) identify the only class of SRS's with $\pi_t > 0$ all t.*

Theorem 3 significantly strengthens and extends the results in Veneziani [66]. Consider economies in which aggregate initial assets are above the minimum barely sufficient to guarantee workers' subsistence ($\gamma_0 > 1$), but below the level that would make aggregate labour demand higher than aggregate labour supply ($\gamma_0 \lambda b \leqq 1$). On the one hand, Theorem 3(i)-(ii) state that the dynamic economy with maximising agents displays persistent exploitation – and possibly persistent unemployment, – if profits are consumed at all t *and* capitalists discount the future.[32] On the other hand, however, this result crucially depends on a strictly positive rate of time preference (Theorem 3(iii)): if capitalists do not discount the future, then there exists no stationary equilibrium with positive profits. There exists, instead, a stationary equilibrium with no profit (and no exploitation) in which capital is scarce in the sense of Definition 2. Further, if $\gamma_0 \lambda b = 1$, the magnitude of inequalities and exploitation will also depend on β.[33]

This suggests that Roemer's [53, 54] claim that a differential distribution of (scarce) property and competitive markets are sufficient institutions to generate persistent UE exploitation may need to be at least be qualified, and it is unclear that UE exploitation can be *reduced* to a focus on asset inequalities. For Theorem 3 proves that, absent time preference, exploitation is not a persistent feature of a competitive economy at a stationary RS, *even if* wealth inequalities endure and capital remains scarce in the sense of Definition 2. An exclusive focus on the differential ownership of (scarce) productive assets seems therefore insufficient to explain the origins of persistent UE exploitation, and asset inequalities alone are not all that matters when evaluating capitalist economies from a normative perspective. Something else is indispensable to generate persistent UE exploitation, which is therefore normatively as important as asset inequalities themselves.

In the next section, we explore further the foundations of persistent UE exploitation, going beyond the stationary equilibria considered in Theorem 3, and examine the relation between growth and inequalities in the distribution of income, and in the exchange of labour.

6 Stable Growth and Distribution

In this section, in order to focus on the key theoretical issues and on macrodynamics, we consider a special case of the n-good economies analysed thus far by setting $n = 1$. The model and notation remain the same, with obvious adaptations and letting ϕ be the identity function.[34] Further, we restrict our attention to the empirically relevant case of economies in which T can be arbitrarily large but remains finite.

Sections 4-5 suggest that asset inequalities (and competitive markets) alone cannot fully explain exploitative relations in dynamic capitalist economies. Persistent growth and exploitation are inconsistent and even if the economy does not grow, persistent exploitation is possible only if $\beta < 1$. This section explores further the relation between exploitation, time preference, and growth, by focusing on stable growth paths in which the economy grows for a certain number of periods and eventually reaches a steady state.[35]

[32] Observe that the proof of existence in cases (i) and (ii) of the theorem are premised on the condition that $c_\beta = kb$ for some $k > 0$. This condition on the agents' optimal consumption bundle is due to the linearity of the model and the presence of a subsistence constraint. None of our conclusions depend on this restriction, and the existence of a SRS can be proved under more general assumptions although at a cost of a significant increase in technicalities.

[33] Theorems 2-3 also characterise inter-capitalist inequalities as a different phenomenon from exploitation. In fact, at a SRS with $\pi_t = \frac{1-\beta}{\beta} > 0$, all t, by Theorem 2 for any two capitalists ν and μ, $C(\omega_0^\nu) > C(\omega_0^\mu)$ if and only if $p_\beta \omega_0^\nu > p_\beta \omega_0^\mu$. Instead, if $\pi_t = 0$, all t, then $C(\omega_0^\nu) = 0$, all ν.

[34] The main conclusions of this section can be extended to n-good economies, albeit at the cost of a significant increase in technicalities. The key definitions and propositions are formulated so as to suggest the relevant n-good extensions.

[35] The notion of stability here does not refer to the concept of asymptotic stability in dynamical systems. The growth paths in Definition 4 are stable in the sense that the economy eventually reaches a stationary state. Observe

Definition 4: A *stable growth path* (SGP) for E_0 is a RS such that there is a period $t' > 0$ such that $\omega_{t+1} = (1 + g_t)\omega_t$, $g_t > 0$, for all $t < t'$, and $\omega_{t+1} = \omega_t$, all $t, T - 1 > t \geqq t'$.

For all t, let $\omega_t = \gamma_t N_w A(1 - A)^{-1}b$, so that any conditions on aggregate capital ω_t can be equivalently expressed as conditions on γ_t. Lemma 4 confirms the relevance of SRS's as a theoretical benchmark: only at a SRS can equilibrium in the labour market and exploitation exist at all t.

Lemma 4: If $(p, \mathbf{1})$ is an unconstrained RS for E_0 such that the economy exhibits capital scarcity at t, then $\gamma_t \lambda b = 1$.

Proof. At a RS with capital scarcity at t, it must be $y_t = A^{-1}\omega_t$. Therefore, $Ly_t = \gamma_t N_w \lambda b$, and since the RS is unconstrained, $Ly_t = z_t = N_w$, which holds if and only if $\gamma_t \lambda b = 1$. ■

In general, if a RS is unconstrained from t' onwards, then $\gamma_t \lambda b = 1$, all $t \geqq t'$, and thus SRS's are a natural benchmark for all accumulation paths with persistent capital scarcity, which lead to a stationary state with equilibrium in the labour market. Instead, if $\gamma_t \lambda b < 1$, the economy is constrained at t. Proposition 2 rules out paths where capital becomes abundant.

Proposition 2: Let $\omega_0 = \gamma_0 N_w A(1 - A)^{-1}b$, $\gamma_0 > 1$, and $\gamma_0 \lambda b \leqq 1$. Suppose $\beta < 1$. Then there is no RS such that there exists a period $\widehat{t}$ such that the economy exhibits capital scarcity at all $t \leqq \widehat{t}$ but $LA^{-1}\omega_{\widehat{t}+1} > N_w$.

Proof: 1. Suppose that there is a RS such that $LA^{-1}\omega_{\widehat{t}} \leqq N_w$ but $LA^{-1}\omega_{\widehat{t}+1} > N_w$, some $\widehat{t}$. Then $\pi_{\widehat{t}} > 0$ but $\pi_{\widehat{t}+1} = 0$ since capital is abundant at $\widehat{t} + 1$.

2. For all $\nu \in \mathcal{N}_c$, $c^{\nu}_{\widehat{t}} = \pi_{\widehat{t}}\omega^{\nu}_{\widehat{t}} - s^{\nu}_{\widehat{t}}$ and $c^{\nu}_{\widehat{t}+1} = -s^{\nu}_{\widehat{t}+1} \geqq 0$. If $s^{\nu}_{\widehat{t}+1} < 0$, some $\nu \in \mathcal{N}_c$, then since $\beta\left(1 + \pi_{\widehat{t}+1}\right) < 1$, there is a feasible perturbation of the savings path with $\mathrm{d}s^{\nu}_{\widehat{t}} = -\mathrm{d}s^{\nu}_{\widehat{t}+1} < 0$, which increases ν's welfare, contradicting optimality.

3. Let $s^{\nu}_{\widehat{t}+1} = 0$, all $\nu \in \mathcal{N}_c$. Since $s_{\widehat{t}+1} = 0$ then $\omega_{\widehat{t}+2} = \omega_{\widehat{t}+1}$, so that $\pi_{\widehat{t}+2} = 0$ and $\beta\left(1 + \pi_{\widehat{t}+2}\right) < 1$. Again, for all $\nu \in \mathcal{N}_c$, $s^{\nu}_{\widehat{t}+2} < 0$ cannot be optimal. Therefore $s^{\nu}_{\widehat{t}+2} = 0$, all $\nu \in \mathcal{N}_c$, and $\pi_{\widehat{t}+3} = 0$; and so on.

4. By construction, $\omega_{\widehat{t}+1} > \omega_0$. Hence, individual optimality implies $\sum_{l=\widehat{t}+1}^{T-1} s^{\nu}_l < 0$, all $\nu \in \mathcal{N}_c$, which contradicts $s^{\nu}_l = 0$, for all $\nu \in \mathcal{N}_c$ and all $T - 1 \geqq l \geqq \widehat{t} + 1$. ■

Proposition 2 shows that overaccumulation is not an equilibrium because the fall of the profit rate to zero would rather lead capitalists to anticipate consumption, if $\beta < 1$. Thus, it confirms the importance of time preference for the persistence of exploitation in Roemer's theory: if $\beta = 1$, overaccumulation and profits falling to zero are not ruled out.

Given Proposition 2, Theorem 4 characterises stable growth paths.

Theorem 4 *Let $\omega_0 = \gamma_0 N_w A(1-A)^{-1}b$ and $\gamma_0 > 1$. Let $(p, \mathbf{1})$ be a SGP for E_0 such that $\gamma_t \lambda b \leqq 1$, all t. At all t, define $g'_t = \frac{[(\gamma_t - 1)N_w b - c^c_t]\widetilde{\pi}}{\gamma_t N_w b}$. Then:*

(i) $\omega_{t+1} = (1 + g'_t)\omega_t$, all $t < t'$, and $p_{t+1} = (1 + g'_t)p_t$, all $t < t' - 1$. Furthermore, if $\beta < 1$ then $g'_t = \pi_t$, all $0 < t < t' - 1$, while if $\beta = 1$ then $g'_t = \pi_t$, all $t < t' - 1$.

(ii) If $\beta < 1$ and $\pi_t > 0$, all t, $T - 2 \geqq t \geqq t'$, then $\beta(1 + \pi_{t+1}) = 1$, all t, $T - 2 \geqq t \geqq t'$. If $\beta = 1$, there is no t, $T - 2 \geqq t \geqq t'$, such that $\pi_t > 0$ and $\pi_{t+j} > 0$, some $j > 0$.

In other words, at any SGP, prior to t', the economy accumulates at the maximal rate, which coincides with the profit rate, and commodity prices grow at the same pace. As p_t and the profit rate increase, the growth rate of the economy is not constant over time. Except for period $t = 0$ the behaviour of the economy in the first phase is the same regardless of whether agents discount

that if $T = 2$, then at any SGP the condition in the second part of Definition 4 is vacuously satisfied.

the future or not (Theorem 4(i)). After t', the economy is stationary: if $\beta < 1$ then the profit rate is positive and equal to the value identified in Theorem 3(ii). If $\beta = 1$ then the only profit rate consistent with a stationary path of capital after t' is $\pi_t = 0$ (Theorem 4(ii)).

At a general level, Theorem 4 provides a clear link between the growth rate and the profit rate: profits are the engine of growth and to the extent that profitability, and the functional distribution of income, are related to income inequalities, Theorem 4 suggests that inequalities are indeed a necessary condition for economic growth in capitalist economies.[36] Further, as in Classical models, Theorem 4 establishes a negative relation between capitalist consumption and growth (for a given workers' subsistence bundle b), and g_t' can be shown to coincide with the growth rate of von Neumann-Sraffa models (e.g., von Neumann [71]; Kurz and Salvadori [34] p.102ff).

The next result characterises capitalists' optimal saving paths with accumulation.

Theorem 5 *Let $(p, \mathbf{1})$ be such that $\pi_t > \pi_\beta$, all $t \leqq \tau$, and $\pi_t = \pi_\beta$, all $T-1 \geqq t \geqq \tau+1$, for some τ, $T-1 \geqq \tau \geqq 0$. Then, for all $\nu \in \mathcal{N}_c$: (i) $\omega_{t+1}^\nu = (1+\pi_t)\omega_t^\nu$, all $t \leqq \tau - 1$, $\omega_{t+1}^\nu = (1+g_t)\omega_t^\nu$, all $g_t \in [0, \pi_\beta]$, all t, $T-2 \geqq t \geqq \tau$, and $\omega_T^\nu = \omega_0^\nu$, is optimal, and (ii) $C(\omega_0^\nu) = \left[\beta^\tau \Pi_{i=0}^\tau (1+\pi_i) - \beta^{T-1}\right]\omega_0^\nu$.*

Let π' be defined by $1 = L[1-(1+\pi')A]^{-1}b$: π' is the profit rate such that a worker supplying one unit of labour would be able to purchase the subsistence bundle b at prices $p' = L[1-(1+\pi')A]^{-1}$.

Recall that by definition the maximum profit rate that can be obtained is given by $\widetilde{\pi} = \frac{1-A}{A}$. Let the sequence $\{\overline{\gamma}_\tau\}_{\tau=0}^{T-1}$ be given by $\overline{\gamma}_0 = \frac{1}{\lambda b}$ and $\overline{\gamma}_{\tau+1} = \frac{(\overline{\gamma}_\tau + \widetilde{\pi})}{1+\widetilde{\pi}}$: given $\lambda b < 1$, the sequence is monotonically decreasing and it defines a corresponding sequence of intervals $[\overline{\gamma}_{\tau+1}, \overline{\gamma}_\tau)$, for all τ. By the productivity of A, the size of the intervals $[\overline{\gamma}_\tau, \overline{\gamma}_{\tau-1})$ decreases with τ and tends to zero, with $\overline{\gamma}_\tau \to 1$ as $\tau \to \infty$. The sequence of intervals $[\overline{\gamma}_{\tau+1}, \overline{\gamma}_\tau)$ identifies a partition of the theoretically relevant set of aggregate productive endowments, given by the interval $\left(1, \frac{1}{\lambda b}\right]$.

Theorem 6 proves the existence of a SGP.

Theorem 6 *Let $\beta \in (\frac{1}{(1+\pi')}, 1]$, $\omega_0 = \gamma_0 N_w A(1-A)^{-1}b$, and $\gamma_0 > 1$. If $\gamma_0 \in [\overline{\gamma}_{\tau+1}, \overline{\gamma}_\tau)$ and $\overline{\gamma}_\tau > \frac{\beta\widetilde{\pi}}{\beta(1+\widetilde{\pi})-1}$, with $\tau \geqq 1$, then the vector $(p, \mathbf{1})$ with $\pi_t = \frac{\widetilde{\pi}(\gamma_t - 1)}{\gamma_t}$, all t, $\tau \geqq t \geqq 0$, with $\gamma_{t+1} = (1+\pi_t)\gamma_t$, all $t \leqq \tau - 1$, and $\pi_t = \pi_\beta$, all t, $T-1 \geqq t \geqq \tau+1$, is a SGP for E_0 with $\omega_{t+1} = (1+\pi_t)\omega_t$, all $t \leqq \tau - 1$, $\omega_{\tau+1} = (1+g_\tau)\omega_\tau$, with $g_\tau \in (0, \pi_\tau]$, and $\omega_{t+1} = \omega_t$, all t, $T-1 \geqq t \geqq \tau+1$.*

By Theorem 6, if initial aggregate endowments are greater than the minimum amount necessary to guarantee workers' subsistence ($\gamma_0 > 1$), then the existence of a SGP can be proved for a range of values of the rate of time preference ($\beta \in (\frac{1}{(1+\pi')}, 1]$). In equilibrium, there exists a period τ such that the economy accumulates and reaches the steady state in τ periods. The value of τ is determined by the initial amount of aggregate capital: if aggregate endowments are such that $\gamma_0 \in [\overline{\gamma}_{\tau+1}, \overline{\gamma}_\tau)$, then it takes τ periods for the economy to reach the steady state.

The dynamic path of the economy can thus be divided into two parts: in the first τ periods, capital grows at the maximum rate, which coincides with the profit rate ($\gamma_{t+1} = (1+\pi_t)\gamma_t$, or, equivalently, $\omega_{t+1} = (1+\pi_t)\omega_t$). By construction, if $\gamma_0 \in [\overline{\gamma}_{\tau+1}, \overline{\gamma}_\tau)$, then $\gamma_1 \in [\overline{\gamma}_\tau, \overline{\gamma}_{\tau-1})$, and so on. In turn, in each period t, the profit rate, and therefore production prices, are determined by aggregate endowments ($\pi_t = \frac{\widetilde{\pi}(\gamma_t - 1)}{\gamma_t}$). Thus, as aggregate capital grows, profits and labour expended increase over time. Yet, by construction $\gamma_t \lambda b < 1$ for all periods $t \leqq \tau - 1$, there is an excess supply of labour, and workers' consumption remains at the subsistence level.[37]

[36] Theorem 4 is reminiscent of the so-called *Cambridge equation* according to which $g_t = s\pi_t$ since, at least in some periods, the capitalists savings rate is equal to one and the growth rate is equal to the profit rate.

[37] Observe that in the first part of the SGP, the economy displays positive profits and accumulates at the maximal rate even if $\gamma_0 \lambda b \leq 1$ and $\beta = 1$. This is consistent with Theorem 3(iii), which rules out positive profits (and exploitation) at stationary equilibria if $\beta = 1$.

Once the steady state is reached, however, full employment prevails ($\gamma_t \lambda b = 1$), profits remain constant at the value that makes capitalists indifferent between accumulating or decumulating ($\pi_t = \pi_\beta$), and workers' consumption exceeds subsistence.[38] If $\beta < 1$, UE exploitation is a persistent phenomenon; if $\beta = 1$, it disappears. By construction, in every period (both when the economy accumulates, and at the steady state), all capitalists use up their entire wealth.

These results confirm the main conclusions of section 5. Only if $\beta < 1$ can overaccumulation – leading to the disappearance of UE exploitation – be ruled out in equilibrium (Proposition 2).[39] Moreover, if $\beta = 1$, inequalities, UE exploitation, and profits may well disappear after a finite number of periods, both at a SRS (Theorem 3) and at a SGP (Theorem 4), *even if capital remains scarce* in the sense of Definition 2. Instead, if agents discount the future, inequalities and UE exploitation can be persistent even in paths with capital accumulation (Theorem 4). The crucial role of time preference, as opposed, e.g., to capital scarcity, is further confirmed by the fact that if $\beta < 1$, then the steady state value of the profit rate (and thus, by Theorem 1, the rate of UE exploitation) is a positive function of β (Theorem 4(ii)).

These results suggest that the role of asset inequalities (and competitive markets) in generating persistent inequalities and persistent UE exploitation in accumulating economies should be reconsidered, *and* they raise some doubts on the claim that exploitation can be reduced – either positively or normatively – to a purely distributive phenomenon.

One may object that Theorems 3 and 4 actually prove that time discounting by capitalists is the missing ingredient necessary to guarantee the persistence of exploitation and inequalities, including in economies that accumulate and eventually reach a steady state. From this perspective, Theorem 3 and 4 can actually be seen as providing a dynamic generalisation of Roemer's theory: provided agents discount the future, asset inequalities and competitive markets generate persistent UE exploitation.

Although this interpretation is certainly legitimate, we are not convinced that an explanation of persistent inequalities (in income and in the exchange of labour) that crucially relies on a strictly positive rate of time preference is the most promising or theoretically satisfactory. Furthermore, as argued in detail in Veneziani [66, 67], if exploitation theory is understood as a distinctive approach in normative economics, then the significance of a purely subjective factor such as time preference is even less obvious both in general (see, for example, the classic analysis by Rawls [50]), and specifically in the context of exploitation theory. At the very least, we would argue, it is worth exploring alternative explanations, focusing on the structural features of capitalist economies.

In the rest of this section, we move a first step in this direction and consider the interaction between technical change *and* labour market conditions and institutions in shaping distributive outcomes and the equilibrium growth paths of capitalist economies.

To be specific, we consider the role of technical change and unemployment in creating the conditions for inequalities and exploitation to persist. As Dosi et al ([15], p.164) have forcefully noted, "Unemployment is a persistent and structural phenomenon of capitalist economies" of such relevance that "other possible stylized facts are in comparison second order ones". Theoretically, in Marx, unemployment is seen as a structural feature of capitalism, whose role is to discipline workers and to restrain wages from rising and, in turn, labour-saving technical change plays a key role in guaranteeing the persistence of a reserve army of the unemployed by increasing labour productivity.

In our model, the disappearance of exploitation derives from an initial excess supply of labour which is rapidly absorbed owing to accumulation. The introduction of labour saving technical progress should avoid this: by increasing labour productivity, technical progress may allow labour supply to be persistently higher than labour demand.[40]

[38] Given $\gamma_0 > 1$, this follows from the assumption that $\beta > \frac{1}{(1+\pi')}$, which implies that $p_\beta b < 1$ and therefore workers can consume above subsistence.

[39] Thus, Devine and Dymsky's [14] result can only be an equilibrium if $\beta = 1$.

[40] The relevance of exogenous growth in the labour force, heterogeneous preferences, and/or labour-saving technical

To be specific, we take "the Schumpeterian view that the creation of new ideas largely occurs at an autonomous rate" (Shell [62], p.67) and assume that the amount of labour directly needed in production declines geometrically over time.[41]

Assumption 2 (A2): At all t, $L_{t+1} = \delta L_t$, $\delta \in (0, 1)$, with $L_0 > 0$ given.

Under (A2), all the results in Section 4 hold, once L_t is substituted for L. Then, Theorem 7 provides sufficient conditions for the existence of a RS with persistent exploitation.

Theorem 7 *Assume (A2). Let $\omega_0 = \gamma_0 N_w A(1-A)^{-1} b$, $\gamma_0 > 1$, and $\gamma_0 \lambda_0 b \leqq 1$. Let $\delta(1+\widetilde{\pi}) \leqq 1$ and $\beta[1 + \frac{\widetilde{\pi}(\gamma_0 - 1)}{\gamma_0}] \geqq 1$. The price vector $(p, \mathbf{1})$ with $\pi_0 = \frac{\widetilde{\pi}(\gamma_0-1)}{\gamma_0}$ and $\pi_{t+1} = \frac{\pi_t(1+\widetilde{\pi})}{(1+\pi_t)}$, all t, $T-2 \geqq t \geqq 0$, is a RS for E_0 with $L_t y_t < N_w$, all $t > 0$, and $\omega_{t+1} = (1+\pi_t)\omega_t$, all t, $T-2 \geqq t \geqq 0$.*

Theorem 7 provides a complete characterisation of dynamic general equilibrium paths with sustained growth *and* persistent – indeed, increasing – UE exploitation and inequality. Consider economies with aggregate initial assets above the minimum barely sufficient to guarantee workers' subsistence ($\gamma_0 > 1$), but below the level that would make aggregate labour demand higher than aggregate labour supply ($\gamma_0 \lambda_0 b \leqq 1$). If technical change is strong enough ($\delta(1+\widetilde{\pi}) \leqq 1$) and agents are not too impatient ($\beta[1 + \frac{\widetilde{\pi}(\gamma_0-1)}{\gamma_0}] \geqq 1$) then there exists an equilibrium with positive – indeed, increasing – profits, and by Theorem 1, UE exploitation in every period, as $\gamma_0 > 1$ implies $\pi_0 > 0$ and $\widetilde{\pi} > \pi_t$, all t, implies $\pi_{t+1} > \pi_t$, all t. Along the equilibrium path, the economy grows at the maximal rate, π_t, but labour demand remains lower than labour supply ($L_t y_t < N_w$).

Theorem 7 highlights an interesting mechanism that may contribute to the persistence of exploitation and inequalities in capitalist economies. For it shows that labour-saving technical progress allows the economy to settle on a "golden rule" growth path with persistent exploitation even if $\beta = 1$. The increase in labour productivity – a long run historical tendency of capitalist economies (Acemoglu [1]) – ensures that labour remains in excess supply even along a growth path with maximal accumulation, thus countering all tendencies for profits and exploitation to disappear. Indeed, as the nominal wage rate is normalised to one and employment $L_t y_t$ either does not grow, or grows more slowly than national income $p_t(1-A)y_t$, Theorem 7 implies that the wage share in national income, $\frac{L_t y_t}{p_t(1-A)y_t}$, tends to fall steadily over time tending to zero in the long run as in Mookherjee and Ray [39].

7 Conclusion

In this paper, an intertemporal model with heterogeneous optimising agents is set up to analyse the relation between inequalities – in income, wealth and labour exchanged, – and accumulation. We have argued that the concept of exploitation as the unequal exchange of labour is well defined, and it can be interpreted as capturing normatively relevant inequalities in well-being freedom. We have generalised the well-known correspondence between the existence of exploitation and positive profits to the dynamic economy. This provides a link between exploitation and the functional income distribution, but it also suggests that inequalities (both in wealth holdings and in the exchange of labour) are necessary for accumulation in capitalist economies.

We have also explored the mechanisms underlying the persistence of inequalities and exploitation in accumulating economies, and the relation between asset inequalities and exploitation. We

progress in making exploitation persistent is stressed by Skillman [63]. Observe that, for the sake of simplicity, and without significant loss of generality we do not assume that technical change is capital using, in addition to being labour saving, and thus the capital coefficient A remains constant.

[41] In other words, we are considering technology as "stemming from advances in science or from the behavior of entrepreneurs driven by a variety of nonprofit motives" (Acemoglu [2], p.11). This is just to focus on *one* key channel through which technical change affects distributive outcomes and growth, namely by preserving capital scarcity. We discuss more complex interactions between technical change and distributive conflict in the concluding section.

have shown that asset inequalities are a fundamental feature of capitalist economies, and a key determinant of its long-run dynamics, but contrary to Roemer's [53, 57, 58] seminal theory, it is unclear that exploitation can be *reduced* to wealth inequalities. For, differential ownership of productive assets is necessary for the emergence of exploitation but it is not sufficient for it to persist in equilibrium, even if capital does not become abundant.

Our analysis thus suggests a more complex role for asset inequalities in generating persistent UE exploitation and inequalities. The relevance of wealth inequalities, we believe, is best understood in conjunction with the asymmetric relations of power that characterise capitalist economies, the mechanisms that ensure the scarcity of capital, and the structural constraints that the differential ownership of productive assets imposes on aggregate investment, technical change, unemployment, and so on. From this perspective, Theorem 7 is the most promising result. For, the analysis of the economy with technical progress highlights a mechanism that may contribute to explain the persistence of UE exploitation and inequalities. In the long-run labour-saving technical progress tends to reduce the demand for labour, thus creating the conditions for an excess supply of labour, which restrains wages from rising, which in turn allows UE exploitation and inequalities to persist.

To be sure, Theorem 7 incorporates a rather simple mechanism through which exploitation and inequalities are reproduced over time, namely *exogenous*, labour-saving technical progress. One promising line for further research, from this perspective, would be the analysis of the endogenous competitive processes leading capitalists to introduce new cost-reducing techniques in general economies. As Bidard [12] has shown, in the multi-sectoral linear production model the adoption of cost-reducing technical change would involve a rather complex process under capitalist competition, which cannot be fully captured in the one-sector model in section 6. Further, Yoshihara and Veneziani [74] have recently proved that some types of cost-reducing technical change do not allow exploitation to persist. Another promising line for further research in this vein would be the introduction of endogenous R&D investment, as in the literature on directed technical change (see Kennedy [32] and, more recently, Acemoglu [1, 3]). One aspect of these strands of literature that is particularly congenial to our research agenda is the explicit relation between the functional income distribution and innovations, and the role of technical change in distributive conflicts.[42]

Another interesting extension of our analysis would be the construction of an index of the intensity of exploitation at the individual level, measuring the discrepancy between labour contributed and labour received by each agent, and then analyse the dynamics of the distribution of the exploitation intensity index over time, as the economy accumulates. Given that our concept of exploitation is based on empirically measurable magnitudes (labour expended, technology, income, and consumption), this approach might yield novel insights on the relation between inequality and growth. We leave these issues for further research.

A Proofs of the Main Theorems

Proof of Theorem 1:

Proof. Part (i). Consider any t. By Definition 1(ii) and (v), at a RS $Ly_t = z_t = p_t c_t^w$. Then, noting that $c_t^w > \mathbf{0}$, by Lemma 2, $Ly_t > \lambda c_t^w$ if and only if $\pi_t > 0$.

Part (ii). The result follows from part (i), since $Ly_t - \lambda c_t^w \geqq 0$, all t.

Part (iii). Let $(\mathbf{p}, 1)$ be a RS for E_0 with $\pi_t > 0$. For this $\pi_t > 0$, the equilibrium price vector $p_t > \mathbf{0}$ is uniquely determined by $p_t \equiv L\left(I - (1+\pi_t)A\right)^{-1}$. Correspondingly, the aggregate demand vector $c_t^w > \mathbf{0}$ is also uniquely determined. As argued in part (i), we observe that $Ly_t = p_t c_t^w$ at the RS. Moreover, $Ly_t = LA^{-1}\,\omega_t$ by Lemma 1. Given $\omega_t > \mathbf{0}$, let $(p_t', 1)$ be an equilibrium price associated with $\pi_t' > 0$, such that the associated y_t, $c_t^{c\prime}$, $c_t^{w\prime}$, and s_t' satisfy the conditions of Definition 1. Then, again $LA^{-1}\omega_t = p_t' c_t^{w\prime}$ holds. Therefore, by Definition 3, the corresponding

[42] We are grateful to an anonymous referee for this suggestion.

exploitation rate is uniquely determined by $e'_t = \frac{p'_t c_t^{w\prime}}{\lambda c_t^{w\prime}} - 1$.

Without loss of generality, let $\pi'_t > \pi_t$. Then, show that $e'_t > e_t$ holds. Note that $\pi'_t > \pi_t$ implies $p'_t \equiv L\left(I - (1+\pi'_t)A\right)^{-1} > p_t$. Then, given that $p_t c_t^w = LA^{-1}\omega_t = p'_t c_t^{w\prime}$, $p'_t > p_t$ implies $c_t^{w\prime} \leq c_t^w$ by (A1). Therefore, $\lambda c_t^{w\prime} < \lambda c_t^w$ holds, which implies that $e'_t > e_t$.

In summary, we can find a monotone increasing function $\pi(e_t) = \pi_t$. ■

Proof of Theorem 2:

Proof. Part (i). Write MP^ν using dynamic recursive optimisation theory. Let $\mathcal{W} \subseteq \mathbb{R}^n_+$ be the state space with generic element ω. For any $(\mathbf{p}, 1)$, let $\Psi : \mathcal{W} \to \mathcal{W}$ be the feasibility correspondence: $\Psi(\omega_t^\nu) = \left\{\omega_{t+1}^\nu \in \mathcal{W} : p_t\omega_{t+1}^\nu \leqq (1+\pi_t)\, p_t\omega_t^\nu\right\}$. Let

$$\Pi(\omega_0^\nu) = \left\{\omega^\nu : \omega_{t+1}^\nu \in \Psi(\omega_t^\nu), \text{ all } t, \omega_T^\nu \geqq \omega_0^\nu, \text{ and } \omega_0^\nu \text{ given}\right\}.$$

Let $\Phi = \left\{\left(\omega_t^\nu, \omega_{t+1}^\nu\right) \in \mathcal{W} \times \mathcal{W} : \omega_{t+1}^\nu \in \Psi(\omega_t^\nu)\right\}$ be the graph of Ψ. By the homogeneity of ϕ, if $\pi_t = \pi_\beta$, all t, then the one-period return function $F : \Phi \to \mathbb{R}_+$ at t is $F\left(\omega_t^\nu, \omega_{t+1}^\nu\right) = \frac{\phi(c_\beta)[\left(1+\pi_\beta\right)p_\beta\omega_t^\nu - p_\beta\omega_{t+1}^\nu]}{p_\beta c_\beta}$. Then, MP^ν can be written as

$$C(\omega_0^\nu) = \max_{\omega^\nu \in \Pi(\omega_0^\nu)} \sum_{t=0}^{T-1} \beta^t \frac{\phi(c_\beta)[(1+\pi_\beta)\, p_\beta\omega_t^\nu - p_\beta\omega_{t+1}^\nu]}{p_\beta c_\beta}.$$

Since $\Psi(\omega_t^\nu) \neq \varnothing$, all $\omega_t^\nu \in \mathcal{W}$, and F is continuous, concave, and bounded below by 0, MP^ν is well defined.

2. By construction, $(1+\pi_\beta)\,\beta = 1$ and MP^ν reduces to

$$C(\omega_0^\nu) = \max_{\omega^\nu \in \Pi(\omega_0^\nu)} \phi(c_\beta)\left[\frac{(1+\pi_\beta)p_\beta\omega_0^\nu}{p_\beta c_\beta} - \beta^{T-1}\frac{p_\beta\omega_T^\nu}{p_\beta c_\beta}\right].$$

Therefore, any $\omega^\nu \in \Pi(\omega_0^\nu)$ such that $\omega_T^\nu = \omega_0^\nu$ is optimal and $C(\omega_0^\nu)$ follows by noting that $\beta < 1$.

Part (ii). The result follows from MP^ν, given that $\omega_T^\nu \geqq \omega_0^\nu$. ■

Proof of Theorem 3:

Proof. Part (i). 1. (Optimal ξ^ν.) By the Perron-Frobenius theorem π' exists and $\pi' \in (0, \widetilde{\pi})$. (The possibility that $\pi' = 0$ is ruled out by the condition $\gamma_0\lambda b = L[I - (1+\pi')A]^{-1}b$ given $\gamma_0 > 1$.) If $\pi' = \pi_\beta$, $c_\beta = kb$, some $k > 0$, and $\pi_t = \pi'$, all t, by Theorem 2, any ξ^ν such that $s_t^\nu = \mathbf{0}$, $p_\beta A y_t^\nu = p_\beta\omega_0^\nu$, and $c_t^\nu = h_t^\nu b$ with $h_t^\nu = \frac{\pi' p_\beta\omega_0^\nu}{p_\beta b}$, all t, solves MP^ν, for all $\nu \in \mathcal{N}_c$.

2. (Capital market.) Hence, it is possible to choose $(y^\nu)_{\nu\in\mathcal{N}_c}$ such that at all t, $p_\beta A y_t^\nu = p_\beta\omega_0^\nu$, all ν, and $y_t = A^{-1}\omega_0$.

3. (Labour market and optimal ξ^η.) Since $Ly_t = \gamma_0\lambda b N_w < N_w$, all t, for all $\eta \in \mathcal{N}_w$ assign actions $z_t^\eta = \widehat{z}_t^\eta = \gamma_0\lambda b$, all t; then by construction $\gamma_0\lambda b = p_\beta b$, and thus $c_t^\eta = b$, all t. Hence, these actions solve MP^η for all η, with $Ly_t = z_t$, all t.

4. (Final goods market.) Definition 1(iii) is satisfied because, at all t: $(I - A)y_t = \gamma_0 N_w b$, $c_t^w = N_w b$, and $c_t^c = h_t^c b$, where $h_t^c = \sum_{\nu\in\mathcal{N}_c} h_t^\nu$, and so $h_t^c p_\beta b = \gamma_0 N_w[p_\beta - \lambda]b$, or $h_t^c = N_w(\gamma_0 - 1)$.

Part (ii). 1. (Optimal ξ^ν.) By the Perron-Frobenius theorem π' exists and $\pi' \in (0, \widetilde{\pi})$. Thus $\pi_\beta \in (0, \widetilde{\pi})$. If $\pi_t = \pi_\beta$, all t, by Theorem 2, any ξ^ν such that $s_t^\nu = \mathbf{0}$, $p_\beta A y_t^\nu = p_\beta\omega_0^\nu$, and $c_t^\nu = h_t^\nu b$ with $h_t^\nu p_\beta b = \pi_\beta p_\beta\omega_0^\nu$, all t, solves MP^ν, for all $\nu \in \mathcal{N}_c$.

2. (Capital market.) Hence, it is possible to choose $(y^\nu)_{\nu\in\mathcal{N}_c}$ such that at all t, $p_\beta A y_t^\nu = p_\beta\omega_0^\nu$, all ν, and $y_t = A^{-1}\omega_0$.

3. (Labour market; optimal ξ^η.) Since $Ly_t = N_w$, all t, assign actions $z_t^\eta = \widehat{z}_t^\eta = 1$ and $c_t^\eta = h_t^\eta b$ with $h_t^\eta = 1/p_\beta b$, all t, to all $\eta \in \mathcal{N}_w$. Since $\pi_\beta \in (0, \pi']$ then $1/\lambda b > h_t^\eta \geqq 1$, all t, η. Hence, these actions solve MP^η for all η, with $Ly_t = z_t$, all t.

4. (Final goods market.) Definition 1(ii) is met because, at all t, $(I - A)y_t = \gamma_0 N_w b$ while $c_t^w = N_w b/p_\beta b$ and $c_t^c = \sum_{\nu\in\mathcal{N}_c} h_t^\nu b$, where $\sum_{\nu\in\mathcal{N}_c} h_t^\nu p_\beta b = \pi_\beta p_\beta \omega_0$, or $\sum_{\nu\in\mathcal{N}_c} h_t^\nu p_\beta b = \gamma_0 N_w [p_\beta - \lambda] b$.

Part (iii). 1. If $\gamma_0 \lambda b = 1$, existence is proved as in part (ii) with $z_t^\eta = \widehat{z}_t^\eta = 1$ and $h_t^\eta = 1/\lambda b$, all $\eta \in \mathcal{N}_w$, and all t. If $\gamma_0 \lambda b < 1$, existence is proved as in part (i) with $y_t = (1/\gamma_0)A^{-1}\omega_0$ and $Ly_t = \lambda b N_w$, all t, $z_t^\eta = \widehat{z}_t^\eta = \lambda b$ and $c_t^\eta = b$, all $\eta \in \mathcal{N}_w$, and all t. In both cases, as shown in parts (i) and (ii), $p_t A y_t^\nu = p_t \omega_t^\nu$, at all t and for all $\nu \in \mathcal{N}_c$.

2. Suppose, by contradiction, that there is a SRS with $\pi_t > 0$, some t. Then, using the same perturbational argument as in the proof of Lemma 3, it is immediate to prove that it must be $\pi_{t-1} = \pi_{t+1} = 0$ (observe that since $\beta = 1$, $\pi_{t+1} > 0$ implies $\beta(1 + \pi_{t+1}) > 1$). Therefore noting that $p_t = (1 + \pi_t)p_t A + w_t L$, and $w_t = 1$ all t, it follows that $p_{t-1} = p_{t+1} = \lambda > \mathbf{0}$, for any $0 \leqq t - 1 < t + 1 \leqq T - 1$. Then, for all $\nu \in \mathcal{N}_c$, there is no optimal ξ^ν such that $s_t^\nu = \mathbf{0}$ and $c_{t-1}^\nu = c_{t+1}^\nu = c_t^\nu$, a contradiction. ■

Proof of Theorem 4:

Proof. Part (ii). 1. Consider capitalist ν's programme MP^ν recursively: at all t, the functional equation is $C_t(\omega_t^\nu) = \max_{\omega_{t+1}^\nu \in \Psi(\omega_t^\nu)} [(1 + \pi_t)\omega_t^\nu - \omega_{t+1}^\nu] + \beta C_{t+1}(\omega_{t+1}^\nu)$. At $T - 1$, since $C_T(\omega_T^\nu) = 0$ for all ω_T^ν, optimality requires $\omega_T^\nu = \omega_0^\nu$ and $C_{T-1}(\omega_{T-1}^\nu) = [(1+\pi_{T-1})\omega_{T-1}^\nu - \omega_0^\nu]$. Therefore at $T-2$, $C_{T-2}(\omega_{T-2}^\nu) = \max_{\omega_{T-1}^\nu \in \Psi(\omega_{T-2}^\nu)} [(1 + \pi_{T-2})\omega_{T-2}^\nu - \omega_{T-1}^\nu] + \beta C_{T-1}(\omega_{T-1}^\nu)$.

2. Suppose $\beta < 1$ and $\pi_t > 0$, all t, $T - 2 \geqq t \geqq t'$. Because $\pi_{T-2} > 0$, if $\beta(1 + \pi_{T-1}) \neq 1$ then $\omega_{T-1}^\nu \neq \omega_{T-2}^\nu$, all $\nu \in \mathcal{N}_c$, and $\omega_{T-1} \neq \omega_{T-2}$. Hence, $\beta(1 + \pi_{T-1}) = 1$ and $C_{T-2}(\omega_{T-2}^\nu) = \left[(1 + \pi_{T-2})\omega_{T-2}^\nu - \beta\omega_0^\nu\right]$. Iterating backwards, if $\omega_{t+1} = \omega_t$, all t, $T-2 \geqq t \geqq t'$, then $\beta(1+\pi_{t+1}) = 1$, all t, $T - 2 \geqq t \geqq t'$, which implies $C_{t'}(\omega_{t'}^\nu) = \left[(1 + \pi_{t'})\omega_{t'}^\nu - \beta^{T-1-t'}\omega_0^\nu\right]$.

3. Suppose $\beta = 1$. Suppose, contrary to the statement, that $\pi_t > 0$ and $\pi_{t+j} > 0$, some t, $T - 2 \geqq t \geqq t'$, and $j > 0$. Since $\pi_t > 0$, then $c_t^\nu = 0$, all $\nu \in \mathcal{N}_c$, is not possible, or else $\omega_{t+1} \neq \omega_t$, and since $\pi_{t+j} > 0$ then $(1 + \pi_{t+j}) > 1$, and there is a feasible perturbation $\mathrm{d}s_t^\nu = -\mathrm{d}s_{t+j}^\nu > 0$, with $\mathrm{d}s_l^\nu = 0$ all $l \neq t, t + j$, that increases ν's welfare, contradicting optimality.

Part (i). 1. Suppose that $(\mathbf{p}, 1)$ is a SGP for E_0. Then by definition there is a $t' > 0$ and a sequence $\{g_t\}_{t=0}^{t'-2}$ such that $\omega_{t+1} = (1 + g_t)\omega_t$, $g_t > 0$, all t, $0 \leqq t < t' - 1$. For all $\nu \in \mathcal{N}_c$, $c_t^\nu = \pi_t\omega_t^\nu - s_t^\nu$, all t. Therefore, summing over ν and noting that by definition $s_t = g_t\omega_t$, all t, it follows that $c_t^c = (\pi_t - g_t)\omega_t$, all t. Since $\omega_t = \gamma_t N_w A(1 - A)^{-1}b$, all t, and noting that in the one good case $\widetilde{\pi} = \frac{1-A}{A}$, then $c_t^c = (\pi_t - g_t)\frac{\gamma_t N_w b}{\widetilde{\pi}}$, all t, or $g_t = [\pi_t - (\frac{c_t^c \widetilde{\pi}}{\gamma_t N_w b})]$, all t.

2. By definition, $(p_t - \lambda) = \pi_t p_t A(1 - A)^{-1}$, all t, or equivalently $\pi_t = \widetilde{\pi}(p_t - \lambda)/p_t$, all t. Hence, $g_t = [\frac{(p_t-\lambda)}{p_t} - (\frac{c_t^c}{\gamma_t N_w b})]\widetilde{\pi}$, all t. Moreover, observe that at a SGP with $LA^{-1}\omega_t = \gamma_t N_w \lambda b \leqq N_w$, all t, it must be $\gamma_t \lambda b < 1$, all $t \leqq t' - 1$. By construction, this implies that at all $t \leqq t' - 1$, $z_t^\eta = \gamma_t \lambda b = p_t b$, for all $\eta \in \mathcal{N}_w$. Therefore $p_t = \gamma_t \lambda$, all $t \leqq t' - 1$, and the first part of the statement follows substituting the latter expression into the equation for g_t, and noting that $\frac{p_{t+1}}{p_t} = \frac{\gamma_{t+1}}{\gamma_t} = \frac{\omega_{t+1}}{\omega_t}$, for all $t < t' - 1$.

3. Suppose $\beta < 1$. If $t' \leqq 2$, then the statement holds vacuously. Hence, assume $t' > 2$. At $t = t' - 1$, $C_{t'-1}(\omega_{t'-1}^\nu) = \max_{\omega_{t'}^\nu \in \Psi(\omega_{t'-1}^\nu)} [(1 + \pi_{t'-1})\omega_{t'-1}^\nu - \omega_{t'}^\nu] + \beta C_{t'}(\omega_{t'}^\nu)$, where $C_{t'}(\omega_{t'}^\nu)$ is as in step 2 of the proof of part (ii) for all $\nu \in \mathcal{N}_c$. Hence, at a SGP $\beta(1 + \pi_{t'}) \geqq 1$, or else $\omega_{t'}^\nu = 0$, all $\nu \in \mathcal{N}_c$. If $\beta(1 + \pi_{t'}) > 1$, then $\omega_{t'}^\nu = (1 + \pi_{t'-1})\omega_{t'-1}^\nu$, all ν, and $g_{t'-1} = \pi_{t'-1}$. If $\beta(1 + \pi_{t'}) = 1$, then $g_{t'-1}$ is undetermined. In either case, $C_{t'-1}(\omega_{t'-1}^\nu) = \left[\beta(1 + \pi_{t'})(1 + \pi_{t'-1})\omega_{t'-1}^\nu - \beta^{T-t'}\omega_0^\nu\right]$, all $\nu \in \mathcal{N}_c$.

4. Consider $t = t' - 2$. Again, at a SGP, it must be $\beta^2(1 + \pi_{t'})(1 + \pi_{t'-1}) \geqq 1$, and $C_{t'-2}(\omega_{t'-2}^\nu)$

$= \Big[\beta^2(1+\pi_{t'})(1+\pi_{t'-1})(1+\pi_{t'-2})\omega^{\nu}_{t'-2} - \beta^{T-t'+1}\omega^{\nu}_0\Big]$, all $\nu \in \mathcal{N}_c$. If $\beta^2(1+\pi_{t'})(1+\pi_{t'-1}) = 1$, then by the previous step $\beta(1+\pi_{t'-1}) \leqq 1$: but then since by step 2 at a SGP $p_{t+1} > p_t$, all $t < t'-1$, by definition it follows that $\beta(1+\pi_{t'-2}) < 1$. However, because $t' > 2$, by considering $C_{t'-3}(\omega^{\nu}_{t'-3})$, it immediately follows that $\omega^{\nu}_{t'-2} = 0$, all $\nu \in \mathcal{N}_c$, violating the definition of SGP. Therefore, it must be $\beta^2(1+\pi_{t'})(1+\pi_{t'-1}) > 1$, $\omega^{\nu}_{t'-1} = (1+\pi_{t'-2})\omega^{\nu}_{t'-2}$, all ν, and $g_{t'-2} = \pi_{t'-2}$. This argument can be iterated backwards for all t, $0 < t < t'-1$, showing that $\omega^{\nu}_{t+1} = (1+\pi_t)\omega^{\nu}_t$, all ν, and all t, $0 < t < t'-1$, and thus $g_t = \pi_t$, all t, $0 < t < t'-1$.

5. Suppose $\beta = 1$. A similar argument as in steps 3 and 4 applies noting that at all $t \leqq t'-1$, $\pi_t > 0$ implies $\beta(1+\pi_t) > 1$, given part (ii). ■

Proof of Theorem 5:
Proof. 1. Take any $\nu \in \mathcal{N}_c$. Consider MP^{ν} recursively. At $T-1$, since $C_T(\omega^{\nu}_T) = 0$, then $\omega^{\nu}_T = \omega^{\nu}_0$ is optimal and $C_{T-1}(\omega^{\nu}_{T-1}) = [(1+\pi_{T-1})\omega^{\nu}_{T-1} - \omega^{\nu}_0]$. At $T-2$, $C_{T-2}(\omega^{\nu}_{T-2}) = \max_{\omega^{\nu}_{T-1}\in\Psi(\omega^{\nu}_{T-2})} [(1+\pi_{T-2})\omega^{\nu}_{T-2} - \omega^{\nu}_{T-1} + \beta C_{T-1}(\omega^{\nu}_{T-1})]$. Hence, if $\pi_{T-1} = \pi_{\beta}$ then any $\omega^{\nu}_{T-1} \geqq \omega^{\nu}_{T-2}$ is optimal and $C_{T-2}(\omega^{\nu}_{T-2}) = [(1+\pi_{T-2})\omega^{\nu}_{T-2} - \beta\omega^{\nu}_0]$. Iterating backwards, if $\pi_t = \pi_{\beta}$, all t, $T-1 \geqq t \geqq \tau+1$, then at all t, $T-2 \geqq t \geqq \tau$, any $\omega^{\nu}_{t+1} \geqq \omega^{\nu}_t$ is optimal and $C_{\tau}(\omega^{\nu}_{\tau}) = [(1+\pi_{\tau})\omega^{\nu}_{\tau} - \beta^{T-\tau-1}\omega^{\nu}_0]$. If $\tau = 0$, the result is proved, noting that $C(\omega^{\nu}_0) = C_0(\omega^{\nu}_0)$.

2. If $\tau > 0$, consider $\tau - 1$. Since $C_{\tau-1}(\omega^{\nu}_{\tau-1}) = \max_{\omega^{\nu}_{\tau}\in\Psi(\omega^{\nu}_{\tau-1})} [(1+\pi_{\tau-1})\omega^{\nu}_{\tau-1} - \omega^{\nu}_{\tau} + \beta C_{\tau}(\omega^{\nu}_{\tau})]$ and $\pi_t > \pi_{\beta}$, at the solution to MP^{ν}, $\omega^{\nu}_{\tau} = (1+\pi_{\tau-1})\omega^{\nu}_{\tau-1}$ and $C_{\tau-1}(\omega^{\nu}_{\tau-1}) = [\beta(1+\pi_{\tau})(1+\pi_{\tau-1})\omega^{\nu}_{\tau-1} - \beta^{T-\tau}\omega^{\nu}_0]$. Iterating backwards, if $\pi_t > \pi_{\beta}$, all $t \leqq \tau$, at the solution to MP^{ν}, $\omega^{\nu}_{t+1} = (1+\pi_t)\omega^{\nu}_t$, all $t \leqq \tau - 1$, and the expression for $C(\omega^{\nu}_0) = C_0(\omega^{\nu}_0)$ follows. ■

Proof of Theorem 6:
Proof. 1. We begin by establishing three properties of the sequence $\{\gamma_t\}_{t=0}^{T-1}$.

1.1. At all $t \leqq \tau$, if $\gamma_t \in [\overline{\gamma}_{\tau+1-t}, \overline{\gamma}_{\tau-t})$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, then $\gamma_{t+1} = (1+\pi_t)\gamma_t$ implies $\gamma_{t+1} \in [\overline{\gamma}_{\tau-t}, \overline{\gamma}_{\tau-t-1})$. To see this, note that at all τ, $\overline{\gamma}_{\tau} = (1+\widetilde{\pi})\overline{\gamma}_{\tau+1} - \widetilde{\pi}$, while $\gamma_{t+1} = (1+\pi_t)\gamma_t$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$ implies $\gamma_{t+1} = (1+\widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t})\gamma_t = (1+\widetilde{\pi})\gamma_t - \widetilde{\pi}$.

1.2. If $\gamma_t \in [\overline{\gamma}_1, \overline{\gamma}_0) = [\overline{\gamma}_1, \frac{1}{\lambda b})$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, then there is a $g_t \in (0, \pi_t]$ such that $\gamma_{t+1} = (1+g_t)\gamma_t$ implies $\gamma_{t+1} = 1/\lambda b$. To see this, note that, as in step 1.1, $\overline{\gamma}_0 = (1+\widetilde{\pi})\overline{\gamma}_1 - \widetilde{\pi}$. Therefore if $\gamma_t = \overline{\gamma}_1$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, then $g_t = \pi_t$ implies $\gamma_{t+1} = \overline{\gamma}_0$, and for all $\gamma_t \in (\overline{\gamma}_1, \overline{\gamma}_0)$, $g_t = \pi_t$ implies $\gamma_{t+1} > \overline{\gamma}_0$, while $g_t = 0$ implies $\gamma_{t+1} < \overline{\gamma}_0$.

1.3. If $\overline{\gamma}_{\tau} > \frac{\beta\widetilde{\pi}}{\beta(1+\widetilde{\pi})-1}$, all $\tau \geqq 1$, then $\pi_1 = \widetilde{\pi}\frac{(\gamma_1-1)}{\gamma_1} > \pi_{\beta}$, for all $\gamma_1 \in [\overline{\gamma}_{\tau}, \overline{\gamma}_{\tau-1})$. To see this, note that if $\gamma_1 = \overline{\gamma}_{\tau}$ then $\pi_1 = \widetilde{\pi}\left(1 - \frac{1}{\overline{\gamma}_{\tau}}\right) > \widetilde{\pi}\left(1 - \frac{\beta(1+\widetilde{\pi})-1}{\beta\widetilde{\pi}}\right) = \pi_{\beta}$, and π_1 is strictly increasing in γ_1.

2. Consider $(\mathbf{p}, 1)$ with $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, all $t \leqq \tau$, where $\gamma_{t+1} = (1+\pi_t)\gamma_t$, all $t \leqq \tau-1$. Then $\pi_0 = \widetilde{\pi}\frac{(\gamma_0-1)}{\gamma_0}$ and $\pi_{t+1} = \widetilde{\pi}\left(1 - \frac{1}{(1+\pi_t)\gamma_t}\right)$, and $(\mathbf{p}, 1)$ is well defined.

3. (Optimal ξ^{ν}; reproducibility.) By step 1.3, and noting that $\gamma_0 > 1$, under the assumptions of the Theorem, we have $\pi_t > \pi_{\beta}$, all $t \leqq \tau$. Hence, by Theorem 5, $\omega^{\nu}_{t+1} = (1+\pi_t)\omega^{\nu}_t$, all $t \leqq \tau-1$, $\omega^{\nu}_{t+1} = (1+g_t)\omega^{\nu}_t$, with $g_t \in [0, \pi_t]$, all t, $T-1 \geqq t \geqq \tau$, and $\omega^{\nu}_T = \omega^{\nu}_0$ is optimal for all $\nu \in \mathcal{N}_c$. Therefore, for all $\nu \in \mathcal{N}_c$, we can choose an optimal ξ^{ν} such that $\omega^{\nu}_{t+1} = (1+\pi_t)\omega^{\nu}_t$, all $t \leqq \tau-1$, $\omega^{\nu}_{\tau+1} = (1+g_{\tau})\omega^{\nu}_{\tau}$, with $g_{\tau} = \left(\frac{1}{\gamma_{\tau}\lambda b} - 1\right) \in (0, \pi_{\tau}]$, $\omega^{\nu}_t = \omega^{\nu}_{\tau+1}$, all t, $T-1 \geqq t \geqq \tau+1$, $\omega^{\nu}_T = \omega^{\nu}_0$; $y^{\nu}_t = A^{-1}\omega^{\nu}_t$, all t; and $c^{\nu}_t = (1+\pi_t)\omega^{\nu}_t - \omega^{\nu}_{t+1}$, all t. (Observe that by steps 1.1 and 1.2, $g_{\tau} = \left(\frac{1}{\gamma_{\tau}\lambda b} - 1\right) \in (0, \pi_{\tau}]$ exists and $\gamma_{\tau+1} = \overline{\gamma}_0$.) Hence, parts (i) and (vi) of Definition 1 are met.

4. (Capital market.) Because $y^{\nu}_t = A^{-1}\omega^{\nu}_t$, all t and all $\nu \in \mathcal{N}_c$, then $y_t = A^{-1}\omega_t$, all t, and Definition 1(iv) is satisfied.

5. (Labour market; optimal ξ^η.) By construction, $\gamma_0 < \overline{\gamma}_1 < \overline{\gamma}_0 = \frac{1}{\lambda b}$ and therefore $Ly_0 = LA^{-1}\omega_0 = \gamma_0 \lambda b N_w < N_w$. By step 3, together with steps 1.1 and 1.2, it follows that $\gamma_t < \overline{\gamma}_0 = \frac{1}{\lambda b}$ for all $t \leqq \tau$, and $\gamma_t = \overline{\gamma}_0 = \frac{1}{\lambda b}$ for all t, $T-1 \geqq t \geqq \tau+1$. Therefore $Ly_t = LA^{-1}\omega_t < N_w$, all $t \leqq \tau$, whereas $Ly_t = LA^{-1}\omega_t = N_w$, all t, $T-1 \geqq t \geqq \tau+1$. Hence, for all $\eta \in \mathcal{N}_w$, assign a vector ξ^η such that $z_t^\eta = \widehat{z}_t^\eta = \gamma_t \lambda b$ and $c_t^\eta = b$, all $t \leqq \tau$, and $z_t^\eta = 1$ and $c_t^\eta = \frac{1}{p_\beta}$, all t, $T-1 \geqq t \geqq \tau+1$. Noting that $p_\beta = L\left[1-(1+\pi_\beta)A\right]^{-1}$ and $\pi_\beta > \pi'$ imply $\frac{1}{p_\beta} > b$, it follows that ξ^η solves MP^η, for all $\eta \in \mathcal{N}_w$. Hence parts (ii) and (v) of Definition 1 are met.

6. (Final goods market.) Consider first all periods $t \leqq \tau$. By construction at all t, $t \leqq \tau$, $c_t^\eta = b$, all $\eta \in \mathcal{N}_w$, and $c_t^\nu + s_t^\nu = \pi_t \omega_t^\nu$, all $\nu \in \mathcal{N}_c$. Therefore $c_t^c + s_t + c_t^w = \pi_t \omega_t + N_w b$, and substituting for $\pi_t = \widetilde{\pi}\frac{(\gamma_t - 1)}{\gamma_t} = \frac{1-A}{A}\frac{(\gamma_t-1)}{\gamma_t}$ and $\omega_t = \gamma_t N_w A(1-A)^{-1} b$, one obtains $c_t^c + s_t + c_t^w = \gamma_t N_w b$. Because $(1-A)y_t = (1-A)A^{-1}\omega_t = \gamma_t N_w b$, it follows that $(1-A)y_t = c_t^c + s_t + c_t^w$, all t, $t \leqq \tau$. Consider next periods t, $T-1 \geqq t \geqq \tau+1$. By construction, at all t, $T-1 \geqq t \geqq \tau+1$, $c_t^\eta = 1/p_\beta$, all $\eta \in \mathcal{N}_w$, and $c_t^\nu + s_t^\nu = \pi_\beta \omega_t^\nu$, all $\nu \in \mathcal{N}_c$, and so $c_t^c + s_t + c_t^w = \pi_\beta \omega_t + \frac{N_w}{p_\beta}$. By substituting for π_β and p_β, and noting that at the proposed path $\gamma_t = 1/\lambda b$, one obtains $c_t^c + s_t + c_t^w = N_w b/\lambda b$. Because $(1-A)y_t = (1-A)A^{-1}\omega_t = \gamma_t N_w b = N_w b/\lambda b$, it follows that $(1-A)y_t = c_t^c + s_t + c_t^w$, all $t \geqq \tau+1$. Therefore, Definition 1(iii) is met. ■

Proof of Theorem 7:

Proof. 1. Consider the sequence of profit rates $\{\pi_t\}_{t=0}^{T-1}$. Since $\gamma_0 > 1$, $\pi_0 \in (0, \widetilde{\pi})$. Moreover, at all t, $\pi_t < \widetilde{\pi}$ implies $\pi_{t+1} > \pi_t$. Therefore given $\beta\left[1+\widetilde{\pi}\frac{(\gamma_0-1)}{\gamma_0}\right] = \beta(1+\pi_0) \geqq 1$, it follows that $\pi_t > \pi_\beta$, all $t > 0$. Finally, we prove that if $\pi_0 = \widetilde{\pi}\frac{(\gamma_0-1)}{\gamma_0}$, $\pi_{t+1} = \frac{\pi_t(1+\widetilde{\pi})}{(1+\pi_t)}$, all t, $T-2 \geqq t \geqq 0$, and $\gamma_{t+1} = (1+\pi_t)\gamma_t$, all t, $T-2 \geqq t \geqq 0$, then $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, all $t > 0$. To see this, suppose the result holds for any $t > 0$. Then $\pi_{t+1} = \frac{\pi_t(1+\widetilde{\pi})}{(1+\pi_t)}$ and $\gamma_{t+1} = (1+\pi_t)\gamma_t$ imply $\pi_{t+1} = \frac{\pi_t\gamma_t(1+\widetilde{\pi})}{\gamma_{t+1}}$. Because $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, the latter expression becomes $\pi_{t+1} = \frac{\widetilde{\pi}(\gamma_t-1)(1+\widetilde{\pi})}{\gamma_{t+1}}$, and the desired result follows noting that $\gamma_{t+1} = (1+\pi_t)\gamma_t$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$ imply $\gamma_{t+1} - 1 = (1+\widetilde{\pi})(\gamma_t - 1)$, as required.

2. (Optimal ξ^ν; reproducibility.) By step 1, $\pi_t > \pi_\beta$, all $t > 0$. Therefore, by Theorem 5, for all $\nu \in \mathcal{N}_c$, the vector ξ^ν with $y_t^\nu = A^{-1}\omega_t^\nu$, all t; $\omega_{t+1}^\nu = (1+\pi_t)\omega_t^\nu$ and $c_t^\nu = 0$, all t, $T-2 \geqq t \geqq 0$; $\omega_T^\nu = \omega_0^\nu$; and $c_{T-1}^\nu = (1+\pi_{T-1})\omega_{T-1}^\nu - \omega_0^\nu$ solves MP^ν. Hence parts (i) and (vi) of Definition 1 are met.

3. (Capital market) Because $y_t^\nu = A^{-1}\omega_t^\nu$, all t and all $\nu \in \mathcal{N}_c$, then $y_t = A^{-1}\omega_t$, all t, and Definition 1(iv) is satisfied.

4. (Labour market; optimal ξ^η) By step 3, $L_t y_t = L_t A^{-1}\omega_t = \gamma_t \lambda_t b N_w$, all t. By (A2), $L_{t+1} = \delta L_t$, all t, $T-2 \geqq t \geqq 0$, and by step 3 $y_{t+1} = y_t(1+\pi_t)$, all t, $T-2 \geqq t \geqq 0$. Hence, $L_{t+1}y_{t+1} = \delta(1+\pi_t)L_t y_t$, all t, $T-2 \geqq t \geqq 0$. Therefore, since $L_0 y_0 = L_0 A^{-1}\omega_0 = \gamma_0 \lambda_0 b N_w \leqq N_w$ and $\delta(1+\widetilde{\pi}) \leqq 1$ by assumption, and $\pi_t < \widetilde{\pi}$, all t, it follows that $L_t y_t \leqq N_w$, all t, and $L_t y_t < N_w$, all $t > 0$. Then, for all $\eta \in \mathcal{N}_w$, let ξ^η be defined by $z_t^\eta = \widehat{z}_t^\eta = \gamma_t \lambda_t b$ and $c_t^\eta = b$, all t. Noting that $\gamma_t \lambda_t b \leqq 1$, all t, and $p_t = L_t\left[1-(1+\pi_t)A\right]^{-1} = L_t\left[1-\left(1+\frac{1-A}{A}\frac{(\gamma_t-1)}{\gamma_t}\right)A\right]^{-1} = \gamma_t\lambda_t$, all t, it follows that ξ^η solves MP^η, all $\eta \in \mathcal{N}_w$. Therefore parts (ii) and (v) of Definition 1 are met.

5. (Final goods market) At the proposed path, $c_t^w = N_w b$ and $c_t^c + s_t = \pi_t\omega_t$, all t, and substituting for $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t} = \frac{1-A}{A}\frac{(\gamma_t-1)}{\gamma_t}$ and $\omega_t = \gamma_t N_w A(1-A)^{-1}b$, one obtains $c_t^c + s_t + c_t^w = \gamma_t N_w b$, all t. Because $(1-A)y_t = (1-A)A^{-1}\omega_t = \gamma_t N_w b$, all t, Definition 1(iii) is satisfied. ■

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