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Growth, Exploitation and Class Inequalities

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Abstract

This paper provides a formal dynamic analysis of exploitation, class inequalities and profits. A stylised model of a capitalist economy with two classes - workers and capitalists - is considered which extends Roemer [26, 27]. First, a dynamic generalisation of a key Marxian insight is provided by proving that the profitability of capitalist production is synonimous with the existence of exploitation. Second, it is shown that, in a competitive environment, asset inequalities are fundamental for the emergence of exploitation, but they are not sufficient for its persistence, both in equilibria with accumulation and growth, and, perhaps more surprisingly, in stationary intertemporal equilibrium paths. Finally, it is shown that labour-saving technical progress may yield persistent exploitation by ensuring the persistent abundance of labour.

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Keywords: Dynamics, accumulation, exploitation, classes.

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1 Introduction

In the Marxian and Kaleckian traditions, capitalism is conceived of as a class system (Sawyer [31]). This is a fundamental and distinctive feature of both approaches, which differentiates them from standard, mainstream macroeconomic models based on the fiction of the representative agent. In Marxian and Kaleckian approaches, the dynamics of capitalist economies cannot be properly understood unless one explicitly analyses their class structure, and the effects that class relations have on behaviour, accumulation and inequalities.[1] It is empirically inaccurate, and analytically misleading, to assume all agents to be fundamentally alike: class positions shape the agents' feasible sets, and influence their attitudes, beliefs and choices. In particular, the dynamics of capitalist economies is determined by inequalities in asset ownership, and the differential control of investment decisions that they entail, *and* by the heterogeneous savings behaviour related to different class positions. In turn, the class-driven dynamics of accumulation tends to reproduce the class structure of capitalist economies, and the associated inequalities.

This paper analyses the dynamics of capitalist economies and the long-run relation between classes, accumulation and inequalities in an abstract model. More precisely, we analyse the relation between classes, profits, accumulation and the exploitation of labour in an intertemporal model with heterogeneous agents in order to explore some fundamental Marxian and Kaleckian themes.[2]

To be sure, the concept of exploitation is seemingly secondary in Kalecki's work, which hardly contains any references to standard Marxian value theory (Sawyer [31], chapter 8; Lopez and Assous [17], chapter 9). Nonetheless, there are deep theoretical and formal affinities between Marx's and Kalecki's theories of growth and distribution. In both Marx and Kalecki, profits are seen as a surplus. In Marx, this surplus depends on the rate of exploitation while in Kalecki, it depends on the firms' degree of monopoly power.[3] But, as Dutt [4] has convincingly argued, at a conceptual level, the rate of exploitation and the degree of monopoly power expressed through the mark-up capture the same relationship between wages and surplus produced by workers.[4] Further, Kalecki's [13] emphasis on the role of class struggle, and the bargaining power of trade unions, in the determination of mark-ups, profits, and income distribution bears a clear conceptual relation with Marxian exploitation theory.[5]

Two main substantive contributions emerge from our analysis. First, we provide a dynamic generalisation of a key Marxian insight by proving that the profitability of capitalist production is synonymous with the existence of exploitation. The existence of a relation between exploitation and profits has a "prominent place in the modern formulation of Marxian economics" (Roemer [26], p.16), and therefore it has been dubbed *Fundamental Marxian Theorem* (henceforth, FMT).[6] This result is important because the rate of profit is one of the key determinants of investment decisions, and of the long-run dynamics of capitalist economies. Thus, the FMT can be interpreted as providing a link between exploitation and growth. But the FMT is important also because it

[1] "[B]oth Rosa Luxemburg and Michal Kalecki took from Marx the very notion of capital, and the conviction that the capitalist system polarized society by two antagonistic classes: the capitalists and the workers. Both were interested more in the dynamics of capitalism than in static theory of value and prices ... both used Marxian reproduction schemata of reproduction to search for the limits of capitalist accumulation." (Kowalik [15], p.111).

[2] Needless to say, our model does not aim to incorporate *all* of Kalecki's or Marx's insights. Important elements of their analysis are missing, such as monopolistic competition and mark-up pricing, because they are not central for the main arguments of this paper.

[3] "In his lectures on the economics of capitalism at the Central School of Planning and Statistics, [Kalecki] used the term 'rate of exploitation' in reference to the relative share of profits in income" (Lopez and Assous [17], p.241).

[4] Kalecki "did not see profits as accruing to capitalists as a reward for waiting or for abstinence ... profits accrued to capitalists on the basis of ownership of wealth (to which there was limited acess) but not as a return for any services rendered" (Sawyer [31], p.148). The affinity is more evident if one notes that modern interpretations of exploitation theory, including the approach adopted in this paper, do not rely on the labour theory of value.

[5] For a thorough discussion, see Rugitsky [30].

[6] See, e.g., the classic contributions by Okishio [22] and Morishima [20]. For a thorough discussion and various generalisations, see Yoshihara and Veneziani [41] and Veneziani and Yoshihara [39].

proves that, given private ownership of productive assets, profits are a counterpart of the transfer of social surplus and social labour from asset-poor agents to the wealthy.

Second, we investigate exploitation, profits and class relations in dynamic capitalist economies. In his seminal *General Theory of Exploitation and Class*, Roemer ([27], p.43) famously proved that "differential distribution of property and competitive markets are sufficient institutions to generate an exploitation phenomenon, under the simplest possible assumptions" and concluded that exploitation, and classes, can be *reduced* to asset inequalities. However, Veneziani [36, 38] has recently argued that this conclusion is unwarranted: Roemer's [27] models are essentially static in that agents face no intertemporal trade-offs - both intertemporal credit markets and savings are ruled out. As a result, they do not seem suitable to analyse exploitation and class as *persistent* features of capitalist economies.[7]

In this paper, we analyse the conditions for both the emergence *and the persistence* of exploitation and classes, and the relation between classes, exploitation, profits and growth, in a dynamic generalisation of Roemer's [26, 27] economies with optimising agents.[8] Unlike in many standard Marxian models, this allows us to explicitly analyse the complex relationship between macroeconomic conditions and behavioural (class-based) relations.[9] But a fully specified intertemporal model also gives us the opportunity to assess the causal and moral relevance of asset inequalities in generating exploitation as a persistent feature of a competitive economy where agents can save and the distribution of productive assets can change over time.[10]

We prove that, contrary to Roemer's claim, in a competitive environment, asset inequalities are indeed fundamental for the emergence of exploitation and classes, but they are not sufficient for their persistence. If unstemmed, the fundamental drive of capitalists to accumulate inexorably leads capital to become abundant and profits and exploitation to disappear.[11] Perhaps more surprisingly, asset inequalities and competitive markets do not guarantee the persistence of classes, profits and exploitation, even in equilibrium paths *with no accumulation.* In competitive economies, it depends on such theoretically objectionable and empirically contingent factors as time preference, rather than the structural characteristics of capitalist economies.

We take these results as suggesting that asset inequalities (and the underlying property relations) are a fundamental feature of capitalist economies, and a key determinant of its long run dynamics, but at the same time the class structure and exploitative nature of capitalism cannot be *reduced* to wealth inequalities in an abstract, power-free competitive setting. Differential ownership of productive assets is causally necessary but normatively secondary in generating exploitation. The central role of asset inequalities can only be understood in conjunction with the asymmetric relations of power that characterise capitalist economies, the mechanisms that ensure the scarcity

[7] Roemer's theory has sparked a vast literature. Several critiques have been expounded on his methodology and conclusions, mainly based on issues of interpretation of Marx's theory, but surprisingly little attention has been devoted to his models. See Veneziani [38] for a survey of Roemer's contributions and the following debate.

[8] For a critical discussion of microfounded models in the analytical Marxist tradition, see Veneziani [37, 38]. Dutt and Sen [32, 5] have developed a sophisticated Kaleckian analysis of income distribution using a microfounded model with optimising agents. Their approach is different from ours in that they use an explicit Nash bargaining model within a static, one-period demand-constrained economy with monopolistic competition.

[9] "Micro questions ... cannot be discussed in the air without any reference to the structure of the economy in which they exist [or] to the process of cyclical and secular change. Equally, macro theories of accumulation and effective demand are generalizations about micro behaviour ... If there is no micro theory, there cannot be any macro either" (Robinson [24], p.4). For a discussion of the role of microfoundations in Kalecki's theory, see also Mott ([21], pp.11ff).

[10] It is important to stress that the purpose of our exercise is not descriptive. Our model is an abstract analytical tool to investigate *some* important causal links characterising capitalist economies. Our methodological approach is similar to Marx's analysis of the schemes of reproduction, which also provide the starting point of Kalecki's theorising. See, for example, Kalecki [11]. For a discussion see Mott [21] and Sawyer [31].

[11] This conclusion is reminiscent of an earlier argument by Devine and Dymski [3], who showed that if Roemer's [27] static model is allowed to run for many periods, capital accumulation eventually drives profits to zero. However, unlike in this paper, they focused on the T-fold iteration of the static model with myopic agents, without explicitly analysing intertemporal decisions.

of capital, and the structural constraints that private ownership of productive assets imposes on aggregate investment, technical change, unemployment, and so on.[12]

As Kalecki [7] has famously argued, for example, the relevance of unemployment in capitalist economies goes beyond its narrowly economic implications, and it has a fundamental political and power-related component. At a broad conceptual level, our paper confirms this intuition, and the centrality of unemployment in capitalist economies. For it proves that labour-saving technical progress may yield persistent exploitation by ensuring persistent abundance of labour. This result does not exhaust the analysis of possible mechanisms guaranteeing the persistence of exploitation: among other things, it depends on the empirically questionable assumption that technical progress is unbounded and reduces the use of labour to zero in the long run throughout the economy.

Yet, in line with Marx's and Kalecki's political-economic approach, and in contrast with Roemer's general equilibrium methodology, it forcefully highlights the relevance of power and unemployment in class relations and income distribution, and the role of technical change to maintain labour abundant. More generally, our analysis suggests that in order to understand classes, exploitation and growth it is necessary to incorporate the multidimensional power relations characteristic of capitalist economies.

2 The Model

The economy consists of a sequence of nonoverlapping generations. In each generation there is a set $\mathcal{N}_c = \{1, \ldots, N_c\}$ of capitalists with generic element ν, and a set $\mathcal{N}_w = \{1, \ldots, N_w\}$ of workers with generic element η. Agents live for T periods, where T can be finite or infinite, and are indexed by the date of birth kT, $k = 0, 1, 2, \ldots$ In every period t, they produce and exchange n commodities and labour. Let (p_t, w_t) denote the $1 \times (n + 1)$ price vector in t, where w_t is the nominal wage.[13]

We analyse a class-divided society and model differences in behaviour starkly.[14] In every t, each capitalist $\nu \in \mathcal{N}_c$ owns a $n \times 1$ nonnegative vector of productive assets ω_t^ν, where ω_{kT}^ν is the vector of endowments inherited, when born in kT. In every t, capitalists do not work but can hire workers in order to operate any activity of a standard Leontief technology (A, L), where A is a $n \times n$ nonnegative, productive and indecomposable matrix of material input coefficients and L is a $1 \times n$ positive vector of direct labour coefficients.[15] For every ν, y_t^ν is the $n \times 1$ vector of activity levels that ν hires workers to operate at t. In every t each capitalist ν has to use her wealth, $p_t\omega_t^\nu$, to obtain the necessary material inputs. At the end of the production period, capitalists use their net income to pay workers and to finance consumption and accumulation. Thus, for each ν, in every t, s_t^ν is the $n \times 1$ vector of net savings and c_t^ν is the $n \times 1$ consumption vector.

The choices available to workers are much more limited. On the one hand, their class position constrains the economic activities they engage in. Let $\mathbf{0}$ be the null vector. Each worker $\eta \in \mathcal{N}_w$ possesses no physical capital, $\omega_t^\eta = \mathbf{0}$ in every t, but is endowed with one unit of (homogeneous) labour. Therefore workers obtain income only by supplying labour, and use their income only to purchase consumption goods. To be precise, at all t, z_t^η is η's labour supply and c_t^η is η's $n \times 1$ consumption vector. On the other hand, the (work and consumption) choices available to workers

[12] The central role of private ownership of the means of production and the concentration of wealth in a few hands in capitalist economies is discussed, for example, in Kalecki [14]. As Mott ([21], p.14) aptly notes "The fact that Kalecki's models are couched in terms of income distribution and based on reproduction schemes ... is no accident. The distribution and reproduction of ownership claims is what governs the macroeconomy". See also Sawyer [31].

[13] Throughout the paper, all variables and vectors are assumed to belong to a Euclidean space $\mathbb{R}^k$ of appropriate dimensionality k.

[14] Given our theoretical focus, and consistently with Marx's analysis of the schemes of reproduction, we abstract from the public sector and foreign trade.

[15] In the basic model, we assume technology to remain unchanged over time. We introduce technical progress in section 5 below. The assumption of constant returns to scale is theoretically appropriate and consistent with both Marx's and Kalecki's analysis (see Dutt and Sen [5]).

are limited by the structural features of capitalist economies and in particular by the presence of structural unemployment. Formally, for all $\eta \in \mathcal{N}_w$, in every t, there exists an upper bound $\widehat{z}_t^\eta$ to η's labour supply, which is determined by demand conditions.

Class differences also affect consumption choices and consumption opportunities. We assume that, at all t, $c_t^\nu \geqq \mathbf{0}$ for all $\nu \in \mathcal{N}_c$,[16] while there exists a reference consumption bundle $b > \mathbf{0}$, such that $c_t^\eta \geqq b$.[17] This incorporates the idea that capitalists are not essential and, together with the assumption that $\omega_{kT}^\eta = \mathbf{0}$, all $\eta \in \mathcal{N}_w$, and classical saving habits, it starkly outlines class differences.[18]

Although some aspects of behaviour are determined by class differences, we rule out heterogeneity in the (subjective or objective) evaluation of individual welfare. Formally, there is a continuous, strictly increasing, strictly quasi-concave, and homogeneous of degree one function $\phi : \mathbb{R}_+^n \to \mathbb{R}_+$, such that $\phi(c_t^h)$ describes agent h's welfare at t, where $h = \nu, \eta$.[19] The function ϕ can be interpreted as an objectivist measure of agents' well-being: given Roemer's emphasis on the normative implications of exploitation theory, for example, the n goods in this economy can be naturally interpreted as Rawlsian primary goods.[20] Alternatively – and equivalently, from a formal viewpoint – it might be interpreted as a neoclassical utility function. In any case, the assumptions on ϕ reflect the theoretical focus on class-related consumption possibilities, rather than individual consumer choice and are consistent with classical savings habits.[21]

Intertemporal trade between agents is ruled out, consistently with the lack of a pure accumulation motive – that is, the desire to maximise capital accumulation per se, which is often assumed in Marxist models (e.g., Morishima [19]; Roemer [26]). Unlike in traditional Marxist models, capitalists do not aim to maximise accumulation of capital per se, and production does not take place "for production's own sake" (Luxemburg [18], p.333). However, Roemer's [26, 27] static models are generalised by allowing for savings and intertemporal trade-offs during an agent's life.

Let $(p, w) = \{p_t, w_t\}_{t=kT,\dots,(k+1)T-1}$ denote the path of the price vector during the lifetime of a generation. Let $y^\nu = \{y_t^\nu\}_{t=kT,\dots,(k+1)T-1}$ denote ν's lifetime plan of activity levels and let a similar notation hold for c^ν, s^ν, ω^ν, z^η, and c^η. As a shorthand notation, let "all t" stand for "all t, $t = kT, \dots, (k+1)T-1$." Let $0 < \beta \leqq 1$ be the discount factor. Capitalist ν is assumed to choose $\xi^\nu = (y^\nu, c^\nu, s^\nu)$ to maximise lifetime welfare subject to the constraint that (1) net revenues are sufficient for consumption and savings, all t; (2) wealth is sufficient for production plans, all t; (3) the dynamics of assets is determined by net savings, all t; (4) ν's descendants receive at least as many resources as she inherited. Formally, agent $\nu \in \mathcal{N}_c$ solves the following maximisation programme (MP^ν), whose value is denoted as $C(\omega_{kT}^\nu)$.

$$MP^\nu\!: \; C(\omega_{kT}^\nu) = \max_{\xi^\nu} \sum_{t=kT}^{(k+1)T-1} \beta^t \phi\left(c_t^\nu\right),$$

[16] For all vectors $x, y \in \mathbb{R}^p$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \dots, p)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \dots, p)$.

[17] The reference vector b does not identify a *physical* subsistence bundle. Rather, we interpret it as a socially-determined basic consumption standard which must be reached in order for workers to supply labour in the capitalist sector. We assume b to be constant over time, but the model can be generalised to incorporate a time-varying b_t reflecting evolving social norms, culture, and so on. See, for example, Cogliano et al [1].

[18] In a less schematic model, if profits fall below some threshold, capitalists would start to work.

[19] We normalise the function ϕ by assuming that $\phi(c_t^h) = 0$ whenever $c_{it}^h = 0$ for some good i.

[20] Silvestre [33] makes similar assumptions in the construction of an index of primary goods.

[21] Further, the assumptions on ϕ make consumption behaviour in our model analogous to that in Sraffian models (e.g. Kurz and Salvadori [16], p.102).

subject to

$$
\begin{aligned}
\left[p_t\left(I-A\right)-w_t L\right] y_t^\nu &\geqq p_t c_t^\nu + p_t s_t^\nu, && (1)\\
p_t A y_t^\nu &\leqq p_t \omega_t^\nu, && (2)\\
\omega_{t+1}^\nu &= \omega_t^\nu + s_t^\nu, && (3)\\
\omega_{(k+1)T}^\nu &\geqq \omega_{kT}^\nu, && (4)\\
y_t^\nu &\geqq \mathbf{0}, \omega_t^\nu \geqq \mathbf{0}, c_t^\nu \geqq \mathbf{0}.
\end{aligned}
$$

Similarly, worker $\eta \in \mathcal{N}_w$ chooses $\xi^\eta = (z^\eta, c^\eta)$ to maximise welfare subject to the constraint that at all t: (5) revenues are sufficient for η's consumption; and (6) subsistence is reached. Furthermore, at all t, (7) workers' labour supply is constrained both by their labour endowment *and* by labour market conditions, as captured by the exogenously given parameter $\widehat{z}_t^\eta$. Formally, agent $\eta \in \mathcal{N}_w$ solves the following maximisation programme (MP^η).

$$MP^\eta\colon \max_{\xi^\eta} \sum_{t=kT}^{(k+1)T-1} \beta^t \phi\left(c_t^\eta\right),$$

subject to

$$
\begin{aligned}
w_t z_t^\eta &\geqq p_t c_t^\eta, && (5)\\
c_t^\eta &\geqq b, && (6)\\
\min\left[1, \widehat{z}_t^\eta\right] &\geqq z_t^\eta \geqq 0. && (7)
\end{aligned}
$$

The optimisation programmes MP^ν and MP^η allow us to investigate Roemer's [26, 27] claim that exploitation can be reduced to asset inequalities in a dynamic context. For, given the absence of capital markets and of any bequest motive, they are a natural generalisation of Roemer's [26, 27] static profit or revenue maximisation programmes.

More generally, the explicit modelling of individual behaviour in MP^ν and MP^η allows us to analyse the behavioural foundations of the macrodynamics, and the interaction between microeconomic conditions and aggregate outcomes.[22] It is worth stressing, however, that the behavioural assumptions incorporated in MP^ν and MP^η are very different from standard neoclassical models. First, as already noted, we drop the representative agent assumption and, in line with the neo-Marxian and Post-Keynesian traditions, explicitly incorporate class-based heterogeneity in consumption and working behaviour, and in particular in saving habits. Our model of workers' behaviour, for example, bears a close conceptual relation with Kalecki's assumptions on the irrelevance of savings out of workers income. Indeed, Kalecki's view of Marx's schemes of reproduction "pushed him to identify the main relations of the circulation of capital as behavioural relations expressing the decision of capitalists while labour is seen as merely passive having no power on the use of productive means. The crucial driving force of economic dynamics is thus seen in the feedback between the accumulation of capital and profits" (Sordi and Vercelli [35], p.148).[23]

Secondly, although MP^ν and MP^η model the behavioural microfoundations of macroeconomic outcomes, there is no unilateral causation from the micro to the macro level. For macroeconomic constraints directly influence individual behaviour: for example, accumulation and labour market conditions determine the set of choices available to workers.

Let $\Omega_{kT} = \left(\omega_{kT}^1, \omega_{kT}^2, ..., \omega_{kT}^{N_c}\right)$. Let $E\left(\left(\mathcal{N}_c, \mathcal{N}_w\right), \left(A, L\right), \Omega_{kT}, \left(\beta, \phi\right)\right)$, or as a shorthand notation E_{kT}, denote the economy with population $(\mathcal{N}_c, \mathcal{N}_w)$, technology (A, L), endowments Ω_{kT}, discount factor β and welfare function ϕ. At all t, let $y_t = \sum_{\nu \in \mathcal{N}_c} y_t^\nu$, $c_t^c = \sum_{\nu \in \mathcal{N}_c} c_t^\nu$, $\omega_t = \sum_{\nu \in \mathcal{N}_c} \omega_t^\nu$, $s_t = \sum_{\nu \in \mathcal{N}_c} s_t^\nu$, $c_t^w = \sum_{\eta \in \mathcal{N}_w} c_t^\eta$, and $z_t = \sum_{\eta \in \mathcal{N}_w} z_t^\eta$. The equilibrium concept can now be defined.

[22] After all, as Kalecki ([10], p.455) famously noted "capitalists do many things as a class, but they certainly do not invest as a class."

[23] For a thorough discussion, see also Sawyer ([31], chapter 8).

Definition 1: A *reproducible solution* (RS) for E_{kT} is a price vector (p, w) and an associated set of actions $((\xi^\nu)_{\nu\in\mathcal{N}_c}, (\xi^\eta)_{\eta\in\mathcal{N}_w})$ such that:

(i) ξ^ν solves MP^ν for all $\nu \in \mathcal{N}_c$;
(ii) ξ^η solves MP^η for all $\eta \in \mathcal{N}_w$;
(iii) $y_t \geqq Ay_t + c_t^c + c_t^w + s_t$, for all t;
(iv) $Ay_t \leqq \omega_t$, for all t;
(v) $Ly_t = z_t$, for all t;
(vi) $\omega_{(k+1)T} \geqq \omega_{kT}$.

Conditions (i) and (ii) require agents to optimise given the individual *and* the aggregate constraints limiting their choices; (iii) and (iv) require that there be enough resources for consumption and saving plans, and for production plans, respectively, at all t; (v) states that the amount of labour performed in the economy must be sufficient for production plans at all t; (vi) requires that resources not be depleted by any given generation.

Definition 1 is an intertemporal generalisation of the concept of RS first defined by Roemer [25]. It provides a general notion of Marxian equilibrium, and may be conceived of as a special type of general equilibrium. However, there are some important differences with standard Walrasian equilibrium concepts. First, as already noted, individual behaviour is different from standard neoclassical models, given the aggregate and class-determined constraints on choices. "[I]n Marx's conception workers had little latitude in making consumption decisions. ... Capitalists optimize, but workers are forced to take what they can get; they live in a world where any optimizing they may do obfuscates the narrow boundaries of their behavior" (Roemer [25], p.509).[24]

Second, the concept of RS does not impose market clearing, and allows for an aggregate excess supply of produced goods, and, crucially, labour. This is consistent with the theoretical emphasis in Marxian and Kaleckian analyses on the conditions for the "reproducibility" of the economic system at the heart, for example, of the Marxian reproduction schema. As Roemer ([25], p.507) put it, in Marxian analysis, "The concern is with whether the economic system can reproduce itself: whether it can produce enough output to replenish the inputs used, and to reproduce the workers for another period of work. ... Marx's investigation of the laws of motion of capitalist society attempts to uncover how capitalist society reproduces itself".

Thus, Definition 1(v) is an ex post condition consistent with the existence of involuntary unemployment. For, although workers choose their labour supply optimally and aggregate labour supply equals labour demand ex post, labour market conditions act as a constraint on workers' choices ex ante in condition (7). In fact, by the monotonicity of ϕ, in our framework, the standard labour market clearing condition at t requires $Ly_t = N_w$, whereas involuntary unemployment occurs at t whenever $Ly_t = z_t < N_w$. Therefore, we say that a RS is *unconstrained* if $Ly_t = z_t = N_w$, for all t, while a RS is *constrained* if there exists some t' such that $N_w > z_{t'} = Ly_{t'}$. Because workers are identical, we assume that at a constrained RS, all of them work an equal amount of time which allows them to reach subsistence. Given the absence of a subsistence sector and of the public sector, this seems an appropriate way of capturing unemployment in this model. Formally, if a RS is constrained at t', then $\widehat{z}_{t'}^\eta = \frac{Ly_{t'}}{N_w}$ and $c_t^\eta = b$, all $\eta \in \mathcal{N}_w$.

Given the focus on the persistence of exploitation and profits, the subset of RSs with stationary capital will be of particular interest. A *stationary reproducible solution* (SRS) for E_{kT} is a RS such that, at all t, $c_t^\nu = c^\nu$ and $s_t^\nu = \mathbf{0}$, all $\nu \in \mathcal{N}_c$, and $c_t^\eta = c^\eta$, all $\nu \in \mathcal{N}_w$.

Definition 2 captures the idea of capital scarcity as requiring that "the total supply of productive assets is limited, relative to current demand" (Skillman [34], p.1, fn.1).[25]

[24] A similar view underlies Kalecki's theorising. See Sawyer ([31], chapter 8) for a thorough discussion.

[25] To be precise, a RS should be denoted as $\left((p, w), ((\xi^\nu)_{\nu\in\mathcal{N}_c}, (\xi^\eta)_{\eta\in\mathcal{N}_w})\right)$. In what follows, we simply write (p, w) for the sake of notational simplicity.

Definition 2: Let (p, w) be a RS for E_{kT}. The economy E_{kT} is said to exhibit *capital scarcity* at (p, w), in period t, if and only if $p_t A y_t^\nu = p_t \omega_t^\nu$, all $\nu \in \mathcal{N}_c$. If $p_t A y_t^\nu < p_t \omega_t^\nu$, some $\nu \in \mathcal{N}_c$, then capital is said to be *abundant* at (p, w), in period t.

3 Exploitation and profits

We begin our analysis by deriving some preliminary results concerning the properties of RS's. Two properties immediately follow from the monotonicity of ϕ. First, because at the solution to MP^ν, $\omega^\nu_{(k+1)T} = \omega^\nu_{kT}$, all $\nu \in \mathcal{N}_c$, if (p, w) is a RS for E_{kT}, then it is also a RS for $E_{(k+1)T}$. Hence, we can interpret (p, w) as a steady state solution and focus on E_0 without loss of generality. Second, at any RS, it must be $p_t > \mathbf{0}$ and $w_t > 0$, all t.

Then, it is immediate to show that at a RS for E_0, in every period, constraints 1 and 2 are binding for all capitalists.

Lemma 1: *Let (p, w) be a RS for E_0. Then, for all t:*
(i) $[p_t (I - A) - w_t L]\, y_t^\nu = p_t c_t^\nu + p_t s_t^\nu$, all $\nu \in \mathcal{N}_c$;
(ii) if $p_t \geq p_t A + w_t L$, then $p_t A y_t^\nu = p_t \omega_t^\nu$, all $\nu \in \mathcal{N}_c$.

Let the profit rate of sector i at t be denoted as $\pi_{it} = \frac{[p_t(I-A)-w_tL]_i}{p_tA_i}$. The next Lemma proves that at a RS, in every period profits are nonnegative and profit rates are equalised across sectors.

Lemma 2: *Let (p, w) be a RS for E_0. Then, at all t, $\pi_{it} \geqq 0$, for at least some i. Furthermore, if either $\pi_{it} > 0$, some i, or $c_t^c + c_t^w + s_t > \mathbf{0}$,* then $\pi_{it} = \pi_t$, all i.

Proof. 1. Suppose that there is some t such that $p_{it} < p_t A_i + w_t L_i$, all i. Then $y_{it}^\nu = 0$, all i, $\nu \in \mathcal{N}_c$, for all ξ^ν that solve MP^ν, and thus $y_t = 0$. By Definition 1(v), this implies $z_t^\eta = 0$, all $\eta \in \mathcal{N}_w$, which violates Definition 1(ii).

2. Suppose that there is some t such that either $\pi_{it} > 0$, some i, or $c_t^c + c_t^w + s_t > \mathbf{0}$, but $\pi_{lt} < \pi_{jt}$, some j, l. Because wealth is used only to activate maximum profit rate processes, it follows that for all $\nu \in \mathcal{N}_c$, $y_{lt}^\nu = 0$, for all ξ^ν that solve MP^ν, and thus $y_{lt} = 0$. However, under the hypotheses stated, $A_l y_t + c_{lt}^c + c_{lt}^w + s_{lt} > \mathbf{0}$, which contradicts Definition 1(iii). ■

By Lemma 2, at a SRS, $\pi_{it} = \pi_t$, all i and all t. More generally, at any RS such that $\pi_{it} = \pi_t$, all i, we can consider price vectors such that $p_t = (1 + \pi_t) p_t A + w_t L$, all t. Furthermore, following standard practice in Kaleckian models, labour can be chosen as the numéraire, setting $w_t = 1$, all t, and in what follows we focus on RS's of the form $(p, \mathbf{1})$, where $\mathbf{1} = (1, ..., 1)'$.

Let $\lambda = (I - A)^{-1}$ be the $1 \times n$ vector of labour values. Let $y = \sum_{t=0}^{T-1} y_t$ and $c^w = \sum_{t=0}^{T-1} c_t^w$. Generalising Roemer [26, 27], *Socially Necessary Labour Time at t* is defined as the amount of labour embodied in workers' consumption, λc_t^w. Similarly, considering the whole life of a generation, *Socially Necessary Labour Time* is defined as λc^w. Then, Roemer's definition of exploitation can be extended to the intertemporal context.[26]

Definition 3: The within-period (WP) exploitation rate at t is $e_t = \frac{(Ly_t - \lambda c_t^w)}{\lambda c_t^w}$ and the whole-life (WL) exploitation rate is $e = \frac{(Ly - \lambda c^w)}{\lambda c^w}$.

As argued in Veneziani [36], both definitions convey morally relevant information, but the WP definition is more pertinent in a Marxian approach and it is more interesting in a dynamic context.

The *Dynamic Fundamental Marxian Theorem* can now be proved.[27]

[26] For a discussion of various definitions of the exploitation rate, see Desai ([2], p.48). If technology changed over time, the definition of WL exploitation rate would need to be adjusted accordingly.

[27] The proofs of all theorems are in Appendix A.

Theorem 1 *Let $(p, \mathbf{1})$ be a RS for E_0 with $\pi_{it} = \pi_t$, all i and all t . Then (i) at all t, $e_t > 0$ if and only if $\pi_t > 0$. Furthermore, (ii) $e > 0$ if and only if $\pi_t > 0$, some t.*

Theorem 1 generalises the FMT in a dynamic context. It shows that, given private ownership of productive assets, profits are a counterpart of the transfer of social surplus and social labour from asset-poor agents to wealthy ones and a general correspondence exists between positive profits and the exploitation of the working class. Thus, Theorem 1 establishes a link between the essence of capitalist social relations - and in particular the wage relation and the existence of profits - and the normative wrongs associated with the exploitation of labour.

Theorem 1 suggests that there is no RS with persistent accumulation and persistent exploitation. In fact, if $e_t > 0$, all t, then by Theorem 1 and Lemma 1(ii), and noting that $p_t > \mathbf{0}$, at a RS $Ly_t = LA^{-1}\omega_t$, all t. By Definition 1(ii) and (v), it must therefore be $LA^{-1}\omega_t \leqq N_w$, all t. Hence, if $\omega_{t+1} > \omega_t$, all t, $T-1 > t \geqq 0$, then $LA^{-1}\omega_t < N_w$, and the RS is constrained at all t, $T-1 > t \geqq 0$. Therefore $c_t^\eta = b$, all η, and $p_t b = \frac{LA^{-1}\omega_t}{N_w}$, all t, $T-1 > t \geqq 0$. By Lemma 1(i), and noting that $p_t > \mathbf{0}$, at a RS $(I-A)y_t = s_t + c_t^c + c_t^w$, all t, which implies $s_t = (I-A)A^{-1}\omega_t - c_t^c - c_t^w$, all t, or by the previous arguments, $\omega_{t+1} = A^{-1}\omega_t - c_t^c - N_w b$, all t, $T-1 > t \geqq 0$.

Given the linearity of MP^ν, there is at most one period in which, for any $\nu \in \mathcal{N}_c$, at the solution to MP^ν, both savings and consumption are positive at a constrained RS with accumulation.[28] Hence, given that capitalists are identical there is a period τ such that $c_t^c = \mathbf{0}$, all $t \geqq \tau$, and $\omega_{t+1} = A^{-1}\omega_t - N_w b$, all $t \geqq \tau$, which implies $\omega_t = \left(A^{-1}\right)^{t-\tau}[\omega_\tau - \omega_S] + \omega_S$, all $t \geqq \tau$, where $\omega_S = N_w A(I-A)^{-1}b$. Thus, by the productivity of A, given that workers' subsistence requires $\omega_t \geqq \omega_S$, all t, if T is sufficiently big, labour demand exceeds supply after a finite number of periods, driving π_t and e_t to zero. This can be summarised as follows.

Proposition 1: *For all $T > 0$, there is a $T' > T$ such that there is no RS with $\omega_{t+1} > \omega_t$, all t, $T'-1 > t \geqq 0$, and $e_t > 0$, all t.*

In other words, persistent accumulation and persistent exploitation and profits are inconsistent. At a broad conceptual level, this conclusion echoes Kalecki's famous argument about capitalists' negative attitudes towards policies that promote growth and full employment. For Proposition 1 suggests that, absent any countervailing measures that preserve their economic and social power, capitalists *as a class* will be concerned with any long-run sustained accumulation that may significantly reduce capital scarcity, even though *individually* they may regard growth paths favourably.

4 Inequalities, Exploitation, and Time Preference

This section analyses the dynamic foundations of exploitative relations, focusing on stationary reproducible solutions. This is due to the theoretical relevance of SRS's, as discussed in Veneziani [36], but also because SRS's represent a benchmark solution whereby the labour market clears at all t. Lemma 3 provides a necessary condition for the existence of a SRS.

Lemma 3: Let $(p, \mathbf{1})$ be a SRS for E_0 with $\pi_t > 0$, all t. Then $\beta(1+\pi_{t+1}) = 1$, all t.

Proof. 1. For all $\nu \in \mathcal{N}_c$, by Lemma 1, at any RS with $\pi_t > 0$, all t, it must be $p_t c_t^\nu = \pi_t p_t \omega_t^\nu - p_t s_t^\nu$, all t. At a SRS, the latter expression becomes $p_t c^\nu = \pi_t p_t \omega_0^\nu$, all t, ν, which implies $c^\nu \geq \mathbf{0}$.

2. Suppose, by way of contradiction, that $\beta(1+\pi_{t'+1}) > 1$, some $t' < T-1$. Take any capitalist $\nu \in \mathcal{N}_c$. Consider a one-period perturbation of ν's optimal choice such that $p_{t'}\mathrm{d}c_{t'}^\nu = -p_{t'}\mathrm{d}s_{t'}^\nu$, $p_{t'+1}\mathrm{d}c_{t'+1}^\nu = \pi_{t'+1}p_{t'+1}\mathrm{d}\omega_{t'+1}^\nu - p_{t'+1}\mathrm{d}s_{t'+1}^\nu$, $\mathrm{d}\omega_{t'+1}^\nu = \mathrm{d}s_{t'}^\nu = -\mathrm{d}s_{t'+1}^\nu$.

3. Since ϕ is homothetic, $c_t^\nu = c^\nu$ implies that at a SRS, at all t it must be $p_{t+1} = k_t p_t$ for some $k_t > 0$. Therefore consider $\mathrm{d}c_{t'}^\nu = h_{t'}c^\nu$ and $\mathrm{d}c_{t'+1}^\nu = h_{t'+1}c^\nu$ for some $h_{t'}, h_{t'+1} \geq 0$, and the one period perturbation can be written as $h_{t'}p_{t'}c^\nu = -p_{t'}\mathrm{d}s_{t'}^\nu$ and $h_{t'+1}p_{t'}c^\nu = \pi_{t'+1}p_{t'}\mathrm{d}s_{t'}^\nu + p_{t'}\mathrm{d}s_{t'}^\nu$.

[28] This is proved rigorously below; see e.g. the analysis of MP^ν in the proof of Theorem 4.

4. By the homogeneity of ϕ it follows that $\phi\left(c^{\nu}+\mathrm{d}c^{\nu}_{t'}\right)+\beta\phi\left(c^{\nu}+\mathrm{d}c^{\nu}_{t'+1}\right)=(1+h_{t'})\,\phi\left(c^{\nu}\right)+(1+h_{t'+1})\,\beta\phi\left(c^{\nu}\right)>\phi\left(c^{\nu}\right)+\beta\phi\left(c^{\nu}\right)$ if and only if $h_{t'}+h_{t'+1}\beta=\left[-1+\beta\left(1+\pi_{t'+1}\right)\right]\frac{p_{t'}}{p_{t'}c^{\nu}}\mathrm{d}s^{\nu}_{t'}>0$. Therefore, if $\beta(1+\pi_{t'+1})>1$, there is a sufficiently small $\mathrm{d}s^{\nu}_{t'}$ with $p_{t'}\mathrm{d}s^{\nu}_{t'}>0$ such that $h_{t'}+h_{t'+1}\beta>0$, a contradiction. A similar argument holds if $\beta(1+\pi_{t'+1})<1$, some $t'<T-1$. ■

Intuitively, if $\beta(1+\pi_{t'+1})>1$, some t', then the cost (in terms of overall welfare) of reducing consumption at t' is lower than the benefit of saving, producing and consuming in $t'+1$, and vice versa if $\beta(1+\pi_{t'+1})<1$. Only if $\beta(1+\pi_{t'+1})=1$ are costs and benefits equal.

Let $\frac{1}{1+\widetilde{\pi}}$ be the Frobenius eigenvalue of A: by the assumptions on A, $\widetilde{\pi}>0$. Let $\pi_{\beta}\equiv\frac{1-\beta}{\beta}$ and let p_{β} denote the solution of $p=(1+\pi_{\beta})pA+L$: for all $\pi_{\beta}\in[0,\widetilde{\pi})$, p_{β} is well defined and strictly positive. By the homotheticity of ϕ, let c_{β} denote a vector identifying the optimal proportions of the different consumption goods corresponding to p_{β}.[29] Theorem 2 analyses MP^{ν}.[30]

Theorem 2 *(i) Let* $1>\beta>\frac{1}{1+\widetilde{\pi}}$. *If* $\pi_t=\pi_{\beta}$, *all* t, *then for all* $\nu\in\mathcal{N}_c$ *there is an optimal* ξ^{ν} *such that* $s^{\nu}_t=\mathbf{0}$, *all* t. *Moreover, if* T *is finite,* $C(\omega^{\nu}_0)=\phi(c_{\beta})(1-\beta^T)\frac{p_{\beta}\omega^{\nu}_0}{\beta p_{\beta}c_{\beta}}$, *while if* $T\to\infty$, $C(\omega^{\nu}_0)=\phi(c_{\beta})\frac{p_{\beta}\omega^{\nu}_0}{\beta p_{\beta}c_{\beta}}$. *(ii) Let* $\beta\leqq 1$. *If* $\pi_t=0$, *all* t, *then for all* $\nu\in\mathcal{N}_c$ *there is an optimal* ξ^{ν} *such that* $s^{\nu}_t=\mathbf{0}$, *all* t, *and* $C(\omega^{\nu}_0)=0$.

Given Theorem 2, the next result proves the existence of a SRS.[31]

Theorem 3 *Let* $\omega_0=\gamma_0 N_w A(I-A)^{-1}b$, $\gamma_0>1$. *Let* $\lambda b<1$. *Let* π' *be defined by* $\gamma_0\lambda b=L[I-(1+\pi')A]^{-1}b$.

(i) Let $\gamma_0\lambda b<1$. *If* $\beta(1+\pi')=1$ *and* $c_{\beta}=kb$ *for some* $k>0$, *there is a SRS for* E_0 *with* $\pi_t=\pi'$, *all* t;

(ii) Let $\gamma_0\lambda b=1$. *Let* $\beta\in[\frac{1}{(1+\pi')},1)$ *be such that* $c_{\beta}=kb$ *for some* $k>0$. *Then there is a SRS for* E_0 *with* $\pi_t=\pi_{\beta}$, *all* t;

(iii) Let $\gamma_0\lambda b\leqq 1$. *If* $\beta=1$, *there is a SRS for* E_0 *with* $\pi_t=0$, *all* t. *Further, there is no SRS with* $\pi_t>0$, *some* t.

Remark 1 *By Lemma 3, Theorem* **3***(i)-(ii) identify the only class of SRS's with* $\pi_t>0$ *all* t.

Theorem 3 significantly strengthens and extends the results in Veneziani [36]. It provides a dynamic generalisation of Roemer's theory of exploitation: provided initial assets are above the minimum barely sufficient to guarantee workers' subsistence ($\gamma_0>1$),[32] the dynamic economy with maximising agents displays persistent exploitation – and possibly persistent unemployment, – if net revenues are consumed at all t *and* capitalists discount future consumption (Theorem 3(i)-(ii)). However, this result crucially depends on a strictly positive rate of time preference (Theorem 3(iii)). Further, if $\gamma_0\lambda b=1$, the magnitude of inequalities and exploitation will also depend on β.[33]

The results presented in this section raise serious doubts on the view that competitive markets and asset inequalities are necessary and sufficient institutions to generate exploitation as a persistent feature of capitalist economies, and therefore exploitation theory reduces to 'a kind of resource

[29] The vector c_{β} is determined up to a scalar transformation. If ϕ'_i denotes the partial derivative of ϕ with respect to the i–th entry, then $\frac{\phi'_i(c_{\beta})}{\phi'_j(c_{\beta})}=\frac{p_{i\beta}}{p_{j\beta}}$, for all i,j.

[30] In the case with $\pi_t=0$, all t, Theorem 2 does not rule out the possibility that for some $\nu\in\mathcal{N}_c$, $s^{\nu}_t\neq\mathbf{0}$, for some t, at the solution to MP^{ν}. However, for all $\nu\in\mathcal{N}_c$ at any ξ^{ν} that solves MP^{ν}, it must be $\lambda s^{\nu}_t=0$, all t.

[31] The restriction $\omega_0=\gamma_0 N_w A(I-A)^{-1}b$ is necessary given the linearity of MP^{ν} and MP^{η}. No theoretical conclusion depends on this restriction, which in any case encompasses a rather large set of economies.

[32] It is not difficult to show that if $\gamma_0=1$, then the only RS for E_0 requires $\pi_t=0$ and $s_t=\mathbf{0}$, all t.

[33] Theorems 2-3 also characterise inter-capitalist inequalities as a different phenomenon from exploitation. In fact, at a SRS with $\pi_t=\frac{1-\beta}{\beta}>0$, all t, by Theorem 2 for any two capitalists ν and μ, $C(\omega^{\nu}_0)>C(\omega^{\mu}_0)$ if and only if $p'\omega^{\nu}_0>p'\omega^{\mu}_0$. Instead, if $\pi_t=0$, all t, then $C(\omega^{\nu}_0)=0$, all t.

egalitarianism' (Roemer [29], p.2). For they prove that, absent time preference, exploitation is not a persistent feature of the economy, *even when* wealth inequalities and capital scarcity endure. Therefore asset inequalities *per se* are not a sufficient statistic of the unfairness of labour/capital relations. Something else is indispensable to make exploitation persist, which is therefore normatively as important as asset inequalities themselves. Formally, one may interpret the above results as suggesting that time preference may be the missing ingredient. Yet, the theoretical and normative relevance of time preference is rather unclear, especially in the context of exploitation theory.

Whether β is interpreted as reflecting subjective time preference, or as incorporating a normative view of intertemporal justice, the general normative significance of time preference has been questioned by many economists and political philosophers (see, for example, the classic analysis by Rawls [23]). In exploitation theory, the significance of time preference seems even more controversial. An explanation of the normative foundations of persistent exploitation based on time preference is far from Marx's own approach. And as Roemer ([28], pp.60ff) himself has noted, the normative relevance of a theory of exploitation critically relying on such exogenous factors would be rather unclear. In Marxian theory, the exploitation of labour is an inevitable consequence of the structural features of capitalist economies rather than empirically contingent features such as time discounting.

In the next section, we explore further the foundations of persistent exploitation and the relation between growth and exploitation in capitalist economies.

5 Stable Growth and Distribution

In this section, in order to focus on the key theoretical issues and on macrodynamics, we consider a special case of the n-good economies analysed thus far by setting $n = 1$. The model and notation remain the same, with obvious adaptations and letting ϕ be the identity function.[34] Further, we restrict our attention to the empirically relevant case of economies in which T can be arbitrarily large but remains finite.

Sections 3-4 suggest that asset inequalities (and competitive markets) cannot fully explain exploitative relations in dynamic capitalist economies. Absent the asymmetric relations of power that arguably characterise capitalist economies, persistent growth and exploitation are inconsistent and even if the economy does not grow, persistent exploitation is possible only if $\beta < 1$. This section explores further the relation between exploitation, time preference, and growth, by focusing on stable growth paths in which the economy grows for a certain number of periods and eventually reaches a steady state.[35]

Definition 4: A *stable growth path* (SGP) for E_0 is a RS such that there is a period $t' > 0$ such that $\omega_{t+1} = (1 + g_t)\omega_t$, $g_t > 0$, for all $t < t'$, and $\omega_{t+1} = \omega_t$, all $t, T - 1 > t \geqq t'$.

For all t, let $\omega_t = \gamma_t N_w A(1 - A)^{-1} b$, so that any conditions on aggregate capital ω_t can be equivalently expressed as conditions on γ_t. Lemma 4 confirms the relevance of SRS's as a theoretical benchmark: only at a SRS can equilibrium in the labour market and exploitation exist at all t.

Lemma 4: If $(p, \mathbf{1})$ is an unconstrained RS for E_0 such that the economy exhibits capital scarcity at t, then $\gamma_t \lambda b = 1$.

Proof. At a RS with capital scarcity at t, it must be $y_t = A^{-1}\omega_t$. Therefore, $Ly_t = \gamma_t N_w \lambda b$, and since the RS is unconstrained, $Ly_t = z_t = N_w$, which holds if and only if $\gamma_t \lambda b = 1$. ■

[34] The main conclusions of this section can be extended to n-good economies, albeit at the cost of a significant increase in technicalities. Indeed, the main definitions and propositions are formulated so as to suggest the relevant n-good extensions.

[35] Observe that if $T = 2$, then at any SGP the condition in the second part of Definition 4 is vacuously satisfied.

In general, if a RS is unconstrained from t' onwards, then $\gamma_t \lambda b = 1$, all $t \geqq t'$, and thus SRS's are a natural benchmark for all accumulation paths with persistent capital scarcity, which lead to a stationary state with equilibrium in the labour market. Instead, if $\gamma_t \lambda b < 1$, the economy is constrained at t. Proposition 2 rules out paths where capital becomes abundant.

Proposition 2: Let $\gamma_0 > 1$ and $\gamma_0 \lambda b \leqq 1$. Suppose $\beta < 1$. Then there is no RS such that there exists a period $\widehat{t}$ such that the economy exhibits capital scarcity at all $t \leqq \widehat{t}$ but $LA^{-1}\omega_{\widehat{t}+1} > N_w$.

Proof: 1. Suppose that there is a RS such that $LA^{-1}\omega_{\widehat{t}} \leqq N_w$ but $LA^{-1}\omega_{\widehat{t}+1} > N_w$, some $\widehat{t}$. Then $\pi_{\widehat{t}} > 0$ but $\pi_{\widehat{t}+1} = 0$ since capital is abundant at $\widehat{t}+1$.

2. For all $\nu \in \mathcal{N}_c$, $c^\nu_{\widehat{t}} = \pi_{\widehat{t}}\omega^\nu_{\widehat{t}} - s^\nu_{\widehat{t}}$ and $c^\nu_{\widehat{t}+1} = -s^\nu_{\widehat{t}+1} \geqq 0$. If $s^\nu_{\widehat{t}+1} < 0$, some $\nu \in \mathcal{N}_c$, then since $\beta\left(1+\pi_{\widehat{t}+1}\right) < 1$, there is a feasible perturbation of the savings path with $ds^\nu_{\widehat{t}} = -ds^\nu_{\widehat{t}+1} < 0$, which increases ν's welfare, contradicting optimality.

3. Let $s^\nu_{\widehat{t}+1} = 0$, all $\nu \in \mathcal{N}_c$. Since $s_{\widehat{t}+1} = 0$ then $\omega_{\widehat{t}+2} = \omega_{\widehat{t}+1}$, so that $\pi_{\widehat{t}+2} = 0$ and $\beta\left(1+\pi_{\widehat{t}+2}\right) < 1$. Again, for all $\nu \in \mathcal{N}_c$, $s^\nu_{\widehat{t}+2} < 0$ cannot be optimal. Therefore $s^\nu_{\widehat{t}+2} = 0$, all $\nu \in \mathcal{N}_c$, and $\pi_{\widehat{t}+3} = 0$; and so on.

4. By construction, $\omega_{\widehat{t}+1} > \omega_0$. Hence, individual optimality implies $\sum_{l=\widehat{t}+1}^{T-1} s^\nu_l < 0$, all $\nu \in \mathcal{N}_c$, which contradicts $s^\nu_l = 0$, for all $\nu \in \mathcal{N}_c$ and all $T-1 \geqq l \geqq \widehat{t}+1$. ■

Proposition 2 shows that overaccumulation is not an equilibrium because the fall of the profit rate to zero would rather lead capitalists to anticipate consumption, if $\beta < 1$. Indeed, Proposition 2 confirms the importance of time preference for the persistence of exploitation in Roemer's theory, and more generally in abstract, power-free competitive settings: if $\beta = 1$, overaccumulation and profits falling to zero are not ruled out.

Given Proposition 2, Theorem 4 characterises stable growth paths.

Theorem 4 *Let $\gamma_0 > 1$. Let $(p, \mathbf{1})$ be a SGP for E_0 such that $\gamma_t \lambda b \leqq 1$, all t. At all t, define $g'_t = \frac{[(\gamma_t - 1)N_w b - c^c_t]\widetilde{\pi}}{\gamma_t N_w b}$. Then:*

(i) $\omega_{t+1} = (1+g'_t)\omega_t$, all $t < t'$, and $p_{t+1} = (1+g'_t)p_t$, all $t < t'-1$. Furthermore, if $\beta < 1$ then $g'_t = \pi_t$, all $0 < t < t'-1$, while if $\beta = 1$ then $g'_t = \pi_t$, all $t < t'-1$.

(ii) If $\beta < 1$ and $\pi_t > 0$, all t, $T-2 \geqq t \geqq t'$, then $\beta(1+\pi_{t+1}) = 1$, all t, $T-2 \geqq t \geqq t'$. If $\beta = 1$, there is no t, $T-2 \geqq t \geqq t'$, such that $\pi_t > 0$ and $\pi_{t+j} > 0$, some $j > 0$.

Theorem 4 shows some interesting links between the present model and the literature on inequalities, classes, and growth. On the one hand, as in neo-Ricardian models, a negative relationship is proved between capitalists' consumption and growth, given workers' subsistence, and g'_t can be shown to coincide with the growth rate of Sraffian models.[36] On the other hand, Theorem 4 proves that the growth rate coincides with the profit rate – at least in some periods – as in the so-called Cambridge equation. Although this is a standard feature of Post-Keynesian models, here it is derived within the context of a RS with explicit behavioural foundations. Indeed, the next result characterises capitalists' optimal saving paths with accumulation.

Theorem 5 *Let $(p, \mathbf{1})$ be such that $\pi_t > \pi_\beta$, all $t \leqq \tau$, and $\pi_t = \pi_\beta$, all $T-1 \geqq t \geqq \tau+1$, for some τ, $T-1 \geqq \tau \geqq 0$. Then, for all $\nu \in \mathcal{N}_c$: (i) $\omega^\nu_{t+1} = (1+\pi_t)\omega^\nu_t$, all $t \leqq \tau - 1$, $\omega^\nu_{t+1} = (1+g_t)\omega^\nu_t$, all $g_t \in [0, \pi_\beta]$, all t, $T-2 \geqq t \geqq \tau$, and $\omega^\nu_T = \omega^\nu_0$, is optimal, and (ii) $C(\omega^\nu_0) = \left[\beta^\tau \Pi^\tau_{i=0}(1+\pi_i) - \beta^{T-1}\right]\omega^\nu_0$.*

Let π' be defined by $1 = L[1-(1+\pi')A]^{-1}b$. Let the sequence $\{\overline{\gamma}_\tau\}^{T-1}_{\tau=0}$ be given by $\overline{\gamma}_0 = \frac{1}{\lambda b}$ and $\overline{\gamma}_{\tau+1} = \frac{(\overline{\gamma}_\tau + \widetilde{\pi})}{1+\widetilde{\pi}}$: if $\lambda b < 1$, then the sequence is decreasing. By the productivity of A, the size

[36] See, for example, Kurz and Salvadori ([16] p.102ff).

of the intervals $[\overline{\gamma}_\tau, \overline{\gamma}_{\tau-1})$ decreases with τ and tends to zero, with $\overline{\gamma}_\tau \to 1$ as $\tau \to \infty$. Theorem 6 proves the existence of a SGP.

Theorem 6 *Let $\lambda b < 1$, $\beta \in (\frac{1}{(1+\pi')}, 1]$ and $\gamma_0 > 1$. If $\gamma_0 \in [\overline{\gamma}_{\tau+1}, \overline{\gamma}_\tau)$ and $\overline{\gamma}_\tau > \frac{\beta\widetilde{\pi}}{\beta(1+\widetilde{\pi})-1}$, with $\tau \geqq 1$, then the vector $(p, \mathbf{1})$ with $\pi_t = \frac{\widetilde{\pi}(\gamma_t-1)}{\gamma_t}$, all t, $\tau \geqq t \geqq 0$, with $\gamma_{t+1} = (1+\pi_t)\gamma_t$, all $t \leqq \tau-1$, and $\pi_t = \pi_\beta$, all t, $T-1 \geqq t \geqq \tau+1$, is a SGP for E_0 with $\omega_{t+1} = (1+\pi_t)\omega_t$, all $t \leqq \tau-1$, $\omega_{\tau+1} = (1+g_\tau)\omega_\tau$, with $g_\tau \in (0, \pi_\tau]$, and $\omega_{t+1} = \omega_t$, all t, $T-1 \geqq t \geqq \tau+1$.*

In other words, the economy accumulates at the maximum rate and reaches the steady state in a finite number of periods. In the first τ periods, profits and labour expended increase over time and workers' consumption remains at the subsistence level. At the steady state, full employment prevails, profits remain constant, and workers' consumption exceeds subsistence. If $\beta < 1$, exploitation is a persistent phenomenon; if $\beta = 1$, it disappears.

These results confirm the main conclusions of section 4. Only if $\beta < 1$ can overaccumulation – leading to the disappearance of exploitation – be ruled out in equilibrium (Proposition 2).[37] Moreover, if $\beta = 1$, exploitation and profits may well disappear after a finite number of periods, both at a SRS (Theorem 3) and at a SGP (Theorem 4), *even if capital remains scarce*. Instead, if agents discount the future, exploitation can be persistent even in paths with capital accumulation (Theorem 4). The crucial role of time preference, as opposed, e.g., to capital scarcity, is further confirmed by the fact that if $\beta < 1$, the steady state value of the profit rate (and thus the rate of exploitation) is a positive function of β (Theorem 4(ii)).

Given the dubious relevance of time preference in Marxian exploitation theory, our results prove that, in a competitive setting, asset inequalities may be both necessary and sufficient for exploitation to emerge, but not for it to persist. This has relevant substantive and methodological implications. Substantively, exploitation cannot be reduced - either positively or normatively - to a purely distributive phenomenon. Methodologically, although our formal framework is both formally and conceptually different from mainstream models, it may be necessary to relax some of the underlying assumptions and explicitly incorporate imperfect competition, collective actors, bargaining and, more generally relations of power.

In the rest of the paper, we move a first step in this direction and consider the role of technical change and unemployment in creating the power conditions for exploitation to persist. In both Marx and Kalecki, unemployment is seen as a structural feature of capitalism, whose role is to discipline workers and to restrain wages from rising,[38] and, in turn, labour-saving technical change plays a key role in guaranteeing the persistence of a reserve army of the unemployed by increasing labour productivity. In our model, the disappearance of exploitation derives from an initial excess supply of labour which is rapidly absorbed owing to accumulation. The introduction of labour saving technical progress should avoid this: by increasing labour productivity, technical progress may allow labour supply to be persistently higher than labour demand.[39]

To be specific, we assume that the amount of labour directly needed in production declines geometrically over time.

Assumption 1 (A1): At all t, $L_{t+1} = \delta L_t$, $\delta \in (0, 1)$, with $L_0 > 0$ given.

Under (A1), all the results in Section 3 hold, once L_t is substituted for L. Then, Theorem 7 provides sufficient conditions for the existence of a RS with persistent exploitation.

Theorem 7 *Assume (A1). Let $\gamma_0 > 1$ and $\gamma_0\lambda_0 b \leqq 1$. Let $\delta(1+\widetilde{\pi}) \leqq 1$ and $\beta[1 + \frac{\widetilde{\pi}(\gamma_0-1)}{\gamma_0}] \geqq 1$. The price vector $(p, \mathbf{1})$ with $\pi_0 = \frac{\widetilde{\pi}(\gamma_0-1)}{\gamma_0}$ and $\pi_{t+1} = \frac{\pi_t(1+\widetilde{\pi})}{(1+\pi_t)}$, all t, $T-2 \geqq t \geqq 0$, is a RS for E_0 with $L_t y_t < N_w$, all $t > 0$, and $\omega_{t+1} = (1+\pi_t)\omega_t$, all t, $T-2 \geqq t \geqq 0$.*

[37] Thus, Devine and Dymsky's [3] result can only be an equilibrium if $\beta = 1$.

[38] For a discussion, see for example Sawyer ([31], chapter 8).

[39] The relevance of exogenous growth in the labour force, heterogeneous preferences, and/or labour-saving technical progress in making exploitation persistent has been stressed by Skillman [34].

Theorem 7 highlights an interesting mechanism that may contribute to the persistence of exploitation in capitalist economies. For it shows that labour-saving technical progress allows the economy to settle on a "golden rule" growth path with persistent exploitation even if $\beta = 1$. The increase in labour productivity - a long run historical tendency of capitalist economies - ensures that labour remains in excess supply even along a growth path with maximal accumulation, thus countering all tendencies for profits and exploitation to disappear.

It is worth noting that, although our analysis focuses on competitive markets, and does not consider the effect of imperfect competition on profits, inequalities and growth, or the divergence between actual and 'equilibrium' profit rates, Theorem 7 bears a broad conceptual similarity with Kalecki's [6, 8, 9, 11] analysis of technical change. On the one hand, according to Kalecki [6], labour-saving technical progress tends to alter the ratio of the maximum capacity of plants to the amount of capital they contain and to increase the degree of monopoly. If these effects are sufficiently strong,[40] then technical progress may yield a rise in the mark up of prices to wages, and a redistribution from wages to profits, which - as we have argued in the Introduction following Dutt [4] - captures the same social and economic relation as an increase in exploitation.[41] On the other hand, in Kalecki's [8, 9, 11] long-run analysis, one of the key roles of technical change is to counter the depressing effects of accumulation on the profit rate and on profit expectations, thus allowing for sustained growth.[42]

6 Conclusion

In this paper, an intertemporal model with heterogeneous agents is set up to analyse the relation between inequalities, classes, exploitation, and accumulation. Two main conclusions emerge. First, asset inequalities (and the underlying property relations) are a fundamental feature of capitalist economies, and a key determinant of its long-run dynamics. Yet contrary to Roemer's [27, 28, 29] seminal theory, the class structure and exploitative nature of capitalism cannot be *reduced* to wealth inequalities. Second, the structural injustices of capitalist economies cannot be properly understood in an abstract, power-free competitive setting. The central role of asset inequalities itself can only be understood in conjunction with the asymmetric relations of power that characterise capitalist economies, the mechanisms that ensure the scarcity of capital, and the structural constraints that private ownership of productive assets imposes on aggregate investment, technical change, unemployment, and so on.

From this perspective, Theorem 7 is the most promising result. For, the analysis of the economy with technical progress highlights a mechanism that may contribute to explain the persistence of exploitation. Consistently with the standard Marxian and Kaleckian view, in the long-run labour-saving technical progress tends to reduce the demand for labour, thus creating the conditions for the existence of a permanent reserve army of labour, which disciplines workers and restrains wages from rising, thus allowing capitalist social relations, and exploitation, to persist.

To be sure, Theorem 7 highlights only *one* possible mechanism through which capitalist social relations are reproduced over time, and a multidimensional analysis of the relations of power that characterise capitalist economies is necessary which incorporates some key insights of Marxian and Kaleckian theory, such as imperfect competition (and market power) in product markets, effective demand issues, and, crucially, a more sophisticated, bargaining-theoretic analysis of the labour market. Yet, hopefully, our analysis forcefully suggests that these are important issues to explore in order to properly understand the relation between classes, power, and exploitation in dynamic capitalist economies.

[40] According to Kalecki [6, p.171], they may be offset by the tendency of labour-saving technical change to lower the general level of prices and to keep the inducement to invest higher than it would be otherwise.

[41] For a different view, see Sawyer [31].

[42] For a detailed analysis see White [40].

A Proofs of the Main Theorems

Proof of Theorem 1:
Proof. Part (i). Consider any t. By Definition 1(ii) and (v), at a RS $Ly_t = z_t = p_t c_t^w$. Then, noting that $c_t^w > \mathbf{0}$, by Lemma 2, $Ly_t > \lambda c_t^w$ if and only if $\pi_t > 0$.

Part (ii). The result follows from part (i), since $Ly_t - \lambda c_t^w \geqq 0$, all t. ■

Proof of Theorem 2:
Proof. Part (i). Write MP^ν using dynamic recursive optimisation theory. Let $\mathcal{W} \subseteq \mathbb{R}_+^n$ be the state space with generic element ω. For any $(\mathbf{p}, 1)$, let $\Psi : \mathcal{W} \to \mathcal{W}$ be the feasibility correspondence: $\Psi(\omega_t^\nu) = \left\{\omega_{t+1}^\nu \in \mathcal{W} : p_t \omega_{t+1}^\nu \leqq (1+\pi_t)\, p_t \omega_t^\nu\right\}$. Let

$$\Pi(\omega_0^\nu) = \left\{\omega^\nu : \omega_{t+1}^\nu \in \Psi(\omega_t^\nu),\ \text{all } t, \omega_T^\nu \geqq \omega_0^\nu, \text{ and } \omega_0^\nu \text{ given}\right\}.$$

Let $\Phi = \left\{\left(\omega_t^\nu, \omega_{t+1}^\nu\right) \in \mathcal{W} \times \mathcal{W} : \omega_{t+1}^\nu \in \Psi(\omega_t^\nu)\right\}$ be the graph of Ψ. By the homogeneity of ϕ, if $\pi_t = \pi_\beta$, all t, then the one-period return function $F : \Phi \to \mathbb{R}_+$ at t is $F\left(\omega_t^\nu, \omega_{t+1}^\nu\right) = \frac{\phi(c_\beta)[(1+\pi_\beta)p_\beta\omega_t^\nu - p_\beta\omega_{t+1}^\nu]}{p_\beta c_\beta}$. Then, MP^ν can be written as

$$C(\omega_0^\nu) = \max_{\omega^\nu \in \Pi(\omega_0^\nu)} \sum_{t=0}^{T-1} \beta^t \frac{\phi(c_\beta)[(1+\pi_\beta)\, p_\beta\omega_t^\nu - p_\beta\omega_{t+1}^\nu]}{p_\beta c_\beta}.$$

Since $\Psi(\omega_t^\nu) \neq \varnothing$, all $\omega_t^\nu \in \mathcal{W}$, and F is continuous, concave, and bounded below by 0, MP^ν is well defined.

2. By construction, $(1+\pi_\beta)\,\beta = 1$ and MP^ν reduces to

$$C(\omega_0^\nu) = \max_{\omega^\nu \in \Pi(\omega_0^\nu)} \phi(c_\beta)\left[\frac{(1+\pi_\beta)p_\beta\omega_0^\nu}{p_\beta c_\beta} - \beta^{T-1}\frac{p_\beta\omega_T^\nu}{p_\beta c_\beta}\right].$$

Therefore, any $\omega^\nu \in \Pi(\omega_0^\nu)$ such that $\omega_T^\nu = \omega_0^\nu$ is optimal and $C(\omega_0^\nu)$ follows by noting that $\beta < 1$.

Part (ii). The result follows from MP^ν, given that $\omega_T^\nu \geqq \omega_0^\nu$. ■

Proof of Theorem 3:
Proof. Part (i). 1. (Optimal ξ^ν.) By the Perron-Frobenius theorem π' exists and $\pi' \in (0, \widetilde{\pi})$. If $\pi' = \pi_\beta$, $c_\beta = kb$, some $k > 0$, and $\pi_t = \pi'$, all t, by Theorem 2, any ξ^ν such that $s_t^\nu = \mathbf{0}$, $p_\beta A y_t^\nu = p_\beta\omega_0^\nu$, and $c_t^\nu = h_t^\nu b$ with $h_t^\nu = \frac{\pi' p_\beta \omega_0^\nu}{p_\beta b}$, all t, solves MP^ν, for all $\nu \in \mathcal{N}_c$.

2. (Capital market.) Hence, it is possible to choose $(y^\nu)_{\nu\in\mathcal{N}_c}$ such that at all t, $p_\beta A y_t^\nu = p_\beta\omega_0^\nu$, all ν, and $y_t = A^{-1}\omega_0$.

3. (Labour market and optimal ξ^η.) Since $Ly_t = \gamma_0\lambda b N_w < N_w$, all t, for all $\eta \in \mathcal{N}_w$ assign actions $z_t^\eta = \widehat{z}_t^\eta = \gamma_0\lambda b$, all t; then by construction $\gamma_0\lambda b = p_\beta b$, and thus $c_t^\eta = b$, all t. Hence, these actions solve MP^η for all η, with $Ly_t = z_t$, all t.

4. (Final goods market.) Definition 1(iii) is satisfied because, at all t: $(I - A)y_t = \gamma_0 N_w b$, $c_t^w = N_w b$, and $c_t^c = h_t^c b$, where $h_t^c = \sum_{\nu\in\mathcal{N}_c} h_t^\nu$, and so $h_t^c p_\beta b = \gamma_0 N_w[p_\beta - \lambda]b$, or $h_t^c = N_w(\gamma_0 - 1)$.

Part (ii). 1. (Optimal ξ^ν.) By the Perron-Frobenius theorem π' exists and $\pi' \in (0, \widetilde{\pi})$. Thus $\pi_\beta \in (0, \widetilde{\pi})$. If $\pi_t = \pi_\beta$, all t, by Theorem 2, any ξ^ν such that $s_t^\nu = \mathbf{0}$, $p_\beta A y_t^\nu = p_\beta\omega_0^\nu$, and $c_t^\nu = h_t^\nu b$ with $h_t^\nu p_\beta b = \pi_\beta p_\beta \omega_0^\nu$, all t, solves MP^ν, for all $\nu \in \mathcal{N}_c$.

2. (Capital market.) Hence, it is possible to choose $(y^\nu)_{\nu\in\mathcal{N}_c}$ such that at all t, $p_\beta A y_t^\nu = p_\beta\omega_0^\nu$, all ν, and $y_t = A^{-1}\omega_0$.

3. (Labour market; optimal ξ^η.) Since $Ly_t = N_w$, all t, assign actions $z_t^\eta = \widehat{z}_t^\eta = 1$ and $c_t^\eta = h_t^\eta b$ with $h_t^\eta = 1/p_\beta b$, all t, to all $\eta \in \mathcal{N}_w$. Since $\pi_\beta \in (0, \pi']$ then $1/\lambda b > h_t^\eta \geqq 1$, all t, η. Hence, these actions solve MP^η for all η, with $Ly_t = z_t$, all t.

4. (Final goods market.) Definition 1(ii) is met because, at all t, $(I-A)y_t = \gamma_0 N_w b$ while $c_t^w = N_w b/p_\beta b$ and $c_t^c = \sum_{\nu\in\mathcal{N}_c} h_t^\nu b$, where $\sum_{\nu\in\mathcal{N}_c} h_t^\nu p_\beta b = \pi_\beta p_\beta \omega_0$, or $\sum_{\nu\in\mathcal{N}_c} h_t^\nu p_\beta b = \gamma_0 N_w [p_\beta - \lambda]b$.

Part (iii). 1. If $\gamma_0 \lambda b = 1$, existence is proved as in part (ii) with $z_t^\eta = \widehat{z}_t^\eta = 1$ and $h_t^\eta = 1/\lambda b$, all $\eta \in \mathcal{N}_w$, and all t. If $\gamma_0 \lambda b < 1$, existence is proved as in part (i) with $y_t = (1/\gamma_0)A^{-1}\omega_0$ and $Ly_t = \lambda b N_w$, all t, $z_t^\eta = \widehat{z}_t^\eta = \lambda b$ and $c_t^\eta = b$, all $\eta \in \mathcal{N}_w$, and all t.

2. Suppose, by contradiction, that there is a SRS with $\pi_t > 0$, some t. By Lemma 3, this implies that $\pi_{t-1} = \pi_{t+1} = 0$, and $p_{t-1} = p_{t+1} = \lambda > \mathbf{0}$, for any $0 \leqq t-1 < t+1 \leqq T-1$. Then, for all $\nu \in \mathcal{N}_c$, there is no optimal ξ^ν such that $s_t^\nu = \mathbf{0}$ and $c_{t-1}^\nu = c_{t+1}^\nu = c_t^\nu$, a contradiction. ■

Proof of Theorem 4:

Proof. Part (ii). 1. Consider capitalist ν's programme MP^ν recursively: at all t, the functional equation is $C_t(\omega_t^\nu) = \max_{\omega_{t+1}^\nu \in \Psi(\omega_t^\nu)} [(1+\pi_t)\omega_t^\nu - \omega_{t+1}^\nu] + \beta C_{t+1}(\omega_{t+1}^\nu)$. At $T-1$, since $C_T(\omega_T^\nu) = 0$ for all ω_T^ν, optimality requires $\omega_T^\nu = \omega_0^\nu$ and $C_{T-1}(\omega_{T-1}^\nu) = [(1+\pi_{T-1})\omega_{T-1}^\nu - \omega_0^\nu]$. Therefore at $T-2$, $C_{T-2}(\omega_{T-2}^\nu) = \max_{\omega_{T-1}^\nu \in \Psi(\omega_{T-2}^\nu)} [(1+\pi_{T-2})\omega_{T-2}^\nu - \omega_{T-1}^\nu] + \beta C_{T-1}(\omega_{T-1}^\nu)$.

2. Suppose $\beta < 1$ and $\pi_t > 0$, all t, $T-2 \geqq t \geqq t'$. Because $\pi_{T-2} > 0$, if $\beta(1+\pi_{T-1}) \neq 1$ then $\omega_{T-1}^\nu \neq \omega_{T-2}^\nu$, all $\nu \in \mathcal{N}_c$, and $\omega_{T-1} \neq \omega_{T-2}$. Hence, $\beta(1+\pi_{T-1}) = 1$ and $C_{T-2}(\omega_{T-2}^\nu) = \left[(1+\pi_{T-2})\omega_{T-2}^\nu - \beta\omega_0^\nu\right]$. Iterating backwards, if $\omega_{t+1} = \omega_t$, all t, $T-2 \geqq t \geqq t'$, then $\beta(1+\pi_{t+1}) = 1$, all t, $T-2 \geqq t \geqq t'$, which implies $C_{t'}(\omega_{t'}^\nu) = \left[(1+\pi_{t'})\omega_{t'}^\nu - \beta^{T-1-t'}\omega_0^\nu\right]$.

3. Suppose $\beta = 1$. Suppose, contrary to the statement, that $\pi_t > 0$ and $\pi_{t+j} > 0$, some t, $T-2 \geqq t \geqq t'$, and $j > 0$. Since $\pi_t > 0$, then $c_t^\nu = 0$, all $\nu \in \mathcal{N}_c$, is not possible, or else $\omega_{t+1} \neq \omega_t$, and since $\pi_{t+j} > 0$ then $(1+\pi_{t+j}) > 1$, and there is a feasible perturbation $\mathrm{d}s_t^\nu = -\mathrm{d}s_{t+j}^\nu > 0$, with $\mathrm{d}s_l^\nu = 0$ all $l \neq t, t+j$, that increases ν's welfare, contradicting optimality.

Part (i). 1. Suppose that $(\mathbf{p}, 1)$ is a SGP for E_0. Then by definition there is a $t' > 0$ and a sequence $\{g_t\}_{t=0}^{t'-2}$ such that $\omega_{t+1} = (1+g_t)\omega_t$, $g_t > 0$, all t, $0 \leqq t < t'-1$. For all $\nu \in \mathcal{N}_c$, $c_t^\nu = \pi_t\omega_t^\nu - s_t^\nu$, all t. Therefore, summing over ν and noting that by definition $s_t = g_t\omega_t$, all t, it follows that $c_t^c = (\pi_t - g_t)\omega_t$, all t. Since $\omega_t = \gamma_t N_w A(1-A)^{-1}b$, all t, and noting that in the one good case $\widetilde{\pi} = \frac{1-A}{A}$, then $c_t^c = (\pi_t - g_t)\frac{\gamma_t N_w b}{\widetilde{\pi}}$, all t, or $g_t = [\pi_t - (\frac{c_t^c \widetilde{\pi}}{\gamma_t N_w b})]$, all t.

2. By definition, $(p_t - \lambda) = \pi_t p_t A(1-A)^{-1}$, all t, or equivalently $\pi_t = \widetilde{\pi}(p_t - \lambda)/p_t$, all t. Hence, $g_t = [\frac{(p_t-\lambda)}{p_t} - (\frac{c_t^c}{\gamma_t N_w b})]\widetilde{\pi}$, all t. Moreover, observe that at a SGP with $LA^{-1}\omega_t = \gamma_t N_w \lambda b \leqq N_w$, all t, it must be $\gamma_t \lambda b < 1$, all $t \leqq t'-1$. By construction, this implies that at all $t \leqq t'-1$, $z_t^\eta = \gamma_t \lambda b = p_t b$, for all $\eta \in \mathcal{N}_w$. Therefore $p_t = \gamma_t \lambda$, all $t \leqq t'-1$, and the first part of the statement follows substituting the latter expression into the equation for g_t, and noting that $\frac{p_{t+1}}{p_t} = \frac{\gamma_{t+1}}{\gamma_t} = \frac{\omega_{t+1}}{\omega_t}$, for all $t < t'-1$.

3. Suppose $\beta < 1$. If $t' \leqq 2$, then the statement holds vacuously. Hence, assume $t' > 2$. At $t = t'-1$, $C_{t'-1}(\omega_{t'-1}^\nu) = \max_{\omega_{t'}^\nu \in \Psi(\omega_{t'-1}^\nu)} [(1+\pi_{t'-1})\omega_{t'-1}^\nu - \omega_{t'}^\nu] + \beta C_{t'}(\omega_{t'}^\nu)$, where $C_{t'}(\omega_{t'}^\nu)$ is as in step 2 of the proof of part (ii) for all $\nu \in \mathcal{N}_c$. Hence, at a SGP $\beta(1+\pi_{t'}) \geqq 1$, or else $\omega_{t'}^\nu = 0$, all $\nu \in \mathcal{N}_c$. If $\beta(1+\pi_{t'}) > 1$, then $\omega_{t'}^\nu = (1+\pi_{t'-1})\omega_{t'-1}^\nu$, all ν, and $g_{t'-1} = \pi_{t'-1}$. If $\beta(1+\pi_{t'}) = 1$, then $g_{t'-1}$ is undetermined. In either case, $C_{t'-1}(\omega_{t'-1}^\nu) = \left[\beta(1+\pi_{t'})(1+\pi_{t'-1})\omega_{t'-1}^\nu - \beta^{T-t'}\omega_0^\nu\right]$, all $\nu \in \mathcal{N}_c$.

4. Consider $t = t'-2$. Again, at a SGP, it must be $\beta^2(1+\pi_{t'})(1+\pi_{t'-1}) \geqq 1$, and $C_{t'-2}(\omega_{t'-2}^\nu) = \left[\beta^2(1+\pi_{t'})(1+\pi_{t'-1})(1+\pi_{t'-2})\omega_{t'-2}^\nu - \beta^{T-t'+1}\omega_0^\nu\right]$, all $\nu \in \mathcal{N}_c$. If $\beta^2(1+\pi_{t'})(1+\pi_{t'-1}) = 1$, then by the previous step $\beta(1+\pi_{t'-1}) \leqq 1$: but then since by step 2 at a SGP $p_{t+1} > p_t$, all $t < t'-1$, by definition it follows that $\beta(1+\pi_{t'-2}) < 1$. However, because $t' > 2$, by considering $C_{t'-3}(\omega_{t'-3}^\nu)$, it immediately follows that $\omega_{t'-2}^\nu = 0$, all $\nu \in \mathcal{N}_c$, violating the definition of SGP. Therefore, it must be $\beta^2(1+\pi_{t'})(1+\pi_{t'-1}) > 1$, $\omega_{t'-1}^\nu = (1+\pi_{t'-2})\omega_{t'-2}^\nu$, all ν, and $g_{t'-2} = \pi_{t'-2}$. This argument can be iterated backwards for all t, $0 < t < t'-1$, showing that $\omega_{t+1}^\nu = (1+\pi_t)\omega_t^\nu$, all ν, and all t, $0 < t < t'-1$, and thus $g_t = \pi_t$, all t, $0 < t < t'-1$.

5. Suppose $\beta = 1$. A similar argument as in steps 3 and 4 applies noting that at all $t \leqq t' - 1$, $\pi_t > 0$ implies $\beta(1 + \pi_t) > 1$, given part (ii). ∎

Proof of Theorem 5:

Proof. 1. Take any $\nu \in \mathcal{N}_c$. Consider MP^ν recursively. At $T-1$, since $C_T(\omega_T^\nu) = 0$, then $\omega_T^\nu = \omega_0^\nu$ is optimal and $C_{T-1}(\omega_{T-1}^\nu) = [(1 + \pi_{T-1})\omega_{T-1}^\nu - \omega_0^\nu]$. At $T-2$, $C_{T-2}(\omega_{T-2}^\nu) = \max_{\omega_{T-1}^\nu \in \Psi(\omega_{T-2}^\nu)} [(1 + \pi_{T-2})\omega_{T-2}^\nu - \omega_{T-1}^\nu + \beta C_{T-1}(\omega_{T-1}^\nu)]$. Hence, if $\pi_{T-1} = \pi_\beta$ then any $\omega_{T-1}^\nu \geqq \omega_{T-2}^\nu$ is optimal and $C_{T-2}(\omega_{T-2}^\nu) = [(1 + \pi_{T-2})\omega_{T-2}^\nu - \beta\omega_0^\nu]$. Iterating backwards, if $\pi_t = \pi_\beta$, all t, $T-1 \geqq t \geqq \tau+1$, then at all t, $T-2 \geqq t \geqq \tau$, any $\omega_{t+1}^\nu \geqq \omega_t^\nu$ is optimal and $C_\tau(\omega_\tau^\nu) = [(1 + \pi_\tau)\omega_\tau^\nu - \beta^{T-\tau-1}\omega_0^\nu]$. If $\tau = 0$, the result is proved, noting that $C(\omega_0^\nu) = C_0(\omega_0^\nu)$.

2. If $\tau > 0$, consider $\tau - 1$. Since $C_{\tau-1}(\omega_{\tau-1}^\nu) = \max_{\omega_\tau^\nu \in \Psi(\omega_{\tau-1}^\nu)} [(1+\pi_{\tau-1})\omega_{\tau-1}^\nu - \omega_\tau^\nu + \beta C_\tau(\omega_\tau^\nu)]$ and $\pi_t > \pi_\beta$, at the solution to MP^ν, $\omega_\tau^\nu = (1+\pi_{\tau-1})\omega_{\tau-1}^\nu$ and $C_{\tau-1}(\omega_{\tau-1}^\nu) = [\beta(1+\pi_\tau)(1+\pi_{\tau-1})\omega_{\tau-1}^\nu - \beta^{T-\tau}\omega_0^\nu]$. Iterating backwards, if $\pi_t > \pi_\beta$, all $t \leqq \tau$, at the solution to MP^ν, $\omega_{t+1}^\nu = (1 + \pi_t)\omega_t^\nu$, all $t \leqq \tau - 1$, and the expression for $C(\omega_0^\nu) = C_0(\omega_0^\nu)$ follows. ∎

Proof of Theorem 6:

Proof. 1. We begin by establishing three properties of the sequence $\{\gamma_t\}_{t=0}^{T-1}$.

1.1. At all $t \leqq \tau$, if $\gamma_t \in [\overline{\gamma}_{\tau+1-t}, \overline{\gamma}_{\tau-t})$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, then $\gamma_{t+1} = (1 + \pi_t)\gamma_t$ implies $\gamma_{t+1} \in [\overline{\gamma}_{\tau-t}, \overline{\gamma}_{\tau-t-1})$. To see this, note that at all τ, $\overline{\gamma}_\tau = (1 + \widetilde{\pi})\overline{\gamma}_{\tau+1} - \widetilde{\pi}$, while $\gamma_{t+1} = (1 + \pi_t)\gamma_t$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$ implies $\gamma_{t+1} = (1 + \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t})\gamma_t = (1 + \widetilde{\pi})\gamma_t - \widetilde{\pi}$.

1.2. If $\gamma_t \in [\overline{\gamma}_1, \overline{\gamma}_0) = [\overline{\gamma}_1, \frac{1}{\lambda b})$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, then there is a $g_t \in (0, \pi_t]$ such that $\gamma_{t+1} = (1 + g_t)\gamma_t$ implies $\gamma_{t+1} = 1/\lambda b$. To see this, note that, as in step 1.1, $\overline{\gamma}_0 = (1 + \widetilde{\pi})\overline{\gamma}_1 - \widetilde{\pi}$. Therefore if $\gamma_t = \overline{\gamma}_1$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, then $g_t = \pi_t$ implies $\gamma_{t+1} = \overline{\gamma}_0$, and for all $\gamma_t \in (\overline{\gamma}_1, \overline{\gamma}_0)$, $g_t = \pi_t$ implies $\gamma_{t+1} > \overline{\gamma}_0$, while $g_t = 0$ implies $\gamma_{t+1} < \overline{\gamma}_0$.

1.3. If $\overline{\gamma}_\tau > \frac{\beta\widetilde{\pi}}{\beta(1+\widetilde{\pi})-1}$, all $\tau \geqq 1$, then $\pi_1 = \widetilde{\pi}\frac{(\gamma_1-1)}{\gamma_1} > \pi_\beta$, for all $\gamma_1 \in [\overline{\gamma}_\tau, \overline{\gamma}_{\tau-1})$. To see this, note that if $\gamma_1 = \overline{\gamma}_\tau$ then $\pi_1 = \widetilde{\pi}\left(1 - \frac{1}{\overline{\gamma}_\tau}\right) > \widetilde{\pi}\left(1 - \frac{\beta(1+\widetilde{\pi})-1}{\beta\widetilde{\pi}}\right) = \pi_\beta$, and π_1 is strictly increasing in γ_1.

2. Consider $(\mathbf{p}, 1)$ with $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, all $t \leqq \tau$, where $\gamma_{t+1} = (1 + \pi_t)\gamma_t$, all $t \leqq \tau - 1$. Then $\pi_0 = \widetilde{\pi}\frac{(\gamma_0-1)}{\gamma_0}$ and $\pi_{t+1} = \widetilde{\pi}\left(1 - \frac{1}{(1+\pi_t)\gamma_t}\right)$, and $(\mathbf{p}, 1)$ is well defined.

3. (Optimal ξ^ν; reproducibility.) By step 1.3, and noting that $\gamma_0 > 1$, under the assumptions of the Theorem, we have $\pi_t > \pi_\beta$, all $t \leqq \tau$. Hence, by Theorem 5, $\omega_{t+1}^\nu = (1 + \pi_t)\omega_t^\nu$, all $t \leqq \tau - 1$, $\omega_{t+1}^\nu = (1 + g_t)\omega_t^\nu$, with $g_t \in [0, \pi_t]$, all t, $T-1 \geqq t \geqq \tau$, and $\omega_T^\nu = \omega_0^\nu$ is optimal for all $\nu \in \mathcal{N}_c$. Therefore, for all $\nu \in \mathcal{N}_c$, we can choose an optimal ξ^ν such that $\omega_{t+1}^\nu = (1 + \pi_t)\omega_t^\nu$, all $t \leqq \tau - 1$, $\omega_{\tau+1}^\nu = (1 + g_\tau)\omega_\tau^\nu$, with $g_\tau = \left(\frac{1}{\gamma_\tau\lambda b} - 1\right) \in (0, \pi_\tau]$, $\omega_t^\nu = \omega_{\tau+1}^\nu$, all t, $T-1 \geqq t \geqq \tau+1$, $\omega_T^\nu = \omega_0^\nu$; $y_t^\nu = A^{-1}\omega_t^\nu$, all t; and $c_t^\nu = (1 + \pi_t)\omega_t^\nu - \omega_{t+1}^\nu$, all t. (Observe that by steps 1.1 and 1.2, $g_\tau = \left(\frac{1}{\gamma_\tau\lambda b} - 1\right) \in (0, \pi_\tau]$ exists and $\gamma_{\tau+1} = \overline{\gamma}_0$.) Hence, parts (i) and (vi) of Definition 1 are met.

4. (Capital market.) Because $y_t^\nu = A^{-1}\omega_t^\nu$, all t and all $\nu \in \mathcal{N}_c$, then $y_t = A^{-1}\omega_t$, all t, and Definition 1(iv) is satisfied.

5. (Labour market; optimal ξ^η.) By construction, $\gamma_0 < \overline{\gamma}_1 < \overline{\gamma}_0 = \frac{1}{\lambda b}$ and therefore $Ly_0 = LA^{-1}\omega_0 = \gamma_0\lambda b N_w < N_w$. By step 3, together with steps 1.1 and 1.2, it follows that $\gamma_t < \overline{\gamma}_0 = \frac{1}{\lambda b}$ for all $t \leqq \tau$, and $\gamma_t = \overline{\gamma}_0 = \frac{1}{\lambda b}$ for all t, $T-1 \geqq t \geqq \tau+1$. Therefore $Ly_t = LA^{-1}\omega_t < N_w$, all $t \leqq \tau$, whereas $Ly_t = LA^{-1}\omega_t = N_w$, all t, $T-1 \geqq t \geqq \tau+1$. Hence, for all $\eta \in \mathcal{N}_w$, assign a vector ξ^η such that $z_t^\eta = \widehat{z}_t^\eta = \gamma_t\lambda b$ and $c_t^\eta = b$, all $t \leqq \tau$, and $z_t^\eta = 1$ and $c_t^\eta = \frac{1}{p_\beta}$, all t, $T-1 \geqq t \geqq \tau+1$. Noting that $p_\beta = L\left[1 - (1 + \pi_\beta)A\right]^{-1}$ and $\pi_\beta > \pi'$ imply $\frac{1}{p_\beta} > b$, it follows that ξ^η solves MP^η, for all $\eta \in \mathcal{N}_w$. Hence parts (ii) and (v) of Definition 1 are met.

6. (Final goods market.) Consider first all periods $t \leqq \tau$. By construction at all t, $t \leqq \tau$, $c_t^\eta = b$, all $\eta \in \mathcal{N}_w$, and $c_t^\nu + s_t^\nu = \pi_t \omega_t^\nu$, all $\nu \in \mathcal{N}_c$. Therefore $c_t^c + s_t + c_t^w = \pi_t \omega_t + N_w b$, and substituting for $\pi_t = \widetilde{\pi}\frac{(\gamma_t - 1)}{\gamma_t} = \frac{1-A}{A}\frac{(\gamma_t-1)}{\gamma_t}$ and $\omega_t = \gamma_t N_w A(1-A)^{-1} b$, one obtains $c_t^c + s_t + c_t^w = \gamma_t N_w b$. Because $(1-A)y_t = (1-A)A^{-1}\omega_t = \gamma_t N_w b$, it follows that $(1-A)y_t = c_t^c + s_t + c_t^w$, all t, $t \leqq \tau$. Consider next periods t, $T-1 \geqq t \geqq \tau+1$. By construction, at all t, $T-1 \geqq t \geqq \tau+1$, $c_t^\eta = 1/p_\beta$, all $\eta \in \mathcal{N}_w$, and $c_t^\nu + s_t^\nu = \pi_\beta \omega_t^\nu$, all $\nu \in \mathcal{N}_c$, and so $c_t^c + s_t + c_t^w = \pi_\beta \omega_t + \frac{N_w}{p_\beta}$. By substituting for π_β and p_β, and noting that at the proposed path $\gamma_t = 1/\lambda b$, one obtains $c_t^c + s_t + c_t^w = N_w b/\lambda b$. Because $(1-A)y_t = (1-A)A^{-1}\omega_t = \gamma_t N_w b = N_w b/\lambda b$, it follows that $(1-A)y_t = c_t^c + s_t + c_t^w$, all $t \geqq \tau+1$. Therefore, Definition 1(iii) is met. ■

Proof of Theorem 7:

Proof. 1. Consider the sequence of profit rates $\{\pi_t\}_{t=0}^{T-1}$. Since $\gamma_0 > 1$, $\pi_0 \in (0, \widetilde{\pi})$. Moreover, at all t, $\pi_t < \widetilde{\pi}$ implies $\pi_{t+1} > \pi_t$. Therefore given $\beta\left[1 + \widetilde{\pi}\frac{(\gamma_0-1)}{\gamma_0}\right] = \beta(1+\pi_0) \geqq 1$, it follows that $\pi_t > \pi_\beta$, all $t > 0$. Finally, we prove that if $\pi_0 = \widetilde{\pi}\frac{(\gamma_0-1)}{\gamma_0}$, $\pi_{t+1} = \frac{\pi_t(1+\widetilde{\pi})}{(1+\pi_t)}$, all t, $T-2 \geqq t \geqq 0$, and $\gamma_{t+1} = (1+\pi_t)\gamma_t$, all t, $T-2 \geqq t \geqq 0$, then $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, all $t > 0$. To see this, suppose the result holds for any $t > 0$. Then $\pi_{t+1} = \frac{\pi_t(1+\widetilde{\pi})}{(1+\pi_t)}$ and $\gamma_{t+1} = (1+\pi_t)\gamma_t$ imply $\pi_{t+1} = \frac{\pi_t \gamma_t (1+\widetilde{\pi})}{\gamma_{t+1}}$. Because $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$, the latter expression becomes $\pi_{t+1} = \frac{\widetilde{\pi}(\gamma_t-1)(1+\widetilde{\pi})}{\gamma_{t+1}}$, and the desired result follows noting that $\gamma_{t+1} = (1+\pi_t)\gamma_t$ and $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t}$ imply $\gamma_{t+1} - 1 = (1+\widetilde{\pi})(\gamma_t - 1)$, as required.

2. (Optimal ξ^ν; reproducibility.) By step 1, $\pi_t > \pi_\beta$, all $t > 0$. Therefore, by Theorem 5, for all $\nu \in \mathcal{N}_c$, the vector ξ^ν with $y_t^\nu = A^{-1}\omega_t^\nu$, all t; $\omega_{t+1}^\nu = (1+\pi_t)\omega_t^\nu$ and $c_t^\nu = 0$, all t, $T-2 \geqq t \geqq 0$; $\omega_T^\nu = \omega_0^\nu$; and $c_{T-1}^\nu = (1+\pi_{T-1})\omega_{T-1}^\nu - \omega_0^\nu$ solves MP^ν. Hence parts (i) and (vi) of Definition 1 are met.

3. (Capital market) Because $y_t^\nu = A^{-1}\omega_t^\nu$, all t and all $\nu \in \mathcal{N}_c$, then $y_t = A^{-1}\omega_t$, all t, and Definition 1(iv) is satisfied.

4. (Labour market; optimal ξ^η) By step 3, $L_t y_t = L_t A^{-1}\omega_t = \gamma_t \lambda_t b N_w$, all t. By (A1), $L_{t+1} = \delta L_t$, all t, $T-2 \geqq t \geqq 0$, and by step 3 $y_{t+1} = y_t(1+\pi_t)$, all t, $T-2 \geqq t \geqq 0$. Hence, $L_{t+1}y_{t+1} = \delta(1+\pi_t)L_t y_t$, all t, $T-2 \geqq t \geqq 0$. Therefore, since $L_0 y_0 = L_0 A^{-1}\omega_0 = \gamma_0 \lambda_0 b N_w \leqq N_w$ and $\delta(1+\widetilde{\pi}) \leqq 1$ by assumption, and $\pi_t < \widetilde{\pi}$, all t, it follows that $L_t y_t \leqq N_w$, all t, and $L_t y_t < N_w$, all $t > 0$. Then, for all $\eta \in \mathcal{N}_w$, let ξ^η be defined by $z_t^\eta = \widehat{z}_t^\eta = \gamma_t \lambda_t b$ and $c_t^\eta = b$, all t. Noting that $\gamma_t \lambda_t b \leqq 1$, all t, and $p_t = L\left[1 - (1+\pi_t)A\right]^{-1} = L\left[1 - \left(1 + \frac{1-A}{A}\frac{(\gamma_t-1)}{\gamma_t}\right)A\right]^{-1} = \gamma_t \lambda_t$, all t, it follows that ξ^η solves MP^η, all $\eta \in \mathcal{N}_w$. Therefore parts (ii) and (v) of Definition 1 are met.

5. (Final goods market) At the proposed path, $c_t^w = N_w b$ and $c_t^c + s_t = \pi_t \omega_t$, all t, and substituting for $\pi_t = \widetilde{\pi}\frac{(\gamma_t-1)}{\gamma_t} = \frac{1-A}{A}\frac{(\gamma_t-1)}{\gamma_t}$ and $\omega_t = \gamma_t N_w A(1-A)^{-1}b$, one obtains $c_t^c + s_t + c_t^w = \gamma_t N_w b$, all t. Because $(1-A)y_t = (1-A)A^{-1}\omega_t = \gamma_t N_w b$, all t, Definition 1(iii) is satisfied. ■

References

[1] Cogliano, J., Veneziani, R., and Yoshihara, N. (2016). "The Dynamics of Exploitation and Class in Accumulation Economies", Metroeconomica, forthcoming.

[2] Desai, M. (1979). Marxian Economics, Oxford: Basil Blackwell.

[3] Devine, J. and Dymski, G. (1991). "Roemer's 'General' Theory of Exploitation is a Special Case", Economics and Philosophy, 7, 235-275.

[4] Dutt, A.K. (1990). Growth, Distribution and Uneven Development, Cambridge: Cambridge University Press.

[5] Dutt, A.K. and Sen, A. (1997). "Union Bargaining Power, Employment, and Output in a Model of Monopolistic Competition with Wage Bargaining", Journal of Economics, 65, 1-17.

[6] Kalecki, M. (1941). "A Theorem on Technical Progress", Review of Economic Studies, 8, 178-184.

[7] Kalecki, M. (1943 [1990]). "Political aspects of full employment" reprinted in (Osiatynski, J., ed.) (1990) Collected Works of Michal Kalecki, Volume I: Capitalism: Business Cycles and Full Employment. Oxford: Clarendon Press.

[8] Kalecki, M. (1943). Studies in Economic Dynamics, London: Allen and Unwin.

[9] Kalecki, M. (1954). Theory of Economic Dynamics, London: Allen and Unwin.

[10] Kalecki, M. (1967 [1991]). "The problem of effective demand with Tugan-Baranovsky and Rosa Luxemburg", reprinted in (Osiatynski, J., ed.) Collected Works of Michal Kalecki, Volume II: Economic Dynamics. Oxford: Clarendon Press.

[11] Kalecki, M. (1968). "The Marxian equations of reproduction and modern economics", Social Science Information, 7, 73-79.

[12] Kalecki, M. (1968). "Trend and business cycles reconsidered", Economic Journal, 78, 263-276.

[13] Kalecki, M. (1970 [1971]). "Class struggle and the distribution of national income", reprinted in (Kalecki, M.) Selected Essays on the Dynamics of the Capitalist Economy, 1933–1970. Cambridge: Cambridge University Press.

[14] Kalecki, M. (1971). Selected Essays on the Dynamics of the Capitalist Economy, 1933-1970, Cambridge: Cambridge University Press.

[15] Kowalik, T. (2009) "Luxemburg's and Kalecki's theories and visions of capitalist dynamics", in (Bellofiore, R., ed.) Rosa Luxemburg and the Critique of Political Economy. London: Routledge.

[16] Kurz, H.D. and Salvadori, N. (1995). Theory of Production, Cambridge: Cambridge University Press.

[17] Lopez, J. and Assous, M. (2010). Michal Kalecki, Basingstoke: Macmillan.

[18] Luxemburg, R. (1951). The Accumulation of Capital. London: Routledge.

[19] Morishima, M. (1969). Theory of Economic Growth. Oxford: Oxford University Press.

[20] Morishima, M. (1973). Marx's Economics: A Dual Theory of Value and Growth, Cambridge: Cambridge University Press.

[21] Mott, T. (1989). "Kaleckianism vs. 'New' Keynesianism", WP 25, Levy Economics Institute.

[22] Okishio, N. (1963) "A Mathematical Note on Marxian Theorems", Weltwirtschaftlisches Archiv, 91, 287-99.

[23] Rawls, J. (1971). A Theory of Justice, Cambridge: Harvard University Press.

[24] Robinson, J. (1977 [1979]) "Michal Kalecki" reprinted in (Robinson, J.) Collected Economic Papers, Vol. V. Oxford: Basil Blackwell.

[25] Roemer, J.E. (1980). "A General Equilibrium Approach to Marxian Economics", Econometrica, 48, 505-530.

[26] Roemer, J.E. (1981). Analytical Foundations of Marxian Economic Theory, Cambridge: Harvard University Press.

[27] Roemer, J.E. (1982). A General Theory of Exploitation and Class, Cambridge: Harvard University Press.

[28] Roemer, J.E. (1988). Free to Lose, Cambridge: Harvard University Press.

[29] Roemer, J.E. (1994). Egalitarian Perspectives: Essays in Philosophical Economics, Cambridge: Cambridge University Press.

[30] Rugitsky, F.M. (2013). "Degree of monopoly and class struggle: political aspects of Kalecki's pricing and distribution theory", Review of Keynesian Economics, 1, 447-464.

[31] Sawyer, M. (1985). The Economics of Michal Kalecki, London: Macmillan.

[32] Sen, A., and Dutt, A.K. (1995). "Bargaining Power, Imperfect Competition and the Markup: Optimizing Microfoundations", Economics Letters, 48, 15-20.

[33] Silvestre, J. (2007). "Intergenerational equity and human development," in (Roemer, J.E. and Suzumura, K., eds.) Intergenerational Equity and Sustainability, London: MacMillan.

[34] Skillman, G.L. (2000). "Accumulation, Time Preference and the Persistence of Exploitation", mimeo, Wesleyan University.

[35] Sordi, S. and Vercelli, A. (2012). "Genesis and Foundations of the Multiplier: Marx, Kalecki and Keynes", History of Economic Ideas, 20,137-155.

[36] Veneziani, R. (2007). "Exploitation and Time", Journal of Economic Theory, 132, 189-207.

[37] Veneziani, R. (2012). "Analytical Marxism", Journal of Economic Surveys, 26, 649-673.

[38] Veneziani, R. (2013). "Exploitation, Inequality, and Power", Journal of Theoretical Politics, 25, 526-545.

[39] Veneziani, R. and Yoshihara, N. (2015). "Exploitation in economies with heterogeneous preferences, skills and assets: An axiomatic approach", Journal of Theoretical Politics, 27, 8-33.

[40] White, G. (1999). "Rethinking Kalecki on the Trend and Cycle", Review of Political Economy, 11, 341-353.

[41] Yoshihara, N. and Veneziani, R. (2012). "Profits and Exploitation: A Reappraisal", Advances in Mathematical Economics, 16, 85-109.