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Partially-honest Nash Implementation with Non-connected Honesty Standards

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Partially-honest Nash implementation with non-connected honesty standards*

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Abstract

An individual may display an honesty standard which allows her to lie a little without that being harmful to her self view as an honest person. On this basis, the paper considers a society with a finite number of individuals involving partially-honest individuals and in which every individual has her own honesty standard. An individual honesty standard is modeled as a subgroup of the society, including the individual herself. A partially-honest individual is an individual who strictly prefers to tell the truth prescribed by her honesty standard whenever lying has no effect on her material well-being. The paper studies the impact of placing honesty standard restrictions on the mechanism designer for Nash implementation problems of that society. It offers a necessary condition for Nash implementation, called partial-honesty monotonicity, and shows that in an independent domain of preferences that condition is equivalent to Maskin monotonicity, provided that honesty standards of society are non-connected. They are non-connected if every individual is excluded from the honesty standard of another individual. Finally, it shows that the limitations imposed by Maskin monotonicity can be circumvented by a q-mechanism (Lombardi and Yoshihara, 2013) provided that there are at least $n - q + 1$ partially-honest individuals in a society and that no participant has a veto-power.

JEL classification: C72; D71; D82.
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Introduction

The implementation problem is the problem of designing a mechanism or game form with the property that for each profile of participants' preferences, the equilibrium outcomes of the mechanism played with those preferences coincide with the recommendations that a given social choice rule (SCR) would prescribe for that profile. If that mechanism design exercise can be accomplished, the SCR is said to be implementable. The fundamental paper on implementation in Nash equilibrium is thanks to Maskin (1999; circulated since 1977), who proves that any SCR that can be Nash implemented satisfies a remarkably strong invariance condition, now widely referred to as Maskin monotonicity. Moreover, he shows that when the mechanism designer faces at least three individuals, a SCR is Nash implementable if it is Maskin monotonic and satisfies the condition of no veto-power, subsequently, *Maskin's theorem*.

Since Maskin's theorem, economists have been interested in understanding how to circumvent the limitations imposed by Maskin monotonicity by exploring the possibilities offered by approximate (as opposed to exact) implementation (Matsushima, 1988; Abreu and Sen, 1991), as well as by implementation in refinements of Nash equilibrium (Moore and Repullo, 1988; Abreu and Sen, 1990; Palfrey and Srivastava, 1991; Jackson, 1992) and by repeated implementation (Kalai and Ledyard, 1998; Lee and Sabourian, 2011; Mezzetti and Renou, 2012). One additional way around those limitations is offered by implementation with partially-honest individuals.

A *partially-honest individual* is an individual who deceives the mechanism designer when the truth poses some obstacle to her material well-being. Thus, she does not deceive when the truth is equally efficacious. Simply put, a partially-honest individual follows the maxim, "Do not lie if you do not have to" to serve your material interest.

In a general environment, a seminal paper on Nash implementation problems involving partially-honest individuals is Dutta and Sen (2012), which shows that for implementation problems involving at least three individuals and in which there is at least one partially-honest individual, the Nash implementability is assured by no veto-power. Similar positive results are uncovered in other environments by Matsushima (2008a,b), Kartik and Tercieux (2012), Kartik et al. (2014), and Ortner (2015). Thus, there are far fewer limitations for Nash implementation when there are partially-honest individuals.[1]

One way to put those studies into perspective of this paper is to recognize that a participant chooses the information about a state of the world as part of her strategy choice. Moreover, a participant's play is honest if she plays a strategy choice which is veracious in its state announcement component. Therefore, their common ground is that the mechanism designer establishes a unique honesty (equivalently, truth-telling) standard which spells out to participants the boundary between an honest and a dishonest play of state announcements and enforces participants to endorse it. Simply put, participants are not free to maintain their own view of honesty through their play.

[1] A pioneering work on the impact of decency constraints on Nash implementation problems is Corchón and Herrero (2004). These authors propose restrictions on sets of strategies available to agents that depend on the state of the world. They refer to these strategies as *decent* strategies and study Nash implementation problems in decent strategies. For a particular formulation of decent strategies, they are also able to circumvent the limitations imposed by Maskin monotonicity.

Most of human behavior, however, is guided by a set of non-material motives, and the actual behavior is often the result of a compromise, an interplay, among them. People want to be honest as well as feel good about themselves. People want to be honest as well as they do not want to needlessly hurt others. People want to be honest as well as they do not want to threaten others' integrity. When social and psychological goals such as these can be attained by a candid communication, then people are fully honest. Sometimes, however, situations are such that veracity thwarts the accomplishment of the goal. In these situations, people may prefer that the truth remain unsaid or partially ignored. Therefore, it seems introspectively plausible that there are situations in which a person may prefer to lie a little about the state of the world without that being harmful to her self view as an honest person.[2]

Furthermore, it also seems plausible that people may have different views of truthful communication about the state of the world which may be based mainly on a categorization caused by the collective decision at hand. Simply put, a person may have truth-telling concerns only for people to whom she feels close, for example, in terms of geographical proximity, socio-economic status, frequency of interaction and so on. For instance, suppose that a government is contemplating the action to implement a trade liberalization policy for a specific agricultural product. Farmers who produce this product could typically have incentives to state that they unanimously agree or disagree on this policy, irrespective of their true individual opinions. A farmer who has a taste for truthful communication and, at the same time, who does not want to betray her fellows, may undergo a mental conflict about whether she should truthfully announce the true opinions of the farmers. However, this same farmer could not have any truth-telling concern for people outside her profession.

The above arguments may suggest that both the uniformity of honesty standards across individuals and their coercion may be strong assumptions and that there is no reason to restrict attention to them. Then, addressing this, we ask the following question: Do personal views of honesty enhance the scope of Nash implementation with partially-honest individuals, or do they hinder it? This is the central question we address in this paper.

This paper models an *individual honesty standard* as a subset of individuals involved with an implementation problem. Our interpretation is that participant i concerns herself with the truth-telling of individuals in the subset specified by her honesty standard when she plays a strategy choice, and such a subset represents the individuals whose truthful information is relevant to retain her self-image as a reasonably honest individual. For instance, it may represent what an individual considers to be good, right, or virtuous to communicate truthfully about individuals involved in an implementation problem. With this interpretation in mind, our definition endorses the view that an individual feels honest as well as good when she is veracious at least about her own self. Then, our study looks at what SCR can be Nash implemented in a society involving partially-honest individuals, in which participants share the responsibility for maintaining their own honesty standards and in which the mechanism designer takes those standards as an institutional constraint.

First, we assume that individual honesty standards are known to the mechanism designer[3] and that he has to respect them. Under this institutional constraint, we show that

[2]In Japan, for instance, the standard of truth-telling is carefully balanced with the value of respect for dignity of others. If the truth brings shame to another person, the social convention is that the truth should be set aside.

[3]Our choice in this paper is motivated by convenience. The reason for it is easy to identify: the fact that

any SCR that can be Nash implemented with partially-honest individuals satisfies a variant of Maskin monotonicity, called *partial-honesty monotonicity*. The idea of this axiom is quite intuitive. If x is one of the outcomes selected by a given SCR at one preference profile but is not selected when there is a monotonic change of preferences around x, then that monotonic change has altered preferences of individuals in the honesty standard of a partially-honest individual.

Second, we consider what we call *non-connected honesty standards*. Simply put, individual honesty standards are connected if there is a participant i with whom all other participants retain their self-image as reasonably honest individuals when they are veracious about her characteristics. When that is not the case, we call it non-connected honesty standards. In other words, they are non-connected if every participant is excluded from the honesty standard of another participant.

In an independent domain of preferences, where the set of the profiles of participants' preferences takes the structure of the Cartesian product of individual preferences, we show that partial-honesty monotonicity is *equivalent* to Maskin monotonicity whenever there exists at least one partially honest individual and all of such individuals share non-connected honesty standards in the society. Thus, under those hypotheses, Maskin's theorem provides an almost complete characterization of SCRs that are Nash implementable in the society with partially-honest individuals.

The above results are derived on the basis that the mechanism designer can structure communication with participants in a way that he forces each participant to report preferences of the entire society as part of her strategy choice. Clearly, there is no reason to restrict attention to such communication schemes. Indeed, often, for practical reasons, the mechanism designer needs to employ simpler communication schemes. However, if the mechanism designer would be forced to structure the communication in a way that would force individuals to behave *as if* their honesty standards were non-connected, though their non-connectedness could be merely an artifact of that communication structure, this would impair his ability to escape the limitations imposed by Maskin monotonicity. Then, addressing this, we ask the following question: Under what conditions would the positive sufficiency result of Dutta and Sen (2012) be restored? Our answer is that the mechanism designer who knows that $\alpha(\geq 1)$ members of society have a taste for honesty can expect to do well if no participant has a veto-power by structuring communication with participants in a way that each of them reports her own preferences and those of other $(n-\alpha)$ 'neighbor' participants.

The remainder of the paper is divided into five sections. Section 2 presents the theoretical framework and outlines the implementation model, with the necessary condition presented in section 3. Section 4 presents the equivalence result. Section 5 presents sufficient conditions for the restoration of Dutta-Sen's theorem. Section 6 concludes.

the mechanism designer knows the honesty standards of individuals can only make implementation simpler than if the actual honesty standards of participants were unknown. Thus, our results also hold when the mechanism designer does *not* know the honesty standards of participants.

Preliminaries

Basic framework

We consider a finite set of individuals indexed by $i \in N = \{1, \cdots, n\}$, which we will refer to as a society. The set of outcomes available to individuals is X. The information held by the individuals is summarized in the concept of a state. Write Θ for the domain of possible states, with θ as a typical state. In the usual fashion, individual i's preferences in state θ are given by a complete and transitive binary relation, subsequently an ordering, $R_i(\theta)$ over the set X. The corresponding strict and indifference relations are denoted by $P_i(\theta)$ and $I_i(\theta)$, respectively. The preference profile in state θ is a list of orderings for individuals in N that are consistent with that state and is denoted by $R_N(\theta)$.

We assume that the mechanism designer does not know the true state. We assume, however, that there is complete information among the individuals in N. This implies that the mechanism designer knows the preference domain consistent with the domain Θ. In this paper, we identify states with preference profiles.

The goal of the mechanism designer is to implement a SCR $F : \Theta \twoheadrightarrow X$ where $F(\theta)$ is non-empty for any $\theta \in \Theta$. We shall refer to $x \in F(\theta)$ as an F-optimal outcome at θ. Given that individuals will have to be given the necessary incentives to reveal the state truthfully, the mechanism designer delegates the choice to individuals according to a mechanism $\Gamma \equiv \left(\prod_{i \in N} M_i, g\right)$, where M_i is the strategy space of individual i and $g : M \rightarrow X$, the outcome function, assigns to every strategy profile $m \in M \equiv \prod_{i \in N} M_i$ a unique outcome in X. We shall sometimes write (m_i, m_{-i}) for the strategy profile m, where $m_{-i} = (m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_n)$.

An *honesty standard of individual* i, denoted by $S(i)$, is a subgroup of society with the property that $i \in S(i)$. Thus, given a state θ, $R_{S(i)}(\theta)$ is a list of orderings consistent with θ for individuals in the honesty standard $S(i)$ of individual i. An *honesty standard of society* is a list of honesty standards for all members of society. Write $S(N)$ for a typical honesty standard of society.

Intrinsic preferences for honesty

An individual who has an intrinsic preference for truth-telling can be thought of as an individual who is torn by a fundamental conflict between her deeply and ingrained propensity to respond to material incentives and the desire to think of herself as an honest as well as a good person. In this paper, the theoretical construct of the balancing act between those contradictory desires is based on two ideas.

First, the triplet $(\Gamma, \theta, S(i))$ acts as a "context" for individuals' conflicts. The reason for this is that an individual who has intrinsic preferences for honesty can categorize her strategy choices as truthful or untruthful relative to her honesty standard $S(i)$, the state θ and the mechanism Γ designed by the mechanism designer to govern the communication with her. That categorization can be captured by the following notion of truth-telling correspondence:

Definition 1 For each Γ and each individual $i \in N$ with an honesty standard $S(i)$, individual i's *truth-telling correspondence* is a (non-empty) correspondence $T_i^{\Gamma}(\cdot; S(i)) : \Theta \twoheadrightarrow M_i$ with the property that for any two states θ and θ', it holds that

$$T_i^{\Gamma}(\theta; S(i)) = T_i^{\Gamma}(\theta'; S(i)) \iff R_{S(i)}(\theta) = R_{S(i)}(\theta').$$

Strategy choices in $T_i^{\Gamma}(\theta; S(i))$ will be referred to as truthful strategy choices for θ according to $S(i)$.

According to the above definition, in a state θ, every truthful strategy choice of individual i is to encode information of individuals' orderings consistent with that state for members of society in her honesty standard $S(i)$. Moreover, if in two different states, say θ and θ', the orderings consistent with those two states for individuals in $S(i)$ are the same, then the sets of individual i's truthful strategy choices for those two states need to be identical according to her honesty standard $S(i)$.

In modeling intrinsic preferences for honesty, we adapt the notion of partially-honest individuals of Dutta and Sen (2012) to our research questions. First, a partially-honest individual is an individual who responds primarily to material incentives. Second, she strictly prefers to tell the truth whenever lying has no effect on her material well-being. That behavioral choice of a partially-honest individual can be modeled by extending an individual's ordering over X to an ordering over the strategy space M, because that individual's preference between being truthful and being untruthful is contingent upon announcements made by other individuals as well as the outcome(s) obtained from them. By following standard conventions of orderings, write $\succcurlyeq_i^{\Gamma,\theta,S(i)}$ for individual i's ordering over M in state θ whenever she is confronted with the mechanism Γ and has set her honesty standard at $S(i)$. Formally, our notion of a partially-honest individual is as follows:

Definition 2 For each Γ, individual $i \in N$ with an honesty standard $S(i)$ is *partially-honest* if, for all $\theta \in \Theta$, her intrinsic preference for honesty $\succcurlyeq_i^{\Gamma,\theta,S(i)}$ on M satisfies the following properties: for all m_{-i} and all $m_i, m_i' \in M_i$, it holds that

(i) If $m_i \in T_i^{\Gamma}(\theta; S(i))$, $m_i' \notin T_i^{\Gamma}(\theta; S(i))$ and $g(m) R_i(\theta) g(m_i', m_{-i})$, then $m \succ_i^{\Gamma,\theta,S(i)} (m_i', m_{-i})$.

(ii) In all other cases, $m \succcurlyeq_i^{\Gamma,\theta,S(i)} (m_i', m_{-i})$ if and only if $g(m) R_i(\theta) g(m_i', m_{-i})$.

Intrinsic preference for honesty of individual i is captured by the first part of the above definition, in that, for a given mechanism Γ, honesty standard $S(i)$ and state θ, individual i strictly prefers the message profile (m_i, m_{-i}) to (m_i', m_{-i}) provided that the outcome $g(m_i, m_{-i})$ is at least as good as $g(m_i, m_{-i})$ according to her ordering $R_i(\theta)$ and that m_i is truthful for θ and m_i' is not truthful for θ, according to $S(i)$.

If individual i is *not* partially-honest, this individual cares for her material well-being associated with outcomes of the mechanism and nothing else. Then, individual i's ordering over M is just the transposition into space M of individual i's relative ranking of outcomes. More formally:

Definition 3 For each Γ, individual $i \in N$ with an honesty standard $S(i)$ is *not partially-honest* if, for all $\theta \in \Theta$, her intrinsic preference for honesty $\succcurlyeq_i^{\Gamma,\theta,S(i)}$ on M satisfies the following property: for all $m, m' \in M$, it holds that

$$m \succcurlyeq_i^{\Gamma,\theta,S(i)} m' \iff g(m)\, R_i(\theta)\, g(m').$$

Implementation problems

In formalizing the mechanism designer's problems, we first introduce our informational assumptions and discuss their implications for our analysis. They are:

Assumption 1 There exists at least one partially-honest individual in a society.

Assumption 2 The mechanism designer knows the honesty standard of a society.

The above two assumptions combined with the assumption that there is complete information among the individuals imply that the mechanism designer only knows the set Θ, the fact that there is at least one partially-honest individual among the individuals and the honesty standard of society, but he does not know either the true state or the identity of the partially-honest individual(s) (or their identities). Indeed, the mechanism designer cannot exclude any member(s) of society from being partially-honest purely on the basis of Assumption 1. Therefore, the following considerations are in order from the viewpoint of the mechanism designer.

An environment is described by three parameters, $(\theta, S(N), H)$: a state θ, an honesty standard of society $S(N)$ and a conceivable set of partially-honest individuals H. We denote by H a typical conceivable set of partially-honest individuals in N, with h as a typical element, and by $\mathcal{H}$ the class of conceivable sets of partially-honest individuals.

A mechanism Γ and an environment $(\theta, S(N), H)$ induce a strategic game $\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$, where

$$\succcurlyeq^{\Gamma,\theta,S(N),H} \equiv \left(\succcurlyeq_i^{\Gamma,\theta,S(i)}\right)_{i \in N}$$

is a profile of orderings over the strategy space M as formulated in Definition 2 and in Definition 3. Specifically, $\succcurlyeq_i^{\Gamma,\theta,S(i)}$ is individual i's ordering over M as formulated in Definition 2 if individual i is in H, whereas it is the individual i's ordering over M as formulated in Definition 3 if individual i is not in H.

A (pure strategy) Nash equilibrium of the strategic game $\left(\Gamma, \succcurlyeq^{\Gamma,\theta,H,S(N)}\right)$ is a strategy profile m such that for all $i \in N$, it holds that

$$\text{for all } m'_i \in M_i : m \succcurlyeq_i^{\Gamma,\theta,S(i)} (m'_i, m_{-i}).$$

Write $NE\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$ for the set of Nash equilibrium strategies of the strategic game $\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$ and $NA\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$ for its corresponding set of Nash equilibrium outcomes.

The following definition is to formulate the mechanism designer's Nash implementation problem involving partially-honest individuals in which the society maintains the standard of honesty summarized in $S(N)$.

Definition 4 Let Assumption 1 and Assumption 2 be given. Let the honesty standard of society be summarized in $S(N)$. A mechanism Γ *partially-honestly Nash implements* the SCR $F : \Theta \twoheadrightarrow X$ provided that for all $\theta \in \Theta$ and $H \in \mathcal{H}$ there exists for any $h \in H$ a truth-telling correspondence $T_h^{\Gamma}(\theta; S(h))$ as formulated in Definition 1 and, moreover, it holds that $F(\theta) = NA\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$. If such a mechanism exists, F is said to be *partially-honestly Nash implementable.*

The objective of the mechanism designer is thus to design a mechanism whose Nash equilibrium outcomes, for each state θ as well as for each conceivable set of partially-honest individuals H, coincide with $F(\theta)$. Note that there is no distinction between the above formulation and the standard Nash implementation problem as long as Assumption 1 is discarded.

A necessary condition

In this section, we discuss a condition that is necessary for the partially-honest Nash implementation where the honesty standard of society is prescribed by $S(N)$.

A condition that is central to the implementation of SCRs in Nash equilibrium is Maskin monotonicity. This condition says that if an outcome x is F-optimal at the state θ, and this x does not strictly fall in preference for anyone when the state is changed to θ', then x must remain an F-optimal outcome at θ'. Let us formalize that condition as follows. For any state θ, individual i and outcome x, the weak lower contour set of $R_i(\theta)$ at x is defined by $L_i(\theta, x) = \{x' \in X | xR_i(\theta)x'\}$. Therefore:

Definition 5 The SCR $F : \Theta \twoheadrightarrow X$ is *Maskin monotonic* provided that for all $x \in X$ and all $\theta, \theta' \in \Theta$, if $x \in F(\theta)$ and $L_i(\theta, x) \subseteq L_i(\theta', x)$ for all $i \in N$, then $x \in F(\theta')$.

An equivalent statement of Maskin monotonicity stated above follows the reasoning that if x is F-optimal at θ but not F-optimal at θ', then the outcome x must have fallen strictly in someone's ordering at the state θ' in order to break the Nash equilibrium via some deviation. Therefore, there must exist some (outcome-)preference reversal if an equilibrium strategy profile at θ is to be broken at θ'.

Our variant of Maskin monotonicity for Nash implementation problems involving partially-honest individuals where the standard of honesty in a society is represented by $S(N)$ can be formulated as follows:

Definition 6 The SCR $F : \Theta \twoheadrightarrow X$ is *partial-honesty monotonic* given the standard $S(N)$ provided that for all $x \in X$, all $H \in \mathcal{H}$ and all $\theta, \theta' \in \Theta$, if $x \in F(\theta) \backslash F(\theta')$ and $L_i(\theta, x) \subseteq L_i(\theta', x)$ for all $i \in N$, then for one $h \in H : R_{S(h)}(\theta) \neq R_{S(h)}(\theta')$.

This says that if x is F-optimal at θ but not F-optimal at θ' and, moreover, there is a monotonic change of preferences around x from θ to θ' (that is, whenever $xR_i(\theta)x'$, one has that $xR_i(\theta')x'$), then that monotonic change has altered preferences of individuals in the honesty standard of a partially-honest individual $h \in H$ (that is, $R_{S(h)}(\theta) \neq R_{S(h)}(\theta')$). Stated in the contrapositive, this says that if x is F-optimal at θ and there is a monotonic change of preferences around x from θ to θ' and, moreover, for any conceivable partially-honest individual h in H that change has not altered preferences of individuals in her honesty standard $S(h)$, then x must continue to be one of the outcomes selected by F at the state θ'. Note that if x is F-optimal at θ but not F-optimal at θ', one has that $R_N(\theta) \neq R_N(\theta')$, and thus any SCR is partial-honesty monotonic whenever the honesty standard of society is such that $S(i) = N$ for all $i \in N$.

The above condition is necessary for partially-honest Nash implementation. This is because if x is F-optimal at θ but not F-optimal at θ' and, moreover, the outcome x has not fallen strictly in any individual's ordering at the state θ', then only a partially-honest individual in the given conceivable set H can break the Nash equilibrium via a unilateral deviation. Therefore, there must exist some strategy-profile-preference reversal for a partially-honest individual $h \in H$ if an equilibrium strategy profile at $(\theta, S(N), H)$ is to be broken at $(\theta', S(N), H)$. Formally:

Theorem 1 Let Assumption 1 and Assumption 2 be given. Let the honesty standard of society be summarized in $S(N)$. The SCR $F : \Theta \twoheadrightarrow X$ is partial-honesty monotonic given the standard $S(N)$ if it is partially-honestly Nash implementable.

Proof. Let Assumption 1 and Assumption 2 be given. Let the honesty standard of society be summarized in $S(N)$. Suppose that $\Gamma \equiv (M, g)$ partially-honest Nash implements the SCR $F : \Theta \twoheadrightarrow X$. For any $x \in X$, consider any environment $(\theta, S(N), H)$ such that $x \in F(\theta)$. Then, there is $m \in NE\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$ such that $g(m) = x$.

Consider any state $\theta' \in \Theta$ such that

$$\text{for all } i \in N \text{ and all } x' \in X : xR_i(\theta)\, x' \implies xR_i(\theta')\, x'. \tag{1}$$

If there exists an individual $i \in N$ such that $g(m_i', m_{-i})\, P_i(\theta')\, g(m)$, then, from (1), $g(m_i', m_{-i})\, P_i(\theta)\, g(m)$, a contradiction of the fact that $m \in NE\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$. Therefore, we conclude that

$$\text{for all } i \in N \text{ and all } m_i' \in M_i : g(m)\, R_i(\theta')\, g(m_i', m_{-i}). \tag{2}$$

Suppose that $x \notin F(\theta')$. Then, the strategy profile m is not a Nash equilibrium of $\left(\Gamma, \succcurlyeq^{\Gamma,\theta',S(N),H}\right)$; that is, there exists an individual $i \in N$ who can find a strategy choice $m_i' \in M_i$ such that $(m_i', m_{-i}) \succ_i^{\Gamma,\theta',S(i)} m$. Given that (2) holds, it must be the case that $i \in H$. From part (i) of Definition 2 we conclude, therefore, that

$$m_i \notin T_i^{\Gamma}(\theta'; S(i)) \text{ and } m_i' \in T_i^{\Gamma}(\theta'; S(i)) \tag{3}$$

and that

$$g(m_i', m_{-i})\, R_i(\theta')\, g(m). \tag{4}$$

Note that (2) and (4) jointly imply that

$$g(m_i', m_{-i})\, I_i(\theta')\, g(m). \tag{5}$$

We show that $R_{S(i)}(\theta) \neq R_{S(i)}(\theta')$. Assume, to the contrary, that

$$\text{for all } h \in H : R_{S(h)}(\theta) = R_{S(h)}(\theta'). \tag{6}$$

Definition 1 implies that

$$\text{for all } h \in H : T_h^{\Gamma}(\theta; S(h)) = T_h^{\Gamma}(\theta'; S(h)). \tag{7}$$

From (3) and (7), it follows that

$$m_i \notin T_i^{\Gamma}(\theta; S(i)) \text{ and } m_i' \in T_i^{\Gamma}(\theta; S(i)). \tag{8}$$

Furthermore, given that $i \in S(i)$, by definition of an individual honesty standard, (5) and (6) jointly imply that

$$g(m_i', m_{-i})\, I_i(\theta)\, g(m). \tag{9}$$

Given (8) and (9) and the fact that $i \in H$, Definition 2 implies that $(m_i', m_{-i}) \succ_i^{\Gamma,\theta,S(i)} m$, which is a contradiction of the fact that $m \in NE\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$. Thus, F is partial-honesty monotonic given the honesty standard $S(N)$. ■

Equivalence result

The classic paper on Nash implementation theory is Maskin (1999), which shows that where the mechanism designer faces a society involving at least three individuals, a SCR is Nash implementable if it is monotonic and satisfies the auxiliary condition of no veto-power.[4]

The condition of no veto-power says that if an outcome is at the top of the preferences of all individuals but possibly one, then it should be chosen irrespective of the preferences of the remaining individual: that individual cannot veto it. Formally:

Definition 7 The SCR $F : \Theta \twoheadrightarrow X$ satisfies *no veto-power* provided that for all $\theta \in \Theta$ and all $x \in X$, if there exists $i \in N$ such that for all $j \in N \setminus \{i\}$ and all $x' \in X : x R_j(\theta) x'$, then $x \in F(\theta)$.

Theorem 2 (Maskin's theorem, 1999) If $n \geq 3$ and $F : \Theta \twoheadrightarrow X$ is a SCR satisfying Maskin monotonicity and no veto-power, then it is Nash implementable.

In a general environment such as that considered here, a seminal paper on Nash implementation problems involving partially-honest individuals is Dutta and Sen (2012). It shows that for Nash implementation problems involving at least three individuals and in which there is at least one partially-honest individual, the Nash implementability is assured by no veto-power (Dutta and Sen, 2012; p. 157). From the perspective of this paper, Dutta-Sen's theorem can be formally restated as follows:

[4] Moore and Repullo (1990), Dutta and Sen (1991), Sjöström (1991) and Lombardi and Yoshihara (2013) refined Maskin's theorem by providing necessary and sufficient conditions for an SCR to be implementable in (pure strategies) Nash equilibrium. For an introduction to the theory of implementation see Jackson (2001), Maskin and Sjöström (2002) and Serrano (2004).

Theorem 3 (Dutta-Sen's theorem, 2012) Let Assumption 1 and Assumption 2 be given. Let the honesty standard of society be summarized in $S(N)$, where $S(i) = N$ for all $i \in N$. If $n \geq 3$ and $F : \Theta \twoheadrightarrow X$ is a SCR satisfying partial-honesty monotonicity for the standard $S(N)$ and no veto-power, then it is partially-honestly Nash implementable.

As already noted in the previous section, any SCR is partial-honesty monotonic whenever the honesty standard of society is such that every individual considers truthful only messages that encode the whole truth about preferences of individuals in society; that is, $S(i) = N$ for all $i \in N$.

Clearly, $S(i) = N$ for all $i \in N$ is a particular kind of honesty standard of individuals, and there is no reason to restrict attention to such standards. Indeed, as discussed in the introduction, people want to be honest as well as feel good about themselves. People want to be honest as well as they do not want to needlessly hurt others. People want to be honest as well as they do not want to threaten others' integrity. When social and psychological goals such as these can be attained by a candid communication, then people are fully honest. Sometimes, however, situations are such that veracity thwarts the accomplishment of the goal. On this basis, we have presented an implementation model which is able to handle such views of honesty and also presented a necessary condition for Nash implementation problems involving partially-honest individuals. In this section, we are interested in understanding the kind of honesty standards of individuals which would make it impossible for the mechanism designer to circumvent the limitations imposed by Maskin monotonicity.

To this end, let us introduce the following notion of standards of honesty of a society.

Definition 8 Given a society N involving at least two individuals, an honesty standard of this society is said to be *non-connected* if and only if for all $i \in N$, $i \notin S(j)$ for some $j \in N$.

Given that the honesty standard of individual i includes the individual herself, by definition of $S(i)$, the honesty standard of society is non-connected whenever every one of its members is excluded from the honesty standard of another member of the society. Simply put, members of a society do not concern themselves with the same individual.

It is self-evident that the kind of honesty standards in Dutta-Sen's theorem are not non-connected, because every individual of the society is interested in telling the truth about the whole society. As another example of honesty standards of a society that are not non-connected, consider a three-individual society where individual 1 concerns herself with herself and with individual 2 (that is, $S(1) = \{1, 2\}$), individual 2 concerns herself with everyone (that is, $S(2) = \{1, 2, 3\}$) and, finally, individual 3 concerns herself with herself and with individual 1 (that is, $S(3) = \{1, 3\}$). The honesty standard of this three-individual society is not non-connected because everyone concerns themselves with individual 1.

Moreover, it is not the case that every non-connected honesty standard of society implies that every individual honesty standard be of the form $S(i) \neq N$, as we demonstrate with the next example. Consider a three-individual society where individual 1 is concerned only with herself (that is, $S(1) = \{1\}$), individual 2 with everyone (that is, $S(2) = \{1, 2, 3\}$) and individual 3 with herself and with individual 2 (that is, $S(3) = \{2, 3\}$). The honesty standard of this society is non-connected given that individual 2 and individual 3 are both excluded from the honesty standard of individual 1 and individual 1 is excluded from the honesty standard of individual 3.

As is the case here, the above definition is a requirement for the honesty standard of a society that is sufficient for partial-honesty monotonicity to be equivalent to Maskin monotonicity when two further assumptions are satisfied. The first assumption requires that the family $\mathcal{H}$ includes singletons. This requirement is innocuous given that the mechanism designer cannot exclude any individual from being partially-honest purely on the basis of Assumption 1.

The second requirement is that the set of states Θ takes the structure of the Cartesian product of allowable independent characteristics for individuals. More formally, the domain Θ is said to be independent if it takes the form

$$\Theta = \prod_{i \in N} \Theta_i,$$

where Θ_i is the domain of allowable independent characteristics for individual i, with θ_i as a typical element. A typical example of independent domain is that each Θ_i simply represents the domain of the preference orderings over X of individual i and so the domain of the profiles of all individuals' preference orderings on X has the structure of the Cartesian product. In such a case, in a state $\theta = (\theta_i)_{i \in N}$, individual i's preference ordering over X depends solely on individual i's independent characteristic θ_i rather than on the profile θ. Given that a characteristic of individual i is independent from those of other individuals, the equivalence result does not hold for the correlated values case.

The latter two requirements and the requirement that the honesty standard of society needs to be non-connected are jointly sufficient for partial-honesty monotonicity to imply Maskin monotonicity. Each of those requirements is indispensable, and this can be seen as follows:

Consider a two-individual society where Θ is the set of states and X is the set of outcomes available to individuals. Let $S(i)$ be the honesty standard of individual $i = 1, 2$. Consider an outcome x and a state θ such that x is an F-optimal outcome at θ. Consider any other state θ' such that individuals' preferences change in a Maskin monotonic way around x from θ to θ'. Maskin monotonicity says that x must continue to be an F-optimal outcome at θ'. To avoid trivialities, let us focus on the case that $\theta \neq \theta'$, which means that $R_N(\theta) \neq R_N(\theta')$, given that we identify states with preference profiles.

If every individual were concerned with the whole society, we could never invoke (the contrapositive of) partial-honesty monotonicity to conclude that x should remain F-optimal at θ' because $R_N(\theta) \neq R_N(\theta')$. Furthermore, consider the case that individual 1 concerns herself with only herself, that is, $S(1) = \{1\}$, while individual 2 with the whole society, that is, $S(2) = \{1, 2\}$. Reasoning such as the one just used shows that partial-honesty monotonicity cannot be invoked if $R_1(\theta) \neq R_1(\theta')$. The argument for honesty standards of the form $S(1) = \{1, 2\}$ and $S(2) = \{2\}$ is symmetric. Thus, the only case left to be considered is the one in which everyone concerns themselves with only themselves, that is, $S(i) = \{i\}$ for $i = 1, 2$. In this situation, the honesty standard of society is reduced to the non-connected one. Note that standards considered earlier were not.

Suppose that preferences of individual 1 are identical in the two states, that is, $R_1(\theta) = R_1(\theta')$. To conclude that x should be F-optimal at θ' by invoking partial-honesty monotonicity we need to find individual 1 in the family $\mathcal{H}$. The argument for the case $R_2(\theta) = R_2(\theta')$

is symmetric. Thus, if $R_i(\theta) = R_i(\theta')$ for one of the individuals, the requirement that the singleton $\{i\}$ is an element of $\mathcal{H}$ is needed for the completion of the argument.

Suppose that preferences of individuals are not the same in the two states, that is, $R_i(\theta) \neq R_i(\theta')$ for every individual i, though they have changed in a Maskin monotonic way around x from the state θ to θ'. In this case, one cannot directly reach the conclusion of Maskin monotonicity by invoking partial-honesty monotonicity. One way to circumvent the problem is to be able to find a feasible state θ'' with the following properties: i) individuals' preferences change in a Maskin monotonic way around x from θ to θ'' and $R_i(\theta) = R_i(\theta'')$ for an individual i, and ii) individuals' preferences change in that way around x from θ'' to θ' and $R_j(\theta') = R_j(\theta'')$ for individual $j \neq i$. A domain Θ that assures the existence of such a state is the independent domain.

Even if one were able to find such a state θ'' by requiring an independent product structure of Θ, one could not invoke partial-honesty monotonicity and conclude that x must continue to be an F-optimal outcome at θ' whenever the family $\mathcal{H}$ did not have the appropriate structure. This can be seen as follows:

Suppose that Θ is an independent domain. Then, states take the form of profiles of individuals' characteristics, that is, $\theta = (\theta_1, \theta_2)$ and $\theta' = (\theta'_1, \theta'_2)$. Moreover, the characteristic of individual i in one state is independent from the characteristic of the other individual. That is, $R_i(\theta) = R_i(\theta_i)$ and $R_i(\theta') = R_i(\theta'_i)$ for every individual i. The product structure of Θ assures that the states (θ_1, θ'_2) and (θ'_1, θ_2) are both available and each of them has the properties summarized above.

Next, suppose that the family $\mathcal{H}$ has a structure given by $\{\{1\}, \{1,2\}\}$. One can invoke partial-honesty monotonicity for $H = \{1\}$ to obtain that x is one of the outcomes chosen by the SCR F at (θ_1, θ'_2) when the state changes from θ to (θ_1, θ'_2), but he cannot conclude that x remains also F-optimal at θ' when it changes from (θ_1, θ'_2) to θ'. The reason is that partial-honesty monotonicity cannot be invoked again for the case $H = \{2\}$ because the structure of the family $\mathcal{H}$ does not contemplate such a case. The argument for the case that $\mathcal{H}$ takes the form $\{\{2\}, \{1,2\}\}$ is symmetric. Thus, each of our requirements is indispensable, and jointly they lead to the following conclusion:

Theorem 4 Let N be a society involving at least two individuals, Θ be an independent domain and $\mathcal{H}$ include singletons. Suppose that the honesty standard of the society is non-connected. Partial-honesty monotonicity is equivalent to Maskin monotonicity.

Proof. Let $n \geq 2$, Θ be an independent domain and $\mathcal{H}$ include singletons. Let $S(N)$ be a non-connected honesty standard of N. One can see that Maskin monotonicity implies partial-honesty monotonicity.

For the converse, consider any SCR $F : \Theta \twoheadrightarrow X$ satisfying partial-honesty monotonicity. Consider any $x \in X$ and any state $\theta \in \Theta$ such that x is an F-optimal outcome at θ. Moreover, consider any state θ' such that individuals' preferences change in a Maskin monotonic way around x from θ to θ', that is,

$$\text{for all } i \in N \text{ and all } x' \in X : xR_i(\theta)\,x' \implies xR_i(\theta')\,x'.$$

We show that x remains F-optimal at θ'.

If characteristics of individuals in the honesty standard of individual $i \in N$ are identical in the two states, that is, $R_{S(i)}(\theta) = R_{S(i)}(\theta')$, partial-honesty monotonicity for the case $H = \{i\}$ assures that x is still F-optimal at θ'. Thus, let us consider the case $R_{S(i)}(\theta) \neq R_{S(i)}(\theta')$ for every individual $i \in N$.

To economize notation, for any subset K of N, write K_C for the complement of K in N. Therefore, for any non-empty subset K of N, we can write any non-trivial combination of the states θ and θ' as $\left(\theta_K, \theta'_{K_C}\right)$, where it is understood that θ_K is a list of characteristics of individuals in K at the state θ and θ'_{K_C} is a list of characteristics of individuals in K_C at θ'. Note that any state that results by that combination is available in Θ because of its product structure.

Given that the honesty standard of society is non-connected, there must be an individual $j(1) \in N$ who does not concern herself with the whole society, that is, $S(j(1)) \neq N$. Consider the state

$$\left(\theta_{K(1)}, \theta'_{K(1)_C}\right) \text{ where } K(1) \equiv S(j(1)),$$

and call it θ^1. By construction, individuals' preferences change in a Maskin monotonic way around x from θ to θ^1 and, moreover, $\theta_{K(1)} = \theta^1_{K(1)}$. Partial-honesty monotonicity for the case $H = \{j(1)\}$ assures that the x remains an F-optimal outcome at θ^1.

If there is an individual $i \in N \setminus \{j(1)\}$ who is not concerned with any of the individuals in the honesty standard of individual $j(1)$, that is, the intersection $S(i) \cap S(j(1))$ is empty, then partial-honesty monotonicity for the case $H = \{i\}$ assures that x is still F-optimal at θ'. This is because, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^1 to θ' and $\theta^1_{S(i)} = \theta'_{S(i)}$.

Thus, consider any individual $j(2) \in N \setminus \{j(1)\}$, and denote by $K(2)$ the set of individuals who jointly concern individual $j(1)$ and individual $j(2)$ according to their individual honesty standards. Furthermore, consider the state

$$\left(\theta_{K(2)}, \theta'_{K(2)_C}\right) \text{ where } K(2) \equiv K(1) \cap S(j(2)),$$

and call it θ^2. By construction, individuals' preferences change in a Maskin monotonic way around x from θ^1 to θ^2 and, moreover, $\theta^1_{S(j(2))} = \theta^2_{S(j(2))}$. Partial-honesty monotonicity for the case $H = \{j(2)\}$ assures that x remains an F-optimal outcome at θ^2.

If there is an individual $i \in N \setminus \{j(1), j(2)\}$ who is not concerned with any of the individuals with whom individuals $j(1)$ and $j(2)$ are jointly concerned, partial-honesty monotonicity for the case $H = \{i\}$ assures that x is also F-optimal at θ'. This is because, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^2 to θ' and $\theta^2_{S(i)} = \theta'_{S(i)}$.

Thus, consider any individual $j(3) \in N \setminus \{j(1), j(2)\}$, and denote by $K(3)$ the set of individuals that jointly concern individuals $j(1)$, $j(2)$ and $j(3)$ according to their individual honesty standards. Furthermore, consider the state

$$\left(\theta_{K(3)}, \theta'_{K(3)_C}\right) \text{ where } K(3) \equiv K(2) \cap S(j(3)),$$

and call it θ^3. By construction, individuals' preferences change in a Maskin monotonic way around x from θ^2 to θ^3 and, moreover, $\theta^2_{S(j(3))} = \theta^3_{S(j(3))}$. Partial-honesty monotonicity for the case $H = \{j(3)\}$ assures that x remains an F-optimal outcome at θ^3.

As above, if there is an individual $i \in N \setminus \{j(1), j(2), j(3)\}$ who is not concerned with any of the individuals with whom individuals $j(1)$, $j(2)$ and $j(3)$ are jointly concerned, partial-honesty monotonicity for the case $H = \{i\}$ assures that x remains also F-optimal at θ', because, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^3 to θ' and $\theta^3_{S(i)} = \theta'_{S(i)}$. And so on.

Since the society N is a finite set and the above iterative reasoning is based on its cardinality, we are left to show that it must stop at most after $n-1$ iterations.

To this end, suppose that we have reached the start of the $n-1$th iteration. Thus, consider any individual $j(n-1) \in N$, with $j(n-1) \neq j(r)$ for $r = 1, \cdots, n-2$, and denote by $K(n-1)$ the set of individuals that jointly concern individuals $j(1)$, $j(2), \cdots, j(n-2)$ and $j(n-1)$ according to their individual honesty standards. Furthermore, consider the state

$$\left(\theta_{K(n-1)}, \theta'_{K(n-1)_C}\right) \text{ where } K(n-1) \equiv K(n-2) \cap S(j(n-1)),$$

and call it θ^{n-1}. As above, by construction, individuals' preferences change in a Maskin monotonic way around x from $\theta^{n-2} \equiv \left(\theta_{K(n-2)}, \theta'_{K(n-2)_C}\right)$ to θ^{n-1} and, moreover, $\theta^{n-2}_{S(j(n-1))} = \theta^{n-1}_{S(j(n-1))}$. Partial-honesty monotonicity for the case $H = \{j(n-1)\}$ assures that x is an F-optimal outcome at θ^{n-1}.

At this stage there is only one individual in N who is left to be considered. Call her $j(n)$. Suppose that this individual is concerned with one of the individuals for whom individuals $j(1)$, $j(2), \cdots, j(n-2)$ and $j(n-1)$ are jointly concerned. In other words, suppose that the intersection $K(n-1) \cap S(j(n))$ is non-empty. Then, the whole society concerns itself with one of its member, and this contradicts the fact that the honesty standard of society is non-connected. Therefore, it must be the case that individual $j(n)$ is not concerned with any of the individuals with whom individuals $j(1)$, $j(2), \cdots, j(n-2)$ and $j(n-1)$ are jointly concerned according to their individual honesty standards. Partial-honesty monotonicity for the case $H = \{j(n)\}$ assures that x remains also F-optimal at θ' given that, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^{n-1} to θ' and $\theta^{n-1}_{S(j(n))} = \theta'_{S(j(n))}$.

The iterative reasoning would stop at the rth ($< n-1$) iteration if there were an individual $i \in N \setminus \{j(1), \cdots, j(r)\}$ who did not concern itself with any of the individuals in $K(r)$, that is, if the intersection $S(i) \cap K(r)$ were empty. If that were the case, then the desired conclusion could be obtained by invoking partial-honesty monotonicity for $H = \{i\}$ because, by construction, it would hold that individuals' preferences change in a Maskin monotonic way around x from θ^r to θ' and that $\theta^r_{S(i)} = \theta'_{S(i)}$. ■

In light of Theorem 1 and Maskin's theorem, the main implications of the above conclusion can be formally stated as follows:

Corollary 1 Let N be a society involving at least two individuals, Θ be an independent domain and $\mathcal{H}$ include singletons. Suppose that the honesty standard of the society is non-connected. Let Assumption 1 and Assumption 2 be given. The SCR $F : \Theta \twoheadrightarrow X$ is Maskin monotonic if it is partially-honestly Nash implementable.

Corollary 2 Let N be a society involving at least three individuals, Θ be an independent domain and $\mathcal{H}$ include singletons. Suppose that the honesty standard of the society is non-connected. Let Assumption 1 and Assumption 2 be given. Any SCR $F : \Theta \twoheadrightarrow X$ satisfying no veto-power is partially-honestly Nash implementable if and only if it is Maskin monotonic.

Restoration of Dutta-Sen's theorem on Nash implementation with strategy space reduction

In an environment in which knowledge is dispersed, how individuals will interact with the mechanism designer is a natural starting point when it comes to Nash implementing a SCR. A particular kind of communication is, as we have done so far, to ask participants to report preferences of the entire society. However, there is no reason to restrict attention to such schemes.

Indeed, there may be sufficiently strong reasons that make it necessary for the mechanism designer to employ communication schemes that are simpler than the type of communication studied so far, and which force individuals to behave *as if* their honesty standards were non-connected. In light of Theorem 4, in cases like this, the predicted result is that the mechanism designer may not be able to escape the limitations imposed by Maskin monotonicity and he is thus expected to do poorly. A natural question that arises immediately is: Under which conditions would the positive sufficiency result of Dutta and Sen (2012) be restored? Given our abstract framework, we answer this question by placing it within the literature on strategy space reduction in Nash implementation. A pioneering work in this respect is Saijo (1988).[5]

The basic idea behind the literature on strategy space reduction is to reduce the informational requirements in the preference announcement component of strategy choices. For example, individual i may be required to choose only her own characteristics as part of her strategy choice, or individual i can be required to choose her own characteristics and those of her neighbor individual $i+1$, and so on. A way to proceed is to arrange individuals in a circular fashion numerically clockwise - facing inward, and to require that each individual i announces her own characteristics together with the characteristics of $q-1$ individuals standing immediately to her left, where $1 \leq q \leq n-1$. Following this literature, a q-mechanism can be defined as follows:

Definition 9 For each $q \in N$, a mechanism $\Gamma_q = (M, g)$ is a q-mechanism if, for each $i \in N$, $M_i \equiv \prod_{k=i}^{q+i-1} \Theta_k \times X \times N$, with the convention that $n + p = p$ for $p \in N$.

In this section, we assume that the actual honesty standard of participant $i \in N$ is $S(i) = N$.

Let us imagine that the mechanism designer knows that participant i feels honest when she is truthful about characteristics of the entire society and that she is forced to govern the communication with individuals by a 2-mechanism; that is, by a communication scheme that requires each individual i to choose her own characteristics and those of her adjacent

[5] See also McKelvey, 1989; Tatamitani, 2001; Lombardi and Yoshihara, 2013.

individual $i+1$. Although the honesty standards of society are connected, this type of communication scheme forces individual i to behave *as if* her honesty standard was of the form $S(i;2)=\{i,i+1\}$, and, thereby, it forces the society to behave *as if* its honesty standard $S(N;2)=(S(i;2))_{i\in N}$ was non-connected. The reason is that individuals cannot articulate the communication according to their actual honesty standards. On this basis, let us formalize the mechanism designer's partially-honest Nash implementation problem by a q-mechanism. Write $S(i;q)$ for a typical honesty standard of participant i that is enforced by a q-mechanism and write $S(N;q)$ for a typical honesty standard of society N that is enforced by a q-mechanism. Therefore:

Definition 10 Let Assumption 1 and Assumption 2 be given. Let $S(i)=N$ for each $i\in N$. A q-mechanism Γ_q *partially-honestly Nash implements* the SCR $F:\Theta\twoheadrightarrow X$ provided that for all $\theta\in\Theta$ and $H\in\mathcal{H}$ there exists for any $h\in H$ a truth-telling correspondence $T_h^{\Gamma_q}(\theta;S(h;q))$ as formulated in Definition 1 and, moreover, it holds that $F(\theta)=NA\left(\Gamma_q,\succsim^{\Gamma_q,\theta,S(N;q),H}\right)$, where $S(N;q)\equiv(S(i;q))_{i\in N}$ is the honesty standard of N enforced by the mechanism. If such a mechanism exists, F is said to be *partially-honestly Nash implementable by a q-mechanism*.

It can be verified by means of Theorem 1 that for any given honesty standard $S(N;q)$, partial-honesty monotonicity with respect to $S(N;q)$ is a necessary condition for partially-honest Nash implementation by a q-mechanism. Furthermore, it can also be verified that the enforced honesty standard $S(N;q)$ of society N is non-connected as long as $q\neq n$. Thus, this enforcement would impair the ability of the mechanism designer to escape the limitations imposed by Maskin monotonicity when some further assumptions of Theorem 4 are met.

Our next result states that the mechanism designer can circumvent the limitations imposed by Theorem 4 and successfully partially-honest Nash implements SCRs that are not Maskin monotonic by a q-mechanism provided that there are at least $n-q+1$ partially-honest individuals in a society and that no participant has a veto-power.[6] The reason is that $n-q+1$ is the minimal number of partially-honest individuals that assures that for every conceivable set H of partially-honest individuals the enforced honesty standard $S(N;q)$ forms a covering of society N; that is, $N\subseteq\bigcup_{h\in H}S(h,q)$.

Put differently, it provides a theoretical reference for the actual mechanism design: If the mechanism designer knows that $\alpha(\geq 1)$ members of society have a taste for honesty, then he can expect to do well by asking each participant to report her own characteristics and those of $n-\alpha$ individuals and achieve, at most, an overall reduction in the size of the strategy space M equal to $n(\alpha-1)$. The following theorem substantiates our discussion:

Theorem 5 Let $n\geq 3$. Let Assumption 1 and Assumption 2 be given. Let Θ be an independent domain and let $S(i)=N$ for each $i\in N$. Suppose that the SCR $F:\Theta\twoheadrightarrow X$ satisfies no veto-power and that it is not Maskin monotonic. Let the class $\mathcal{H}$ include singletons whenever it is possible. Then, for any $q\in N\setminus\{1\}$ and any environment $(\theta,S(N;q),H)$, the

[6] Recall that the importance of Dutta-Sen's theorem for Nash implementation is that SCRs that are not Maskin monotonic can be partially-honestly implemented if there is at least one individual who is partially-honest in a society.

SCR F is partially-honestly implementable by a q-mechanism if and only if the number of partially-honest individuals in N is at least $n-q+1$.

Proof. Let the premises hold. Let us first show the "if" part of the statement. Suppose that the number of partially-honest individuals in society is at least $n-q+1$. We show that the SCR F is partially-honestly implementable by a q-mechanism if it satisfies no veto-power. A typical strategy played by individual i is denoted by $m_i = \left(\boldsymbol{\theta}^i, x^i, z^i\right)$.For each $(m,\theta,x) \in M \times \Theta \times X$, we say that m is:

(i) *consistent* with (θ,x) if $x^j = x$ and $\boldsymbol{\theta}^j = (\theta_j, \theta_{j+1}, \cdots, \theta_{q+j-1})$ for each $j \in N$.

(ii) for all $i \in N$, m_{-i} *consistent* with (θ,x) if $x^j = x$ and $\boldsymbol{\theta}^j = (\theta_j, \theta_{j+1}, \cdots, \theta_{q+j-1})$ for each $j \in N \backslash \{i\}$, and $x^i \neq x$ or $\boldsymbol{\theta}^i \neq (\theta_i, \theta_{i+1}, \cdots, \theta_{q+i-1})$.

In other words, a message profile m is consistent with (x,θ) if there is no break in the cyclic announcement of characteristics and all individuals announce the outcome x. On the other hand, it is m_{-i} consistent with (x,θ) *either* if there are, at most, q consecutive breaks in the cyclic announcement of characteristics such that these breaks happen in correspondence of the characteristics announced by individual i, and x is unanimously announced *or* if individual i announces an outcome different from the outcome x announced by the others, and there are no more than q consecutive breaks in the cyclic announcement of characteristics such that these breaks (if any) happen in correspondence of the characteristics announced by individual i.

For each individual i, i's truth-telling correspondence is defined as follows: For all $\theta \in \Theta$,

$$\left(\boldsymbol{\theta}^i, x^i, z^i\right) \in T_i^{\Gamma_q}(\theta, S(i;q)) \quad \text{if and only if} \quad \boldsymbol{\theta}^i = (\theta_i, \theta_{i+1}, \cdots, \theta_{q+i-1}), \text{ with } n+p=p.$$

As in Lombardi and Yoshihara (2013)'s 2-mechanism, in our q-mechanism individuals make a cyclic announcement of strategies while the profile of characteristics, that is, the state, is determined without relying upon the deviator's announcement. Thus, the outcome function g is defined with the following three rules: For each $m \in M$,

Rule 1: If m is consistent with $\left(\bar{\theta}, x\right)$ and $x \in F\left(\bar{\theta}\right)$, then $g(m) = x$.

Rule 2: If for some $i \in N$, m_{-i} is consistent with $\left(\bar{\theta}, x\right)$ and $x \in F\left(\bar{\theta}\right)$, then $g(m) = x$.

Rule 3: Otherwise, a modulo game is played: identify the individual $i = \sum_{j \in N} z^j \pmod{n}$. This individual is declared the winner of the game, and the alternative implemented is the one she selects.

Let us check that the above q-mechanism partially-honest implements F. Suppose that $\theta \in \Theta$ is the "true" state and that $H \in \mathcal{H}$ is the "true" set of partially-honest individuals. Suppose that $x \in F(\theta)$. Let $m_i = \left(\boldsymbol{\theta}^i, x, z^i\right)$ for each $i \in N$ such that the corresponding strategy profile m is consistent with (θ, x). Then, Rule 1 implies that $g(m) = x$. Note that no unilateral deviation can change the outcome. Also, note that individual i is truthful in the preference announcement component $\boldsymbol{\theta}^i$ of her strategy. Therefore, $x \in NA\left(\Gamma_q, \succcurlyeq^{\Gamma_q, \theta, S(N;q), H}\right)$.

We now show that $NA\left(\Gamma_q, \succsim^{\Gamma_q,\theta,S(N;q),H}\right) \subseteq F(\theta)$. Let $m \in NE\left(\Gamma_q, \succsim^{\Gamma_q,\theta,S(N;q),H}\right)$. To avoid triviality, suppose that $|X| \geq 2$.[7] We distinguish three cases.

Suppose that m falls into Rule 1. Take any partially-honest individual $h \in H$. Suppose that $m_h \notin T_h^{\Gamma_q}(\theta, S(h,q))$. Then, it is the case that $\boldsymbol{\theta}^h \neq (\theta_h, \theta_{h+1}, \cdots, \theta_{q+h-1})$. By changing m_h into $m'_h \in T_h^{\Gamma_q}(\theta, S(h,q))$, agent h can induce Rule 2 and obtain $g(m'_h, m_{-h}) = x$. Given that $g(m) = g(m'_h, m_{-h})$, that $m'_h \in T_h^{\Gamma_q}(\theta, S(h,q))$ and that $m_h \notin T_h^{\Gamma_q}(\theta, S(h,q))$, it follows from part (i) of Definition 2 that $(m'_h, m_{-h}) \succ_h^{\Gamma_q,\theta,S(h,q)} m$, which is a contradiction. Therefore, we have established that $m_h \in T_h^{\Gamma_q}(\theta, S(h,q))$, and so $\boldsymbol{\theta}^h = (\theta_h, \theta_{h+1}, \cdots, \theta_{q+h-1})$. Finally, we need to show that $\bar{\theta} = \theta$. Assume, to the contrary, that $\bar{\theta} \neq \theta$. Thus, $\bar{\theta}_j \neq \theta_j$ for some $j \in N$, and so individual j is not truthful in her announcement $\boldsymbol{\theta}^j$. Since every partially-honest individual is truthful, individual j is not a partially-honest individual, that is, $j \notin H$. Furthermore, given that $\bar{\theta}_j \neq \theta_j$, it also follows that at least $q-1$ individuals standing immediately to her right are not partially-honest.[8] Thus, there must be at least q individuals in N who are not partially-honest, which contradicts the fact that there are at least $n-q+1$ partially-honest individuals, and so it must be the case that there are most $q-1$ individuals who are not partially-honest. We conclude that $\bar{\theta} = \theta$.

Suppose that m falls into Rule 2. We proceed according to whether $x^i \neq x$ or not.

Case 1: $x^i \neq x$.

- Suppose that $|X| \neq 2$ or $n \neq 3$. If $|X| > 2$, then every individual $j \neq i$ can induce Rule 3. Thus, we have that $X \subseteq g(M_j, m_{-j})$ for each individual $j \in N \backslash \{i\}$. Otherwise, let us suppose that $|X| = 2$ and that $n \neq 3$. By replacing x with $x^j = x^i$, individual j can make $\left|\left\{\ell \in N | x^\ell = x\right\}\right| \geq 2$ and $\left|\left\{\ell \in N | x^\ell \neq x\right\}\right| \geq 2$. Since the outcome is determined by Rule 3, individual j can attain any outcome in X by appropriately choosing z^j. Again, we have that $X \subseteq g(M_j, m_{-j})$ for any $j \in N \backslash \{i\}$. Finally, let us consider the case that $|X| = 2$ and that $n = 3$. Then, let $N = \{i-1, i, i+1\}$, with $n+1 = 1$ and $1-1 = n$. We proceed according to whether or not there exist two distinct individuals $\ell, \ell' \in N$ such that $|\Theta_\ell| \neq 1$ and $|\Theta_{\ell'}| \neq 1$ hold.

- Suppose that there are two distinct individuals $\ell, \ell' \in N$ such that $|\Theta_\ell| \neq 1$ and $|\Theta_{\ell'}| \neq 1$. In this case, individual $i-1$ (resp., $i+1$) can always induce Rule 3 by appropriately changing the announcement of her own characteristics or that of her successor, and by carefully choosing the outcome announcement. To attain x^i, individual $i-1$ (resp., $i+1$) has only to adjust the integer index.

- Suppose that for all two distinct individuals $\ell, \ell' \in N$, it holds that $|\Theta_\ell| = 1$ or $|\Theta_{\ell'}| = 1$.

  Suppose that $|\Theta_k| = 1$ for all $k \in N$. Since m falls into Rule 2, it follows that $x \in F(\theta)$, as desired.

  Suppose that there exists $k \in \{i-1, i, i+1\}$ such that $|\Theta_k| \neq 1$.

[7] For a set S, we write $|S|$ to denote the number of elements in S.

[8] Recall that individuals are arranged in a circular fashion clockwise facing inward, and each i is required to announce her own cahracteristics together with the characteristics of $q-1$ individuals standing immediately to her left.

If either $|\Theta_{i-1}| > 1$ or $|\Theta_i| > 1$, then individual $i-1$ can induce Rule 3 by changing m_{i-1} to either $m'_{i-1} = \left(\left(\theta^{i-1}_{i-1}, \boldsymbol{\theta}^{i-1}_{-(i-1)}\right), x, z^{i-1}\right)$ with $\theta^{i-1}_{i-1} \neq \bar{\theta}_{i-1}$ (if $|\Theta_{i-1}| > 1$) or $m'_{i-1} = \left(\left(\theta^{i-1}_{i}, \boldsymbol{\theta}^{i-1}_{-i}\right), x^i, z^{i-1}\right)$ with $\theta^{i-1}_{i} \neq \bar{\theta}_{i}$ (if $|\Theta_i| > 1$). Individual $i-1$ can attain x^i by announcing z^{i-1} by which individual i becomes the winner of the modulo game. Therefore, we have that $X \subseteq g\left(M_{i-1}, m_{-(i-1)}\right)$. Suppose that $|\Theta_{i-1}| = |\Theta_i| = 1$. If $q = n$, individual $i-1$ can induce Rule 3 by changing m_{i-1} into $m'_{i-1} = \left(\left(\boldsymbol{\theta}^{i-1}_{-(i+1)}, \theta^{i-1}_{i+1}\right), x, z^{i-1}\right)$, with $\theta^{i-1}_{i+1} \neq \bar{\theta}_{i+1}$. Individual $i-1$ can attain x^i by announcing z^{i-1} by which individual i becomes the winner of the modulo game. Suppose that $q \neq n$. Individual $i-1$ can change m_{i-1} into $m'_{i-1} = \left(\boldsymbol{\theta}^{i-1}, x^i, z^{i-1}\right)$. Note that $\left(m'_{i-1}, m_i\right)$ is consistent with $\left(x^i, \left(\boldsymbol{\theta}^{i-1}, \theta^i_{i+1}\right)\right)$ given that $\theta^{i-1}_i = \theta^i_i$. If $x^i \in F\left(\boldsymbol{\theta}^{i-1}, \theta^i_{i+1}\right)$, then $\left(m'_{i-1}, m_{-(i-1)}\right)$ falls into Rule 2, and so $g\left(m'_{i-1}, m_{-(i-1)}\right) = x^i$. If $x^i \notin F\left(\boldsymbol{\theta}^{i-1}, \theta^i_{i+1}\right)$, then $\left(m'_{i-1}, m_{-(i-1)}\right)$ falls into Rule 3. Individual $i-1$ can attain x^i by announcing z^{i-1} by which she becomes the winner of the modulo game. We have established that $X \subseteq g\left(M_{i-1}, m_{-(i-1)}\right)$ if $|\Theta_k| \neq 1$ for some $k \in \{i-1, i, i+1\}$. Reasoning like that used for individual $i-1$ shows that $X \subseteq g\left(M_{i+1}, m_{-(i+1)}\right)$ if $|\Theta_k| \neq 1$ for some $k \in \{i-1, i, i+1\}$. Thus, $X \subseteq g\left(M_j, m_{-j}\right)$ for each individual $j \in \{i-1, i+1\}$.

From the above arguments, we obtained that $X \subseteq g\left(M_j, m_{-j}\right)$ for each individual $j \in N\backslash\{i\}$. Given that $m \in NE\left(\Gamma_q, \succsim^{\Gamma_q, \theta, S(N), H}\right)$, it follows that $g\left(M_j, m_{-j}\right) = X \subseteq L_j\left(\theta, x\right)$ for each $j \in N\backslash\{i\}$. No veto-power implies that $x \in F\left(\theta\right)$.

Case 2: $x^i = x$.

Then, it is the case that $\boldsymbol{\theta}^i \neq \left(\bar{\theta}_i, \bar{\theta}_{i+1}, \cdots, \bar{\theta}_{q+i-1}\right)$ given m_{-i} is consistent $\left(x, \bar{\theta}\right)$ and $x^i = x$. We proceed according to whether $q = 2$ or not.

- Suppose that $q \neq 2$. Thus, individual i is a unique deviator. By altering her strategy choice m_j into $m'_j = \left(\boldsymbol{\theta}^j, x^j, z^j\right)$, with $x^j \neq x$, individual j can induce Rule 3. Then, to attain x^j, individual j has only to announce z^j by which she becomes the winner of the modulo game. Since $g\left(m\right) = x$ and since, moreover, the choice of $x^j \neq x$ was arbitrary, we have that $X \subseteq g\left(M_j, m_{-j}\right)$. Thus, we have established that $X \subseteq g\left(M_j, m_{-j}\right)$ for each individual $j \in N\backslash\{i\}$. Given that $m \in NE\left(\Gamma_q, \succsim^{\Gamma_q, \theta, S(N), H}\right)$, it follows that $g\left(M_j, m_{-j}\right) = X \subseteq L_j\left(\theta, x\right)$ for each $j \in N\backslash\{i\}$. No veto-power implies that $x \in F\left(\theta\right)$.

- Suppose that $q = 2$. We proceed according to the following sub-cases: 1) $\theta^i_i \neq \bar{\theta}_i$ and $\theta^i_{i+1} \neq \bar{\theta}_{i+1}$, and 2) $\theta^i_i \neq \bar{\theta}_i$ and $\theta^i_{i+1} = \bar{\theta}_{i+1}$.[9]

  Suppose that $\theta^i_i \neq \bar{\theta}_i$ and $\theta^i_{i+1} \neq \bar{\theta}_{i+1}$. Thus, individual i is a unique deviator. Reasoning like that used for the case $q \neq 2$ shows that $x \in F\left(\theta\right)$.

  Suppose that $\theta^i_i \neq \bar{\theta}_i$ and $\theta^i_{i+1} = \bar{\theta}_{i+1}$. Suppose that $x \notin F\left(\bar{\theta}, \theta^i_i\right)$. Thus, individual i is a unique deviator. Reasoning like that used for the case $q \neq 2$ shows that $x \in F\left(\theta\right)$. Let us consider the case that

$$x \in F\left(\bar{\theta}_{-i}, \theta^i_i\right) \bigcap F\left(\bar{\theta}\right). \tag{10}$$

[9] The sub-case $\theta^i_i = \bar{\theta}_i$ and $\theta^i_{i+1} \neq \bar{\theta}_{i+1}$ is not explicitly considered, since it can be proved similarly to the sub-case 2 shown below.

Then, $i-1$ and i are both deviators.

Suppose that individual $i \in H$ and that $m_i \notin T_i^{\Gamma_q}(\theta, S(i;q))$. If $(\theta_i, \theta_{i+1}) = (\bar{\theta}_i, \bar{\theta}_{i+1})$, then individual i can induce Rule 1 by changing m_i into $m_i' = ((\bar{\theta}_i, \bar{\theta}_{i+1}), x, z^i) \in T_i^{\Gamma_q}(\theta, S(i;q))$. Given that $g(m) = g(m_i', m_{-i})$, that $m_i' \in T_i^{\Gamma_q}(\theta, S(i;q))$ and that $m_i \notin T_i^{\Gamma_q}(\theta, S(i;q))$, it follows from part (i) of Definition 2 that $(m_i', m_{-i}) \succ_i^{\Gamma_q,\theta,S(i;q)} m$, which is a contradiction. Suppose that $(\theta_i, \theta_{i+1}) \neq (\bar{\theta}_i, \bar{\theta}_{i+1})$. By changing m_i into $m_i' = ((\theta_i, \theta_{i+1}), x, z^i) \in T_i^{\Gamma_q}(\theta, S(i;q))$, individual i can induce Rule 2, thus $g(m_i', m_{-i}) = x$. Given that $g(m) = g(m_i', m_{-i})$, that $m_i' \in T_i^{\Gamma_q}(\theta, S(i;q))$ and that $m_i \notin T_i^{\Gamma_q}(\theta, S(i;q))$, it follows from part (i) of Definition 2 that $(m_i', m_{-i}) \succ_i^{\Gamma_q,\theta,S(i;q)} m$, which is a contradiction. We conclude that $m_i \in T_i^{\Gamma_q}(\theta, S(i;q))$ if $i \in H$.

Suppose that individual $i-1 \in H$ and that $m_{i-1} \notin T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$. If $(\theta_{i-1}, \theta_i) = (\bar{\theta}_{i-1}, \theta_i^i)$, then individual $i-1$ can induce Rule 1 by changing m_{i-1} into $m_{i-1}' = ((\theta_{i-1}, \theta_i), x, z^{i-1}) \in T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$. Given that $g(m) = g(m_{i-1}', m_{-(i-1)})$, that $m_{i-1}' \in T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$ and that $m_{i-1} \notin T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$, it follows from part (i) of Definition 2 that $(m_{i-1}', m_{-(i-1)}) \succ_{i-1}^{\Gamma_q,\theta,S(i-1;q)} m$, which is a contradiction. Suppose that $(\theta_{i-1}, \theta_i) \neq (\bar{\theta}_{i-1}, \theta_i^i)$. By changing m_{i-1} into $m_{i-1}' = ((\theta_{i-1}, \theta_i), x, z^{i-1}) \in T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$, individual $i-1$ can induce Rule 2, thus $g(m_{i-1}', m_{-(i-1)}) = x$. Given that $g(m) = g(m_{i-1}', m_{-(i-1)})$, that $m_{i-1}' \in T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$ and that $m_{i-1} \notin T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$, it follows from part (i) of Definition 2 that $(m_{i-1}', m_{-(i-1)}) \succ_{i-1}^{\Gamma_q,\theta,S(i-1;q)} m$, which is a contradiction. We conclude that $m_{i-1} \in T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$ if $i-1 \in H$.

Suppose that individual $j \in H \setminus \{i-1, i\}$ and that $m_j \notin T_j^{\Gamma_q}(\theta, S(j;q))$. Take any $x^j \neq x$. By changing m_j into $m_j' = ((\theta_j, \theta_{j+1}), x^j, z^j) \in T_j^{\Gamma_q}(\theta, S(j;q))$, individual j can induce Rule 3, where z^j satisfies $i = \sum_{k \in N} z^k \pmod{n}$, and thus $g(m_j', m_{-j}) = x$. Given that $g(m) = g(m_j', m_{-j})$, that $m_j' \in T_j^{\Gamma_q}(\theta, S(j;q))$ and that $m_j \notin T_j^{\Gamma_q}(\theta, S(j;q))$, it follows from part (i) of Definition 2 that $(m_j', m_{-j}) \succ_j^{\Gamma_q,\theta,S(j;q)} m$, which is a contradiction. We conclude that $m_j \in T_j^{\Gamma_q}(\theta, S(j;q))$ if $j \in H \setminus \{i-1, i\}$.

From the above arguments, we obtain that $m_h \in T_h^{\Gamma_q}(\theta, S(h,q))$ for all $h \in H$. Also, note that $i-1 \notin H$ if $i \in H$, given that $\theta_i^i \neq \bar{\theta}_i$. For the same reason, it holds that $i \notin H$ if $i-1 \in H$. Given that there are at least $n-q+1 = n-1$ partially-honest individuals, it is the case that at least one of the deviators is a partially-honest individual. If both deviators are partially-honest, then arguments like those used above for the case $q=2$ shows that either i or $i-1$ can find a profitable unilateral deviation from the profile $m \in NE(\Gamma_q, \succsim^{\Gamma_q,\theta,S(N),H})$, which is a contradiction. Thus, it is the case that only one of the deviators can be a partially-honest individual. Given that all partially-honest individuals are truthful and given that (10) holds, it follows that $x \in F(\theta)$, as we sought.

Suppose that m falls into Rule 3. By the definition of the outcome function, we have that for each individual i, $g(M_i, m_{-i}) = X$. Given that $m \in NE(\Gamma_q, \succsim^{\Gamma_q,\theta,S(N),H})$, it follows that $g(M_i, m_{-i}) = X \subseteq L_i(\theta, x)$ for each $i \in N$. No veto-power implies that $x \in F(\theta)$.

Let us show the "only if" part of the statement. Suppose that the SCR F is partially-honestly implementable by a q-mechanism. Assume, to the contrary, the number of partially-honest individuals in N is lower than $n-q+1$. Given that Assumption 1 assures that there is at least one individual who is partially-honest, an immediate contradiction is obtained if $q=n$. Thus, let us consider the case that $q \neq n$. Furthermore, the class $\mathcal{H}$ includes singletons given that the largest admissible family of conceivable sets of partially-honest individuals is

$$\mathcal{H} \equiv \{H \subseteq N | \, |H| < n-q+1\}.$$

Finally, given that $S(N;q)$ is non-connected, Corollary 2 implies that the SCR F is Maskin monotonic, which is a contradiction. ■

We make several remarks below regarding Theorem 5.

Remark 1 It is known that Maskin's theorem is robust against Saijo (1988)'s simplification of Maskin's communication scheme. Indeed, the class of Nash implementable SCRs is equivalent to the class of SCRs that are Nash implementable by a q-mechanism provided that $q \geq 2$ (Lombardi and Yoshihara, 2013). In light of Theorem 5, this equivalence relationship no longer holds if there are less than $n-q+1$ individuals who have a taste for honesty.

Remark 2 The "only if" part of the statement continues to hold if $q=1$, that is, when every individual $i \in N$ is required to choose only her own characteristics as part of her strategy choice, like a self-relevant mechanism (Tatamitani, 2001). It means that if a non-Maskin monotonic SCR F is partially-honestly Nash implementable by this type of mechanism, then all individuals in a society need to be partially-honest. However, if the requirement $q \neq 1$ is dropped, the "if" part of the statement fails to hold. The reason is that the mechanism constructed to prove Theorem 5 detects a participant's lie by relying on the play of other participants. This type of detection is not possible in the case of a self-relevant mechanism. A sufficient condition for the SCR F to be partially-honest Nash implementable by a self-relevant mechanism is that F satisfies no veto-power as well as there is a worst outcome in X for any individual i.

Remark 3 Note that Theorem 5 holds as long as $S(i;q) \subseteq S(i)$ for each $i \in N$. Our choice of $S(i) = N$ for each individual i is simply motivated by convenience. Moreover, when the honesty standard of participant i that is enforced by a q-mechanism is not a subset of her actual honesty standards, that is, $S(i;q) \nsubseteq S(i)$, then Maskin monotonicity may become again a necessary condition for Nash implementation, though there is at least $n-q+1$ partially-honest individuals in N. For instance, consider a society with $n=3$ participants and $q=2$ for the q-mechanism. By Theorem 5, any SCR satisfying the no veto-power condition is partially-honestly Nash implementable if and only if there are at least two partially-honest individuals, provided that $S(i) = S(i;2)$ for each participant $i \in N$. However, we can show that any SCR satisfying partial-honesty monotonicity should also satisfy Maskin monotonicity in a society with at least two partially-honest individuals if every participant i's honesty standard is $S(i) = \{i\}$.

Concluding remarks

The assumption that the mechanism designer knows the honesty standard of a society is often not met in reality, although it may be plausible in societies with a small number of individuals in which the mechanism designer knows their sensitivity to honesty. Outside of cases like those, we view as more plausible the assumption that the mechanism designer only knows the type of honesty standards shared by individuals. Does the conclusion change in this case? The answer is no. After all, if individuals are honesty-sensitive, the mechanism designer can test for connectedness of their honesty standards. If the test fails, it would be vain for him to attempt to Nash implement any SCR that is not Maskin monotonic. The reason for it is easy to identify: the fact that he solely knows that the honesty standard of a society is non-connected can only make implementation harder than if the actual non-connected honesty standards of participants were known. Moreover, the assumption is naturally met if we place our contribution within the literature on Nash implementation with strategy space reduction.

In an environment in which knowledge is dispersed, how individuals will interact with the mechanism designer is a natural starting point when it comes to Nash implementing a SCR. A particular kind of communication is, as we have done in the first part of this paper, to ask participants to report preferences of the entire society. However, there is no reason to restrict attention to such schemes.

A simpler way to go about it is to ask individuals to report only their own preferences. An obvious advantage of such a type of communication is that the mechanism designer does not need to understand the psychological motivations of the individuals, beyond a basic self-interest. However, if the mechanism designer structured the communication in this way, he would then force individuals to behave *as if* their honesty standards were non-connected, though their non-connectedness could be merely an artifact of that communication structure. The reason is that individuals cannot articulate the communication according to their actual honesty standards. And, from the perspective of this paper, this would impair the ability of the mechanism designer to escape the limitations imposed by Maskin monotonicity. Not surprisingly, in an independent domain of strict preferences, Saporiti (2014) shows that any social choice function that can be securely implemented is Maskin monotonic, though all participants are partially-honest.[10]

There are multiple other ways for the mechanism designer to structure the exchange of information with individuals, and there is no limit to how imaginative he can be. This paper offers this guidance on how to go about it in environments involving partially-honest individuals: If the honesty standards of participants are connected, the informational requirements need not force individuals to behave *as if* their honesty standards were not.

Postlewaite and Schmeidler (1986), Palfrey and Srivastava (1989) and Jackson (1991) have shown that Maskin's theorem can be generalized to Bayesian environments. A necessary condition for Bayesian Nash implementation is Bayesian monotonicity. In a Bayesian environment involving at least three individuals, Bayesian monotonicity combined with no veto-power is sufficient for Bayesian Nash implementation provided that a necessary condi-

[10]Secure implementation is implementation in Nash equilibrium and in dominant strategies. Recall that in an independent domain of strict preferences, strategy-proofness implies Maskin monotonicity (Dasgupta et al., 1979).

tion called closure and the Bayesian incentive compatibility condition are satisfied (Jackson, 1991). Although the implementation model developed in this paper needs to be modified to handle Bayesian environments, we believe a similar equivalence result holds in those environments for suitably defined non-connected honesty standards. This subject is left for future research.

Based upon the view that people may have different honesty standards, we identified conditions for Nash implementation with partially-honest individuals which, if satisfied, send us back to the limitations imposed by Maskin's theorem. Thus, the exploration of the possibilities offered by that implementation needs to move away from those properties. As yet, where the exact boundaries of those possibilities lay for general environments and economic environments is far from known.[11]

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[11] An exception is represented by Lombardi and Yoshihara (2014), in which problems in pure exchange economies by price-quantity mechanisms are studied. They also take care of the boundary problem that is prominent in Nash implementation literature and show that in the case that there are at least three agents, the Walrasian rule is Nash implementable by a natural price-quantity mechanism when at least one agent is partially-honest and the mechanism designer does not know the identity of the partially-honest agent(s).

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