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A Progressive Report on Marxian Economic Theory: On the Controversies in Exploitation Theory since Okishio (1963)

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A Progressive Report on Marxian Economic Theory: On the Controversies in Exploitation Theory since Okishio (1963)*

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Abstract

This report explores the development of exploitation theory in mathematical Marxian economics by reviewing the main controversies surrounding the definition of exploitation since the contribution of Okishio (1963). The report first examines the robustness and economic implications of the debates on the Fundamental Marxian Theorem, developed mainly in the 1970s and 1980s, followed by the property relation theory of exploitation by Roemer (1982). Then, the more recent exploitation theory proposed by Vrousalis (2013) and Wright (2000) is introduced, before examining its economic implications using a simple economic model. Finally, the report introduces and comments on recent axiomatic studies of exploitation by focusing on the work of Veneziani and Yoshihara (2013a).

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1 Introduction

In Marxian economics, the capitalist economy is depicted as an exploitative system. The validity of this basic Marxian insight has been recognized since the work of Okishio (1963), and proved by the so-called Fundamental Marxian Theorem (FMT), which assumes a simple Leontief economic model and uses Okishio's definition of exploitation. However, the FMT loses robustness once a more complex economic model is considered. Moreover, the Generalized Commodity Exploitation Theorem indicates that the definition of exploitation à la Okishio (1963)–Morishima (1973) does not properly capture the core feature of exploitation as a concept of social relations. Instead, it simply represents the productiveness of the economic system as a whole.

Given these two criticisms, Roemer (1982, 1994) proposed the property relational definition of exploitation (PR-exploitation), which recognizes exploitation as a concept of social relations, as stipulated by the ownership structure of productive assets. Though PR-exploitation has nothing to do with the classical labor theory of value, it is a mathematical extension of the Okishio definition. Moreover, it is generally true that, under the definition of PR-exploitation, the capitalist economy can be conceived of as exploitative. However, the PR theory of exploitation denies the relevance of exploitation as a primary normative concern: Roemer (1994) argued that the primary normative concern should be the injustice of the unequal distribution of productive assets, rather than exploitation *per se*. His criticism of exploitation was so influential that the Marxian theory of exploitation was almost dismissed, in that, until recently, there had been no substantial studies in this field since that of Roemer (1994).

However, the Marxian notion of exploitation has now been revived, and there have been some significant recent developments in the theory of exploitation as the social relations of the unequal exchange of labor (UE-exploitation). This report examines, among others, the proper conceptual definitions of exploitation developed by Vrousalis (2013), in political philosophy, and by Wright (2000) in sociology. Both approaches address the systematic generation of an unequal exchange of labor due to the asymmetric power relations embedded in the trading structure. Interestingly, using the new approach to exploitation à la Vrousalis (2013)–Wright (2000), Roemer's claim that the theory of exploitation is reduced to a theory of distributive injustice can be invalidated. As a result, the notion of UE-exploitation has been restored as a primary normative concern.

Given this new trend, one of the relevant subjects for Marxian exploitation theory is to properly formulate UE-exploitation, which has developed significantly as a result of an axiomatic theory of exploitation initiated by Veneziani and Yoshihara. Among their works, this report examines their Profit-Exploitation Correspondence Principle (PECP) [Veneziani and Yoshihara (2013a)] which is proposed to characterize axiomatically the eligible definitions of exploitation. Then, an extension of the exploitation form à la "New Interpretation" is shown to be uniquely eligible among the main definitions provided by current literature.

In the following discussion, section 2 examines the development of exploitation theory in mathematical Marxian economics, from the contribution of Okishio (1963) until the 1990s. First, this section discusses the robustness and economic implications of the debates on the Fundamental Marxian Theorem, developed mainly in the 1970s and 1980s, followed by a discussion of Roemer's (1982) property relation theory of exploitation. Section 3 introduces the recent trend in exploitation theory initiated by Vrousalis (2013) and Wright (2000), and then examines its economic implications using a simple economic model. Section 4 provides an overview of the recent axiomatic studies of exploitation by focusing on Veneziani and Yoshihara (2013a). Finally, section 5 concludes the report and provides a perspective of the remaining subjects on exploitation theory in mathematical Marxian economics.

2 The main developments in mathematical Marxian economics from the 1970s until the 1990s

In this section, we provide an overview of the main arguments in mathematical Marxian economics developed up to the 1990s. We begin with the significant contribution by Nobuo Okishio, known for the Fundamental Marxian Theorem, and then discuss the successive developments and relevant debates on this theorem, mainly initiated by Michio Morishima and John Roemer during the 1970s and 1980s.

2.1 The formulation of labor exploitation by Okishio and the Fundamental Marxian Theorem

Let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}_+$ (*resp.* $\mathbb{R}_-$) the set of non-negative (*resp.* non-positive) real numbers. For all $x, y \in \mathbb{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, n)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; and $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, n)$. For any sets, X and Y, $X \subseteq Y$ if and only if for any $x \in X$, $x \in Y$; $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$; $X \subsetneq Y$ if and only if $X \subseteq Y$ and $X \neq Y$.

An economy comprises a set of agents, $\mathcal{N} = \{1, .., N\}$, with generic element $\nu \in \mathcal{N}$. Denote the cardinal number of this set by N. Similarly, the cardinal number for any subset, $\mathcal{S} \subseteq \mathcal{N}$, is denoted by S. There are n types of (purely private) commodities that are transferable in markets. The production technology, commonly accessible by any agent, is represented by a Leontief production technology, (A, L), where A is an $n \times n$ non-negative square matrix of material input coefficients, and L is a $1 \times n$ positive vector of labor input coefficients. Here, A is assumed to be productive and indecomposable. For the sake of simplicity, let us assume that for each production period, the maximal amount of labor supply by every agent is equal to unity and there is no difference in labor skills (human capital) among agents. Let $b \in \mathbb{R}^n_+$ be the basic consumption bundle, which is the minimum consumption necessary for every agent when supplying one unit of labor. Let $\omega \in \mathbb{R}^n_+ \setminus \{\mathbf{0}\}$ be the social endowments of commodities.

Assuming a private ownership economy, let ω^ν be the initial endowment of commodities owned by agent $\nu \in \mathcal{N}$. In the following discussion, let $\mathcal{W} \equiv \{\nu \in \mathcal{N} \mid \omega^\nu = \mathbf{0}\}$ be the set of propertyless agents. Typically, $\mathcal{W}$ would represent the set of workers who own no material means of production. In summary, one *capitalist economy* is described by a profile $\left\langle \mathcal{N}; (A, L); (\omega^\nu)_{\nu \in \mathcal{N}} \right\rangle$.

Let v represent a vector of each commodity's labor value. Note that, according to the classical economics and Marx, the labor value of commodity i, v_i, is defined as the sum of the amount of labor directly and/or indirectly input to produce one unit of this commodity. Therefore, this value is mathematically formulated by the solution of the system of equations, $v = vA + L$. Here, since the matrix A is productive and the vector L is positive, $v \in \mathbb{R}^n_{++}$ is the unique solution of the system of equations. Then, the labor value of any commodity vector $c \in \mathbb{R}^n_+$ is given by $vc \geqq 0$.

Let $w \in \mathbb{R}_+$ represent a wage rate. Assume that any agent, $\nu \in \mathcal{W}$, can purchase the consumption vector, b, with wage revenue, w, per working day. Moreover, let $p \in \mathbb{R}^n_+ \backslash \{\mathbf{0}\}$ represent a vector of market prices for n types of commodities. Then:

Definition 1: A *balanced-growth equilibrium for a capitalist economy* $\left\langle \mathcal{N}; (A, L); (\omega^\nu)_{\nu \in \mathcal{N}} \right\rangle$ is a profile $(p, w) \in \mathbb{R}^{n+1}_+ \backslash \{\mathbf{0}\}$ that satisfies the following:

$$p = (1 + \pi) [pA + wL] \quad \& \quad w = pb,$$

where the scalar $\pi \geqq 0$ represents the equal profit rate.

Definition 2 [Okishio (1963)]: In a capitalist economy $\left\langle \mathcal{N}; (A, L); (\omega^\nu)_{\nu \in \mathcal{N}} \right\rangle$, *labor exploitation exists* if and only if $vb < 1$.

That is, within one working day, normalized to unity, vb corresponds to the necessary labor hours for each $\nu \in \mathcal{W}$, so that $1 - vb$ represents the surplus labor hours. Therefore, the existence of labor exploitation is none other than the existence of positive surplus labor.

Under Definition 2, Okishio proves the validity of the basic Marxian view, which conceives the capitalist economy as exploitative, by the Fundamental Marxian Theorem, as follows:

Fundamental Marxian Theorem (FMT) [Okishio (1963)]: Let (p, w) be a balanced-growth equilibrium associated with equal profit rate π for capitalist economy $\left\langle \mathcal{N}; (A, L); (\omega^\nu)_{\nu \in \mathcal{N}} \right\rangle$. Then:

$$\pi > 0 \Leftrightarrow vb < 1.$$

Morishima's (1973) introduction of this theorem prompted hot debate on its robustness and implications. There have been many studies on the robustness of the FMT. Of these, we review the work of Morishima (1974) and Roemer (1980). Both works discuss the generalization of the FMT to a more general model than that of the Leontief type, in order to show the robustness of the

FMT in economies with fixed capital, joint production, and the possibility of technical choices.[1]

To formulate such economies, the von Neumann production technology, represented by a profile (A, B, L), is introduced. Note that A is an $n \times m$ matrix, the generic component of which, $a_{ij} \geqq 0$, represents the amount of commodity i used as an input to operate one unit of the j-th production process; B is an $n \times m$ matrix, the generic component of which, $b_{ij} \geqq 0$, represents the amount of commodity i produced as an output by operating one unit of the j-th production process; and L is a $1 \times m$ positive row vector of direct labor input coefficients. Let $x_j \geqq 0$ represent an activity level of the j-th production process, so that a profile of social production activities is represented by a non-negative $m \times 1$ column vector, $x \equiv (x_j)_{j=1,\ldots,m}$. In the following discussion, we will sometimes use the notation A_i (resp. B_i) to refer to the i-th row vector of A (resp. B).

For a von Neumann capitalist economy, $\langle \mathcal{N}; (A, B, L); (\omega^\nu)_{\nu \in \mathcal{N}} \rangle$, we can respectively define the notions of balanced-growth equilibrium, labor values, and labor exploitation as follows:

Definition 3: A *balanced-growth equilibrium for a capitalist economy,* $\langle \mathcal{N}; (A, B, L); (\omega^\nu)_{\nu \in \mathcal{N}} \rangle$ is a profile of non-negative and non-zero vectors, $((p, w), x) \in \mathbb{R}_+^{n+1} \times \mathbb{R}_+^m$, that satisfy the following:

$$pB \leqq (1+\pi)[pA + wL]; \;\; Bx \geqq (1+\pi)[A + bL]x; \;\; pBx > 0; \;\; \& \;\; w = pb.$$

Definition 4 [Morishima (1974)]: Given a capitalist economy, $\langle \mathcal{N}; (A, B, L); (\omega^\nu)_{\nu \in \mathcal{N}} \rangle$, the *labor value of a consumption bundle,* $c \in \mathbb{R}_+^n$, is the solution, Lx^c, of the following constrained optimization program:

$$\min_{x \geqq \mathbf{0}} Lx \quad s.t. \;\; [B - A]x \geqq c.$$

Definition 5 [Morishima (1974)]: In a capitalist economy, $\langle \mathcal{N}; (A, B, L); (\omega^\nu)_{\nu \in \mathcal{N}} \rangle$, *labor exploitation exists* if and only if $Lx^b < 1$.

Morishima (1974) shows that, under the balanced-growth equilibrium, the equivalence between the existence of labor exploitation and the positive equal profit rate is preserved, even in the von Neumann capitalist economy. This is formalized in the following theorem:

Generalized Fundamental Marxian Theorem (GFMT) [Morishima (1974)]: Let $((p, w), x)$ be a balanced-growth equilibrium associated with the equal profit rate, π, for capitalist economy $\langle \mathcal{N}; (A, B, L); (\omega^\nu)_{\nu \in \mathcal{N}} \rangle$. Then:

$$\pi > 0 \Leftrightarrow Lx^b < 1.$$

[1] There is also another generalization of the FMT to a Leontief economy with heterogeneous labor, as proposed by Morishima (1973), Bowles and Gintis (1977, 1978), and Krauze (1981). The focus of this line of research was to solve the reduction problem of heterogeneous labor into one common unit, and/or to solve the dilemma of the heterogeneity of labor and the respective rates of exploitation. So far, the robustness of the FMT in this line of generalization has remained firm.

In contrast, Roemer (1980) defines an alternative equilibrium notion, called a *reproducible solution*, which is defined to preserve its coherency with the profit-maximizing behavior of every capital owner, $\nu \in \mathcal{N} \backslash \mathcal{W}$. It then examines the robustness of the FMT under this equilibrium. That is:

Definition 6 [Roemer (1980)]: A *reproducible solution for a capitalist economy*, $\left\langle \mathcal{N}; (A, B, L); (\omega^\nu)_{\nu \in \mathcal{N}} \right\rangle$, is a profile of non-negative and non-zero vectors, $((p^*, w^*), x^*) \in \mathbb{R}_+^{n+1} \times \mathbb{R}_+^m$, that satisfies the following:[2]
(a) $x^{\nu *} \in \arg\max_{x^\nu \geqq \mathbf{0}} p^* [B - A] x^\nu - w^* L x^\nu$, such that $[p^* A + w^* L] x^\nu \leqq p^* \omega^\nu$ $(\forall \nu \in \mathcal{N} \backslash \mathcal{W})$, where $x^* \equiv \sum_{\nu \in \mathcal{N} \backslash \mathcal{W}} x^{\nu *}$;
(b) $[B - A] x^* \geqq b L x^*$;
(c) $w^* = p^* b$;
(d) $[A + bL] x^* \leqq \omega$.

Roemer's Fundamental Marxian Theorem (RFMT) [Roemer (1980)]: For any capitalist economy, $\left\langle \mathcal{N}; (A, B, L); (\omega^\nu)_{\nu \in \mathcal{N}} \right\rangle$, and any reproducible solution, $((p^*, w^*), x^*)$, the following two statements are equivalent:
(1) $p^* [B - A] x^* - w^* L x^* > 0 \Leftrightarrow L x^b < 1$;
(2) $\forall x, x' \geqq \mathbf{0}$, $Lx = Lx'$, $[\exists i \in \{1, \ldots, n\} : (B_i - A_i) x > (B_i - A_i) x'] \Rightarrow \exists x'' \geqq \mathbf{0} : Lx'' = Lx', (B_i - A_i) x'' = (B_i - A_i) x', \& \ \exists i' \in \{1, \ldots, n\} : (B_{i'} - A_{i'}) x'' > (B_{i'} - A_{i'}) x$.

In the above theorem, statement (1) implies the equivalence between the positivity of total profit, $p^* [B - A] x^* - w^* L x^*$, at the reproducible solution and the existence of labor exploitation in terms of Definition 5. In contrast, statement (2) characterizes the necessary and sufficient condition for statement (1) to hold. Suppose two production activities, say x and x', that have the same corresponding labor inputs. Then, according to statement (2), if the net output of some commodity, say i, via activity x is strictly greater than that via activity x', then there is another commodity, say i', such that the net output of i' via some suitable production activity x'', which may be identical to or different from x', is strictly greater than that via x. This statement can be interpreted as a condition that excludes the possibility of production via an *inferior process*. Thus, the RFMT implies that, in any capitalist economy, the equivalence relationship between positive profits and the existence of labor exploitation holds for any reproducible solution if and only if there is no inferior production process (in terms of the condition (2)) in this economy.

To see the difference between the GFMT and RFMT, consider the following example.

Example 1: Consider a von Neumann economy, $\left\langle \mathcal{N}; (A, B, L); (\omega^\nu)_{\nu \in \mathcal{N}} \right\rangle$, such

[2] As Roemer (1980) explicitly shows, there is essential no difference between the balanced-growth equilibrium (given as Definition 1) and the reproducible solution in capitalist economies with a Leontief production technology. However, these two notions of equilibrium are different whenever a more general model of capitalist economies is considered.

that

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}, \ L = (1,1), \ b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \text{ and } \omega = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

In this economy, condition (2) of the RFMT is violated, because $B - A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $L = (1,1)$, which implies that the 1st production process is inferior to the 2nd production process.

Note that in this economy, the set of the balanced-growth equilibria is characterized by

$$\left\{ \left(\left(p, w \right), x \right) \in \left\{ \left(\left(0, 1 \right), 1 \right) \right\} \times \mathbb{R}^2_+ \mid x \neq \mathbf{0} \right\},$$

where all balanced-growth equilibria are associated with $\pi = 0$. In contrast, the set of reproducible solutions is characterized by

$$\left\{ \left(\left(p^*, w^* \right), x^* \right) \in \mathbb{R}^2_+ \times \{1\} \times \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \mid p^*_1 + p^*_2 = 1 \right\},$$

where, if $p^*_1 > 0$, then $\pi^* = \frac{p^*_1}{2p^*_1 + p^*_2} > 0$; while, if $p^*_1 = 0$, then $\pi^* = 0$.

Next, in this economy, the labor value of the commodity bundle b is $Lx^b = 1$, where x^b is any non-negative vector satisfying $x^b_1 + x^b_2 = 1$. Thus, according to Definition 5, there is no exploitation in this economy.

Therefore, the GFMT holds in this economy, since in any balanced growth-equilibrium, the corresponding profit rate is $\pi = 0$, while there is no exploitation. However, we can find a reproducible solution $\left(\left(p^*, w^* \right), x^* \right)$, with $p^*_1 > 0$, whose corresponding profit rate is $\pi^* > 0$. Thus, if the economy arrives at this equilibrium, then positive profits are generated in conjunction with no exploitation, which violates condition (1) of the RFMT. This contrast between the GFMT and RFMT can be observed when the economy does not satisfy condition (2) of the RFMT. ■

Next, we comment briefly on the Okishio–Morishima proposal for the formulation of labor exploitation given in Definitions 2, 4, and 5. Firstly, these definitions presume the employment relation of capital and labor in the production process. Secondly, these are faithful to the Marxian theory of surplus value in that labor exploitation is defined as the existence of positive surplus value. In other words, the supply of labor time exceeds the necessary labor time (the value of labor power). Thirdly, these definitions are consistent with the basic perception of the labor theory of value, since they are formulated completely independently of price information. As a result of these properties, the Okishio–Morishima formulation of labor exploitation is conceivably faithful to the conceptual definition of exploitation given in Marx's *Das Kapital*.

The works developed by Morishima and Roemer indicate that we cannot generally confirm the basic Marxian perception of the capitalist economy as exploitative. Firstly, according to Morishima's GFMT, it is true even under a general economic environment with the possibility of fixed capital, joint production, and technical choices that the necessary and sufficient condition for positive profits is the existence of labor exploitation, as long as only a balanced-growth equilibrium is assumed. However, secondly, the RFMT suggests that, given the Okishio–Morishima definition of exploitation, once we extend our concern from the balanced-growth equilibrium to the reproducible solution, the equivalence between positive profits and the existence of exploitation no longer holds in a general economic environment with the possibility of fixed capital, joint production, and technical choices. Note that this extension of the notion of equilibrium seems reasonable whenever we view a capitalist economy as a resource allocation mechanism working via the capitalists' profit-seeking motivation under market competition.

There is another, even more serious criticism of the FMT, which raises doubt about the FMT characterizing a capitalist economy as an exploitative system. This criticism is based on the **Generalized Commodity Exploitation Theorem (GCET)**, as shown by Bowles and Gintis (1981), as well as Samuleson (1982) and Roemer (1982). To see the GCET, we return to a Leontief capitalist economy, $\left\langle \mathcal{N}; (A, L); (\omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$. Then, take any commodity, k, and let $v_i^{(k)}$, for each commodity i, be the aggregate amount of commodity k directly and/or indirectly input to produce one unit of the commodity i. Let $v^{(k)} \equiv \left(v_i^{(k)} \right)_{i \in \{1,\ldots,n\}}$ be a vector of *commodity k-values*. Analogical to the case of the vector of labor values, $v^{(k)}$ can be defined as the solution of the following system of equations:

$$v^{(k)} = v^{(k)} \left[A + bL \right] + \left(1 - v_k^{(k)} \right) \left[A_k + b_k L \right],$$

where A_k is the k-th row vector of the matrix A. Then:

Definition 7 [Bowles & Gintis (1981)]: In a capitalist economy, $\left\langle \mathcal{N}; (A, L); (\omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$, the *exploitation of commodity k exists* if and only if $v_k^{(k)} < 1$.

Generalized Commodity Exploitation Theorem (GCET) [Bowles & Gintis (1981)]: Let (p, w) be a balanced growth-equilibrium associated with the equal profit rate, π, for capitalist economy $\left\langle \mathcal{N}; (A, L); (\omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$. Then:

$$\pi > 0 \Leftrightarrow vb < 1 \Leftrightarrow v_k^{(k)} < 1.$$

Establishing the GCET leads us to see the Okishio–Morishima definition of labor exploitation as representing the productiveness of an overall economic system, which uses labor power as a factor of production in a technologically efficient way to guarantee the possibility of surplus products. This is because the existence of commodity k's exploitation is the exact numerical representation of the productiveness of an overall economic system if we select commodity k as the numéraire, in the sense that the overall economic system is productive enough to guarantee the possibility of surplus products via the technologically efficient use of commodity k as a factor of production. Analogically, we can interpret the existence of labor exploitation as the numerical representation of the productiveness of an overall economic system by selecting labor as the numéraire. Therefore, the equivalence between the FMT and GCET indicates that the necessary and sufficient condition for positive profits is that the whole economic system is sufficiently productive to guarantee the possibility of surplus products, which is a trivial proposition. This view prompted the criticism of Okishio's original motivation and interpretation of the FMT: that it may simply affirm the productiveness of the capitalist economy, rather than the Marxian perception of the capitalist economy as an exploitative system.[3]

2.2 The property relations definition of exploitation by Roemer (1982)

Recall that the Okishio–Morishima definition of labor exploitation is a formulation of the unequal exchange of labor (UEL), presuming that the UEL represents an essential feature of the notion of exploitation. In contrast, John Roemer (1994) argues that exploitation as the UEL should be replaced with exploitation as the distributional consequences of an unjust inequality in the distribution of productive assets and resources. What constitutes unjust inequality? Roemer (1994) argues that this is the unequal distribution of alienable assets, which is unjust in capitalist societies.[4]

Based on this view, Roemer (1994) proposes the *property relational definition of exploitation* (*PR-exploitation*). That is, *a group or individual* (*capitalistically*) *exploits another group or individual* if and only if the following three conditions hold: (i) were the latter to withdraw from the society, endowed with his/her per capita share of social alienable goods and with his/her own labor and skill, then he/she would be better off in his/her welfare than at the present allocation; (ii) were the former to withdraw under the same conditions, then he/she would be worse off in his/her welfare than at the present allocation; and (iii) were the latter to withdraw from the society, endowed with his/her own endowments, then the former would be worse off than at present.

Such a definition can be formulated within the framework of cooperative game theory. Let $(V^1, \ldots, V^N) \in \mathbb{R}^N_+$ be a profile of each agent's welfare level in the present society. Let $P(\mathcal{N})$ be the power set of $\mathcal{N}$ and let $K : P(\mathcal{N}) \to \mathbb{R}_+$ be a *characteristic function* of the society, which assigns to every coalition $\mathcal{S} \subseteq \mathcal{N}$, with S agents, an aggregate payoff, $K(\mathcal{S})$, if it withdraws from the economy. Then:

Definition 8 [Roemer (1982)]: At a welfare allocation $(V^1, \ldots, V^N)$ of the present society, *coalition $\mathcal{S} \subseteq \mathcal{N}$ is exploited (resp. exploiting) with respect*

[3] For a more detailed discussion on the implications of the GCET, see Roemer (1982) and Yoshihara and Veneziani (2010a, b). In addition, some recent literature criticizes the GCET, supporting the Okishio–Morishima definition of labor exploitation. For more information, see Fujimoto and Fujita (2008) and Matsuo (2009).

[4] Alienable assets are typically financial assets and/or material capital goods. In contrast, inalienable assets are typically talents and/or skills immanent in individuals.

to alternative K if and only if the complement $\mathcal{T} \equiv \mathcal{N}\backslash\mathcal{S}$ is in a relation of dominance to $\mathcal{S}$,[5] and the following two conditions hold:
(i) $\sum_{\nu\in\mathcal{S}} V^{\nu} < K(\mathcal{S})$ (resp. $\sum_{\nu\in\mathcal{S}} V^{\nu} > K(\mathcal{S})$);
(ii) $\sum_{\nu\in\mathcal{T}} V^{\nu} > K(\mathcal{T})$ (resp. $\sum_{\nu\in\mathcal{T}} V^{\nu} < K(\mathcal{T})$).

That is, condition (i) of Definition 8 states that an exploited coalition would be improved with respect to its aggregate payoff by withdrawing from the present society to the alternative society characterized by the allocation rule K. Condition (ii) implies that the complement of the exploited coalition would become worse off by withdrawing from the present society to the alternative society. It may be supposed that there exists a sub-coalition of the complement that exploits the exploited coalition in the present society.

What kinds of features would characteristic function K have to include as a welfare allocation rule of the alternative society? This depends on what the alternative society would be. For instance, if the present society is a capitalist society, function K would be defined in terms of the welfare allocation implementable from the equal distribution of alienable assets. That is, firstly, let $u : \mathbb{R}^n_+ \times [0,1] \to \mathbb{R}_+$ be the welfare function of each agent that associates a non-negative real number, $u(c,l)$, with each $c \in \mathbb{R}^n_+$ of consumption vectors and each $l \in [0,1]$ of labor supply. Secondly, define feasible allocations for an economic environment with a Leontief production technology:

Definition 9: Given a Leontief production economy, $\langle\mathcal{N};(A,L);\omega\rangle$, *profile* $\left((c^{\nu},l^{\nu})_{\nu\in\mathcal{N}},x\right) \in \left(\mathbb{R}^n_+ \times [0,1]\right)^N \times \mathbb{R}^n_+$ *constitutes a feasible allocation* if and only if the following conditions hold for this profile:
(i) $Ax \leqq \omega$;
(ii) $Lx = \sum_{\nu\in\mathcal{N}} l^{\nu}$;
(iii) $(I-A)x \geqq \sum_{\nu\in\mathcal{N}} c^{\nu}$.

Denote the set of feasible allocations for economy $\langle\mathcal{N};(A,L);\omega\rangle$ by $Z(\omega)$. If a feasible allocation $\left((c^{*\nu},l^{*\nu})_{\nu\in\mathcal{N}},x^*\right) \in Z(\omega)$ is implemented as a reproducible solution for the capitalist economy $\left\langle\mathcal{N};(A,L);(\omega^{\nu})_{\nu\in\mathcal{N}}\right\rangle$, then its corresponding welfare allocation is denoted by $(V^{*1},\ldots,V^{*N})$, where $V^{*\nu} \equiv u(c^{*\nu},l^{*\nu})$, for each $\nu\in\mathcal{N}$.

Now, we denote a welfare allocation rule of an alternative society to the capitalist society by $K^{CE} : P(\mathcal{N}) \to \mathbb{R}_+$. For each coalition, $\mathcal{S} \subseteq \mathcal{N}$, consider the following optimization program (CE):

$$\max_{\left((c^{\nu},l^{\nu})_{\nu\in\mathcal{S}},x\right)} \sum_{\nu\in\mathcal{S}} u(c^{\nu},l^{\nu})$$
$$\text{s.t. } (I-A)x \geqq \sum_{\nu\in\mathcal{S}} c^{\nu};\ Lx = \sum_{\nu\in\mathcal{S}} l^{\nu} \leqq S;\ \&\ Ax \leqq \frac{S}{N}\omega. \qquad (CE)$$

[5] Note that there is no explicit formal definition of the dominance relation of coalition $\mathcal{S}$ and coalition $\mathcal{T}$. Roemer (1982) simply states that the notion of 'the dominance relation' here is given mainly based on a sociological concept.

Denote the solution of program (CE) by $\left(\left(c^{**\nu}, l^{**\nu}\right)_{\nu\in\mathcal{S}}, x^{\mathcal{S}}\right)$. Then, the characteristic function, K^{CE}, is defined by $K^{CE}(\mathcal{S}) \equiv \sum_{\nu\in\mathcal{S}} u\left(c^{**\nu}, l^{**\nu}\right)$, for each $\mathcal{S} \subseteq \mathcal{N}$.

The program (CE) presumes a counterfactual situation in which a group, $\mathcal{S}$, withdraws from the capitalist society to form a *commune* comprising the members of this group, and then investigates the expected sum of the welfare levels achievable in that alternative society. That is, the program maximizes the aggregate of the welfare levels attainable by the group $\mathcal{S}$ endowed with its accessible aggregate capital stock, $\frac{S}{N}\omega$. Here, $\frac{S}{N}\omega$ is the sum of the capital stocks of all members in $\mathcal{S}$ derived from the counterfactual equal distribution of the overall material means of production, ω. It is the solution of this program that constitutes the value $K^{CE}(\mathcal{S})$ as the total payoff attainable by the group $\mathcal{S}$ if it forms a communal society by withdrawing from the present society. Following Roemer (1982), the **property-relational exploitation of a capitalist society** (**capitalist PR-exploitation**) is defined by means of this K^{CE}, as follows:

Definition 10 [Roemer (1982)]: At a welfare allocation $(V^{*1}, \ldots, V^{*N})$ of a capitalist economy, $\left\langle \mathcal{N}; (A, L); (\omega^{\nu})_{\nu\in\mathcal{N}} \right\rangle$, *coalition* $\mathcal{S} \subseteq \mathcal{N}$ *is capitalistically exploited (resp. capitalistically exploiting)* if and only if the complement $\mathcal{T} \equiv \mathcal{N} \backslash \mathcal{S}$ is in a relation of dominance to $\mathcal{S}$, and the following two conditions hold:
(i) $\sum_{\nu\in\mathcal{S}} V^{*\nu} < K^{CE}(\mathcal{S})$ (resp. $\sum_{\nu\in\mathcal{S}} V^{*\nu} > K^{CE}(\mathcal{S})$);
(ii) $\sum_{\nu\in\mathcal{T}} V^{*\nu} > K^{CE}(\mathcal{T})$ (resp. $\sum_{\nu\in\mathcal{T}} V^{*\nu} < K^{CE}(\mathcal{T})$).

That is, condition (i) of Definition 10 states that a capitalistically exploited coalition is worse off in terms of its attainable payoff in the capitalist society than in the communal society endowed with an equal distribution of material means of production. Moreover, condition (ii) of Definition 10 states that the complement of the capitalistically exploited coalition would be better off in terms of its attainable payoff in the capitalist society than in the communal society of this complement. It would be expected that a capitalistically exploiting coalition would exist within this complement. In addition to the definition given in Roemer (1982), Roemer (1994) introduces a third condition, as noted above. This condition suggests that the aggregate welfare of group $\mathcal{T}$ would be worse off if group $\mathcal{S}$ withdraws, taking $\omega^{\mathcal{S}} \equiv \sum_{\nu\in\mathcal{S}} \omega^{\nu}$ with it from the capitalist society. This condition would naturally follow whenever the welfare allocation $(V^{*1}, \ldots, V^{*N})$ is derived from the reproducible solution in our setting of the Leontief capitalist economy.

A non-exploitative society in terms of Definition 10 can be formulated as a society without an unequal distribution of material capital goods, as confirmed by the following definition.

Definition 11 [Roemer (1982)]: For any Leontief production economy, $\langle \mathcal{N}; (A, L); \omega \rangle$, a *welfare allocation* $(V^{*1}, \ldots, V^{*N})$ *lies in a communal core* if and only if any coalition $\mathcal{S} \subseteq \mathcal{N}$ is not capitalistically exploited by the allocation.

Definition 11 implies that the core property of a communal society is equivalent to the non-existence of capitalist exploitation in terms of Definition 10.

What types of feasible allocations can a communal core contain? To answer this question, consider the program (CE) in the case of $\mathcal{S} = \mathcal{N}$, and denote the corresponding optimal solution by $\left(\left(c^{**\nu}, l^{**\nu}\right)_{\nu\in\mathcal{N}}, x^{\mathcal{N}}\right)$. Note that, each agent has a common welfare function and common level of labor skill. Therefore, for the sake of simplicity, we can restrict our attention to the symmetric allocation in which all agents consume the same consumption bundle, $c^{**\mathcal{N}}$, and supply the same labor hours, $l^{**\mathcal{N}}$. Hence, the corresponding welfare allocation, $(V^{\mathcal{N}*\nu})_{\nu\in\mathcal{N}}$, has the property that, for any $\nu, \nu' \in \mathcal{N}$, $V^{\mathcal{N}*\nu} = V^{\mathcal{N}*\nu'}$ holds, where $V^{\mathcal{N}*\nu} \equiv u\left(c^{**\mathcal{N}}, l^{**\mathcal{N}}\right)$. In this case, for any coalition, $\mathcal{S} \subsetneq \mathcal{N}$, $\sum_{\nu\in\mathcal{S}} V^{\mathcal{N}*\nu} = Su\left(c^{**\mathcal{N}}, l^{**\mathcal{N}}\right) = Su\left(c^{**\mathcal{S}}, l^{**\mathcal{S}}\right) = K^{CE}(\mathcal{S})$ holds, which implies that the welfare allocation $(V^{\mathcal{N}*\nu})_{\nu\in\mathcal{N}}$ lies in the communal core. That is, the welfare allocation lies in the communal core if, (i) it is generated from the situation in which all individuals in $\mathcal{N}$ constitute a communal society, (ii) all individuals engage in a cooperative production activity using the overall set of material capital goods, ω, (iii) all individuals share the reward of the activity equally. Such an allocation is a non-exploitative allocation in terms of Definition 10.[6]

Unlike the traditional Marxian theory of exploitation, the capitalist PR-exploitation formulated in Definition 10 never refers to the UEL. Rather, it straightforwardly refers to the unequal distribution of material means of production as the basic feature of exploitation in the capitalist economy. However, Definition 10 is an extension of the Okishio–Morishima definition of labor exploitation, as pointed out by Roemer (1982). Indeed, given the reproducible solution $((p^*, w^*), x^*)$ in the capitalist economy $\left\langle \mathcal{N}; (A, L); (\omega^\nu)_{\nu\in\mathcal{N}} \right\rangle$ discussed in the last section, if any worker, $\nu \in \mathcal{W}$, is identified as an exploited agent by the Okishio–Morishima definition of exploitation, then he/she would be a member of an exploited coalition in terms of Definition 10. This is because, while each worker, $\nu \in \mathcal{W}$, receives, at most, the welfare level $u(b, 1)$ in the present reproducible solution, $((p^*, w^*), x^*)$, they can all enjoy a higher level of welfare than $u(b, 1)$ by withdrawing, with the material capital goods $\frac{W}{N}\omega$, from the capitalist economy to form their own commune: each worker in the commune of $\mathcal{W}$ can access a welfare level available from the revenue, $\pi^* \frac{p^*\omega}{N} + w^*$, while his/her welfare level, $u(b, 1)$, under the capitalist economy is simply derived from his/her wage revenue, $w^* = p^*b$. Thus, every exploited agent in terms of the Okishio–Morishima definition is a member of an exploited group in terms of Definition 10. Furthermore, Definition 10 allows us to identify all exploited agents beyond the members of $\mathcal{W}$, as well as all members of the exploiters. Henceforth, the PR theory of exploitation provides a finer definition of exploitation than do the theories of the UEL, such as the Okishio–Morishima approach. In summary, whenever we are interested in exploitation as a feature of social relations, Roemer (1994) concludes that we should discuss it based on the PR definition rather than the UEL definition of exploitation.

[6] For a more detailed discussion, see Veneziani and Yoshihara (2013b; section 4.4).

Given this alternative definition of exploitation, Roemer (1994) questions whether the issue of exploitation is an intrinsic normative problem worth discussing in the context of contemporary societies. That is, he argues that exploitation is *per se*, at best a morally secondary phenomenon. Instead, he believes that the normatively primary concern that we should be addressing is the injustice of property relations. For instance, according to Definition 10, capitalist PR-exploitation exists whenever alienable capital goods are unequally distributed. However, though inequality in the distribution of alienable resources could be conceived of as unjust when all agents are homogenous in their welfare functions and skills, the issue is less straightforward when these functions and skills are heterogeneous and diverse. Given that the heterogeneity and diversity of agents are generic features of contemporary societies, it seems necessary for us to develop a more comprehensive theory of distributive justice, which should be the subject of normatively primary concern in contemporary societies, rather than the development of exploitation theory.

So, what kinds of theories of distributive justice should be addressed? As a solution, Roemer (1994, 1998) has developed a *theory of equality of opportunity*, based on the debates on equality by Dworkin (1981), Arneson (1989), and Cohen (1989). His theory can be summarized by the following axiom:

Principle of voluntary disadvantage: The distribution of alienable resources between any agents, $\nu \in \mathcal{N}$ and $\nu' \in \mathcal{N}$, is *just* if and only if any difference in ν's and ν''s enjoyment of the resources reflects a difference in their choices, desserts, or faults.

Any inequality violating this principle implies *involuntary disadvantage*, which should be deemed distributive injustice.

Note that involuntary disadvantage implies disadvantages due to circumstantial factors for which individuals should not be deemed responsible, such as those due to household environments, native talents, disaster, and so on. It is reasonable to regard an agent's disadvantage in private ownership of material capital goods as involuntary, at least in his/her initial stage of economic activities. For instance, in the above-mentioned capitalist economy, $\left\langle \mathcal{N}; (A, L); (\omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$, there is supposedly no difference in agents' native talents, and the possibility of disaster is not considered. Therefore, it is the inequality in private ownership of material capital goods that is the sole source of involuntary disadvantages in this economy. In this respect, an equilibrium allocation in the economy $\left\langle \mathcal{N}; (A, L); (\omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$ implies involuntary disadvantages if and only if it entails capitalist PR-exploitation in terms of Definition 10.

In summary, given the above arguments, the existence of exploitation, à la Roemer's theory of PR-exploitation, is equivalent to distributive injustice, à la Roemer's theory of equality of opportunity, at least in any Leontief capitalist economy with no heterogeneity or diversity of agents. Hence, in such homogeneous societies, it is sufficient to argue distributive injustice in terms of the theory of equality of opportunity. Moreover, the theory of equality of opportunity can diagnose allocations of alienable resources as unjust, even in societies with heterogeneity and/or diversity among agents. Therefore, the issue of exploitation can be replaced with, or be reduced to the issue of distributive injustice due to the theory of equality of opportunity. Remember that the Okishio–Morishima theory of exploitation is contained within Roemer's PR theory of exploitation. Given this point, the Marxian theory of exploitation, as represented by the Okishio–Morishima formulation, is no longer *per se* the subject of normatively primary concern. Rather, it is sufficient that we diagnose societies using the theory of equal opportunity, which is the main message derived from Roemer's PR theory of exploitation in conjunction with the theory of equality of opportunity.

3 Recent trends of exploitation theory in political philosophy and sociology

This section introduces new trends in exploitation theory, mainly developed in the fields of political philosophy and sociology. Here, the work of Vrousalis (2013) provides a remarkable new development of exploitation theory in political philosophy, while in sociology, the work of Wright (2000) has made a significant contribution to the recent revival of exploitation theory.

3.1 A conceptual definition of Vrousalis (2013) in political philosophy

Nicholas Vrousalis (2013) gives the following argument for the general conceptual definition of exploitation:

Definition 12 [Vrousalis (2013)]: An agent, ν, *exploits* an agent, μ, if and only if ν and μ are embedded in a systematic relationship in which (a) ν *instrumentalizes* μ*'s vulnerability to* ν in order to (b) *extract a net benefit from* μ.

To make this definition understandable, we examine each concept in Definition 12 individually.

First, *instrumentalization* of a subject implies that the subject is being used as a means to an end. Note that, according to Vrousalis (2013), neither unfairness nor intentionality of instrumentalization is necessary for the definition of exploitation. Others, such as Roberto Goodin, define exploitation as unfairly taking advantage of another's attributes. For example, one might take advantage of a person's honesty or blindness to steal from him/her. This would constitute exploiting that person. However, as we will see, Vrousalis (2013) provides examples of the "non-unfair" utilization of others' attributes, which is still deemed to be exploitative. Vrousalis (2013) also discusses that one can unintentionally or unknowingly instrumentalize another's vulnerability, and thereby exploit that person.

Second, Vrousalis (2013) describes two types of vulnerability: absolute and relational. An agent suffers *absolute vulnerability* when he/she is at substantial risk of a significant loss in the relevant metric (welfare, resources, capabilities, etc.). The absence of absolute vulnerability is guaranteed by security, which implies such losses will not occur. However, absolute vulnerability does not refer to an agent's power over another person. In contrast, the notion of *relational vulnerability* is defined as follows: μ *is relationally vulnerable to* ν if ν has some sort of power over μ in that, (i) μ lacks something that he/she wants/needs, F, that is a requirement for μ to flourish; (ii) μ can only obtain F from ν; and (iii) ν has it within his/her discretion to withhold F from μ.[7]

Given the above discussion, Vrousalis (2013) derives the notion of *economic vulnerability*, defined as follows: μ *is economically vulnerable to* ν if and only if μ is relationally vulnerable to ν by virtue of μ's position relative to ν in the relations of production. Here, the relations of production refers to systematic relations of effective ownership, and therefore, of power over human labor power and means of production in society. For instance, suppose that ν owns a water-producing well and ν's ownership is fully enforced. If μ needs water, but has no independent access to water, then μ is economically vulnerable to ν. The implication of this notion in a capitalist economy is that μ is relationally vulnerable to ν by virtue of ν's ownership of a means of production and μ's lack thereof (or, μ's ownership is substantially less than ν's). This gives ν *economic power over* μ, regardless of whether μ is forced by economic circumstances to supply labor power to ν.

The definition of *economic power over* is as follows: ν has *economic power over* μ if and only if ν has the relevant ability and opportunity to get μ to do something by virtue of control over a greater share of resources than μ. Therefore, if $\omega^{\nu} > \omega^{\mu}$, then ν has economic power over μ.

Summarizing the above arguments, we can derive the following logical implication:

Proposition 1 [Vrousalis (2013)]: If ν instrumentalizes μ's economic vulnerability to ν, then in doing so, ν takes advantage of his/her economic power over μ.

Here, under capitalism, if μ does not have any means of production but ν does, or μ owns substantially less than ν, then μ is economically vulnerable to ν. This is because ν is given economic power over μ and can get μ to supply his/her labor power to ν. Indeed, assuming an equal distribution of internal resources,[8] the wealth owned by capitalists (or agent ν) systematically gives them a decisive bargaining advantage over workers (or agent μ). This means capitalists always

[7] Note, that Vrousalis (2013) does not consider condition (iii) of relational vulnerability to be a necessary condition for exploitation, for there is nothing contradictory in the thought that ν is forced to exploit μ, and therefore lacks the said discretion.

[8] Internal resources imply talents and/or skills inherited in individuals. In contrast, any other types of resources that are transferrable are often called external resources. For a more detail argument on these concepts, see Cohen (1995).

have an advantage of economic power over workers, but never the other way around.

Given the above discussion, Vrousalis (2013) derives the notion of *economic exploitation* by applying the general definition of exploitation in Definition 12 to economic problems, as follows:

Definition 13 [Vrousalis (2013)]: Agent ν *economically exploits* agent μ if and only if ν and μ are embedded in a systematic relationship in which, (a) ν *instrumentalizes* μ's *economic vulnerability to* ν in order to (b) *appropriate (the fruits of) μ's labor*.

Here, condition (b) of Definition 13 needs clarification: ν appropriates μ's labor when μ toils for H hours, and ν appropriates a use-value of $H - G$ hours of toil, where G can be any number satisfying $H > G \geqq 0$.

It is worth noting that, in Definition 13, an unequal exchange of labor (UEL) is simply a necessary condition for economic exploitation. Unequal exchange occurs when there is an unreciprocated net transfer of goods or labor time from one party to another. According to Definition 13, condition (b) implies an unequal exchange of (the fruits of) labor. Hence, the UEL is necessary for economic exploitation. However, the UEL *per se* is not sufficient for economic exploitation, as economic exploitation requires both conditions (a) and (b) of Definition 13. For instance, gift-giving implies an unequal exchange, but no one thinks of (even systematic) gift-giving as exploitative. If one party freely decides to pass on a large part of whatever use-value he/she creates (with his/her own labor power) to another party of society, the resulting inequality in the consumption of (surplus) labor need not be objectionable.

3.2 A conceptual definition of exploitation by Wright (2000) in sociology

Eric Ohlin Wright (2000) defines exploitation as follows:

Definition 14 [Wright (2000)]: Exploitation exists if the following three criteria are satisfied:
(1) *The inverse interdependent welfare principle*: The material welfare of exploiters causally depends upon the reduction of material welfare of the exploited;
(2) *The exclusion principle:* This inverse interdependence of the welfare of exploiters and the exploited depends upon the exclusion of the exploited from access to certain productive resources;
(3) *The appropriation principle:* The exclusion generates a material advantage to exploiters because it enables them to appropriate the labor effort of the exploited.

In a market economy, both parties to an exchange gain relative to their condition before making the exchange: both workers and capitalists gain when an exchange of labor power for a wage occurs. Such mutual gains from trade can occur, but it can still be the case that the magnitude of the gain by one party is at the expense of another party. Thus, criterion (1) should be satisfied and, according to Wright (2000), we should not assume that market exchanges do not satisfy (1) because of mutual gains from trade.

So far, Wright (2000) argues that exploitation is the process through which certain inequalities in income are generated by inequalities in rights and powers over productive resources. Such inequalities in income occur through the ways in which exploiters, by virtue of their exclusionary rights and powers over productive resources, are able to appropriate the labor effort of the exploited.

Before closing this subsection, it is worth noting the following with regard to Definition 14. I do not believe that Definition 14 is sufficient as a definition of exploitation, nor is it as elaborate a conceptual configuration as Definition 13. Definition 14 looks to simply list the indispensable principles of exploitation as its essential features, although the three principles are intuitively appealing and well acknowledged. Moreover, it is easy to check that Definition 13 satisfies all three principles in Definition 14. Indeed, the appropriation principle is obviously satisfied, and the exclusion principle is satisfied by the definition of economic vulnerability. Finally, Definition 13 also satisfies the inverse interdependence welfare principle as long as the fruit of labor is defined as a use-value contributing to human welfare.

3.3 Relations of exploitation with forced transfers, economic oppression, and distributive injustice

This subsection examines the logical relation of exploitation to similar notions of economic oppression and/or distributive injustice using the conceptual definition of exploitation developed by Vrousalis (2013) and Wright (2000).

3.3.1 Exploitation and forced transfer

First, we refer to the definitions of exploitation given by well-known Marxists, such as Nancy Holmstrom, R. G. Peffer, and Jeffrey Reiman. According to their arguments, agent ν exploits agent μ if and only if ν extracts forced, unpaid surplus labor from μ. That is, according to their definitions, *forced transfer of (the fruit of) labor* is an indispensable condition for exploitation. In contrast, neither Definition 13 nor Definition 14 includes any condition related to forced transfer.

Indeed, forced transfer does not constitute a sufficient condition for exploitation. For instance, societies with welfare states generally provide for the sick and disabled, among others. Those welfare beneficiaries receive a net transfer of labor time from able-bodied tax payers. These able-bodied are also forced to engage in these net transfers by the state. However, no one would say that the disabled or the sick exploit the able-bodied.

Moreover, forced transfer is not necessary for exploitation. For instance, assume that both agents ν and μ have the same welfare function with respect to the consumption of coconuts, and ν is wealthier than μ in terms of the ownership of land. Then, ν's land is more productive than μ's land in terms of coconuts. However, without any input of labor, both have access to $\overline{g}$ coconuts because of the natural productivity of the palms in each of their lands. Suppose that both agents can enjoy a decent level of welfare by consuming $\overline{g}$ coconuts. Now, ν offers μ the option of working on ν's land, which is much more productive than μ's land when he/she works on it by himself/herself. If they agree on the contract of ν's offer, this implies that μ will produce H coconuts and consume $\widehat{g}$, where $\widehat{g} > \overline{g}$ and is also sufficiently large to compensate for his/her disutility of labor, if any. As a consequence, μ accepts this contract, and therefore ν consumes $H - \widehat{g}$ $(> \widehat{g})$ coconuts without working at all. This is an example of economic exploitation, according to Definition 13, but μ is not being forced by economic circumstances or by a third party to enter into this agreement. This implies that forced transfer is not necessary for exploitation.

3.3.2 Exploitation and distributive injustice

Based on the notion of economic exploitation in Definition 13, Roemer's claim that the issue of exploitation can be reduced to that of distributive injustice is not valid. To argue this point, Vrousalis (2013) introduces the notion of *cleanly generated capitalism*; that is, "a form of capitalism that does not arise from 'primitive accumulation' through massacre, plunder, forced extraction, or, more generally, by transgressing some norm of distributive justice. Rather, it arises from 'clean' social interactions: a laborer, or class of laborers, manages to accumulate significant quantities of capital through toil and savings, thereby turning himself/herself into a capitalist."

In considering cleanly generated capitalism, Vrousalis (2013) provides us with the following example:

Example of Grasshopper and Ant: Grasshopper spends the summer months singing, whereas Ant spends all her time working. When the winter comes, Grasshopper needs shelter, which he presently lacks. Ant has three options:
(i) she can do nothing to help Grasshopper, in which case, the corresponding payoff allocation, (V^{*An}, V^{*Gh}) is $(V^{*An}, V^{*Gh}) = (10, 1)$;
(ii) she can offer Grasshopper costless shelter on the condition that he signs a sweatshop contract, in which case, the corresponding payoff allocation is $(V^{*An}, V^{*Gh}) = (12, 2)$;
(iii) she can offer Grasshopper her shelter, which costs her nothing, in which case, the corresponding payoff allocation is $(V^{*An}, V^{*Gh}) = (10, 3)$.

Now, it is plausible to think that Ant has an obligation to help Grasshopper. However, one need not have a view on this to believe that (ii) is morally worse than (iii), in part because the choice of (ii) constitutes exploitation. Indeed, according to the Roemerian principle of voluntary disadvantages discussed in the last section, (i), (ii), and (iii) are equally acceptable. This implies that, even if it is agreed that the option (ii) involves exploitation, it cannot be condemned as distributive injustice by means of Roemer's theory of equality of opportunity.

The above argument suggests that Roemer's claim that exploitation implies distributive injustice cannot be validated, as long as Definition 13 is presumed. The reason why exploitation survives in the absence of distributive injustice is that, in Definition 13, the notion of exploitation is to diagnose the structure of an economic transaction involving an asymmetric power relation that systematically generates an unequal exchange of labor. In other words, exploitation constitutes a procedural injury to status, which is not reducible to distributive injury.

3.3.3 Exploitation and non-exploitative economic oppression

Exploitation is nothing but a category of *economic oppression*. Generally speaking, economic oppression could be conceived of as social relations satisfying the inverse interdependence welfare principle and the exclusion principle in Definition 14. According to Wright (2000), various forms of economic oppression can be categorized into the following two notions: exploitation and *non-exploitative economic oppression*.

In non-exploitative oppression, the advantaged group does not itself need the excluded group. The welfare of the advantaged does depend on the exclusion principle, but there is no ongoing interdependence between their activities and those of the disadvantaged. However, in exploitation, exploiters depend upon the effort of the exploited for their own welfare. Hence, exploiters depend upon and need the exploited.

We can find a sharp contrast between these two notions by considering the difference in the treatment of indigenous people in North America (non-exploitative economic oppression) and South Africa (exploitation) by European settlers. First, in both cases, we can find a causal relationship between the material advantage to the settlers and the material disadvantage to the indigenous people. This implies that both cases satisfy the inverse interdependence welfare principle. Second, in both cases, this causal relation is rooted in processes by which indigenous people were excluded from a crucial productive resource, namely land. Hence, both cases satisfy the exclusion principle.

However, in South Africa, the settlers appropriated the fruits of labor of the indigenous population, first as agricultural labor, and later as mine workers. This implies that the relation between the settlers and the indigenous people in South Africa is characterized as exploitative.

In contrast, in North America, the labor effort of the indigenous people was generally not appropriated. The indigenous people were simply excluded from capitalistic economic activities developed by the settlers. This implies that the settlers in North America could adopt a strategy of genocide in response to the conflict generated by this exclusion, because they did not need the labor effort of Native Americans. Thus, the relation between the settlers and the indigenous people in North America is as an example of non-exploitative economic oppression.

3.4 Implications of exploitation theory à la Vrousalis (2013)-Wright (2000) from the standpoint of economic theory

The Vrousalis (2013)–Wright (2000) theory of exploitation, unlike the Okishio–Morishima theory, constitutes the notion of exploitation without relying on the basic framework of the classical labor theory of value or the classical Marxian theory of surplus value. However, it also treats the UEL as an indispensable component of exploitation. In this respect, unlike the Roemer theory of PR-exploitation, it inherits a traditional feature from the classical Marxian exploitation theory.

As a result of their independence from the classical framework of the surplus value theory, their arguments make it possible for us to infer exploitation even in economies with no relationship of capital and labor. To see this point, we define a model of Heckscher–Ohlin international economies with two nations and two commodities. We then examine whether a free trade equilibrium between the North and the South involves exploitation. Since the Heckscher-Ohlin model does not have international factor markets, our question here is beyond the traditional subject of the classical Marxian theory and the Okishio–Morishima exploitation theory, which address the existence of exploitation in the production process in terms of employment relations between capital and labor.

Following the notation used in section 2, assume $\mathcal{N} \equiv \{Nh, Sh\}$ and $n = 2$. Let $b \in \mathbb{R}^2_{++}$ be the *subsistence consumption bundle*, which every citizen in every nation must consume for his/her survival in one period of production, regardless of whether he/she supplies labor. For the sake of simplicity, each nation has the same size population, normalized to unity. In addition, as in section 2, the maximal labor supply of each agent is equal to unity and there is no difference in labor skills (human capital) among agents. Let $\overline{\omega} \in \mathbb{R}^2_{++}$ be the world endowments of material capital goods at the beginning of the initial period of production. For the sake of simplicity, assume $\overline{\omega} \equiv A\left[I - A\right]^{-1}(Nb)$, where $N = 2$ in this section. Every national economy has the common consumption space, $C \equiv \left\{c \in \mathbb{R}^2_+ \mid c \geqq b\right\} \times [0, 1]$ and the common welfare function, $u : C \to \mathbb{R}$, defined as follows: for each $(c, l) \in C$,

$$u\left(c, l\right) = 1 - l.$$

That is, no nation is concerned by an increase in consumption goods beyond the subsistence level, b, but they evaluate their social welfare in terms of the increase in free hours (leisure time), once b is guaranteed. An *international economy* is thus defined by the profile $\langle \mathcal{N}, (A, L, b), \overline{\omega} \rangle$, which we call a *subsistent (international) economic environment*.

In an economic model with two nations and two goods, the input coefficient matrix, A, is given by:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} > \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

where $1 - a_{11} > 0$, $1 - a_{22} > 0$, and $(1 - a_{11})(1 - a_{22}) > 0$. The labor coefficient vector, L, is given by $L = (L_1, L_2) > (0, 0)$. Denote each nation's capital endowments in period t by $\omega_t^{Nh} = \left(\omega_{1t}^{Nh}, \omega_{2t}^{Nh}\right) > (0,0)$ and $\omega_t^{Sh} = \left(\omega_{1t}^{Sh}, \omega_{2t}^{Sh}\right) > (0,0)$. In addition, assume that $\omega_0^{Nh} > \omega_0^{Sh}$.

In the following discussion, unlike in the neoclassical Heckscher-Ohlin model of international trade, we assume there are multiple types of capital goods, each of which is reproducible by an overall economic system. In contrast, labor is a primary productive factor in that it is not reproducible, and is indispensable as a factor in any production activity.

We explicitly take the time structure of production. Hence, the capital goods available in the present period of production cannot exceed the amount of capital goods accumulated until the end of the preceding period of production. Moreover, the time structure of production is given as follows:

(1) Given the market prices $p_{t-1} = (p_{1t-1}, p_{2t-1}) \geq (0,0)$ at the beginning of period t, each nation, $\nu = Nh, Sh$, purchases, under the constraint of its wealth endowment, $p_{t-1}\omega_t^\nu$, capital goods Ax_t^ν as inputs for the production in the present period. Each nation also purchases the commodities δ_t^ν to sell, for speculative purposes, at the end of the present period;

(2) Each nation is engaged in the production activity of the period t by inputting labor, Lx_t^ν, and the purchased capital goods, Ax_t^ν;

(3) The production activity is completed and x_t^ν is produced as an output at the end of this period. Then, in goods markets with market prices $p_t \geq (0,0)$, each nation earns the revenue $(p_t x_t^\nu + p_t \delta_t^\nu)$ derived from the output x_t^ν, as well as the speculative commodity bundle δ_t^ν. The nation uses the revenue to purchase the bundle b for consumption at the end of this period and the capital stock ω_{t+1}^ν for production in the next period. Therefore, the wealth endowment carried over to the next period, $t+1$, is $p_t\omega_{t+1}^\nu$.

A model of international trade endowed with the above-mentioned time structure is called a *Marxian Heckscher-Ohlin model of international trade.*

Let (w_t^ν, r_t^ν) be the profile of prices in the domestic factor markets of nation ν in period t. That is, this is the profile of the wage rate and interest rate in ν's domestic markets. Given a price system, $\left\langle \{p_{t-1}, p_t\}; (w_t^\nu, r_t^\nu)_{\nu\in\mathcal{N}} \right\rangle$, in period t, each nation, $\nu\,(=Nh, Sh)$, solves the following optimization program:

$$
\begin{aligned}
&\min_{x_t^\nu, \delta_t^\nu} l_t^\nu \\
\text{s.t.}\quad p_t x_t^\nu + p_t\delta_t^\nu &\geqq p_t b + p_t\omega_{t+1}^\nu; \\
p_t x_t^\nu - p_{t-1}Ax_t^\nu &= w_t^\nu Lx_t^\nu + r_t^\nu p_{t-1}Ax_t^\nu; \\
l_t^\nu &= Lx_t^\nu \leqq 1; \\
p_{t-1}\delta_t^\nu + p_{t-1}Ax_t^\nu &\leqq p_{t-1}\omega_t^\nu, \text{ where } \delta_t^\nu \in \mathbb{R}_+^2; \\
p_t\omega_{t+1}^\nu &\geqq p_{t-1}\omega_t^\nu.
\end{aligned}
$$

We denote the set of solutions to the optimization program of each nation, ν, in period t by $\boldsymbol{O}_t^\nu\left(\{p_{t-1}, p_t\}; (w_t^\nu, r_t^\nu)_{\nu\in\mathcal{N}}\right)$.

For the sake of simplicity, we focus on the case of stationary equilibrium prices (i.e., $p_t = p_{t-1} = p^*$). In this case, for any optimal solution, $(x_t^{*\nu}, \delta_t^{*\nu}) \in \boldsymbol{O}_t^{\nu}\left(p^*; (w_t^{\nu*}, r_t^{\nu*})_{\nu\in\mathcal{N}}\right)$, it follows that $p^* x_t^{*\nu} - p^* A x_t^{*\nu} = p^* b$.

Definition 15: For a subsistence international economy, $\left\langle \mathcal{N}, (A, L, b), \left(\omega_0^{Nh}, \omega_0^{Sh}\right)\right\rangle$, where $\omega_0^{Nh} + \omega_0^{Sh} = \overline{\omega}$, an *international reproducible solution (IRS)* is a profile of a price system $\left\langle p^*; (w_t^{\nu*}, r_t^{\nu*})_{\nu\in\mathcal{N}}\right\rangle$ and production activities $(x_t^{*\nu})_{\nu\in\mathcal{N}}$ $(\forall t)$ that satisfies the following conditions:
(i) $(x_t^{*\nu}, \delta_t^{*\nu}) \in \boldsymbol{O}_t^{\nu}\left(p^*; (w_t^{\nu*}, r_t^{\nu*})_{\nu\in\mathcal{N}}\right)$ $(\forall t)$ (each nation's welfare optimization);
(ii) $2b \leqq [I - A]\left(x_t^{*Nh} + x_t^{*Sh}\right)$ $(\forall t)$ (the demand-supply matching at the end of each period);
(iii) $A\left(x_t^{*Nh} + x_t^{*Sh}\right) + \left(\delta_t^{*Nh} + \delta_t^{*Sh}\right) \leqq \omega_t^{Nh} + \omega_t^{Sh}$ $(\forall t)$ (the social feasibility of production at the beginning of each period).

In addition to the above definition, we focus on the following subset of the IRS: An *international reproducible solution is imperfectly specialized* if and only if $x_t^{*\nu} \in \mathbb{R}^2_{++}$ and $\delta_t^{*\nu} = \mathbf{0}$ $(\forall t)$, for each $\nu \in \mathcal{N}$. By the property of imperfect specialization of the IRS, it follows that $p^* \in \mathbb{R}^2_{++}$ and $[I - A]\left(x_t^{*Nh} + x_t^{*Sh}\right) = 2b$. The latter equation implies $\left(x_t^{*Nh} + x_t^{*Sh}\right) = [I - A]^{-1}(2b)$. Therefore, $A\left(x_t^{*Nh} + x_t^{*Sh}\right) = A[I - A]^{-1}(2b) = \overline{\omega} = \omega_0^{Nh} + \omega_0^{Sh}$ holds.

It is well known that, in the so-called neoclassical Heckscher–Ohlin model of international trade, the factor price equalization theorem and the Heckscher-Ohlin theorem hold. Even in the Marxian Heckscher-Ohlin model of international trade presented here, we can verify the factor price equalization theorem, as follows:

Theorem 1 (Factor price equalization theorem in subsistence economies): For any subsistence international economy, $\left\langle \mathcal{N}, (A, L, b), \left(\omega_0^{Nh}, \omega_0^{Sh}\right)\right\rangle$, with $\omega_0^{Nh} + \omega_0^{Sh} = \overline{\omega}$, let $\left\langle p^*; (w_t^{\nu*}, r_t^{\nu*})_{\nu\in\mathcal{N}}, (x_t^{*\nu})_{\nu\in\mathcal{N}}\right\rangle$ be an imperfectly specialized IRS. Then, if $\frac{p^* A\mathbf{e}_1}{L_1} \neq \frac{p^* A\mathbf{e}_2}{L_2}$, where $\mathbf{e}_i$ is the i-th unit vector (only the i-th component is unity, and any other is zero), then $\left(w_t^{Nh*}, r_t^{Nh*}\right) = \left(w_t^{Sh*}, r_t^{Sh*}\right)$ holds.

Proof. By the property of an imperfectly specialized IRS, the following equation holds for each nation, $\nu = Nh, Sh$:

$$p^*[I - A] = r_t^{\nu*} p^* A + w_t^{\nu*} L. \quad (2.1)$$

Let $H \equiv A[I - A]^{-1}$ and $v \equiv L[I - A]^{-1}$. Then, by (2.1), we have:

$$p^* = r_t^{\nu*} p^* H + w_t^{\nu*} v.$$

Note that by the indecomposability of matrix A and the hypothesis of the Hawkins–Simon condition, $[I - A]^{-1}$ is a positive matrix. This implies that H is also a positive matrix and v is a positive vector. Therefore:

$$\left(r_t^{Nh*} - r_t^{Sh*}\right) p^* H + \left(w_t^{Nh*} - w_t^{Sh*}\right) v = \mathbf{0}.$$

To establish $\left(w_t^{Nh*}, r_t^{Nh*}\right) = \left(w_t^{Sh*}, r_t^{Sh*}\right)$, it is sufficient to confirm that the row vectors p^*H and v are linearly independent. By $\frac{p^*A\mathbf{e}_1}{L_1} \neq \frac{p^*A\mathbf{e}_2}{L_2}$, it follows that $p^*A\mathbf{e}_1 \cdot L_2 - p^*A\mathbf{e}_2 \cdot L_1 \neq 0$, which implies the matrix $\begin{bmatrix} p^*A \\ L \end{bmatrix}$ is non-singular, and therefore the row vector p^*A and the row vector L are linearly independent. Now, assume that the row vectors p^*H and v are linearly dependent. Then, there exists a positive scalar, $\varsigma > 0$, such that $\varsigma p^*H = v$. Multiplying both sides of this equation by $[I - A]$, from the right, we obtain $\varsigma p^*A = L$, which contradicts the fact that p^*A and L are linearly independent. Thus, p^*H and v are linearly independent. ■

Theorem 2 ("Quasi-Heckscher–Ohlin theorem" in subsistence economies): For any subsistence international economy, $\left\langle \mathcal{N}, (A, L, b), \left(\omega_0^{Nh}, \omega_0^{Sh}\right)\right\rangle$, with $\omega_0^{Nh} + \omega_0^{Sh} = \overline{\omega}$, let $\left\langle p^*; (w_t^*, r_t^*), (x_t^{*\nu})_{\nu \in \mathcal{N}}\right\rangle$ be an imperfectly specialized IRS with $\frac{p^*A\mathbf{e}_1}{L_1} > \frac{p^*A\mathbf{e}_2}{L_2}$. Then, if $p^*\omega_t^{Nh} > p^*\omega_t^{Sh}$, the wealthier nation, Nh, exports the more capital-intensive good, good 1, and imports the more labor-intensive good, good 2. Correspondingly, the poorer nation, Sh, exports the more labor-intensive good, good 2, and imports the more capital-intensive good, good 1.

Proof. Firstly, we show that, in equilibrium, it follows that:

$$\frac{p^*A\mathbf{e}_1}{p^*A\mathbf{e}_2} > \frac{p^*\left[I - A\right]\mathbf{e}_1}{p^*\left[I - A\right]\mathbf{e}_2} \geqq \frac{L_1}{L_2}.$$

Assume that $\frac{p^*A\mathbf{e}_1}{p^*A\mathbf{e}_2} \leqq \frac{p^*[I-A]\mathbf{e}_1}{p^*[I-A]\mathbf{e}_2}$. Then, since $\frac{p^*A\mathbf{e}_1}{p^*A\mathbf{e}_2} > \frac{L_1}{L_2}$, both nations have the optimal solution:

$$x_t^{*\nu} = \left(\min\left\{\frac{p^*b}{p^*\left[I - A\right]\mathbf{e}_1}, \frac{p^*\omega_t^\nu}{p^*A\mathbf{e}_1}\right\}, 0\right).$$

This implies that $(x_t^{*\nu})_{\nu \in \mathcal{N}}$ violates condition (ii) of Definition 15, which is a contradiction. Likewise, if we assume that $\frac{p^*[I-A]\mathbf{e}_1}{p^*[I-A]\mathbf{e}_2} < \frac{L_1}{L_2}$, then both nations have the optimal solutions:

$$x_t^{*\nu} = \left(0, \min\left\{\frac{p^*b}{p^*\left[I - A\right]\mathbf{e}_2}, \frac{p^*\omega_t^\nu}{p^*A\mathbf{e}_2}\right\}\right),$$

which again violates condition (ii) of Definition 15, producing a contradiction. In summary, we must have $\frac{p^*A\mathbf{e}_1}{p^*A\mathbf{e}_2} > \frac{p^*[I-A]\mathbf{e}_1}{p^*[I-A]\mathbf{e}_2} \geqq \frac{L_1}{L_2}$ in equilibrium.

In this case, the optimal production activity, $x_t^{*\nu}$, of each nation, ν, has the properties $p^*b = p^*\left[I - A\right]x_t^{*\nu}$ and $p^*Ax_t^{*\nu} = p^*\omega_t^\nu$. Moreover, by condition (ii) of Definition 15, we have $\left[I - A\right]^{-1}b = \frac{1}{2}\left(x_t^{*Nh} + x_t^{*Sh}\right)$. Then, since $p^*\omega_t^{Nh} > p^*\omega_t^{Sh}$, we have $x_1^{*Nh} > \mathbf{e}_1\left[I - A\right]^{-1}b > x_1^{*Sh}$ and $x_2^{*Nh} < \mathbf{e}_2\left[I - A\right]^{-1}b < x_2^{*Sh}$, which implies that

$$\begin{aligned} &(1 - a_{11})\, x_1^{*Nh} - a_{12}x_2^{*Nh} > b_1 > (1 - a_{11})\, x_1^{*Sh} - a_{12}x_2^{*Sh}; \\ &(1 - a_{22})\, x_2^{*Nh} - a_{21}x_1^{*Nh} < b_2 < (1 - a_{22})\, x_2^{*Sh} - a_{21}x_1^{*Sh}. \end{aligned}$$

That implies that nation Nh exports good 1 and imports good 2, whereas nation Sh imports good 1 and exports good 2. Since $\frac{p^* A \mathbf{e}_1}{L_1} > \frac{p^* A \mathbf{e}_2}{L_2}$, we can say that good 1 is more capital-intensive and good 2 is more labor-intensive. ■

Note that, unlike the standard Heckscher–Ohlin theorem derived from the neoclassical Heckscher–Ohlin model, Theorem 2 is not necessary to explain the mechanism of free trade as the principle of comparative advantage. The good 1 industry is, in the present equilibrium, incidentally more capital intensive than the good 2 industry. This allows the possibility that, in a transient price system before arriving at the equilibrium, the good 2 industry would be more capital intensive than the good 1 industry. Likewise, nation Nh is incidentally wealthier than nation Sh in the present equilibrium price system, which allows the possibility that, in a transient price system before arriving at the equilibrium, the monetary value of Sh's capital endowments is larger than that of Nh. Therefore, it is difficult to preserve the implication of the standard Heckscher–Ohlin theorem, which states that a free trade equilibrium is established in international markets through the mechanism of international division of labor. In other words, each nation chooses its own production activity, following the principle of comparative advantage, to specialize in the industry that uses this nation's relatively abundant factor of production more intensively. In this way, a free trade equilibrium is established, according to the standard Heckscher-Ohlin theorem.

The notion of labor exploitation under subsistence economies is formally defined as follows:

Definition 16: For any subsistence economy, $\langle \mathcal{N}, (A, L, b), \overline{\omega} \rangle$, let $\left\langle p^*; (w_t^{\nu *}, r_t^{\nu *})_{\nu \in \mathcal{N}}, (x_t^{*\nu})_{\nu \in \mathcal{N}} \right\rangle$ be an IRS. Then, the amount of socially necessary labor required to produce b as a net output is:

$$\frac{1}{2} L \left(x^{*Nh} + x^{*Sh} \right) = vb = L \left[I - A \right]^{-1} b.$$

Moreover, for each nation, $\nu = Nh, Sh$, the supply of labor hours to earn revenue $p^* b$ for its own survival is $Lx^{*\nu}$, which implies:

$$\nu \text{ is an } \textit{exploiting} \text{ nation} \Longleftrightarrow Lx^{*\nu} < vb;$$
$$\nu \text{ is an } \textit{exploited} \text{ nation} \Longleftrightarrow Lx^{*\nu} > vb.$$

Under the assumption of Definition 16, the following theorem indicates that if the quasi-Hecksher–Ohlin international division of labor is generated in the international relation between the North and South, it is characterized as an exploitative relation:

Theorem 3 (The generation of exploitative relations in subsistence economies): For any subsistence international economy, $\left\langle \mathcal{N}, (A, L, b), \left(\omega_0^{Nh}, \omega_0^{Sh} \right) \right\rangle$, with $\omega_0^{Nh} + \omega_0^{Sh} = \overline{\omega}$, let $\left\langle p^*; \left(w_t^*, r_t^*\right), \left(x_t^{*\nu}\right)_{\nu \in \mathcal{N}} \right\rangle$ be an imperfectly specialized IRS with $\frac{p^* A \mathbf{e}_1}{L_1} > \frac{p^* A \mathbf{e}_2}{L_2}$. Then, if $r_t^* > 0$ and $p^* \omega_t^{Nh} > p^* \omega_t^{Sh}$, then the wealthier nation, Nh, is exploiting, and the poorer nation, Sh, is exploited, in terms of Definition 16. Conversely, if $r_t^* = 0$ or $p^* \omega_t^{Nh} = p^* \omega_t^{Sh}$ holds, then there is no exploitative relation.

Proof. Following the proof of Theorem 2, we can confirm that at the IRS, the optimal production activity, $x_t^{*\nu}$, for each nation, ν, is selected to satisfy $p^* b = p^* \left[I - A\right] x_t^{*\nu}$ and $p^* A x_t^{*\nu} = p^* \omega_t^{\nu}$. Then, by $p^* \omega_t^{Nh} > p^* \omega_t^{Sh}$, the hyperplane $p^* A x_t^{*Nh} = p^* \omega_t^{Nh}$ is placed above the hyperplane $p^* A x_t^{*Sh} = p^* \omega_t^{Sh}$. Moreover, by $p^* \left[I - A\right] x_t^{*Nh} = p^* b = p^* \left[I - A\right] x_t^{*Sh}$, the point x_t^{*Nh} is placed more to the right than point x_t^{*Sh}. Recall that the condition

$$\frac{p^* A \mathbf{e}_1}{p^* A \mathbf{e}_2} > \frac{p^* \left[I - A\right] \mathbf{e}_1}{p^* \left[I - A\right] \mathbf{e}_2} \geqq \frac{L_1}{L_2}$$

holds in equilibrium, with $p^* \omega_t^{Nh} > p^* \omega_t^{Sh}$, which implies that the normal vector, L, has a gentler slope than the normal vector $p^* \left[I - A\right]$, or that the slopes of both normal vectors are identical. Therefore, since the point x_t^{*Nh} is placed more to the right than point x_t^{*Sh}, $\frac{p^*[I-A]\mathbf{e}_1}{p^*[I-A]\mathbf{e}_2} > \frac{L_1}{L_2}$ implies

$$L x_t^{*Nh} < L \left[I - A\right]^{-1} b < L x_t^{*Sh}.$$

In contrast, $\frac{p^*[I-A]\mathbf{e}_1}{p^*[I-A]\mathbf{e}_2} = \frac{L_1}{L_2}$ implies

$$L x_t^{*Nh} = L \left[I - A\right]^{-1} b = L x_t^{*Sh}.$$

Note that the property $\frac{p^*[I-A]\mathbf{e}_1}{p^*[I-A]\mathbf{e}_2} > \frac{L_1}{L_2}$ is confirmed as follows: first, by $p^* = r_t^* p^* H + w_t^* v$, if $r_t^* > 0$, then

$$\frac{p^*}{w_t^*} > v \Leftrightarrow \frac{p^*}{w_t^*} \left[I - A\right] > L.$$

If there exists a positive number, $\varsigma > 1$, such that $\frac{p^*}{w_t^*} \left[I - A\right] = \varsigma L$ holds, then $\frac{p^*}{w_t^*} = \varsigma v$ holds. However, according to the proof of Theorem 1, the linear independence of the vectors $p^* H$ and v is confirmed by $\frac{p^* A \mathbf{e}_1}{L_1} > \frac{p^* A \mathbf{e}_2}{L_2}$. Therefore, there is no positive number $\varsigma > 1$ such that $\frac{p^*}{w_t^*} = \varsigma v$. Thus, there is no $\varsigma > 1$ such that $\frac{p^*}{w_t^*} \left[I - A\right] = \varsigma L$. In summary, we have $\frac{p^*}{w_t^*} \left[I - A\right] > L$ since $r_t^* > 0$, and $\frac{p^*[I-A]\mathbf{e}_1}{p^*[I-A]\mathbf{e}_2} \geqq \frac{L_1}{L_2}$ holds by the property of an imperfectly specialization equilibrium, both of which imply $\frac{p^*[I-A]\mathbf{e}_1}{p^*[I-A]\mathbf{e}_2} > \frac{L_1}{L_2}$.

In contrast, if $r_t^* = 0$, then

$$\frac{p^*}{w_t^*} = v \Leftrightarrow p^* \left[I - A\right] = w_t^* L,$$

which corresponds to the case of $\frac{p^*[I-A]\mathbf{e}_1}{p^*[I-A]\mathbf{e}_2} = \frac{L_1}{L_2}$. ■

In Theorem 3, the inequality $Lx_t^{*Nh} < vb < Lx_t^{*Sh}$ that represents the *unequal exchange of labor* implies the *generation of exploitative relation*. In an IRS for a subsistence economic environment, both Nh and Sh earn the minimal income required to purchase the subsistence bundle, b. However, there is a difference between the two nations in terms of their labor supply, which means nation Nh enjoy more hours of freedom from the necessary labor for survival than does nation Sh. Based on Definition 13, this phenomenon is not simply an issue of wealth inequality, but implies the existence of an exploitative relation.

Firstly, nation Sh is economically vulnerable to nation Nh. This is because nation Nh has sufficient wealth, $p^*\omega_t^{Nh}$, that it can survive autarkically, but Sh cannot do so given its level of wealth, $p^*\omega_t^{Sh}$, evaluated at the present international market equilibrium prices. Therefore, the survival of nation Sh can be guaranteed trading with Nh. In other words, given the present equilibrium price system, though Nh can withdraw from the trade relation with Sh at the expense of its economic rationality, it is substantially impossible for Sh to withdraw from the trade relation with Nh.

Secondly, Nh can instrumentalize the economic vulnerability of Sh, which gives Nh a bargaining advantage over Sh in their trade relation. As a consequence, the trade relation between Nh and Sh is characterized by the systematic feature that Sh cannot but accept the appropriation of the fruits of its labor by Nh. This is the structure of the trade relation between Nh and Sh generated systematically in the imperfectly specialized IRS.

This phenomenon obviously implies that the inverse interdependent welfare principle of Definition 14 is satisfied. Moreover, this inverse interdependency occurs because Sh does not have sufficient access to capital goods because of a lack of wealth. This implies that the exclusion principle of Definition 14 is also satisfied. Indeed, if Sh were to own sufficient wealth that it was able to purchase the capital goods, $A\,[I-A]^{-1}\,b$, necessary for its autarkic survival, then Nh could not appropriate the fruit of the labor of Sh, $Lx_t^{*Sh} - L\,[I-A]^{-1}\,b$.

Finally, for Nh to enjoy more free hours depends heavily on the trade relation with Sh. Indeed, without the trade with Sh, Nh should invest $L\,[I-A]^{-1}\,b$ of labor, even if it owns $\omega_t^{Nh} > A\,[I-A]^{-1}\,b$. As a result of the trade relation with Sh, in which the mutual gains from trade exist, Nh can guarantee its own survival by appropriating the fruit of labor from Sh, $Lx_t^{*Sh} - L\,[I-A]^{-1}\,b$, even with fewer labor hours than $L\,[I-A]^{-1}\,b$. Thus, the inequality of the UEL, $Lx_t^{*Nh} < vb < Lx_t^{*Sh}$, also implies an exploitative relation, even in terms of Definition 14.

Note that the neoclassical international trade theory is ignorant of the generation of exploitation in free trade equilibria, since it usually evaluates the performance of the free trade in terms of the mutual gains from trade. Here, free trade is praised for its mechanism that enables both parties to increase their welfare from their autarkic activities. Furthermore, according to this theory, another virtue of free trade is that South's gain from the trade is typically greater than that of North. These features are also found in the imperfectly specialized IRS of Marxian Heckscher–Ohlin international trade. Therefore, the mutual gains from trade and the generation of exploitative relations are completely compatible in a free trade equilibrium under the Marxian Heckscher–Ohlin framework. This is because the viewpoint of the mutual gains from trade is not concerned about the asymmetric structure of the initial endowments or the corresponding asymmetric power relations, both of which are primary concerns of the Vrousalis (2013)–Wright (2000) viewpoint of exploitation.

4 Recent developments of exploitation theory in economics: an axiomatic approach to exploitation theory

According to the Vrousalis (2013)–Wright (2000) theory, exploitation should be conceptualized as the systematic structure of economic transactions characterized by the UEL. Here, part of the fruits of the labor of the exploited agents is appropriated by the exploiters under the institutional framework of asymmetric power relations resulting from private ownership. In contrast, the formal definition of exploitation in economic theory has been discussed mainly as the formulation of the UEL feature of exploitation.

The issue of how to formally define the UEL is not difficult to fix, as long as we assume the simple Leontief types of production economies. For instance, the formulation given by Definition 16 would be the unique, proper definition of the UEL whenever the economies are restricted to subsistence economies with a Leontief production technology. However, once we extend our perspective beyond the simple Leontief production economies to more general economic environments, it becomes more difficult to formally define the UEL. Many proper formal definitions of exploitation as the UEL have been proposed, such as those of Morishima (1974), Roemer (1982; chapter 5), Foley (1982), and so on. These proposals are essentially equivalent within the class of simple Leontief production economies, but behave differently whenever the class of economies is extended to contain a more general type of economic model.

Note that if a definition of exploitation as the UEL is appropriate, it should point out the existence of a transfer mechanism by which the UEL is mediated: the UEL is implemented by a mechanism that transfers (a part of) the productive fruits from the exploited to the exploiter. In perfectly competitive markets, where neglecting the issue of rent, net outputs are distributed into two categories of income: wage income and profit income. Moreover, every party receives an equal wage per unit of (effective) labor. Therefore, the appropriation of more of the productive fruits by exploiters must be explained by a source of income other than wages, which implies the necessity of profit income. In other words, a valid formal definition of exploitation as the UEL should be able to verify the correspondence between the UEL and profits.

Summarizing the above argument leads to the following logical implication as our desideratum:

(a) the formal definition of exploitation as the UEL is valid $\Rightarrow$ (b) in any economic equilibrium, the generation of positive profits must be equivalent to the state that at least each of propertyless worker is exploited, according to the presumed definition of exploitation.

Statement (b) is referred to as the **Profit-Exploitation Correspondence Principle (PECP)**.

The PECP looks similar to the FMT, but they are both conceptually and formally different. Conceptually, the FMT, in general, refers to the (average) rate of exploitation (= the rate of surplus value) for the working class as a whole. Therefore, the FMT would be unsatisfactory if we are interested in each individual worker's exploitation status in an economy with heterogeneity and diversity of individual agents.[9] In contrast, the PECP requires the equivalence between the generation of positive profits and the situation in which each propertyless worker is identified as exploited, even if the economic environments have heterogenous and diverse agents and a more general production technology. However, formally speaking, this does not necessarily imply that the PECP is a stronger condition than the FMT. Indeed, as discussed in detail later, the PECP and FMT are logically independent in that the former allows for zero profits in conjunction with a positive average rate of exploitation, which would violate the FMT.

Veneziani and Yoshihara (2013a) axiomatically characterize the definitions of exploitation that satisfy the PECP, and so shed new light on the debate about the proper definition of exploitation in Marxian economics. Firstly, they propose a general model of capitalist economies that allows for heterogeneity in each agent's preferences for consumption goods and leisure, heterogeneity in their endowments of material and human capital, and a general closed-convex cone type of production set. Secondly, given such a general model, they axiomatically characterize the formulations of exploitation as the UEL in which the PECP is preserved in any equilibrium. As a result, most definitions of exploitation proposed in the literature, such as those of Morishima (1974) and Roemer (1982; chapter 5) do not preserve the PECP, with only the definition à la New Interpretation [Duménil (1980), Foley (1982)] doing so. In addition, Yoshihara and Veneziani (2013c) define exploitation in terms of a general commodity analogically to the New Interpretation definition of labor exploitation. They then show that, given such a definition, the equivalence between positive profits and the existence of exploitation in terms of a general commodity is not established.

[9] For instance, as shown by Yoshihara and Veneziani (2012), in a von Neumann economy with heterogeneity of propertyless workers' welfare functions, the positivity of the average rate of exploitation coexists with the non-exploitation of some propertyless workers, simply because of their consumption choices. This implies that, even if the FMT holds in such economies, it may be that some propertyless workers are not exploited.

In the following subsections, we introduce the main arguments developed by Veneziani and Yoshihara (2013a) by restricting our focus to the von Neumann production technology, (A, B, L), introduced in section 2.

4.1 Model

We define a production possibility set derived from a von Neumann production technology, (A, B, L), as follows:

$$P_{(A,B,L)} \equiv \left\{\alpha \equiv (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists x \in \mathbb{R}^m_+ : \alpha \leqq (-Lx, -Ax, Bx)\right\}.$$

Here, by the definition of $\alpha \in P_{(A,B,L)}$, let $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$ represent the vector of net outputs corresponding to α. Given $P_{(A,B,L)}$, we define the set of production activities feasible with k units of labor inputs by:

$$P_{(A,B,L)}(\alpha_l = k) \equiv \left\{(-\alpha'_l, -\underline{\alpha}', \overline{\alpha}') \in P_{(A,B,L)} \mid \alpha'_l = k\right\}.$$

The frontier of the production possibility set $P_{(A,B,L)}$ is given by:

$$\partial P_{(A,B,L)} \equiv \left\{\alpha \in P_{(A,B,L)} \mid \nexists \alpha' \in P_{(A,B,L)} : \alpha' > \alpha\right\}.$$

Moreover, for any bundle $c \in \mathbb{R}^n_+$, the production possibility set to produce c as a net output is given by:

$$\phi(c) \equiv \left\{\alpha \in P_{(A,B,L)} \mid \widehat{\alpha} \geqq c\right\}.$$

To characterize the types of agents in the von Neumann capitalist economies defined in section 2, we assume in this section that each agent can be heterogeneous in terms of their capital endowments $(\omega^\nu_0)_{\nu \in \mathcal{N}}$, welfare functions, and labor skills. That is, for each $\nu \in \mathcal{N}$, $s^\nu > 0$ represents his/her skill level. Moreover, let $C \subseteq \mathbb{R}^n_+ \times [0,1]$ be the consumption space common to all agents, and for each $\nu \in \mathcal{N}$, let $u^\nu : C \to \mathbb{R}_+$ be his/her welfare function. All available welfare functions are assumed to be increasing in consumption bundles and decreasing in the supply of labor hours. Thus, one capitalist economy is defined by the list $\mathcal{E} \equiv \left\langle \mathcal{N}; P_{(A,B,L)}; (u^\nu, s^\nu, \omega^\nu_0)_{\nu \in \mathcal{N}}\right\rangle$.

Assuming the same time structure of production as in section 3.4, and given a price system $\langle \{p_{t-1}, p_t\}, w_t \rangle$ in period t, each agent $\nu \in \mathcal{N}$ engages in an optimal choice of production plan $\alpha^\nu_t \in P_{(A,B,L)}$. Here, each agent, (i) purchases a bundle of capital goods $\underline{\alpha}^\nu_t$ under his/her wealth constraint, $p_{t-1}\omega^\nu_t$, and employs labor power, α^ν_{lt}, at the beginning of this period; (ii) purchases an optimal amount of commodity bundle δ^ν_t under budget constraint $p_{t-1}(\omega^\nu_t - \underline{\alpha}^\nu_t)$ for speculative purposes, to be sold at the end of the period; and (iii) chooses an optimal labor supply and consumption plan, $(c^\nu_t, l^\nu_t) \in C$, where c^ν_t will be purchased at the end of this period under the budget constraint of his/her revenue from both production and speculation. This choice behavior is determined as a solution to the optimization problem (MP^ν_t), as follows:

$$MP^\nu_t : \max_{(c^\nu_t, l^\nu_t) \in C;\ \delta^\nu_t \in \mathbb{R}^n_+;\ \alpha^\nu_t \in P_{(A,B,L)}} u^\nu(c^\nu_t, l^\nu_t)$$

$$\text{s.t.}\quad [p_t\overline{\alpha}_t^\nu - w_t\alpha_{lt}^\nu] + w_t\Lambda_t^\nu + p_t\delta_t^\nu \geqq p_t c_t^\nu + p_t\omega_{t+1}^\nu,\ \text{where } \Lambda_t^\nu \equiv s^\nu l_t^\nu;$$
$$p_{t-1}\delta_t^\nu + p_{t-1}\underline{\alpha}_t^\nu \leqq p_{t-1}\omega_t^\nu;$$
$$p_t\omega_{t+1}^\nu \geqq p_{t-1}\omega_t^\nu.$$

Then, denote the set of solutions to the problem (MP_t^ν) by $\boldsymbol{O}_t^\nu(\{p_{t-1}, p_t\}, w_t)$.

As in section 3.4, we focus on the stationary equilibrium price vector, $p^* = p_{t-1} = p_t$ $(\forall t)$. Moreover, we focus on the non-trivial equilibrium satisfying $\max_{\alpha' \in P_{(A,B,L)}} \frac{p^*\overline{\alpha}' - p^*\underline{\alpha}' - w_t\alpha_l'}{p^*\underline{\alpha}'} \geqq 0$. In this case, by the monotone increasing characteristic of u^ν at c_t^ν, there always exists an optimal solution having $\delta_t^\nu = \mathbf{0}$. By focusing on this optimal solution, we can remove the description of δ_t^ν without loss of generality. Henceforth, we consider the following equilibrium notion:

Definition 17: For a capitalist economy, $\mathcal{E}$, a *reproducible solution (RS)* is a profile $\left((p^*, w_t^*); ((c_t^{*\nu}, l_t^{*\nu}); \alpha_t^{*\nu})_{\nu\in\mathcal{N}}\right)$ of a price system and economic activities in each period, t, satisfying the following conditions:
(i) $((c_t^{*\nu}, l_t^{*\nu}); \alpha_t^{*\nu}) \in \boldsymbol{O}_t^\nu(p^*, w_t^*)$ $(\forall t)$ (each agent's optimization);
(ii) $\sum_{\nu\in\mathcal{N}} \widehat{\alpha}_t^{*\nu} \geqq \sum_{\nu\in\mathcal{N}} c_t^{*\nu}$ $(\forall t)$ (demand-supply matching at the end of each period);
(iii) $\sum_{\nu\in\mathcal{N}} \alpha_{lt}^{*\nu} = \sum_{\nu\in\mathcal{N}} \Lambda_t^{*\nu}$ $(\forall t)$ (the labor market equilibrium);
(iv) $\sum_{\nu\in\mathcal{N}} \underline{\alpha}_t^{*\nu} \leqq \sum_{\nu\in\mathcal{N}} \omega_t^\nu$ $(\forall t)$ (social feasibility of production at the beginning of each period).

In the following section, we assume the stationary state on economic activities of agents and delete the time description, t.

4.2 Alternative definitions of exploitation and the domain axiom of admissible definitions of exploitation

Recall that the model of capitalist economies considered in section 2 assumes there is no difference in agents' labor skills or consumption preferences. In this section, we assume a more general model of a capitalist economy, $\mathcal{E} = \left\langle \mathcal{N}; P_{(A,B,L)}; (u^\nu, s^\nu, \omega_0^\nu)_{\nu\in\mathcal{N}} \right\rangle$, that includes heterogeneity of labor skills and preferences. Here, discuss an axiom proposed by Veneziani and Yoshihara (2013a), which represents the minimal necessary condition for admissible definitions of exploitation as the UEL. Then, we introduce some alternative definitions of exploitation proposed in the literature on mathematical Marxian economics.

Any definition of exploitation should be able to identify, associated with each equilibrium allocation, the set of exploiting agents, $\mathcal{N}^{ter} \subseteq \mathcal{N}$, and the set of exploited agents, $\mathcal{N}^{ted} \subseteq \mathcal{N}$, such that $\mathcal{N}^{ter} \cap \mathcal{N}^{ted} = \varnothing$ holds. Moreover, it should capture the feature of the UEL as the difference between the amount of labor supplied by each agent and the amount of labor "received" via each agent's income. In particular, it should have the form that the supplied labor amount is greater than the received labor amount for each exploited agent. Such properties should be preserved as a core feature of exploitation, regardless of the way in which exploitation as the UEL is defined.

Note that, for capitalist economies considered herein, each agent's supply of labor is identified by Λ^{ν}. In contrast, what remains open to debate is how to formulate the labor amount that each agent can "receive" via his/her earned income. According to the forms of the "received" labor, there are a number of possible definitions of exploitation.

Summarizing the above arguments, Veneziani and Yoshihara (2013a) propose an axiom that represents the minimal necessary condition for any definition of exploitation, whenever it is deemed admissible as the form of the UEL:

Labor Exploitation (LE) [Veneziani and Yoshihara (2013a)]: Given any definition of exploitation, for any capitalist economy $\mathcal{E}$ and any RS $\left((p,w);((c^{\nu},l^{\nu});\alpha^{\nu})_{\nu\in\mathcal{N}}\right)$, the set of exploited agents, $\mathcal{N}^{ted}\subseteq\mathcal{N}$, should have the following property: there exists a profile of consumption bundles, $(c_e^{\nu})_{\nu\in\mathcal{W}}\in\mathbb{R}_+^{nW}$, such that, for any $\nu\in\mathcal{W}$, $pc_e^{\nu}=w\Lambda^{\nu}$ holds, and for some production point, $\alpha^{c_e^{\nu}}\in\phi(c_e^{\nu})\cap\partial P$ with $\widehat{\alpha}^{c_e^{\nu}}\not>c_e^{\nu}$:

$$\nu\in\mathcal{N}^{ted}\Leftrightarrow\alpha_l^{c_e^{\nu}}<\Lambda^{\nu}.$$

That is, axiom **LE** requires that any admissible definition of exploitation must identify whether each propertyless agent is exploited for each reproducible solution under any economy. More specifically, the axiom stipulates that the set of propertyless exploited agents be identified as follows: according to each specific admissible definition, there should be a profile, $(c_e^{\nu})_{\nu\in\mathcal{W}}$, for each propertyless agent's consumption bundle affordable by that agent's revenue, and its corresponding profile $\left(\alpha^{c_e^{\nu}}\right)_{\nu\in\mathcal{W}}$ of production activities, where each $\alpha^{c_e^{\nu}}$ can produce the corresponding consumption bundle c_e^{ν} as a net output in a technologically efficient way. Then, the exploitation status of each propertyless agent can be identified by comparing the amount of his/her labor supply Λ^{ν} to the amount of labor input $\alpha_l^{c_e^{\nu}}$ that he/she is able to "receive" via his/her income $w\Lambda^{\nu}$.

Axiom **LE** is a rather weak condition in that it only refers to the exploitation status of propertyless agents in each reproducible solution. This should be reasonable as a minimal necessary condition for the admissible domain. In other words, a definition of exploitation is not necessarily deemed proper, even if it satisfies **LE**. In fact, there are potentially infinitely many definitions of exploitation that satisfy **LE**, and all the main definitions proposed in mathematical Marxian economics literature satisfy this axiom.[10]

The following three definitions all satisfy **LE**. Note that, in the following definitions, the labor value of any commodity $c\in\mathbb{R}_+^n$ given in Definition 4 is represented by $l.v.(c)\equiv\min\{\alpha_l\mid\alpha=(-\alpha_l,-\underline{\alpha},\overline{\alpha})\in\phi(c)\}$. Then, Definition 18 is a natural extension of the Morishima's (1974) own definition of economies with homogeneous agents to economies with possibly heterogeneous agents:

[10]Of course, this does not imply that the axiom **LE** is trivial. For instance, the definition proposed by Matsuo (2008) does not satisfy **LE.**

Definition 18 [Morishima (1974)]**:** For any capitalist economy, $\mathcal{E}$, and any $\nu \in \mathcal{W}$, who supplies Λ^{ν} and consumes $c^{\nu} \in \mathbb{R}^n_+$, $\nu \in \mathcal{N}^{ted}$ if and only if $\Lambda^{\nu} > l.v.(c^{\nu})$.

Then, we can naturally extend the Roemer (1982; chapter 5) definition of exploitation, given for economies with homogeneous agents, to economies with possibly heterogeneous agents, which also satisfies **LE**. For any price system $(p, w) \in \mathbb{R}^{n+1}_+$ and any $c \in \mathbb{R}^n_+$, let $\phi(c; p, w) \equiv \left\{ \alpha \in \arg\max_{\alpha' \in P_{(A,B,L)}} \frac{p\overline{\alpha}' - w\alpha'_l}{p\underline{\alpha}'} \mid \widehat{\alpha} \geqq c \right\}$ and

$$l.v.(c; p, w) \equiv \min \{ \alpha_l \mid \alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \phi(c; p, w) \}.$$

Then:

Definition 19 [Roemer (1982; chapter 5)]**:** For any capitalist economy, $\mathcal{E}$, any RS, $\left((p, w); ((c^{\nu}, l^{\nu}); \alpha^{\nu})_{\nu \in \mathcal{N}}\right)$, and any $\nu \in \mathcal{W}$, who supplies Λ^{ν} and consumes $c^{\nu} \in \mathbb{R}^n_+$, $\nu \in \mathcal{N}^{ted}$ if and only if $\Lambda^{\nu} > l.v.(c^{\nu}; p, w)$.

Finally, for any capitalist economy, $\mathcal{E}$, and any RS, $\left((p, w); ((c^{\nu}, l^{\nu}); \alpha^{\nu})_{\nu \in \mathcal{N}}\right)$, let $\alpha^{p,w} \equiv \sum_{\nu \in \mathcal{N}} \alpha^{\nu}$. Moreover, for any $c \in \mathbb{R}^n_+$, we define a non-negative number, $\tau^c \in \mathbb{R}_+$, as satisfying $\tau^c p\widehat{\alpha}^{p,w} = pc$. Then:

Definition 20 [Veneziani and Yoshihara (2013a)]**:** For any capitalist economy, $\mathcal{E}$, any RS, $\left((p, w); ((c^{\nu}, l^{\nu}); \alpha^{\nu})_{\nu \in \mathcal{N}}\right)$, and any $\nu \in \mathcal{W}$, who supplies Λ^{ν} and consumes $c^{\nu} \in \mathbb{R}^n_+$, $\nu \in \mathcal{N}^{ted}$ if and only if $\Lambda^{\nu} > \tau^{c^{\nu}} \alpha^{p,w}_l$.

Definition 20 is also an extension of the New Interpretation definition of exploitation à la Duménil (1980)–Foley (1982), which was originally defined in Leontief economies with homogeneous agents, then extended to economies with possibly heterogeneous agents.

4.3 Profit-Exploitation Correspondence Principle

Now, we are ready to formulate Profit-Exploitation Correspondence Principle, given as follows:

Profit-Exploitation Correspondence Principle (PECP) [Veneziani and Yoshihara (2013a)]: For any capitalist economy, $\mathcal{E}$, and any RS, $\left((p, w); ((c^{\nu}, l^{\nu}); \alpha^{\nu})_{\nu \in \mathcal{N}}\right)$:

$$\left[p\widehat{\alpha}^{p,w} - w\alpha^{p,w}_l > 0 \Leftrightarrow \mathcal{N}^{ted} \supseteq \mathcal{W}_+\right],$$

where $\mathcal{W}_+ \equiv \{\nu \in \mathcal{W} \mid \Lambda^{\nu} > 0\} \neq \varnothing$.

That is, whatever the definition of exploitation, it must follow that for any capitalist economy and any reproducible solution, total profits are positive if and only if any propertyless employee is exploited in terms of this definition, assuming the definition of exploitation is deemed appropriate. This is required by **PECP**.

Note that for the available class of capitalist economies considered here, there is no requirement of a restriction that excludes the existence of fixed capital goods, the possibility of joint production, or of technical changes. In addition, unlike in condition (2) of the RFMT discussed in section 2, there is no restriction that excludes the existence of inferior production processes. Moreover, the heterogeneity of agents' preferences and skills is also available. The equilibrium notion presumed here is also sufficiently general that there is no requirement of a subsistence wage condition. Therefore, the correspondence between profits and exploitation is required for a large class of economic environments, as assumed by the standard general equilibrium theory.

However, **PECP** *per se* is not so strong. Indeed, **PECP** even allows for a situation in which some propertyless employees are exploited in equilibrium, with zero total profit.[11] This implies that, at least within the class of economies with homogeneous agents, **PECP** is logically weaker than the statement of the FMT. For, within the class of such economies, the FMT implies that no propertyless employee is exploited in any equilibrium with zero profit.

As noted at the start of this section, we can derive the following lesson from the recent developments in exploitation theory in political philosophy and sociology: if a definition of exploitation satisfying axiom **LE** is proper, it must satisfy **PECP**. Based on this perspective, Veneziani and Yoshihara (2013a) studied the necessary and sufficient condition for **PECP**, as stated in the following theorem:

Theorem 4 [Veneziani and Yoshihara (2013a)]: For any definition of exploitation satisfying **LE**, the following two statements are equivalent for any capitalist economy, $\mathcal{E}$, and any RS, $\left((p,w);((c^\nu,l^\nu);\alpha^\nu)_{\nu\in\mathcal{N}}\right)$:
(1) **PECP** holds under this definition of exploitation;
(2) If $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$, then for any $\nu \in \mathcal{W}_+$, there exists a production activity $\alpha_\pi^\nu \in P(\alpha_l = \Lambda^\nu) \cap \partial P$ such that $\widehat{\alpha}_\pi^\nu \in \mathbb{R}_+^n$, $p\widehat{\alpha}_\pi^\nu > w\Lambda^\nu$, and $(\alpha_{\pi l}^\nu, \underline{\alpha}_\pi^\nu, \overline{\alpha}_\pi^\nu) \geqq \eta^\nu \left(\alpha_l^{c_e^\nu}, \underline{\alpha}^{c_e^\nu}, \overline{\alpha}^{c_e^\nu}\right)$ hold for some $\eta^\nu > 1$.

That is, condition (2) of Theorem 4 is the necessary and sufficient condition for any definition of exploitation satisfying **LE** to preserve **PECP**. Condition (2) states that, if total profits are positive in the present equilibrium, then for each propertyless employee, $\nu \in \mathcal{W}_+$, there exists a suitable efficient production point, α_π^ν, activated by the present amount of labor supply, Λ^ν, which in conjunction with production activity, $\alpha^{c_e^\nu}$, can verify that this agent is being exploited. Recall that, according to axiom **LE**, production activity $\alpha^{c_e^\nu}$ is identified by the presumed definition of exploitation, and the corresponding labor input $\alpha_l^{c_e^\nu}$ represents agent ν's "received" labor. Production activity $\alpha_\pi^\nu \in P(\alpha_l = \Lambda^\nu) \cap \partial P$ is defined as the proportional expansion of production point $\alpha^{c_e^\nu}$ up to the point of his/her present labor supply, Λ^ν, and that produces a non-negative net output, $\widehat{\alpha}_\pi^\nu \in \mathbb{R}_+^n$, that is non-affordable by ν at the present equilibrium because $p\widehat{\alpha}_\pi^\nu > w\Lambda^\nu$. Therefore, since $\Lambda^\nu = \alpha_{\pi l}^\nu > \alpha_l^{c_e^\nu}$ holds for such a selection of α_π^ν,

[11] However, any definition of exploitation satisfying **LE** does not allow the existence of exploited propertyless employees in conjunction with zero profit.

we can confirm that agent $\nu \in \mathcal{W}_+$ is exploited at this RS, according to the given definition satisfying **LE**.

Theorem 4 does not provide a normative characterization of the presumed definition of exploitation, but rather a demarcation line (condition (2)) by which one can test which of infinitely many potential definitions preserves the essential relation of exploitation and profits in capitalist economies. Thus, if a definition of exploitation satisfying **LE** does not generally meet condition (2), then it will not satisfy **PECP**, which implies that it is not a proper definition of exploitation as the UEL.

Some may criticize the methodological positions of **PECP** and Theorem 4, claiming that **PECP** should be proved as a theorem rather than treated as an axiom. In fact, as Okishio and Morishima did, the methodological standpoint of the FMT was, assuming a specific definition of exploitation, to verify that a capitalist economy can be conceived of as exploitative by establishing the equivalence between exploitation and positive profits.

In contrast, Theorem 4 presumes a correspondence between positive profits and exploitation for every propertyless employee as an axiom, and then tests the validity of each alternative definition of exploitation by checking whether it satisfies this axiom. This methodological standpoint is more likely to be approved, since **PECP** should be positioned as a necessary condition for any proper definition of exploitation, as argued above. Such a methodology has been implicitly adopted within the debates on the FMT. Typically, whenever a counterexample was raised against the FMT with a major definition of exploitation by generalizing the model of economic environments, this criticism was resolved by proposing an alternative definition and proving that the FMT is held with this alternative form under the generalized economic model. This implicitly suggests that, in the overall debate over the FMT, the validity of each exploitation form has been tested by the robustness of the equivalence between exploitation and positive profits. However, even if such an interpretation is acceptable, the structure of the debate over the FMT could not function as a test of the validity of a form of exploitation, because it may involve an infinite repetition of "counterexample and alternate proposal." In contrast, by providing an axiomatic characterization, such as Theorem 4, the validity of every form of exploitation is testable simply by checking condition (2).

There is another argument to justify the treatment of **PECP** as an axiom. It can be shown that in any Leontief economic environment, regardless of whether the heterogeneity of preferences and skills is involved, the equivalence of positive profits and the exploitation of each propertyless employee and the equivalence of zero profit and no exploitation are preserved for any definition of exploitation, as long as it satisfies **LE**.

Theorem 5 [Veneziani and Yoshihara (2013a)]: For any capitalist economy, $\left\langle \mathcal{N}; P_{(A,L)}; (u^\nu, s^\nu, \omega_0^\nu)_{\nu\in\mathcal{N}} \right\rangle$, and any RS, $\left((p,w); ((c^\nu, l^\nu); \alpha^\nu)_{\nu\in\mathcal{N}}\right)$, **PECP** holds for any definition of exploitation satisfying **LE**.

Proof. Take any definition of exploitation that satisfies **LE**. Then, for any

Leontief economy and any RS, (p, w), we can find a profile of reference consumption bundles, $(c_e^\nu)_{\nu \in \mathcal{W}} \in \mathbb{R}_+^{nW}$. Then, regardless of the heterogeneity of welfare functions and skills, the corresponding profile of production activities, $\left(\alpha^{c_e^\nu}\right)_{\nu \in \mathcal{W}}$, is uniquely given by

$$\alpha^{c_e^\nu} \equiv \left(-vc_e^\nu, -A\left(I-A\right)^{-1} c_e^\nu, \left[I + A\left(I-A\right)^{-1}\right] c_e^\nu\right) \text{ for each } \nu \in \mathcal{W}.$$

Thus, $\alpha_l^{c_e^\nu} = vc_e^\nu$. Let $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$ for this RS. This implies that, under the Leontief economy

$$p = (1+\pi)\, pA + wL \text{ for some } \pi > 0.$$

Then, as shown in the proof of Theorem 3, $\frac{p}{w} > v$. Thus, by $w\Lambda^\nu = pc_e^\nu$ from **LE**, we have $\Lambda^\nu = \frac{p}{w} c_e^\nu > vc_e^\nu$, for any $\nu \in \mathcal{W}_+$. Therefore, according to **LE**, any propertyless employee is exploited in terms of the presumed definition of exploitation. ■

However, once the production technology of economic environments is replaced by a more general type, such as the von Neumann production technology, some definitions of exploitation violate **PECP**, even if they satisfy **LE**. Does this suggest that the validity of the basic Marxian perception of capitalist economies as exploitative crucially depends on the degree of the complexity of the production technology? Or, does it suggest that such counterexamples are generated because of *incoherency* in these definitions, in that they cannot properly identify the set of exploited agents whenever a more complex production technology is applied? Veneziani and Yoshihara (2013a) take the latter view. That is, they believe that the complexity of the production technology, such as the existence of fixed capital and the possibility of joint production, should not be essential to determining the exploitation status of each agent. Rather, these counterexamples should be viewed as representing the non-validity of the presumed definitions of exploitation.

Theorem 4 does not identify a unique definition that meets **PECP**, but rather a class of definitions that satisfy condition (2). Yet, Veneziani and Yoshihara (2013a, Corollary 1) show that it has surprising implications concerning the main approaches in exploitation theory. There are economies in which, for all $\nu \in \mathcal{W}_+$, condition (2) is never satisfied if $\alpha^{c_e^\nu}$ is given by Definition 18 or 19, and so **PECP** does not hold. In contrast, Definition 20 satisfies condition (2), and thus **PECP** holds *for all* $\mathcal{E}$ *and all* RS:

Corollary 1 [Veneziani and Yoshihara (2013a)]: There exists a capitalist economy, $\mathcal{E}$, and an RS for this economy such that neither Definition 18 nor Definition 19 satisfies **PECP**.

The proof of Corollary 1 is given by using the economy defined in Example 1 of section 2.1. In that economy, assume an RS $(p^*, 1)$ with $p_1^* > 0$. Then, every agent, $\nu \in \mathcal{W}_+$, consumes $c^\nu = b$ and $l.v.(b) = l.v.(b; p^*, 1) = 1 = \Lambda^{*\nu}$, while $\pi^* > 0$. This implies that neither Definition 18 nor Definition 19 satisfies **PECP**.

Corollary 2 [Veneziani and Yoshihara (2013a)]: For any capitalist economy, $\mathcal{E}$, and any RS, Definition 20 satisfies **PECP**.

These corollaries suggest that, at least among the main competing proposals of exploitation forms, Definition 20 is the sole appropriate form.

There is another interesting argument to support the New Interpretation definition of exploitation. Though Definition 20 formulates exploitation as the unequal exchange of labor, it is also possible to formulate the unequal exchange of any commodity, k, which is analogical to Definition 20. In this case, is an argument such as the GCET again established by using such a definition of unequal exchange? The answer is negative, according to Veneziani and Yoshihara (2013c).

Let us define exploitative relations as an unequal exchange of commodity k, analogical to Definition 20, given as follows:

Definition 21 [Veneziani and Yoshihara (2013c)]: For any capitalist economy, $\mathcal{E}$, and any RS, $\left((p,w)\,;((c^{\nu},l^{\nu})\,;\alpha^{\nu})_{\nu\in\mathcal{N}}\right)$, any agent, $\nu \in \mathcal{N}$, supplies some amount of commodity k, $\omega_k^{\nu} \geqq 0$, as a factor of production, and consumes $c^{\nu} \in \mathbb{R}_+^n$. Then, agent ν is k-exploited if and only if $\omega_k^{\nu} > \tau^{c^{\nu}} \underline{\alpha}_k^{p,w}$.

Our concern is whether the equivalence between positive profits and the existence of k-exploited agents in terms of Definition 21 can be established for any reproducible solution. Veneziani and Yoshihara (2013c) prove that such an equivalence does not hold. For instance, assuming an economy with homogeneity of welfare functions and labor skills, consider a reproducible solution with zero profit. In such an RS, it follows that, for any $\nu \in \mathcal{N}$, $\tau^{c^{\nu}} = \frac{1}{N}$. In contrast, whenever the initial endowment of capital good k is unequal, there generically exists an agent, ν', endowed with $\omega_k^{\nu'} > \frac{1}{N}\omega_k$. Then, it is not difficult to construct an equilibrium with zero profit under which this agent is deemed to be k-exploited, which violates the equivalence of k-exploitation with positive profits in terms of Definition 21.

Summarizing these arguments, if we take the New Interpretation definition of exploitation, such as in Definition 20, it follows that the unequal exchange of any productive factor other than labor and the UEL are not logically equivalent. Therefore, there can be no room for criticism against this definition by means of an analogical argument of the GCET, unlike the criticism of the Okishio–Morishima definition.

5 Concluding remarks

One of the most prominent contributions of Okishio (1963) is that he inspired research beyond the classical Marxian theory of surplus value to the great controversies on the proper definitions of exploitation. Though the Okishio definition of exploitation (Definition 2 in this paper) was essentially faithful to the labor theory of value and theory of surplus value, the sequence of the later controversies suggests the limitation and noneligibility of such a classical definition. It was the property relation theory of exploitation by Roemer (1982, 1994), proposed as an alternative to the Okishio–Morishima approach, that was to prove influential in the fields of economics and political philosophy beyond the Marxian camp. However, recent developments of exploitation theory, such as Vrousalis (2013) and Wright (2000), have successfully defined the notion of exploitation as social relations of the UEL independently of the classical labor theory of value and theory of surplus value. According to these new arguments, the primary normative concern of exploitation has been restored. Along with this recent trend, the present controversy regarding the proper definitions of exploitation as the UEL is the New Interpretation type (Definition 20 in this paper). Though this definition is also independent of the classical framework of labor value and surplus value, the validity of this definition is verified by Veneziani and Yoshihara (2013a,c) through an axiomatic analysis of **PECP**.

Note that there are other axiomatic analyses to support the New Interpretation definition, such as those of Yoshihara (2010) and Yoshihara and Veneziani (2009). Yoshihara (2010) formulates **Class-Exploitation Correspondence Principle** (**CECP**), another important argument in exploitation theory, as an axiom that any proper definition of exploitation should meet, and then characterizes the class of proper definitions of exploitation satisfying this axiom. As a result, the New Interpretation definition has been shown to be the unique and proper definition among the current definitions. Yoshihara and Veneziani (2009) introduce an axiom called **Relational Exploitation** to capture the social relational feature of exploitation as the UEL, and then show that a small number of rather weak axioms, including **Relational Exploitation**, can completely characterize the New Interpretation definition.

Given this current standpoint, it may be concluded that the New Interpretation definition is appropriate as a form of exploitation with which to conceive a capitalist economy as exploitative within a rather broad class of economic environments. Then, it remains to examine whether the New Interpretation definition can be deemed appropriate even in economic environments with heterogeneous labor. However, as a prerequisite of this subject, we may have to identify a proper measure with which to aggregate each vector of multiple heterogeneous labor contents. This problem is discussed by Veneziani and Yoshihara (2013d), who axiomatically derive one proper measure.

Secondly, even if it is shown to be valid to conceive the capitalist economy as exploitative, it would be more desirable in terms of economics to study the degree of seriousness of the exploitation in each society. Proceeding with this line of research would require a new subject to identify the *proper measure of the degree of exploitation*.

Thirdly, the New Interpretation definition of exploitation, such as that shown in Definition 20 in this paper, suggests that the non-exploitative resource allocations should be nothing but the *proportional solution* proposed by Roemer and

Silvestre (1993). Though it is not eligible to reduce the issues of exploitation to the issues of distributive injustice, as argued in section 3.3, it is still an intrinsically interesting problem to study the ethical properties of non-exploitative allocations. With regard to this point, Roemer (2010; 2013) recently proved that the proportional solution, that is, the allocation rule of non-exploitation, would be implementable in a moral state of society in which every citizen behaves in accordance with the Kantian categorical imperative. Such a moral state of society is formulated by Roemer (2010) as a social state of Kantian equilibrium. This line of research would be interesting for Marxian economists to study further.

Lastly, this paper has mainly discussed the generation of exploitative relations in perfectly competitive equilibria in a capitalist economy. However, we have not addressed the persistency of exploitative relations,[12] nor the generation of exploitative relations under capitalist economies with imperfect labor contracts.[13] The former problem would be relevant, in a broader sense, to the controversies over the Okishio Theorem [Okishio (1961)], another significant contribution by Nobuo Okishio. We leave this point as a topic of further research.

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[12] For the current standpoint of this subject, refer to Veneziani (2007, 2013) and Veneziani and Yoshihara (2013b; section 4).

[13] For more information on this line of research, refer to Yoshihara (1998) and Skillman (2013).

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