Investment Complementarities, Coordination Failure, and the Role and Effects of Public Investment Policy

Tetsuya Kasahara
(College of Economics, Rikkyo University)

June 2013

Investment complementarities, coordination failure, and the role and effects of public investment policy[*]

Tetsuya Kasahara[†]

Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-8603, Japan

College of Economics, Rikkyo University, 3-34-1, Nishi-Ikebukuro, Toshima-ku, Tokyo 171-8501 Japan

June 2013

Abstract

This paper analyzes the role and effects of public investment policy when coordination problems among agents can result in individually rational but socially inefficient investment decisions. Developing a coordination investment model in which individuals simultaneously and independently determine whether to undertake a risky but potentially more profitable investment project or an alternative with safe but lower returns, we first show that the risk of coordination failure can in equilibrium result in socially inefficient investment and small consumption. We then investigate the role and effects of a public investment policy designed to help mitigate inefficiency. In our model, the size of a feasible public investment policy is determined endogenously. Our numerical results show that the divisibility of investment projects, the presence of financial constraints, the productivity of public investments, and the relative precision of public and private information, as well as the relative tax rates imposed on risky investments and safe investments, have complex effects on the effectiveness of public investment policy and welfare. In particular, we demonstrate that a public investment policy of a larger size and the availability of more precise information do not necessarily increase welfare.

1. Introduction

In a highly decentralized economy, there are many situations in which coordination problems among agents can be sources of economic inefficiency and instability. A well-known example is the possibility of bank runs resulting from coordination failure among multiple depositors. In their seminal paper, Diamond and Dybvig (1983) show that uncoordinated early withdrawals by depositors who fear other depositors' pre-emptive early withdrawals can lead to a socially inefficient bank run. A large recent literature also argues that serious financial crises, great investment fluctuations, and large booms and bursts of the markets can be caused by coordination problems among self-interested and individually rational agents (e.g., Kiyotaki 1988; Lamont 1995; Obstfeld 1996; Morris and Shin 1998; Cooper and Ross 1999; Cooper 1999; Chui *et al.* 2002; Abreu and Brunnermeier 2003; Oyama 2004).

In general, grave coordination problems are prone to arise when numerous agents are involved and strong strategic complementarities exist among their activities. This fundamental feature of coordination problems implies that potential losses resulting from an incidence of coordination failure can become enormous and wide ranging. In fact, taking the examples of bank runs and financial crises, significantly large social losses have been caused by the occurrence of runs and crises. Many economists have therefore directed attention to how serious coordination problems and associated inefficiency and instability can be avoided in various situations. The introduction of deposit insurance into the banking system is a typical example of such social measures.

The objective of this paper is to contribute to understanding the role and effects of a public investment policy when socially inefficient investments and small consumption can result from coordination problems among multiple agents. To this end, we develop an investment coordination model incorporated into a simple two-period economy ($t = 0, 1$). In our model, individuals born in period 0 live for two periods (period 0 and period 1). Each individual is born with an initial endowment and faces a choice in his or her first period of whether to invest in a risky but potentially more profitable investment project or an alternative with safe but lower returns. The rate of return of the safe investment project is certain and constant, whereas that of the risky investment project depends on the unobservable quality (fundamentals) of the project and the behavior of other agents. In particular, we assume that strategic

3

complementarities exist in the payoffs from the risky investment, such that the profitability of the risky investment project increases with the level of other agents' activities. Because of this strategic complementarity in individuals' investment activities, coordination problems can arise in our model. Each individual chooses between the safe investment project and the risky investment project so as to maxmize the (expected) amount of consumption in old age.

By employing the analytical frameworks developed by Morris and Shin (2004), we first show that the risk of coordination failure among individuals can cause socially inefficient investments and small consumption in equilibrium. In Morris and Shin (2004), they apply the equilibrium selection framework of global games to a creditor coordination game. Global games, pioneered by Carlsson and van Damme (1993) and further extended by Morris and Shin (1998) and others, are incomplete information games where individual players receive noisy private signals about the underlying payoff-relevant state of nature that is assumed unobservable. Typically, coordination games under complete information have multiple equilibria, which makes it difficult to conduct rigorous policy analysis and other comparative statics exercises since the effects of marginal changes in the policy variables or other model parameters on equilibrium outcomes cannot be definitely and meaningfully determined under multiplicity. One of the advantages of using the global games approach is that it may enable us to obtain a unique equilibrium and thus conduct rigid policy analysis under more realistic conditions. In the global games method, the presence of noisy, privately observed signals creates heterogeneity among individuals and, under some conditions, generates uniqueness. In the global games models, if information precision of private signals is sufficiently high, individual agents come to 'coordinate' on the risk-dominant equilibrium which is uniquely selected by the iterated deletion of dominated strategies.

Having shown that socially inefficient investments can arise both in multiple equilibria under complete information and in a uniquely determined equilibrium under incomplete information, we specify a welfare function and analyze the role and effects of a public investment policy in the investment coordination problem. In particular, we consider the case in which the government can influence the unobservable quality (fundamentals) of risky investment projects through its public investment policy. In this analysis, we take into consideration various different conditions through which the government can influence individuals' investment decisions. Firstly, we deal with the case in which investment projects are divisible

and the government can levy taxes on the initial endowments of individuals. In this case, the size of a public investment policy by the government can be exogenously determined. Next, we treat the case of the indivisibility of risky investment projects. In this case, it is suboptimal for the government to levy taxes on the initial endowments of individuals since ex ante taxes on an initial endowment make it impossible for individuals to undertake risky but potentially more profitable investment projects. Therefore, to implement its investment policy, the government needs to raise the necessary funds from another source. We consider the following two cases, one in which the government can borrow from the international financial market and the other in which the government faces financial constraints and cannot borrow. In the former case, the size of borrowing and thus the size of a public investment policy are endogenously determined. A notable point in the latter case is that the government can no longer implement ex ante a public investment policy because of the impossibility of separating the timing of taxes and expenditure. As a result, in this case, it is only the ex post redistribution policy that the government can implement to mitigate inefficiency. We consider that an ex post redistribution policy is implemented so that individuals who chose the risky investment projects and whose projects end in failure can receive some subsidy from the government, and the necessary funds for this redistribution policy are collected from individuals who chose the safe investment projects and obtained constant and certain returns.

Providing various numerical results corresponding to each case above, we show that the divisibility of the investment projects, the presence of financial constraints, the productivity of public investments, and the relative size of public and private information precision, as well as the relative size of tax rates imposed on risky investments and safe investments, complexly affect the effects of a public investment policy and thus welfare. In particular, we demonstrate that a public investment policy of a larger size and the availability of more precise information do not necessarily increase expected welfare. Rather, it is shown that there is the possibility that a public investment policy of a larger size and greater transparency can decrease individuals' consumption and thus welfare.

The rest of the paper is organized as follows. In Section 2, we provide the basic framework of the model. Section 3 analyzes, as the benchmark case, the optimal strategy of individuals under complete information. Section 4 investigates the optimal strategy of individuals under incomplete information and derives a unique Bayesian Nash equilibrium. In Section 5, presenting various numerical results, we

address extensively policy issues and their implications. Section 6 concludes the paper.

2. The model

We consider a two-period economy in which a continuum of risk-neutral individuals indexed by the unit interval $[0, 1]$ lives for two periods ($t = 0, 1$). Individuals are born with initial endowment w and have an opportunity to undertake an investment project in the first period (hereafter called the investment period). Two kinds of investment projects are available for the individuals. One is the safe investment project that yields a certain unit of capital goods R per investment at the end of the investment period. That is, the gross rate of return of the safe investment project is R. The other project is the risky investment project whose gross rate of return R_r is uncertain as of the date of investment and, as specified below, the realized value of the gross rate of return is assumed to depend on the quality (fundamentals) of the project and the behavior of other agents. Individuals rent capital goods produced by their investment to competitive firms in the second period and consume the obtained interest and principal at the end of second period. For simplicity, it is assumed that there is no depreciation and no discounting. Furthermore, individuals are assumed to be technologically and financially constrained. This assumption means that in our model no individual can be a 'large player' who is free from coordination problems. Since individuals are assumed to be risk neutral, they choose between the safe investment project and the risky investment project in a manner that maximizes the (expected) amount of consumption in old age.

In the model, following Morris and Shin (2004), we assume that the gross rate of return of the risky investment project undertaken in period 0 is determined as follows:

$$
R_r(\theta, \lambda) = \begin{cases} R_s & if \ \theta \geq \lambda a, \\ R_f & if \ \theta < \lambda a, \end{cases} \tag{1}
$$

where θ is a randomly determined value of the quality (fundamentals) of the risky investment project undertaken in period 0, λ denotes the proportion of individuals who choose the safe investment project in their investment period 0, $a \ (> 0)$ is a parameter that captures the severity of the coordination problem, and $R_s > R > R_f$ are assumed to be satisfied. In other words, Eq. (1) means that a risky investment

6

project undertaken in period 0 can succeed with a higher return R_s if and only if its quality θ is so large that it can resist the influence of uncoordinated behavior (i.e., the behavior of abstaining from choosing the risky but potentially more profitable investment project), which is captured by the value of λa. Eq. (1) also states that if θ is smaller than λa, then the risky investment project with such low quality will end in failure with a lower return R_f.[1] Therefore, strategic complementarities among individuals' activities exist in our model.

In our model, the value of the quality (fundamentals) of a risky investment project undertaken in period 0, θ, is assumed to be an unobservable random variable for individuals and has a normal prior distribution with mean of $\bar{\theta}$ and variance $1/\gamma$ (i.e., precision γ). Each individual i, however, receives the following two kinds of signals as to the value of θ before their investment decisions in period 0: one is a private signal x_i and the other is a public signal y.

$$x_i \;=\; \theta + \epsilon_i, \quad \epsilon_i \sim N(0, 1/\gamma_x), \tag{2}$$

$$y \;=\; \theta + \xi, \quad\; \xi \sim N(0, 1/\gamma_y), \tag{3}$$

where γ_x and γ_y denote the precision of the private signal and the public signal, respectively. The signal y is public in the sense that it is commonly observable to all individuals. On the other hand, the private signal x_i can differ among individuals. In the following analysis, the noise parameters of the private and public signals are assumed to be independent of each other and of the value of project quality. In addition, the distributional properties of the signals are presumed to be common knowledge among individuals.

There is a continuum of competitive firms and they have an identical linear production function $Y_i = Ak_i$ (i.e., the production function of AK type). Here, Y_i is the output of production of firm i, k_i is the amount of capital employed by firm i, and A denotes productivity (which is equal to the marginal product of capital in this case).[2] The firms can employ capital at the (gross) interest rate r, which is equal

---

[1]Note that Eq. (1) implies that a risky investment project with sufficiently large fundamentals can potentially succeed with a higher return, even in the limit case where $\lambda \to 1$. This appears to be somewhat unrealistic, but we use this specification to avoid a more complicated classification. In fact, as illustrated in the following analysis, such an unrealistic case does not emerge in equilibrium in our model. Note also that we assume that the rate of return of the risky investment projects depends only on the *proportion* of the agents who choose the safe (risky) investment projects.

[2]If we assume the decreasing (concave) production function, we would be able to analyze the situation in which both

to $1 + A$ in equilibrium.

Under these circumstances, each individual with initial endowment w and the signals x_i and y first makes an investment decision in period 0 and then rents capital and consumes the obtained interest and principal at the end of period 1.

3. The optimal investment strategy under complete information

In this section, as a benchmark case, we analyze the optimal investment strategy of the individuals when the realized value of θ is common knowledge. A feature in the case of complete information is that, as in the currency attack model in Obstfeld (1996), self-fulfilling multiple equilibria can arise because of a coordination problem among individuals.

First, we suppose that θ is small such that $\theta < 0$ is satisfied. It is then always optimal for each individual to choose the safe investment project irrespective of other agents' behavior. This is because a risky investment project with such small fundamentals will necessarily fail even if all individuals choose the risky investment in period 0. If so, since $R > R_f$, choosing the safe investment project in period 0 brings higher returns than the risky investment project and thus choosing the safe investment project becomes the optimal strategy for each individual. In contrast, if θ is large enough to satisfy $\theta \geq a$, then choosing the risky investment project becomes the optimal strategy for each individual. This is because a risky investment project with such large fundamentals will succeed even if all other agents choose the safe investment project. As long as the project succeeds, the risky investment project brings higher returns ($R_s > R$). Therefore, choosing the risky investment project becomes the optimal strategy in the case of $\theta \geq a$.

An interesting case is when θ lies in $[0, a)$. In such a medium region of the fundamentals, a coordination problem will arise. For a risky investment project with medium fundamental like these, choosing the risky investment project becomes optimal for an individual only when the proportion of the other individuals who also choose the risky investment project is sufficiently large, such that $\theta \geq \lambda a$ is satisfied.

strategic complementarities and strategic substitutes coexist. For example, we assume that the return form the risky investment is specified as $R_s(\lambda)$ where $R'_s(\lambda) < 0$ and the production function is concave (i.e., the production function with decreasing returns), the marginal product of capital *decreases* with the proportion of the agents who choose the risky investment projects.

Because of this strategic complementarity between individuals' activities, as described in Diamond and Dybvig (1983) and Obstfeld (1996), self-fulfilling multiple equilibria can arise in the case of $\theta \in [0, a)$. In our model, two (Pareto-ranked) pure strategy Nash equilibria can arise: one is the Pareto-superior equilibrium where every individual chooses the more profitable risky investment project and the other is the Pareto-inferior equilibrium where every individual chooses the safe investment project with constant but lower returns. It is important to note that in the latter equilibrium, inefficiently small investment occurs in the sense that socially more profitable projects are not chosen as a result of a coordination failure among individuals.

In the equilibrium where $\lambda = 1$, the amount of aggregated capital is equal to wR and the amount of output produced in period 1 is AwR. On the other hand, in the equilibrium where $\lambda = 0$, the amount of aggregated capital is equal to wR_s and the amount of output produced in period 1 is AwR_s. The amount of consumption of the individuals in period 1 is given by $(1 + A)wR$ in the equilibrium where $\lambda = 1$ and by $(1 + A)wR_s$ in the equilibrium where $\lambda = 0$.

4. Unique equilibrium under incomplete information and the possibility of inefficient investments

In this section, we analyze the optimal investment strategy of individuals under incomplete information and derive a unique Bayesian Nash equilibrium. In the incomplete information game, the fundamentals value is not common knowledge among individuals and they can have different information. Because of this heterogeneity among individuals' information sets, the possibility exists that a uniquely determined equilibrium can be obtained under the incomplete information game. The relative size of the precision between the two signals, i.e., the public signal and the private signal, plays an important role in the possibility of uniqueness.

After receiving the two signals at the beginning of period 0, each individual makes his or her investment decision. Since the public signal y is commonly observed by all individuals, their information sets differ only in terms of private signals. Accordingly, as demonstrated in Heinemann and Illing (2002), Morris and Shin (2004), Corsetti et al. (2004) and others, a strategy for individual i can be given by

a decision rule that maps each realization of his or her private signal x_i to one of two choices, i.e., a risky investment or a safe investment. On the other hand, an equilibrium can be characterized by two critical values: a critical value of θ, θ^*, such that a risky investment project with fundamentals larger than or equal to this value will necessarily succeed, and a critical switching value of x_i, x^*, such that every individual who receives a private signal smaller than or equal to this value will always choose the safe investment project. In what follows, we sketch the derivation of a unique equilibrium characterized by θ^* and x^*, and then show that inefficiently small investments and consumption levels can arise in the uniquely determined equilibrium under incomplete information.

First, we consider the critical value θ^*, given a switching threshold value of the private signal x^*. Suppose now that the true value of the fundamentals is θ and each individual follows a switching strategy around x^*. Then, since the noise terms in private signals are assumed to be independently and identically distributed, the proportion of the individuals who choose the safe investment project in period 0, λ corresponds to the probability that any particular individual receives a private signal smaller than or equal to x^*:

$$\lambda = \Pr(x_i \leq x^* \mid \theta) = \Phi\left(\sqrt{\gamma_x}(x^* - \theta)\right), \tag{4}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function for the standard normal. From Eq. (1), a risky investment project succeeds if and only if its fundamental value satisfies the successful condition $\theta \geq \lambda a$. Thus, given a switching strategy around x^*, the critical value θ^* must satisfy the following critical mass condition:

$$\theta^* = \lambda \cdot a = \Phi\left(\sqrt{\gamma_x}(x^* - \theta^*)\right) \cdot a. \tag{5}$$

Second, we consider the optimal switching strategy of the individuals, given the values of θ^*, x_i, and y. From Eq. (2) and Eq. (3) and given the assumptions about the prior distribution of θ and the distribution of each signal, the posterior distribution of θ for individual i receiving a private signal x_i and a common signal y is given by a normal distribution with the following mean and variance:

$$E(\theta \mid x_i, y) = \frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x_i + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y, \tag{6}$$

$$\text{Var}(\theta \mid x_i, y) = 1/(\gamma + \gamma_x + \gamma_y). \tag{7}$$

10

Therefore, under the given values of θ^*, x_i, and y, the conditional probability of success of the risky investment project for this individual is given by:

$$\Pr(\theta \geq \theta^* \mid x_i, y) = \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x_i + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y - \theta^*\right)\right).$$

(8)

Each individual chooses the risky investment project at the beginning of period 0 only when the expected returns from the risky investment project exceed the certain returns from the safe investment project. Accordingly, given θ^*, the optimal switching threshold value x^* such that individuals receiving a private signal smaller than or equal to this value will always choose the safe investment should satisfy the following optimal cutoff condition:

$$wR_s \cdot \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^* + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y - \theta^*\right)\right)$$
$$+ wR_f \cdot \left(1 - \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^* + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y - \theta^*\right)\right)\right)$$
$$= wR.$$

(9)

We now have a pair of equations (Eq. (5) and Eq. (9)) in terms of θ^* and x^*. By solving this pair of equations, we can obtain the unknown critical values, θ^* and x^*. Solving for θ^*, we have

$$\theta^* = a \cdot \Phi\left(\frac{\gamma + \gamma_y}{\sqrt{\gamma_x}}\left(\theta^* - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y + \frac{\sqrt{\gamma + \gamma_x + \gamma_y}}{\gamma + \gamma_y}\Phi^{-1}\left(\frac{R - R_f}{R_s - R_f}\right)\right)\right).$$

(10)

Since the right-hand side of the above equation is a scaled-up cumulative normal distribution with mean $\frac{\gamma}{\gamma + \gamma_y}\bar{\theta} + \frac{\gamma_y}{\gamma + \gamma_y}y - \frac{\sqrt{\gamma + \gamma_x + \gamma_y}}{\gamma + \gamma_y}\Phi^{-1}\left(\frac{R - R_f}{R_s - R_f}\right)$ and variance $\gamma_x/(\gamma + \gamma_y)^2$, the critical value θ^* is determined at the intersection between this distribution and the 45-degree line. On the other hand, the critical switching point x^* can be obtained by substituting this value into Eq. (9).

Note that, as illustrated in Morris and Shin (2003, 2004), the critical value θ^* is uniquely determined if the scaled-up cumulative normal distribution in Eq. (10) has a slope that is less than one throughout the whole range of possible values. The slope is given by $a\Phi'(\gamma + \gamma_y)/\sqrt{\gamma_x}$ and the maximum value of $\Phi'(\cdot)$, i.e., the density of the standard normal, does not exceed $1/\sqrt{2\pi}$. Therefore, a sufficient condition for uniqueness is given by:

$$\frac{\gamma + \gamma_y}{\sqrt{\gamma_x}} < \frac{\sqrt{2\pi}}{a}.$$

(11)

In other words, the equilibrium derived here is unique if, under the given values of γ, γ_y, and a, the precision of the private signals γ_x is relatively high (or if the precision of the common signal γ_y is relatively low under the given values of γ, γ_x, and a).[3]

In the unique equilibrium derived above, it is optimal for each individual to choose the safe investment project if it receives a private signal that is smaller than or equal to the switching threshold value x^*. On the other hand, under such a switching strategy by individuals, a risky investment project whose fundamentals are lower than the critical value θ^* will always end in failure. Since $0 \leq \Phi(\cdot) \leq 1$ in Eq. (10), the critical value θ^* lies in the interval $[0, a]$. As shown in the previous sections, however, choosing the risky investment project in period 0 whose fundamentals lie in this interval should be socially efficient. Therefore, we can say that the interval $[0, \theta^*]$ represents the occurrence of inefficient small investments in the uniquely determined equilibrium under incomplete information. Needless to say, a rise in the value of θ^* means an increase in the likelihood of small investments in the risky but potentially more profitable investment projects.

Now that we specify the condition for uniqueness, we can examine how the critical value θ^* is influenced by the changes of the model parameters. From the signs of the partial derivatives of θ^* with respect to R_s, R_f, R, $\bar{\theta}$, y, and a, we can show that the probability of inefficient small investments is decreasing in R_s, R_f, $\bar{\theta}$, and y, and is increasing in R and a under the uniqueness condition.[4] In other words, other things being equal, the higher the expected value of the returns from the risky investment, the lower the

---

[3]So far, we have examined how a uniquely determined equilibrium, if any, can be obtained in a game of incomplete information by supposing that individuals follow a switching strategy around a critical value of the private signals, x^*. In fact, as explained in Morris and Shin (2004), Heinemann and Illing (2002), and others, it can be shown that the switching strategy around x^* is the only strategy that survives the iterated deletion of dominated strategies in our model. Hence, there is no loss of generality by restricting our analysis to a switching strategy around x^*. For further details, see Morris and Shin (2003, 2004) and Heinemann and Illing (2002).

[4]For example, the partial derivative of θ^* with respect to R_s is given as $\partial\theta^*/\partial R_s = \left(a\Phi' \cdot \sqrt{\frac{\gamma+\gamma_x+\gamma_y}{\gamma_x}} \frac{\partial\Phi^{-1}\left(\frac{R-R_f}{R_s-R_f}\right)}{\partial R_s} \right) / \left(1 - a\Phi' \cdot \frac{\gamma+\gamma_y}{\sqrt{\gamma_x}} \right)$. The numerator of this expression is negative since $\partial\Phi^{-1}\left(\frac{R-R_f}{R_s-R_f}\right)/\partial R_s < 0$. On the other hand, the denominator is positive if the condition for uniqueness is satisfied. Hence, it can be shown that θ^* is decreasing in R_s. In the same manners, $\partial\theta^*/\partial R_f < 0$, $\partial\theta^*/\partial\bar{\theta} < 0$, $\partial\theta^*/\partial y < 0$, $\partial\theta^*/\partial R > 0$, and $\partial\theta^*/\partial a \geq 0$ can be shown.

likelihood of inefficiently small investments, while the higher returns from the safe investment project increase the possibility of inefficiently small investments. Note also that an increase in a raises the value of θ^*. That is, the greater the degree of disruptive influences among individuals, the higher the likelihood of inefficiently small investments in the potentially more profitable projects.

We finally specify the ex ante expected aggregated capital stock and welfare function under incomplete information. When individuals choose the safe investment project, they can obtain wR at the end of period 0 and thus consume $(1 + A)wR$ at the end of period 1, irrespective of the realization of θ. In the case of the risky investment project, individuals can obtain wR_s at the end of period 0 when their investment projects succeed (i.e., $\theta \geq \theta^*$) and wR_f when their investment projects end in failure (i.e., $\theta < \theta^*$). The proportion of individuals who choose the safe investment project is given by $\Phi\left(\sqrt{\gamma_x}(x^* - \theta)\right)$ when each individual follows the switching strategy around x^* and the fundamentals are θ. Similarly, the proportion of individuals who choose the risky investment project is given by $1 - \Phi\left(\sqrt{\gamma_x}(x^* - \theta)\right) = \Phi\left(\sqrt{\gamma_x}(\theta - x^*)\right)$. Therefore, the ex ante expected aggregated capital stock K and the expected welfare W (evaluated by public information) can be respectively specified as follows:

$$
\begin{aligned}
K = {} & wR_s \int_{\theta^*}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(\theta - x^*\right)\right) \sqrt{\gamma + \gamma_y} \, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y\right)\right) d\theta \\
& + wR_f \int_{-\infty}^{\theta^*} \Phi\left(\sqrt{\gamma_x}\left(\theta - x^*\right)\right) \sqrt{\gamma + \gamma_y} \, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y\right)\right) d\theta \\
& + wR \int_{-\infty}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(x^* - \theta\right)\right) \sqrt{\gamma + \gamma_y} \, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y\right)\right) d\theta, \quad (12)
\end{aligned}
$$

$$
\begin{aligned}
W = {} & (1 + A)wR_s \int_{\theta^*}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(\theta - x^*\right)\right) \sqrt{\gamma + \gamma_y} \, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y\right)\right) d\theta \\
& + (1 + A)wR_f \int_{-\infty}^{\theta^*} \Phi\left(\sqrt{\gamma_x}\left(\theta - x^*\right)\right) \sqrt{\gamma + \gamma_y} \, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y\right)\right) d\theta \\
& + (1 + A)wR \int_{-\infty}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(x^* - \theta\right)\right) \sqrt{\gamma + \gamma_y} \, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y\right)\right) d\theta, \quad (13)
\end{aligned}
$$

where $\phi(\cdot)$ denotes the probability density function for the standard normal. In the limit where $x^* \to \infty$, $\theta^* \to a$ and K and W converge to wR $(< wR_s)$ and $(1 + A)wR$ $(< (1 + A)wR_s)$, respectively.

Under the parameter values listed in Table 1, we calculate the expected welfare for different values of y. The expected welfare when $y = -1.5$, $y = 0.1$ and $y = 1.5$ is 40.0000, 40.0006, 40.7948,

respectively.[5]

<< **Table 1 goes about here.** >>

5. The effects of public investment policy

In the previous sections, we have shown that inefficiently small investments and consumption levels can arise in both multiple equilibria under complete information and a unique equilibrium under incomplete information as a result of the rational behavior of individuals. In this section, we address the policy issues and analyze how the government can mitigate inefficiency arising from coordination failures among individuals.

As the government policy that would help mitigate inefficiency, we can consider several types of policy in our framework. For example, the government may be able to increase the fundamentals of the risky investment projects through its public investment or reduce the severity of the coordination problem among individuals (which is captured by the parameter a in our model). In what follows, we will focus on the effects of a public investment policy where the government's ex ante investment expenditure can influence the fundamentals of the risky investment projects θ.

We suppose the situation where public investment expenditure g at the beginning of period 0 (before individuals make their investment decisions but after the two signals are realized) can increase the fundamentals of the risky investment projects from θ to $\theta + h(g)$. The function $h(g)$ specifies the productivity of a public investment policy and is assumed to satisfy the conditions $h'(\cdot) > 0$ and $h''(\cdot) < 0$.[6] Then, the important problem for the government is how to finance its investment expenditure. In the following analysis, we consider three cases. The first is the case in which investment projects are divisible and the government can levy a tax on individuals' initial endowments. In this case, there is no financial constraint for the government. The second is the case in which risky investment projects are indivisible but the gov-

---

[5]These results mean that almost all individual agents choose the safe investment projects and the coordination problem is severe.

[6]When we consider the situation where the government investment expenditure can reduce the severity of the coordination problem among individuals, we can specify this alternative situation by assuming that public investment expenditure g at the beginning of period 0 can decrease the severity of the coordination problem from a to $a/(1 + h(g))$.

ernment can raise the necessary funds in the international financial markets at the (gross) interest rate r per period. The third is the case in which risky investment projects are indivisible and the government cannot borrow in the international financial markets. In this third case, because of the impossibility of separation of the timing of taxes and expenditure, the feasible public policy for the government is only an ex post policy such as an ex post redistribution policy. A crucial difference among these three cases is that while the size of the public investment policy in the first case is determined exogenously, the feasible size of the public investment policy and the size of the ex post redistribution policy in the second and third cases are determined endogenously.

5.1. The effects of a public investment policy under divisibility

We first investigate the effects of a public investment policy when the investment projects are divisible. In this case, the government can levy a tax on individuals' initial endowments to finance the necessary funds for its investment expenditure. Letting $\tau \in [0, \bar{\tau}]$ denote the tax rate imposed on individuals' initial endowments, the size of the public investment g is given by τw and the feasible investment size of individuals reduces to $(1 - \tau)w$. Following this change, the optimal cutoff condition (Eq. (9)) and the threshold value of θ, θ^*, are also modified as follows[7]:

$$
(1 - \tau)wR_s \cdot \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}\hat{x}^* + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y + h(g) - \hat{\theta}^*\right)\right)
$$
$$
+ (1 - \tau)wR_f \cdot \left(1 - \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}\hat{x}^* + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y + h(g) - \hat{\theta}^*\right)\right)\right)
$$
$$
= (1 - \tau)wR, \tag{14}
$$

$$
\hat{\theta}^* = a \cdot \Phi\left(\frac{\gamma + \gamma_y}{\sqrt{\gamma_x}}\left(\hat{\theta}^* - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y - \frac{\gamma + \gamma_x + \gamma_y}{\gamma + \gamma_y}h(g) + \frac{\sqrt{\gamma + \gamma_x + \gamma_y}}{\gamma + \gamma_y}\Phi^{-1}\left(\frac{R - R_f}{R_s - R_f}\right)\right)\right), \tag{15}
$$

where $\hat{\theta}^*$ and $\hat{x}^*$ are respectively the threshold value of θ and the optimal switching threshold of x_i under divisibility of investment projects.

---

[7]We assume that the tax rate imposed on individuals who choose the risky investment project and on individuals who choose the safe investment project is the same. This assumption is reasonable since if the tax rates are different among them, each individual has an incentive to make a mendacious report about his or her choice to avoid a higher tax rate.

On the other hand, the ex ante expected aggregated capital stock $\hat{K}$ and the expected welfare $\hat{W}$ under divisibility are given by:

$$\hat{K} = (1-\tau)wR_s \int_{\hat{\theta}^*}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(\theta - \hat{x}^*\right)\right) \sqrt{\gamma + \gamma_y}\, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y - h(g)\right)\right) d\theta$$

$$+ (1-\tau)wR_f \int_{-\infty}^{\hat{\theta}^*} \Phi\left(\sqrt{\gamma_x}\left(\theta - \hat{x}^*\right)\right) \sqrt{\gamma + \gamma_y}\, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y - h(g)\right)\right) d\theta$$

$$+ (1-\tau)wR \int_{-\infty}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(\hat{x}^* - \theta\right)\right) \sqrt{\gamma + \gamma_y}\, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y - h(g)\right)\right) d\theta,$$

(16)

$$\hat{W} = (1-\tau)(1+A)wR_s \int_{\hat{\theta}^*}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(\theta - \hat{x}^*\right)\right) \sqrt{\gamma + \gamma_y}\, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y - h(g)\right)\right) d\theta$$

$$+ (1-\tau)(1+A)wR_f \int_{-\infty}^{\hat{\theta}^*} \Phi\left(\sqrt{\gamma_x}\left(\theta - \hat{x}^*\right)\right) \sqrt{\gamma + \gamma_y}\, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y - h(g)\right)\right) d\theta$$

$$+ (1-\tau)(1+A)wR \int_{-\infty}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(\hat{x}^* - \theta\right)\right) \sqrt{\gamma + \gamma_y}\, \phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y - h(g)\right)\right) d\theta.$$

(17)

In Figure 1, we calculate the expected welfare $\hat{W}$ in the case of $h(g) = d\sqrt{g}$ for different values of τ, y and d, where d is the parameter that specifies the productivity of a public investment policy. When the productivity of a public investment policy is medium size ($d = 0.1$), public investment increases welfare only when the public signal y is relatively large ($y = 1.5$) (see Fig.1(a)). In fact, the expected welfare monotonically decreases with τ when the public signal is not so large ($y = -1.5$ or $y = 0.1$), while it nonmonotonically increases with τ when the public signal is relatively large. This is because when the public signal y is small, the coordination problem is so severe (i.e., $\hat{\theta}^*$ is so high) that a public investment policy that is not so productive cannot serve as an effective coordinator. A notable point is that even if the public signal y is large, a public investment of a larger size does not necessarily increase the expected welfare; in other words, there is an optimal tax rate that maximizes expected welfare. The optimal tax rate when $y = 1.5$ and $d = 0.1$ is 0.16 (i.e., $g = 1.6$) and the expected welfare at this tax rate is about 53.66. For tax rates larger than this optimal value, a public investment of a larger size decreases welfare. This nonmonotonicity stems from the nonlinearity of the effect of public investment in our model. Since the productivity function of a public investment $h(\cdot)$ has decreasing returns ($h''(\cdot) < 0$) and $\hat{\theta}^*$, $\hat{x}^*$, and

the conditional probability of success (failure) of the risky investment projects nonlinearly change with $h(\cdot)$, a public investment policy affects the expected welfare in a nonmonotonic manner.

When the productivity of a public investment policy is high ($d = 0.4$), public investment can increase welfare even when the public signal y is not so large ($y = 0.1$) (Fig.1(b)). The optimal tax rate when $y = 0.1$ is 0.08 (i.e., $g = 0.8$) and the expected welfare at this tax rate is about 56.12. When y is large ($y = 1.5$), expected welfare sharply increases and reaches its maximum level 71.56 when $\tau = 0.04$ ($g = 0.4$). On the other hand, when the productivity of a public investment policy is low ($d = 0.01$), the public investment policy decreases welfare irrespective of its size (Fig.1(c)). Therefore, the optimal tax rate in this case is zero (i.e., no public investment policy).[8]

$<<$ **Figure 1 goes about here.** $>>$

5.2. The effects of a public investment policy under indivisibility (the case of no financial constraints)

We next investigate the effects of a public investment policy when the risky investment projects are indivisible but the government faces no financial constraints. If the investment projects are indivisible, it is suboptimal for the government to levy taxes on the initial endowments of individuals since such ex ante taxes on initial endowments make it impossible for individuals to undertake risky but potentially more profitable investment projects. Therefore, to implement its investment policy, the government needs to raise the necessary funds from another source. We suppose that the government can issue debt in the international financial markets at the (gross) interest rate r.[9]

Letting $\tau_r \in [0, \bar{\tau}_r]$ and $\tau_s \in [0, \bar{\tau}_s]$ be the tax rates imposed on the return of the risky investment projects when they succeed and the return of the safe investment projects, respectively, the optimal cutoff

---

[8]When the productivity of a public investment policy is very high, public investment can increase welfare even when the public signal is small ($y = -1.5$). For example, when d is 0.9, the expected welfare nonmonotonically increases and reaches its maximum level 44.06 when $\tau = 0.28$ ($g = 2.8$).

[9]We assume that individuals do not purchase government debt. In other words, we continue to assume that individuals can invest their initial endowments only in the risky investment project or the safe investment project. This assumption for simplicity would be reasonable in our model since the return from the government bond is weakly dominated by the returns of the risky investment projects or the safe investment projects.

condition and the threshold value of θ, θ^*, are modified as follows:

$$(1-\tau_r)wR_s \cdot \Phi\left(\sqrt{\gamma+\gamma_x+\gamma_y}\left(\frac{\gamma}{\gamma+\gamma_x+\gamma_y}\bar{\theta}+\frac{\gamma_x}{\gamma+\gamma_x+\gamma_y}\hat{\hat{x}}^*+\frac{\gamma_y}{\gamma+\gamma_x+\gamma_y}y+h(g)-\hat{\hat{\theta}}^*\right)\right)$$

$$+wR_f\cdot\left(1-\Phi\left(\sqrt{\gamma+\gamma_x+\gamma_y}\left(\frac{\gamma}{\gamma+\gamma_x+\gamma_y}\bar{\theta}+\frac{\gamma_x}{\gamma+\gamma_x+\gamma_y}\hat{\hat{x}}^*+\frac{\gamma_y}{\gamma+\gamma_x+\gamma_y}y+h(g)-\hat{\hat{\theta}}^*\right)\right)\right)$$

$$=(1-\tau_s)wR, \tag{18}$$

$$\hat{\hat{\theta}}^* = a\cdot\Phi\left(\frac{\gamma+\gamma_y}{\sqrt{\gamma_x}}\left(\hat{\hat{\theta}}^*-\frac{\gamma}{\gamma+\gamma_y}\bar{\theta}-\frac{\gamma_y}{\gamma+\gamma_y}y-\frac{\gamma+\gamma_x+\gamma_y}{\gamma+\gamma_y}h(g)+\frac{\sqrt{\gamma+\gamma_x+\gamma_y}}{\gamma+\gamma_y}\Phi^{-1}\left(\frac{(1-\tau_s)R-R_f}{(1-\tau_r)R_s-R_f}\right)\right)\right), \tag{19}$$

where $\hat{\hat{\theta}}^*$ and $\hat{\hat{x}}^*$ are respectively the threshold value of θ and the optimal switching threshold of x_i under indivisibility of the risky investment projects and no financial constraints. It should be noted that we assume that the tax rates on the risky investment projects and the safe investment projects can be different and there is no tax on the returns from the risky investment projects when they fail. On the other hand, the ex ante expected aggregated capital stock $\hat{\hat{K}}$ and the expected welfare $\hat{\hat{W}}$ under indivisibility and no financial constraints are given by:

$$\hat{\hat{K}} = (1-\tau_r)wR_s\int_{\hat{\hat{\theta}}^*}^{\infty}\Phi\left(\sqrt{\gamma_x}\left(\theta-\hat{\hat{x}}^*\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta-\frac{\gamma}{\gamma+\gamma_y}\bar{\theta}-\frac{\gamma_y}{\gamma+\gamma_y}y-h(g)\right)\right)d\theta$$

$$+wR_f\int_{-\infty}^{\hat{\hat{\theta}}^*}\Phi\left(\sqrt{\gamma_x}\left(\theta-\hat{\hat{x}}^*\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta-\frac{\gamma}{\gamma+\gamma_y}\bar{\theta}-\frac{\gamma_y}{\gamma+\gamma_y}y-h(g)\right)\right)d\theta$$

$$+(1-\tau_s)wR\int_{-\infty}^{\infty}\Phi\left(\sqrt{\gamma_x}\left(\hat{\hat{x}}^*-\theta\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta-\frac{\gamma}{\gamma+\gamma_y}\bar{\theta}-\frac{\gamma_y}{\gamma+\gamma_y}y-h(g)\right)\right)d\theta, \tag{20}$$

$$\hat{\hat{W}} = (1-\tau_r)(1+A)wR_s\int_{\hat{\hat{\theta}}^*}^{\infty}\Phi\left(\sqrt{\gamma_x}\left(\theta-\hat{\hat{x}}^*\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta-\frac{\gamma}{\gamma+\gamma_y}\bar{\theta}-\frac{\gamma_y}{\gamma+\gamma_y}y-h(g)\right)\right)d\theta$$

$$+(1+A)wR_f\int_{-\infty}^{\hat{\hat{\theta}}^*}\Phi\left(\sqrt{\gamma_x}\left(\theta-\hat{\hat{x}}^*\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta-\frac{\gamma}{\gamma+\gamma_y}\bar{\theta}-\frac{\gamma_y}{\gamma+\gamma_y}y-h(g)\right)\right)d\theta$$

$$+(1-\tau_s)(1+A)wR\int_{-\infty}^{\infty}\Phi\left(\sqrt{\gamma_x}\left(\hat{\hat{x}}^*-\theta_t\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta-\frac{\gamma}{\gamma+\gamma_y}\bar{\theta}-\frac{\gamma_y}{\gamma+\gamma_y}y-h(g)\right)\right)d\theta, \tag{21}$$

where g must satisfy the following feasibility condition:

$$g \leq \frac{1}{r} \left(\tau_r w R_s \int_{\hat{\hat{\theta}}^*}^{\infty} \Phi \left(\sqrt{\gamma_x} \left(\theta - \hat{\hat{x}}^* \right) \right) \sqrt{\gamma + \gamma_y} \, \phi \left(\sqrt{\gamma + \gamma_y} \left(\theta - \frac{\gamma}{\gamma + \gamma_y} \bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y} y - h(g) \right) \right) d\theta \right.$$

$$\left. + \tau_s w R \int_{-\infty}^{\infty} \Phi \left(\sqrt{\gamma_x} \left(\hat{\hat{x}}^* - \theta \right) \right) \sqrt{\gamma + \gamma_y} \, \phi \left(\sqrt{\gamma + \gamma_y} \left(\theta - \frac{\gamma}{\gamma + \gamma_y} \bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y} y - h(g) \right) \right) d\theta \right).$$

$$(22)$$

Eq. (22) states that although the public investment expenditure g is a given variable for individuals when making their investment decisions, it must be such that it does not exceed the expected present value of tax revenues collected at the end of period 0.[10] Since it is inefficient and suboptimal for the government to levy superfluous taxes, we suppose that the condition Eq. (22) is satisfied in equality.

In Table 2, we calculates expected welfare $\hat{\hat{W}}$ when $h(g) = d\sqrt{g}$ and τ_r and τ_s are optimally chosen so as to maximize expected welfare. When the public signal y is very low ($y = -1.5$), the optimal tax rates on the risky investment and the safe investment are zero and no public investment policy is optimal, as in the case of divisibility. This is because for such a low public signal, pessimism among individuals is so deep that the government cannot serve as an effective coordinator. On the other hand, when the public signal is not too low ($y = 0.1$ or $y = 1.5$), the optimal tax rates are positive and the public investment policy serves as an effective confidence builder (at least when the productivity of the public investment is not so small). In fact, when $y = 0.1$, the optimal tax rates on the safe investment project τ_s^* are 0.4 (which is equal to the upper limit of the imposable tax rate) and 0.16 and the optimal tax rates on the risky investment project τ_r^* are 0.00 and 0.08 when $d = 0.1$ and $d = 0.4$, respectively. Similarly, when $y = 1.5$, the optimal tax rates on the safe investment project τ_s^* are 0.4, 0.4, and 0.32 and the optimal tax rates on the risky investment project τ_r^* are 0.00, 0.00, and 0.04 when $d = 0.01$, $d = 0.1$, and $d = 0.4$, respectively.

As compared with the case of divisibility, the expected welfare $\hat{\hat{W}}$ is larger than $\hat{W}$ in most cases

---

[10]If we assume that the government repays at the end of period 1, the feasibility condition becomes $g \leq \frac{1}{r^2} \left(\tau_r (1 + A) w R_s \int_{\hat{\theta}^*}^{\infty} \Phi \left(\sqrt{\gamma_x} \left(\theta - \hat{\hat{x}}^* \right) \right) \sqrt{\gamma + \gamma_y} \phi \left(\sqrt{\gamma + \gamma_y} \left(\theta - \frac{\gamma}{\gamma + \gamma_y} \bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y} y - h(g) \right) \right) d\theta + \tau_s (1 + A) w R \int_{-\infty}^{\infty} \Phi \left(\sqrt{\gamma_x} \left(\hat{\hat{x}}^* - \theta \right) \right) \sqrt{\gamma + \gamma_y} \phi \left(\sqrt{\gamma + \gamma_y} \left(\theta - \frac{\gamma}{\gamma + \gamma_y} \bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y} y - h(g) \right) \right) d\theta \right)$, which is fundamentally equivalent to Eq.(4-22) in our model. That is, as long as $r = (1 + A)$, repayment at the end of period 0 and repayment at the end of period 1 are indifferent for the government. If $r < (1 + A)$, it would be optimal for the government to repay at the end of period 1.

(except for the cases where d and y are sufficiently small), while the size of the public investment policy under indivisibility and no financial constraints is larger (smaller) than that under divisibility when $d = 0.01$, $y = 1.5$, when $d = 0.1$, $y = 0.1$, when $d = 0.4$, $y = 0.1$, and when $d = 0.4$, $y = 1.5$ (when $d = 0.1$, $y = 1.5$). In other words, the results here demonstrate that the ex ante tax on the initial endowment under divisibility is not necessarily optimal and the availability of access to financial markets is beneficial.[11]

<< **Table 2 goes about here.**>>

5.3. Information precision and welfare

In this subsection, we analyze how the information precision of the two signals influences the effects of public investment policy and welfare. In general, for a decision maker facing a choice under uncertainty, the availability of more precise information is considered beneficial, leading to better decision making. In our framework, the problem is whether indeed an increase in transparency increases individuals' welfare. We first investigate how different values of the information precision of the public signal γ_y affect expected welfare $\hat{\hat{W}}$, given the information precision of the private signals γ_x.

Figure 2 plots the expected welfare $\hat{\hat{W}}$ for different values of γ_y (from 0.0 to 2.0), y, and d. When the productivity of a public investment policy is medium and small size ($d = 0.1$ and $d = 0.01$), more precise public information increases the expected welfare only when the public signal is relatively high ($y = 1.5$) (see Fig.2(a) and Fig.2(c)). An intuitive explanation of this result is as follows. As known from Eq. (6), when the relative precision of the public signal against that of the private signals increases, individuals come to attach more weight to the information content of the former in calculating the expected value of the unknown fundamentals. Accordingly, other things being equal, if the information content conveyed

---

[11]When the productuvuty of a public investment policy is very high, as in the case of disivility in the previous subsection, we confirm that public investment can increase welfare even when the public signal is small ($y = -1.5$). In fact, when d is 0.9 and $y = -1.5$, the expected welfare $\hat{\hat{W}}$ is 52.4847 under the values of $\tau_s^* = 0.4$, $\tau_r^* = 0.24$, and $g = 4.00$ (although we receive warnings of getting the "Inf" or "Nan" values in the calculation process). From the results here, we can state that (1) the repayments of public investment policy (if any) are basically and mainly made by the tax revenues from the safe investment projects and (2) as the productivity of public investment becomes higher, the tax on the risky investment projects is also positive.

by precise public information is favorable (unfavorable), then the expected value of the fundamentals of the risky investment projects becomes relatively high (low) for individuals. But, as inferred from Eqs (18) and (5), this high (low) expected value of the project fundamentals in turn leads to relatively low (high) values of $\hat{\hat{x}}^*$ and $\hat{\hat{\theta}}^*$. Furthermore, more directly, since expected welfare is evaluated by public information, high (low) values of y under relatively large values of γ_y increase (decrease) expected welfare. Consequently, more precise public information increases (decreases) the expected welfare when the public signal y is relatively high (low). In fact, the expected welfare in both cases ($d = 0.1$ and $d = 0.01$) when $y = 1.5$ monotonically increases with γ_y and reaches its maximum level (about 75.0). In contrast, in these cases, when the public signal is not high ($y = 0.1$ and $y = -1.5$), the expected welfare gradually and sharply decreases and approaches its minimum level (about 40.00) (Fig.2(a) and Fig.(c)).[12]

On the other hand, when the productivity of a public investment policy is large ($d = 0.4$), more precise public information slightly increases the expected welfare even when the public signal is not so high ($y = 0.1$),[13] although the expected welfare sharply decreases with γ_y when the public signal is low ($y = -1.5$) as in the cases of $d = 0.01$ and $d = 0.1$ (Fig.2(b)). Therefore, it can be concluded that more precise public information is beneficial, particularly when the public signal and the productivity of the public investment policy are relatively high.[14]

<< **Figure 2 goes about here.**>>

We next investigate how different values of the information precision of private signals γ_x affect the expected welfare $\hat{\hat{W}}$, given the information precision of the public signal γ_y. Figure 3 plots the

---

[12]In the currency crises model, Metz (2002) theoretically analyzes how an increase in the relative precision of public information affects the threshold value of the fundamentals at which a currency crisis can be caused.

[13]As γ_y becomes larger, the conditional distribution of θ becomes more thin-tailed. Therefore, when the productivity of a public investment policy d is large and thus $\hat{\hat{\theta}}$ is relatively low, more precise public information can increase the conditional probability of success of the risky investment projects, other things being equal. As a result, the expected welfare slightly increases when the productivity of a public investment policy is relatively large.

[14]From the results here, the government may have an incentive (if possible) to announce the large values of y and γ_y as much as possible. But this government's incentive may be penetrated by the individual agents and thus the government's initial intention would end in failure.

expected welfare $\hat{\hat{W}}$ for different values of γ_x (from 10.0 to 40.0), y, and d. When the productivity of a public investment policy is medium and large size ($d = 0.1$ and $d = 0.4$), the expected welfare lines are almost flat for high and low values of the public signal (i.e., $y = -1.5$ and $y = 1.5$), although the levels are not necessarily the same between them (Fig.3(a) and Fig.3(b)). In contrast, for these values of the productivity of a public investment policy, the expected welfare gradually increases with γ_x when $y = 0.1$. As the relative precision of the private signals against that of the public signal increases, individuals come to attach less weight to the information content of the public signal in calculating the expected value of unknown fundamentals. Accordingly, when the information content of the public signal is not so favorable, an increase in the precision of the private signals can decrease the optimal switching threshold $\hat{\hat{x}}^*$ in Eq. (18) and thus $\hat{\hat{\theta}}^*$ so as to satisfy the critical mass condition. As a result, when these changes are significant, increasing the relative precision of the private signals increases the expected welfare. When the information content of the public signal is very bad ($y = -1.5$), such a mechanism does not work as well and the expected welfare lines are almost flat at about 40.0.[15] On the other hand, when the productivity of a public investment policy is low ($d = 0.01$), the expected welfare does not significantly change with the increase of γ_x (Fig.3(c)). Rather, when the public signal is high ($y = 1.5$), the expected welfare slightly decreases with γ_x.

<< **Figure 3 goes about here.**>>

5.4. Financial constraints and the effects of an ex post redistribution policy

We analyze finally the case in which risky investment projects are indivisible and the government faces financial constraints. Under indivisibility of investment projects and financial constraints, the government can no longer implement any ex ante public investment policy because of the impossibility of separating the timing of taxes and expenditure. In fact, in our model, the only implementable public policy of the government is an ex post policy, such as an ex post redistribution policy. Therefore, in what follows, we investigate the effects of the ex post redistribution policy when the government levies a tax on incomes

---

[15]For sufficiently large values of γ_x, however, the expected welfare can increase with γ_x even when the public signal is very low. In fact, we confirm that the expected welfare gradually increases for sufficiently large values of γ_x when the productivity of a public investment policy is large ($d = 0.4$).

of individuals who chose the safe investment project in period 0 and redistributes tax revenues among individuals in period 1 who chose the risky investment project in period 0 and whose projects ended in failure.

Letting $\tau_s^{fc} \in [0, \bar{\tau}_s^{fc}]$ and $D_1(< (1+A)w(R - R_f))$ denote respectively the tax rate on incomes of the individuals who chose the safe investment project in period 0 and the amount of per capita redistribution in period 1 for the individuals who chose the risky investment projects in period 0 and whose projects ended in failure, D_1 must satisfy the feasibility constraints $(1 - \lambda^{fc})D_1 \leq \lambda^{fc}\tau_s^{fc}(1+A)wR$, where λ^{fc} is the proportion of individuals who choose the safe investment projects under the ex post redistribution policy. More specifically, we assume that D_1 can be specified as follows:

$$D_1 = \min\left(\frac{\lambda(x^{*fc})}{1 - \lambda(x^{*fc})}\tau_s^{fc}(1+A)wR, \ \bar{D}\right), \tag{23}$$

where x^{*fc} is the optimal switching threshold of x_i under indivisibility and financial constraints. In other words, we assume that there is an upper limit $\bar{D}$ for the amount of per capita ex post redistribution in period 1 and the feasibility constraint is satisfied in equality when the amount of per capita redistribution is smaller than the upper limit $\bar{D}$.

Under this ex post redistribution policy, the optimal cutoff condition is given by:

$$
\begin{aligned}
&(1+A)wR_s \cdot \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^{*fc} + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y - \theta^{*fc}\right)\right) \\
&+ \int_{-\infty}^{\theta^{*fc}}((1+A)wR_f + D_1)\sqrt{\gamma + \gamma_x + \gamma_y}\,\phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} - \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^{*fc}\right.\right. \\
&\left.\left. - \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y\right)\right)d\theta = (1+A)wR \cdot \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^{*fc} + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y - \theta^{*fc}\right)\right) \\
&+ (1 - \tau_s^{fc})(1+A)wR \cdot \left(1 - \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^{*fc} + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y - \theta^{*fc}\right)\right)\right),
\end{aligned}
\tag{24}
$$

where θ^{*fc} is the threshold value of θ under indivisibility of investment projects and financial constraints and $x^{*fc} = \Phi^{-1}\left(\frac{\theta^{*fc}}{a}\right)\frac{1}{\sqrt{\gamma_x}} + \theta^{*fc}$. We now consider a single individual who uses a switching strategy around x^{fc}, while all other agents use a switching strategy around $\tilde{x}^{fc}$. Furthermore, we define the net

expected utility $u(x^{fc}, \tilde{x}^{fc})$ as follows:

$$u(x^{fc}, \tilde{x}^{fc}) = (1+A)wR_s \cdot \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^{fc} + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y - \theta^{fc}\right)\right)$$

$$+ \int_{-\infty}^{\theta^{fc}} ((1+A)wR_f + D_1)\sqrt{\gamma + \gamma_x + \gamma_y}\,\phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} - \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^{fc}\right.\right.$$

$$\left.\left. - \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y\right)\right)d\theta - (1+A)wR \cdot \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^{fc} + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y - \theta^{fc}\right)\right)$$

$$- (1 - \tau_s^{fc})(1+A)wR \cdot \left(1 - \Phi\left(\sqrt{\gamma + \gamma_x + \gamma_y}\left(\frac{\gamma}{\gamma + \gamma_x + \gamma_y}\bar{\theta} + \frac{\gamma_x}{\gamma + \gamma_x + \gamma_y}x^{fc} + \frac{\gamma_y}{\gamma + \gamma_x + \gamma_y}y - \theta^{fc}\right)\right)\right),$$

(25)

where θ^{fc} is the failure point defined as the solution to the equation (the critical mass condition) $\theta^{fc} = a\Phi\left(\sqrt{\gamma_x}\left(\tilde{x}^{fc} - \theta^{fc}\right)\right)$. Then, as explained in Morris and Shin (2004), if $u(x^{fc}, \tilde{x}^{fc})$ satisfies the following three properties, there exists a unique switching strategy that survives the iterated deletion of dominated strategies. (1) Continuity: u is continuous with respect to x^{fc}. (2) Full Range: For any $\tilde{x}^{fc} \in \mathbb{R} \cup \{-\infty, \infty\}$, $u(x^{fc}, \tilde{x}^{fc}) < 0$ for $x^{fc} \to -\infty$ and $u(x^{fc}, \tilde{x}^{fc}) > 0$ for $x^{fc} \to \infty$. (3) Monotonicity: u is strictly increasing in its first argument, and is strictly decreasing in its second argument.

The first two properties are satisfied in our model for not so large values of D_1. On the other hand, whether the third property (monotonicity) is satisfied is not necessarily obvious since while the increase of $\tilde{x}^{fc}$ decreases the probability of success of the risky investment projects, it increases the amount of ex post redistribution when the risky investment projects end in failure. In other words, for monotonicity to be satisfied, the amount of ex post redistribution must be relatively small. We have numerically confirmed that for relatively small values of $\bar{D}$, monotonicity is satisfied and a unique pair of x^{*fc} and θ^{*fc} satisfying the critical mass condition and the optimal cutoff condition simultaneously can be obtained. Under a unique pair of x^{*fc} and θ^{*fc}, the ex ante expected aggregated capital stock and the expected welfare under indivisibility and financial constraints are given by:

$$K^{fc} = wR_s \int_{\theta^{*fc}}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(\theta - x^{*fc}\right)\right)\sqrt{\gamma + \gamma_y}\,\phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y\right)\right)d\theta$$

$$+ wR_f \int_{-\infty}^{\theta^{*fc}} \Phi\left(\sqrt{\gamma_x}\left(\theta - x^{*fc}\right)\right)\sqrt{\gamma + \gamma_y}\,\phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y\right)\right)d\theta$$

$$+ wR \int_{-\infty}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(x^{*fc} - \theta\right)\right)\sqrt{\gamma + \gamma_y}\,\phi\left(\sqrt{\gamma + \gamma_y}\left(\theta - \frac{\gamma}{\gamma + \gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma + \gamma_y}y\right)\right)d\theta,$$

(26)

$$W^{fc} = (1+A)wR_s \int_{\theta^{*fc}}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(\theta - x^{*fc}\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta - \frac{\gamma}{\gamma+\gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma+\gamma_y}y\right)\right)d\theta$$

$$+ \int_{-\infty}^{\theta^{*fc}}\left((1+A)wR_f + D_1\right)\Phi\left(\sqrt{\gamma_x}\left(\theta - x^{*fc}\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta - \frac{\gamma}{\gamma+\gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma+\gamma_y}y\right)\right)d\theta$$

$$+ (1+A)wR \int_{\theta^{*fc}}^{\infty} \Phi\left(\sqrt{\gamma_x}\left(x^{*fc} - \theta\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta - \frac{\gamma}{\gamma+\gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma+\gamma_y}y\right)\right)d\theta$$

$$+ (1-\tau_s^{fc})(1+A)wR \int_{-\infty}^{\theta^{*fc}} \Phi\left(\sqrt{\gamma_x}\left(x^{*fc} - \theta\right)\right)\sqrt{\gamma+\gamma_y}\,\phi\left(\sqrt{\gamma+\gamma_y}\left(\theta - \frac{\gamma}{\gamma+\gamma_y}\bar{\theta} - \frac{\gamma_y}{\gamma+\gamma_y}y\right)\right)d\theta.$$

$$(27)$$

In Figure 4, we calculate W^{fc} for different values of τ_s^{fc}, y, and $\bar{D}$ (2.0, 6.0, 15.0). When the public signal y is relatively high ($y = 1.5$), an ex post redistribution policy increases the expected welfare irrespective of the value of $\bar{D}$. Considering that the proportion of individuals who choose the risky investment projects $(1 - \lambda)$ is relatively large when the public signal is high, this result implies that even an ex post redistribution policy of a small amount of per capita redistribution is effective when y is high. In contrast, when the public signal y is medium size ($y = 0.1$), the expected welfare nonmonotonically changes with τ_s^{fc} when $\bar{D}$ is high (15.0) (Fig.4(b)) or medium (6.0) (Fig.4(a)), while it monotonically decreases with τ_s^{fc} when $\bar{D}$ is small (2.0) (Fig.4(c)). In particular, when $\bar{D}$ is large, the expected welfare decreases for small values of τ_s^{fc} but it ultimately increases for large values of τ_s^{fc} and the optimal tax rate is its upper limit ($\bar{\tau}_s^{fc} = 0.4$) (Fig.4(b)). Therefore, it can be concluded that a large size of an ex post redistribution policy is effective when the public signal y is medium size ($y = 0.1$). On the other hand, when the public signal is low ($y = -1.5$), an ex post redistribution policy decreases the expected welfare irrespective of the value of $\bar{D}$. For such a low value of the public signal, an ex post redistribution policy cannot become an effective coordination device even if the amount of per capita redistribution is relatively large.

<< **Figure 4 goes about here.**>>

6. Conclusion

In a highly decentralized economy, strategic complementarities among individuals' activities can be potential sources of economic inefficiency and instability. In this paper, we analyzed the role and effects

of public investment policy when inefficiently small investments and consumption levels can arise as a result of a coordination failure among individuals. In particular, we investigate how divisibility of investment projects, the presence of financial constraints, and the relative size of public and private information precision affect the investment decisions of individuals and the effects of public investment policy. Having shown that socially inefficient investment can arise in both multiple equilibria under complete information and a uniquely determined equilibrium under incomplete information, we specified a welfare function and analyzed the effects of a public investment policy that is considered to help mitigate inefficiency. Our main results are:

- An ex ante tax on the initial endowment of individuals under divisibility is not necessarily optimal and the accessibility of financial markets helps a public investment policy for the effective mitigation of the coordination problems facing individuals.

- More precise information does not necessarily increase welfare. In particular, more precise public information can decrease welfare when the information content of the public signal is relatively low.

- An ex post redistribution policy under financial constraints is effective for mitigating coordination problems, particularly when information content of the public signal is relatively high and the size of the per capita redistribution is sufficiently large when the information content of the public signal is not so good.

Finally, we suggest some possible directions for future research. First, in this paper, we have assumed that individuals are risk neutral and obtain no utility from consumption in their investment period. Modifying these assumptions and addressing the optimal portfolio selection problem is a possible and important issue for future research. Next, in this paper, we have assumed symmetric individuals for analyzing their investment coordination problem. A recent paper by Corsetti et al. (2004) analyzes the coordination game when agents are asymmetric and there is a large player. Extending our analysis to include the case of asymmetric individuals is also an important topic for future research. Furthermore, in this paper, we did not explicitly analyze the 'game' between the government and individuals. A recent

paper by Angeletos et al. (2006) analyzes the signaling effect of government intervention in a global game model in which the type of government, which is unobservable for agents, is revealed through the policymaker's strategic decision. On the other hand, Morris and Shin (2006) and Corsetti et al. (2006) analyze the problem of potential trade-offs between public intervention and moral hazard of agents and derive some important policy implications for the role and effects of public policy in coordination problems. Including such strategic situations and moral hazard problems in our analysis is also an additional and important avenue for future research. Finally, our result that the government (without so large public investment productivity) should not intervene when its public information is too low largely depends on our model assumption of the simple two-period economy with initial endowments and safe investment projects and the assumption that the government's investment policy is effective only in one period. In a more sophisticated, realistic economy with dynamic structures, however, the role and effects of public investment policy under coordination problems could be more complicated. Extending our simple two-period economy structure into a more sophisticated structure is also an important issue in the future research.

References

Abreu, D., Brunnermeier, M, K., 2003. Bubbles and crashes. Econometrica 71, 173–204.

Angeletos G, M., Hellwig, C., Pavan, A, 2006. Signaling in a global game: Coordination and policy traps. Journal of Political Economy 114, 452–484.

Carlsson, H., van Damme, E., 1993. Global games and equilibrium selection. Econometrica 61, 989–1018.

Chui, M., Gai, P., Haldane, A., 2002. Sovereign liquidity crises: Analytics and implications for public policy. Journal of Banking and Finance 26, 519–546.

Cooper, R., 1999. *Coordination Games: Complementarities and Macroeconomics*. Cambridge, Cambridge University Press.

Cooper, R., Ross, T., 1999. Public and private guarantee funds with market fragility. Journal of Risk and Insurance, 66, 163–184.

Corsetti, G., Dasgupta, A., Morris, S., Shin, H., 2004. Does one Soros make a difference? A theory of currency crises with large and small traders. Review of Economic Studies 71, 87–113.

Corsetti, G., Guimarães, B., Roubini, N., 2006. International lending of last resort and moral hazard: A model of IMF's catalytic finance. Journal of Monetary Economics 53, 441–471.

Diamond, D. W., Dybvig, P., 1983. Bank runs, deposit insurance and liquidity. Journal of Political Economy 91, 401–419.

Heinemann, F., Illing, G., 2002. Speculative attacks: unique equilibrium and transparency. Journal of International Economics 58, 429–450.

Kiyotaki, N., 1988. Multiple expectational equilibria under monopolistic competition. Quarterly Journal of Economics 103, 695–713.

Lamont, O., 1995. Corporate-debt overhang and macroeconomic expectations. American Economic Review 85, 1106–1117.

Metz, C., 2002. Private and public information in self-fulfilling currency crises. Journal of Economics 76, 65–85.

Morris, S., Shin, H., 1998. Unique equilibrium in a model of self-fulfilling currency attacks. American Economic Review 88, 587–597.

Morris, S., Shin, H., 2003. Global games: Theory and applications. In M. Dewatripont, L. Hansen, and S. Turnovsky (Eds.), Advances in Economics and Econometrics (8th World Congress of the Econometric Society), Cambridge University Press, Cambridge.

Morris, S., Shin, H., 2004. Coordination risk and the price of debt. European Economic Review 48, 133–153.

Morris, S., Shin, H., 2006. Catalytic finance: When does it work? Journal of International Economics 70, 161–177.

Obstfeld, M., 1996. Models of currency crisis with self-fulfilling features. European Economic Review 40, 1037–1047.

Oyama, D., 2004. Booms and slumps in a game of sequential investment with the changing fundamentals. The Japanese Economic Review 55, 311–320.

Table 1

Parameter values for the numerical calculations

Parameter	Parameter value	Parameter	Parameter value
w	10.0	γ	0.5
a	4.0	γ_x	20.0
R_s	4.0	γ_y	1.0
R_f	0.5	$\bar{\tau}$	0.4
R	2.0	$\bar{\tau}_s$	0.4
A	1.0	$\bar{\tau}_r$	0.4
r	2.0	$\bar{\tau}_s^{fc}$	0.4
$\bar{\theta}$	0.0	d	$0.01, \mathbf{0.1}, 0.4$
y	$-1.5, \mathbf{0.1}, 1.5$		

Table 2

Expected welfare and the optimal tax rates under indivisibility and no financial constraints

	$d = 0.01$	$d = 0.1$	$d = 0.4$
$y = -1.5$	$\hat{\hat{W}} = 40.0000$ $\tau_r^* = 0.00 \quad \tau_s^* = 0.00$ $g = 0.00$	$\hat{\hat{W}} = 40.0000$ $\tau_r^* = 0.00 \quad \tau_s^* = 0.00$ $g = 0.00$	$\hat{\hat{W}} = 40.0000$ $\tau_r^* = 0.00 \quad \tau_s^* = 0.00$ $g = 0.00$
$y = 0.1$	$\hat{\hat{W}} = 40.0006$ $\tau_r^* = 0.00 \quad \tau_s^* = 0.00$ $g = 0.00$	$\hat{\hat{W}} = 46.1955$ $\tau_r^* = 0.00 \quad \tau_s^* = 0.40$ $g = 1.64$	$\hat{\hat{W}} = 58.9413$ $\tau_r^* = 0.08 \quad \tau_s^* = 0.16$ $g = 1.32$
$y = 1.5$	$\hat{\hat{W}} = 61.5095$ $\tau_r^* = 0.00 \quad \tau_s^* = 0.40$ $g = 0.84$	$\hat{\hat{W}} = 68.5824$ $\tau_r^* = 0.00 \quad \tau_s^* = 0.40$ $g = 0.40$	$\hat{\hat{W}} = 73.5737$ $\tau_r^* = 0.04 \quad \tau_s^* = 0.32$ $g = 0.76$

In this table, we calculate the expected welfare $\hat{\hat{W}}$, the optimal tax rates τ_r^* and τ_s^*, and the size of public investment policy g. The parameter values given in Table 1 are used in generating these values.
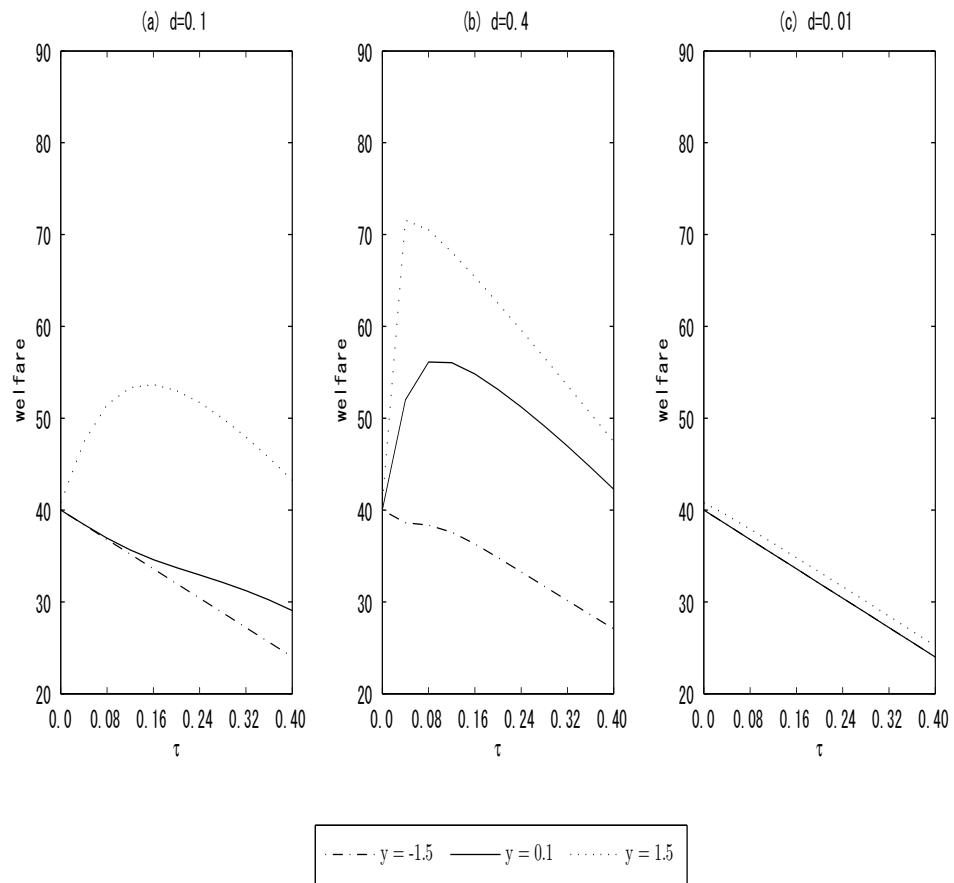
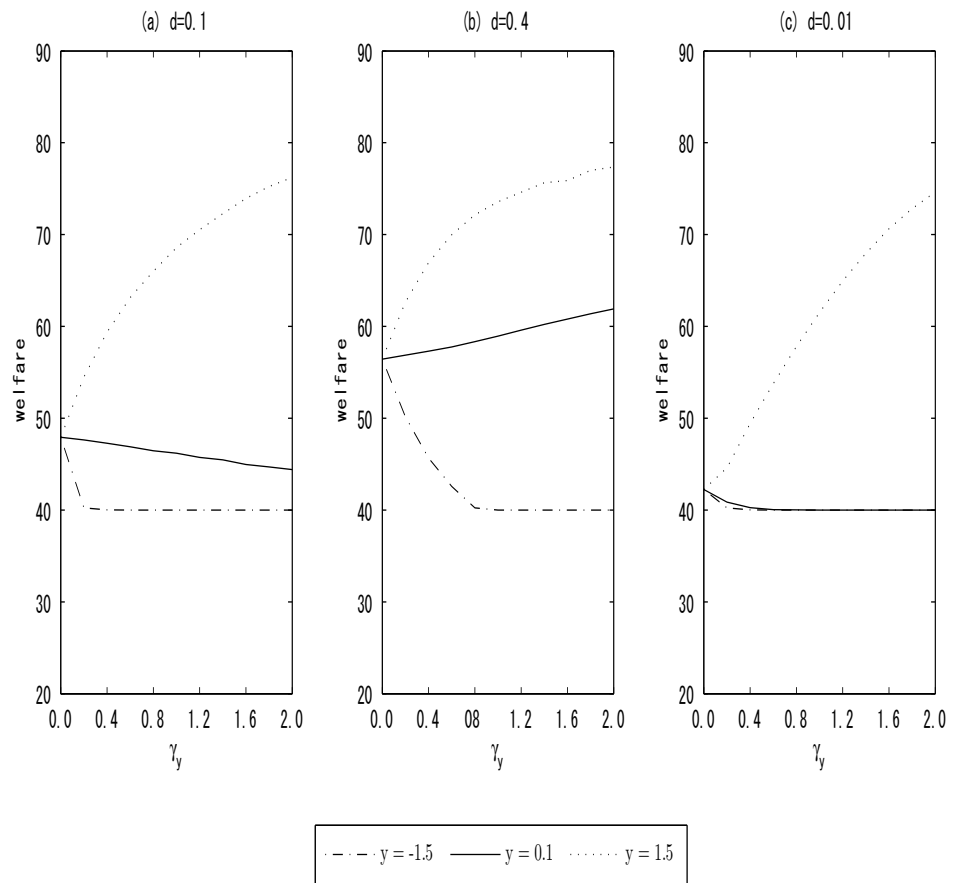Figure 1: The case of divisibility of the investment projects.

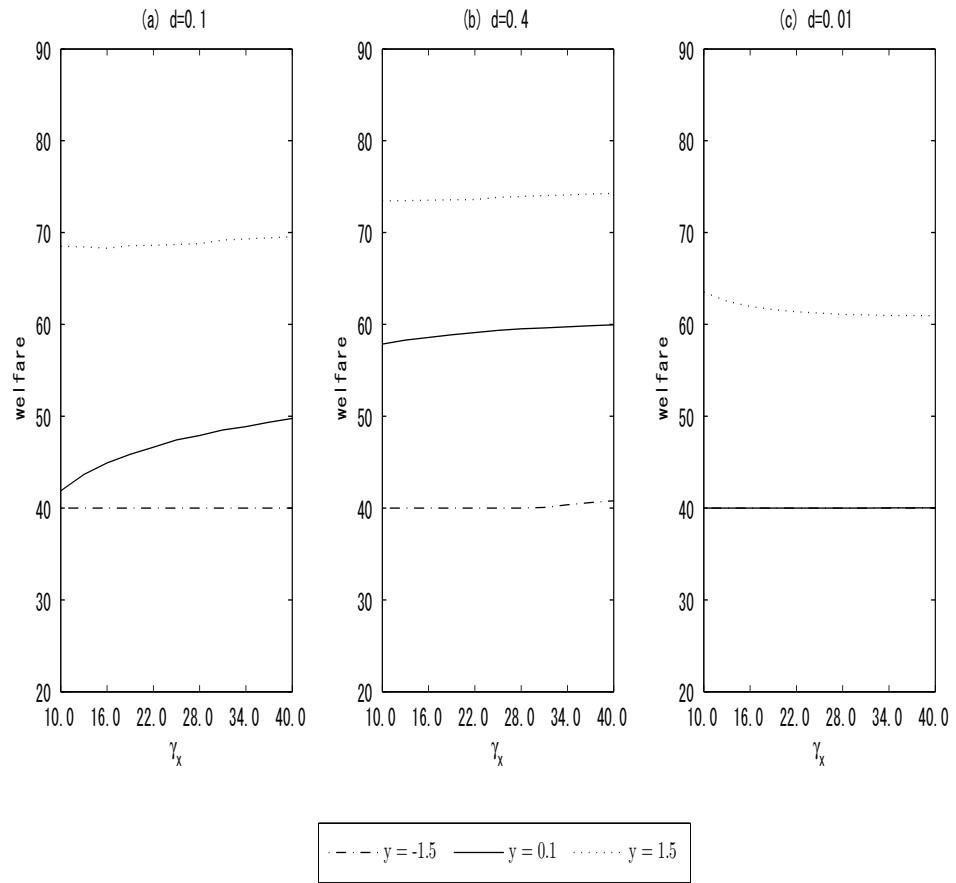Figure 2: The effect of the changes of the information precision (public signal).

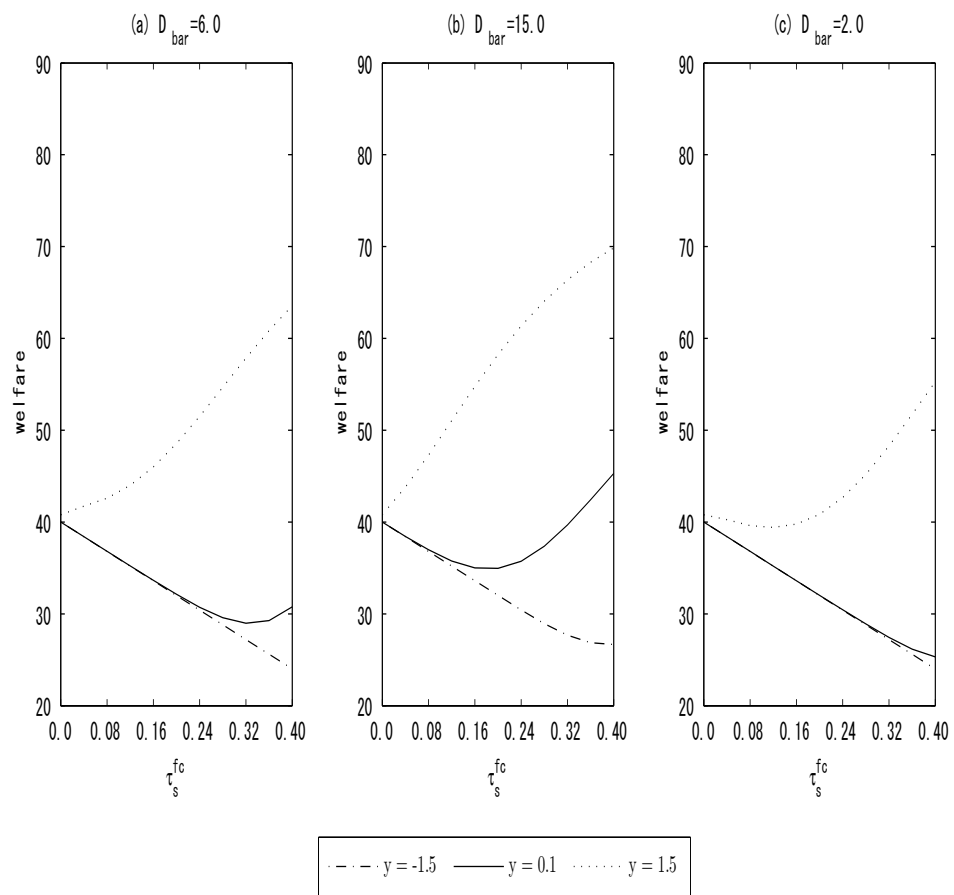Figure 3: The effect of the changes of the information precision (private signal).

Figure 4: The effect of an ex post redistribution policy.