Discussion Paper Series A No.475

Triple Implementation by Sharing Mechanisms in Production Economics with Unequal Labor Skill

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February 2006

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Triple Implementation by Sharing Mechanisms in Production Economies with Unequal Labor Skills*

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First: December 2004, This Version: December 2005

Abstract

In production economies with unequal labor skills, we study axiomatic characterizations of Pareto subsolutions which are implementable by *sharing mechanisms* in Nash, strong Nash, and subgame perfect equilibria. The sharing mechanism allows agents to work freely and distributes the produced output to the agents, according to the profile of labor hours and the information on demands, prices, and labor skills. Based on the characterizations, we find that most fair allocation rules, which embody the ethical principles of *responsibility and compensation,* cannot be implemented when individuals' labor skills are private information.

Journal of Economic Literature Classification Numbers: C72, D51, D78, D82

Keywords: labor sovereignty, triple implementation, different labor skills

*We are grateful to William Thomson for his kind and detailed advice on editing the paper, as well as to the two anonymous referees of this journal for their helpful comments on improving the paper. In addition, we appreciate the careful and detailed comments provided by Yoshikatsu Tatamitani on an earlier draft. An earlier version of this paper was presented at the Second World Congress of the Game Theory Society held at Marseille in July 2004, the Seventh International Meeting of the Society for Social Choice and Welfare held at Osaka in July 2004, and the Spring Annual Meeting of the Japanese Economic Association held at Kyoto in June 2005.

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1 Introduction

We consider the implementation of allocation rules in production economies with possibly unequal labor skills among individuals. Varian (1994), Hurwicz et al. (1995), Hong (1995), Suh (1995), Tian (1999, 2000), Yoshihara (1999), and Kaplan and Wettstein (2000) have proposed simple or natural mechanisms (game forms) to implement particular rules such as the Walrasian solution and the proportional solution (Roemer and Silvestre (1993)). In contrast, a few works such as Shin and Suh (1997) and Yoshihara (2000) have discussed characterizations of allocation rules implementable by such simple or natural mechanisms. However, in these works, there are two implicit assumptions about the basic information structure among individuals and the social planner (or mechanism coordinator).

The first implicit assumption is that the planner knows every individual's level of labor skill, or alternatively that all individuals have the same skill level. Thus, the main problem of asymmetric information in this structure is reduced to the possibility of misrepresenting each individual's preference ordering,[1] and at most, the possibility of understating each individual's endowment of material goods.[2] However, if individual skill levels differ, it is more natural to consider an informational structure such that the planner cannot know each individual's true skill, which gives the individual an incentive to *overstate,* or to understate, his own skill. Note that the possibility of overstating individual skill is an essential feature of production economies with asymmetric information, because the planner cannot require individuals to "place the claimed endowments on the table" (Hurwicz et al. (1995)) in advance of production. Thus, taking this feature of the problem into consideration, our concern in this paper is to characterize the class of allocation rules, each of which assigns a subset of Pareto efficient allocations to each economic environment, that are implementable by a *natural mechanism* even when individuals' skills are unknown to the planner.

What kind of mechanism should we take as a natural one in this context? This issue is relevant to our discussion of the second implicit assumption in the present literature on implementation in production economies. Although Shin and Suh (1997) and Yoshihara (2000) define the conditions for

[1] For instance, Varian (1994), Suh (1995), Shin and Suh (1997), Yoshihara (1999, 2000), and Kaplan and Wettstein (2000) discussed this type of problem.

[2] For instance, Hurwicz et al. (1995), Hong (1995), and Tian (1999, 2000) discussed this type of problem.

characterizing "natural mechanisms" in production economies, the list of those conditions[3] is not yet satisfactory, because they omit another important feature of production economies with asymmetric information. Usually, the mechanisms in the implementation literature consist of pairs of strategy spaces and outcome functions, where each agent is required to state some information, and the outcome function assigns an allocation to each profile of individuals' strategies. So, in production economies where one of the main productive factors is labor, it is implicitly assumed that the planner is authorized to force individuals to provide the amount of labor assigned by the outcome function of the mechanism.[4] However, the planner may not have such authority.

To solve this problem, we introduce another condition, *labor sovereignty* (Kranich (1994)), for characterizing "natural mechanisms" in production economies, and we propose *sharing mechanisms* as a type of game form satisfying labor sovereignty. Labor sovereignty requires that every individual should have a right to choose his own labor time. In a sharing mechanism, each individual can freely supply his labor time, and the individual is asked to give information concerning his skill and demand for consumption goods. The outcome function simply distributes the produced output to agents, according to the information they provided and the record of their labor hours completed.

Thus, the question this paper addresses is: what kinds of rules that assign some Pareto efficient allocations are implementable by sharing mechanisms, even when individuals' skills are unknown to the planner? We consider three equilibrium notions, *Nash*, *strong Nash*, and *subgame perfect Nash*, for the non-cooperative games defined by sharing mechanisms.[5] We identify two axioms that characterize rules *triply implementable by sharing mechanisms*. The two axioms are relevant to the ethical principles of *responsibility and compensation* (Fleurbaey (1998)) in fair allocation problems. Thus, our characterization provides an insight into the implementability of fair allocation

[3] Those conditions are *feasibility*, *forthrightness*, *best response property*, and *simple strategy spaces*, which were originally proposed by Dutta, Sen, and Vohra (1995) and Saijo, Tatamitani, and Yamato (1996) to characterize "natural mechanisms" in pure exchange economies.

[4] Roemer (1989) pointed out this implicit assumption explicitly.

[5] Yamada and Yoshihara (2002) proposed a sharing mechanism that triply implements the proportional solution in these three equilibria, when the production function is differentiable.

rules in terms of responsibility and compensation.

In the following discussion, the model is defined in Section 2. Section 3 provides a characterization of triple implementation by sharing mechanisms. Section 4 gives some samples of implementable and non-implementable rules. Concluding remarks appear in Section 5. All proofs of the theorems are relegated to the Appendix.

2 The Basic Model

There are two goods, one of which is an input (labor time) $x \in \mathbb{R}_+$ to be used to produce the other good $y \in \mathbb{R}_+$.[6] The population is given by the set $N = \{1, \ldots, n\}$, where $2 \leq n < +\infty$. Each agent i's consumption is denoted by $z_i = (x_i, y_i)$, where x_i denotes his labor time, and y_i denotes the amount of his output. All the agents are assumed to face a common upper bound of labor time $\bar{x}$, where $0 < \bar{x} < +\infty$, so that they have the same consumption set $[0, \bar{x}] \times \mathbb{R}_+$. Each agent i's preference is defined on $[0, \bar{x}] \times \mathbb{R}_+$ and represented by a utility function $u_i : [0, \bar{x}] \times \mathbb{R}_+ \to \mathbb{R}$ which is continuous and quasi-concave on $[0, \bar{x}] \times \mathbb{R}_+$, and strictly monotonic (decreasing in labor time and increasing in the share of output) on $[0, \bar{x}) \times \mathbb{R}_{++}$.[7] We use $\mathcal{U}$ to denote the class of such utility functions. In addition, each agent i is characterized by a *labor skill*, which is represented by a positive real number $s_i^t \in \mathbb{R}_{++}$. The superscript t on s_i^t indicates "true," so that s_i^t denotes *agent i's true skill*. The universal set of skills for all agents is denoted by $\mathcal{S} = \mathbb{R}_{++}$.[8] The labor skill $s_i^t \in \mathcal{S}$ is i's *labor supply* per hour measured in efficiency units. It can also be interpreted as i's *labor intensity* exercised in the production process.[9] Thus, if the agent's *labor time* is $x_i \in [0, \bar{x}]$ and his labor intensity

[6] The symbol $\mathbb{R}_+$ denotes the set of non-negative real numbers.

[7] The symbol $\mathbb{R}_{++}$ denotes the set of positive real numbers.

[8] For any two sets X and Y, $X \subseteq Y$ whenever any $x \in X$ also belongs to Y, and $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$.

[9] It might be more natural to define labor skill and labor intensity in a discriminative way: for example, if $\overline{s}_i^t \in \mathcal{S}$ is i's labor skill, then i's labor intensity is a variable s_i^t, where $0 < s_i^t \leq \overline{s}_i^t$. In such a formulation, we may view the amount of s_i^t as being determined endogenously by the agent i. In spite of this more natural view, we will assume in the following discussion that the labor intensity is a constant value, $s_i^t = \overline{s}_i^t$, for the sake of simplicity. The main theorems in the following discussion would remain valid with a few changes in the settings of the economic environments even if the labor intensity were assumed to be varied.

is $s_i^t \in \mathcal{S}$, then $s_i^t x_i \in \mathbb{R}_+$ denotes the agent's *labor contribution* to the production process measured in efficiency units. The production technology is a function $f : \mathbb{R}_+ \to \mathbb{R}_+$, assumed to be continuous, strictly increasing, concave, and $f(0) = 0$. For simplicity, we fix this f for all economies. Thus, the economy is characterized by a pair of profiles $\mathbf{e} \equiv (\mathbf{u}, \mathbf{s}^t)$ with $\mathbf{u} = (u_1, \dots, u_n) \in \mathcal{U}^n$ and $\mathbf{s}^t = (s_1^t, \dots, s_n^t) \in \mathcal{S}^n$. Denote the class of such economies by $\mathcal{E} \equiv \mathcal{U}^n \times \mathcal{S}^n$.

Given $\mathbf{s}^t = (s_1^t, \dots, s_n^t) \in \mathcal{S}^n$, an allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is *feasible for* $\mathbf{s}^t$ if $\sum y_i \leq f(\sum s_i^t x_i)$. We denote by $Z(\mathbf{s}^t)$ the set of feasible allocations for $\mathbf{s}^t \in \mathcal{S}^n$. An allocation $\mathbf{z} = (z_1, \dots, z_n) \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is *Pareto efficient for* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ if $\mathbf{z} \in Z(\mathbf{s}^t)$ and there does not exist $\mathbf{z}' = (z_1', \dots, z_n') \in Z(\mathbf{s}^t)$ such that for all $i \in N$, $u_i(z_i') \geq u_i(z_i)$, and for some $i \in N$, $u_i(z_i') > u_i(z_i)$. We use $P(\mathbf{e})$ to denote the set of Pareto efficient allocations for $\mathbf{e} \in \mathcal{E}$. A *solution* is a correspondence $\varphi : \mathcal{E} \twoheadrightarrow ([0, \bar{x}] \times \mathbb{R}_+)^n$ such that for each $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\varphi(\mathbf{e}) \subseteq Z(\mathbf{s}^t)$. A solution is called a *Pareto subsolution* if for each $\mathbf{e} \in \mathcal{E}$, $\varphi(\mathbf{e}) \subseteq P(\mathbf{e})$. Given a Pareto subsolution φ, an allocation $\mathbf{z} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is *φ-optimal for* $\mathbf{e} \in \mathcal{E}$ if $\mathbf{z} \in \varphi(\mathbf{e})$.

2.1 Sharing Mechanisms

We are interested in mechanisms having the property of *labor sovereignty*, under which every agent can choose his own labor time freely. In particular, we focus on *sharing mechanisms* that simply distribute output among the agents according to their announcements on private information and their supplied labor time.

Definition 1. *A* sharing mechanism *is a function $g : M \times [0, \bar{x}]^n \to \mathbb{R}_+^n$ such that for any $(\mathbf{m}, \mathbf{x}) \in M \times [0, \bar{x}]^n$, $g(\mathbf{m}, \mathbf{x}) = \mathbf{y}$ for some $\mathbf{y} \in \mathbb{R}_+^n$, where $\mathbf{m} = (m_1, \dots, m_n)$ denotes the agents' messages.*

A sharing mechanism g is *feasible* if for each $\mathbf{s}^t \in \mathcal{S}^n$, each $\mathbf{m} \in M$, and each $\mathbf{x} \in [0, \bar{x}]^n$, we have $(\mathbf{x}, g(\mathbf{m}, \mathbf{x})) \in Z(\mathbf{s}^t)$. Note that a feasible sharing mechanism g needs not refer to $\mathbf{s}^t$ in dividing the total output $f\left(\sum s_j^t x_j\right)$. We denote by $\mathcal{G}$ the class of all (feasible sharing) mechanisms.

Given a mechanism $g \in \mathcal{G}$, a (*feasible*) *sharing game* is defined for each economy $\mathbf{e} \in \mathcal{E}$ as a non-cooperative game $(N, M \times [0, \bar{x}]^n, g, \mathbf{e})$. Fixing the set of players N and their strategy sets $M \times [0, \bar{x}]^n$, we simply denote a feasible sharing game $(N, M \times [0, \bar{x}]^n, g, \mathbf{e})$ by $(g, \mathbf{e})$.

Given a strategy profile $(\mathbf{m}, \mathbf{x}) \in M \times [0, \bar{x}]^n$, let $\left(\mathbf{m}_{m_i'}, \mathbf{x}_{x_i'}\right) \in M \times [0, \bar{x}]^n$ be another strategy profile that is obtained by replacing the i-th component (m_i, x_i) of $(\mathbf{m}, \mathbf{x})$ with (m_i', x_i'). A strategy profile $(\mathbf{m}^*, \mathbf{x}^*) \in M \times [0, \bar{x}]^n$ is a *(pure-strategy) Nash equilibrium of the sharing game* $(g, \mathbf{e})$ if for any $i \in N$ and any $(m_i, x_i) \in M_i \times [0, \bar{x}]$, $u_i(x_i^*, g_i(\mathbf{m}^*, \mathbf{x}^*)) \geq u_i\left(x_i, g_i\left(\mathbf{m}^*_{m_i}, \mathbf{x}^*_{x_i}\right)\right)$. Denote by $NE(g, \mathbf{e})$ the set of Nash equilibria of $(g, \mathbf{e})$. An allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is a *Nash equilibrium allocation of the sharing game* $(g, \mathbf{e})$ if there exists $\mathbf{m} \in M$ such that $(\mathbf{m}, \mathbf{x}) \in NE(g, \mathbf{e})$ and $\mathbf{y} = g(\mathbf{m}, \mathbf{x})$, where $\mathbf{x} = (x_i)_{i \in N}$ and $\mathbf{y} = (y_i)_{i \in N}$. We use $NA(g, \mathbf{e})$ to denote the set of Nash equilibrium allocations of $(g, \mathbf{e})$. A mechanism $g \in \mathcal{G}$ *implements* a solution φ on $\mathcal{E}$ *in Nash equilibria* if for all $\mathbf{e} \in \mathcal{E}$, $NA(g, \mathbf{e}) = \varphi(\mathbf{e})$.

A strategy profile $(\mathbf{m}^*, \mathbf{x}^*) \in M \times [0, \bar{x}]^n$ is a *(pure-strategy) strong (Nash) equilibrium of the sharing game* $(g, \mathbf{e})$ if for any $T \subseteq N$ and any $(m_i, x_i)_{i \in T} \in (M_i)_{i \in T} \times [0, \bar{x}]^{\#T}$, there exists $j \in T$ such that

$$u_j\left(x_j^*, g_j(\mathbf{m}^*, \mathbf{x}^*)\right) \geq u_j\left(x_j, g_j\left((m_i, x_i)_{i \in T}, (m_k^*, x_k^*)_{k \in T^c}\right)\right).[10]$$

We use $SNE(g, \mathbf{e})$ to denote the set of strong equilibria of $(g, \mathbf{e})$. An allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is a *strong equilibrium allocation of the sharing game* $(g, \mathbf{e})$ if there exists $\mathbf{m} \in M$ such that $(\mathbf{m}, \mathbf{x}) \in SNE(g, \mathbf{e})$ and $\mathbf{y} = g(\mathbf{m}, \mathbf{x})$. We use $SNA(g, \mathbf{e})$ to denote the set of strong equilibrium allocations of $(g, \mathbf{e})$. A mechanism $g \in \mathcal{G}$ *implements* a solution φ on $\mathcal{E}$ *in strong equilibria,* if for all $\mathbf{e} \in \mathcal{E}$, $SNA(g, \mathbf{e}) = \varphi(\mathbf{e})$.

2.2 The Timing Problem with Sharing Mechanisms

We should mention here that $\mathbf{m}$ and $\mathbf{x}$ represent the agents' different kinds of strategic actions: $\mathbf{m}$ indicates the agents' announcements on their private information, whereas $\mathbf{x}$ indicates their production activity from supplying labor time. Thus, there may be a difference between the point in time when $\mathbf{m}$ is announced and the time when $\mathbf{x}$ is exercised. It implies that there may be at least *two polar opposite time sequences of decision making*: the agents may announce $\mathbf{m}$ before they engage in production, or they may announce $\mathbf{m}$ after supplying $\mathbf{x}$. The former enables each agent i to decide his supply of labor time with knowledge of the announcements $\mathbf{m}$, whereas the latter

[10] For any $T \subseteq N$, $\#T$ denotes the number of agents in T. For any $T \subseteq N$, T^c denotes the complement of T in N.

enables each agent i to decide his announcement m_i with knowledge of the agents' actions $\mathbf{x}$ in the production process.

Additionally, we should consider at least two types of two-stage game forms:

(1) In the first type of two-stage game form, every agent i simultaneously makes an announcement, m_i, in the first stage on his private information, and in the second stage, every agent i engages in production and provides his labor time, x_i. After the production process, the outcome function assigns a distribution of the output produced.

(2) The second type of two-stage game form has the reverse sequence of strategic actions. In the first stage, every agent i engages in production and provides his labor time, x_i, and after production takes place, every agent i simultaneously makes an announcement, m_i, on his private information in the second stage. Finally, the outcome function assigns a distribution of the output produced.

Given a feasible sharing mechanism $g \in \mathcal{G}$, the first type of the *g-implicit* two-stage extensive game form is a feasible mechanism $\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}$ in which the first stage consists of selecting $\mathbf{m} \in M$, the second stage consists of selecting $\mathbf{x} \in [0, \bar{x}]^n$, and the final stage assigns an outcome $(\mathbf{x}, g(\mathbf{m}, \mathbf{x}))$. Given $g \in \mathcal{G}$, the second type of the *g-implicit* two-stage extensive game form is a feasible mechanism $\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}$ in which the first stage consists of selecting $\mathbf{x} \in [0, \bar{x}]^n$, the second stage consists of selecting $\mathbf{m} \in M$, and the final stage assigns an outcome $(\mathbf{x}, g(\mathbf{m}, \mathbf{x}))$.

Given a two-stage game $\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right)$ and a strategy profile $\mathbf{m} \in M$ in the first stage of the game, let us denote its corresponding second-stage subgame by $\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{m}), \mathbf{e}\right)$. Let $\mathbf{x}^{\mathbf{s}} : M \to [0, \bar{x}]^n$ be a *strategy mapping* such that for each $\mathbf{m} \in M$, $\mathbf{x}^{\mathbf{s}}(\mathbf{m})$ is a strategy profile of the subgame $\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{m}), \mathbf{e}\right)$. Denote the set of such strategy mappings of $\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right)$ by $\mathbf{X}$. A strategy profile $(\mathbf{m}^*, \mathbf{x}^{\mathbf{s}*}) \in M \times \mathbf{X}$ is a *(pure-strategy) subgame perfect (Nash) equilibrium of* $\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right)$ if for any $i \in N$, any $m_i \in M_i$, any $\mathbf{x}^{\mathbf{s}} \in \mathbf{X}$ with $\mathbf{x}^{\mathbf{s}} = \left(x_i^s, \mathbf{x}_{-i}^{\mathbf{s}*}\right)$, and any $\mathbf{m} \in M$,

$$\begin{aligned} u_i\left(x_i^{s*}(\mathbf{m}^*), g_i\left(\mathbf{m}^*, \mathbf{x}^{s*}(\mathbf{m}^*)\right)\right) &\geq u_i\left(x_i^{s*}(\mathbf{m}_{m_i}^*), g_i\left(\mathbf{m}_{m_i}^*, \mathbf{x}^{s*}(\mathbf{m}_{m_i}^*)\right)\right) \\ \text{and } u_i\left(x_i^{s*}(\mathbf{m}), g_i\left(\mathbf{m}, \mathbf{x}^{s*}(\mathbf{m})\right)\right) &\geq u_i\left(x_i^{s}(\mathbf{m}), g_i\left(\mathbf{m}, \mathbf{x}^{s}(\mathbf{m})\right)\right), \end{aligned}$$

where $x_i^{s*}(\mathbf{m})$ (*resp.* $x_i^s(\mathbf{m})$) is the i-th component of the strategy profile $\mathbf{x}^{s*}(\mathbf{m})$ (*resp.* $\mathbf{x}^s(\mathbf{m})$) in the second-stage subgame induced by the strategy choice $\mathbf{m}$ in the first stage.

Given a two-stage game $\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$ and a strategy profile $\mathbf{x} \in [0, \bar{x}]^n$ in the first stage of the game, let us denote its corresponding second stage subgame by $\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$. Let $\mathbf{m}^{\mathbf{s}} : [0, \bar{x}]^n \to M$ be a *strategy mapping* such that for each $\mathbf{x} \in [0, \bar{x}]^n$, $\mathbf{m}^{\mathbf{s}}(\mathbf{x})$ is a strategy profile of the subgame $\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$. Denote the set of such strategy mappings of $\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$ by $\mathbf{M}^{\mathbf{s}}$. A strategy profile $(\mathbf{m}^{\mathbf{s}*}, \mathbf{x}^*) \in \mathbf{M}^{\mathbf{s}} \times [0, \bar{x}]^n$ is a (*pure-strategy*) *subgame perfect (Nash) equilibrium of the game* $\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$ if for any $i \in N$ and any $x_i \in [0, \bar{x}]$, any $\mathbf{m}^{\mathbf{s}} \in \mathbf{M}^{\mathbf{s}}$ with $\mathbf{m}^{\mathbf{s}} = \left(m_i^s, \mathbf{m}_{-i}^{\mathbf{s}*}\right)$, and any $\mathbf{x} \in [0, \bar{x}]^n$,

$$\begin{aligned} u_i\left(x_i^*, g_i\left(\mathbf{m}^{\mathbf{s}*}\left(\mathbf{x}^*\right), \mathbf{x}^*\right)\right) &\geq u_i\left(x_i, g_i\left(\mathbf{m}^{\mathbf{s}*}\left(\mathbf{x}_{x_i}^*\right), \mathbf{x}_{x_i}^*\right)\right), \\ \text{and } u_i\left(x_i, g_i\left(\mathbf{m}^{\mathbf{s}*}\left(\mathbf{x}\right), \mathbf{x}\right)\right) &\geq u_i\left(x_i, g_i\left(\mathbf{m}^{\mathbf{s}}\left(\mathbf{x}\right), \mathbf{x}\right)\right). \end{aligned}$$

We use $SPE\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right)$ (resp. $SPE\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$) to denote the set of subgame perfect equilibria of $\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right)$ (resp. $\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$). An allocation $\mathbf{z} = (x_i, y_i)_{i\in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is a *subgame perfect equilibrium allocation of* $\left(\Gamma_g^1, \mathbf{e}\right)$ (resp. $\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$) if there exists $(\mathbf{m}, \mathbf{x}^{\mathbf{s}}) \in SPE\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right)$ (resp. $(\mathbf{m}^{\mathbf{s}}, \mathbf{x}) \in SPE\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$) such that $\mathbf{x}^{\mathbf{s}}(\mathbf{m}) = \mathbf{x}$ and $\mathbf{y} = g\left(\mathbf{m}, \mathbf{x}^{\mathbf{s}}(\mathbf{m})\right)$ (resp. $\mathbf{y} = g\left(\mathbf{m}^{\mathbf{s}}(\mathbf{x}), \mathbf{x}\right)$). Denote by $SPA\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right)$ (resp. $SPA\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$) the set of subgame perfect equilibrium allocations of $\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right)$ (resp. $\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$). Given $g \in \mathcal{G}$, the two-stage mechanism $\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}$ (resp. $\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}$) *implements* a solution φ on $\mathcal{E}$ *in subgame perfect equilibria* if for all $\mathbf{e} \in \mathcal{E}$, $SPA\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right) = \varphi(\mathbf{e})$ (resp. $SPA\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right) = \varphi(\mathbf{e})$). Given $g \in \mathcal{G}$, the two-stage mechanism $\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}$ (resp. $\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}$) *triply implements* a solution φ on $\mathcal{E}$ *in Nash, strong, and subgame perfect equilibria* if for all $\mathbf{e} \in \mathcal{E}$, $NA(g, \mathbf{e}) = SNA(g, \mathbf{e}) = SPA\left(\Gamma_g^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right) = \varphi(\mathbf{e})$ (resp. $NA(g, \mathbf{e}) = SNA(g, \mathbf{e}) = SPA\left(\Gamma_g^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right) = \varphi(\mathbf{e})$).

3 Implementation by Sharing Mechanisms

Throughout our discussion, we assume that each agent prefers consumption vectors with positive amounts of output and leisure, over consumption vectors without output or leisure.

Assumption 1 (boundary condition of utility functions):

$$\forall i \in N, \forall z_i \in [0, \bar{x}) \times \mathbb{R}_{++}, \forall z_i' \in \partial\left([0, \bar{x}] \times \mathbb{R}_+\right), u_i(z_i) > u_i(z_i').[11]$$

[11] $\partial\left([0, \bar{x}] \times \mathbb{R}_+\right) \equiv \left([0, \bar{x}] \times \mathbb{R}_+\right) \setminus \left([0, \bar{x}) \times \mathbb{R}_{++}\right)$.

Here, let us introduce a new notation. We denote the set of price vectors by the unit simplex $\Delta \equiv \{p = (p_x, p_y) \in \mathbb{R}_+ \times \mathbb{R}_+ : p_x + p_y = 1\}$, where p_x represents the price of labor (measured in efficiency units) and p_y is the price of output. The message space M of the mechanism considered in the present paper is defined by $M \equiv \Delta^n \times \mathcal{S}^n \times \mathbb{R}^n_+$, with generic element $(\mathbf{p}, \mathbf{s}, \mathbf{w})$, where $\mathbf{p} = (p^1, \ldots, p^n)$, in which p^i denotes i's reported price vector, $\mathbf{s} = (s_1, \ldots, s_n)$, in which s_i denotes i's reported amount of skill, and $\mathbf{w} = (w_1, \ldots, w_n)$, in which w_i denotes i's desired amount of output. Moreover, we define efficiency prices as follows.

Definition 2. *A price vector* $p = (p_x, p_y) \in \Delta$ *is* an efficiency price for $\mathbf{z} = (\mathbf{x}, \mathbf{y}) \in [0, \bar{x}]^n \times \mathbb{R}^n_+$ at $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *iff*

(i) $(x_i)_{i \in N} \in \arg \max_{(x'_i)_{i \in N}} p_y f\left(\sum s^t_i x'_i\right) - p_x \sum s^t_i x'_i$;

(ii) *for all* $i \in N$ *and all* $z'_i \in [0, \bar{x}] \times \mathbb{R}_+$, *if* $u_i(z'_i) \geq u_i(z_i)$, *then* $p_y y'_i - p_x s^t_i x'_i \geq p_y y_i - p_x s^t_i x_i$.

The set of efficiency prices for $\mathbf{z}$ at $\mathbf{e}$ is denoted by $\Delta^P(\mathbf{e}, \mathbf{z})$.

We define implementability by sharing mechanism as follows.

Definition 3. *A Pareto subsolution* φ *is* triply labor sovereign implementable, *if there exists a feasible sharing mechanism* $g \in \mathcal{G}$ *such that*:

(i) $\Gamma_g^{\mathbf{m} \vdash \mathbf{x}}$ (resp. $\Gamma_g^{\mathbf{x} \vdash \mathbf{m}}$) triply implements φ *on* $\mathcal{E}$ in Nash, strong, and subgame perfect equilibria;

(ii) g *is* forthright*: for all* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *and all* $(\mathbf{x}, \mathbf{y}) \in \varphi(\mathbf{e})$, *there exists* $p \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{y})$ *such that* $(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) \in NE(g, \mathbf{e})$ *and* $g(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) = \mathbf{y}$, *where* $\mathbf{p} = (p^i)_{i \in N}$ *with* $p^i = p$ *for all* $i \in N$;

(iii) *for all* $\mathbf{e} = (u_i, s^t_i)_{i \in N} \in \mathcal{E}$, *if* $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in NE(g, \mathbf{e})$ *and* $g(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \mathbf{y}$, *where* $\mathbf{p} = (p^i)_{i \in N}$ *with* $p^i = p \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{y})$ *for all* $i \in N$, $\mathbf{x} = (x_i)_{i \in N}$, *and* $\mathbf{y} = (y_i)_{i \in N}$, *then*

$$g_i\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s'_i}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s^t_i (x'_i - x_i)\right\}$$

for all $i \in N$ *and all* $(p^{i\prime}, s'_i, x'_i, w'_i) \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$;

(iv) *for all* $\mathbf{e} = (u_i, s^t_i)_{i \in N} \in \mathcal{E}$, *if* $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in NE(g, \mathbf{e})$, *then* $(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w}) \in NE(g, \mathbf{e})$ *and* $g(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = g(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w})$ *whenever* $s_i = s'_i$ for all $i \in N$ *with* $x_i > 0$.

Definition 3 (ii) was first introduced by Dutta et al. (1995) and discussed by Saijo et al. (1996). It requires that if a strategy profile is consistent with a φ-optimal allocation, then it is a Nash equilibrium and the outcome coincides with the φ-optimal allocation. That is, any φ-optimal allocation should be realizable as an equilibrium outcome in a straightforward way.

Definition 3 (iii) requires a kind of informational efficiency of the mechanism. It says that in equilibrium, each agent's attainable set is included in a half space, which is included in the lower contour set of the agent's utility function when the equilibrium allocation is Pareto efficient. The point is that this half space is defined only by the information on the production point and the production possibility set. Owing to this condition, the mechanism coordinator does not need to know all the information on the agents' preferences in order to obtain φ-optimal allocations as equilibrium allocations.

Definition 3 (iv) is another requirement of informational efficiency. It says that the distribution of output by the mechanism would not change regardless of any change in skill information stated by "non-working" agents. That is, unexercised labor skills should be equally taken into account in the determination of distribution. Owing to this condition, the mechanism coordinator need not consider degenerative labor skills.

We introduce two axioms as necessary conditions for labor sovereign implementation. The first axiom requires that any φ-optimal allocation should remain φ-optimal if the profile of the utility functions changes, without the Pareto efficiency of this allocation being affected. It is a condition of informational efficiency, because it only requires local information on individuals' preference orderings.

Supporting Price Independence (SPI) (Yoshihara (1998), Gaspart (1998)). *For all* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *and all* $\mathbf{z} \in \varphi(\mathbf{e})$, *there exists* $p \in \Delta^P(\mathbf{e}, \mathbf{z})$ *such that for all* $\mathbf{e}' = (\mathbf{u}', \mathbf{s}^t) \in \mathcal{E}$, *if* $p \in \Delta^P(\mathbf{e}', \mathbf{z})$, *then* $\mathbf{z} \in \varphi(\mathbf{e}')$.

Let $\Delta^{SPI}(\mathbf{u}, \mathbf{s}, \mathbf{z}) \equiv \{p \in \Delta^P(\mathbf{u}, \mathbf{s}, \mathbf{z}) : \forall \mathbf{u}' \in \mathcal{U}^n \text{ s.t. } p \in \Delta^P(\mathbf{u}', \mathbf{s}, \mathbf{z}), \mathbf{z} \in \varphi(\mathbf{u}', \mathbf{s})\}$.

The second axiom requires that any φ-optimal allocation should remain φ-optimal if the labor skills of non-working agents in this allocation change, without the Pareto efficiency of this allocation being affected. In addition, this axiom is also a condition of informational efficiency, because it admits ignorance of the information on the skills of non-working agents.

Independence of Unused Skills (IUS). *For all* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) = (u_i, s_i^t)_{i \in N} \in$

$\mathcal{E}$ *and all* $\mathbf{z} = (x_i, y_i)_{i \in N} \in \varphi(\mathbf{e})$*, there exists* $p \in \Delta^P(\mathbf{e}, \mathbf{z})$ *such that for all* $\mathbf{e}' = (\mathbf{u}, \mathbf{s}^{t\prime}) = (u_i, s_i^{t\prime})_{i \in N} \in \mathcal{E}$ *where* $s_i^t = s_i^{t\prime}$ *for all* $i \in N$ *with* $x_i > 0$*, if* $p \in \Delta^P(\mathbf{e}', \mathbf{z})$*, then* $\mathbf{z} \in \varphi(\mathbf{e}')$.

Let $\Delta^{IUS}(\mathbf{u}, \mathbf{s}, \mathbf{z}) \equiv \{p \in \Delta^P(\mathbf{u}, \mathbf{s}, \mathbf{z}) : \forall \mathbf{s}' \in \mathcal{S}^n \text{ s.t. } p \in \Delta^P(\mathbf{u}, \mathbf{s}', \mathbf{z}), \mathbf{z} \in \varphi(\mathbf{u}, \mathbf{s}'),$ where $s_i' = s_i$ for all $i \in N$ with $x_i > 0\}$, where $\mathbf{z} = (x_i, y_i)_{i \in N}$.

Furthermore, the two axioms can be interpreted in terms of *responsibility* and *compensation* (Fleurbaey (1998)) in fair allocation problems. The axiom SPI represents a "stronger" condition of responsibility, because it requires independence of the *particular* change of individuals' utility functions which are interpreted as responsible factors.[12] It is "stronger" because SPI is stronger than *Maskin Monotonicity* (Maskin (1999)), which was seen as a relatively strong axiom of responsibility by Fleurbaey and Maniquet (1996). In contrast, the axiom IUS can be interpreted as a weaker condition of compensation, because it requires independence of the *particular* change of individuals' labor skills, which are interpreted as non-responsible factors. It is "weaker" because IUS is weaker than the axiom of *Independence of Skill Endowments* (Yoshihara (2003)), which was seen as a relatively weak axiom of compensation.

Note that a Pareto subsolution φ satisfies SPI and IUS if and only if for all $\mathbf{e} \in \mathcal{E}$, all $\mathbf{z} \in \varphi(\mathbf{e})$, there exist $p \in \Delta^{SPI}(\mathbf{e}, \mathbf{z})$ and $p' \in \Delta^{IUS}(\mathbf{e}, \mathbf{z})$. In general, $\Delta^{SPI}(\mathbf{e}, \mathbf{z}) \neq \Delta^{IUS}(\mathbf{e}, \mathbf{z})$. However, there exists some intersection between the two sets as the following lemma shows.

Lemma 0: *Let* φ *satisfy SPI and IUS. Then, for all* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *and all* $\mathbf{z} \in \varphi(\mathbf{e})$*,* $\Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \mathbf{z}) \subseteq \Delta^{IUS}(\mathbf{u}, \mathbf{s}^t, \mathbf{z})$.

Proof. Given $(\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, let $\mathbf{z} = (x_i, y_i)_{i \in N} \in \varphi(\mathbf{u}, \mathbf{s}^t)$ and $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \mathbf{z})$. Without loss of generality, suppose that $x_1 = 0$ and $x_i > 0$ for any $i \neq 1$. Let $\mathbf{s}^{t\prime} = (s_1^{t\prime}, \mathbf{s}_{-1}^t)$ be such that $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{z})$. If $\mathbf{z} \in \varphi(\mathbf{u}, \mathbf{s}^{t\prime})$, then by the definition of Δ^{IUS}, we have that $p \in \Delta^{IUS}(\mathbf{u}, \mathbf{s}^t, \mathbf{z})$.

Let $\mathbf{u}^* = (u_2^*, \mathbf{u}_{-2})$, where $u_2^*(x, y) = (\overline{x} - x)^{1-\alpha} \cdot y^\alpha$ such that $-\frac{\partial u_2^*(x_2, y_2)/\partial x}{\partial u_2^*(x_2, y_2)/\partial y} = \frac{p_x s_2^t}{p_y}$. Note there exists such $\alpha \in (0, 1)$, because $-\frac{\partial u_2^*(x_2, y_2)/\partial x}{\partial u_2^*(x_2, y_2)/\partial y} = \frac{y_2}{\overline{x} - x_2}\left(\frac{1}{\alpha} - 1\right)$, and $\frac{y_2}{\overline{x} - x_2}\left(\frac{1}{\alpha} - 1\right) \to 0$ as $\alpha \to 1$, whereas $\frac{y_2}{\overline{x} - x_2}\left(\frac{1}{\alpha} - 1\right) \to \infty$ as $\alpha \to 0$, which guarantees the desired result from the intermediate value theorem. Thus, u_2^*

[12] Note that there is another axiom closely related to SPI, *Local Independence* (Nagahisa (1991)), although it is applied only to economies with differentiable utility functions.

is compatible with **Assumption 1**. As $\Delta^P(\mathbf{u}^*, \mathbf{s}^t, \mathbf{z}) = \{p\}$ and φ satisfies SPI, $\mathbf{z} \in \varphi(\mathbf{u}^*, \mathbf{s}^t)$ and $\Delta^{SPI}(\mathbf{u}^*, \mathbf{s}^t, \mathbf{z}) = \{p\}$. Consider moving from $(\mathbf{u}^*, \mathbf{s}^t)$ to $(\mathbf{u}^*, \mathbf{s}^{t\prime})$. From the definition of $\mathbf{u}^*$, $\Delta^P(\mathbf{u}^*, \mathbf{s}^{t\prime}, \mathbf{z}) = \{p\}$. As φ satisfies IUS, $\mathbf{z} \in \varphi(\mathbf{u}^*, \mathbf{s}^{t\prime})$ and $\Delta^{IUS}(\mathbf{u}^*, \mathbf{s}^{t\prime}, \mathbf{z}) = \{p\}$. Consider moving from $(\mathbf{u}^*, \mathbf{s}^{t\prime})$ to $(\mathbf{u}, \mathbf{s}^{t\prime})$. As $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{z})$ and φ satisfies SPI, $\mathbf{z} \in \varphi(\mathbf{u}, \mathbf{s}^{t\prime})$. ■

SPI and IUS are necessary conditions for the labor sovereign triple implementation.

Theorem 1. *If a Pareto subsolution φ is* triply labor sovereign implementable*, then φ satisfies SPI.*

Theorem 2. *If a Pareto subsolution φ is* triply labor sovereign implementable*, then φ satisfies IUS.*

Next, we show that SPI and IUS together are sufficient for labor sovereign implementation. First, to construct our mechanism, let us introduce two mechanisms, defined as follows:

- $g^{\mathbf{w}}$ is such that for each $\mathbf{s}^t \in \mathcal{S}^n$ and each strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$, and for all $i \in N$,
$$g_i^{\mathbf{w}}(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \begin{cases} f\left(\sum s_k^t x_k\right) & \begin{array}{l}\text{if } x_i = \mu(\mathbf{x}_{-i}) \text{ and} \\ w_i > \max\left\{f\left(\sum s_k \bar{x}\right),\ \max_{j \neq i}\{w_j\}\right\},\end{array} \\ 0 & \text{otherwise,} \end{cases}$$
where $\mu(\mathbf{x}_{-i}) \equiv \max_{x_j < \bar{x}, j \neq i}\left\{\frac{x_j + \bar{x}}{2}\right\}$.

- $g^{\mathbf{s}}$ is such that for each $\mathbf{s}^t \in \mathcal{S}^n$ and each strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$, and for all $i \in N$,
$$g_i^{\mathbf{s}}(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \begin{cases} f\left(\sum s_k^t x_k\right) & \text{if } x_i = 0,\ w_i = 0, \text{ and } s_i > s_j \text{ for all } j \neq i, \\ 0 & \text{otherwise.} \end{cases}$$

The mechanism $g^{\mathbf{w}} \in \mathcal{G}$ assigns all of the produced output[13] to only one agent who provides the maximal *interior* amount of labor time and reports a maximal amount of demand for the output, where $\mu(\mathbf{x}_{-i})$ is a scheme to have agents find their best response strategies. The mechanism $g^{\mathbf{s}} \in \mathcal{G}$ assigns all

[13]Note that we implicitly assume that the mechanism coordinator can hold all of the produced output after the production process, although he may not monitor that process perfectly.

of the produced output to only one agent who demands no putput, reports the highest labor skill, and does not work at all.

Given $p \in \Delta$ and $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$, let $\varphi(p, \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} \equiv \left\{\mathbf{u} \in \mathcal{U}^n : (\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s}) \text{ and } p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w})\right\}$. Given $p \in \Delta$ and $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$, let $N(p, \mathbf{s}, \mathbf{x}, \mathbf{w}) \equiv \{i \in N : \exists (x'_i, w'_i) \in [0, \bar{x}) \times \mathbb{R}_{++}$ s.t. $\varphi\left(p, \mathbf{s}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right)^{-1} \neq \varnothing\big\}$.

Given a strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ such that $p^i = p$ for all i, an agent $i \in N(p, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is called a "*potential deviator*." Let us discuss the meaning of "potential deviators." Consider $\varphi(p, \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} = \varnothing$ and $N(p, \mathbf{s}, \mathbf{x}, \mathbf{w}) \neq \varnothing$. The first equation implies that the strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is inconsistent with the solution φ. The second equation $N(p, \mathbf{s}, \mathbf{x}, \mathbf{w}) \neq \varnothing$ implies that there is an agent i who can change his strategy to another one (p^i, s_i, x'_i, w'_i) so that the new strategy profile $\left(\mathbf{p}, \mathbf{s}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right)$ would become consistent with φ. That is, it may be this agent i who makes the current strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ inconsistent with φ. This means that $i \in N(p, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is a "potential deviator."

We propose a feasible sharing mechanism $g^* \in \mathcal{G}$ that works in each given $\mathbf{s}^t \in \mathcal{S}^n$ as follows:

For any $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = (p^i, s_i, x_i, w_i)_{i \in N} \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$,
Rule 1: if $f(\sum s_k x_k) = f(\sum s^t_k x_k)$, then
1-1: if there exists $p \in \Delta$ such that $p^i = p$ for all $i \in N$ and $\varphi(p, \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} \neq \varnothing$, then $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \mathbf{w}$,
1-2: if there exists $j \in N$ such that for some $p \in \Delta$, $p^i = p$ for all $i \neq j$, $\varphi(p^j, \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} = \varnothing$, and $j \in N(p, \mathbf{s}, \mathbf{x}, \mathbf{w})$, then $g^*_j(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) =$

$$\begin{cases} \max\left\{0, \min\left\{w'_j + \frac{p_x}{p_y}\left(s_j x_j - s_j x'_j\right), f\left(\sum s^t_k x_k\right)\right\}\right\} & \text{if } w_j > f\left(\sum s_k \bar{x}\right) \\ 0 & \text{otherwise,} \end{cases}$$

and $g^*_i(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$ for all $i \neq j$, where (x'_j, w'_j) s.t. $\varphi\left(p, \mathbf{s}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j}\right)^{-1} \neq \varnothing$,
1-3: for any other case, $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = g^{\mathbf{w}}(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$,

Rule 2: if $f(\sum s_k x_k) \neq f(\sum s^t_k x_k)$, then $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = g^{\mathbf{s}}(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$.

It is easy to see that g^* satisfies *forthrightness* (Saijo et al. (1996)) and the *best response property* (Jackson et al. (1994)). Moreover, it is a *price-quantity* type, and so it is *self-relevant* (Hurwicz (1960)). In addition, it is feasible.

Note that the total amount of output $f\left(\sum s_k^t x_k\right)$ is observable at the end of the production process, even without the true information on labor skills, because the coordinator can hold all of the produced output.

A strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$ is called φ-*consistent* if for some $p \in \Delta$, $p^i = p$ for all $i \in N$ and $\varphi\left(p, \mathbf{s}, \mathbf{x}, \mathbf{w}\right)^{-1} \neq \varnothing$. Given a strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$, g^* works as follows: First, g^* computes the expected amount of output $f\left(\sum s_k x_k\right)$ from $(\mathbf{s}, \mathbf{x})$ and compares this amount with the amount $f\left(\sum s_k^t x_k\right)$. Suppose that these two values coincide. Then, if the strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is φ-consistent, then g^* distributes $f\left(\sum s_k^t x_k\right)$ in accordance with $\mathbf{w}$ under Rule 1-1. If $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is not φ-consistent, and there is a unique potential deviator, then g^* punishes the agent according to Rule 1-2. For any other case of $f\left(\sum s_k x_k\right) = f\left(\sum s_k^t x_k\right)$, g^* assigns the same value as $g^{\mathbf{w}}$ under Rule 1-3. If $f\left(\sum s_k x_k\right) \neq f\left(\sum s_k^t x_k\right)$, then g^* assigns the same value as $g^{\mathbf{s}}$ under Rule 2.

We briefly explain how the mechanism g^* induces true information on labor skills at least for working agents, and how it attains desirable allocations, in the following parts (A) and (B), respectively:

(A) g^* distributes the total amount $f\left(\sum s_k^t x_k\right)$ of output among agents according to the agents' supplies of labor time $\mathbf{x}$, reported price vectors $\mathbf{p}$, reported labor skills $\mathbf{s}$, and demands for the output $\mathbf{w}$. The problem is that the agents' true labor skills are not observable and they may misrepresent their labor skills to increase their share of output. To solve this problem, a scheme of reward and punishment is set up in the mechanism as follows. First, if $f\left(\sum s_k x_k\right) \neq f\left(\sum s_k^t x_k\right)$, then clearly $\mathbf{s} \neq \mathbf{s}^t$ holds, and there must be at least one agent, say $j \in N$, who has misrepresented his labor skill, $s_j \neq s_j^t$, and supplied a positive amount of labor time $x_j > 0$. Then, this agent is definitely punished under the application of Rule 2.

Second, consider the case where $f\left(\sum s_k x_k\right) = f\left(\sum s_k^t x_k\right)$ but $\mathbf{s} \neq \mathbf{s}^t$. Then, there are at least two agents who have misrepresented their labor skills while supplying positive amounts of labor time, or someone, say j, has chosen "non working" while misrepresenting his labor skill. Let us put aside the latter case for the moment. In the former case, suppose that one such misrepresenting agent, say $j \in N$, changes from $x_j > 0$ to $x_j' = 0$, while reporting a sufficiently high level of labor skill. Then, the situation $f\left(\sum s_k x_k\right) = f\left(\sum s_k^t x_k\right)$ shifts to $f\left(s_j x_j' + \sum_{i \neq j} s_i x_i\right) \neq f\left(s_j^t x_j' + \sum_{i \neq j} s_i^t x_i\right)$. In this case, j may be better off under the application of Rule 2. Thus, the case may not correspond to an equilibrium situation. The following lemma actually confirms this insight.

Lemma 1: *Let Assumption 1 hold. Let the feasible sharing mechanism* $g^* \in \mathcal{G}$ *be given as above. Given an economy* $(\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$*, let a strategy profile* $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$ *be a Nash equilibrium of the game* $(g^*, \mathbf{u}, \mathbf{s}^t)$ *such that* $f\left(\sum s_k x_k\right) = f\left(\sum s_k^t x_k\right)$*. Then, it follows that* $s_i = s_i^t$ *for all* $i \in N$ *with* $x_i > 0$*.*

(B) We still need to explain how the mechanism implements the Pareto subsolution φ when all agents report their true labor skills, $\mathbf{s} = \mathbf{s}^t$. To do this, we adopt a scheme developed by Yoshihara (2000a).

As $\mathbf{s} = \mathbf{s}^t$, the strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$ corresponds only to Rule 1. Note that among the three subrules of Rule 1, only Rule 1-1 can realize a desirable allocation in equilibrium, while the other two are to punish agents who have deviated from the situation of Rule 1-1. Suppose that $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is φ-consistent. Then, $(\mathbf{x}, \mathbf{w})$ becomes the outcome from Rule 1-1, which is a φ-optimal allocation for some economy with $\mathbf{s} = \mathbf{s}^t$. However, this does not necessarily imply that $(\mathbf{x}, \mathbf{w})$ is φ-optimal for the actual economy. If $(\mathbf{x}, \mathbf{w})$ is not Pareto efficient for the actual economy, $(\mathbf{x}, \mathbf{w})$ should not be an equilibrium allocation. Rule 1-2 is necessary for solving this problem: if $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to Rule 1-1 and results in the allocation $(\mathbf{x}, \mathbf{w})$, any agent is able to benefit from another consumption vector on the budget line, determined by a supporting price at $(\mathbf{x}, \mathbf{w})$, by deviating to induce Rule 1-2. Therefore, if $(\mathbf{x}, \mathbf{w})$ is an equilibrium allocation, then $(\mathbf{x}, \mathbf{w})$ must be Pareto efficient.

We are ready to discuss the full characterizations of labor sovereignty triple implementation by examining the performance of g^*.

Theorem 3. *Let Assumption 1 hold. Then, if a Pareto subsolution* φ *satisfies SPI and IUS, then* φ *is* triply labor sovereign implementable by g^**.*

Note that this result does not depend upon the number of agents: any Pareto subsolution satisfying SPI and IUS can be triply implementable by g^* even in economies of *two agents.*

Corollary 1. *Let Assumption 1 hold. Then, a Pareto subsolution* φ *is* triply labor sovereign implementable *if and only if* φ *satisfies SPI and IUS.*

Note that even if any agent i can control his contribution by selecting $\widetilde{s}_i \in [0, s_i^t]$, Corollary 1 remains robust. In such a situation, the observable total amount of output is given not by $f\left(\sum s_k^t x_k\right)$, but by $f\left(\sum \widetilde{s}_k x_k\right)$, where $\widetilde{s}_k \in [0, s_k^t]$ for any $k \in N$. Then, the coordinator can still compare the expected amount $f\left(\sum s_k x_k\right)$ with $f\left(\sum \widetilde{s}_k x_k\right)$, and there is no difficulty in g^* functioning.

From Corollary 1, we gain two new insights on the implementability of Pareto subsolutions in production economies with unequal skills. First, we can classify which solutions remain implementable, if the profile of labor skills becomes unknown to the coordinator, compared with the situation when the profile was known to the coordinator. Note that it is easy to see that any Pareto subsolution is labor sovereign implementable if and only if it satisfies SPI, whenever the labor skills are known to the coordinator. Second, because the two axioms, SPI and IUS, can be seen as the axioms of responsibility and compensation, Corollary 1 indicates that the implementable solutions should have a rather strong property on responsibility, and a rather weak property on compensation.

4 Characterizations

By applying Corollary 1, let us examine which Pareto subsolutions are implementable. First, we discuss the three variations of the *Walrasian solution*:

Definition 4. *Given a profit share* $\theta = (\theta_1, \ldots, \theta_n) \in [0,1]^n$ *with* $\sum \theta_i = 1$, *a solution* φ^W *is* Walrasian *if for any* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\mathbf{z} \in \varphi^W(\mathbf{e})$ *implies that there exists* $p = (p_x, p_y) \in \Delta$ *such that:*
(i) *for all* $\mathbf{z}' \in Z(\mathbf{s}^t)$, $\sum \left(p_y y_i' - p_x s_i^t x_i'\right) \leq \sum \left(p_y y_i - p_x s_i^t x_i\right)$*;*
(ii) *for all* $i \in N$ *and any* $(x, y) \in [0, \bar{x}] \times \mathbb{R}_+$, *if* $u_i(x, y) > u_i(z_i)$, *then* $p_y y - p_x s_i^t x > \theta_i \sum \left(p_y y_i - p_x s_i^t x_i\right)$.

Definition 5 (Roemer and Silvestre (1989, 1993))**.** *A solution* φ^{PR} *is the* proportional solution *if for any* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\mathbf{z} \in \varphi^{PR}(\mathbf{e})$ *implies that:*

(i) $\mathbf{z}$ *is Pareto efficient for* $\mathbf{e}$*;*
(ii) *for all* $i \in N$, $y_i = \frac{s_i^t x_i}{\sum s_j^t x_j} \sum y_j$.

Definition 6 (Roemer and Silvestre (1989))**.** *A solution* φ^{EB} *is the* equal benefit solution *if for any* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\mathbf{z} \in \varphi^{EB}(\mathbf{e})$ *implies that:*

(i) $\mathbf{z}$ *is Pareto efficient for* $\mathbf{e}$*;*
(ii) *there exists an efficiency price* $p = (p_x, p_y) \in \Delta$ *for* $\mathbf{z}$ *at* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *such that for all* $i \in N$, $p_y y_i - p_x s_i^t x_i = \frac{1}{n} \sum \left(p_y y_j - p_x s_j^t x_j \right)$.

From Yoshihara (2000), all three solutions above are known to satisfy SPI. Thus, to confirm the implementability, it suffices to examine IUS. Then:

Lemma 6. *The Walrasian solution* φ^W *satisfies IUS.*

Lemma 7. *The proportional solution* φ^{PR} *satisfies IUS.*

Lemma 8. *The equal benefit solution* φ^{EB} *satisfies IUS.*[14]

Thus, all three solutions above are implementable by sharing mechanisms.

Corollary 2. *Let Assumption 1 hold. Then,* φ^W *is* triply labor sovereign implementable*.*

Corollary 3. *Let Assumption 1 hold. Then,* φ^{PR} *is* triply labor sovereign implementable*.*

Corollary 4. *Let Assumption 1 hold. Then,* φ^{EB} *is* triply labor sovereign implementable*.*

Is there any non-Walrasian type of allocation rule that is implementable? To discuss this, let us consider the following types of allocation rules:

Definition 7 (Yoshihara (2000a))**.** *Let* $\lambda \in [0, 1]$*. Then, a solution* $\varphi^{\lambda ER}$ *is the* λ-effort-reward solution *if for any* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\mathbf{z} \in \varphi^{\lambda ER}(\mathbf{e})$ *implies:*

(i) $\mathbf{z}$ *is Pareto efficient for* $\mathbf{e}$*;*
(ii) *for all* $i \in N$, $y_i = \left\{ \lambda \frac{x_i}{\sum x_j} + (1 - \lambda) \frac{1}{n} \right\} f \left(\sum s_j^t x_j \right)$.

The solution $\varphi^{\lambda ER}$ is well defined for any $\lambda \in [0, 1]$, according to Yoshihara (2000a). In addition, it satisfies the *equal-reward-for-equal-labor-time* (**EREL**) principle (Kranich (1994)). It is easy to see that $\varphi^{\lambda ER}$ satisfies SPI for any $\lambda \in [0, 1]$. Moreover, since $\varphi^{\lambda ER}$ distributes output completely independently of skills, it is obvious that $\varphi^{\lambda ER}$ satisfies IUS for any $\lambda \in [0, 1]$. Thus:

[14] The proof of **Lemma 8** would be a variation of that of **Lemma 6**.

Corollary 5. *Let Assumption 1 hold. Then, for any* $\lambda \in [0,1]$, $\varphi^{\lambda ER}$ *is* triply labor sovereign implementable.

As $\varphi^{\lambda ER}$ is not a variant of the Walrasian rules, but an equitable allocation rule in terms of EREL, Corollary 5 indicates the existence of equitable and implementable Pareto subsolutions.

Let us consider allocation rules that meet the *equal-opportunity-for-budget-set* (**EOB**) principle. Van Parijs (1995) argued that this principle was a condition for the basic income policy. The EOB principle may have the following formulation:

Set-inclusion Undomination (SIU).[15] *For all* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *and all* $\mathbf{z} \in \varphi(\mathbf{e})$, *there exists* $p \in \Delta^P(\mathbf{e}, \mathbf{z})$ *such that for any* $i, j \in N$, $B(p, s_i^t, z_i) \nsubseteq B(p, s_j^t, z_j)$ *and* $B(p, s_i^t, z_i) \nsupseteq B(p, s_j^t, z_j)$, *where*
$B(p, s_i^t, z_i) \equiv \{(x, y) \in [0, \bar{x}] \times \mathbb{R}_+ : p_y y - p_x s_i^t x \leq p_y y_i - p_x s_i^t x_i\}$.

Note that Pareto efficiency and SIU are compatible. For instance, the *$\widetilde{u}$-reference welfare equivalent budget solution* (Fleurbaey and Maniquet (1996)) is a Pareto subsolution satisfying SIU. In addition, such a solution is desirable in terms of responsibility and compensation. However, any Pareto subsolution satisfying SIU cannot be implementable, as shown in the following:

Corollary 6. *Any Pareto subsolution satisfying SIU is not* triply labor sovereign implementable.

In summary, the above characterizations give us a new insight on implementation of Pareto subsolutions in production economies. It has been already shown by Dutta, et. al (1995), Saijo, et. al (1999), and Yoshihara (2000), under the implicit assumption that the production skills of agents are known to the coordinator, that the three variations of the Walrasian types are implementable by natural mechanisms, whereas the *no-envy and efficient solution* is not implementable. In this paper, we have seen that the three variations of the Walrasian types and $\varphi^{\lambda ER}$ are implementable by sharing mechanisms even if the skills of agents are private information. In contrast, any Pareto subsolution satisfying SIU cannot be implementable once the skills are private information. Among such unimplementable solutions, note that the $\widetilde{u}$-reference welfare equivalent budget solution can be

[15] Note that van Parijs (1995) formulated the EOB principle as Undominated Diversity (Parijs (1995)), which is stronger than SIU.

implementable by sharing mechanisms whenever the skills are known to the coordinator, because it satisfies SPI.

5 Concluding Remarks

We characterized implementation by sharing mechanisms in production economies with unequal labor skills. The class of Pareto subsolutions implementable by sharing mechanisms is characterized by two axioms, Supporting Price Independence and Independence of Unused Skills. Based upon this characterization, we found that the Walrasian, proportional, equal benefit, and λ-effort-reward solutions were implementable, whereas any Pareto subsolution satisfying Set-inclusion Undomination failed to be implementable if individuals' labor skills were unknown to the coordinator. This result indicates the impossibility of implementing a Pareto subsolution that provides every agent with the equal opportunity for budget sets, whenever labor skills are private information.

The workability of our proposed feasible sharing mechanism depends on two implicit but reasonable assumptions: first, although every individual i's labor performance, $s_i x_i$, measured in efficiency units, is imperfectly observable and unverifiable by the coordinator, his working time, x_i, is perfectly observable. Second, despite such imperfect observability, the coordinator can observe the real amount of outputs produced in the economy, so that he can compare this amount with the expected amount of outputs based on the announcements of the individuals. We believe that these implicit assumptions are reasonable enough to formulate the essential aspect of informational asymmetry in production economies. However, it is open discussion whether implementation of Pareto subsolutions by natural mechanisms in production economies with possibly unequal labor skills holds without these implicit assumptions.

6 Appendix

6.1 Proofs of Theorems 1 and 2

Proof of Theorem 1. Suppose that a Pareto subsolution φ is triply labor sovereign implementable. Then, there exists a feasible sharing mechanism $g \in \mathcal{G}$ that satisfies conditions (i)-(iv) in Definition 3. For any $\mathbf{z} = (x_i, y_i)_{i \in N} \in$

$[0,\bar{x}]^n \times \mathbb{R}^n_+$ and any $\mathbf{e} = (u_i, s^t_i)_{i\in N}$, $\mathbf{e}' = (u'_i, s^{t\prime}_i)_{i\in N} \in \mathcal{E}$ where $s^t_i = s^{t\prime}_i$ ($\forall i \in N$), suppose that $\mathbf{z} \in \varphi(\mathbf{e})$ and that there exists a price $p \in \Delta^P(\mathbf{e},\mathbf{z}) \cap \Delta^P(\mathbf{e}',\mathbf{z})$. From (ii), $(\mathbf{p},\mathbf{s}^t,\mathbf{x},\mathbf{y}) \in NE(g,\mathbf{e})$ and $g(\mathbf{p},\mathbf{s}^t,\mathbf{x},\mathbf{y}) = \mathbf{y}$, where $\mathbf{p} = (p)_{i\in N}$, $\mathbf{s}^t = (s^t_i)_{i\in N}$, $\mathbf{x} = (x_i)_{i\in N}$, and $\mathbf{y} = (y_i)_{i\in N}$. Therefore, from (iii), $g_i\left(\mathbf{p}_{p^{i'}}, \mathbf{s}^t_{s'_i}, \mathbf{x}_{x'_i}, \mathbf{y}_{w'_i}\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s^t_i (x'_i - x_i)\right\}$ for all $i \in N$ and all $(p^{i'}, s'_i, x'_i, w'_i) \in \Delta\times\mathcal{S}\times[0,\bar{x}]\times\mathbb{R}_+$. As $p \in \Delta^P(\mathbf{e}',\mathbf{z})$, this implies $(\mathbf{p},\mathbf{s}^t,\mathbf{x},\mathbf{y}) \in NE(g,\mathbf{e}')$ and $(\mathbf{x},\mathbf{y}) \in NA(g,\mathbf{e}')$. Hence, $\mathbf{z} \in \varphi(\mathbf{e}')$ by (i). ■

Proof of Theorem 2. Suppose that a Pareto subsolution φ is triply labor sovereign implementable. Then, there exists a feasible sharing mechanism $g \in \mathcal{G}$ that satisfies conditions (i)-(iv) in Definition 3. For any $\mathbf{z} = (x_i, y_i)_{i\in N} \in [0,\bar{x}]^n \times \mathbb{R}^n_+$ and any $\mathbf{e} = (u_i, s^t_i)_{i\in N}$, $\mathbf{e}' = (u'_i, s^{t\prime}_i)_{i\in N} \in \mathcal{E}$, where $u_i = u'_i$ for all $i \in N$ and $s^t_i = s^{t\prime}_i$ for all $i \in N$ with $x_i > 0$, suppose that $\mathbf{z} \in \varphi(\mathbf{e})$ and that there exists a price $p \in \Delta^P(\mathbf{e},\mathbf{z}) \cap \Delta^P(\mathbf{e}',\mathbf{z})$. From (ii), $(\mathbf{p},\mathbf{s}^t,\mathbf{x},\mathbf{y}) \in NE(g,\mathbf{e})$ and $g(\mathbf{p},\mathbf{s}^t,\mathbf{x},\mathbf{y}) = \mathbf{y}$, where $\mathbf{p} = (p)_{i\in N}$, $\mathbf{s}^t = (s^t_i)_{i\in N}$, $\mathbf{x} = (x_i)_{i\in N}$, and $\mathbf{y} = (y_i)_{i\in N}$, which implies $(\mathbf{p},\mathbf{s}^{t\prime},\mathbf{x},\mathbf{y}) \in NE(g,\mathbf{e})$ and $g(\mathbf{p},\mathbf{s}^{t\prime},\mathbf{x},\mathbf{y}) = g(\mathbf{p},\mathbf{s}^t,\mathbf{x},\mathbf{y}) = \mathbf{y}$ by (iv). Then, from (iii), $g_i\left(\mathbf{p}_{p^{i'}}, \mathbf{s}^{t\prime}_{s'_i}, \mathbf{x}_{x'_i}, \mathbf{y}_{w'_i}\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s^t_i (x'_i - x_i)\right\}$ for all $i \in N$ and all $(p^{i'}, s'_i, x'_i, w'_i) \in \Delta \times \mathcal{S} \times [0,\bar{x}] \times \mathbb{R}_+$. As $p \in \Delta^P(\mathbf{e}',\mathbf{z})$, this implies that $(\mathbf{p},\mathbf{s}^{t\prime},\mathbf{x},\mathbf{y}) \in NE(g,\mathbf{e}')$ and $(\mathbf{x},\mathbf{y}) \in NA(g,\mathbf{e}')$. Hence, $\mathbf{z} \in \varphi(\mathbf{e}')$ from (i). ■

6.2 Proof of Theorem 3

Proof of Lemma 1. Suppose that there exists $j \in N$ with $s_j \neq s^t_j$ and $x_j > 0$. Let $N(\mathbf{s},\mathbf{x})$ be the set of all such j. As $f(\sum s_i x_i) = f(\sum s^t_i x_i)$, $N(\mathbf{s},\mathbf{x})$ is not a singleton. Moreover, under Rule 2, any $j \in N(\mathbf{s},\mathbf{x})$ can obtain $y'_j = f\left(\sum_{i\neq j} s^t_i x_i\right) > 0$ with $s'_j > \max_{i\neq j}\{s_i\}$, $x'_j = 0$, and $w'_j = 0$. Note that:

$$\begin{aligned}\sum_{j\in N(\mathbf{s},\mathbf{x})} y'_j &= \sum_{j\in N(\mathbf{s},\mathbf{x})} f\left(\sum_{i\neq j} s^t_i x_i\right) = \sum_{j\in N(\mathbf{s},\mathbf{x})} f\left(\sum_{i\in N(\mathbf{s},\mathbf{x})\setminus\{j\}} s^t_i x_i + \sum_{k\notin N(\mathbf{s},\mathbf{x})} s^t_k x_k\right)\\ &\geq \sum_{j\in N(\mathbf{s},\mathbf{x})} f\left(s^t_j x_j + \sum_{k\notin N(\mathbf{s},\mathbf{x})} s^t_k x_k\right) \text{ (as } N(\mathbf{s},\mathbf{x}) \text{ is not a singleton)}\end{aligned}$$

$$
\begin{aligned}
&\geq f\left(\sum_{j\in N(\mathbf{s},\mathbf{x})}\left(s_j^t x_j+\sum_{k\notin N(\mathbf{s},\mathbf{x})} s_k^t x_k\right)\right) \text{ (as } f \text{ is concave and } f(0)\geq 0)\\
&\geq f\left(\sum_{j\in N(\mathbf{s},\mathbf{x})} s_j^t x_j+\sum_{k\notin N(\mathbf{s},\mathbf{x})} s_k^t x_k\right)\\
&= f\left(\sum s_k^t x_k\right)\geq \sum_{j\in N(\mathbf{s},\mathbf{x})} y_j \equiv \sum_{j\in N(\mathbf{s},\mathbf{x})} g_j^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}).
\end{aligned}
$$

Hence, there is $j \in N(\mathbf{s},\mathbf{x})$ with $y_j' \geq y_j$. Note that for $x_j' = 0$, $u_j(x_j', y_j') \geq u_j(x_j', y_j) \geq u_j(x_j, y_j)$ holds, and $u_j(x_j', y_j') > u_j(x_j', y_j)$ if $y_j = 0$ by Assumption 1, whereas $u_j(x_j', y_j) > u_j(x_j, y_j)$ if $y_j > 0$ from the strict monotonicity of u_j. Hence, agent j has an incentive to change from x_j to $x_j' = 0$ to obtain y_j'. Thus, $(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})$ does not constitute a Nash equilibrium. ■

Lemma 2: *Let Assumption 1 hold. Let the feasible sharing mechanism $g^* \in \mathcal{G}$ be given as above. Then,* g^* implements *any Pareto subsolution φ satisfying SPI and IUS on $\mathcal{E}$* in Nash equilibria*.*

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ be given.

(1) First, we show that $\varphi(\mathbf{e}) \subseteq NA(g^*, \mathbf{e})$. Let $\mathbf{z} = (\mathbf{x}, \mathbf{y}) \in \varphi(\mathbf{e})$. Let the strategy profile be $(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) = (p^i, s_i^t, x_i, y_i)_{i\in N} \in (\Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^n$ such that $p^i = p$ for all $i \in N$, where $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \mathbf{z})$. Then, $g^*(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) = \mathbf{y}$ from Rule 1-1. Suppose that an individual $j \in N$ deviates from $\left(p^j, s_j^t, x_j, y_j\right)$ to $\left(p^{j\prime}, s_j', x_j', w_j'\right) \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$. Note from Assumption 1 and the continuity of utility functions that if $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}^t_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) = 0$, then the deviation provides no reward to j.

As every $i \neq j$ truly reports his skill, $s_j' = s_j^t$ is necessary to induce some subrule of Rule 1 with $x_j' > 0$. That is, the deviation cannot induce Rule 1-3 as long as $x_j' > 0$, which is a necessary condition for the deviator to consume a positive output under Rule 1-3. If the deviation induces Rule 2, then $x_j' > 0$, so that $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}^t_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) = 0$. In fact, if $x_j' = 0$, then $\sum_{i\neq j} s_i^t x_i + s_j' x_j' = \sum_{i\neq j} s_i^t x_i + s_j^t x_j'$, so that $f\left(\sum_{i\neq j} s_i^t x_i + s_j' x_j'\right) = f\left(\sum_{i\neq j} s_i^t x_i + s_j^t x_j'\right)$, which contradicts the fact that Rule 2 is induced.

Suppose that the deviation induces Rule 1-2. If $x_j' > 0$, then $s_j' = s_j^t$ and $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}^t_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) \leq \max\left\{0, y_j + \frac{p_x}{p_y}\left(s_j^t x_j' - s_j^t x_j\right)\right\}$, which implies that j cannot gain from his deviation. Let $x_j' = 0$. The application of Rule 1-2 implies that there exist x_j'' and w_j'' such that $\varphi\left(p, \mathbf{s}^t_{s_j'}, \mathbf{x}_{x_j''}, \mathbf{y}_{w_j''}\right)^{-1} \neq \varnothing$. Note that the information of $\mathbf{p}$ induces $w_j'' + \frac{p_x}{p_y}\left(s_j' x_j' - s_j' x_j''\right) = y_j + \frac{p_x}{p_y}\left(s_j' x_j' - s_j^t x_j\right)$. Thus, $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}^t_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) \leq \max\left\{0, y_j + \frac{p_x}{p_y}\left(s_j' x_j' - s_j^t x_j\right)\right\}$, which again implies that j cannot gain from his deviation.

Finally, if the deviation induces Rule 1-1, then $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}^t_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) = w_j' = f\left(\sum\limits_{i \neq j} s_i^t x_i + s_j^t x_j'\right) - \sum\limits_{i \neq j} y_i$. Thus, $\mathbf{z} \in P(\mathbf{e})$ implies that there is no additional benefit for j.

(2) Second, we show that $NA(g^*, \mathbf{e}) \subseteq \varphi(\mathbf{e})$. Let $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = (p^i, s_i, x_i, w_i)_{i \in N} \in NE(g^*, \mathbf{e})$.

Suppose that $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ induces Rule 2. If $N^0(\mathbf{x}) \equiv \{i \in N : x_i = 0\} = \varnothing$, then $g_i^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$ for all $i \in N$. Suppose that for any $j \in N$, $\sum_{i \neq j} s_i x_i = \sum_{i \neq j} s_i^t x_i$ holds. Then, $(n-1) \cdot (\sum s_i x_i) = (n-1) \cdot (\sum s_i^t x_i)$, which contradicts the fact that Rule 2 is induced. Thus, there exists at least one individual $j \in N$ such that $\sum_{i \neq j} s_i x_i \neq \sum_{i \neq j} s_i^t x_i$, who obtains $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{w}_{w_j'}\right) > 0$ under Rule 2 by $s_j' > \max\{s_i\}_{i \in N}$, $x_j' = 0$, and $w_j' = 0$.

Let $N^0(\mathbf{x}) \neq \varnothing$. Then, for any $j \in N^0(\mathbf{x})$, if the agent's deviating strategy $\left(s_j', x_j', w_j'\right)$ is such that $s_j' > s_i$ for all $i \neq j$ and $\left(x_j', w_j'\right) = (0, 0)$, then the agent obtains all $f(\sum s_k^t x_k)$ under Rule 2. Thus, if $\#N^0(\mathbf{x}) \geq 2$, then such a strategy profile cannot constitute a Nash equilibrium under Rule 2.

Let $\#N^0(\mathbf{x}) = 1$ and $\#N \backslash N^0(\mathbf{x}) \geq 2$. Then, if for any $j \in N \backslash N^0(\mathbf{x})$, $\sum_{i \in N \backslash (N^0(\mathbf{x}) \cup \{j\})} s_i x_i = \sum_{i \in N \backslash (N^0(\mathbf{x}) \cup \{j\})} s_i^t x_i$ holds, then $(n-2) \cdot \left(\sum_{i \in N \backslash N^0(\mathbf{x})} s_i x_i\right) = (n-2) \cdot \left(\sum_{i \in N \backslash N^0(\mathbf{x})} s_i^t x_i\right)$, which contradicts the fact that Rule 2 is induced. Thus, there exists at least one individual $j \in N \backslash N^0(\mathbf{x})$ such that $\sum_{i \in N \backslash (N^0(\mathbf{x}) \cup \{j\})} s_i x_i \neq \sum_{i \in N \backslash (N^0(\mathbf{x}) \cup \{j\})} s_i^t x_i$, who obtains $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{w}_{w_j'}\right) > 0$ under Rule 2 by $s_j' > \max\{s_i\}_{i \in N}$, $x_j' = 0$, and $w_j' = 0$.

Let $\#N^0(\mathbf{x}) = 1$ with $N^0(\mathbf{x}) = \{i\}$ and $\#N \backslash N^0(\mathbf{x}) = 1$ with $N \backslash N^0(\mathbf{x}) =$

$\{j\}$. If $w_i > 0$, then i can enjoy $g_i^*\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) = f\left(s_j^t x_j\right)$ under Rule 2 by $x_i' = 0$ and $w_i' = 0$. If $w_i = 0$, then j can enjoy $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{w}_{w_j'}\right) = f\left(s_j^t x_j'\right)$ under Rule 1-3 by $p^{j\prime} = p^i$, $s_j' = s_j^t$, $x_j' = \frac{\overline{x}}{2}$, and $w_j' > f\left(s_j^t \overline{x} + s_i \overline{x}\right)$. Thus, if $\# N^0(\mathbf{x}) = 1$, such a strategy profile cannot constitute a Nash equilibrium.

Suppose that $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ induces Rule 1-2 or 1-3. Then, there exists $j \in N$ such that $g_j^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$. For this j, $s_j = s_j^t$ or $x_j = 0$ by Lemma 1. Thus, if j deviates to $x_j' > 0$ and $s_j' = s_j^t$, then the agent induces either Rule 1-1, Rule 1-2, or Rule 1-3. Let $p^k \neq p^l$ for some $k, l \neq j$. Then, j obtains $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{w}_{w_j'}\right) > 0$ under Rule 1-3 by $p^k \neq p^{j\prime} \neq p^l$, $s_j' = s_j^t$, $x_j' = \mu(\mathbf{x}_{-j}) < \overline{x}$, and $w_j' > \max\left\{f\left(\sum_{i\neq j} s_i \bar{x} + s_j' \bar{x}\right), \max_{i\neq j}\{w_i\}\right\}$. Let $p^k = p^l = p$ for all $k, l \neq j$. In addition, let $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ be such that Rule 1-2 applies. Then, $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}, \mathbf{x}_{x_j'}, \mathbf{w}_{w_j'}\right) > 0$ under Rule 1-2 by $p^{j\prime} = p$, $s_j' = s_j^t$, $w_j' > \max\left\{f\left(\sum_{i\neq j} s_i \bar{x} + s_j' \bar{x}\right), \max_{i\neq j}\{w_i\}\right\}$, and an appropriate x_j'. By the same type of deviating strategy, j obtains a positive amount of output when $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is such that Rule 1-3 applies.

Thus, $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ should be such that Rule 1-1 applies, and $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \mathbf{w}$. Then, there exists $\mathbf{u}' \in \mathcal{U}^n$ such that $p \in \Delta^{SPI}(\mathbf{u}', \mathbf{s}, \mathbf{x}, \mathbf{w})$ where $p^i = p$ for all $i \in N$. Moreover, $p \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{w})$ and $(\mathbf{x}, \mathbf{w}) \in P(\mathbf{u}, \mathbf{s}^t)$, because otherwise, some j has an incentive to deviate to Rule 1-2. Let $(\mathbf{u}', \mathbf{s}') \in \mathcal{E}$ be such that $s_i' = \min\{s_i, s_i^t\}$ for each $i \in N$ with $x_i = 0$ and $s_i' = s_i$ ($= s_i^t$ by Lemma 1) for every $i \in N$ with $x_i > 0$. First, from the definition of $\mathbf{s}'$, $(\mathbf{x}, \mathbf{w}) \in P(\mathbf{u}', \mathbf{s})$ implies $(\mathbf{x}, \mathbf{w}) \in P(\mathbf{u}', \mathbf{s}')$. Hence, from IUS, $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}', \mathbf{s})$ implies $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}', \mathbf{s}')$. Next, from the definition of $\mathbf{s}'$, $(\mathbf{x}, \mathbf{w}) \in P(\mathbf{u}, \mathbf{s}^t)$ implies $(\mathbf{x}, \mathbf{w}) \in P(\mathbf{u}, \mathbf{s}')$. Note here $p \in \Delta^{SPI}(\mathbf{u}', \mathbf{s}, \mathbf{x}, \mathbf{w}) \cap \Delta^P(\mathbf{u}', \mathbf{s}', \mathbf{x}, \mathbf{w})$ and $p \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{w}) \cap \Delta^P(\mathbf{u}, \mathbf{s}', \mathbf{x}, \mathbf{w})$. Thus, from SPI, $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}', \mathbf{s}')$ implies $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s}')$. Finally, as $(\mathbf{x}, \mathbf{w}) \in P(\mathbf{u}, \mathbf{s}^t)$ and $s_i' = s_i^t$ for all $i \in N$ with $x_i > 0$, $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s}')$ implies $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s}^t)$ from IUS. ■

Lemma 3: *Let Assumption 1 hold. Let the feasible sharing mechanism $g^* \in \mathcal{G}$ be given as above. Then, g^** implements *any Pareto subsolution φ satisfying SPI and IUS on $\mathcal{E}$* in strong equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ be given. As $NA(g^*, \mathbf{e}) = \varphi(\mathbf{e})$, we have only to show that

$NA(g^*, \mathbf{e}) \subseteq SNA(g^*, \mathbf{e})$. Suppose that there exists $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in NE(g^*, \mathbf{e})$ such that for some $T \subseteq N$ with $2 \leq \#T < n$ and some $(p^{i\prime}, s_i', x_i', w_i')_{i \in T} \in (\Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^{\#T}$,

$$u_j \left(x_j, g_j^* (\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})\right) < u_j \left(x_j', g_j^* \left((p^{i\prime}, s_i', x_i', w_i')_{i \in T}, \left(p^k, s_k, x_k, w_k\right)_{k \in T^c}\right)\right)$$

holds for all $j \in T$. Note that $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is such that Rule 1-1 applies and $g^* (\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \mathbf{w}$, as is shown in the proof of Lemma 2. Moreover, $(\mathbf{x}, \mathbf{w}) \in P(\mathbf{u}, \mathbf{s}^t)$.

From the construction of g^*, there is at most one agent who obtains a positive amount of output under Rules 1-2, 1-3, and Rule 2. Thus, from Assumption 1, the deviation by T should induce Rule 1-1. Then:

$$g^* \left((p^{i\prime}, s_i', x_i', w_i')_{i \in T}, \left(p^k, s_k, x_k, w_k\right)_{k \in T^c}\right) = \left((w_i')_{i \in T}, (w_k)_{k \in T^c}\right).$$

Hence, the supposition of the deviation by T contradicts $(\mathbf{x}, \mathbf{w}) \in P(\mathbf{u}, \mathbf{s}^t)$. Thus, $NA(g^*, \mathbf{e}) \subseteq SNA(g^*, \mathbf{e})$. ■

Lemma 4: *Let Assumption 1 hold. Let the feasible sharing mechanism* $g^* \in \mathcal{G}$ *be given as above. Then,* $\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}$ implements *any Pareto subsolution* φ *satisfying SPI and IUS on* $\mathcal{E}$ in subgame perfect equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ be given. From Lemma 2, $NA(g^*, \mathbf{e}) = \varphi(\mathbf{e})$. Moreover, $SPA\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}, \mathbf{e}\right) \subseteq NA(g^*, \mathbf{e})$. Hence, we have only to show that $\varphi(\mathbf{e}) \subseteq SPA\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}, \mathbf{e}\right)$.

First, we show that in every second-stage subgame, there is at least one Nash equilibrium strategy. We use $NE\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$ to denote the set of Nash equilibria of $\left(\Gamma_{g}^{\mathbf{x} \vdash \mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$. Let $\mathbf{m}^{\mathbf{s}*} : [0, \bar{x}]^n \to \Delta^n \times \mathcal{S}^n \times \mathbb{R}_+^n$ be such that for each second-stage subgame $\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$, $\mathbf{m}^{\mathbf{s}*}(\mathbf{x}) = (\mathbf{p}, \mathbf{s}, \mathbf{w})$, where for all $i \in N$

$$\left(p^i, s_i, w_i\right) = \begin{cases} \left((0,1), s_i^t, f\left(\sum s_k^t \bar{x}\right) + 1\right) & \text{if } x_i \neq \mu(\mathbf{x}_{-i}) \\ \left((0,1), s_i^t, f\left(\sum s_k^t \bar{x}\right) + 2\right) & \text{otherwise.} \end{cases}$$

Then, $(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$. Note that $g^* (\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to Rule 1-3. As $p^i = (0, 1)$ for all $i \in N$, no individual can induce Rule 1-1 by deviating his strategy. Inducing Rule 1-2 with $\left(p^{j\prime}, s_j', w_j'\right)$ would not be beneficial for any $j \in N$, as $p^i = (0, 1)$ for another $i \neq j$, and there does not exist $\left(x_j'', w_j''\right) \in [0, \bar{x}] \times \mathbb{R}_{++}$ such that $\varphi\left(p^i, \mathbf{s}_{s_j'}, \mathbf{x}_{x_j''}, \mathbf{w}_{w_j''}\right)^{-1} \neq \varnothing$.

Moreover, if any $i \in N$ can induce Rule 2 by deviating his strategy, then it must be the case that $x_i > 0$, which implies that this deviation is not beneficial for i. Finally, if any $i \in N$ can deviate to induce Rule 1-3 again, then such a deviation does not give the agent any additional benefit, because the individual i who has $x_i = \mu(\mathbf{x}_{-i})$ is already fixed in the first-stage game. Thus, $(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$.

Now, we show that for this $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, if $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}}, \widehat{\mathbf{y}}) \in \varphi(\mathbf{e})$, then there exists a subgame perfect equilibrium whose corresponding outcome is $\widehat{\mathbf{z}}$. We define a strategy profile of the game $\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}, \mathbf{e}\right)$ as follows.
(1) In Stage 1, every individual i supplies $\widehat{x}_i$.
(2) In Stage 2, the agents have $\mathbf{m}^{\mathbf{s}} : [0, \bar{x}]^n \rightarrow \Delta^n \times \mathcal{S}^n \times \mathbb{R}_+^n$ such that $\mathbf{m}^{\mathbf{s}}(\mathbf{x}) = (\mathbf{p}, \mathbf{s}, \mathbf{w})$, which is defined as follows:
(2-1): if $\mathbf{x} = \widehat{\mathbf{x}}$ in Stage 1, then for any $i \in N$, $m_i^{\mathbf{s}}(\mathbf{x}) = (p^i, s_i, w_i) = (p, s_i^t, \widehat{y}_i)$ where $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}})$;
(2-2): if $\mathbf{x} = \widehat{\mathbf{x}}_{x'_j}$, where $x'_j \neq \widehat{x}_j$, in Stage 1, then for this $j \in N$:

$$m_j^{\mathbf{s}}(\mathbf{x}) = \left(p^j, s_j, w_j\right) = \left((0,1), s_j^t, f\left(\sum s_k^t \bar{x}\right) + 1\right),$$

and for all $i \neq j$:

$$m_i^{\mathbf{s}}(\mathbf{x}) = \left(p^i, s_i, w_i\right) = \begin{cases} (p, s_i^t, \widehat{y}_i) & \text{if } x_i \neq \mu(\mathbf{x}_{-i}), \\ ((1,0), s_i^t, f(\sum s_k^t \bar{x}) + 2) & \text{otherwise,} \end{cases}$$

where $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}})$;
(2-3): in any other case, $\mathbf{m}^{\mathbf{s}}(\mathbf{x}) = \mathbf{m}^{\mathbf{s}*}(\mathbf{x})$.

Note that for the subgame of (2-1), $\mathbf{m}^{\mathbf{s}}(\mathbf{x}) = (\mathbf{p}, \mathbf{s}^t, \widehat{\mathbf{y}}) \in NE\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$, because $(\mathbf{p}, \mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}}) \in NE(g^*, \mathbf{e})$. In addition, $\mathbf{m}^{\mathbf{s}}(\mathbf{x}) \in NE\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$ for any subgame $\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$ of the case (2-3), as we have already shown.

In addition, we show that for the subgame of (2-2), $\mathbf{m}^{\mathbf{s}}(\mathbf{x}) = (\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^{\mathbf{x} \vdash \mathbf{m}}(\mathbf{x}), \mathbf{e}\right)$. As $\mathbf{s} = \mathbf{s}^t$, if an agent deviates to induce Rule 2, then he has to supply a positive amount of labor time, which implies that the deviation is not beneficial for the agent by the construction of Rule 2. For all $i \neq j$, let $x_i \neq \mu(\mathbf{x}_{-i})$ in (2-2). Then, $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to Rule 1-2. No $i \neq j$ can induce Rule 1-1. Nor can any agent other than j consume a positive output under Rule 1-2, because $p^j = (0,1)$ and $\varphi\left(p^j, \mathbf{s}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right)^{-1} = \varnothing$ for any $(x'_i, w'_i) \in [0, \bar{x}] \times \mathbb{R}_{++}$. Moreover, no $i \neq j$ can consume a positive output under Rule 1-3, because $x_i \neq \mu(\mathbf{x}_{-i})$. In contrast, j can deviate to induce Rule 1-1 or Rule 1-2. When Rule 1-1 is induced by j, his output consumption is $f\left(s_j^t x_j' + \sum_{k\neq j} s_k^t \widehat{x}_k\right) - \sum_{k\neq j}\widehat{y}_k$, which is no more than $\widehat{y}_j + \frac{p_x}{p_y} s_j^t\left(x_j' - \widehat{x}_j\right)$, because $p \in \Delta^{SPI}\left(\mathbf{u}, \mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}}\right)$. When Rule 1-2 is induced by j, his output consumption remains no more than $\max\left\{0, \min\left\{\widehat{y}_j + \frac{p_x}{p_y}\left(s_j^t x_j' - s_j^t \widehat{x}_j\right), f\left(\sum s_k^t x_k\right)\right\}\right\}$. Moreover, if j deviates to induce Rule 1-3, then he has no positive output consumption.

Let there exist $i \neq j$ with $x_i = \mu\left(\mathbf{x}_{-i}\right)$ in (2-2). Then, $g^*\left(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}\right)$ corresponds to Rule 1-3. Any deviation cannot induce Rule 1-1, because $p^j = (0,1)$ and $p^i = (1,0)$. Agent j's deviation to induce Rule 1-2 results in $g_j^*\left(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}\right) = 0$, because $p^i = (1,0)$ for $i \neq j$ and $\varphi\left((1,0), \mathbf{s}, \mathbf{x}, \mathbf{w}\right)^{-1} = \varnothing$ for any $(\mathbf{s}, \mathbf{x}, \mathbf{w})$. The same is true for i. Moreover, if any $j \in N$ induces Rule 1-3 by changing his strategy, then such a deviation does not give the agent any additional benefit, because the individual i who has $x_i = \mu\left(\mathbf{x}_{-i}\right)$ is already fixed in Stage 1. Thus, $\mathbf{m}^{\mathbf{s}}\left(\mathbf{x}\right) \in NE\left(\Gamma_{g^*}^{\mathbf{x}\vdash\mathbf{m}}\left(\mathbf{x}\right), \mathbf{e}\right)$ for the subgame of (2-2).

We show that this $(\widehat{\mathbf{x}}, \mathbf{m}^{\mathbf{s}})$ constitutes a subgame perfect equilibrium of the game $\left(\Gamma_{g^*}^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$. In accordance with (1)-(2-1) of $(\widehat{\mathbf{x}}, \mathbf{m}^{\mathbf{s}})$, the outcome is $(\widehat{\mathbf{x}},\widehat{\mathbf{y}})$. Suppose that some j has an incentive to deviate from $\widehat{x}_j$ to $x_j' \neq \widehat{x}_j$ in Stage 1. Then, from (2-2), he obtains only:

$$g_j^*\left(\mathbf{m}^{\mathbf{s}}\left(\widehat{\mathbf{x}}_{x_j'}\right), \widehat{\mathbf{x}}_{x_j'}\right) \leq \max\left\{0, \min\left\{\widehat{y}_j + \tfrac{p_x}{p_y}\left(s_j^t x_j' - s_j^t \widehat{x}_j\right), f\left(\textstyle\sum s_k^t x_k\right)\right\}\right\},$$

which contradicts $\widehat{\mathbf{z}} \in P\left(\mathbf{e}\right)$. Thus, $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}}, \widehat{\mathbf{y}}) \in SPA\left(\Gamma_{g^*}^{\mathbf{x}\vdash\mathbf{m}}, \mathbf{e}\right)$. ■

Lemma 5: *Let Assumption 1 hold. Let the feasible sharing mechanism $g^* \in \mathcal{G}$ be given as above. Then,* $\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}$ implements *any Pareto subsolution φ satisfying SPI and IUS on $\mathcal{E}$* in subgame perfect equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ be given. From Lemma 2, $NA\left(g^*, \mathbf{e}\right) = \varphi\left(\mathbf{e}\right)$. Moreover, $SPA\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right) \subseteq NA\left(g^*, \mathbf{e}\right)$. Hence, we have only to show that $\varphi\left(\mathbf{e}\right) \subseteq SPA\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}, \mathbf{e}\right)$.

First, we show that in every second-stage subgame, there exists at least one Nash equilibrium. We use $NE\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}\left(\mathbf{m}\right), \mathbf{e}\right)$ to denote the set of Nash equilibria of $\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}\left(\mathbf{m}\right), \mathbf{e}\right)$. Let:

$$I\left(p, \mathbf{s}, \mathbf{0}, \mathbf{w}\right) \equiv \left\{i \in N : \exists x_i' \text{ s.t. } \varphi\left(p, \mathbf{s}, \mathbf{0}_{x_i'}, \mathbf{w}\right)^{-1} \neq \varnothing\right\}.$$

Let $\mathbf{x}^{\mathbf{s}*} : \Delta^n \times \mathcal{S}^n \times \mathbb{R}_+^n \to [0, \bar{x}]^n$ be such that for each second-stage subgame $\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}\left(\mathbf{p}, \mathbf{s}, \mathbf{w}\right), \mathbf{e}\right)$, $\mathbf{x}^{\mathbf{s}*}\left(\mathbf{p}, \mathbf{s}, \mathbf{w}\right) \in [0, \bar{x}]^n$ is defined as follows. For all $i \in N$:

(i) if $s_i = s_i^t$ and $\exists p$ s.t. $p^j = p$ for all $j \in N$ and $i = \min I(p, \mathbf{s}, \mathbf{0}, \mathbf{w})$, then $x_i^{\mathbf{s}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) = x_i'$ such that $\varphi\left(p, \mathbf{s}, \mathbf{0}_{x_i'}, \mathbf{w}\right)^{-1} \neq \varnothing$;
(ii) if $s_i = s_i^t$, $w_i > f(\sum s_k \bar{x})$, and $\exists p$ s.t. $p^j = p$ $(\forall j \neq i)$ and $i \in N(p, \mathbf{s}, \mathbf{0}, \mathbf{w})$, then
$x_i^{\mathbf{s}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) = \arg\max_{x_i} u_i\left(x_i, \max\left\{0, \min\left\{w_i' + \frac{p_x}{p_y} s_i^t (x_i - x_i'), f(s_i^t x_i)\right\}\right\}\right)$,
where (x_i', w_i') comes from $\varphi\left(p, \mathbf{s}, \mathbf{0}_{x_i'}, \mathbf{w}_{w_i'}\right)^{-1} \neq \varnothing$;
(iii) if $s_i = s_i^t, w_i > \max\{f(\sum s_k \bar{x}), \max_{j \neq i}\{w_j\}\}$, and $[\{\exists p \text{ s.t. } p^j = p \ (\forall j \neq i)\} \Rightarrow i \notin N(p, \mathbf{s}, \mathbf{0}, \mathbf{w})]$, then $x_i^{\mathbf{s}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) = \frac{\bar{x}}{2}$;
(iv) otherwise, $x_i^{\mathbf{s}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) = 0$.
Note that $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^{\mathbf{s}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{w})$ corresponds to one of the subrules of Rule 1, because $x_i = 0$ for all i with $s_i \neq s_i^t$. Then, we can see that $\mathbf{x}^{\mathbf{s}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^{\mathbf{m} \vdash \mathbf{x}}(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$. To simplify the notation, let us use $\mathbf{x}^*$ to denote $\mathbf{x}^{\mathbf{s}*}(\mathbf{p}, \mathbf{s}, \mathbf{w})$ in the following discussion.

As $x_i = 0$ for all i with $s_i \neq s_i^t$, if any agent deviates to induce Rule 2, then he has to supply a positive amount of labor time. This implies that such a deviation is not beneficial for the agent from the construction of Rule 2.

Suppose that $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ corresponds to Rule 1-1. Then, as $(\mathbf{p}, \mathbf{s}, \mathbf{w})$ is already fixed, no unilateral deviation from $\mathbf{x}^*$ can induce Rule 1-1. Moreover, $w_i \leq f(\sum s_k \bar{x})$ for any i when $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ corresponds to Rule 1-1, which implies that no individual would gain under Rule 1-2 or Rule 1-3.

Suppose that $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ corresponds to Rule 1-2. Then, we first show that there exists a unique agent $j \in N$ who selects (ii) of the strategy mapping $x_j^{\mathbf{s}*}$, whereas any other $i \neq j$ selects (iv) of the strategy $x_i^{\mathbf{s}*}$. From the definition of Rule 1-2, there exists $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ for $p = p^i$ $(\forall i \neq j)$, which implies $w_i \leq f(\sum s_k \bar{x})$ for any $i \neq j$. Thus, no $i \neq j$ can select (ii) and (iii) of $x_i^{\mathbf{s}*}$ under Rule 1-2. In addition, when $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ corresponds to Rule 1-2, no agent should select (i) of $x_i^{\mathbf{s}*}$. Thus, any $i \neq j$ should select (iv) of $x_i^{\mathbf{s}*}$, whereas $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ should select (ii) of $x_j^{\mathbf{s}*}$ so as to induce Rule 1-2. In this strategy profile, no $i \neq j$ can gain by any deviation to induce Rules 1-2, 1-3, or 2. In addition, as $w_j > f(\sum s_k \bar{x})$, no $i \neq j$ can induce Rule 1-1. Furthermore, $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ cannot induce Rule 1-1 by any deviation, because $w_j > f(\sum s_k \bar{x})$. Moreover, as $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ implies $j \in N\left(p, \mathbf{s}, \mathbf{x}^*_{x_j'}, \mathbf{w}\right)$, the agent cannot induce Rule 1-3. Finally, j cannot gain by deviation to induce Rule 1-2. Thus, having (ii) of $x_j^{\mathbf{s}*}$ is the best response for $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$, which implies that $\mathbf{x}^{\mathbf{s}*}$ is a Nash equilibrium.

Suppose that $g^*(\mathbf{p},\mathbf{s},\mathbf{x}^*,\mathbf{w})$ corresponds to Rule 1-3. Then, $\mathbf{x}^*$ consists of $x_j^* = \frac{\overline{x}}{2}$ and $x_i^* = 0$ for any $i \neq j$, or $x_i^* = 0$ for all $i \in N$. In both cases, $\mathbf{x}^* \in NE\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$. In the latter case, any i selects (iv) of $x_i^{\mathbf{s}*}$, and there is no other better strategy for i on his labor choice, given $(\mathbf{p},\mathbf{s},\mathbf{w})$ and $\mathbf{x}_{-i}^* = \mathbf{0}_{-i}$. In the former case, $x_j^* = \frac{\overline{x}}{2}$ is the best response for $j \in N$ to $\mathbf{x}_{-j}^* = \mathbf{0}_{-j}$. In fact, $j \in N$ cannot induce Rules 1-1 or 1-2, but he cannot gain by any deviation to Rule 2, given $\mathbf{x}_{-j}^* = \mathbf{0}_{-j}$. In contrast, given $x_j^* = \frac{\overline{x}}{2}$ and $\mathbf{x}_{-\{i,j\}}^* = \mathbf{0}_{-\{i,j\}}$, no $i \neq j$ can gain by any deviation to Rules 1-3 or 2. In addition, i cannot induce Rules 1-1 or 1-2, because $w_j > f\left(\sum s_k \bar{x}\right)$ implies $i \notin N\left(p,\mathbf{s},\left(\mathbf{0}_{-j},\frac{\overline{x}}{2}\right),\mathbf{w}\right)$ even if $p = p^k$ $(\forall k \neq i)$. Thus, $\mathbf{x}^{\mathbf{s}*}(\mathbf{p},\mathbf{s},\mathbf{w}) \in NE\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$.

Now, we show that for $\mathbf{e} = (\mathbf{u},\mathbf{s}^t) \in \mathcal{E}$, if $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}},\widehat{\mathbf{y}}) \in \varphi(\mathbf{e})$, then there exists a subgame perfect equilibrium whose corresponding outcome is $\widehat{\mathbf{z}}$. Consider the following strategy profile of the game $\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}},\mathbf{e}\right)$.

(1) In Stage 1, every individual i reports $(p^i, s_i, w_i) = (p, s_i^t, \widehat{y}_i)$, where $p \in \Delta^{SPI}(\mathbf{u},\mathbf{s}^t,\widehat{\mathbf{x}},\widehat{\mathbf{y}})$.

(2) In Stage 2, $\mathbf{x} = \mathbf{x}^{\mathbf{s}}(\mathbf{p},\mathbf{s},\mathbf{w})$ is given as follows:

(2-1): if $(\mathbf{p},\mathbf{s},\mathbf{w}) = \left((p^i)_{i\in N},\mathbf{s}^t,\widehat{\mathbf{y}}\right)$ with $p^i = p$ for all $i \in N$ is the action profile in Stage 1, then any $i \in N$ supplies $x_i = \widehat{x}_i$;

(2-2): if $(\mathbf{p},\mathbf{s},\mathbf{w}) = \left(\left(p^{j\prime},(p^i)_{i\neq j}\right),\mathbf{s}^t,\widehat{\mathbf{y}}_{w_j'}\right)$, where $p^i = p$ for all $i \neq j$ and $w_j' > f\left(\sum s_k^t \bar{x}\right)$, is the action profile in Stage 1, then for this $j \in N$:

$$x_j = \arg\max_{x_j'} u_j\left(x_j', \min\left\{\widehat{y}_j + \frac{p_x}{p_y}s_j^t\left(x_j' - \widehat{x}_j\right), f\left(\sum_{i\neq j} s_i^t \widehat{x}_i + s_j^t x_j'\right)\right\}\right),$$

and for all $i \neq j$, $x_i = \widehat{x}_i$;

(2-3): if $(\mathbf{p},\mathbf{s},\mathbf{w}) = \left(\left(p^{j\prime},(p^i)_{i\neq j}\right),\mathbf{s}^t,\widehat{\mathbf{y}}_{w_j'}\right)$, where $p^i = p$ for all $i \neq j$, $\left(p^{j\prime},w_j'\right) \neq (p,\widehat{y}_j)$ and $w_j' \leq f\left(\sum s_k^t \bar{x}\right)$, is the action profile in Stage 1, then $\mathbf{x} = \overline{\mathbf{x}}$;

(2-4): if $(\mathbf{p},\mathbf{s},\mathbf{w}) = \left(\left(p^{j\prime},(p^i)_{i\neq j}\right),\mathbf{s}_{s_j'}^t,\widehat{\mathbf{y}}_{w_j'}\right)$, where $p^i = p$ for all $i \neq j$ and $s_j' \neq s_j^t$, is the action profile in Stage 1, then for this $j \in N$, $x_j = \frac{\overline{x}}{2}$, and for all $i \neq j$, $x_i = 0$;

(2-5): in any other case, $\mathbf{x}^{\mathbf{s}}(\mathbf{p},\mathbf{s},\mathbf{w}) = \mathbf{x}^{\mathbf{s}*}(\mathbf{p},\mathbf{s},\mathbf{w})$.

Note for the subgame of (2-1), $\mathbf{x} = \widehat{\mathbf{x}} \in NE\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$, because $w_i \leq f\left(\sum s_k \bar{x}\right)$ for any $i \in N$, and no deviator can obtain a positive amount of output under Rule 1-2. In addition, $\mathbf{x} \in NE\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$ for the subgame of (2-5), as we have already shown. Moreover, we see that for the subgames (2-2) to (2-4), $\mathbf{x} \in NE\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$.

Note $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})$ corresponds to Rule 1-2 if $\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$ corresponds to (2-2). In this case, no agent can induce Rules 1-1 or 2. Moreover, no $i \neq j$ can obtain a positive amount of output under Rules 1-2 or 1-3, because $w_i = \widehat{y}_i \leq f\left(\sum s_k \bar{x}\right)$. Finally, j cannot induce Rule 1-3 simply by changing his labor time. Thus, $\mathbf{x} \in NE\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$.

If $\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$ corresponds to (2-3), $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})$ corresponds to Rule 1-3, because $\mathbf{x} = \overline{\mathbf{x}}$, where no agent can obtain a positive output consumption from Rule 1-3. In this case, some unilateral deviation may induce Rules 1-2 or 1-3, but it is not beneficial for any $i \in N$ because $w_i \leq f\left(\sum s_k \bar{x}\right)$. Thus, $\mathbf{x} \in NE\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$.

Finally, $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})$ corresponds to Rule 2 if $\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$ corresponds to (2-4). In this case, any $x_j' > 0$ induces Rule 2 again and $x_j' = 0$ makes the total output zero. Thus, j cannot obtain a positive output consumption in any case. As for $i \neq j$, any $x_i' \neq x_i$ induces Rule 2 again, but the deviation does not bring any additional benefit for i, because $x_i = 0$ and $x_i' > 0$. Thus, $\mathbf{x} \in NE\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$.

Show that the above strategy profile (1)-(2) constitutes a subgame perfect equilibrium of the game $\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}},\mathbf{e}\right)$. From the strategy profile (1)-(2) of the game $\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}},\mathbf{e}\right)$, $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) = g^*\left((p)_i,\mathbf{s}^t,\widehat{\mathbf{x}},\widehat{\mathbf{y}}\right) = \widehat{\mathbf{y}}$. Suppose that some j has an incentive to deviate from (p^j, s_j, w_j) to $\left(p^{j\prime}, s_j, w_j'\right)$ in Stage 1. Then, from (2-2) and (2-3), he obtains only $g_j^*\left(\mathbf{p}_{p^{j\prime}},\mathbf{s},\mathbf{x}^{\mathbf{s}}\left(\mathbf{p}_{p^{j\prime}},\mathbf{s},\mathbf{w}_{w_j'}\right),\mathbf{w}_{w_j'}\right) \leq \widehat{y}_j + \frac{p_x}{p_y} s_j^t\left(x_j - \widehat{x}_j\right)$, where $x_j = x_j^{\mathbf{s}}\left(\mathbf{p}_{p^{j\prime}},\mathbf{s},\mathbf{w}_{w_j'}\right)$. This contradicts $\widehat{\mathbf{z}} \in P(\mathbf{e})$. Suppose that some j has an incentive to deviate from (p^j, s_j, w_j) to $\left(p^{j\prime}, s_j', w_j'\right)$ with $s_j \neq s_j'$ in Stage 1. Then from (2-4), the agent cannot obtain a positive output consumption. Thus, $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}},\widehat{\mathbf{y}}) \in SPA\left(\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}},\mathbf{e}\right)$. ■

Proof of Theorem 3. Let Assumptions 1 hold. Let φ be a Pareto subsolution satisfying SPI and IUS. From Lemmas 2, 3, 4, and 5, $\Gamma_{g^*}^{\mathbf{m}\vdash\mathbf{x}}$ (resp. $\Gamma_{g^*}^{\mathbf{x}\vdash\mathbf{m}}$) triply implements φ *on* $\mathcal{E}$ in Nash, strong, and subgame perfect equilibria. Moreover, g^* is forthright, as is shown in the former half of the proof of Lemma 2. Thus, it suffices to show that g^* meets (iii) and (iv) of Definition 3.

1. Definition 3 (iii). The latter half of the proof of Lemma 2 shows that if $(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) \in NE(g^*,\mathbf{e})$, then Rule 1-1 applies and $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) = \mathbf{w}$. For any $i \in N$ and any $(p^{i\prime}, s_i', x_i', w_i') \in \Delta \times \mathcal{S} \times [0,\bar{x}] \times \mathbb{R}_+$, if $\left(\mathbf{p}_{p^{i\prime}},\mathbf{s}_{s_i'},\mathbf{x}_{x_i'},\mathbf{w}_{w_i'}\right)$ is such that Rules 1-1 or 1-2 apply, then:

$g_i^*\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s_i^t \left(x_i' - x_i\right)\right\}$. If $\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ is such that Rule 1-3 applies, this implies either: (i) $\varphi\left(p^{i\prime}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) \neq \varnothing$ and $i \in N\left(p, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$; or (ii) $i \notin N\left(p, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$, where $p = p^k$ for all $k \neq i$. Case (i) implies that $g_i^*\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) = 0$ by $w_i' \leq f\left(\sum s_k^t \overline{x}\right)$. Consider case (ii). As from Lemma 2, $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{e})$ and $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ hold, $i \in N\left(p, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ whenever $s_i' = s_i = s_i^t$. Thus, $s_i' \neq s_i$, which implies that $x_i' = 0$ because $\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ is such that Rule 1-3 applies. Then, $g_i^*\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) = 0$. If $\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ is such that Rule 2 applies, then $x_i' > 0$ by Lemma 1 and $g_i^*\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) = 0$. Thus, g^* meets Definition 3 (iii).

2. Definition 3 (iv). Note again that if $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in NE(g^*, \mathbf{e})$, then it is such that Rule 1-1 applies, or there exists $\mathbf{u} \in \mathcal{U}^n$ such that $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s})$ and for all $i \in N$, $p^i = p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w})$. Moreover, if $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s})$ for some $\mathbf{u} \in \mathcal{U}^n$, then $p \in \Delta^P(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ implies that for any $\mathbf{s}' \in \mathcal{S}^n$ such that $s_i' = s_i$ for all $i \in N$ with $x_i > 0$, there exists some $\mathbf{u}' \in \mathcal{U}^n$ such that $p \in \Delta^P(\mathbf{u}', \mathbf{s}, \mathbf{x}, \mathbf{w}) \cap \Delta^P(\mathbf{u}', \mathbf{s}', \mathbf{x}, \mathbf{w})$. By SPI, $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s})$ and $p \in \Delta^P(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \cap \Delta^P(\mathbf{u}', \mathbf{s}, \mathbf{x}, \mathbf{w})$ together imply $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}', \mathbf{s})$. By IUS, $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}', \mathbf{s})$ and $p \in \Delta^P(\mathbf{u}', \mathbf{s}, \mathbf{x}, \mathbf{w}) \cap \Delta^P(\mathbf{u}', \mathbf{s}', \mathbf{x}, \mathbf{w})$ together imply $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}', \mathbf{s}')$. Thus, $(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w})$ is such that Rule 1-1 applies. Hence, $g^*(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w}) = g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \mathbf{w}$, and $(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w}) \in NE(g^*, \mathbf{e})$. ■

6.3 Proofs of Lemmas in Section 4

Proof of Lemma 6. Let $(\mathbf{u}, \mathbf{s}^t)$ be an economy such that $(\mathbf{x}, \mathbf{y}) \in \varphi^W(\mathbf{u}, \mathbf{s}^t)$. Let $p = (p_x, p_y)$ be a competitive equilibrium price corresponding to $(\mathbf{x}, \mathbf{y})$ at $(\mathbf{u}, \mathbf{s}^t)$. Suppose that the economy $(\mathbf{u}, \mathbf{s}^t)$ changes to $(\mathbf{u}, \mathbf{s}^{t\prime})$ so that $\mathbf{s}_i^{t\prime} = \mathbf{s}_i^t$ for all $i \in N$ with $x_i > 0$, but still $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{x}, \mathbf{y})$. Then, by the definition of efficiency prices and the strict monotonicity of utility functions, it holds that:

(i) for all $\mathbf{z}' \in Z(\mathbf{s}^{t\prime})$, $\sum(p_y y_i' - p_x s_i^{t\prime} x_i') \leq \sum(p_y y_i - p_x s_i^{t\prime} x_i)$;

(ii) for all $i \in N$, $z_i \in \arg\max_{(x,y) \in B\left(p, s_i^{t\prime}, z_i, \theta_i\right)} u_i(x, y)$ where $B(p, s_i^{t\prime}, z_i, \theta_i) \equiv \left\{(x, y) \in [0, \bar{x}] \times \mathbb{R}_+ : p_y y - p_x s_i^{t\prime} x \leq \theta_i \sum\left(p_y y_j - p_x s_j^{t\prime} x_j\right)\right\}$. Therefore, $(\mathbf{x}, \mathbf{y}) \in \varphi^W(\mathbf{u}, \mathbf{s}^{t\prime})$. Thus, φ^W satisfies IUS. ■

Proof of Lemma 7. Let $(\mathbf{u}, \mathbf{s}^t)$ be an economy such that $(\mathbf{x}, \mathbf{y}) \in \varphi^{PR}(\mathbf{u}, \mathbf{s}^t)$.

Let $p = (p_x, p_y) \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{y})$. Suppose that the economy $(\mathbf{u}, \mathbf{s}^t)$ changes to $(\mathbf{u}, \mathbf{s}^{t\prime})$ so that $\mathbf{s}_i^{t\prime} = \mathbf{s}_i^t$ for all $i \in N$ with $x_i > 0$ but still $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{x}, \mathbf{y})$. Then, as $(\mathbf{x}, \mathbf{y}) \in \varphi^{PR}(\mathbf{u}, \mathbf{s}^t)$ and $\mathbf{s}_j^{t\prime} = \mathbf{s}_j^t$ for all $j \in N$ with $x_j > 0$, for each $i \in N$, $y_i = \frac{s_i^t x_i}{\sum s_j^t x_j} \sum y_j = \frac{s_i^t x_i}{\sum_{j \in N, x_j > 0} s_j^t x_j} \sum y_j = \frac{s_i^{t\prime} x_i}{\sum_{j \in N, x_j > 0} s_j^{t\prime} x_j} \sum y_j = \frac{s_i^{t\prime} x_i}{\sum s_j^{t\prime} x_j} \sum y_j$. Therefore, $(\mathbf{x}, \mathbf{y}) \in \varphi^{PR}(\mathbf{u}, \mathbf{s}^{t\prime})$. Thus, φ^{PR} satisfies IUS. ■

Proof of Corollary 6. Let φ be a Pareto subsolution satisfying SIU. Let $(\mathbf{u}, \mathbf{s}^t)$ be an economy such that $\mathbf{z} \in \varphi(\mathbf{u}, \mathbf{s}^t)$, where $x_i = 0$ for some $i \in N$. W.l.o.g., suppose that φ satisfies SPI. Let $p = (p_x, p_y) \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \mathbf{z})$ and for any $j \in N$, $B(p, s_i^t, z_i) \nsubseteq B(p, s_j^t, z_j)$ and $B(p, s_i^t, z_i) \nsupseteq B(p, s_j^t, z_j)$.

If $s_i^t > \min_{k \in N}\{s_k^t \mid k \in N\}$, then consider $(\mathbf{u}, \mathbf{s}^{t\prime})$ such that: $s_i^{t\prime} = \min_{k \in N}\{s_k^t \mid k \in N\}$ and $\mathbf{s}_{-i}^{t\prime} = \mathbf{s}_{-i}^t$. Let $\min_{k \in N}\{s_k^t \mid k \in N\} = s_j^t$. Then, $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{z})$, but $B(p, s_i^{t\prime}, z_i) \subsetneq B(p, s_j^t, z_j)$. Thus, $\mathbf{z} \notin \varphi(\mathbf{u}, \mathbf{s}^{t\prime})$, which implies that φ dose not satisfy IUS.

If $s_i^t \leq \min_{k \in N}\{s_k^t \mid k \in N\}$, then consider $(\mathbf{u}, \mathbf{s}^{t\prime})$ such that $s_i^{t\prime} > \min_{k \in N}\{s_k^t \mid k \in N\} \setminus \{s_i^t\}$ and $\mathbf{s}_{-i}^{t\prime} = \mathbf{s}_{-i}^t$. In addition, consider $\mathbf{u}' \in \mathcal{U}^n$ such that $p \in \Delta^P(\mathbf{u}', \mathbf{s}^{t\prime}, \mathbf{z})$. Then, $p \in \Delta^P(\mathbf{u}', \mathbf{s}^t, \mathbf{z})$, which implies that $\mathbf{z} \in \varphi(\mathbf{u}', \mathbf{s}^t)$, because φ satisfies SPI. Note that there exists $\min_{k \in N}\{s_k^t \mid k \in N\} \setminus \{s_i^t\} = s_j^t$. Then, $B(p, s_i^{t\prime}, z_i) \supsetneq B(p, s_j^t, z_j)$, so that $\mathbf{z} \notin \varphi(\mathbf{u}', \mathbf{s}^{t\prime})$. As $p \in \Delta^P(\mathbf{u}', \mathbf{s}^{t\prime}, \mathbf{z})$, this implies that φ dose not satisfy IUS. ■

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