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Fundamental Incompatibility among Economic Efficiency, Interngenerational Equity,and Sustainability

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Fundamental Incompatibility among Economic Efficiency, Intergenerational Equity, and Sustainability*

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Abstract

We discuss intergenerational resource allocations in production economies with long-run negative externality. The long-run negative externality implies that the emission of public bads by the current generation only affects his future generation's welfare, and the current generation only suffers from the negative externality accumulated by his past generations. We introduce the basic axioms of economic efficiency, intergenerational equity, and environmental sustainability, and examine whether there exists an allocation rule which satisfies these basic axioms. Unfortunately, our answer to this question is negative.

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1 Introduction

In this paper, we consider intergenerational resource allocations in production economies with joint production of public bads. The technological character

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in this production model is that the emission of the public bad is unavoidable in production of private goods. Then, the emitted public bads are accumulated through every generation's economic activity, and the accumulated public bads stay in the stratosphere or under the ground over a long period of time. Thus, the current emission of public bads by the current generation may effect the living conditions of succeeding generations rather than itself. A typical example of such public bads is the green house gases (primarily carbon dioxide) in *global warming*.

Given this kind of technological structure, intergenerational resource allocations are discussed. There are possibly infinite streams of population, and each individual represents one generation. We assume that each generation exists on the earth in one particular period of time, and there is no structure of overlapping generations. Each generation represented by an individual engages in economic activity. So, he produces some private good by utilizing his labor as well as emitting a public bad. Moreover, he not only enjoys consumption of his leisure hours and the private good produced by himself, but also should be confronted with the negative effect of the accumulated public bads emitted by his ancestor generations. However, he is not affected by the current emission of the public bad which he produces. He can also invest some amount of the produced private good for *education*, which improves the technological knowledge of production utilized in the ages of his descendent generations.[1]

Thus, each *temporary allocation* by each generation consists of his consumption vector (a profile of his working hours, his consumption of private goods, and the accumulation of public bads confronted by him), his emission of public bads, and his investment of the private good for education. Then, an *intergenerationally feasible allocation* is defined by a historical sequence of each generation's temporary allocation, which is dynamically consistent with the technological condition of production.

Our concern in this model is to examine the existence of *allocation rules*, which satisfy the three basic normative criteria, each of which is *economic efficiency*, *intergenerational equity*, and *environmental sustainability* respec-

[1] Asheim, Buchholz, and Tunggodden (2001) and Asheim and Buchholz (2005) considered a similar type of intergenerational resource allocations to ours, although they discussed bequest of the natural resources, which is characterized as positive externality rather than negative externality. Also, their economic models are much simpler than ours: they do not have the component of educational investment that our model has.

tively.[2] Economic efficiency is represented by the axiom of *Pareto efficiency*. In our model, every generation's preference is defined only on his own consumption space, so there is no altruistic aspect in each generation's rational choice behavior. So, the definition of Pareto efficiency is a standard one except that the set of social alternatives is given by that of intergenerational allocations which is an infinite dimensional space. The criterion of environmental sustainability concerns improving the natural environment or leaving it as close to its initial condition as possible for the future generations.

As the axioms of intergenerational equity, we start from introducing the two traditional notions of equity: *equity as no-envy* (Foley (1967)) and *egalitarian equivalence* (Pazner and Schemeidler (1978)). Moreover, as weaker variants of the no-envy axiom, we define the axioms of *responsibility and compensation*, which were originally discussed and defined by Dworkin (1981) and Fleurbaey (1994, 1995) in the context of intragenerational resource allocations. Motivated by Suzumura and Tadenuma (2005), our axioms of responsibility and compensation represent the value judgements such that "Any generations should be equally responsible for their descendent generations' living environments" and "The generation who is more handicapped in his living environment should be compensated, being permitted to exploit the natural environment more intensively." Both of them seem to be reasonable requirements in the context of intergenerational resource allocations with long-run negative externality.

It would be desirable to have an allocation rule satisfying the axioms, each of which respectively represents one of the above three basic normative criteria. Unfortunately, our answer to this question is essentially negative: Almost all of our main theorems discuss incompatibility between Pareto efficiency and any one of the axioms of intergenerational equity as well as the axiom of environmental sustainability.

In the following discussion, section 2 provides the basic model of this paper. Section 3 introduces the basic axioms of allocation rules in this economic model. Section 4 characterizes Pareto efficient allocations in this economic model, and section 5 discusses the main theorems in this paper. Finally,

[2]Asheim et. al (2001), Asheim and Buchholz (2005), and Roemer (2005) also discuss intergenerational equity and sustainability in production economies with externality, non-overlapping generation, and non-utility discounting. As intergenerational equity criteria, Asheim et. al (2001) and Asheim and Buchholz (2005) discuss the Suppes-Sen grading principle and Roemer (2005) takes the 'maximin welfare' principle, while this paper starts from discussing the no-envy and the egalitarian equivalence principles.

section 6 gives us a short concluding remarks.

2 The Basic Model

There is an infinite sequence of periods, $\mathbb{T} = \{1, 2, \ldots, t, \ldots\}$ with generic element t. Consider an economy at period t with one generation. To simplify the argument, we assume that every generation can live only in one period. Also, to focus on the intergenerational resource allocations, each generation is represented by one individual. Thus, the period t also implies "*generation* t."

There is a (private) good $y \in \mathbb{R}_+$ which is produced from the labor input $l \in \mathbb{R}_+$. The production process also involves emission of one public bad $z \in \mathbb{R}_+$. Thus, the production technology is represented by a function $f : \mathbb{R}_+^2 \to \mathbb{R}_+$ which is defined as: for any labor input $l \in \mathbb{R}_+$ and any public bad emissions $z \in \mathbb{R}_+$, $f(l, z) = y$. This function is assumed to be continuous, strictly increasing, concave, and $f(0, z) = 0$ & $f(l, 0) = 0$ for any $z \in \mathbb{R}_+$ and any $l \in \mathbb{R}_+$.

We impose an additional assumption on the technological progress in the production process. The function f is rewritten as: there exists a continuous, strictly increasing, and concave function $g : \mathbb{R}_+^2 \to \mathbb{R}_+$ and a positive number $h \in \mathbb{R}_{++}$ such that for any $l \in \mathbb{R}_+$ and any $z \in \mathbb{R}_+$, $f(l, z) = g(h \cdot l, z)$. Note that the number h indicates a level of *technological knowledge* or "*human capital.*" We consider that the value h is valuable in each generation. Thus, the production technology at generation t is denoted by: $f^t(l, z) = g(h^t \cdot l, z)$. The value h^t is determined by the combination of the knowledge h^{t-1} for any $t \in \mathbb{T} \backslash \{1\}$, which is accumulated from 1-st generation to $(t-1)$-th generation, and the investment for education $w^{t-1} \in \mathbb{R}_+$ by the generation $t-1$. For $t = 1$, $h^1 = h^0 \in \mathbb{R}_{++}$, where h^0 is the level of human capital given at the age of pre-history. Thus, there is an *increasing* and *continuous* function H such that $h^t = H(h^{t-1}, w^{t-1})$ for any $t \in \mathbb{T} \backslash \{1\}$. The investment w^{t-1} is financed from the private good produced by the generation $t-1$. Suppose in the following discussion, that there is an *upper bound* $\overline{z}$ of the public bad emission. Also, suppose that $h^t = h^{t-1} = H(h^{t-1}, 0)$ for any $t \in \mathbb{T} \backslash \{1\}$. Sometimes, we will discuss the case that H is constant. Note that the function H is *constant* if for any $h \in \mathbb{R}_{++}$ and any $w \in \mathbb{R}_+$, $h = H(h, w)$.

Each and every generation $t \in \mathbb{T}$ has the common consumption space $X \equiv [0, \overline{l}] \times \mathbb{R}_+ \times \mathbb{R}_+$, where $0 < \overline{l} < +\infty$ is the common upper bound of labor-leisure time, with the following generic formulas of consumption vector: $x^t = (l^t, y^t, Z^{t-1})$. The first two components of the vector x_t indicates that the person in the generation t supplies l_t amount of labor hour, and consumes y_t amount of private good produced at period t, which is actually a standard argument. In contrast, the last component needs additional explanation: Z^{t-1} indicates the accumulated amount of public bads emitted at each period from $1 \in \mathbb{T}$ until $t-1 \in \mathbb{T}$. That is, if $z^{t'}$ is the amount of public bads emitted by the generation $t' = 1, \ldots, t-1$, then the consumption level of public bads by the generation t is defined by

$$Z^{t-1} \equiv \delta^{t-1} \cdot Z^0 + \sum_{t'=1}^{t-1} \delta^{(t-1)-t'} \cdot z^{t'}$$

where $\delta \in (0,1)$ is the natural rate of depletion at each period, and Z^0 is the *initial endowment of public bads* which was provided in the age of pre-history. Note that every generation t does not suffer from the current emission of the public bad z^t. He suffers from the accumulated amount of the public bad emitted by his ancestors up to the previous generation. This is actually the stylized fact of public bads consumption such as the issue of Global Warming.

Each and every generation $t \in \mathbb{T}$ is characterized by his preference relation R_t on X. This relation is complete and transitive on X. For any $x^t, \widetilde{x}^t \in X$, $(x^t, \widetilde{x}^t) \in R_t$ means that x^t is at least as good as $\widetilde{x}^t$ according to t's preference. $P(R_t)$ and $I(R_t)$ denote, respectively, the strict preference relation and the indifference relation corresponding to R_t, viz., $(x^t, \widetilde{x}^t) \in P(R_t)$ if and only if $[(x^t, \widetilde{x}^t) \in R_t \ \& \ (\widetilde{x}^t, x^t) \notin R_t]$, and $(x^t, \widetilde{x}^t) \in I(R_t)$ if and only if $[(x^t, \widetilde{x}^t) \in R_t \ \& \ (\widetilde{x}^t, x^t) \in R_t]$. Also, R_t is assumed to be *strictly monotonic* (decreasing in labor time and public bad, and increasing in the share of output) on $[0, \bar{l}) \times \mathbb{R}_{++} \times \mathbb{R}_+$,[3] *continuous* and *convex* on X. The universal class of such preference relations is denoted by $\mathcal{R}$.

Given a stock of the public bad Z^{t-1}, an accumulated knowledge h^{t-1}, and an investment w^{t-1}, a pair $\mathbf{a}^t = (x^t, z^t, w^t) \in X \times \mathbb{R}_+ \times \mathbb{R}_+$ is a *temporarily feasible allocation for generation* t *in* $(Z^{t-1}, h^{t-1}, w^{t-1})$ if $x^t = (l^t, y^t, Z^{t-1})$, $h^t = H(h^{t-1}, w^{t-1})$, and $g(h^t \cdot l^t, z^t) \geq y^t + w^t$. The set of temporarily feasible allocations for generation t in $(Z^{t-1}, h^{t-1}, w^{t-1})$ is denoted by $A^t(Z^{t-1}, h^{t-1}, w^{t-1})$.

[3] In what follows, $\mathbb{R}_+$, $\mathbb{R}^n_+$ and $\mathbb{R}^n_{++}$ denote, respectively, the set of non-negative real numbers, the non-negative orthant and the positive orthant in the Euclidean n-space.

Given an initial stock of the public bad Z^0 and an initial endowment of human capital h^0, a historical sequence $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in (X \times \mathbb{R}_+ \times \mathbb{R}_+)^\infty$ is an *(intergenerationally) feasible allocation in* (Z^0, h^0) if for all $t \in \mathbb{T}$, the stock of the public bad in this period is characterized by $Z^{t-1} \equiv \delta^{t-1} \cdot Z^0 + \sum_{t'=1}^{t-1} \delta^{(t-1)-t'} \cdot z^{t'}$, and $\mathbf{a}^t$ is the temporarily feasible allocation for generation t in $(Z^{t-1}, h^{t-1}, w^{t-1})$ at the period t, and the component of the public bad consumption of x^t is Z^{t-1}. Fixing (Z^0, h^0) in the following discussion, the set of feasible allocations is denoted by A.

Let us define an *economy* by a historical sequence of all generations' preferences $\mathbf{R}^{\mathbb{T}} = (R_t)_{t\in\mathbb{T}} \in \mathcal{R}^\infty$. An *allocation rule* is a correspondence $\varphi : \mathcal{R}^\infty \twoheadrightarrow A$ which associates to each $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^\infty$, a non-empty subset $\varphi(\mathbf{R}^{\mathbb{T}})$ of A.

3 Basic Axioms for Allocation Rules

In this section, we will introduce basic axioms, each of which should represent an indispensable value in the problem of intergenerational resource allocations under economies with the long-run influence of negative externality. Those axioms are categorized into the three normative viewpoints: *economic efficiency*, *intergenerational equity*, and *environmental sustainability*.

3.1 Axioms of Economic Efficiency

First, we discuss axioms of economic efficiency. Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^\infty$, for each $t \in \mathbb{T}$ and each $(Z^{t-1}, h^{t-1}, w^{t-1})$, a temporarily feasible allocation $\mathbf{a}^t = (x^t, z^t, w^t) \in A^t(Z^{t-1}, h^{t-1}, w^{t-1})$ for generation t is *temporary Pareto efficient at* $\mathbf{R}^{\mathbb{T}}$ if there is no temporarily feasible allocation $\widetilde{\mathbf{a}}^t = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t) \in A^t(Z^{t-1}, h^{t-1}, w^{t-1})$ for generation t such that $(\widetilde{x}^t, x^t) \in P(R_t)$. Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^\infty$, a feasible allocation $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ is *Pareto efficient at* $\mathbf{R}^{\mathbb{T}}$ if there is no feasible allocation $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t\in\mathbb{T}} = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)_{t\in\mathbb{T}} \in A$ such that for any $t \in \mathbb{T}$, $(\widetilde{x}^t, x^t) \in R_t$ holds, and there exists $t \in \mathbb{T}$ such that $(\widetilde{x}^t, x^t) \in P(R_t)$.

Now, we are ready to introduce a well-known axiom on allocation rules:

Pareto Efficiency (PE): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^\infty$ *and* $\mathbf{a} \in \varphi(\mathbf{R}^{\mathbb{T}})$, $\mathbf{a}$ *is* Pareto efficient at $\mathbf{R}^{\mathbb{T}}$.

Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, for each $t \in \mathbb{T}$ and each $(Z^{t-1}, h^{t-1}, w^{t-1})$, a temporarily feasible allocation $\mathbf{a}^t = (x^t, z^t, w^t) \in A^t(Z^{t-1}, h^{t-1}, w^{t-1})$ for generation t is *temporarily selfish* if $(z^t, w^t) = (\overline{z}, 0)$ and there is no other $\widetilde{\mathbf{a}}^t = (\widetilde{x}^t, \overline{z}, 0) \in A^t(Z^{t-1}, h^{t-1}, w^{t-1})$ such that $(\widetilde{x}^t, x^t) \in P(R_t)$. Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, a feasible allocation $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ is *selfish* if for each $t \in \mathbb{T}$, $(z^t, w^t) = (\overline{z}, 0)$ and there is no other $\widetilde{\mathbf{a}}^t = (\widetilde{x}^t, \overline{z}, 0) \in A^t(Z^{t-1}, h^{t-1}, 0)$ such that $(\widetilde{x}^t, x^t) \in P(R_t)$.

Lemma 1: *Given an economy* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$*, a* selfish allocation $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, \overline{z}, 0)_{t\in\mathbb{T}} \in A$ *is* Pareto efficient at $\mathbf{R}^{\mathbb{T}}$.

Proof. Consider $t = 1$. Clearly, there is no other temporarily feasible allocation $\widetilde{\mathbf{a}}^1 = (\widetilde{x}^1, \widetilde{z}^1, \widetilde{w}^1) \in A^1(Z^0, h^0, 0)$ such that $(\widetilde{x}^1, x^1) \in P(R_1)$. Suppose $\widetilde{\mathbf{a}}^1 \in A^1(Z^0, h^0, 0)$ such that $(\widetilde{x}^1, x^1) \in I(R_1)$. Since $z^1 = \overline{z}$, it should be $\widetilde{z}^1 = \overline{z}$. Then, any feasible allocation $\widetilde{\mathbf{a}} \in A$ with $\widetilde{\mathbf{a}}^1$ at the 1-st period should be unable to have $\widetilde{\mathbf{a}}^2 = (\widetilde{x}^2, \widetilde{z}^2, \widetilde{w}^2) \in A^2(Z^0, h^0, 0)$ such that $(\widetilde{x}^2, x^2) \in P(R_2)$. By repeating this procedure infinitely, we can see that there is no other feasible allocation which Pareto-dominates $\mathbf{a}$. ■

Although **PE** is a fundamental requirement of economic efficiency, **Lemma 1** indicates that the set of Pareto efficient allocations contains selfish allocations in this model. Note that implementation of selfish allocations cannot resolve the issue of negative externality. In particular, if the negative externality by the emissions of public bads leads the society to the crisis of human's sustainability, the implementation of selfish allocations indicates an undesirable situation. Thus, we would like to require:

Non-Selfish Pareto Efficiency (NSPE): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ *and* $\mathbf{a} \in \varphi(\mathbf{R}^{\mathbb{T}})$, $\mathbf{a}$ *is* non-selfish Pareto efficient at $\mathbf{R}^{\mathbb{T}}$.

Thus, the axiom **NSPE** requires not only efficient allocations of resources among generations, but also implementation of some policies for regulating the public bads emissions and of educational investments for future generations.

We will also discuss a second best notion of efficiency. This notion requires a constrained efficient allocation of resources by some policies of regulation and investment. Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, for each $t \in \mathbb{T}$ and each $(Z^{t-1}, h^{t-1}, w^{t-1})$, a temporarily feasible allocation $\mathbf{a}^t = (x^t, z^t, w^t) \in A^t(Z^{t-1}, h^{t-1}, w^{t-1})$ for generation t is (z^t, w^t)*-constrained temporary Pareto* *efficient at* $\mathbf{R}^{\mathbb{T}}$ if there is no temporarily feasible allocation $\widetilde{\mathbf{a}}^t = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t) \in A^t(Z^{t-1}, h^{t-1}, w^{t-1})$ for generation t such that $(\widetilde{z}^t, \widetilde{w}^t) = (z^t, w^t)$ and $(\widetilde{x}^t, x^t) \in P(R_t)$. It is clear that if $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ is Pareto efficient at $\mathbf{R}^{\mathbb{T}}$, then for any $t \in \mathbb{T}$, $\mathbf{a}^t = (x^t, z^t, w^t)$ is (z^t, w^t)-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$.

Given a feasible allocation $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$, $(z^t, w^t)_{t\in\mathbb{T}}$ is a *feasible sequence of public bads provisions and investments for new technological knowledge*. Given such a feasible sequence $(z^t, w^t)_{t\in\mathbb{T}}$, let $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t\in\mathbb{T}} = (\widetilde{x}^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ be another feasible allocation whose components of public bads and investments are $(z^t, w^t)_{t\in\mathbb{T}}$. Let us denote the set of such feasible allocations by $A(z^t, w^t)_{t\in\mathbb{T}}$ when the feasible sequence $(z^t, w^t)_{t\in\mathbb{T}}$ is given. Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^\infty$, and given a feasible sequence $(z^t, w^t)_{t\in\mathbb{T}}$, a feasible allocation $\mathbf{a}^* = (\mathbf{a}^{*t})_{t\in\mathbb{T}} = (x^{*t}, z^t, w^t)_{t\in\mathbb{T}} \in A$ is $(z^t, w^t)_{t\in\mathbb{T}}$-*constrained Pareto efficient at* $\mathbf{R}^{\mathbb{T}}$ if there is no other feasible allocation $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t\in\mathbb{T}} = (\widetilde{x}^t, z^t, w^t)_{t\in\mathbb{T}} \in A(z^t, w^t)_{t\in\mathbb{T}}$ such that for any $t \in \mathbb{T}$, $(\widetilde{x}^t, x^t) \in R_t$ holds, and there exists $t' \in \mathbb{T}$ such that $\left(\widetilde{x}^{t'}, x^{t'}\right) \in P(R_{t'})$. Note that for any feasible allocation $\mathbf{a} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$, it is $(z^t, w^t)_{t\in\mathbb{T}}$-constrained Pareto efficient at $\mathbf{R}^{\mathbb{T}}$ if and only if for any $t \in \mathbb{T}$, $\mathbf{a}^t = (x^t, z^t, w^t)$ is (z^t, w^t)-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$.

3.2 Axioms on Intergenerational Equity

Here we discuss axioms of intergenerational equity. At the first place, the following axiom is relevant to *equity* in terms of subjective well-being. This is an extension of the no-envy principle (Foley (1967)) to the problem of intergenerational resource allocations:

No-Envy (NE): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^\infty$ *and* $\mathbf{a} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *we have that: for any* $t \in \mathbb{T}$, $\left(x^t, x^{t'}\right) \in R_t$ *holds for any* $t' \in \mathbb{T}$.

The following five axioms are weaker versions of the no-envy axiom:

Equal Welfare for Equal Preferences (EWEP): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^\infty$ *and* $\mathbf{a} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *we have that: for any* $t, t' \in \mathbb{T}$, *if* $R_t = R_{t'}$, *then one of the following three holds*: $\left(x^t, x^{t'}\right) \in I(R_t)$, $\left(x^t, x^{t'}\right) \in R_t$ *for* $x^t = (l^t, 0, Z^{t-1})$, *and* $\left(x^{t'}, x^t\right) \in R_t$ *for* $x^{t'} = \left(l^{t'}, 0, Z^{t'-1}\right)$.

Equal Welfare for Uniform Preferences (EWUP): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^\infty$ *and* $\mathbf{a} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *we have that: if for any* $t, t' \in \mathbb{T}$, $R_t = R_{t'}$, *then for any* $t, t' \in \mathbb{T}$, *one of the following three holds*: $\left(x^t, x^{t'}\right) \in I(R_t)$, $\left(x^t, x^{t'}\right) \in R_t$ *for* $x^t = (l^t, 0, Z^{t-1})$, *and* $\left(x^{t'}, x^t\right) \in R_t$ *for* $x^{t'} = \left(l^{t'}, 0, Z^{t'-1}\right)$.

The above two axioms are originated from Fleurbaey (1994, 1995), which discussed intragenerational resource allocations under pure exchange economies. These are axioms of *compensation for "more handicapped generations."* Note, here the "more handicapped generations" means the generations endowed with more amount of accumulated public bads and/or less accumulation of human capital.

It is easy to see that **EWUP** is a weaker variant of **EWEP**. The next axiom is another weaker variant of **EWEP**:

Undomination among Equal Preferences (**UNEP**): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ *and* $\mathbf{a} = (x^t, z^t, w^t)_{t \in \mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *we have that: for any* $t, t' \in \mathbb{T}$, *if* $R_t = R_{t'}$, *then* $(-Z^{t-1}, h^t) > \left(-Z^{t'-1}, h^{t'}\right) \Rightarrow \left[z^t < z^{t'} \text{ or } w^t > w^{t'}\right]$.[4]

This axioms says that the "more handicapped generation" has a right to produce and utilize more resources for only his own consumption.

The next three axioms are of *responsibility* axioms. The first two are a variation of the "No-envy among equal skills," which was originally discussed by Fleurbaey and Maniquet (1996) in the context of intragenerational resource allocations under production economies:

No-Envy among Equal-Endowed Generations (**NEEG**): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ *and* $\mathbf{a} = (x^t, z^t, w^t)_{t \in \mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *we have that: for any* $t, t' \in \mathbb{T}$, *if* $Z^{t-1} = Z^{t'-1}$ *and* $h^t = h^{t'}$, *where* Z^{t-1} *(resp.* $Z^{t'-1}$*) is the third component of the consumption vector* x^t *(resp.* $x^{t'}$*), then* $\left(x^t, x^{t'}\right) \in R_t$ *and* $\left(x^{t'}, x^t\right) \in R_{t'}$ *hold.*

No-Envy among Uniform-Endowed Generations (**NEUG**): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ *and* $\mathbf{a} = (x^t, z^t, w^t)_{t \in \mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *we have that: if for any* $t, t' \in \mathbb{T}$, $Z^{t-1} = Z^{t'-1}$ *and* $h^t = h^{t'}$, *where* Z^{t-1} *(resp.* $Z^{t'-1}$*) is the third component of the consumption vector* x^t *(resp.* $x^{t'}$*), then for any* $t, t' \in \mathbb{T}$, $\left(x^t, x^{t'}\right) \in R_t$ *and* $\left(x^{t'}, x^t\right) \in R_{t'}$ *hold.*

The third axiom of responsibility is introduced as follows:

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[4] Note that the vector inequalities are defined as follows: for any $\mathbf{a}, \mathbf{b} \in \mathbb{R}^q$ with $q > 1$, $\mathbf{a} \geq \mathbf{b}$ if and only if $a_i \geq b_i$ for all $i = 1, 2, \ldots, q$; $\mathbf{a} > \mathbf{b}$ if and only if $\mathbf{a} \geq \mathbf{b}$ and $a_i > b_i$ for some $i = 1, 2, \ldots, q$; and $\mathbf{a} \gg \mathbf{b}$ if and only if $\mathbf{a} \geq \mathbf{b}$ and $a_i > b_i$ for all $i = 1, 2, \ldots, q$.

Responsibility for Future Generations (RFG): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ *and* $\mathbf{a} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *we have that: for any* $t, t' \in \mathbb{T}$, *if* $Z^{t-1} = Z^{t'-1}$ *and* $h^t = h^{t'}$, *where* Z^{t-1} *(resp.* $Z^{t'-1}$*) is the third component of the consumption vector* x^t *(resp.* $x^{t'}$*), then* $\left[z^t > z^{t'}\right] \Rightarrow \left[w^t > w^{t'}\right]$.

This axiom requires responsibility of the current generation to keep the "living environment" as well as possible for future generations. Note that **NEEG** implies **RFG**.

Although the above axioms are of equity as no-envy and its weaker variations, we can also discuss a variation of the egalitarian-equivalent principle (Pazner and Schmeidler (1978)) in this problem of intergenerational resource allocations. When we discuss the egalitarian-equivalence axiom here, let us assume that $(1-\delta) Z^0 \leq \overline{z}$. Let $z^* \leq \overline{z}$ be a social reference level of public bads emission. Then, let us define (Z^0, h^0, z^*) as a *social reference level of "natural environments."* Given any generation $t \in \mathbb{T}$ with his preference R_t, let $x^t(R_t; Z^0, h^0, z^*)$ be t's ideal consumption vector which is maximal w.r.t. R_t whenever he is faced with production condition $g(h^0 l, z^*)$ and the stock of public bads Z^0. Now, we are ready to discuss a variation of the egalitarian-equivalent principle in this context:

z^*-**Egalitarian Equivalence (**z^***-EE):** *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ *and* $\mathbf{a} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *we have that: for any* $t \in \mathbb{T}$, $(x^t, x^t(R_t; Z^0, h^0, z^*)) \in I(R_t)$ *holds.*

This axiom is a requirement of equal opportunity for welfare among generations. In particular, if z^* is given by $z^* = (1-\delta) Z^0$, then the social reference profile (Z^0, h^0, z^*) indicates that every generation is guaranteed an initial natural environment (Z^0, h^0) by his preceding generations, and he also guarantees this environment for his descendant generation by restricting the public bads emission to z^*. Thus, the axiom z^*-**EE** guarantees every generation equally the welfare level which is maximal under the constraint (Z^0, h^0, z^*).

3.3 Axioms of Environmental Sustainability

We can also consider other axioms to judge the wellness of intergenerational resource allocations, which are relevant to *sustainability*. By sustainability, we may consider at least two meanings in the environmental literature. One is of the 'natural environmentalist' who insists the intrinsic value of natural environments, where such a value is not necessarily relevant to the welfare of human beings. So, sustainability should mean for the 'natural environmentalist,' that the historical sequence of stocks of public bads is non-increasing in $\mathbb{T}$.

As the axiom of sustainability for the 'natural environmentalist,' we define the followings:

Public Bads Monotonicity (PBM): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ *and* $\mathbf{a} = (x^t, z^t, w^t)_{t \in \mathbb{T}} = ((l^t, y^t, Z^{t-1}), z^t, w^t)_{t \in \mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *it holds that* $Z^{t'-1} \geq Z^{t''-1}$ *whenever* $t' \leq t''$.

This axiom is well-defined. Take a historical sequence of public bad provision $\mathbf{z} = (z^t)_{t \in \mathbb{T}}$ such that $z^1 \leq (1-\delta) \cdot Z^0$, and for any other $t \in \mathbb{T} \backslash \{1\}$, $z^t = (1-\delta) \cdot [\delta Z^0 + z^1]$. Then, the emission of z^1 is compatible with temporarily feasible allocations, and for any $t \in \mathbb{T}$, $Z^{t-1} = Z^1 = \delta Z^0 + z^1 \leq Z^0$ holds. These facts imply that it is possible to construct an allocation rule which satisfies the axiom **PBM**.

The other meaning is from the standpoint of 'humanist.' So, sustainability means for the 'humanist,' that the historical sequence of *human's welfare* is non-increasing in $\mathbb{T}$.[5] In this approach, an important issue is how to measure human's welfare. Since each generation's preference is ordinally measurable and intergenerationally noncomparable in this model, we cannot use it as for measuring each generation's welfare: the requisite of the non-increasing of human's welfare over periods implicitly assumes the existence of intergenerationally comparable welfare units.

In this paper, we assume the existence of an objective welfare measure. A typical example of such a measure can be found in the theory of *functionings and capability* developed by Sen (1980; 1985): the welfare measure is a *representation of some ordering relation defined over alternative capabilities* that human beings can enjoy. This is formulated by an ordering J defined over $X \times \mathbb{R}_+$, where this $\mathbb{R}_+$ is the space for human capital. Thus, for any $(x, h), (x', h') \in X \times \mathbb{R}_+$, $((x, h), (x', h')) \in J$ implies that having the consumption vector x and the knowledge h is at least as desirable for human beings as having the consumption vector x' and the knowledge h'. Then:

J-Reference Human Development (J-HD): *For all* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ *and* $\mathbf{a} =$

[5] Asheim et. al (2001), Asheim and Buchholz (2005), and Roemer (2005) adopted this approach for defining sustainability. See also Silvestre (2002; 2005).

$(x^t, z^t, w^t)_{t\in\mathbb{T}} = ((l^t, y^t, Z^{t-1}), z^t, w^t)_{t\in\mathbb{T}} \in \varphi(\mathbf{R}^{\mathbb{T}})$, *it holds that* $\left(\left(x^{t''}, h^{t''}\right), \left(x^{t'}, h^{t'}\right)\right) \in J$ *whenever* $t' \leq t''$.

Note that the meaning of human development is based upon the property of the ordering J. In the following discussion, we naturally assume that J is *continuous* and *strictly monotonic in* $X \times \mathbb{R}_+$ (decreasing in labor hours and public bads, and increasing in the share of output and level of knowledge), and *convex on* $X \times \{h\}$ for any $h \in \mathbb{R}_+$. Thus, for every generation, inherited a higher level of knowledge and a lower level of the stock of public bads can enhance his objective welfare, while bequeathing a lower level of educational investment and a higher level of public bads emission can make his descendent generations worse off in terms of the objective welfare measure J.

4 Characterizations of Intergenerational Pareto Efficiency

Before examining the possibility of allocation rules satisfying the axioms relevant to economic efficiency, intergenerational equity, and sustainability, we would like to characterize Pareto efficient allocations in this model. At the first place, the following lemma gives us a necessary and sufficient condition for Pareto efficiency.

Lemma 2: *Given an economy* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, *a feasible allocation* $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ *is* Pareto efficient at $\mathbf{R}^{\mathbb{T}}$ *if and only if the following condition holds*:

($\star$) *For any* $\bar{t} \in \mathbb{T}$ *and any* $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t\in\mathbb{T}} = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)_{t\in\mathbb{T}} \in A$ *with* $(x^t, \widetilde{x}^t) \in I(R_t)$ *for any* $t < \bar{t}$, *if* $\left(\widetilde{x}^{\bar{t}}, x^{\bar{t}}\right) \in P(R_{\bar{t}})$, *then there exists* $\bar{t}' \in \mathbb{T}$ *such that* $\bar{t}' > \bar{t}$ *and* $\left(x^{\bar{t}'}, \widetilde{x}^{\bar{t}'}\right) \in P(R_{\bar{t}'})$.

This lemma is almost the definition of Pareto efficiency, so we will skip the proof. We can also have the necessary and sufficient condition for Pareto efficiency, in the specific case of the constant H.

Lemma 3: *Assume* H *is constant. Given an economy* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, *a feasible allocation* $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ *is* Pareto efficient at $\mathbf{R}^{\mathbb{T}}$ *if and only if the following condition holds*:

(*) *For any* $\overline{t} \in \mathbb{T}$ *and any* $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t \in \mathbb{T}} = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)_{t \in \mathbb{T}} \in A$ *with* $(x^t, z^t, w^t) = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)$ *for any* $t < \overline{t}$*, if* $\left(\widetilde{x}^{\overline{t}}, x^{\overline{t}}\right) \in P(R_{\overline{t}})$*, then there exists* $\overline{t}' \in \mathbb{T}$ *such that* $\overline{t}' > \overline{t}$ *and* $\left(x^{\overline{t}'}, \widetilde{x}^{\overline{t}'}\right) \in P(R_{\overline{t}'})$*.*

Proof. Let us examine if $\mathbf{a}$ is Pareto efficient at $\mathbf{R}^{\mathbb{T}}$, then $\mathbf{a}$ meets the condition (*). Suppose $\mathbf{a} = (x^t, z^t, w^t)_{t \in \mathbb{T}} \in A$ violates (*). Then, there exists $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t \in \mathbb{T}} = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)_{t \in \mathbb{T}} \in A$ with some $\overline{t} \in \mathbb{T}$ such that $(x^t, z^t, w^t) = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)$ for any $t < \overline{t}$, $\left(\widetilde{x}^{\overline{t}}, x^{\overline{t}}\right) \in P(R_{\overline{t}})$, and $(\widetilde{x}^t, x^t) \in R_t$ for any $t > \overline{t}$. This implies $\mathbf{a}$ is Pareto-dominated by $\widetilde{\mathbf{a}}$.

Consider the inverse relation. Suppose $\mathbf{a} = (x^t, z^t, w^t)_{t \in \mathbb{T}} \in A$ is not Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. Then, there is a feasible allocation $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t \in \mathbb{T}} = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)_{t \in \mathbb{T}} \in A$ which Pareto-dominates $\mathbf{a}$ at $\mathbf{R}^{\mathbb{T}}$. Suppose there exists $\overline{t} \in \mathbb{T} \backslash \{1\}$ such that $(x^t, \widetilde{x}^t) \in I(R_t)$ for any $t < \overline{t}$, $\left(\widetilde{x}^{\overline{t}}, x^{\overline{t}}\right) \in P(R_{\overline{t}})$, and $(\widetilde{x}^t, x^t) \in R_t$ for any $t > \overline{t}$.

First, consider the case that for any $t < \overline{t}$, (x^t, z^t, w^t) is (z^t, w^t)-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. Then, for $t = 1$, $(\widetilde{x}^1, \widetilde{z}^1, \widetilde{w}^1) \neq (x^1, z^1, w^1)$ and $(x^1, \widetilde{x}^1) \in I(R_1)$ imply either $w^1 > 0$ or $\widetilde{z}^1 > z^1$. Since H is constant, if $w^1 > 0$, then the new allocation $\widehat{\mathbf{a}} \in A$ such that $\widehat{\mathbf{a}}^1 = ((l^1, y^1 + w^1, Z^0), z^t, 0^1)$ and $\widehat{\mathbf{a}}^t = (x^t, z^t, w^t)$ for any $t \in \mathbb{T} \backslash \{1\}$ has the property to violate the condition (*) for $\mathbf{a}$ at $\mathbf{R}^{\mathbb{T}}$. If $\widetilde{z}^1 > z^1$, then $(x^1, \widetilde{x}^1) \in I(R_1)$ implies that $(\widetilde{x}^1, \widetilde{z}^1, \widetilde{w}^1)$ is not $(\widetilde{z}^1, \widetilde{w}^1)$-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. Thus, the new allocation $\widehat{\mathbf{a}} \in A$ such that $\widehat{\mathbf{a}}^1 = (\widehat{x}^1, \widetilde{z}^1, \widetilde{w}^1)$, where $\widehat{\mathbf{a}}^1$ is $(\widetilde{z}^1, \widetilde{w}^1)$-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$, and $\widehat{\mathbf{a}}^t = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)$ for any $t \in \mathbb{T} \backslash \{1\}$ has the property to violate the condition (*) for $\mathbf{a}$ at $\mathbf{R}^{\mathbb{T}}$.

If $(\widetilde{x}^1, \widetilde{z}^1, \widetilde{w}^1) = (x^1, z^1, w^1)$ and $(\widetilde{x}^2, \widetilde{z}^2, \widetilde{w}^2) \neq (x^2, z^2, w^2)$, then by a similar discussion to that for $(\widetilde{x}^1, \widetilde{z}^1, \widetilde{w}^1) \neq (x^1, z^1, w^1)$ in the previous paragraph, we can construct a new allocation $\widehat{\mathbf{a}} \in A$ which Pareto-dominates $\mathbf{a}$ at $\mathbf{R}^{\mathbb{T}}$ such that $\widehat{\mathbf{a}}^1 = (x^1, z^1, w^1)$ and $\widehat{\mathbf{a}}^2$ with $(\widehat{x}^2, x^2) \in P(R_2)$. This $\widehat{\mathbf{a}} \in A$ has the property to violate the condition (*) for $\mathbf{a}$ at $\mathbf{R}^{\mathbb{T}}$. In such a way, we can show that if $(x^t, \widetilde{x}^t) \in I(R_t)$ holds for any $t < \overline{t}$, then $(x^t, z^t, w^t) = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)$ holds for any $t < \overline{t}$. In this case, $\widetilde{\mathbf{a}}$ is the desired allocation which has the property to violate the condition (*) for $\mathbf{a}$ at $\mathbf{R}^{\mathbb{T}}$.

Second, consider the case that there exists $t' < \overline{t}$ such that $\left(x^{t'}, z^{t'}, w^{t'}\right)$ is not $\left(z^{t'}, w^{t'}\right)$-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. Then, it is easy to construct an alternative allocation $\widehat{\mathbf{a}} \in A$ such that $\left(\widehat{x}^{t'}, \widehat{z}^{t'}, \widehat{w}^{t'}\right)$ is $\left(z^{t'}, w^{t'}\right)$-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$, and $(\widehat{x}^t, \widehat{z}^t, \widehat{w}^t) = (x^t, z^t, w^t)$

holds for any other $t \neq t'$. Then, this allocation $\widehat{\mathbf{a}}$ violates the condition (*) for $\mathbf{a}$ at $\mathbf{R}^{\mathbb{T}}$. ■

Let a feasible allocation $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ be called a *no-investment allocation* if for any $t \in \mathbb{T}$, $w^t = 0$.

Lemma 4: *Assume H is constant. Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, let a feasible allocation $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, 0)_{t\in\mathbb{T}} \in A$ be a no-investment allocation. Suppose for this allocation, there is any integer $k > 0$ such that*
(1) $z^t < \overline{z}$ for any $t < k$;
(2) $z^t < \overline{z}$ for any $t \in \mathbb{T}$ such that there exists a positive integer $n > 0$ with $nk < t < (n+1)k$; and
(3) $z^t = \overline{z}$ for any $t = nk$ and for any positive integer $n = 1, 2, \ldots,$.
*Moreover, for any $t \in \mathbb{T}$, $\mathbf{a}^t$ is $(z^t, 0)$-*constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. *Then,* $\mathbf{a}$ *is* Pareto efficient at $\mathbf{R}^{\mathbb{T}}$.

Proof. Suppose that there exists an alternative allocation $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t\in\mathbb{T}} = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)_{t\in\mathbb{T}} \in A$ and a generation $\overline{t} \in \mathbb{T}$ such that $(x^t, z^t, w^t) = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)$ for any $t < \overline{t}$, and $\left(\widetilde{x}^{\overline{t}}, x^{\overline{t}}\right) \in P(R_{\overline{t}})$. Then, by definition of $\mathbf{a}$, $\widetilde{z}^{\overline{t}} > z^{\overline{t}}$ and $\widetilde{w}^{\overline{t}} = 0$. Note that there exists a positive integer $n > 0$ such that $(n-1)k < \overline{t} < nk$. Then, $z^{nk} = \overline{z}$. Suppose that for any $t \in \mathbb{T}$ with $\overline{t} < t < nk$, $(\widetilde{x}^t, x^t) \in R_t$. Then, $\widetilde{z}^{\overline{t}+1} > z^{\overline{t}+1}$ and $\widetilde{w}^{\overline{t}+1} = 0$, since $\widetilde{Z}^{\overline{t}} > Z^{\overline{t}}$. Thus, to keep $(\widetilde{x}^t, x^t) \in R_t$ for $t = \overline{t}+2$, it follows $\widetilde{z}^{\overline{t}+2} > z^{\overline{t}+2}$ and $\widetilde{w}^{\overline{t}+2} = 0$, since $\widetilde{Z}^{\overline{t}+1} > Z^{\overline{t}+1}$. In a similar way, $\widetilde{z}^t > z^t$ and $\widetilde{w}^t = 0$ for any $t \in \mathbb{T}$ with $\overline{t} < t < nk$. Thus, $\widetilde{Z}^{nk-1} > Z^{nk-1}$ and $z^{nk} = \overline{z}$ imply $\left(x^{nk}, \widetilde{x}^{nk}\right) \in P(R_{nk})$. By **Lemma 3**, $\mathbf{a}$ is Pareto efficient. ■

The next lemma shows that if every generation is assigned a temporarily non-selfish allocation, such a feasible allocation cannot be Pareto efficient.

Lemma 5: *Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, let a feasible allocation $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ be $(z^t, w^t)_{t\in\mathbb{T}}$-*constrained Pareto efficient at $\mathbf{R}^{\mathbb{T}}$ *such that $z^t < \overline{z}$ for any $t \in \mathbb{T}$. Then,* $\mathbf{a}$ *is not* Pareto efficient at $\mathbf{R}^{\mathbb{T}}$.

Proof. Suppose that R_t is representable by a continuous real-valued function u^t. Consider an alternative allocation $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t\in\mathbb{T}} = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)_{t\in\mathbb{T}} \in A$, which is $(\widetilde{z}^t, \widetilde{w}^t)_{t\in\mathbb{T}}$-constrained Pareto efficient at $\mathbf{R}^{\mathbb{T}}$ and is defined as follows:
(1) $\widetilde{w}^t = w^t$ for any $t \in \mathbb{T}$;

(2) $\widetilde{z}^t = z^t + \triangle z^t$ for $t = 1$, where $\triangle z^t > 0$ is small enough;
(3) $\widetilde{z}^t = z^t + \triangle z^t$ for $t \in \mathbb{T} \setminus \{1\}$ such that

$$\begin{aligned} & u^t\left(l^t, g\left(h^t l^t, z^t\right) - w^t, Z^{t-1}\right) - u^t\left(l^t, g\left(h^t l^t, z^t\right) - w^t, \widetilde{Z}^{t-1}\right) \\ \leq \; & u^t\left(l^t, g\left(h^t l^t, \widetilde{z}^t\right) - w^t, \widetilde{Z}^{t-1}\right) - u^t\left(l^t, g\left(h^t l^t, z^t\right) - w^t, \widetilde{Z}^{t-1}\right), \end{aligned}$$

where $\triangle Z^{t-1} = \widetilde{Z}^{t-1} - Z^{t-1} > 0$. We can find an appropriate $\left(\triangle z^t\right)_{t\in\mathbb{T}}$ which guarantees $z^t + \triangle z^t \leq \overline{z}$ for any $t \in \mathbb{T}$.

Let us show this. Given any small enough $\triangle z^1 > 0$, let $\triangle \mathbf{z}^{\mathbb{T}}\left(\triangle z^1\right) \in \mathbb{R}^{\infty}_{++}$ be a vector $\left(\triangle z^t\right)_{t\in\mathbb{T}}$ satisfying the above (2) and (3) with $\triangle z^1$ as its first component, such that for any other $\left(\triangle \widehat{z}^t\right)_{t\in\mathbb{T}}$ satisfying (2) and (3) with $\triangle \widehat{z}^1 = \triangle z^1$, $\triangle z^t \leq \triangle \widehat{z}^t$ holds for any $t \in \mathbb{T}$. By the continuity of u^t and g, the existence of such $\triangle \mathbf{z}^{\mathbb{T}}\left(\triangle z^1\right)$ is guaranteed for any small enough $\triangle z^1 > 0$. Note that each component of the vector $\triangle \mathbf{z}^{\mathbb{T}}\left(\triangle z^1\right)$ increases when $\triangle z^1$ increases. The mapping $\triangle \mathbf{z}^{\mathbb{T}}$ is also continuous at every small enough $\triangle z^1$.[6]

Suppose for some $\triangle \widehat{z}^1 > 0$, $\triangle \mathbf{z}^{\mathbb{T}}\left(\triangle \widehat{z}^1\right)$ has a subset $\mathbb{N}$ of $\mathbb{T}$ such that for any $t \in \mathbb{N}$, t has $\triangle z^t\left(\triangle \widehat{z}^1\right) > \overline{z} - z^t$ in this vector, where $\triangle z^t\left(\triangle \widehat{z}^1\right)$ means the t-th component of the vector $\triangle \mathbf{z}^{\mathbb{T}}\left(\triangle \widehat{z}^1\right)$. However, since $\triangle \mathbf{z}^{\mathbb{T}}\left(\triangle z^1\right) \to \mathbf{0} \in \mathbb{R}^{\infty}_{+}$ as $\triangle z^1 \to 0$, we find an appropriately small enough $\triangle z^{*1} > 0$ such that for any $t \in \mathbb{T}$, $\triangle z^t\left(\triangle z^{*1}\right) \leq \overline{z} - z^t$ by the increasing and continuous property of the mapping $\triangle \mathbf{z}^{\mathbb{T}}$.

By construction, $\left(\widetilde{x}^1, x^1\right) \in P\left(R_1\right)$. Moreover, $\left(\widetilde{x}^t, x^t\right) \in R_t$ for any $t \in \mathbb{T} \setminus \{1\}$. This implies $\widetilde{\mathbf{a}}$ Pareto dominates $\mathbf{a}$. ■

Lemma 5 deserves some comment. It shows that if generations fail to emit to the maximal public bads, then a Pareto improving allocation can be constructed by increasing the sequence of public bads $\{\widetilde{z}^t\}_{t\in\mathbb{T}}$ in such a way that each generation compensates the increased inherited public bads, in turn by appropriately increasing the amount of public bads bequeathed. This type of situation would not work in a finite-horizon economy.

Throughout the above arguments on Pareto efficiency in this intergenerational resource allocations, we can summarize as in the following:

Proposition 1: *Given an economy* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$*, let a feasible allocation* $\mathbf{a} = \left(\mathbf{a}^t\right)_{t\in\mathbb{T}} = \left(x^t, z^t, w^t\right)_{t\in\mathbb{T}} \in A$ *be* $\left(z^t, w^t\right)_{t\in\mathbb{T}}$-constrained Pareto efficient at $\mathbf{R}^{\mathbb{T}}$.

[6] To define the continuity of the mapping $\triangle \mathbf{z}^{\mathbb{T}}$, we may adopt the *sup metric* as the topology of $\mathbb{R}^{\infty}_{+}$.

Then, $\mathbf{a}$ *is* Pareto efficient at $\mathbf{R}^{\mathbb{T}}$ *only if for any* $t \in \mathbb{T}$, $[z^t < \overline{z}\ ]$ *implies* $\left[\exists t' \in \mathbb{T} \text{ s.t. } t' > t,\ z^{t'} = \overline{z} \text{ and } w^{t'} = 0\right]$.

Proof. Suppose that there exists $\overline{t} \in \mathbb{T}$ such that $z^{\overline{t}} = \overline{z}$ holds, and for any $t' \in \mathbb{T}$ with $t' > \overline{t}$, $z^{t'} < \overline{z}$ or $w^{t'} > 0$ holds. Then, consider another feasible allocation $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t\in\mathbb{T}} = (\widetilde{x}^t, \widetilde{z}^t, \widetilde{w}^t)_{t\in\mathbb{T}} \in A$, which is $(\widetilde{z}^t, \widetilde{w}^t)_{t\in\mathbb{T}}$-constrained Pareto efficient at $\mathbf{R}^{\mathbb{T}}$ and is defined as follows:
(1) for any $t < \overline{t}$, $\widetilde{\mathbf{a}}^t = \mathbf{a}^t$;
(2) for $t = \overline{t}$, $\widetilde{z}^t = z^t + \triangle z^t$ & $\widetilde{w}^t = w^t$, where $\triangle z^t > 0$ is small enough;
(3) for $t \in \mathbb{T} \backslash \left\{\overline{t}\right\}$ with $t > \overline{t}$, either (i) $\widetilde{z}^t = z^t + \triangle z^t$ & $\widetilde{w}^t = w^t$, or (ii) $\widetilde{z}^t \geq z^t$ & $\widetilde{w}^t = w^t - \triangle w^t$ if $w^t > 0$, such that

$$\begin{aligned} & u^t\left(l^t, g\left(h^t l^t, z^t\right) - w^t, Z^{t-1}\right) - u^t\left(l^t, g\left(\widetilde{h}^t l^t, z^t\right) - w^t, \widetilde{Z}^{t-1}\right) \\ \leq\ & u^t\left(l^t, g\left(\widetilde{h}^t l^t, \widetilde{z}^t\right) - \widetilde{w}^t, \widetilde{Z}^{t-1}\right) - u^t\left(l^t, g\left(\widetilde{h}^t l^t, z^t\right) - w^t, \widetilde{Z}^{t-1}\right), \end{aligned}$$

where $\widetilde{Z}^{t-1} = Z^{t-1} + \triangle Z^{t-1}$ for $\triangle Z^{t-1} = \sum_{t'=1}^{t-1} \delta^{(t-1)-t'} \cdot \triangle z^{t'}$, $\widetilde{h}^{\overline{t}+1} = H\left(h^{\overline{t}}, \widetilde{w}^{\overline{t}}\right) = H\left(h^{\overline{t}}, w^{\overline{t}}\right) = h^{\overline{t}+1}$, and $\widetilde{h}^t = H\left(\widetilde{h}^{t-1}, \widetilde{w}^{t-1}\right)$ for any $t \in \mathbb{T} \backslash \left\{\overline{t}+1\right\}$ with $t > \overline{t}$. Through an argument similar to that in the proof of Lemma 5, we can confirm that $\triangle z^t \leq \overline{z} - z^t$ and $\triangle w^t \in [0, w^t]$ for any $t \in \mathbb{T} \backslash \left\{\overline{t}+1\right\}$ with $t > \overline{t}$. By construction, $\left(\widetilde{x}^{\overline{t}+1}, x^{\overline{t}+1}\right) \in P\left(R_{\overline{t}+1}\right)$. Moreover, $(\widetilde{x}^t, x^t) \in R_t$ for any $t \in \mathbb{T} \backslash \left\{\overline{t}+1\right\}$ with $t > \overline{t}$. This implies $\widetilde{\mathbf{a}}$ Pareto dominates $\mathbf{a}$. ■

Thus, **Proposition 1** shows that every Pareto efficient allocation needs an infinite subset of generations who enjoy a "selfish" consumptions. Otherwise, there exists a Pareto improving allocation with an increased sequence of public bads emissions as discussed in the comment for **Lemma 5**.

Proposition 2: *Assume H is constant. Given an economy* $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, *let a feasible allocation* $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, 0^t)_{t\in\mathbb{T}} \in A$ *be* $(z^t, 0^t)_{t\in\mathbb{T}}$-constrained Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. *Then,* $\mathbf{a}$ *is* Pareto efficient at $\mathbf{R}^{\mathbb{T}}$ *if and only if the following condition holds:*

($*$) *For any* $t \in \mathbb{T}$, $[z^t < \overline{z}\ ]$ *implies* $\left[\exists t' \in \mathbb{T} \text{ s.t. } t' > t,\ z^{t'} = \overline{z} \text{ and } w^{t'} = 0\right]$.

Proof. Suppose that for any $t \in \mathbb{T}$, $[z^t < \overline{z}\ ]$ implies

$$\left[\exists t' \in \mathbb{T} \text{ s.t. } t' > t,\ z^{t'} = \overline{z} \text{ and } w^{t'} = 0\right].$$

Suppose that there exists an alternative allocation $\widetilde{\mathbf{a}} = (\widetilde{\mathbf{a}}^t)_{t \in \mathbb{T}} = (\widetilde{x}^t, \widetilde{z}^t, 0^t)_{t \in \mathbb{T}} \in A$ and a generation $\overline{t} \in \mathbb{T}$ such that $(x^t, z^t, 0^t) = (\widetilde{x}^t, \widetilde{z}^t, 0^t)$ for any $t < \overline{t}$, and $\left(\widetilde{x}^{\overline{t}}, x^{\overline{t}}\right) \in P(R_{\overline{t}})$. Then, by definition of $\mathbf{a}$, $\widetilde{z}^{\overline{t}} > z^{\overline{t}}$. Note there exists $t' \in \mathbb{T}$ such that $t' > \overline{t}$, $z^{t'} = \overline{z}$ and $w^{t'} = 0$. Then, following the proof of **Lemma 4**, we can see that if $(\widetilde{x}^t, x^t) \in R_t$ for any $t \in \mathbb{T}$ with $\overline{t} < t < t'$, then $\left(x^{t'}, \widetilde{x}^{t'}\right) \in P(R_{t'})$. Thus, by **Lemma 3**, $\mathbf{a}$ is Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. ■

The above two characterization results indicate that any regulation policy for the public bad emissions is incompatible with Pareto efficiency, whenever it requires $z^t < \overline{z}$ for any $t \in \mathbb{T}$. However, it seems to be reasonable, from the viewpoint of intergenerational equity, to require $z^t = z^{t'} < \overline{z}$ for any t, $t' \in \mathbb{T}$. Thus, **Proposition 1** implies that such a requirement of intergenerational equity is inconsistent with Pareto efficiency. To be Pareto efficient, the feasible allocation should have temporary selfish allocations, as **Proposition 2** shows for the case of constant H.

5 Main Theorems

In this section, we argue the fundamental incompatibility between Pareto efficiency and intergenerational equity. First, we will focus on the case $(1-\delta) Z^0 \leq \overline{z}$ in the following discussion. This assumption is reasonable, since the case $(1-\delta) Z^0 > \overline{z}$ implies that even the maximal emissions of public bads by all generations decrease the accumulated amount of public bads, which is counterintuitive. Second, as an efficiency requirement, we think **NSPE** is more reasonable than **PE**, since **PE** permits selfish allocations. In fact, it is not so desirable even if **PE** and an intergenerational equity axiom are compatible only at selfish allocations, because such allocations do not resolve the issue of negative externality.

Let us examine the compatibility between Pareto efficiency and intergenerational equity in terms of no-envy, given the above two reasonable restrictions. We arrive at the following fundamental impossibility theorem:

Theorem 1: *Suppose $(1-\delta) Z^0 \leq \overline{z}$. Then, there is no allocation rule φ which satisfies* **NSPE** *and* **NE**.

Proof. Let us consider an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ such that for any $t, t' \in \mathbb{T}$, $R_t = R_{t'}$. Moreover, in this economy $\mathbf{R}^{\mathbb{T}}$, we will suppose that every generation's preference R_t is not so much sensitive to the change of accumulated public bads.

Case 1: Let us take any Pareto efficient allocation $\mathbf{a} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ in which there exists a generation $\bar{t} \in \mathbb{T}$ such that $\left(z^{\bar{t}}, w^{\bar{t}}\right) = (\overline{z}, 0)$ and $\left(z^{\bar{t}+1}, w^{\bar{t}+1}\right) \neq (\overline{z}, 0)$ for the generation $\bar{t}+1 \in \mathbb{T}$. The existence of the generation $\bar{t}$ is guaranteed by **Lemma 5** and **Proposition 1**. Consider $Z^{\bar{t}-1}$ and $Z^{\bar{t}}$ in this allocation. Note that $Z^{\bar{t}-1}$ is consumed by the generation $\bar{t}$ as the third component of the vector $x^{\bar{t}}$, while $Z^{\bar{t}}$ is consumed by the generation $\bar{t}+1$ as the third component of the vector $x^{\bar{t}+1}$. Consider

$$Z^{\bar{t}} - Z^{\bar{t}-1} = -(1-\delta)\cdot Z^{\bar{t}-1} + \overline{z}.$$

We will show $Z^{\bar{t}} - Z^{\bar{t}-1} > 0$. Compare $Z^{\bar{t}-1}$ with $\frac{\overline{z}}{1-\delta}$. Note

$$\begin{aligned}
Z^{\bar{t}-1} &= \delta^{\bar{t}-1} Z^0 + \sum_{t'=1}^{\bar{t}-1} \delta^{\bar{t}-1-t'} z^{t'}, \\
\text{while } \frac{\overline{z}}{1-\delta} &= \delta^{\bar{t}-1}\overline{z} + \sum_{t'=1}^{\bar{t}-1} \delta^{\bar{t}-1-t'}\overline{z} + \frac{\delta^{\bar{t}}\overline{z}}{1-\delta}.
\end{aligned}$$

Also note that

$$\begin{aligned}
\delta^{\bar{t}-1}\overline{z} + \frac{\delta^{\bar{t}}\overline{z}}{1-\delta} - \delta^{\bar{t}-1}Z^0 &= \delta^{\bar{t}-1}\left(\overline{z} + \frac{\delta\overline{z}}{1-\delta} - Z^0\right) \\
&= \frac{\delta^{\bar{t}-1}}{1-\delta}\left[(1-\delta)\,\overline{z} + \delta\overline{z} - (1-\delta)\,Z^0\right] \\
&= \frac{\delta^{\bar{t}-1}}{1-\delta}\left[\overline{z} - (1-\delta)\,Z^0\right] \geq 0
\end{aligned}$$

by the assumption. Thus, $\frac{\overline{z}}{1-\delta} \geq Z^{\bar{t}-1}$ holds, since $\sum_{t'=1}^{\bar{t}-1}\delta^{\bar{t}-1-t'}\overline{z} \geq \sum_{t'=1}^{\bar{t}-1}\delta^{\bar{t}-1-t'}z^{t'}$. This implies $Z^{\bar{t}} - Z^{\bar{t}-1} = -(1-\delta)\cdot Z^{\bar{t}-1} + \overline{z} \geq -(1-\delta)\cdot\frac{\overline{z}}{1-\delta} + \overline{z} = 0$. Thus, $Z^{\bar{t}} \geq Z^{\bar{t}-1}$ and $\left(z^{\bar{t}}, w^{\bar{t}}\right) = (\overline{z}, 0) \neq \left(z^{\bar{t}+1}, w^{\bar{t}+1}\right)$ imply that $\left(x^{\bar{t}}, x^{\bar{t}+1}\right) \in P\left(R_{\bar{t}+1}\right)$, since $x^{\bar{t}}$ is temporarily selfish.

Case 2: Let us take any Pareto efficient allocation $\mathbf{a} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ in which there exists a generation $\bar{t} \in \mathbb{T}$ such that $\left(z^{\bar{t}}, w^{\bar{t}}\right) = (\overline{z}, 0)$ and

$(z^t, w^t) = (\overline{z}, 0)$ for any $t \in \mathbb{T}$ with $t > \overline{t}$. In this case, compare $x^{\overline{t}}$ with $x^{\overline{t}+1}$. By the same argument on $Z^{\overline{t}} - Z^{\overline{t}-1}$ as Case 1, $Z^{\overline{t}} \geq Z^{\overline{t}-1}$. By the definition of $\mathbf{a}$, $\left(z^{\overline{t}-1}, w^{\overline{t}-1}\right) \neq (\overline{z}, 0)$, since $\mathbf{a}$ is non-selfish. In particular, if $z^{\overline{t}-1} < \overline{z}$, then $Z^{\overline{t}} > Z^{\overline{t}-1}$. Thus, $\left(x^{\overline{t}}, x^{\overline{t}+1}\right) \in P\left(R_{\overline{t}+1}\right)$, since $x^{\overline{t}}$ is temporarily selfish. If $z^{\overline{t}-1} = \overline{z}$, then compare $x^{\overline{t}}$ with $x^{\overline{t}-1}$. Since $\left(z^{\overline{t}-1}, w^{\overline{t}-1}\right) \neq (\overline{z}, 0)$, $w^{\overline{t}-1} > 0$. Note it may be the case that $Z^{\overline{t}-1} \geq Z^{\overline{t}-2}$. However, in this economy $\mathbf{R}^{\mathbb{T}}$, every generation is not so much sensitive to the change of accumulated public bads. Thus, the effect of $w^{\overline{t}-1} > 0$ can cancel out the effect of $Z^{\overline{t}-1} \geq Z^{\overline{t}-2}$, so that $\left(x^{\overline{t}}, x^{\overline{t}-1}\right) \in P\left(R_{\overline{t}-1}\right)$, since $x^{\overline{t}}$ is temporarily selfish.

In summary, if a Pareto efficient allocation is non-selfish, then it does not meet the no-envy condition. Thus, there is no allocation rule satisfies **NSPE** and **NE.** ■

The above theorem implies that any policy for regulating the emissions of public bads and promoting education for human capital is Pareto inefficient whenever it cares about intergenerational equity in terms of no-envy. However, if we give up any of such policy, is it possible to implement efficient and equitable allocations in this model? The answer is still negative in general, as the following theorem suggests:

Theorem 2: *Suppose $(1-\delta) Z^0 \leq \overline{z}$. Then, there exists an allocation rule φ which satisfies selfish-**PE** and **NE** if and only if $(1-\delta) Z^0 = \overline{z}$.*

Proof. Suppose $(1-\delta) Z^0 = \overline{z}$. Then, a historical sequence $(z^{*t}, w^{*t})_{t\in\mathbb{T}}$ with $(z^{*t}, w^{*t}) = (\overline{z}, 0)$ for any $t \in \mathbb{T}$ constitutes a feasible sequence of emitted public bads and investments for human capital, and it holds that $Z^{t-1} = Z^0$ for any $t \in \mathbb{T}$. Given any economy $\mathbf{R}^{\mathbb{T}}$ and this sequence $(z^{*t}, w^{*t})_{t\in\mathbb{T}}$, let us consider a feasible allocation $\mathbf{a}^* = (x^{*t}, z^{*t}, w^{*t})_{t\in\mathbb{T}}$ as follows: for any $t \in \mathbb{T}$, the third component of x^{*t} is given by Z^0; and the first and the second components (l^{*t}, y^{*t}) of x^{*t} are given by: $\left(\left(l^{*t}, y^{*t}, Z^0\right), \left(l^t, y^t, Z^0\right)\right) \in R_t$ where (l^t, y^t) satisfies $g\left(h^0 \cdot l^t, \overline{z}\right) \geq y^t$. Thus, the allocation $\mathbf{a}^*$ is selfish, so that it is Pareto efficient by **Lemma 1**. Moreover, since in this allocation, every generation chooses his selfish action under the same components of the public bads accumulation Z^0 and the human capital accumulation h^0. Thus, $\mathbf{a}^*$ is no-envy.

Consider $(1-\delta) Z^0 < \overline{z}$. Then, in the selfish sequence of emitted pubic bads and investments for human capital $(z^{*t}, w^{*t})_{t\in\mathbb{T}}$, $Z^{t-1} > Z^0$ holds for any $t \in \mathbb{T} \backslash \{1\}$. Consider an economy $\mathbf{R}'^{\mathbb{T}} \in \mathcal{R}^{\infty}$ such that for any $t, t' \in \mathbb{T}$, $R'_t = R'_{t'}$. Consider any selfish allocation $\mathbf{a}' = (x'^t, z^{*t}, w^{*t})_{t \in \mathbb{T}}$ which is consistent with $(z^{*t}, w^{*t})_{t \in \mathbb{T}}$ in this economy $\mathbf{R}'^{\mathbb{T}}$. Then, every generation $t \in \mathbb{T} \backslash \{1\}$ except the generation 1 in this selfish allocation strictly prefers x'^1 to x'^t under the economy $\mathbf{R}'^{\mathbb{T}}$.

Consider $(1 - \delta) Z^0 > \overline{z}$. Then, in the selfish sequence of emitted public bads and investments for human capital $(z^{*t}, w^{*t})_{t \in \mathbb{T}}$, $Z^{t-1} < Z^0$ holds for any $t \in \mathbb{T} \backslash \{1\}$. Consider an economy $\mathbf{R}''^{\mathbb{T}} \in \mathcal{R}^{\infty}$ such that for any $t, t' \in \mathbb{T}$, $R''_t = R''_{t'}$. Consider any selfish allocation $\mathbf{a}'' = (x''^t, z^{*t}, w^{*t})_{t \in \mathbb{T}}$ which is consistent with $(z^{*t}, w^{*t})_{t \in \mathbb{T}}$ in this economy $\mathbf{R}''^{\mathbb{T}}$. Then, $(x'^t, x'^1) \in P(R_1)$ holds for any generation $t \in \mathbb{T} \backslash \{1\}$. ■

The implication of the above theorem is incompatibility between Pareto efficiency and intergenerational equity in terms of no-envy even over selfish allocations. This is because selfish allocations can be no-envy if and only if $(1 - \delta) Z^0 = \overline{z}$, but the occurrence of this equation is almost improbable. In fact, the most probable setting is $(1 - \delta) Z^0 < \overline{z}$, which implies the situation that the negative externality to the future generation becomes more serious whenever the current generation emits the maximal amount of public bads.

If the no-envy axiom is replaced by the responsibility and compensation axioms, is it possible to have a better result? Unfortunately, the following theorem gives us a negative answer:

Theorem 3: *Suppose* $(1 - \delta) Z^0 \leq \overline{z}$. *Then, there is no allocation rule* φ *which satisfies* **NSPE** *and* **EWUP**.

Proof. Let us consider an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ such that for any $t, t' \in \mathbb{T}$, $R_t = R_{t'}$. Moreover, in this economy $\mathbf{R}^{\mathbb{T}}$, we suppose the following type of preference:
(i) every generation's preference R_t is not so much sensitive to the change of accumulated public bads;
(ii) every generation's preference R_t meets the *boundary condition* in the sense that for any (l^t, Z^t), $\left(\widetilde{l}^t, \widetilde{Z}^t\right) \in \left[0, \overline{l}\right] \times \mathbb{R}_+$, and for any $y^t \in \mathbb{R}_{++}$, $\left((l^t, y^t, Z^t), \left(\widetilde{l}^t, 0, \widetilde{Z}^t\right)\right) \in P(R_t)$ holds.

Thus, in this economy $\mathbf{R}^{\mathbb{T}}$, any feasible allocation $\mathbf{a} = (x^t, z^t, w^t)_{t \in \mathbb{T}} \in A$ satisfying **EWUP** has the property that for any $t, t' \in \mathbb{T}$, $\left(x^t, x^{t'}\right) \in I(R_t)$. In other words, **EWUP** is equivalent to **NE** at $\mathbf{R}^{\mathbb{T}}$. Thus, following the proof of **Theorem 1**, we can see that every non-selfish Pareto efficient allocation at $\mathbf{R}^{\mathbb{T}}$ has a pair of generations $t, t' \in \mathbb{T}$ such that $\left(x^t, x^{t'}\right) \in P\left(R_{t'}\right)$, which indicates the violation of **EWUP**. ■

Corollary 1: *Suppose* $(1-\delta)\, Z^0 \leq \overline{z}$. *Then, there is no allocation rule* φ *which satisfies* **NSPE**, **EWUP**, *and* **RFG**.

Proof. It is obvious from **Theorem 3**. ■

The above impossibility result in **Corollary** 1 comes from the inconsistency of **EWUP** with **NSPE**. Thus, the axioms of responsibility and compensation cannot constitute an efficient allocation rule whenever **EWUP** is taken as the weaker variant of the basic axiom of compensation. By the way, if we take **UNEP** as another weaker axiom of the compensation principle, is it possible to make an efficient allocation rule satisfying the principles of responsibility and compensation? The following theorem still gives us a negative answer whenever **NEEG** is required as the basic axiom of responsibility:

Theorem 4: *Suppose* $(1-\delta)\, Z^0 < \overline{z}$ *and* H *is constant. Then, there is no allocation rule* φ *which satisfies* **PE**, **UNEP**, *and* **NEEG**.

Proof. Take any feasible allocation $\mathbf{a} = \left(\mathbf{a}^t\right)_{t\in\mathbb{T}} = \left(x^t, z^t, w^t\right)_{t\in\mathbb{T}} \in A$ such that there exists two generations $\overline{t}, \overline{t}' \in \mathbb{T}$ such that $\overline{t} < \overline{t}'$ and $Z^{\overline{t}-1} = Z^{\overline{t}'-1}$. Since we consider the case that H is constant, $h^{\overline{t}} = h^{\overline{t}'} = h^0$. W. l.o.g., let us suppose that $(1-\delta)\, Z^{\overline{t}-1} < \overline{z}$. Let $z^* \equiv (1-\delta)\, Z^{\overline{t}-1}$.

Consider the following economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$: for any $t \in \mathbb{T}$ such that $t \geq \overline{t}$, his preference R_t is represented by the following utility function u_t: for any $\left(\widetilde{l}^t, \widetilde{y}^t, \widetilde{Z}^{t-1}\right) \in \left[0, \overline{l}\right] \times \mathbb{R}_+ \times \mathbb{R}_+$,

$$u_t\left(\widetilde{l}^t, \widetilde{y}^t, \widetilde{Z}^{t-1}\right) = \widetilde{y}^t - g\left(h^0 \cdot \widetilde{l}^t, z^*\right) - \widetilde{Z}^{t-1}.$$

Suppose $\mathbf{a}$ has the property of **UNEP** and **NEEG**. Thus, $\left(x^{\overline{t}}, x^{\overline{t}'}\right) \in I\left(R_{\overline{t}}\right) = I\left(R_{\overline{t}'}\right)$. Moreover, it follows that the generation $\overline{t}$ emits $z^{\overline{t}} = z^*$ in the allocation $\mathbf{a}$, which we will show now. Suppose $z^{\overline{t}} > z^*$. Then, $Z^{\overline{t}} > Z^{\overline{t}-1}$. Then, $z^{\overline{t}+1} > z^{\overline{t}}$ by **UNEP**. Thus, $Z^{\overline{t}+1} > Z^{\overline{t}-1}$. So, $z^{\overline{t}+2} > z^{\overline{t}}$ by **UNEP**, which implies $Z^{\overline{t}+2} > Z^{\overline{t}-1}$. By repeating this process up to $\overline{t}' - 1$, we conclude that $Z^{\overline{t}'-1} > Z^{\overline{t}-1}$, which is a contradiction. By applying the similar argument for the case of $z^{\overline{t}} < z^*$, we can arrive at $Z^{\overline{t}'-1} < Z^{\overline{t}-1}$, which is also a contradiction. Thus, $z^{\overline{t}} = z^*$ holds in the allocation $\mathbf{a}$.

Note that to make $\mathbf{a}$ Pareto efficient at $\mathbf{R}^{\mathbb{T}}$, we have to require that each temporary allocation $\mathbf{a}^t$ is at least $(z^t, 0^t)$-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. Thus, the consumption vector $x^{\overline{t}} = \left(l^{\overline{t}}, y^{\overline{t}}, Z^{\overline{t}-1}\right)$ in the temporary allocation $\mathbf{a}^{\overline{t}} = \left(x^{\overline{t}}, z^{\overline{t}}, 0^{\overline{t}}\right)$ should have the property that $y^{\overline{t}} = g\left(h^0 l^{\overline{t}}, z^*\right)$. Then, by **NEEG**, we will show that for the generation $\overline{t}' \in \mathbb{T}$, $z^{\overline{t}'} = z^{\overline{t}} = z^*$ and $y^{\overline{t}'} = g\left(h^0 l^{\overline{t}'}, z^*\right)$ hold.

First, if $z^{\overline{t}'} = z^{\overline{t}} = z^*$ and $y^{\overline{t}'} = g\left(h^0 l^{\overline{t}'}, z^*\right)$ in the temporary allocation $\mathbf{a}^{\overline{t}'} = \left(x^{\overline{t}'}, z^{\overline{t}'}, 0^{\overline{t}'}\right)$, then $\left(x^{\overline{t}}, x^{\overline{t}'}\right) \in I\left(R_{\overline{t}}\right) = I\left(R_{\overline{t}'}\right)$ holds and $\mathbf{a}^{\overline{t}'}$ is $\left(z^{\overline{t}'}, 0^{\overline{t}'}\right)$-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. Second, if $z^{\overline{t}'} > z^{\overline{t}} = z^*$, then to keep $\left(x^{\overline{t}}, x^{\overline{t}'}\right) \in I\left(R_{\overline{t}}\right) = I\left(R_{\overline{t}'}\right)$, it follows that $l^{\overline{t}'} < l^{\overline{t}}$. This is because $y^{\overline{t}'} = g\left(h^0 l^{\overline{t}'}, z^{\overline{t}'}\right) > g\left(h^0 l^{\overline{t}'}, z^*\right) \geq g\left(h^0 l^{\overline{t}}, z^*\right) = y^{\overline{t}}$ whenever $l^{\overline{t}'} \geq l^{\overline{t}}$. Then, however, $x^{\overline{t}'}$ cannot be $\left(z^{\overline{t}'}, 0^{\overline{t}'}\right)$-constrained temporary Pareto efficient at $\mathbf{R}^{\mathbb{T}}$, since

$$\begin{aligned} u_{\overline{t}'}\left(\widetilde{l}^{\overline{t}'}, \widetilde{y}^{\overline{t}'}, Z^{\overline{t}'-1}\right) &= g\left(h^0 \widetilde{l}^{\overline{t}'}, z^{\overline{t}'}\right) - g\left(h^0 \widetilde{l}^{\overline{t}'}, z^*\right) - Z^{\overline{t}'-1} \\ &> y^{\overline{t}} - g\left(h^0 l^{\overline{t}}, z^*\right) - Z^{\overline{t}'-1} = u_{\overline{t}'}\left(l^{\overline{t}'}, y^{\overline{t}'}, Z^{\overline{t}'-1}\right) \end{aligned}$$

for any $\widetilde{l}^{\overline{t}'} \geq l^{\overline{t}}$. Third, if $z^{\overline{t}'} < z^{\overline{t}} = z^*$, then $\left(x^{\overline{t}}, x^{\overline{t}'}\right) \in P\left(R_{\overline{t}}\right) = P\left(R_{\overline{t}'}\right)$ holds. Thus, **NEEG** implies that $z^{\overline{t}'} = z^{\overline{t}}$.

The above argument implies that $Z^{\overline{t}'-1} = Z^{\overline{t}'}$, so that the property of **NEEG** should be applied for the generations $\overline{t}'$ and $\overline{t}' + 1$. Thus, following the above argument, we arrive at $z^{\overline{t}'+1} = z^{\overline{t}'} < \overline{z}$. In such a way, $z^t = z^{\overline{t}'} < \overline{z}$ holds for any $t \in \mathbb{T}$ with $t > \overline{t}'$, since $\mathbf{a}$ has the property of **NEEG**. By **Proposition 1**, $\mathbf{a}$ cannot be Pareto efficient. ■

Let us also examine the compatibility between Pareto efficiency and intergenerational equity in terms of egalitarian-equivalence. Unfortunately, the following theorem gives us a negative answer:

Theorem 5: *Suppose* $(1-\delta) Z^0 < \overline{z}$ *and let* $z^* \equiv (1-\delta) Z^0$. *Then, there is no allocation rule* φ *which satisfies* **PE** *and* z^*-**EE**.

Proof. Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, let us take any feasible allocation $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ which is a z^*-egalitarian equivalent allocation. Thus, for any $t \in \mathbb{T}$ and any $\mathbf{a}^t$, there exists a corresponding ideal allocation $\mathbf{a}_0^t = (x_0^t, z^*, 0^t)$ such that $x_0^t \equiv (l_0^t, g(h^0 l_0^t, z^*), Z^0)$ is the maximizer of R_t under the constraint (Z^0, h^0, z^*). Since $z^* = (1-\delta) Z^0$, the allocation $\mathbf{a}_0 = (\mathbf{a}_0^t)_{t\in\mathbb{T}} = (x_0^t, z^*, 0^t)_{t\in\mathbb{T}}$ becomes a feasible allocation. Since $z^* < \overline{z}$, the feasible allocation $\mathbf{a}_0 \in A$ is $(z^*, 0)$-constrained Pareto efficient at $\mathbf{R}^{\mathbb{T}}$, but not Pareto efficient at $\mathbf{R}^{\mathbb{T}}$ by **Proposition 1**. Thus, there exists an alternative feasible allocation $\mathbf{a}' \in A$ which Pareto-dominates $\mathbf{a}_0$. By the way, since $\mathbf{a}$ is Pareto indifferent to $\mathbf{a}_0$, $\mathbf{a}$ is Pareto-dominated by $\mathbf{a}'$. ■

Finally, we can also obtain the incompatibility between Pareto efficiency and environmental sustainability. The first incompatibility relevent to **PBM** is given by the following theorem:

Theorem 6: *Suppose $(1-\delta) Z^0 < \overline{z}$. Then, there is no allocation rule φ which satisfies* **PE** *and* **PBM**.

Proof. Given an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$, let us take any feasible allocation $\mathbf{a} = (\mathbf{a}^t)_{t\in\mathbb{T}} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ which has a monotone non-increasing sequence of public bads consumptions $(Z^{t-1})_{t\in\mathbb{T}}$. Since $(1-\delta) Z^0 < \overline{z}$, and $(Z^{t-1})_{t\in\mathbb{T}}$ is a monotone non-increasing sequence, $z^t < \overline{z}$ holds for any $t \in \mathbb{T}$. Thus, by **Proposition 1**, $\mathbf{a}$ cannot be Pareto efficient at $\mathbf{R}^{\mathbb{T}}$. ■

The second incompatibility is relevent to **J-HD**.

Theorem 7: *Suppose $(1-\delta) Z^0 < \overline{z}$. Then, there is no allocation rule φ which satisfies* **NSPE** *and* **J-HD**.

Proof. Let us consider an economy $\mathbf{R}^{\mathbb{T}} \in \mathcal{R}^{\infty}$ such that for any $t, t' \in \mathbb{T}$, $R_t = R_{t'}$. Moreover, in this economy $\mathbf{R}^{\mathbb{T}}$, we will suppose that every generation's preference R_t is consistent with J in the sense that for any $x^{t'}, x^{t''} \in X$, $(x, x') \in R_t$ holds if and only if there exists $h \in \mathbb{R}_+$ such that $((x, h), (x', h)) \in J$.

Let us take any Pareto efficient allocation $\mathbf{a} = (x^t, z^t, w^t)_{t\in\mathbb{T}} \in A$ in which there exists a generation $\overline{t} \in \mathbb{T}$ such that $\left(z^{\overline{t}}, w^{\overline{t}}\right) = (\overline{z}, 0)$. Then, $h^{\overline{t}} = h^{\overline{t}+1}$. This case corresponds to either **Case 1** or **Case 2** in the proof of **Theorem 1**. Without loss of generality, let us assume that $\left(x^{\overline{t}}, x^{\overline{t}+1}\right) \in R_t$

if and only if $\left(\left(x^{\bar{t}}, h^{\bar{t}}\right), \left(x^{\bar{t}+1}, h^{\bar{t}}\right)\right) \in J$, and $\left(x^{\bar{t}+1}, x^{\bar{t}}\right) \in R_t$ if and only if $\left(\left(x^{\bar{t}+1}, h^{\bar{t}}\right), \left(x^{\bar{t}}, h^{\bar{t}}\right)\right) \in J$ for this $h^{\bar{t}}$. Consider $Z^{\bar{t}-1}$ and $Z^{\bar{t}}$ in this allocation as in the proof of **Theorem 1**. Then, since $(1-\delta)\, Z^0 < \overline{z}$, $\frac{\overline{z}}{1-\delta} > Z^{\bar{t}-1}$ holds, and so $Z^{\bar{t}} - Z^{\bar{t}-1} = -(1-\delta) \cdot Z^{\bar{t}-1} + \overline{z} > -(1-\delta) \cdot \frac{\overline{z}}{1-\delta} + \overline{z} = 0$. Thus, $Z^{\bar{t}} > Z^{\bar{t}-1}$ and $\left(z^{\bar{t}}, w^{\bar{t}}\right) = (\overline{z}, 0)$ imply that $\left(\left(x^{\bar{t}}, h^{\bar{t}}\right), \left(x^{\bar{t}+1}, h^{\bar{t}+1}\right)\right) \in P(J)$, since $h^{\bar{t}} = h^{\bar{t}+1}$ and $x^{\bar{t}}$ is temporarily selfish for $R_{\bar{t}}$. This implies that **a** violates **J-HD**. ■

This theorem implies that non-selfish Pareto efficiency leads to the violation of human development in terms of the objective welfare measure J.

6 Concluding Remarks

The main theorems put forward in section 5 indicate that Pareto efficiency is not so attractive in this context of resource allocations. In this model, the more efficient production of private goods by one generation involves the more emission of the public bad, which this generation does not suffer from. Thus, from the point of this generation's rational choice, he has no motivation to regulate the emission of the public bad. However, from the point of sustainability of human beings as well as the point of intergenerational equity, each generation should implement some policy for regulating the public bad emissions. In contrast, Pareto efficiency requires that there should be generations who never implement any policy for regulating the public bad emissions. Facing with these two mutually incompatible judgements, I believe that the judgement derived from the axioms of sustainability and intergenerational equity should be given a priority to the judgement derived from Pareto efficiency. So, at the expense of Pareto efficiency, we should consider the existence of second best allocation rules which meet the axioms of sustainability and intergenerational equity as well as the second best efficiency axiom, that would be an open question.

It would be worth commenting on another type of intergenerational equity. Here, we only discussed the types of welfaristic equity axioms, where the main informational basis for measuring individual's wellness was individual's subjective preference. However, it is possible to discuss intergenerational equity by adopting some *objective well-being measure*. For instance, we may consider "J-Reference Maximin principle," as an intergenerational equity axiom, where J was introduced in axioms of sustainability. Then, it would be interesting to consider the compatibility of the J-Reference Maximin principle with J-Reference Human Development in this context.[7]

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[7] A similar type of problem was addressed and solved by Roemer (2005) and Silvestre (2005) in different types of models respectively.

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