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INTERNATIONAL TRADE AND INVESTMENT UNDER DIFFERENT RATES OF TIME PREFERENCE

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ABSTRACT

This paper attempts to integrate the theory of trade with that of capital movements, and to study the two country world where each nation has a different rate of time preference. It resolves the indeterminacy problem intrinsic in the Heckscher-Ohlin model where trade and factor movements coexist by assuming that capital movements are infinitesimally more costly than trade in goods. Under certain assumptions, one can dichotomize the behavior of asset accumulation from the dynamic pattern of trade specialization.

Complete specialization will take place most likely in the country with a higher rate of time preference, which specializes into the more labor-intensive sector. It is shown that a single-commodity model does exaggerate the amount of capital movements, but that the qualitative nature of asset accumulation patterns obtained in a single-commodity model of capital movements holds intact in the model that incorporates trade. This paper offers another explanation to the Feldstein-Horioka paradox that domestic investment responds more closely to increasing savings than capital outflows do. If an economy is imperfectly specialized, increased savings will be absorbed in capital deepening rather than in capital outflow.

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I. INTRODUCTION

In his ingenious article, Stiglitz (1970) considered the consequence of different rates of time preference and resulting saving behavior between two trading countries. He demonstrated in a two–country model of trade that at least one country must completely specialize unless the two nations have identical long–run interest rates, i.e. identical rates of time preference. His model, however, allowed no factor movements, perhaps following the tradition of international trade theory. A natural extension of his model in light of the present world where capital movements are active would be to inquire what happens if one allows factor movements in addition to trade in the presence of different rates of time preference.

Another line of research on capital movements under different savings behavior has focused on the study of a single commodity model of economic growth (Bardhan 1967; Buiter 1981; Hamada 1966; and Ruffin 1979). Under the single commodity assumption, no factor-price equalization takes place through trade and "equilibrium would be attained by a division of society into two classes, the thrifty enjoy bliss and the improvident at the subsistence level" (Ramsey 1928). This type of single commodity model (except Buiter (1981) who relies on the overlapping generation model) exaggerates the amount of capital movements because of the absence of factor–price equalizing forces through trade.[1] Moreover, from trade–theorists' perspective, there is nothing in this type of model that

[1]Existence of non–traded goods or consumption bias toward domestic goods also decreases the amount of capital movements. See the comments by Krugman (1989) on Hamada and Iwata (1989) that claim the third of the U.S. capital will be owned either by West Germany or Japan in 2010. Also see Engel and Kletzer (1989).

distinguishes itself from a closed economy with heterogeneous consumers (e.g., Becker 1980). Ideally, intra–temporal terms of trade as well as intertemporal terms of trade should be considered in a unified framework.

This paper is an attempt to integrate these two lines of research. In order to integrate trade theory and the theory of factor movements, we must cope with the "indeterminacy" problem. In the absence of transportation or adjustment costs, as already noted by many authors (Chipman, 1971; Kemp 1969; and Wong 1986), trade and factor movements become substitutes as equalizers of the factor price as long as both economies are imperfectly specialized. Therefore, we need some additional assumptions to prevent the system from being indetermined. Conforming to the tradition of international trade theory, we resolve this indeterminacy issue by assuming that international capital flows are slightly more costly than movements of two consumption goods, though transportation costs could be negligible in the actual calculation of the development path. This assumption may seem artificial, but it is one reasonable way to choose an equilibrium path from multiple indeterminate equilibria. It will also enable us to depict explicitly the development of trade and factor movements and to delineate the limitation of the single–good capital–movements model as well as of trade model without factor movements. Moreover, by doing this we are now able to develop a model of trade and investment based on a solid microeconomic foundation. We may also note that this assumption is conceivably the least favorable assumption for capital movements. By assuming this, we can assess the upper bound of the degree of exaggeration of capital flows in a single–commodity model. (An alternative approach to resolve this indeterminacy problem is suggested by Koch (1989).)[2]

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[2]In a small country with static expectations, Onitsuka (1975a,b) attempts to resolve this issue by introducing an investment function with adjustment cost. Probably the ideal way to cope with this indeterminacy problem would be to introduce an explicit structure for the cost of trade and that of factor movements along the line suggested, for example, by Mussa (1974). This would be, however, another research topic. Also, we did not get into the more general models (e.g. Brecher and Feenstra 1983; Chipman 1971; Uekawa 1972) that allow the differences in production technology to resolve the indeterminacy.

The main conclusions of the paper are summarized as follows: First, in the model with capital movements as well as trade commodity the rate of returns to capital are always equalized, and the structure of production in both countries can be easily analyzed. By some additional assumptions, we can dichotomize the development pattern of ownership and that of trade specialization. Similar to Stiglitz (1970) the world economy that starts from incomplete specialization will eventually approach a long–run equilibrium where one of them is completely specialized, but in contrast to his result without capital movements that allows substantial possibility of complete specialization, in our setting it is unlikely except for a coincidence that both countries are completely specialized.

Secondly, the conclusion of a single–good growth model certainly exaggerates the magnitude of capital movements compared to present model where trade is assumed to be always easier than the transportation of capital goods. However, the qualitative nature of the Ramseyan conclusion that a more patient country will eventually own a major part of world capital (Hamada and Iwata 1989) still holds true in a model that takes account of factor equalization through trade even under our most favorable assumption against factor movements. Also, the same structure could be superimposed on the model of evolution with nonlinear saving functions (cf. Fukao and Hamada 1989). This analysis shows that, even in the world economy where trade works towards equalization of factor prices, a long-term accumulation of foreign indebtedness can easily occur provided that a nation has a less time-patient attitude in its saving behavior.

Thirdly, the change of specialization patterns depends crucially on the spending propensity, i.e., the slope of the Engle curve and relative factor intensity of production. Suppose the home country is more time patient, and the propensities to spend on two goods are not very different. Then it is more likely that the less time–patient foreign country will end up being perfectly specialized in the labor intensive industry. In other words, an empire with the diversified industrial structure will coexist with a colony with the monolithic labor–intensive industry.

Finally, this model sheds some light on the controversy between Feldstein and Horioka (1970) and their critiques (e.g., Obstfeld 1986; Roubini 1989). The weak response of capital outflow over excess savings may well be the result of the capital deepening effect that excess savings are invested in domestic capital–intensive sector rather than exported abroad in the economy that is incompletely specialized.

The plan of this paper is as follows: In the next section, we develop a two–country model with trade and investment. In Section III, we discuss the short–run determination of the specialization patterns, and in Section IV the process of capital accumulation with changing patterns of trade specialization. We sketch some possibility for generalization of the model in Section V and conclude with the discussion of Feldstein–Horioka controversy in the final section.

II. A DYNAMIC MODEL OF INTERNATIONAL TRADE AND CAPITAL FLOWS

Let us first present a standard Heckscher–Ohlin model of international trade and capital flows on which we conduct our analysis. Consider two countries, H (home) and F (foreign) with identical technology and identical population. Each country can produce two goods, X and Y, by well–behaved homogeneous production functions:

$$X^j = F_X(K_X^j, L_X^j) = L_X^j f_X(k_X^j) ,$$

$$Y^j = F_Y(K_Y^j, L_Y^j) = L_Y^j f_Y(k_Y^j) , \ j = H, F ,$$

where X^j is the output of goods X in country j, Y^j that of goods Y, K_X^j and L_X^j the capital and labor input in X sector of country j, and k_X^j capital–labor ratio, and similarly for good Y. X sector is assumed to be more capital–intensive than Y sector without any factor–intensity reversal $(k_X^j > k_X^j)$. The aggregate capital–labor ratio of county j is denoted by $k^j = K^j/L^j$. We assume

$$f_i'(\cdot) \geq 0\ ,\ f_i''(\cdot) < 0\ ,\ f_i'(0) = +\infty\ ,\ \text{and}\ f_i'(+\infty) = 0\ ,\ \text{for}\ i = X \text{ and } Y\ .$$

Labor is assumed to be growing at the same, constant rate, n, in both countries, that is

$$L^j(t) = L^j(0)\, e^{nt}\ ,\ j = H, F\ . \tag{1}$$

The scale of the two economies is identical, so that $L^H(0) = L^F(0)$.[3]

There can be many alternative assumptions on the way investment processes take place in a two–sector model of trade with capital accumulation. In most studies the dynamic aspects of the Heckscher–Ohlin model have been analyzed with Meade–Uzawa type two–sector growth model,[4] in which two goods are assumed to be a consumption good and a capital good. Instead, we assume that households consume both types of goods and that capital accumulation requires both goods as input. In order to make a global analysis of the capital accumulation tractable, we will assume later that the relative composition of inputs in the investment process is identical to the relative composition of goods in consumption given a relative price.

Each country can accumulate the physical capital, K^j, which can be used in either sector, using linear homogeneous capital accumulation function satisfying concavity, differentiability and the Inada conditions:

$$\dot{K}^j(t) = H(I_X^j(t), I_Y^j(t)) = I_Y^j h\left[\frac{I_X^j(t)}{I_Y^j(t)}\right], \tag{2}$$

where we assume

[3] A more general case with different relative scale between the populations of the two countries and with different, constant, rates of population growth could be handled without much difficulty.

[4] See Fischer and Frenkel (1974), Kemp (1969), Meade (1961), Oniki and Uzawa (1961), Stiglitz (1970), and Uzawa (1965).

$$h'(\cdot) \geq 0 \,, \quad h''(\cdot) < 0 \,,$$

$$h'(0) = +\infty \,, \text{ and } h'(+\infty) = 0 \,.$$

$I_X^j(t)$ and $I_Y^j(t)$ are respectively quantities of good X and Y used for investment in country j. For cost minimizing of investors, the input ratio $I_X^j(t)/I_Y^j(t)$ is uniquely determined by the relative price of X and Y.

$$\frac{I_X^j(t)}{I_Y^j(t)} = \varphi(p(t)) \,, \tag{3}$$

where $p(t)$ is the price of goods Y in terms of goods X. The investment ratio function $\varphi(\ )$ satisfies

$$\varphi'(\cdot) > 0 \,, \quad \varphi(0) = 0 \,, \text{ and } \varphi(+\infty) = +\infty \,.$$

The optimization of investors also requires

$$q(t) = \frac{1}{h'(\varphi(p(t)))} \tag{4}$$

where $q(t)$ is the price of a claim to one unit of physical capital in terms of goods X. Unless this condition is not satisfied, the optimal investment will become plus or minus infinite.

Labor and physical capital are immobile between the countries, while they are completely mobile within each country. Not only the two goods, X and Y, but also the ownership claim to the physical capital are traded internationally, so that the current account balance of each country need not always be zero. Due to the linear homogeneity of the capital formation function, the assumption of free trade of the two goods implies that free international transaction of the ownership claim to physical capital is equivalent to perfect international mobility of physical capital. As is well known, the trade of goods and the factor mobility are complete substitutes,[5] if both countries are imperfectly specialized. In such a situation, the trade pattern and the international capital flows will be indeterminate. Thus, in order to analyze the interaction between the international trade and the capital flows, we need additional assumptions regarding the question which of the trades occur first, trade of two goods, or trade of the claims to capital with one good.[6]

In the real world, both transactions encounter many obstacles. Tariffs and sectoral adjustment cost reduce the volume of goods traded. Official restrictions on international capital flows and difficulty in getting information about production and management in foreign countries reduce the volume of international capital flows. In this paper we will analyze the case in which capital flows occur only when there is perfect specialization. We assume not only that no obstacles exist in the trade of goods, but also that costs in international capital flows are so slight that we can disregard them as if the factor price equalization were to hold even after perfect specialization occurs. In a way, our model reflects the traditional attitude of international trade economists who emphasize the analysis of trade flows compared to that of factor movements. Whether or not our assumption can be justified is naturally an empirical question. A promising alternative way of resolving this indeterminacy is proposed by Koch (1989), in which he analyzes a similar model (but with constant saving ratios) by assuming some portfolio preference between home and foreign investment.

In order to analyze the global rather than local properties of the dynamical system, we shall make the following Kaldorian saving assumptions. In fact, Stiglitz was assuming a constant propensity to save for profit incomes when he analyzed the transition path to the steady–state equilibrium.

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[5]See Chipman (1971), Mundell (1957), Ohlin (1933), Samuelson (1949), and Wong (1986).

[6]Oniki and Uzawa (1965) and Stiglitz (1970) assume that there is trade in both consumption and investment goods, but not in securities. Fischer and Frenkel (1974) assume that there is trade in consumption goods and securities, but not in investment goods.

ASSUMPTION (i) All the wages are consumed. Only the rentiers save in such a way to maximize the utility to be derived from their consumption.

(ii) The population of rentiers is growing at the same rate as that of labor.

(iii) The instantaneous utility function of both labor and rentiers satisfies

$$U(c^j_{X,i}, c^j_{Y,i}) = \ln(H(c^j_{X,i}, c^j_{Y,i})) \text{ for } j = H, F \text{ and } i = 1, 2 ,$$

where subscript 1 indicates the labor and subscript 2 does the rentier so that $c^j_{X,1}$ and $c^j_{Y,1}$ are the labor's per–capita consumption of goods X and goods Y in the j'th country, $c^j_{X,2}$ and $c^j_{Y,2}$, the rentier's per–capita consumption of goods X and goods Y. The function $H(\cdot)$ is identical with the capital accumulation function defined above.

Under Assumptions (i) to (iii), the maximization problems that the labor in the j'th country face is:

$$\max_{\{c^j_{X,1}(t),\ c^j_{Y,1}(t)\}} H(c^j_{X,1}(t), c^j_{Y,1}(t))$$

subject to

$$c^j_{X,1}(t) + p(t)c^j_{Y,1}(t) = w(t) ,$$

$$c^j_{X,1}(t) \geq 0 ,$$

$$c^j_{Y,1}(t) \geq 0 ,$$

where $w(t)$ is the wage rate in terms of goods X. The maximization problems that the rentiers in the j'th country face is:

$$\max_{\{c^j_{X,2}(t),\ c^j_{Y,2}(t)\}} \int_0^\infty e^{-\beta^j t} \ln\left[H(c^j_{X,2}(t),\, c^j_{Y,2}(t))\right] dt ,$$

subject to

$$\dot{a}^j(t) = \frac{1}{q(t)}\left\{r(t)a^j(t) - c^j_{X,2}(t) - p(t)c^j_{Y,2}(t)\right\} - n \cdot a^j(t) , \tag{5}$$

$$a^j(0) = \bar{a}^j , \tag{6}$$

$$a^j(t) \geq 0 , \tag{7}$$

$$c^j_{X,2}(t) \geq 0 , \tag{8}$$

$$c^j_{Y,2}(t) \geq 0 , \tag{9}$$

where $a^j(t)$ is per–capita physical capital owned by the rentiers in country j. β^j is the time preference rate of the rentiers in country j. $r(t)$ is the rental to the capital in terms of goods X. The time–preference rate β^j can be different between the two countries.

Utility maximization of both labor and rentiers requires that the marginal rate of substitution of $c^j_{X,i}(t)$ and $c^j_{Y,i}(t)$ equal to the relative price $p(t)$. Because of the homotheticity and quasi–concavity of $U(\ )$, the optimal $c^j_{X,i}(t)/c^j_{Y,i}(t)$ is an increasing function of $p(t)$.

Under Assumption (iii), this function is identical with the investment ratio function $\varphi(\ )$:

$$\frac{c^j_{X,i}(t)}{c^j_{Y,i}(t)} = \varphi(p(t)) ,\ \ \varphi'(\ ) > 0 ,\ \text{for}\ \ j = H, F\ \ \text{and}\ \ i = 1, 2 , \tag{10}$$

that is, at any relative price $p(t)$ the consumption ratio of the two goods is identical with the input ratio of the two goods in investment.

Admittedly, Assumption (iii) is quite strong. However, the alternative assumption taken by the Meade–Uzawa model is also a stringent one.[7] Besides, the two–country version of their model assumed some simple saving behavior usually with constant saving ratio, when the microeconomic foundation was also given, that was restricted to the stationary states.[8] In contrast to the Meade–Uzawa model, we can study the consequence of the intertemporal optimization behavior partly due to the fact that short–run equilibria are independent of consumers' intertemporal optimization under our assumptions.

Using the function $\varphi(\cdot)$, the labor's optimal consumption levels $c^j_{X,1}(t)$ and $c^j_{Y,1}(t)$ can be written as

$$c^j_{X,1}(t) = \frac{\varphi(p(t))}{\varphi(p(t)) + p(t)}\, w(t) ,$$

$$c^j_{X,2}(t) = \frac{1}{\varphi(p(t)) + p(t)}\, w(t) .$$

Under Assumption (iii), the foregoing rentier's maximization problem can be transformed into

$$\max_{\{c^j_2(t)\}} \int_0^\infty e^{-\beta^j t} \ell n(c^j_2(t))dt ,$$

subject to

$$\dot{a}^j(t) = \left\{\frac{r(t)}{q(t)} - n\right\} a^j(t) - c^j_2(t) , \tag{11}$$

$$a^j(0) = \bar{a}^j , \tag{6}$$

$$a^j(t) \geq 0 , \tag{7}$$

$$c^j_2(t) \geq 0 . \tag{12}$$

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[7]In our notation, they assume $I^j_X(t) = 0$ and $C^j_{Y,i}(t) = 0$ regardless of the level of relative price.

[8]Stiglitz (1970) studies the consumers' intertemporal optimization behavior. But only the property of long–run stationary equilibria is analyzed in a two–country case.

Here $c_2^j(t)$ is the rentier's per-capita spending in terms of real capital unit. By equation (4) and the definition of $\varphi(\ )$, $c_2^j(t)$ can be written,

$$c_2^j(t) \equiv [c_{X,2}^j(t) + p(t)c_{Y,2}^j(t)]/q(t)$$

$$= c_{Y,2}^j(t)h(c_{X,2}^j(t)/c_{Y,2}^j(t))$$

$$= H(c_{X,2}^j(t), c_{Y,2}^j(t)) . \tag{13}$$

The optimal savings behavior is given by budget constraint (11), initial condition (6), and proportional consumption behavior

$$c_2^j(t) = \beta^j \cdot a^j(t) . \tag{14}$$

The rentier's optimal consumption levels of the goods, $c_{X,2}^j(t)$, $c_{Y,2}^j(t)$, can be written as

$$c_{X,2}^j(t) = \frac{\varphi(p(t))}{\varphi(p(t)) + p(t)} \beta^j q(t)a^j(t) ,$$

$$c_{Y,2}^j(t) = \frac{1}{\varphi(p(t)) + p(t)} \beta^j q(t)a^j(t) .$$

III. THE DETERMINATION OF THE SHORT-RUN EQUILIBRIUM

We shall first describe briefly the determination of the equilibrium at each moment of time, given two countries' physical capital ownership, $A^H(t)$ and $A^F(t)$. For brevity, we denote all variables without explicitly referring to time t in this section.

From the factor price equalization mechanism and capital movements under perfect specialization, the rentals to capital, r, in two countries are equalized at any time. under imperfect specialization, we have

$$r \equiv f_X'(k_X^H) = f_X'(k_X^F) = Pf_Y'(k_Y^H) = Pf_Y'(k_Y^F) , \tag{15}$$

$$k_X \equiv k_X^H = k_X^F , \text{ and } k_Y \equiv k_Y^H = k_Y^F . \tag{16}$$

If perfect specialization takes place, then the variables corresponding to the disappeared sector will become meaningless. For example, if the foreign country perfectly specializes in goods Y production, (15) and (16) will become as follows:

$$r \equiv f'_X(k^H_X) = Pf'_Y(k^H_Y) = pf'_Y(k^F_Y)\ , \tag{15'}$$

$$k_X \equiv k^H_X\ , \text{ and } k_Y \equiv k^H_Y = k^F_Y\ . \tag{16'}$$

Because of the factor–price equalization mechanism and capital movements under perfect specialization, the equilibrium price–factor–reward structure and the equalized capital–labor ratios, k_X and k_Y, are not affected by the distribution of the world endowments among the countries. They are determined in the same way as in the closed economy that is endowed capital K and labor L as much as

$$K = K^H + K^F = A^H + A^F \text{ and } L = L^H + L^F$$

respectively (cf. Jones, 1965; Uzawa, 1961). A^H (K^H) and A^F (K^F) are the amount of physical capital owned by (located in) the home and the foreign country.

From profit–maximization conditions, we obtain

$$\frac{f_X(k_X) - f'_X(k_X)k_X}{f'_X(k_X)} = \frac{f_Y(k_Y) - f'_Y(k_Y)k_Y}{f'_Y(k_Y)} = \frac{w}{r} \equiv \omega\ , \tag{17}$$

$$p = \frac{f'_X(k_X)}{f'_Y(k_Y)}\ , \tag{18}$$

where w is the wage rate, ω the wage–rentals ratio. Given a price of goods Y in terms of goods X p, the wage–rentals ratio ω as well as the capital–labor ratio in each sector k_X, k_Y is uniquely determined from equations (17) and (18).[9] As p rises, the price of the factor service that is intensively used in the goods Y sector becomes more expensive

[9]The short–run equilibrium of our model is identical with the equilibrium of the two–sector model of Jones (1965). For detail of comparative statistics, see Jones (1965).

compared with other factor prices. Under our assumption that the goods Y sector is more labor intensive, ω rises:

$$\omega'(p) > 0 .$$

As ω rises, capital is substituted for labor in each production sector:

$$k_X'(p) > 0 , \ k_Y'(p) > 0 .$$

In Figure 1, the vertical axis indicates the price of goods Y in terms of goods X . The horizontal axis indicates the share of the world labor that is allocated to the Y sector, $\ell_Y \equiv (L_Y^H + L_Y^F)/(L^H + L^F)$.

The SS curve denotes the locus of $\ell_Y - p$ pairs that satisfies equilibrium conditions of factor markets:

$$(1 - \ell_Y)k_X(p) + \ell_Y k_Y(p) = k , \tag{19}$$

where k is the world capital–labor ratio, i.e., $k \equiv (K^H + K^F)/(L^H + L^F) = (k^H + k^F)/2$. Given a world capital–labor ratio, k , if the price of labor intensive goods Y in terms of goods X rises, then k_X and k_Y will increase and the labor share of the labor intensive sector, ℓ_Y , will need to expand to keep the factor markets in equilibrium (see the upward–sloping SS curve in Figure 1). The world economy specializes in X ($\ell_Y = 0$) for a sufficient low p , and in Y ($\ell_Y = 1$) for a sufficiently high p . If the world capital labor ratio, k , increases, the SS curve shifts upward.

The DD curve denotes the locus of $\ell_Y - p$ pairs that satisfies equilibrium conditions of goods markets. Equilibrium of goods markets requires

$$\frac{c_{X,1}}{c_{Y,1}} \equiv \frac{c_{X,2}}{c_{Y,2}} \equiv \frac{I_X}{I_Y} \equiv \varphi(p) = \frac{(1 - \ell_Y)f_X(k_X)}{\ell_Y f_Y(k_Y)} \tag{20}$$

From (17), (18), and (20), we get the demand–side relationship of ℓ_Y and p :

$$\ell_Y = \frac{f_X(k_X(p))}{\varphi(p)f_Y(k_Y(p)) + f_X(k_X(p))} \tag{21}$$

The DD curve shows this relationship. The slope of the DD curve may be either positive or negative. As p rises, the demand shifts from goods Y to goods X. If the labor was a unique production factor, then ℓ_Y would decrease as p rises. Because the capital also moves between sectors, the response of ℓ_Y to p is indefinite. Nevertheless the reciprocal of the slope of the DD curve is always smaller than that of the SS curve.[10] And $0 < \ell_Y < 1$ for all p. Therefore there exists for any given k, a unique intersection of the two curves which determines the short–run equilibrium. If the world capital–labor ratio, k, increases as time elapses, the SS curve shifts upward, raising p. As p rises k_X and k_Y all increase. Due to the assumption that the consumption ratios of the two

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[10]The reciprocal of the slope of the SS curve is

$$\left.\frac{d\ell_Y}{dp}\right|_{SS} = \frac{\ell_Y}{k_X - k_Y}\frac{dk_Y}{dp} + \frac{1 - \ell_Y}{k_X - k_Y}\frac{dk_X}{dp}.$$

The reciprocal of the slope of the DD curve is

$$\left.\frac{d\ell_Y}{dp}\right|_{DD} = \ell_Y(1-\ell_Y)\left\{\frac{k_X}{\omega+k_X}\frac{1}{k_X}\frac{dk_X}{dp} - \frac{k_Y}{\omega+k_Y}\frac{1}{k_Y}\frac{dk_Y}{dp} + \frac{p\varphi'}{\varphi}\left[\frac{1}{\omega+k_X} - \frac{1}{\omega+k_Y}\right]\frac{d\omega}{dp}\right\}$$

where

$$\frac{k_X}{\omega + k_X} < 1,$$

and

$$\frac{1 - \ell_Y}{k_X - k_Y}\frac{dk_X}{dp} - \ell_Y(1 - \ell_Y)\frac{1}{k_X}\frac{dk_X}{dp} = (1 - \ell_Y)\frac{(1 - \ell_Y)k_X + \ell_Y k_Y}{k_X(k_X - k_Y)}\frac{dk_X}{dp} > 0.$$

Therefore,

$$\left.\frac{d\ell_Y}{dp}\right|_{SS} > \left.\frac{d\ell_Y}{dp}\right|_{DD}$$

goods are identical with the input ratio of the two goods in investment, the short–run equilibrium is independent of the dynamics of the model. If the consumption ratio of the two goods was different from the input ratio in investment, then the DD curve in Figure 1 would depend on households' decision on intertemporal consumption pattern.

Let us now turn to the specialization pattern of each country and the state of international lending in short–run equilibria. In the short–run, not only the world capital-labor ratio, k, but also the levels of per–capita physical capital owned by the two, a^H and a^F, are given, and they satisfy

$$k = (a^H + a^F)/2 . \tag{22}$$

If a^H and a^F lie between the equilibrium capital–labor ratios of X and Y sector k_X and k_Y, i.e., $k_Y < a^j < k_X$, $j = H, F$, then neither perfect specialization nor international capital flow will occur in the world under our assumption of the relative ease of trade compared to capital movements. If a^H or a^F lies outside of k_X and k_Y, then international capital flows will take place.

We can analyze by Figure 2[11] the specialization pattern in more detail. The horizontal axis indicates the distribution of the labor force between the two countries (L^H, L^F). The vertical axis denotes the distribution of the ownership claim to the physical capital (A^H, A^F). In a short–run equilibrium, the wage–rental ratio, p and ω are determined as in Figure 1. The line XO^H in Figure 2 denotes the locus of cost-minimizing capital–labor pairs at the goods X sector in the home country under the equilibrium wage–rental ratio ω, that is, the locus of tangency between isoquant curves of the goods X sector and iso–cost lines, the slope of which is ω. Similarly the line YO^H for the goods Y sector in the home country, and the line X^*O^H and Y^*O^H for the foreign country, X^*O^H and Y^*O^H are the symmetrical images of XO^H and YO^H

[11]This figure is frequently used in international trade theory (cf. Dixit and Norman 1980; Helpman and Krugman 1985; and Travis 1964).

respectively rotated symmetric to the center of the figure. Due to the homotheticity of the production functions, these lines are straight.

In general, if the distribution of the labor force and the capital ownership is located in the parallelogram $O^H F O^F E$, like point G, then neither specialization nor international capital flow will occur, and factor prices of the two countries will be equalized by trade alone. The location of the world capital, (K^H, K^F) will be identical with the ownership pattern (A^H, A^F). The production and the trade pattern of two countries is also determined by Figure 2. The labor allocation between two sectors in each country is determined by

$$\frac{L_X^H}{L_Y^H} = \frac{O^H L_X^H}{O^H L_Y^H}$$

$$\frac{L_X^F}{L_Y^F} = \frac{O^F L_X^F}{O^F L_Y^F}$$

Consistent with the standard theory, a capital abundant country relatively specializes in capital intensive goods and exports them. In fact, because of our assumption of the identical size of the population, we need to consider points like G, the points on TT', the dividing segment of the box into halves.

Suppose the capital ownership pair (A^H, A^F) is located as point G'. Then the capital rentals of the foreign country will tend to be higher than those of the home country even under free trade. The minimum amount of capital flows that equalizes two countries' factor prices is expressed as the vertical distance between point G' and the parallelogram, i.e., $G'Q$. As capital inflows this amount into the foreign country, the location of the world capital, (K^H, K^F), is determined as point Q, on which the production and the trade pattern in the two countries depend. The foreign country perfectly specializes in the production of goods Y. The home country produces both goods.

The relative size of k_X to k_Y determines which country perfectly specializes under the unbalanced capital ownership pattern. If k_X is smaller than that in Figure 2 (XO^H is flatter) and point R is located below point Q , then the home country will perfectly specialize with asset endowment pattern G′ . In Figure 2,

$$\frac{TQ}{TR} = \frac{k - \frac{1}{2} k_Y}{\frac{1}{2} k_X} .$$

From this equation and $k = (1 - \ell_Y)k_X + \ell_Y k_Y$, it is clear that TQ/TR is less than unity if and only if $\ell_Y > 1/2$. Thus we obtain the following

<u>PROPOSITION I</u>. *Assume the ownership pattern is so unbalanced that perfect specialization and international capital flows take place. Perfect specialization will occur in a labor abundant country (in a capital abundant country), if and only if the fraction of the world labor allocated to the labor intensive industry* ℓ_Y , *is greater than* 0.5 (*smaller than* 0.5).

Under our assumption that two countries have same amount of labor force, if one country perfectly specializes in goods Y production (in goods X production), ℓ_Y will certainly be greater than 0.5 (smaller than 0.5).

Note that if the ownership pattern is unbalanced enough, that is $A^H/(A^H + A^F)$ is close to 0 or 1, then perfect specialization will certainly occur, because k_Y is positive in any short–run equilibrium. In a short–run equilibrium, ℓ_Y is determined as Figure 1 displays. If the equilibrium point, E , is located on the right of the $\ell_Y = 1/2$ line, perfect specialization will occur in a labor abundant country.

IV. CAPITAL ACCUMULATION AND DYNAMICS OF INTERNATIONAL LENDING

Let us now analyze capital accumulation process in the growing world economy.

The rate of returns to real capital in terms of composite capital goods can be written by equation (4),

$$\frac{r(t)}{q(t)} = f_X'(k_X)\cdot h'(\varphi(p)) \,. \tag{22}$$

At any instant of time the world average capital stock $k(t) = (a^H(t) + a^F(t))/2$ is given so that from short–run equilibrium conditions (equations (17), (18), (19), (21)) ω, p and k_X will be determined. Accordingly $r(t)/q(t)$ is the function of $k(t)$ or $(a^H(t) + a^F(t))/2$.

Let us define this function by $\mu(\cdot)$ such that

$$\frac{r(t)}{q(t)} = \mu\left[\frac{1}{2}(a^H(t) + a^H(t))\right] . \tag{23}$$

It is easily seen

$$\mu'(\,) = f_X''h'\frac{dk_X}{d\omega} + f_X'h''\varphi'p\left[\frac{1}{\omega + k_Y} - \frac{1}{\omega + k_X}\right]\frac{d\omega}{dk}$$

$$< 0 \,.$$ [12]

Also we have

$$\lim_{k\to+\infty} \mu(k) = 0 \text{ and } \lim_{k\to 0} \mu(k) = +\infty$$

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[12] This inequality follows $f_X'' < 0$, $h' > 0$, $dk_X/d\omega > 0$, $f_X' > 0$, $h'' < 0$, $\varphi' > 0$, $k_X > k_Y$ and $d\omega/dk > 0$.

by Inada conditions on the production functions.[13]

Hence the dynamical equations of per–capita net assets can be expressed as

$$\dot{a}^H(t) = \{\mu((a^H(t) + a^F(t))/2) - n - \beta^H\}a^H(t) \tag{24}$$

$$\dot{a}^F(t) = \{\mu((a^H(t) + a^F(t))/2) - n - \beta^F\}a^F(t) \tag{25}$$

Therefore we can analyze the long–run path of capital accumulation exactly in the same way as we did in a single commodity model (Fukao and Hamada 1989). Owing to the assumption that the consumption ratio of the two goods is identical with the input ratio of the two goods in investment at any relative price, we can analyze the dynamics in terms of single composite good, i.e., the real capital. Figure 3 illustrates this. If the home country is more time patient, i.e. $\beta^H < \beta^F$, then there will be three long–run equilibria O, E and B. Only E is stable. At point E $a^F = 0$ and a^H is determined by $\mu(a^H(t)/2) = n + \beta^H$. If the rate of time preference is identical between the countries, namely $\beta^F = \beta^F \equiv \beta$, then the long–run equilibria are all the points on line CD, which is defined by the pair (a^H, a^F) satisfying

$$\mu((a^H + a^F)/2) = n + \beta .$$

What is the relationship between the trade specialization pattern and capital movements? Figure 4 illustrates this. The horizontal axis indicates the amount of per–capita capital owned by the home country, a^H, and the amount of capital located in the home country, k^H; similarly the vertical axis a^F and k^F. Given the pair of initial capital ownership (a^H, a^F), world capital–labor ratio k is given so that by the short–run equilibrium conditions p and ω. Given the pair of initial capital stock, both $(a^H + a^F)$ and $(k^H + k^F)$ are given so that (a^H, a^F), (k^H, k^F) are on the negative 45 degree line

[13]Since $h''(\ ) < 0$, $h'(\ )$ declines when k approaches infinity. $k \to +\infty$ implies $k_X \to +\infty$, which in turn implies $f_X'(k_X) \to 0$. This proves the first property. The second property also comes from the fact that $k \to 0$ implies $k_X \to 0$, and accordingly $f_X'(k_X) \to +\infty$.

$T^*T'^*$. If (a^H, a^F) is inside the intersection set of the diversification cones that will be shortly defined, the allocation pair (k^H, k^F) and the ownership pair (a^H, a^F) will coincide each other. If the pair is outside the intersection set then one country will be perfectly specialized and capital movements will take place. In Figure 4, the critical points where perfect specialization starts are indicated by P^* and Q^* , the border of the negative 45 degree line $T^*T'^*$ with the intersection set of diversification cones H^XOH^Y and F^XOF^Y .

The location of H^XOH^Y and F^XOF^Y will be determined as follows. Suppose (a^H, a^F) and accordingly $k = (a^H + a^F)/2$ are given. Then, p and ω will be determined so that the angles of OH^X , OH^Y , OF^X , OF^Y will also be given by the mechanism that was illustrated by Figure 2. Those rays determine the critical levels of diversification for a^H and a^F such as Q^* and P^* . Q in Figure 2, where the foreign country starts specializing in the production of Y , corresponds to point Q^* in Figure 4. Similarly, R , P , and S in Figure 2 correspond to R^* , P^* , and S^* in Figure 4. P^* indicates the point where the home country begins to perfectly specialize in Y . In other words, given k , the pair (a^H, a^F) is on $T^*T'^*$, and the value of a^H , a^F calculated by the value of A^H and A^S at the critical points of complete specialization in Figure 2 and labor force will determine the location of R^* , Q^* , P^* , and S^* . By moving the value of k , we obtain the loci of OH^X , OH^Y , OF^X , and OF^Y . It can be easily seen that OH^X and OF^Y are the mirror images of OF^X and OH^Y respectively reflected by the 45 degree line.

If the two countries' ownership pair (a^H, a^F) is located like point G^* within the intersection of two cones, H^XOH^Y and F^XOF^Y , then no specialization will take place. Factor prices of the two countries will be equalized by trade alone and no international capital flow will need to occur. Therefore the location of the world capital, (k^H, k^F) is identical to the ownership pattern (a^H, a^F) . The production and trade pattern of two countries is also determined by Figure 4. The labor allocation between two sectors in each country is determined by

$$\frac{\ell_X^H}{\ell_Y^H} = \frac{P^*G^*}{G^*R^*}, \text{ and } \frac{\ell_X^F}{\ell_Y^F} = \frac{Q^*G^*}{G^*S^*}.$$

Consistent with the standard theory, the capital abundant country relatively specializes in and exports more capital–intensive goods.

Suppose the capital ownership pair (a^H, a^F) is located like point N. Then the foreign country will perfectly specialize in the labor–intensive goods Y, and the capital rentals of the foreign country will tend to be higher than those of the home country under the free trade without capital movements. The minimum amount of capital movements that equalizes two countries' factor prices, is expressed as the vertical distance of point Q^* and point N, that is, $k^F - a^F$. Thus in the presence of capital movements, the pair of location of capital (k^F, k^F) will be at point Q^*, and will determine the production and trade patterns. The foreign country perfectly specializes in and exports goods Y, and the home country produces both goods. The labor allocation between two sectors in the home country is given by

$$\frac{\ell_X^H}{\ell_Y^H} = \frac{P^*Q^*}{Q^*R^*}.$$

Notice that the amount of international capital movements is reduced by trade compared with that in a single–good economy, where the location of the world capital would be determined by point I and the amount of international capital movements equals the vertical distance between point I and point N.

Now let us leave the static analysis of constant (a^H, a^F) and k, and consider the dynamics of capital accumulation and trade specialization. As explained above, the asset accumulation path in this model is determined exactly by the same dynamical system (24) and (25) as in the single–goods growth model (cf. Fukao and Hamada 1989). Thus the phase–diagram analysis in Figures 3(a) and 3(b) is valid as well. One can visualize the accumulation process and specialization pattern by superimposing phase diagram, Figure 3, on the diversification cones in Figure 4. Figure 5 is the superimposition of 3(a) on Figure 4.

Suppose the initial ownership pair is inside the intersection of the diversification cones like C . Then capital ownership pattern moves towards E according to the phase diagram 3(a). Both the ownership pair (a^H, a^F) and the allocation pair (k^H, k^F) start from C , following the phase diagram as drawn in Figure 3(a). When they hit the border of the intersection set, in this case OF^Y , at D , (a^H, a^F) will still follow the path depicted in the phase diagram, but (k^H, k^F) will depart from the path and proceed along DM . The foreign country will be specialized in labor–intensive goods. A similar story applies to the world economy where both (a^H, a^F) and (k^H, k^F) start from a point V . First, both pairs move towards south–west direction. Either they cross $\dot{a}^H = 0$ line and diverge from one another when they hit segment DM ; or as indicated in Figure 5, they keep moving towards southwest, hit OF^Y at W where (k^H, k^F) starts moving towards M on OF^Y while (a^H, a^F) still keeps moving towards E .

If the initial ownership pair is like point U , then, under our assumption of positive but infinitesimal cost of capital movements, the capital allocation will be instantaneously adjusted at the moment when capital moments are liberalized, and (a^H, a^F) will move along UE , and (k^H, k^F) along JM .

It is clear from the construction that, except for the measure–zero case where two cones H^XOH^Y , F^XOF^Y exactly coincides each other, only one country completely specializes in production of one good, and the other keeps producing both goods. If we define state E as *the Ramsey equilibrium with the home country as the empire*, we can summarize our analysis as

<u>PROPOSITION II</u>. *Under the assumptions of the model, if the rate of time preference of the home country is even slightly less than that of the foreign country, then the capital ownership pair* (a^H, a^F) *will approach the Ramsey state with the home country as the empire. Almost surely, either of the following two cases will take place after a while: (i) the foreign country will perfectly specialize in labor–intensive goods, or (ii) the home country will perfectly specialize in capital–intensive goods.*

This result that the complete specialization of both countries is a measure–zero phenomenon is a distinct departure from the result in Stiglitz (1970) where such a case is generally possible. The channel of factor–price equalization through factor movements presumably mitigates the need for perfect specialization in both sides.

The reader is now curious which alternatives (i) or (ii) will take place in the development path. Which of (i) or (ii) occur depends on whether OF^Y is inside OH^X (i.e. close to the 45 degree line) or OH^X is inside OF^Y. This question can be answered, given k and accordingly ω, by drawing Figure 2 and by asking whether R is above Q (or S is below P). Without specifying utility function on commodities, the answer to this question is not so obvious.[14]

From Proposition I, under a given value of k, the specialization pattern depends on the value of ℓ_Y. In other words, labor–abundant country (the foreign country in this case) specializes if $\ell_Y > 1/2$. From equation (21), one obtains

$$\ell_Y = \left\{\frac{\varphi(p)}{p}\frac{1 + k_Y(p)/\omega}{1 + k_X(p)/\omega} + 1\right\}^{-1}, \tag{26}$$

where $\varphi(p)/p$ is the relative share of expenditures on x to that on Y, and k_X/ω and k_Y/ω are respectively the relative share of capital income to labor income in X and Y

[14]From the relationships such as $k_Y < k_X$, $k_Y \le k \le k_X$, $\lim_{k\to 0} k_X = 0$, $\lim_{k\to\infty} k_Y = 0$, it is easy to see that OH^X and OF^Y start from the origin and stay below the positive 45 degree line.

sector. Therefore ℓ_Y depends on the elasticity of substitution in expenditures in consumption and investment

$$\eta_E \equiv \frac{pc_Y}{c_X}\frac{d(c_X/c_Y)}{dp}$$

and the elasticities of substitution in production

$$\eta_X \equiv \frac{\omega}{k_X}\frac{dk_X}{d\omega}, \quad \eta_Y = \frac{\omega}{k_Y}\frac{dk_Y}{d\omega}.$$

Assume that, for simplicity, $\eta_X = \eta_Y = 1$, i.e., production functions are Cobb-Douglas. We can discuss the determinants of specialization pattern depending on the elasticity of substitution in expenditures η_E.

(i) $\eta_E = 1$. Then all the relative shares in (26) becomes constant, and OH^X, OH^Y, OF^X, OF^Y become all straight lines. If we denote the capital share in X and Y by α_X and α_Y respectively, and the share of expenditure on X by γ, then one can easily see:

(a) If $(1-\alpha_X)/(1-\alpha_Y) < (1-\gamma)/\gamma$, labor–abundant country will specialize in labor–intensive sector when the perfect specialization takes place.

(b) If $(1-\alpha_X)/(1-\alpha_Y) > (1-\gamma)/\gamma$, then capital–abundant country will specialize.

(c) If $(1-\alpha_X)/(1-\alpha_Y) = (1-\gamma)/\gamma$, then both countries specialize at the same time.

(ii) $\eta_E < 1$. Then the specialization of capital–abundant country will occur when k is small, and the specialization of labor–abundant country will occur when k is large. This can be seen from the fact that DD line in Figure 1, which was vertical when $\eta_E = 1$, becomes upward sloping and ℓ_Y increases when k increases. The diversification cones are no more linear cones as are shown in Figure 6(a).

(iii) $\eta_E > 1$. Then the specialization of labor–abundant country will occur when k is small, and the specialization of capital–abundant country will occur when k is large. See Figure 6(b).

Even in this rather simple case with unitary elasticity of substitution in production, if we have non–unitary elasticity of substitution in expenditures, the dynamics of trade and capital flows will take various patterns. Suppose, for example, $\eta_E > 1$, $\beta^H < \beta^F$ and the initial configuration (a^H, a^F) starts from Region II [with $k < k^*$ where k^* is defined by $\mu(k^*) = \beta^H + n$] in 6(b). Then, first the foreign country will specialize in labor–intensive goods Y , and next the home country will specialize in capital–intensive goods X as the path moves into Region I. Also, if $\eta_E > 1$, $\beta^H = \beta^F$ and the initial (a^H, a^F) is in Region II. Then the following path will be possible. First the foreign country specializes in Y with international borrowing, next both countries incompletely specialize without international borrowing, and finally the home country specializes in X with international lending.

Let us return to the case of (i) where $\eta_E = 1$. Since $\alpha_X > \alpha_Y$ by our assumption, for a value of γ that is sufficiently close to 0.5, Case (i) holds. Thus one can state

<u>PROPOSITION III</u>. *Suppose both elasticities of substitution in productions and that in expenditures are unity. Then there exists an open interval on the expenditure share* γ , $I = (\underline{\gamma}, \overline{\gamma})$ *such that* $1/2 \in I$ *and that for any* γ *inside* I *the foreign country will be specialized in producing labor intensive goods* Y .

This proposition indicates that if the expenditure pattern towards both goods is more or less balanced, then the borrowing country will specialize in labor–intensive goods. When the Ramsey state E is attained, the more patient country owns the world as an empire, and the less patient country will be left as a colony with monolithic production of solely labor–intensive goods. (We have to notice that this result depends on the assumption of equal sizes of the two countries. If there is a difference in scales of population, the smaller country will be more likely to specialize.)

If we relax the assumption of unitary elasticity of substitution in production, then the story will be even more involved. We do not attempt to analyze this case here because non–unitary production elasticity would easily lead to the factor–intensity reversal that we excluded from our model at the outset of this paper.

Now we are ready to compare the amount of international lending or borrowing in this model with trade to that in a single good model without trade. It will suffice to compare the relative magnitude of international credit at the Ramsey equilibria at E. In a single good model where one abstracts from trade, the amount of per–capita international credit equals the distance between A and E in Figure 5. In the model with factor–price equalization through trade, the per–capita credit equals the horizontal distance between M and E. In particular, in this Cobb–Douglas example with $(1-\alpha_X)/(1-\alpha_Y) < (1-\gamma)/\gamma$

$$\frac{ME}{ZE} = \frac{k_Y}{k} = \frac{\alpha_Y}{1-\alpha_Y}\,\frac{\gamma(1-\alpha_X) + (1-\gamma)(1-\alpha_Y)}{\gamma\alpha_X + (1-\gamma)\alpha_Y}$$

$$= \frac{\gamma\,\dfrac{1-\alpha_X}{1-\alpha_Y} + (1-\gamma)}{\gamma\,\dfrac{\alpha_X}{\alpha_Y} + (1-\gamma)} < 1\,. \tag{27}$$

This ratio varies with the difference in capital share between the two sectors and it is not much less the unity for reasonable values of α_X and α_Y.[15] For example, if $\alpha_X = 1/3$, $\alpha_Y = 1/4$, $\gamma = 1/2$ then $ME/ZE = 17/21$. Only for the extreme case where $\alpha_X = 2/5$, $\alpha_Y = 1/5$, $\gamma = 1/2$ $ME/ZE = 7/12$.

[15]If $(1-\alpha_X)/(1-\alpha_Y) > (1-\gamma)/\gamma$, the ratio will become again

$$\frac{\gamma + (1-\gamma)(\alpha_Y - \alpha_X)/[\alpha_X(1-\alpha_X)]}{\gamma + (1-\gamma)\alpha_Y/\alpha_X} < 1\,.$$

This is, at least, a partial answer to the criticism on predictions made by a single–commodity model (Hamada and Iwata 1989). A single–commodity model may exaggerate the amount of indebtedness. The degree of exaggeration depends on the broadness of divergence cones, and may not be too great if α_X is close to α_Y. At the same time, one should note that this model makes the conceivably most unfavorable assumption against capital movements by maximizing the factor-price equalization role of trade in goods. If we resolve the indeterminacy issue by assuming that international trade of final goods X and Y is slightly more costly than movements of real capital and ownership claim to capital, then capital–labor ratios of two countries will always be identical, i.e., $k^H = k^F$, and the amount of indebtedness will be identical with that in a single-commodity model. At any rate, qualitative conclusions obtained for the long–run capital-ownership pattern hold intact even in a model with trade.

We summarize as

PROPOSITION IV. *A single–good model exaggerates the amount of international indebtedness in the long run. The degree of overestimate increases with the divergence between α_X and α_Y.*

V. POSSIBLE GENERALIZATIONS

We will briefly discuss the possible consequences of relaxing some of our assumptions and some further extensions.

First, one may wish to relax the assumption that only rentiers save and that workers do not. Then one has to take account of the development of not only physical but also human capital. The system becomes intractable by analytical solutions. We have tried some simulations that seem to indicate that most of qualitative results are similar to the case with our Kaldorian saving assumptions. For example if $\beta^H = \beta^F$, $a^H(t)/a^F(t)$ will be kept constant and the final equilibrium will be reached such that $r/q = \beta^H = \beta^F$

and $k = (a^H + a^F)/2$. If the rates of time preference are different, the less patient country may end up with the situation where not only its physical capital is totally owned by the other country but also the future earnings from its human capital become collateral to its further indebtedness. In general, neoclassical foundations for possible different attitudes to profit and wage income seem to be an interesting question.

Secondly, if we allow a general functional form rather than the logarithmic form in the instantaneous utility, then $c^j(t)$ will depend not only on the present physical asset $a^j(t)$ but also on the whole future stream of rate of returns to capital.[16] This will make the analysis quite difficult.

Thirdly, one can superimpose more general nonlinear saving behavior into this trade model. We have considered in another paper (Fukao and Hamada 1989) the consequence of non–linear saving behavior such that β^H or β^F depend non–monotonically on the value of wealth a^H or a^F reflecting the fact that people will not wish to save much if they are very poor. Thus we could analyze three types of equilibria, co–property equilibria C where both economies grow harmoniously without substantial international credit, imperialism equilibria (the Ramsey states) E and F, and starvation equilibria O. This dynamical system can be superimposed on the diversification cones as shown in Figure 7.[17] Similar stories can be told on the development of specialization patterns.

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[16]This can be seen from the fact that in such a case, the optimum condition becomes instead of (14)

$$\dot{c}^j(t) = \frac{\psi'(c^j(t))}{\psi''(c^j(t))}\left[\beta^j - \frac{r(t)}{q(t)} + n\right], \tag{14'}$$

where $\psi(\cdot)$ denotes the felicity function.

[17]To avoid excessive complexity, here we assume the unitary elasticity of substitution between expenditures on the two goods so that all the diversification cones become linear cones, i.e., OH^X, OH^Y, etc. are all straight lines.

VI. CONCLUDING REMARKS

By integrating the model of trade with capital movements, we have extended a two-country model with different saving behavior between the nations to a more realistic situation where capital movements can take place. We have resolved the indeterminacy problem intrinsic in the model that allow the coexistence of trade and factor movements by assuming that the flow of capital is infinitesimally more costly than the flow of goods but that transactions costs can be neglected in the calculation of equilibrium paths. Thus we have integrated the tradition of trade theory without factor movements and factor movements theory without trade. By additional assumptions, on instantaneous utility functions of rentiers and the composite bundle of goods for investment, we have obtained a model where the behavior of asset accumulation can be dichotomized from the pattern of trade specialization. Moreover, all the behavior in our model, with some exceptions in Section V, has a complete microeconomic foundation.

We have shown that complete specialization will take place in one country, and most likely in the country with a higher rate of time preference into the more labor-intensive sector, but that almost surely only one of the two countries will completely specialize. We have also shown that a single–country model does exaggerate the amount of capital movements and that the degree of exaggeration depends on the difference of factor shares between the two sectors. However, we have also found that the qualitative nature of asset accumulation patterns obtained in a single–commodity model holds intact in the model that incorporates trade.

In this sense, we have made both trade theory and factor movements theory more realistic. Needless to say, there is still a wide gap between reality and such a level of economic analysis as has been conducted in this paper. The distance between actual trade practices and the standard trade theory could be compared to the distance between actual gardens and artificial miniature gardens. As Leamer (1984) clearly demonstrates, the

Heckscher–Ohlin trade paradigm can only explain very little part of trading patterns in the world economy. Further empirical investigations are needed to make our theory applicable to the real world.

Another interesting application of this model may be found in the controversy around the Feldstein and Horioka (1980) article on the degree of capital movements (cf. Dooley *et al.* 1987). Their claim that international capital mobility is imperfect because increased savings do not create capital outflows is under scrutiny from the standpoint of intertemporal optimization, nature of shocks and econometric methodology (e.g., Obstfeld 1986; Roubini 1989). This paper offers another explanation as to why investment responds to increasing savings. If the economy is perfectly specialized, then increased savings will create immediate capital outflows. If the economy is imperfectly specialized, however, increased savings will be absorbed in capital deepening to change the industrial composition between the sectors. This theory would predict little response of capital outflows to increased savings for a while. Then after the complete specialization takes place, capital flows will respond to increased savings one by one. Whether or not this explanation is applicable should be carefully analyzed by empirical studies. The recent burst of capital outflows from West Germany and Japan to the United States, however, seems to suggest that this may be one of the plausible explanations.

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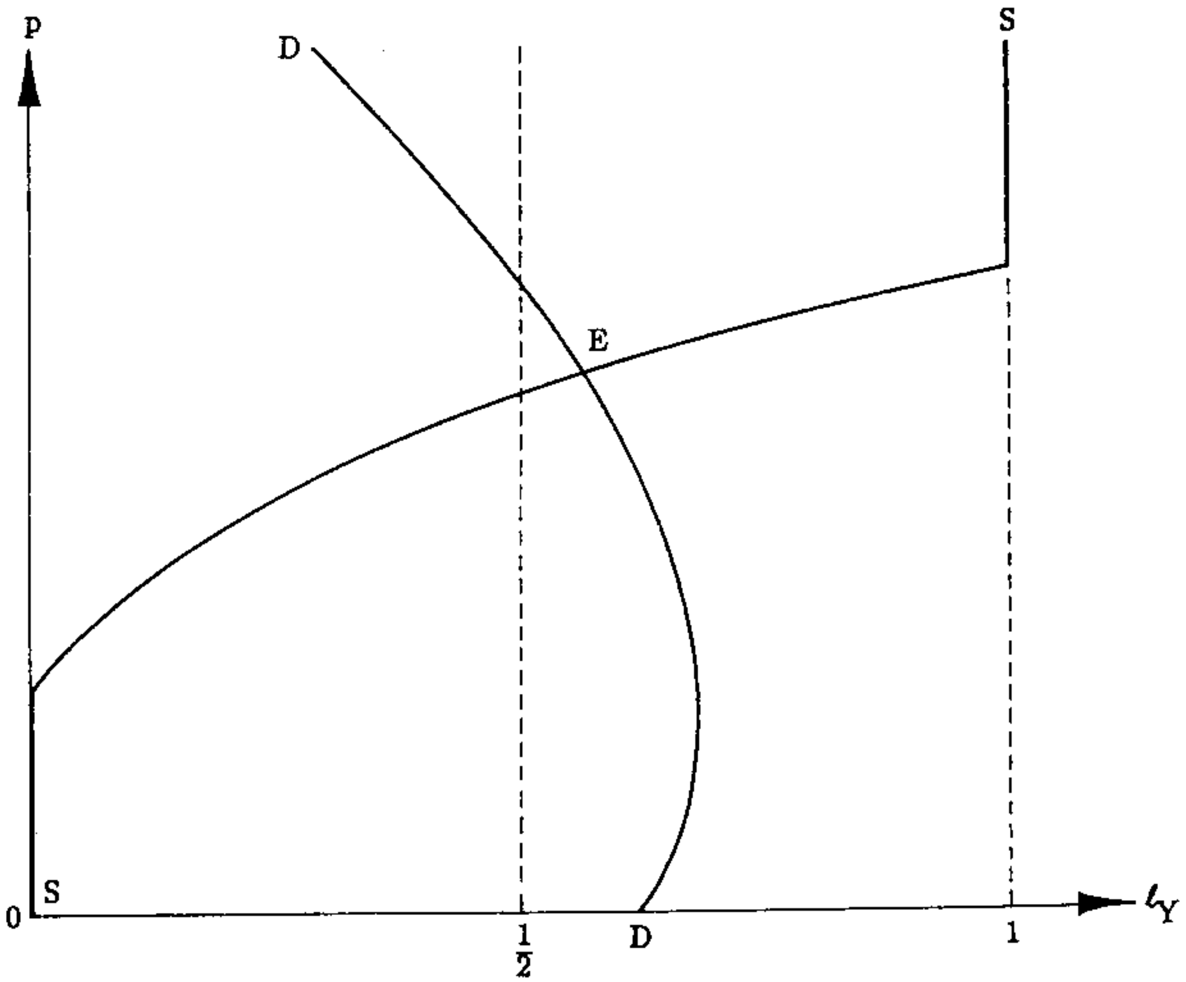


FIGURE 1

Determination of ℓ_Y and p in a Short–run Equilibrium

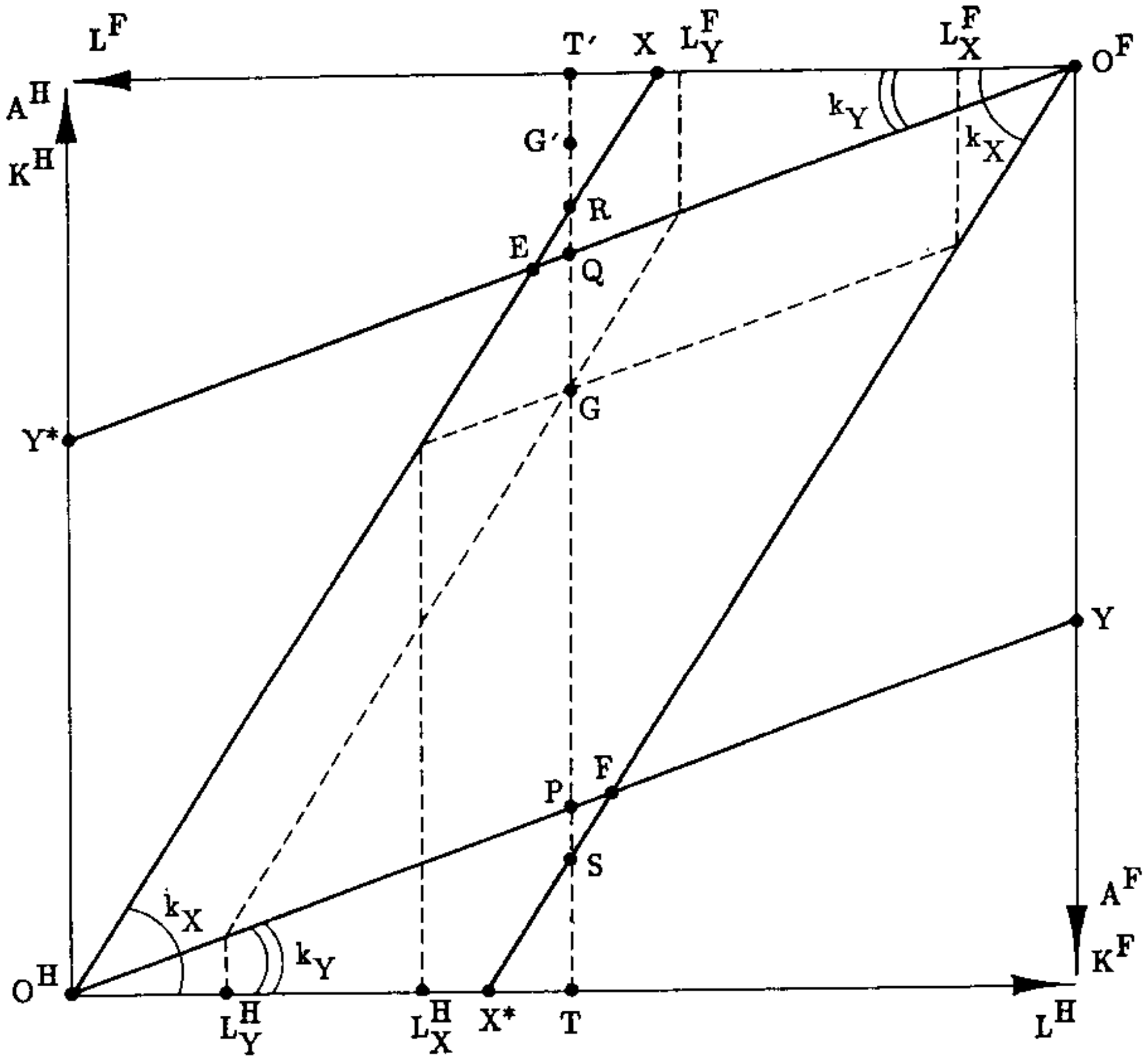
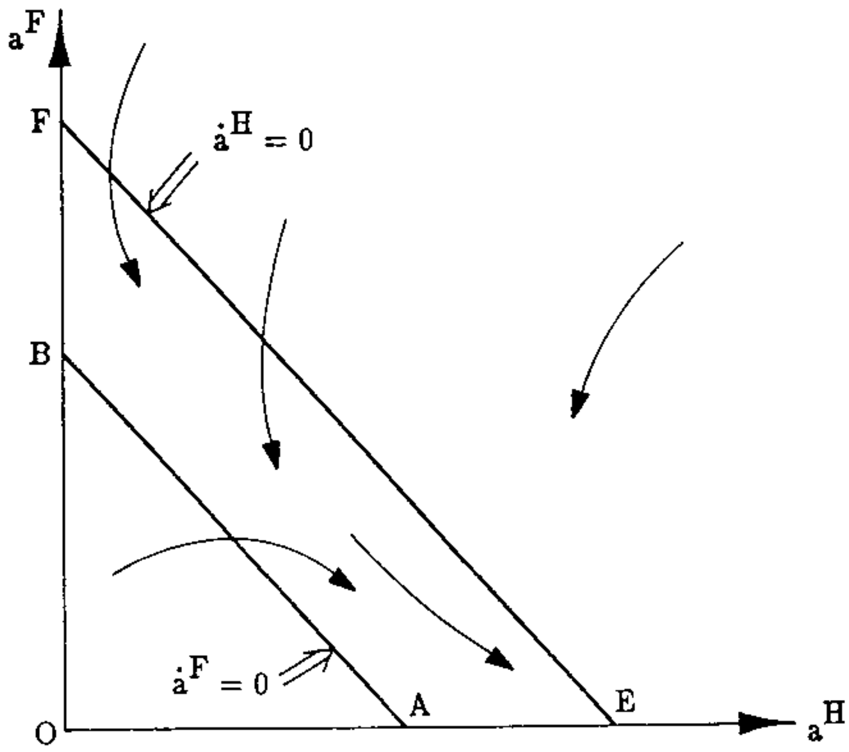


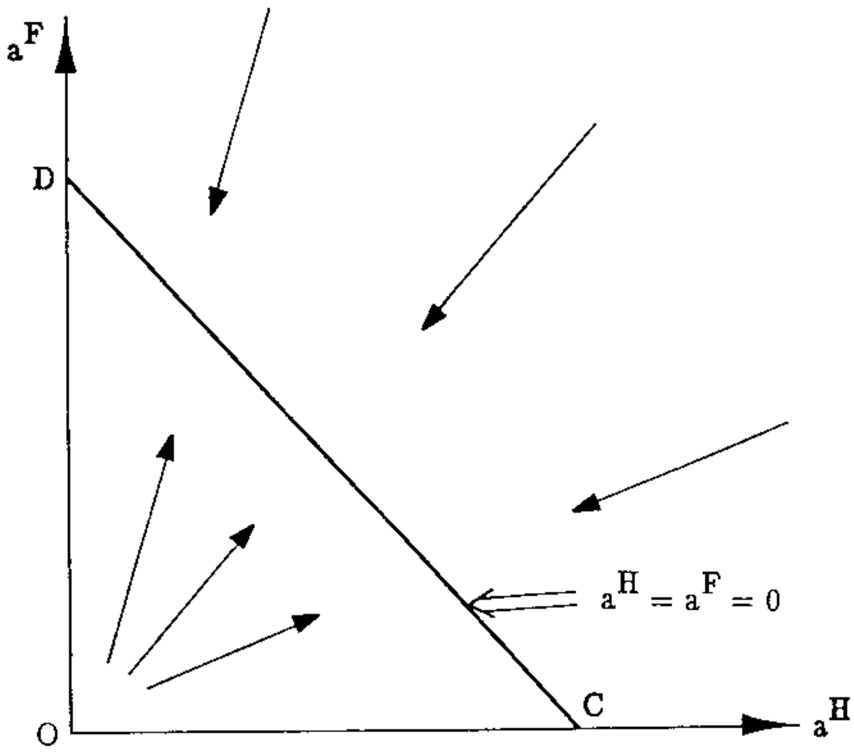
FIGURE 2

Determination of Specialization and International Capital Flow

FIGURE 3



(a) Dynamics of Capital Ownerships, $\beta^H < \beta^F$



(b) Dynamics of Capital Ownerships, $\beta^H = \beta^F$

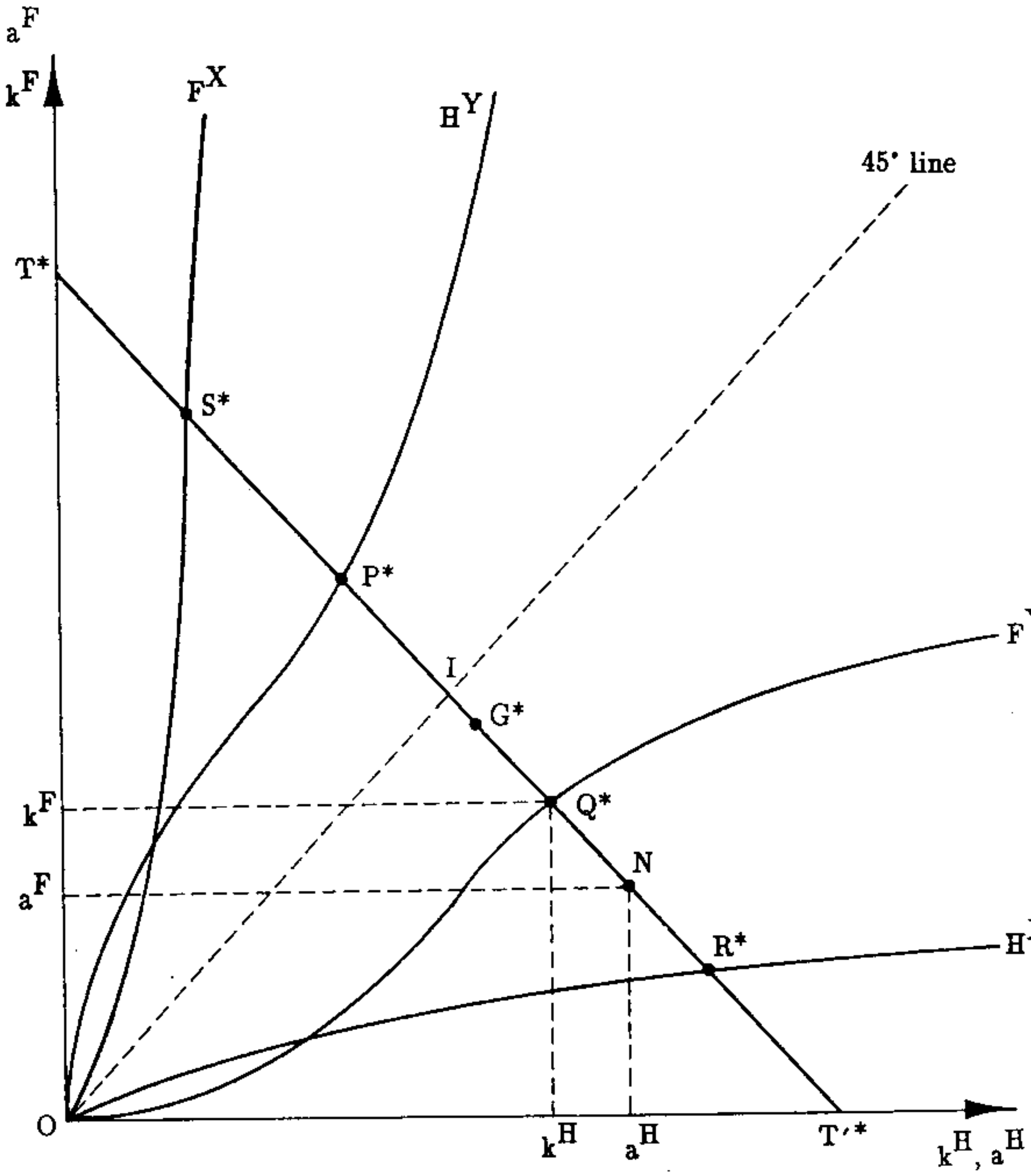


FIGURE 4
Determination of Specialization Pattern

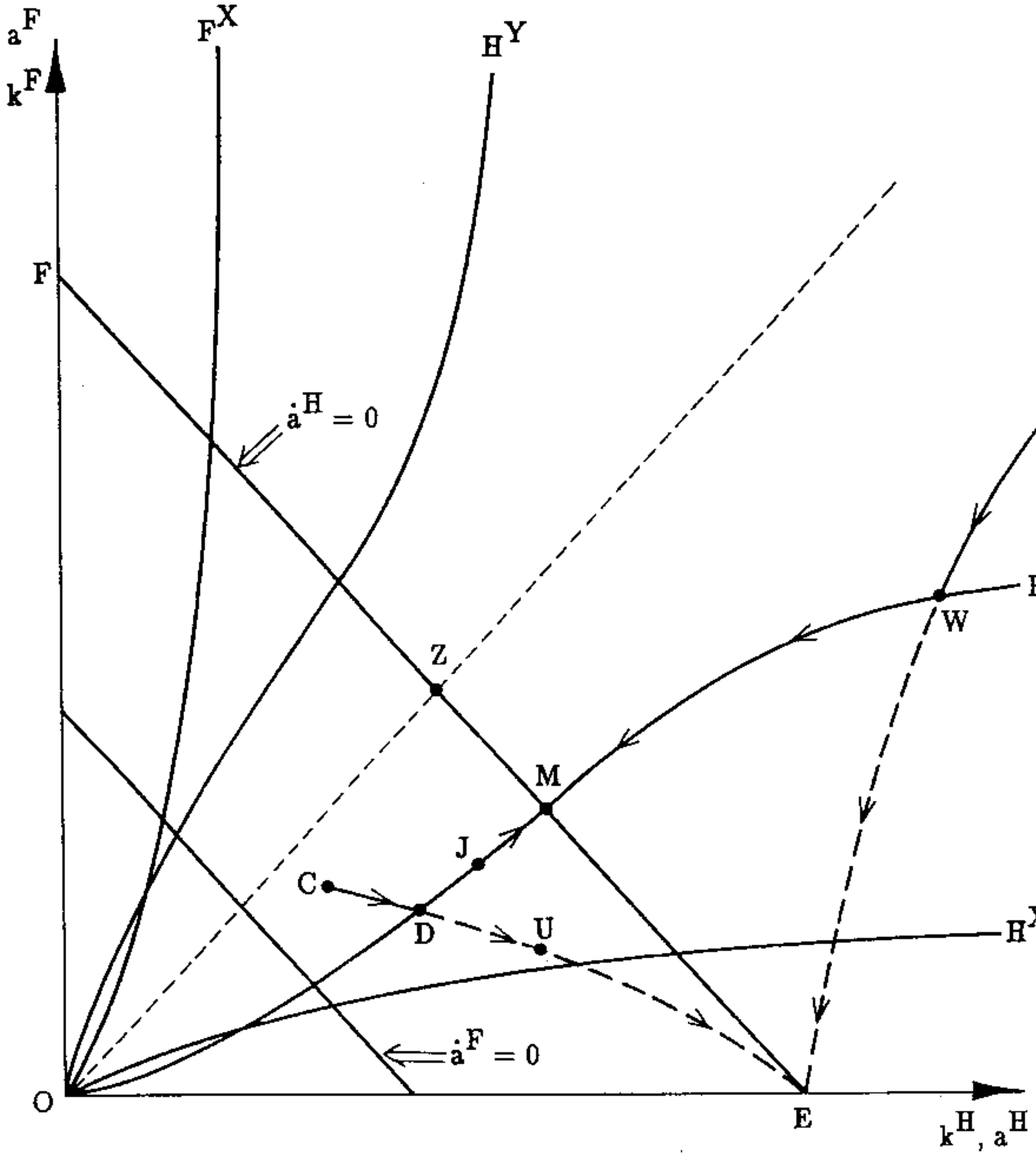
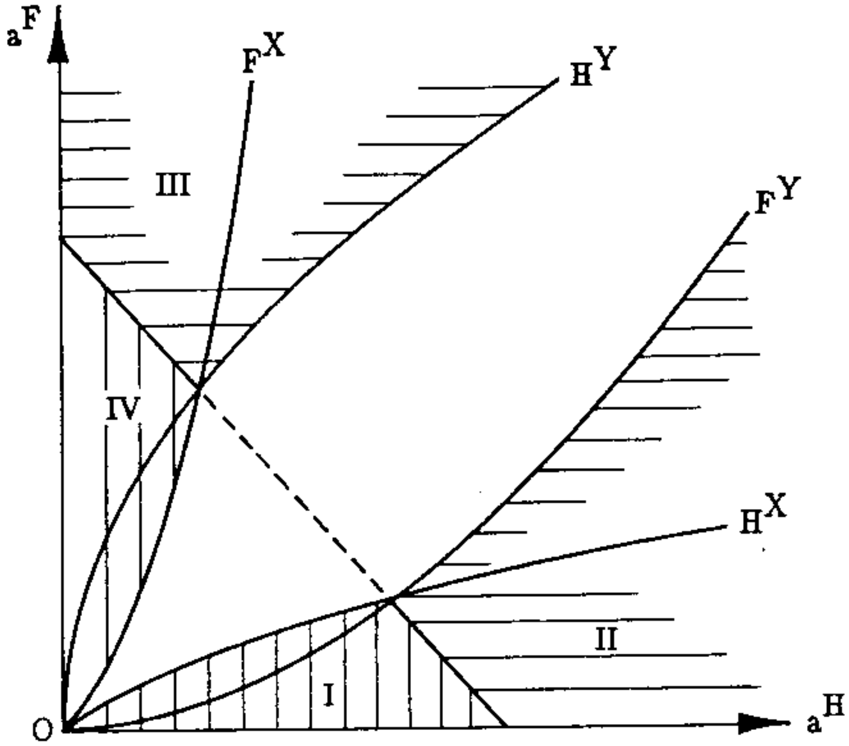


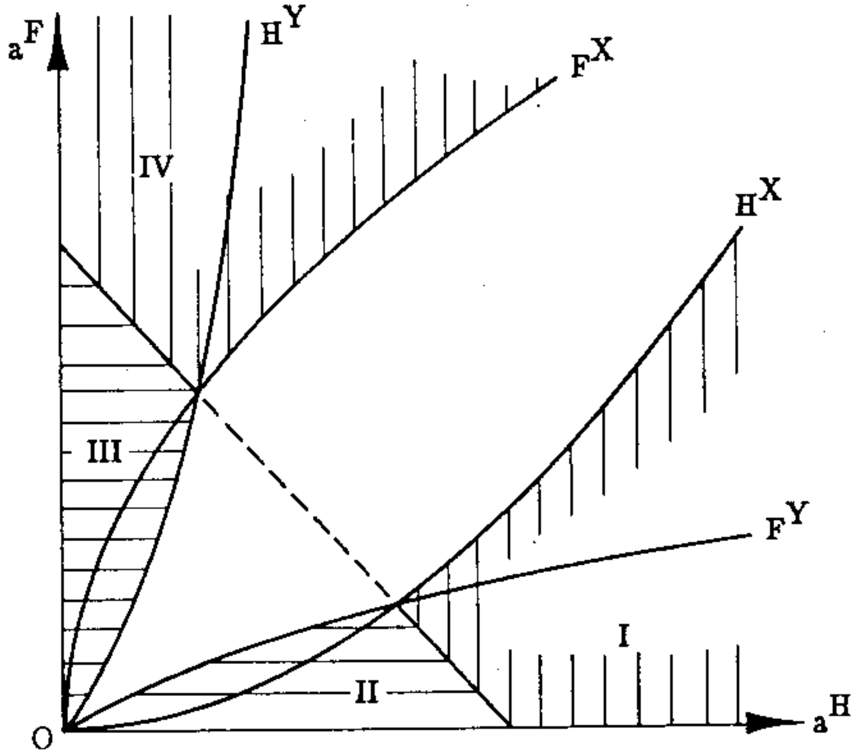
FIGURE 5

Dynamics of Trade Pattern and International Lending;
The Time Preference Rate of the Home Country Is Smaller Than That of the Foreign Country

FIGURE 6



(a) Specialization Pattern, $\eta_E < 1$



(b) Specialization Pattern, $\eta_E > 1$

I: The home country perfectly specializes in goods X .
II: The foreign country perfectly specializes in goods Y .
III: The home country perfectly specializes in goods Y .
IV: The foreign country perfectly specializes in goods X .

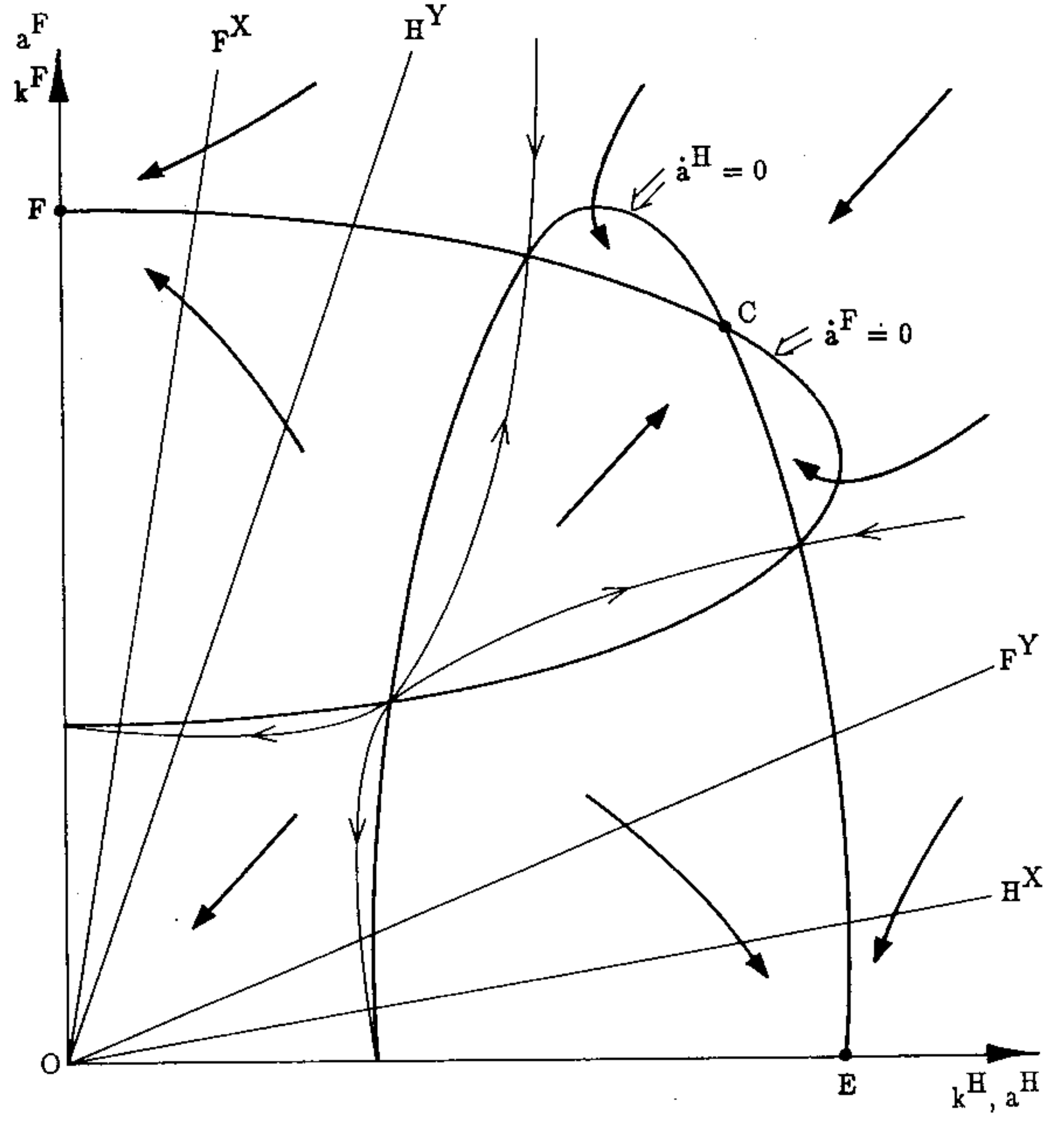


FIGURE 7

Dynamics of Trade Pattern and International Lending under the Fisherian Time Preference