Class and Exploitation in General Convex Cone Economies*

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Abstract

In this paper, we examine what appropriate formulations for labor exploitation are, in order to explain the emergence of class and exploitation status in capitalist economies. Given the well-known controversy pertaining to plausible formulations for labor exploitation in joint production economies, we propose an axiom, *Axiom for Labor Exploitation* (**LE**), which every 'appropriate' formulation of labor exploitation should satisfy. Using this axiom, the necessary and sufficient condition for plausible formulations of labor exploitation is characterized to verify Class-Exploitation Correspondence Principle (CECP) [Roemer (1982)]. According to this, CECP no longer holds in general convex cone economies if the well-known formulations of labor exploitation such as Morishima (1974) and Roemer (1982; Chapter 5) are applied. Therefore, we propose two new definitions of labor exploitation, each of which verifies CECP.

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1 Introduction

Exploitation of labor is the difference between labor hours an individual provides and the labor hours necessary to produce commodities the individual can purchase via his income. This notion has been one of the prominent key-concepts relevant to capitalist economic systems, particularly in a number of debates, ranging from analyses of labour relations, especially focusing on the weakest segments of the labour force, such as women, children, and migrants (see, e.g., ILO, 2005; 2005a; 2006). This has been seen as the cornerstone of Marxist social theory, but it is also extensively discussed in normative theory and political philosophy (see, e.g., Wertheimer, 1996; Wolff, 1999; Bigwood, 2003; and Sample, 2003). In addition, this notion deserves recognition in contemporary economics for understanding one of the *essential* characteristics of market economies with private ownership of wealth, given the recent and common trends of growing disparity in income and wealth and the increase in poverty among advanced countries. In fact, though, traditionally, the Marxian social theory has described capitalist society as that the capitalist class *exploits* the working class, it is Roemer (1982) who discusses, by applying the standard general equilibrium analysis, that this phenomenon is shown as a formal *theorem* called *Class-Exploitation Correspondence Principle* (CECP),[1] as opposed to a mere descriptive theory.

[1]Before this argument, during the 1970's and 1980's, there were remarkable developments in the debate about this concept in mathematical Marxian economics. Fundamental Marxian Theorem (FMT) was originally proved by Okishio (1963) and later named as such by Morishima (1973). FMT shows a correspondence between the existence of positive profit and the existence of labor exploitation. It gives us a useful characterization for *non-trivial* equilibria, where a trivial equilibrium is such that its social production point is zero.

Note that FMT was originally considered to prove the classical Marxian argument that the exploitation of labor is the sole source of positive profits in a capitalist economy. However, the exploitation of labor is not the unique source of positive profits. The reason being that any commodity can be shown to be exploited in a system with positive profits whenever the exploitation of labor exists. This observation was pointed out by Brody (1970), Bowles and Gintis (1981), Samuelson (1982), and was named "Generalized Commodity Exploitation Theorem (GCET)" by Roemer (1982).

After the seminal work by Morishima (1973), there were many generalizations and discussions of FMT. While the original FMT is discussed in simple Leontief economies with homogeneous labor, the generalization of FMT to Leontief economies with heterogeneous labor was made by Fujimori (1982), Krause (1982), etc. The problem of generalizing FMT to von Neumann economies was discussed by Steedman (1977) and one solution was proposed by Morishima (1974). Furthermore, Roemer (1980) generalized the theorem to

Given this CECP, an appropriate formulation of exploitation can be seen as relevant to the inequality of opportunities which are generic in capitalist society. This is because CECP shows that the wealthier agents are *exploiters*, and they can rationally choose from all classes of society to belong to the *capitalist class*, whereas the least wealthy agents are *exploited* and relegated to the *working class* being that there is no other available option. Thus, the exploiting agents of the capitalist class tend to have a myriad of options, whereas the exploited agents of the working class have far fewer options: the existence of labor-exploiters and labor-exploited agents reflects *unequal opportunity of life options*, due to unequal access to productive assets.[2]

Furthermore, interestingly, an appropriate formulation of exploitation can perhaps be seen as an index for capturing 'unjust' distribution of *well-being freedom* in the following sense: As discussed by Rawls (1971), Sen (1985, 1985a), etc., well-being freedom implies an individual's ability to choose to pursue the life she values.[3] There are two crucial factors which stipulate the degree of an individual's well-being freedom: one is the amount of income she can spend to purchase the commodities necessary to achieve her goals, and the other is the amount of time she has to sacrifice as labor supply in order to purchase such commodities. Then, the rate of labor exploitation can represent her degree of well-being freedom, or indeed *un*freedom, since it measures the difference of these two factors by using labor hours as a *numéraire*: if this value is negative for the individual, she is *exploiting* the free hours that some other agents sacrificed as labor supply for the production of the commodities she can purchase. If the value is positive, she is *exploited* in the sense that some of the free hours she sacrificed as labor supply to purchase the commodities are appropriated by somebody else.

convex cone economies. These arguments may reflect the robustness of FMT.

[2] This argument was criticized by Bowles and Gintis (1990) and Devine and Dymski (1991, 1992), since it assumed a standard neoclassical labor market, which was regarded as not a *real* model, but an *ideal* one of capitalist economies by these critics. However, as Yoshihara (1998) showed, CECP essentially holds true even if the neoclassical labor market is replaced by a non-neoclassical labor market with *efficiency wage* contracts, which was interpreted as a more realistic aspect of capitalist economies by those same critics.

[3] In the Rawls-Sen theory, inequalities in the distribution of well-being freedom are formulated as *inequalities of capabilities*, whereas they are formulated as *inequalities of (comprehensive) resources* in Dworkin's theory [Dworkin (2000)]. The resource allocation problem, in terms of equality of capability, is explicitly analyzed in Gotoh and Yoshihara (2003), whereas this problem, in terms of equality of resources, is explicitly analyzed in Roemer (1986) and Yoshihara (2003).

Granting the normative relevance of labor exploitation, the well-known formulation of this notion was proposed by Okishio (1963) in the simple Leontief-type model, and given this model, CECP holds true under this definition of exploitation, as Roemer (1982; Chapter 4) illustrated. However, once a more general convex cone model such as the von Neumann model is applied, the Okishio (1963) formulation is known to be ill-defined, as Morishima (1973) and Steedman (1977) argued. Given this difficulty, two alternative formulations were respectively proposed by Morishima (1974) and Roemer (1982; Chapter 5), which are indeed well-defined in more general models. However, as this paper will point out below, neither the Morishima (1974) nor the Roemer (1982; Chapter 5) definition could preserve CECP as a theorem in general convex cone models. This is indeed problematic for both the definitions, since, as Roemer (1982) forcefully argued, the central relevance of CECP in exploitation theory implies that it should be epistemologically considered as a *postulate*, by requiring that any satisfactory definition of exploitation preserves CECP.

Given this background, our main concern in this paper is to discuss what formulations of exploitation are appropriate. Regarding this issue, previous publications on exploitation theory, including Morishima (1974) and Roemer (1982), have repeated the process of criticizing the existing formulations while proposing an alternative one that to some, could be seen as more appealing. However, since there is potentially an infinite number of formulations for exploitation, we may find it difficult to come to a consensus on which formulations could be deemed appropriate if we were to continue this argument. In contrast, this paper introduces an *axiomatic method* which is a completely new approach to the exploitation theory. By taking an axiomatic approach, this paper suggests to start from the very basic principles which represent the normative intuitions behind exploitation theory, thus explicitly identifying the class of proper formulations for exploitation.

To be precise, in this paper we first propose a plausible axiom, *Axiom for Labor Exploitation* (**LE**), which is a *minimal* necessary condition for any formulation to be considered as an appropriate one. By using this axiom, we characterize what kinds of formulations can verify CECP as a theorem in general convex cone economies. Based upon this characterization, we show that, in contrast to the above mentioned two traditional definitions, the two new definitions of exploitation satisfy **LE** and preserve CECP: one is a refinement of Roemer's (1982; Chapter 5) formulation and the other is an extension of the so-called "New Interpretation" [Dumenil (1980); Foley (1982)] formulation of exploitation (originally defined in Leontief models) to general convex cone models. Both of these definitions formulate the exploitation index as the difference between one unit of labor supplied by an agent per day and the minimal amount of labor socially necessary to achieve the agent's *income* per day. We could also resolve, by using the two new definitions, most of the difficulties that Marxian economic theory has faced. That is, the difficulty of *Fundamental Marxian Theorem* (FMT) in the general convex cone economy, that was discussed by Petri (1980) and Roemer (1980), is resolved.

In the following paper, section 2 defines a basic economic model with convex cone production technology, and an equilibrium notion. Section 3 introduces alternative formulations, including our new definitions, for labor exploitation. Section 4 discusses the robustness of CECP in general convex cone economies by using the various definitions of labor exploitation, while section 5 discusses the performance of the new definitions in terms of FMT. Finally, section 6 provides some concluding remarks.

2 The Basic Model

Let P be the production set. P has elements of the form $\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha})$ where $\alpha_0 \in \mathbb{R}_+$, $\underline{\alpha} \in \mathbb{R}^m_+$, and $\overline{\alpha} \in \mathbb{R}^m_+$. Thus, elements of P are vectors in $\mathbb{R}^{2m+1}$. The first component, $-\alpha_0$, is the direct labor input of process α; and the next m components, $-\underline{\alpha}$, are the inputs of goods used in the process; and the last m components, $\overline{\alpha}$, are the outputs of the m goods from the process. We denote the net output vector arising from α as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. We assume that P is a closed convex cone containing the origin in $\mathbb{R}^{2m+1}$. Moreover, it is assumed that:

A 1. $\forall \alpha \in P$ s.t. $\alpha_0 \geq 0$ and $\underline{\alpha} \geqq 0$, $[\overline{\alpha} \geq 0 \Rightarrow \alpha_0 > 0]$;[4] and
A 2. $\forall$ commodity m vector $c \in \mathbb{R}^m_+$, $\exists \alpha \in P$ s.t. $\widehat{\alpha} \geqq c$.
A 3. $\forall \alpha \in P, \forall (-\underline{\alpha}', \overline{\alpha}') \in \mathbb{R}^m_- \times \mathbb{R}^m_+$, $[(-\underline{\alpha}', \overline{\alpha}') \leqq (-\underline{\alpha}, \overline{\alpha}) \Rightarrow (-\alpha_0, -\underline{\alpha}', \overline{\alpha}') \in P]$.

A1 implies that labor is indispensable to produce any non-negative output vector; A2 states that any non-negative commodity vector is producible as a net output; and A3 is a *free disposal* condition, which states that, given any

[4]For all vectors $x = (x_1, \ldots, x_p)$ and $y = (y_1, \ldots, y_p) \in \mathbb{R}^p$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, p)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, p)$.

feasible production process α, any vector producing (weakly) less net output than α is also feasible using the same amount of labour as α itself.

Given such P, we will sometimes use the following notations:

$$P(\alpha_0 = 1) \equiv \{(-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \mid \alpha_0 = 1\},$$
$$\widehat{P}(\alpha_0 = 1) \equiv \{\widehat{\alpha} \in \mathbb{R}^m \mid \exists \alpha = (-1, -\underline{\alpha}, \overline{\alpha}) \in P \text{ s.t. } \overline{\alpha} - \underline{\alpha} \geqq \widehat{\alpha}\}.$$

As a notation, we use, for any set $S \subseteq \mathbb{R}^m$, $\partial S \equiv \{x \in S \mid \nexists x' \in S \text{ s.t. } x' > x\}$.

Given a market economy, any price system is denoted by $p \in \mathbb{R}^m_+$, which is a price vector of m commodities. Moreover, a *subsistence vector* of commodities $b \in \mathbb{R}^m_+$ is also necessary in order to supply one unit of labor per day. We assume that the nominal wage rate is normalized to unity when it purchases the subsistence consumption vector only, so that $pb = 1$ holds.

For the sake of simplicity, we follow the same setting as that in Roemer (1982; Chapter 5). That is, we consider a temporary equilibrium framework, in which there is no financial capital market and time is essential in the production process in the sense that outputs are available tomorrow from today's production. Moreover, all agents are assumed to be accumulators who seek to expand the value of their endowments as rapidly as possible. Let us denote the set of agents by N with generic element ν. All agents have access to the same technology P, but they possess different endowments ω^ν, whose distribution in the economy is given by $(\omega^\nu)_{\nu \in N} \in \mathbb{R}^{Nm}_+$. An agent $\nu \in N$ with ω^ν can engage in the following three types of economic activities: she can sell her labor power γ^ν_0; she can hire the labor powers of others to operate $\beta^\nu = \left(-\beta^\nu_0, -\underline{\beta}^\nu, \overline{\beta}^\nu\right) \in P$; or she can work for herself to operate $\alpha^\nu = (-\alpha^\nu_0, -\underline{\alpha}^\nu, \overline{\alpha}^\nu) \in P$. Given a price vector $p \in \mathbb{R}^m_+$ and a nominal wage rate w, it is assumed that each agent chooses her activities, α^ν, β^ν, and γ^ν_0, in order to maximize the revenue *subject to the constraints of her capital and labor endowments.* This is because she must be able to afford to lay out the operating costs in advance for the activities she chooses to operate, either with her own labor or hired labor, funded by the value of her capital endowment, $p\omega^\nu$. Such a constraint is reasonable in this kind of temporary equilibrium setting with no financial capital market, where an agent has to pay today for inputs before receiving revenues from production tomorrow.

Thus, given (p, w), each ν solves the following program MP^ν:

$$\max_{\left(\alpha^\nu;\, \beta^\nu;\, \gamma^\nu_0\right) \in P \times P \times \mathbb{R}_+} \left[p\left(\overline{\alpha}^\nu - \underline{\alpha}^\nu\right)\right] + \left[p\left(\overline{\beta}^\nu - \underline{\beta}^\nu\right) - w\beta^\nu_0\right] + \left[w\gamma^\nu_0\right]$$

subject to

$$\begin{aligned} p\underline{\alpha}^{\nu} + p\underline{\beta}^{\nu} &\leqq p\omega^{\nu} \equiv W^{\nu}, \\ \alpha_0^{\nu} + \gamma_0^{\nu} &\leqq 1. \end{aligned}$$

Given (p, w), let $\mathcal{A}^{\nu}(p, w)$ be the set of actions $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu}) \in P \times P \times [0, 1]$ which solve MP^{ν} at prices (p, w), and let

$$\Pi^{\nu}(p, w) \equiv \max_{(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu}) \in P \times P \times \mathbb{R}_+} [p(\overline{\alpha}^{\nu} - \underline{\alpha}^{\nu})] + \left[p\left(\overline{\beta}^{\nu} - \underline{\beta}^{\nu}\right) - w\beta_0^{\nu}\right] + [w\gamma_0^{\nu}].$$

Based on Roemer (1982; Chapter 5), the equilibrium notion for this model is given as follows:

Definition 1: A *reproducible solution* (RS) for the economy specified above is a pair $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu \in N}\right)$, where $p \in \mathbb{R}_+^m$, $w \geqq pb = 1$, and $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu}) \in P \times P \times [0, 1]$, such that:

(a) $\forall \nu \in N$, $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu}) \in \mathcal{A}^{\nu}(p, w)$ (revenue maximization);

(b) $\underline{\alpha} + \underline{\beta} \leqq \omega$ (social feasibility),
where $\underline{\alpha} \equiv \sum_{\nu \in N} \underline{\alpha}^{\nu}$, $\underline{\beta} \equiv \sum_{\nu \in N} \underline{\beta}^{\nu}$, and $\omega \equiv \sum_{\nu \in N} \omega^{\nu}$;

(c) $\beta_0 \leqq \gamma_0$ (labor market equilibrium)
where $\beta_0 \equiv \sum_{\nu \in N} \beta_0^{\nu}$ and $\gamma_0 \equiv \sum_{\nu \in N} \gamma_0^{\nu}$; and

(d) $\widehat{\alpha} + \widehat{\beta} \geqq \alpha_0 b + \beta_0 b$ (reproducibility),
where $\widehat{\alpha} \equiv \sum_{\nu \in N}(\overline{\alpha}^{\nu} - \underline{\alpha}^{\nu})$, $\widehat{\beta} \equiv \sum_{\nu \in N}(\overline{\beta}^{\nu} - \underline{\beta}^{\nu})$, and $\alpha_0 \equiv \sum_{\nu \in N} \alpha_0^{\nu}$.[5]

The three parts except (a) need some comments. Part (d) states that net outputs should at least replace employed workers' total consumption. This is equivalent to requiring that the vector of social endowments does not decrease in terms of components, because (d) is equivalent to $\omega - (\underline{\alpha} + \alpha_0 b) + \overline{\alpha} \geqq \omega$, where the right hand side is the social stocks at the beginning of this period,

[5] The notion of RS is a refinement of the competitive equilibrium (CE) [Roemer (1981; Chapter 1; p.29)] notion. Here, since we do not take into account consumers' behaviors explicitly, CE is defined by the following three conditions: Definition 1(a), 1(c), and the *excess demand condition*, $\alpha_0 b + \beta_0 b + \underline{\alpha} + \underline{\beta} \leqq \widehat{\alpha} + \widehat{\beta} + \omega$. Then, the set of RSs is a subset of CEs, since Definitions 1(b) and 1(d) imply this excess demand condition. Thus, any RS is a CE such that the vector of social stocks endowed at the beginning of the period can be reproduced at the end of this period.

the left hand side is the stocks at the beginning of the next period. Part (b) says that intermediate inputs must be available from current stocks. This condition is reasonable in this model with no financial capital market, where the production process takes time.[6] Here, we assume that wage goods are dispensed at the end of each production period, therefore stocks need not to be sufficient to accommodate them as well. Finally, (c) is the condition of labor market equilibrium. This condition allows strict inequality between labor demand β_0 and labor supply γ_0. If it holds in strict inequality, then the nominal wage rate is driven down to the subsistence wage $w = pb = 1$. If it holds in equality, then it might hold that $w \geqq pb = 1$.

Given an RS, $\left((p,w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}\right)$, let $\alpha^{p,w} \equiv \sum_{\nu \in N} \alpha^\nu + \sum_{\nu \in N} \beta^\nu$, which is the aggregate production activity *actually* accessed in this RS. Thus, the pair $((p,w), \alpha^{p,w})$ is the summary information of this RS. In the following, we sometimes use $((p,w), \alpha^{p,w})$ or only (p,w) for the representation of the RS, $\left((p,w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}\right)$.

The existence of RS depends on the position of the initial endowment vectors; For instance, if the endowment vector, ω, lies on the *balanced growth path*, then an RS exists. In the following, we will show the characterization of the initial endowment domain, under which an RS exists. Let $\alpha_0(\omega) \equiv \max\{\alpha_0 \mid \exists \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \text{ s.t. } \underline{\alpha} \leqq \omega\}$. Given P, let $\widetilde{P} \equiv \{\overline{\alpha} - \underline{\alpha} - \alpha_0 b \in \mathbb{R}^m \mid (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P\}$. Let

$$\mathbb{C}^* \equiv \left\{\omega \in \mathbb{R}^m_+ \mid \exists \alpha \in P : \underline{\alpha} \leqq \omega,\ \underline{\alpha} \not< \omega,\ \overline{\alpha} - \alpha_0 b \geqq \underline{\alpha}\ \&\ \overline{\alpha} - \omega - \alpha_0 b \in \partial \widetilde{P}\right\}.$$

Note this $\mathbb{C}^*$ is a non-empty and closed convex cone. Then:

Proposition 1: *Let $b \in \mathbb{R}^m_{++}$, $\omega \in \mathbb{R}^m_{++}$, and $\alpha_0(\omega) \leqq |N|$. Under A1∽A3, a* reproducible solution (RS) *exists for the economy above if and only if $\omega \in \mathbb{C}^*$.*

Proof. ($\Rightarrow$): Let $((p,1), \alpha^{p,1})$ be an RS. At the RS, $p\underline{\alpha}^{p,1} = p\omega$. Thus, $\underline{\alpha}^{p,1} \leqq \omega$ by Definition 1(b), but $\underline{\alpha}^{p,1} \not< \omega$. From Definition 1(d), $\overline{\alpha}^{p,1} - \alpha_0^{p,1} b \geqq \underline{\alpha}^{p,1}$. Also, from Definition 1(a), $\alpha^{p,1} \in \sum_{\nu \in N} \mathcal{A}^\nu(p,1)$, so that $\frac{p\left(\overline{\alpha}^{p,1} - \omega\right) - \alpha_0^{p,1}}{p\omega}$ is the maximal profit rate at $(p,1)$. Thus, for any other $\alpha' \in P$

[6] That is, given no financial capital market, if the society needs more inputs than what the current capital stocks can supply, the additional input goods must be produced in the current period, but those goods are available for use as inputs in the next period.

with $p\underline{\alpha}' = p\omega$, $p\overline{\alpha}^{p,1} - p\omega - w\alpha_0^{p,1} \geqq p\overline{\alpha}' - p\underline{\alpha}' - w\alpha_0'$ holds. This implies that $\overline{\alpha}^{p,1} - \omega - \alpha_0^{p,1}b \in \partial\widetilde{P}$ and $p \in S \equiv \left\{p \in \mathbb{R}_+^m \mid pb = 1\right\}$ is a supporting price of it. Note that $\left(-\alpha_0^{p,1}, -\omega, \overline{\alpha}^{p,1}\right) \in P$ by $\alpha^{p,1} \in P$, $\underline{\alpha}^{p,1} \leqq \omega$, and A3.

($\Leftarrow$): Let $\omega \in \mathbb{C}^*$. Then, there exists $\alpha^* = (-\alpha_0^*, -\underline{\alpha}^*, \overline{\alpha}^*) \in P$ such that $\underline{\alpha}^* \leqq \omega$ with $\underline{\alpha}^* \nless \omega$, $\overline{\alpha}^* - \alpha_0^* b \geqq \underline{\alpha}^*$, and $\overline{\alpha}^* - \omega - \alpha_0^* b \in \partial\widetilde{P}$. Since $\overline{\alpha}^* - \omega - \alpha_0^* b \in \partial\widetilde{P}$, by the supporting hyperplane theorem, there exists $p^* \in S$ which supports $\overline{\alpha}^* - \omega - \alpha_0^* b$ in $\widetilde{P}$. Thus, for any $\overline{\alpha} - \underline{\alpha} - \alpha_0 b \in \widetilde{P}$ with $p^*\underline{\alpha} = p^*\omega$, $p^*\overline{\alpha}^* - p^*\omega - \alpha_0^* \geqq p^*\overline{\alpha} - p^*\underline{\alpha} - \alpha_0$ holds. This implies that α^* realizes the maximal profit rate at $(p^*, 1)$. Thus, since $\mathbf{0} \in \widetilde{P}$, $p^*\overline{\alpha}^* - p^*\omega - \alpha_0^* \geqq 0$ holds. Let $(\omega^\nu)_{\nu \in N}$ be a distribution of initial endowments such that $\sum_{\nu \in N} \omega^\nu = \omega$. Then, by the cone property of P, $\alpha^{*\nu} + \beta^{*\nu} \equiv \frac{p^*\omega^\nu}{p^*\omega}\alpha^*$ and $\gamma^{*\nu} = 1$ constitute an individually optimal solution for each $\nu \in N$. Also, by the property of ω and α^*, Definition 1(b) and 1(d) hold. Finally, since $\alpha_0(\omega) \leqq |N|$, Definition 1(c) holds. Thus, $\left((p^*, 1), (\alpha^{*\nu};\ \beta^{*\nu};\ \gamma_0^{*\nu})_{\nu \in N}\right)$ constitutes an RS. ■

The above proof develops the existence proof of Karlin (1959) and Roemer (1980; Appendix II). Proposition 1 shows that for any $(\omega^\nu)_{\nu \in N} \in \mathbb{R}_+^{nm}$ with $\sum_{\nu \in N} \omega^\nu = \omega \in \mathbb{C}^*$ and $\alpha_0(\omega) \leqq |N|$, an RS exists. Thus, the existence of an RS is shown independently of distribution of endowment vectors, under the assumption of *capital-limited* economies, $\alpha_0(\omega) \leqq |N|$.

Roemer (1980, 1981) provides a similar type of characterization for the domain of initial endowment vectors under the assumption of Leontief technology model. We can see that by using our characterization in a convex cone model, Roemer's characterization in a Leontief model is also derived as a corollary of our Proposition 1. The condition $\overline{\alpha} - \omega - \alpha_0 b \in \partial\widetilde{P}$ is necessary in general convex cone economies, because by this, the society can access the most profitable production processes with its endowment vector ω. Note, in Leontief production models such as in Roemer (1980, 1981), every production process is as profitable as any other, so in that case, the condition $\overline{\alpha} - \omega - \alpha_0 b \in \partial\widetilde{P}$ is trivially satisfied.

Before closing this section, let us briefly argue the basic characterizations of RSs which were examined by Roemer (1982). Given $\alpha_0(\omega) \leqq |N|$ for the economy, let $\left((p, w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}\right)$ be an RS. Then:
(1) $p\left(\underline{\alpha}^\nu + \underline{\beta}^\nu\right) = p\omega^\nu$ for all $\nu \in N$;
(2) if this RS is coupled with full employment (that is, Definition 1(c) holds in equality), then $\Pi^\nu(p, w) = \pi(p, w)p\omega^\nu + w$ such that $1 \leqq w \leqq \frac{p\widehat{\alpha}^{p,w}}{\alpha_0^{p,w}}$ for

all $\nu \in N$, where

$$\pi(p,w) \equiv \max\left\{\frac{p\overline{\alpha} - (p\underline{\alpha} + w\alpha_0)}{p\underline{\alpha}} \mid \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P\right\}.$$

Note that, if an RS is together with full employment, its nominal wage rate is essentially indeterminate under such an RS in the sense that any $w \in \left[1, \frac{p\widehat{\alpha}^{p,w}}{\alpha_0^{p,w}}\right]$ could be an equilibrium wage rate.

3 Various Formulations for Exploitation of Labor

In this section, we discuss a general condition that every formulation for labor exploitation has to satisfy to be considered appropriate. Then, by this condition, the class of plausible formulations for labor exploitation is identified. We show that both the Morishima (1974) and the Roemer (1982; Chapter 5) definitions meet this condition. We also introduce three alternative definitions for labor exploitation, which also meet the condition.

In the following, we assume an RS with full employment (that is, Definition 1(c) holds in equality) for simplicity. Given any economy $\langle N; (P,b); (\omega^{\nu})_{\nu \in N}\rangle$, and any RS, $((p,w), \alpha^{p,w})$, let $N^{ter} \subseteq N$, $N^{ted} \subseteq N$, and $N^{ter} \cap N^{ted} = \varnothing$. Also, let $B(p, \Pi^{\nu}(p,w)) \equiv \left\{f^{\nu} \in \mathbb{R}^m_+ \mid pf^{\nu} = \Pi^{\nu}(p,w)\right\}$, $B_+(p, \Pi^{\nu}(p,w)) \equiv \left\{f^{\nu} \in \mathbb{R}^m_+ \mid pf^{\nu} \geqq \Pi^{\nu}(p,w)\right\}$, and $B_-(p, \Pi^{\nu}(p,w)) \equiv \left\{f^{\nu} \in \mathbb{R}^m_+ \mid pf^{\nu} \leqq \Pi^{\nu}(p,w)\right\}$. Let $c \in \mathbb{R}^m_+$ be a vector of produced commodities. Let $\phi(c) \equiv \{\alpha \in P \mid \widehat{\alpha} \geqq c\}$, which is the set of the production points producing, as net output vectors, at least c. Let $\zeta \equiv \partial\widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$. Then:

Axiom for Labor Exploitation (LE): *Let $((p,w), \alpha^{p,w})$ be an RS. Two subsets N^{ter} and N^{ted} constitute* the set of exploiters *and* the set of exploited agents *if and only if there exist $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ such that $p\overline{c} \geqq p\underline{c}$, and for any $\nu \in N$,*

$$\nu \in N^{ter} \Leftrightarrow \exists c^{\nu} \in B_-(p, \Pi^{\nu}(p,w)) \text{ s.t. } c^{\nu} \geq \overline{c} \text{ and } \exists \alpha \in \phi(c^{\nu}) \text{ with } \alpha_0 > 1;$$
$$\nu \in N^{ted} \Leftrightarrow \exists c^{\nu} \in B_+(p, \Pi^{\nu}(p,w)) \text{ s.t. } c^{\nu} \leq \underline{c} \text{ and } \exists \alpha \in \phi(c^{\nu}) \text{ with } \alpha_0 < 1.$$

Note that the above $\overline{c}$ and $\underline{c}$ need not be uniquely fixed. Given any formulation of labor exploitation, the *set of exploiters* N^{ter} and the *set of exploited agents* N^{ted} are defined. Then, the axiom **LE** requires that for any RS, the two reference commodity vectors $\overline{c}, \underline{c} \in \mathbb{R}_+^m$ can be identified, corresponding to this given formulation of labor exploitation, thus characterizing N^{ter} and N^{ted}. Both $\overline{c}, \underline{c}$ can be produced as net outputs by supplying one unit of labor, which are to identify the income range of *non-exploited non-exploiting* agents: any agent $\nu \in N$ with income $p\underline{c} \leqq \Pi^{\nu}(p, w) \leqq p\overline{c}$, who supplies one unit of labor, is regarded as neither exploited nor exploiting, since the amount of socially necessary labor that she can receive from consumption through her income is exactly one unit. Thus, if an agent $\nu \in N$ supplies one unit of labor and receives $\Pi^{\nu}(p, w) < p\underline{c}$, then she has a consumption bundle $c^{\nu} \in B_+(p, \Pi^{\nu}(p, w))$ with $c^{\nu} \leq \underline{c}$ such that c^{ν} is produced as a net output with less than one unit of labor. The axiom **LE** requires that the set of such agents coincides with N^{ted}.[7] The parallel argument can be also applied to the case of N^{ter}. We think all potential formulations for the classical notion of labor exploitation should have this property.

We can see that both the Morishima (1974) and the Roemer (1982; Chapter 5) definitions of labor exploitation, which we will provide below, satisfy this axiom. First:

Definition 2: The Morishima (1974) labor value of commodity vector c, $l.v.(c)$, is given by

$$l.v.(c) \equiv \min \left\{ \alpha_0 \mid \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in \phi(c) \right\}.$$

It is easy to see that $\phi(c)$ is non-empty by A2. Also,

$$\left\{ \alpha_0 \mid \alpha = (-\alpha_0; -\underline{\alpha}; \overline{\alpha}) \in \phi(c) \right\}$$

is bounded from below by 0, by the assumption $\mathbf{0} \in P$ and A1. Thus, $l.v.(c)$ is well-defined since P is compact. Moreover, by A1, $l.v.(c)$ is positive whenever $c \neq \mathbf{0}$.

Then:

[7] This argument is independent of whether he really supplies one unit of labor or not, though we now assume full employment. Even if the economy is in equilibrium with unemployment, and an agent does not work at all, we can still apply the same argument to identify whether he is exploited or not. In fact, we can see that if he were to supply one unit of labor, he would receive his income $\Pi^{\nu}(p, w)$ from which the amount of 'socially necessary labor' for his income would be identified. This is a simple way to justify the assumption of full employment, and there are other ways to do likewise.

Definition 3: A producer $\nu \in N$ is *exploited in the Morishima (1974) sense* if and only if:

$$\max_{f^\nu \in B(p, \Pi^\nu(p,w))} l.v.\left(f^\nu\right) < 1,$$

and she is an *exploiter in the Morishima (1974) sense* if and only if:

$$\min_{f^\nu \in B(p, \Pi^\nu(p,w))} l.v.\left(f^\nu\right) > 1.$$

Given an RS, (p, w), let $\overline{c} \in \zeta$ be such that $p\overline{c} \geqq pc$ for all $c \in \zeta$. Also, let $\underline{c} \in \zeta$, which is a subset of $\widehat{P}\left(\alpha_0 = 1\right) \cap \mathbb{R}^m_+$, be such that $p\underline{c} \leqq pc$ for all $c \in \zeta$. We can check that ν is an exploiter in the Morishima (1974) sense if and only if $\Pi^\nu\left(p, w\right) > p\overline{c}$. Also, ν is exploited in the Morishima (1974) sense if and only if $\Pi^\nu\left(p, w\right) < p\underline{c}$. This argument implies that Definition 3 satisfies **LE**.

In contrast to the Morishima (1974) labor value, the definition of labor value in Roemer (1982; Chapter 5) depends, in part, on the particular equilibrium the economy is in. Given a price system (p, w), let

$$\overline{P}\left(p, w\right) \equiv \left\{\alpha = \left(-\alpha_0, -\underline{\alpha}, \overline{\alpha}\right) \in P \mid \frac{p\overline{\alpha} - \left(p\underline{\alpha} + w\alpha_0\right)}{p\underline{\alpha}} = \pi\left(p, w\right)\right\}.$$

Then, let

$$\phi\left(c; p, w\right) \equiv \left\{\alpha \in \overline{P}\left(p, w\right) \mid \widehat{\alpha} \geqq c\right\},$$

which is the set of those profit-rate-maximizing actions which produce, as net output vectors, at least c. Then:

Definition 4: The Roemer (1982; Chapter 5) labor value of commodity vector c, $l.v.\left(c; p, w\right)$, is given by

$$l.v.\left(c; p, w\right) \equiv \min\left\{\alpha_0 \mid \alpha = \left(-\alpha_0, -\underline{\alpha}, \overline{\alpha}\right) \in \phi\left(c; p, w\right)\right\}.$$

Then:

Definition 5: Let (p, w) be an RS. A producer $\nu \in N$ is *exploited in the Roemer (1982; Chapter 5) sense* if and only if:

$$\max_{f^\nu \in B(p, \Pi^\nu(p,w))} l.v.\left(f^\nu; p, w\right) < 1,$$

and she is an *exploiter in the Roemer (1982; Chapter 5) sense* if and only if:

$$\min_{f^{\nu}\in B(p,\Pi^{\nu}(p,w))} l.v.\left(f^{\nu};p,w\right) > 1.$$

It is easy to verify that $l.v.\,(c;p)$ is well-defined, and has a positive value whenever $c \neq \mathbf{0}$. Also, $l.v.\,(c;p) \geqq l.v.\,(c)$ holds.

To see that Definition 5 satisfies **LE**, let us define for any (p,w),

$$\theta_{(p,w)} \equiv \left\{c \in \mathbb{R}^{m}_{+} \mid \exists \alpha \in \phi\left(c;(p,w)\right) : \alpha_0 = 1 \text{ \& } \alpha_0 \text{ is minimized over } \phi\left(c;(p,w)\right)\right\}.$$

Then, given an RS, (p,w), let $\overline{c} \in \theta_{(p,w)}$ be such that $p\overline{c} \geqq pc$ for all $c \in \theta_{(p,w)}$. Also, let $\underline{c} \in \theta_{(p,w)}$ be such that $p\underline{c} \leqq pc$ for all $c \in \theta_{(p,w)}$. We can check that ν is an exploiter in the Roemer (1982; Chapter 5) sense if and only if $\Pi^{\nu}(p,w) > p\overline{c}$. Also, ν is exploited in the Roemer (1982; Chapter 5) sense if and only if $\Pi^{\nu}(p,w) < p\underline{c}$. This argument implies that Definition 5 satisfies **LE**.

In addition to the above two definitions of labor exploitation, we also propose two new definitions. Following Roemer (1982; Chapter 5), we still adopt the definition of labor value of commodities as in Definition 4. However, we refine the definition of labor exploitation from Roemer's (1982; Chapter 5). The first new definition is given as follows:

Definition 6: Let $((p,w), \alpha_{p,w})$ be an RS. A producer $\nu \in N$ is *exploited* if and only if:

$$\min_{f^{\nu}\in B(p,\Pi^{\nu}(p,w))} l.v.\left(f^{\nu};p,w\right) < 1,$$

and he is an *exploiter* if and only if:

$$\min_{f^{\nu}\in B(p,\Pi^{\nu}(p,w))} l.v.\left(f^{\nu};p,w\right) > 1.$$

We can see that Definition 6 satisfies **LE** by choosing $\overline{c} \in \theta_{(p,w)}$ as $p\overline{c} \geqq pc$ for all $c \in \theta_{(p,w)}$, and $\underline{c} = \overline{c}$.

Note that $\min_{f^{\nu}\in B(p,\Pi^{\nu}(p,w))} l.v.\left(f^{\nu};p,w\right)$ in Definition 6 can be regarded as the *indirect labor value of ν's income*. This implies that the labor value in Definition 6 is concerned *not* with an agent's *consumption vector*, *but* rather with an agent's *income* earned. Thus, this new definition implies the following: Suppose an economy is under a reproducible solution $((p,w), \alpha^{p,w})$. Then, if the *minimal expenditure* of labor socially necessary to reach an agent

ν's income $\Pi^{\nu}(p,w)$ under the RS, $((p,w),\alpha^{p,w})$, is less (*resp.* more) than unity, then ν is exploited (*resp.* exploiter). [8]

The second new definition is now ready to be discussed. Given any RS, $((p,w),\alpha^{p,w})$, let $\widehat{\alpha}_{p,w}^{N} \equiv \frac{\widehat{\alpha}^{p,w}}{\alpha_{0}^{p,w}}$. Moreover, for any $\nu \in N$, let $t_{p,w}^{\nu} > 0$ be such that $pt^{\nu}\widehat{\alpha}_{p,w}^{N} = \Pi^{\nu}(p,w)$. Note that the labor input corresponding to the net output $\widehat{\alpha}_{p,w}^{N}$ is exactly one unit. Then, the following is an extension of the so-called "New Interpretation" which was originally defined in Leontief models by Dumenil (1980) and Foley (1982):[9]

Definition 7: Let $((p,w),\alpha_{p,w})$ be an RS. A producer $\nu \in N$ is *exploited* if and only if:

$$t_{p,w}^{\nu} < 1,$$

and she is an *exploiter* if and only if:

$$t_{p,w}^{\nu} > 1.$$

We can see that Definition 7 satisfies **LE** by choosing $\overline{c} = \widehat{\alpha}_{p,w}^{N}$ and $\underline{c} = \widehat{\alpha}_{p,w}^{N}$.

Definition 7 is also concerned *not* with an agent's *consumption vector*, *but* rather with an agent's *income* earned. The difference of Definition 7 from Definition 6 is that the minimal expenditure of labor socially necessary to reach an agent ν's income $\Pi^{\nu}(p,w)$ is given by examining the ray passing through the *actually accessed* social production point $\alpha^{p,w}$ solely, rather than the minimizer over $\overline{P}(p,w)$. Under this definition, the following relationship holds:

$$\text{total labor employed} = \text{labor value of national income } (= \text{net product}).$$

This *macroeconomic identity* has been required as a basic property of labor value in Marxian economic theory.[10]

[8] This interpretation of $\min_{f^{\nu} \in B(p,\Pi^{\nu}(p,w))} l.v.(f^{\nu};p,w)$ is analogous to the notion of the minimal expenditure of wealth required to reach a given utility level in the expenditure minimization problem of the standard micro theory of consumer behavior.

[9] See also Lipietz (1982).

[10] The macroeconomic identity is also satisfied by the labor value formulation of Flaschel (1983), although his method to derive labor values is extremely different from that of Definition 7: in Flaschel (1983), *additive labor values* are derived from the *square* matrices of input and output coefficients, which are defined by the maximally profitable production processes at an RS. In contrast, the labor value formulation in Definition 7 is given by Definition 4. Based on Flaschel's (1983) labor value formulation, we can consider another formulation of labor exploitation which satisfies **LE** with $\overline{c} = \widehat{\alpha}_{p,w}^{N} = \underline{c}$.

We may also consider a more subjective notion of labor exploitation. Suppose that there is a representative agent of this economy, and introduce this agent's welfare function $U : \mathbb{R}^m_+ \to \mathbb{R}$. This U is continuous and strictly monotonic on $\mathbb{R}^m_+$, and it should have the following property: for any RS, $((p, w), \alpha^{p,w})$, $\widehat{\alpha}^N_{p,w}$ is the maximizer of $U(c)$ over $B\left(p, p\widehat{\alpha}^N_{p,w}\right)$. Given this welfare function U, let $c^{\max}_U \in \mathbb{R}^m_+$ be the maximizer of $U(c)$ over ζ. Then:

Definition 8: Let $((p, w), \alpha^{p,w})$ be an RS. A producer $\nu \in N$ is *exploited* if and only if:

$$\Pi^\nu(p, w) < pc^{\max}_U,$$

and she is an *exploiter* if and only if:

$$\Pi^\nu(p, w) > pc^{\max}_U.$$

We can see that Definition 8 satisfies **LE** by choosing $\overline{c} = c^{\max}_U$ and $\underline{c} = c^{\max}_U$. This definition is extended from Matsuo (2008), although Matsuo provides only the definition of exploited agents in order to discuss FMT.

4 CECP in Accumulation Economies

In the following discussion, we will examine the viability of the above five definitions of labor exploitation respectively by checking whether CECP [Roemer (1982; Chapter 5)] holds true under each of these definitions, and show that only Definitions 6 and 7 verify CECP in general convex cone economies.

Following Roemer (1982; Chapter 5), let us define possible *classes*. At every RS in the model of section 2, different producers relate differently to the means of production. An individually optimal solution for an agent ν at the RS consists of three vectors $(\alpha^\nu; \beta^\nu; \gamma^\nu_0)$. Therefore, let (a_1, a_2, a_3) be a vector where $a_i \in \{+, \mathbf{0}\}$, $i = 1, 2, 3$, where "+" means a non-zero vector in the appropriate place. Agent ν is said to be a member of class (a_1, a_2, a_3), if there is an individually optimal $(\alpha^\nu; \beta^\nu; \gamma^\nu_0)$ which has the form (a_1, a_2, a_3). The notation $(+, +, 0)$ implies, for instance, that an agent works in her own 'shop' and hires others to work for her; $(+, 0, +)$ implies that an agent works both in her own 'shop' and for others, etc. Although there are seven possible classes in the accumulation economy, it can be proved that at an RS, the set of producers N can be partitioned into the following four, theoretically relevant classes.

$$\begin{aligned}
C^H &= \{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,+,0) \setminus (+,0,0)\}, \\
C^{PB} &= \{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,0,0)\}, \\
C^S &= \{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,0,+) \setminus (+,0,0)\}, \\
C^P &= \{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (0,0,+)\}.
\end{aligned}$$

The notation $(+,+,0) \setminus (+,0,0)$ means that agent ν is a member of class $(+,+,0)$ but not of class $(+,0,0)$, and likewise for the other classes. As a first step in the analysis of classes, we can see that at any RS, no agent is a member of class $(0,+,0)$. This is because any agent with the economic activity of the form $(0,+,0)$ can increase her revenue by selling her labor, which implies that the activity vector $(0,+,0)$ cannot constitute an optimal solution, thus a contradiction. As a second step, Lemma 1 proves that $(+,+,+)$ and $(0,+,+)$ are indeed redundant.

Lemma 1: *Let (p,w) be a given price vector. Let ν be such that $W^\nu > 0$. Let ν belong to either $(+,+,+)$ or $(0,+,+)$. Then, exactly one of the following statements holds*:

$$\begin{aligned}
\text{if } \gamma_0^\nu &< \beta_0^\nu \text{ for all optimal } (\alpha^\nu;\beta^\nu;\gamma_0^\nu), \text{ then } \nu \in (+,+,0)\setminus(+,\mathbf{0},0); \\
\text{if } \gamma_0^\nu &= \beta_0^\nu \text{ for some optimal } (\alpha^\nu;\beta^\nu;\gamma_0^\nu), \text{ then } \nu \in (+,\mathbf{0},0); \\
\text{if } \gamma_0^\nu &> \beta_0^\nu \text{ for all optimal } (\alpha^\nu;\beta^\nu;\gamma_0^\nu), \text{ then } \nu \in (+,\mathbf{0},+)\setminus(+,\mathbf{0},0).
\end{aligned}$$

Thus, by Lemma 1 and the above first step analysis of classes, the above mentioned four classes, C^H, C^{PB}, C^S, and C^P, are actually relevant. Then, the following proposition shows that the four classes are pairwise disjoint and exhaustive,[11] and wealthier agents belong to the upper classes:

Proposition 2 [Roemer (1982; Chapter 5)]**:** *Let (p,w) be an* RS *with $\pi(p,w) > 0$. Then,*

$$\nu \in C^H \Leftrightarrow W^\nu > \max_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right],$$

[11] The partition of N into C^H, C^{PB}, C^S, and C^P is independent of whether the corresponding RS is with full employment or not. In fact, even if the economy is in equilibrium with unemployment, and an agent does not supply one unit of labor at all, we can still develop a hypothetical argument that indicates what class he would rationally choose to belong to if he were to supply one unit of labor.

$$\nu \in C^{PB} \Leftrightarrow \min_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0} \right] \leqq W^{\nu} \leqq \max_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0} \right],$$
$$\nu \in C^{S} \Leftrightarrow 0 < W^{\nu} < \min_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0} \right],$$
$$\nu \in C^{P} \Leftrightarrow W^{\nu} = 0.$$

Now, **CECP**, which is a principle we would like to verify.

Class-Exploitation Correspondence Principle (CECP)[Roemer (1982)]**:** *Given an economy defined as in section 2, for any reproducible solution, it holds that*:

(A) every member of C^H is an exploiter.

(B) every member of $C^S \cup C^P$ is exploited.

First, we discuss that under any definition of labor exploitation which satisfies **LE**, **CECP** holds true if the production possibility set is given by *Leontief technology.* Let A be an $m \times m$ non-negative, indecomposable square matrix with input-output coefficients $a_{ij} \geqq 0$ for any $i, j = 1, \ldots, m$, and L be a positive $1 \times m$ vector with labor input coefficients $L_j > 0$ for any $j = 1, \ldots, m$. Then, let $P_{(A,I,L)} \equiv \left\{ \alpha \in \mathbb{R}_{-} \times \mathbb{R}^m_{-} \times \mathbb{R}^m_{+} \mid \exists x \in \mathbb{R}^m_{+} : \alpha \leqq (-Lx, -Ax, x) \right\}$. Then:

Theorem 1: *Under A1, A2, let (p, w) be an* RS *with $\pi(p, w) > 0$ for an economy $\langle N; \left(P_{(A,I,L)}, b\right); (\omega^{\nu})_{\nu \in N} \rangle$. Then, under any definition of labor exploitation satisfying **LE**, **CECP** holds true if and only if $\underline{c} \in \zeta$.*

The complete proof of this theorem will be given after Theorem 2 is discussed.

Note that **CECP** holds under any of the five definitions of labor exploitation in economies with Leontief technology, since any of them satisfies **LE** with $\underline{c} \in \zeta$ in those economies. Note also that in economies with Leontief technology, Definitions 3 and 5 are equivalent.

Insert Figure 1 around here.

Figure 1 illustrates that **CECP** holds under Definitions 3 and 5 in a two-goods economy with Leontief technology.

Second, we characterize, in general convex cone economies, what types of definitions of labor exploitation satisfying **LE** can preserve **CECP** as a theorem. Let $\overline{\Gamma}(p, w) \equiv \left\{ \alpha \in \overline{P}(p, w) \mid \alpha_0 = 1 \right\}$ and $\widehat{\overline{\Gamma}}(p, w) \equiv \left\{ \widehat{\alpha} \in \mathbb{R}^m_{+} \mid \alpha \in \overline{\Gamma}(p, w) \right\}$.

For any set $S \subseteq \mathbb{R}_{+}^{m}$, let $co\{S\}$ denote the convex hull of S, and $comp\{S\}$ denote the *comprehensive hull* of S. Given any economy $\langle N; (P,b); (\omega^{\nu})_{\nu\in N}\rangle$, and any RS, $((p,w), \alpha^{p,w})$, note that $\pi(p,w) = \frac{p\widehat{\alpha}^{p,w} - w\alpha_0^{p,w}}{p\underline{\alpha}^{p,w}}$ follows from the definition of RS. There exists $\alpha^{p,w*} \in \overline{\Gamma}(p,w)$ such that for some $t > 0$, $t\alpha^{p,w*} = \alpha^{p,w}$. Moreover, there exists $c^{p,w} \in \zeta$ such that $pc^{p,w} \geqq pc$ for any $c \in \zeta$. Since $\widehat{\alpha}^{p,w*} \in \zeta$ by Definition 1(d), we have $pc^{p,w} \geqq p\widehat{\alpha}^{p,w*}$. Then:

Lemma 2: *Under A1∼A3, there exists an economy* $\langle N; (P,b); (\omega^{\nu})_{\nu\in N}\rangle$ *which has an* RS, $((p,1), \alpha^{p,1})$, *such that* $pc^{p,1} > p\widehat{\alpha}$ *for any* $\alpha \in \overline{\Gamma}(p,1)$.

Proof. Let us consider the following von Neumann system:

$$B = \begin{bmatrix} 5 & 3 & 9.8 & 0 \\ 5.25 & 4.5 & 0 & 5.25 \end{bmatrix}, A = \begin{bmatrix} 3.5 & 2 & 8 & 0 \\ 4.5 & 3 & 0 & 3.5 \end{bmatrix}, L = \begin{pmatrix} 0.75 & 1 & 0.6 & 1 \end{pmatrix}.$$

Define a production possibility set $P_{(A,B,L)}$ by

$$P_{(A,B,L)} \equiv \left\{\alpha \in \mathbb{R}_{-} \times \mathbb{R}_{-}^{2} \times \mathbb{R}_{+}^{2} \mid \exists x \in \mathbb{R}_{+}^{4} : \alpha \leqq (-Lx, -Ax, Bx)\right\}.$$

This $P_{(A,B,L)}$ is a closed convex cone in $\mathbb{R}_{-} \times \mathbb{R}_{-}^{m} \times \mathbb{R}_{+}^{m}$ with $\mathbf{0} \in P_{(A,B,L)}$. Moreover, $P_{(A,B,L)}$ is shown to satisfy A1 and A2.

Let $\mathbf{e}_j \in \mathbb{R}_{+}^{m}$ be a unit column vector with 1 in the j-th component and 0 in any other component. Then, $\alpha^1 \equiv (-L\mathbf{e}_1, -A\mathbf{e}_1, B\mathbf{e}_1)$, $\alpha^2 \equiv (-L\mathbf{e}_2, -A\mathbf{e}_2, B\mathbf{e}_2)$, $\alpha^3 \equiv (-L\mathbf{e}_3, -A\mathbf{e}_3, B\mathbf{e}_3)$, and $\alpha^4 \equiv (-L\mathbf{e}_4, -A\mathbf{e}_4, B\mathbf{e}_4)$. Moreover,

$$\begin{aligned} \widehat{\alpha}^1 &\equiv (B-A)\mathbf{e}_1 = \begin{pmatrix} 1.5 \\ 0.75 \end{pmatrix}, \widehat{\alpha}^2 \equiv (B-A)\mathbf{e}_2 = \begin{pmatrix} 1 \\ 1.5 \end{pmatrix}, \\ \widehat{\alpha}^3 &\equiv (B-A)\mathbf{e}_3 = \begin{pmatrix} 1.8 \\ 0 \end{pmatrix}, \widehat{\alpha}^4 \equiv (B-A)\mathbf{e}_4 = \begin{pmatrix} 0 \\ 1.75 \end{pmatrix}. \end{aligned}$$

Also, we have $\widehat{P}(\alpha_0 = 1) = co\{(2,1), (1,1.5), (3,0), (0,1.75), \mathbf{0}\}$.

Let $b = (1,1)$, and the social endowment of capital be given by $\omega = (2|N|, 3|N|)$. Then, for any economy $\langle N; (P_{(A,B,L)}, b); (\omega^{\nu})_{\nu\in N}\rangle$ with $\sum_{\nu\in N}\omega^{\nu} = \omega$, a pair $((p,1), |N|\alpha^2)$ with $p = (0.5, 0.5)$ constitutes an RS. Note that

$$\begin{aligned} \frac{[p(B-A)-L]\mathbf{e}_1}{pA\mathbf{e}_1} &= \frac{3}{32}; \frac{[p(B-A)-L]\mathbf{e}_2}{pA\mathbf{e}_2} = \frac{1}{10}; \\ \frac{[p(B-A)-L]\mathbf{e}_3}{pA\mathbf{e}_3} &= \frac{3}{40}; \frac{[p(B-A)-L]\mathbf{e}_4}{pA\mathbf{e}_4} = \frac{-1}{14}. \end{aligned}$$

This implies that $\overline{\Gamma}(p,1) = \{\alpha^2\}$ and $\theta_{(p,1)} = \partial comp\left\{\widehat{\alpha}^2\right\}$. Thus,

$$\min_{\alpha \in \overline{P}(p,1)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right] = \min_{\alpha \in \overline{\Gamma}(p,1)} p\underline{\alpha} = \max_{\alpha \in \overline{\Gamma}(p,1)} p\underline{\alpha} = \max_{\alpha \in \overline{P}(p,1)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right] = p\underline{\alpha}^2.$$

Let $H\left(p,\widehat{\alpha}^2\right) \equiv \left\{c \in \mathbb{R}^2_+ \mid pc = p\widehat{\alpha}^2\right\}$, $H_+\left(p,\widehat{\alpha}^2\right) \equiv \left\{c \in \mathbb{R}^2_+ \mid pc > p\widehat{\alpha}^2\right\}$, and $H_-\left(p,\widehat{\alpha}^2\right) \equiv \left\{c \in \mathbb{R}^2_+ \mid pc < p\widehat{\alpha}^2\right\}$. Moreover, $\zeta_+ \equiv \zeta \cap H_+\left(p,\widehat{\alpha}^2\right)$, $\zeta_- \equiv \zeta \cap H_-\left(p,\widehat{\alpha}^2\right)$. Note that $\zeta_+ = [co\{(1,1.5),(2,1)\} \cup co\{(2,1),(3,0)\}] \setminus \{(1,1.5)\}$, $\zeta_- = co\{(0,1.75),(1,1.5)\} \setminus \{(1,1.5)\}$, and $\widehat{\alpha}^2 = (1,1.5)$. Since $c^{p,1} \in \zeta$ implies $c^{p,1} = (2,1)$, we have $pc^{p,1} > p\widehat{\alpha}^2$. Thus, we obtain a desired result. ■

Insert Figure 2 around here.

Then, the following theorem gives us a necessary and sufficient condition for formulations of labor exploitation satisfying **LE** in order to preserve **CECP** as a theorem:

Theorem 2: *Under A1∼A3, let* $\langle N;(P,b);(\omega^\nu)_{\nu\in N}\rangle$ *be an economy with an* RS, $((p,w),\alpha^{p,w})$, *with* $\pi(p,w) > 0$. *Then, for any definition of labor exploitation satisfying* ***LE***, ***CECP*** *holds true under this definition if and only if its corresponding* $\overline{c} \in \zeta$ *and* $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ *imply* $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p,w)$.

Proof. Let $\langle N;(P,b);(\omega^\nu)_{\nu\in N}\rangle$ be an economy with an RS, $((p,w),\alpha^{p,w})$, such that $pc^{p,w} > p\widehat{\alpha}$ for any $\alpha \in \overline{\Gamma}(p,w)$. Let

$$\alpha^{\max(p,w)} \equiv \arg\max_{\alpha\in\overline{\Gamma}(p,w)} p\underline{\alpha}; \text{ and } \alpha^{\min(p,w)} \equiv \arg\min_{\alpha\in\overline{\Gamma}(p,w)} p\underline{\alpha}.$$

Then, by Proposition 2, we have

$$\begin{aligned}
C^H &= \left\{\nu \in N \mid \Pi^\nu(p,w) > \pi(p,w)\, p\underline{\alpha}^{\max(p,w)} + w\right\}; \\
C^{PB} &= \left\{\nu \in N \mid \pi(p,w)\, p\underline{\alpha}^{\min(p,w)} + w \leqq \Pi^\nu(p,w) \leqq \pi(p,w)\, p\underline{\alpha}^{\max(p,w)} + w\right\}; \\
C^S &= \left\{\nu \in N \mid 1 < \Pi^\nu(p,w) < \pi(p,w)\, p\underline{\alpha}^{\min(p,w)} + w\right\}; \\
C^P &= \left\{\nu \in N \mid \Pi^\nu(p,w) = w\right\}.
\end{aligned}$$

Insert Figure 3 around here.

Let $H(p,\widehat{\alpha}) \equiv \left\{c \in \mathbb{R}^m_+ \mid pc = p\widehat{\alpha}\right\}$, $H_+(p,\widehat{\alpha}) \equiv \left\{c \in \mathbb{R}^m_+ \mid pc > p\widehat{\alpha}\right\}$, and $H_-(p,\widehat{\alpha}) \equiv \left\{c \in \mathbb{R}^m_+ \mid pc < p\widehat{\alpha}\right\}$. Moreover, $\zeta_+ \equiv \zeta \cap H_+\left(p, \widehat{\alpha}^{\max(p,w)}\right)$, $\zeta_- \equiv \zeta \cap H_-\left(p, \widehat{\alpha}^{\min(p,w)}\right)$. Then, $\zeta = \zeta_+ \cup \widehat{\widehat{\Gamma}}(p,w) \cup \zeta_-$.

1. Proof of the necessity.

Case 1): Consider any definition of labor exploitation satisfying **LE**, and for this definition, let its corresponding $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ have the property that $\overline{c} \in \zeta_+$. Thus, $p\overline{c} > p\widehat{\alpha}^{\max(p,w)}$. Since $p\widehat{\alpha}^{\max(p,w)} = \pi(p,w)\, p\underline{\alpha}^{\max(p,w)} + w$, we can construct an economy $\langle N; (P,b); (\widetilde{\omega}^\nu)_{\nu \in N}\rangle$ with $\sum_{\nu \in N} \widetilde{\omega}^\nu = \omega$, such that for some $\nu \in N$, $p\overline{c} > \pi(p,w)\, p\widetilde{\omega}^\nu + w > p\widehat{\alpha}^{\max(p,w)}$ holds. This agent ν belongs to C^H, as per Proposition 2. However, $p\overline{c} > \pi(p,w)\, p\widetilde{\omega}^\nu + w = \Pi^\nu(p,w)$ implies that ν is not an exploiter.

Case 2): Consider any definition of labor exploitation satisfying **LE**, and for this definition, let its corresponding $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ have the property that $\overline{c} \in \zeta_-$. Then, since $p\overline{c} \geqq p\underline{c}$ by **LE**, $p\underline{c} < p\widehat{\alpha}^{\min(p,w)}$ holds. Then, we can construct an economy $\langle N; (P,b); (\widetilde{\omega}^\nu)_{\nu \in N}\rangle$ with $\sum_{\nu \in N} \widetilde{\omega}^\nu = \omega$, such that for some $\nu \in N$, $p\underline{c} < \pi(p,w)\, p\widetilde{\omega}^\nu + w < p\widehat{\alpha}^{\min(p,w)}$ holds. Note that this agent ν belongs to C^S, as per Proposition 2. However, $p\underline{c} < \pi(p,w)\, p\widetilde{\omega}^\nu + w = \Pi^\nu(p,w)$ implies that ν is not exploited.

Case 3): Finally, consider any definition of labor exploitation satisfying **LE**, and for this definition, let its corresponding $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ have the property that $\overline{c} \in \widehat{\widehat{\Gamma}}(p,w)$ and $p\overline{c} > p\underline{c}$. If $p\underline{c} < p\widehat{\alpha}^{\min(p,w)}$, then the argument of **Case 2)** can be applied.

In summary, the arguments of the above three cases imply that if a definition of labor exploitation satisfying **LE** preserves **CECP** as a theorem, then its corresponding $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ imply $\overline{c}, \underline{c} \in \widehat{\widehat{\Gamma}}(p,w)$.

2. Proof of the sufficiency.

Since the definition of labor exploitation satisfies **LE**, there are $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ such that $p\overline{c} \geqq p\underline{c}$ under the RS, $((p,w), \alpha^{p,w})$. Note that if $\overline{c}, \underline{c} \in \widehat{\widehat{\Gamma}}(p,w)$ under this definition of labor exploitation, then

$$\pi(p,w)\, p\underline{\alpha}^{\min(p,w)} + w \leqq p\underline{c} \leqq p\overline{c} \leqq \pi(p,w)\, p\underline{\alpha}^{\max(p,w)} + w.$$

By **LE**, any agent $\nu \in N$ with W^ν such that $\Pi^\nu(p,w) < p\underline{c}$ is exploited, whereas any agent $\nu \in N$ with W^ν such that $\Pi^\nu(p,w) > p\overline{c}$ is an exploiter. Thus, any $\nu \in C^H$ becomes an exploiter, whereas any $\nu \in C^S \cup C^P$ is exploited in this economy. Thus, **CECP** holds under this definition of labor exploitation. ■

Proof of Theorem 1: In economies with Leontief technology, $\widehat{\overline{\Gamma}}(p,w) = \zeta$ holds. If a definition of labor exploitation satisfies **LE** with $\underline{c} \in \zeta$, then there exists $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p,w)$ such that $p\overline{c} \geqq p\underline{c}$ under the RS, $((p,w), \alpha^{p,w})$. Thus, by Theorem 2, **CECP** holds under this definition. ■

Theorem 1 implies that any formulation of labor exploitation satisfying **LE** should have $\overline{c}, \underline{c} \in \zeta$ in order to verify **CECP** in models with Leontief technology, in which $\zeta = \widehat{\overline{\Gamma}}(p,w)$ holds. Since $\overline{c}, \underline{c} \in \zeta$ is independent of the information about market equilibria, this characterization looks to justify a *price-independent* formulation of labor exploitation in capitalist economies with Leontief technology. In contrast, according to Theorem 2, price-independent formulations can no longer be valid in models with general convex cone technology, and any plausible formulation of labor exploitation must be *price-dependent* in order to verify **CECP**. This is because in general convex cone economies, $\widehat{\overline{\Gamma}}(p,w)$ is generally a (proper) *subset of* ζ, and what subset of ζ constitutes $\widehat{\overline{\Gamma}}(p,w)$ is varied according to the information of equilibrium price (p,w). Note that in any Leontief economy, if its input-output matrix A is indecomposable, then its associating Frobenius eigenvector is positive, which implies that all production processes are operated as profit-maximizers at the RS (p,w), as shown in Roemer (1981; Chapter 1). Thus, $\widehat{\overline{\Gamma}}(p,w)$ coincides with ζ in the Leontief economy.

By the above Theorem 2, we can show that both the Morishima (1974) and the Roemer (1982) formulations for labor exploitation cannot preserve **CECP** as a theorem:

Corollary 1: *Under A1∼A3, there is an economy in which* ***CECP*** *is violated under Definition 3.*

Proof. Let $\langle N; (P,b); (\omega^{\nu})_{\nu \in N} \rangle$ be the economy constructed in Lemma 2. Then, this economy has an RS, $((p,1), \alpha^{p,1})$, such that $pc^{p,1} > p\widehat{\alpha}$ for any $\alpha \in \overline{\Gamma}(p,1)$. In this economy, if the Morishima (1974) formulation of labor exploitation (Definition 3) is applied, then $\underline{c} = \widehat{\alpha}^4$ and

$$\overline{c} \in \left\{ c \in \mathbb{R}^2_{++} \mid \exists t \in [0,1] : c = t\,(2,1) + (1-t)\,(3,0) \right\}.$$

Insert Figure 4 around here.

Note $\widehat{\widehat{\Gamma}}(p,1) = \{\widehat{\alpha}^2\}$. Then, by Theorem 2, **CECP** is violated under Definition 3. ■

Corollary 2: *Under A1∼A3, there is an economy in which **CECP** is violated under Definition 5.*

Proof. Let $\langle N; (P,b); (\omega^\nu)_{\nu \in N}\rangle$ be the economy constructed in Lemma 2 as in the proof of Corollary 1. In this economy, if the Roemer (1982) formulation of labor exploitation (Definition 5) is applied, then $\underline{c} = (1,0)$ and $\overline{c} = \widehat{\alpha}^2$.

Insert Figure 5 around here.

Then, since $\widehat{\widehat{\Gamma}}(p,1) = \{\widehat{\alpha}^2\}$, by Theorem 2, **CECP** is violated under Definition 5. ■

We can also show that even Definition 8 cannot preserve **CECP** as a theorem.

Corollary 3: *Under A1∼A3, there is an economy in which **CECP** is violated under Definition 8.*

Proof. Let $\langle N; (P,b); (\omega^\nu)_{\nu \in N}\rangle$ be the economy constructed in Lemma 2 as in the proof of Corollary 1. In this economy, if Definition 8 is applied as a formulation of labor exploitation, then the welfare function U of the representative agent has the following properties: $\widehat{\alpha}^N_{p,1} = \widehat{\alpha}^2$ and $c^{\max}_U = (2,1)$. Thus, $\underline{c} = (2,1)$ and $\overline{c} = (2,1)$.

Insert Figure 6 around here.

Then, since $\widehat{\widehat{\Gamma}}(p,1) = \{\widehat{\alpha}^2\}$, by Theorem 2, **CECP** is violated under Definition 8. ■

Next, we show that in general convex cone economies, **CECP** holds true under Definitions 6 and 7:

Corollary 4: *Under A1∼A3, for any economy with an* RS, $((p,w), \alpha^{p,w})$, *with* $\pi(p,w) > 0$, ***CECP** holds true under Definition 6.*

Proof. Given an RS, (p, w), let $\widetilde{c} \in \theta_{(p,w)}$ be such that $p\widetilde{c} \geqq pc$ for all $c \in \theta_{(p,w)}$. Note that in Definition 6, $\overline{c} = \widetilde{c}$ and $\underline{c} = \widetilde{c}$. Since $\widetilde{c} \in \widehat{\overline{\Gamma}}(p, w)$ by definition, the desired result follows from Theorem 2. ■

Insert Figure 7 around here.

Corollary 5: *Under A1∼A3, for any economy with an* RS, $((p, w), \alpha^{p,w})$, *with* $\pi(p, w) > 0$, ***CECP*** *holds true under Definition 7.*

Proof. Note that in Definition 7, $\overline{c} = \widehat{\alpha}^N_{p,w}$ and $\underline{c} = \widehat{\alpha}^N_{p,w}$. Since $\widehat{\alpha}^N_{p,w} \in \widehat{\overline{\Gamma}}(p, w)$ by definition, the desired result follows from Theorem 2. ■

Insert Figure 8 around here.

There may potentially be another formulation of labor exploitation which satisfies **LE** and the condition $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p, w)$. For instance, we can consider a formulation of labor exploitation based on the labor value formulation of Flaschel (1983), as discussed in footnote 10. In this case, since the labor value of $\widehat{\alpha}^N_{p,w}$ is *unity* under Flaschel's (1983) formulation, the corresponding labor exploitation can be formulated to satisfy **LE** with $\overline{c} = \widehat{\alpha}^N_{p,w} = \underline{c}$. Then, **CECP** holds under this formulation. However, at least to the best of my knowledge in current literature, excepting this formulation, there are no other explicit formulations than Definitions 6 and 7, which satisfy **LE** with $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p, w)$. In this sense, each of Definitions 6 and 7 could represent one of the most plausible formulations for labor exploitation.

5 FMT in general convex cone economies

In the last section, we showed that if the maximal profit rate is positive in the RS, then **CECP** holds under Definitions 6 and 7 of labor exploitation. Here, we will discuss the necessity of the positive profit rate for **CECP**: if the maximal profit rate is non-positive in the RS, then **CECP** no longer holds under Definitions 6 and 7. This statement can be verified by examining the so-called Fundamental Marxian Theorem (**FMT**) in convex-cone economies with Definitions 6 and 7. **FMT** states that every agent in the working class C^P is exploited if and only if the uniform profit rate prevailed in the economy is positive. Thus, the necessary part of **FMT** is sufficient in showing the necessity of the positive profit rate for **CECP**: the necessary part of **FMT** is to show that the non-positive maximal profit rate implies that an agent in the working class is not exploited, which implies a violation of **CECP**.

By the above mentioned reason, we show that under Definitions 6 and 7, **FMT** holds true, as in the following:

Theorem 3: *Consider any definition of exploitation which satisfies* **LE**. *For any economy* $\langle N; (P,b); (\omega^{\nu})_{\nu \in N} \rangle$, *let* $((p,w), \alpha^{p,w})$ *be an* RS, *and let* $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p,w)$ *for this definition of exploitation. Then, the following three statements are equivalent:*
(1) $\pi(p,w) > 0$;
(2) ***CECP*** *holds; and*
(3) *every worker in* C^P *is exploited.*

Proof. First, (1)⇒(2) follows from Theorem 2. Second, it is clear that (2)⇒(3).

Finally, on (3)⇒(1). Since there is no RS with a negative total profit, it suffices to discuss only a zero profit case. Let $((p,w), \alpha^{p,w})$ be an RS with a zero total profit. Thus, $p\widehat{\alpha}^N_{p,w} - w = 0$. Note that for any $\widehat{\alpha} \in \widehat{\overline{\Gamma}}(p,w)$, $p\widehat{\alpha} = p\widehat{\alpha}^N_{p,w}$ holds, since $\pi(p,w) = 0$ and $\widehat{\alpha}^N_{p,w} \in \widehat{\overline{\Gamma}}(p,w)$. Thus, for any $\widehat{\alpha} \in \widehat{\overline{\Gamma}}(p,w)$, $p\widehat{\alpha} = w$. Since $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p,w)$, $p\overline{c} = p\underline{c} = w$. Since $\pi(p,w) = 0$, $\Pi^{\nu}(p,w) = w$ holds for any $\nu \in N$. Thus, for any $\nu \in N$, $p\overline{c} = \Pi^{\nu}(p,w) = p\underline{c}$ holds, which implies from **LE** that every agent is neither an exploiter nor exploited. This implies that if $\pi(p,w) = 0$, then every agent ν with $\omega^{\nu} = 0$ is non-exploited. ■

This theorem has the following three implications: for the whole class of definitions of exploitation satisfying axiom **LE** with $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p,w)$, firstly, the equivalence between **FMT** and **CECP** holds; secondly, **CECP** holds if and only if the maximal profit rate at the RS is positive; and finally, the well-known difficulty of **FMT** in convex cone economies is resolved. Note that, by Corollaries 4 and 5, all of these three implications are applied to both Definitions 6 and 7.

The third implication should be commented on. There exists a convex cone economy with an RS such that the *maximal profit rate* is positive whereas the rate of the Morishima (1974) labor exploitation (Definition 3 in this paper) is zero, which is the difficulty of **FMT** pointed out by Petri (1980) and Roemer (1980).[12] The same difficulty appears when the Roemer (1982; Chapter 5) exploitation (Definition 5 in this paper) is used.[13] In contrast, Definitions 6 and 7 resolve such a difficulty, as Theorem 3 shows.[14]

6 Concluding Remarks

We have characterized the condition for the plausible formulation of labor exploitation to verify CECP, as well as proposed two new definitions of labor exploitation, each of which performed well in terms of both FMT and CECP. However, the new definitions have exclusively distinct characteristics in comparison with the previous definitions such as those proposed by Morishima (1974) and Roemer (1982; Chapter 5), which may give us new insights on theories of labor exploitation.

First, Roemer (1982) claimed that prices should emerge logically prior to labor values so as to preserve CECP as a theorem in general convex cone economies. Though he failed in proving this claim with his own price-dependent labor value formulation (Definition 4 in this paper), Theorem 2 in this paper proves that his claim itself is true. In fact, in order to verify CECP as a theorem, any formulation of labor exploitation satisfying **LE** must be price-dependent, as we discussed in section 3. This implies that the classic transformation problem in Marxian economic theory is no longer worth investigating, since any price-independent labor value formulation causes the failure of CECP. In other words, according to Theorems 1 and 2, the scope of

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[12]Note that Morishima (1974) showed that if the economy is under the von Neumann balanced growth equilibrium, then the *warranted profit rate* is positive if and only if the Morishima (1974) labor exploitation is positive, where the *warranted profit rate* is the minimal value of uniform profit rates. See Morishima (1974) for a more detailed discussion.

[13]This is confirmed by examining the economy constructed in the proof of Lemma 1. (See Figure 2.) In that economy, we can see that $l.v.\,(b; p, 1) = 1$, which implies every worker is not exploited in the sense of Roemer (1982; Chapter 5), though the maximal profit rate is positive under the RS of that economy.

[14]Note that, using the Morishima (1974) labor exploitation, Roemer (1980; 1981) showed that **FMT** holds at an RS of a convex cone economy if and only if this economy satisfies the following assumption:

Independence of Production: $\forall\,(-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P$, $\forall \widehat{\alpha} \geqq 0$, and $\forall c \in \mathbb{R}^m_+$ with $c \leq \widehat{\alpha}$, $\exists\,(-\alpha'_0, -\underline{\alpha}', \overline{\alpha}') \in P$ s.t. $\overline{\alpha}' - \underline{\alpha}' \geqq c$ and $\alpha'_0 < \alpha_0$,

which implies that every production set has no inferior production process.

the classical Marxian perspective on labor exploitation (that the exploitative relationship between capital and labor was considered to be *logically prior to* the prices that constitute an equilibrium in the capitalist economy[15]) must be limited to models with Leontief technology.

Second, in the classical Marxian argument, labor exploitation was explained by using the concept of the labor value of labor power, which was defined in the Morishima (1974) framework as the *technologically* minimal amount of direct labor necessary to produce the subsistence consumption vector as a net output. In such an argument, the subsistence consumption vector plays a crucial role. In Definition 6 of this paper, however, the labor value of labor power might be defined as the minimal amount of direct labor socially necessary to achieve workers' *income* by which they can respectively purchase at least the subsistence consumption vector. Also, in Definition 7, the labor value of labor power might be defined as the minimal amount of direct labor socially necessary to achieve workers' *income*, which is evaluated via the *actually used* social production point. In both of these formulations, the subsistence consumption vector is used, *at most indirectly*, to define the labor value of labor power. Thus, the labor value of labor power also no longer emerges logically prior to the price of labor power (wage income). Hence, the concepts of labor value in these new definitions are irrelevant to theories of *exchange values of commodities and labor power*.

In spite of such a significant difference of these new definitions from the classical Marxian notion of labor exploitation, they would be justified, according to the scenario Roemer (1982) offered, since both FMT and CECP hold true for these new definitions. Note that we still need further argument as to which of Definitions 6 and 7 is more appropriate. This problem is relevant to the issue of characterizing the unique appropriate formulation of exploitation by applying the axiomatic method. Recently, Yoshihara and Veneziani (2008) proposed, in *subsistence convex cone economies*, four axioms as the necessary conditions that any appropriate formulation of exploitation should satisfy, and then showed that the Definition 7 formulation is the unique one satisfying all of these axioms. Since this result is obtained from subsistence economies, it could not be directly applied to the accumulation economies studied in this paper. Indeed, it is an ongoing problem in

[15]In the classical Marxian perspective on the capitalist economy, the phenomenon of market movements was regarded as one reflection of the so-called *class struggle* between capital and labor, and the rate of labor exploitation was considered to measure the strength of the class struggle.

the model of accumulation economies to identify the unique formulation for exploitation by utilizing the axioms. We leave this issue for future occasions.

Note that there have been recently some papers published, such as Skillman (1995) and Veneziani (2007), which address the issue of whether the class and exploitation structure is (logically) *persistent* or not in the long run. They argue that if the possibility of savings is introduced in intertemporal models, then positive profits and exploitation tend to disappear over time. Though we did not address this issue in this paper, because our objective was to discuss proper definitions of exploitation for explaining the emergence of class and exploitation in a simple temporary equilibrium model, it is worth commenting on. To be precise, Veneziani (2007) showed, assuming an intertemporal Leontief model of subsistence economies, that any price vector of the *stationary equilibrium* converges to the vector of labor value, which drives the rate of exploitation to zero over time. This result indicates that if all agents are *indifferent* between the current consumption and the future consumption in the sense that the *discount factor* is equal to unity, then the class and exploitation structure disappears in the long run, due to the profit rate converging to zero. In other words, it may be possible that if the discount factor of all agents is *less than* unity, then the positive profit rate of the stationary equilibrium as well as the class and exploitation structure persist. This is indeed true even in more general intertemporal models with convex cone production sets, as Veneziani and Yoshihara (2009) show. In contrast, although these results focus on the stationary equilibrium where any agent no longer has an incentive to save, it is easily conjectured that in the out-of-stationary-equilibrium state, the positive profit rate and the class and exploitation structure are not persistent. This is due to, as Skillman (1995) and Veneziani (2007) pointed out, the introduction of savings without population growth easily diminishes the scarcity of capital relative to labor in the process of capital accumulation, which drives profits and the rate of exploitation to zero over time. To take this issue seriously, we should introduce, in addition to savings, the factor of population growth explicitly in an intertemporal model. This kind of perspective is also shared with the classical Marxian argument for the *progressive production of a relative surplus population*, which is beyond the scope of this paper.

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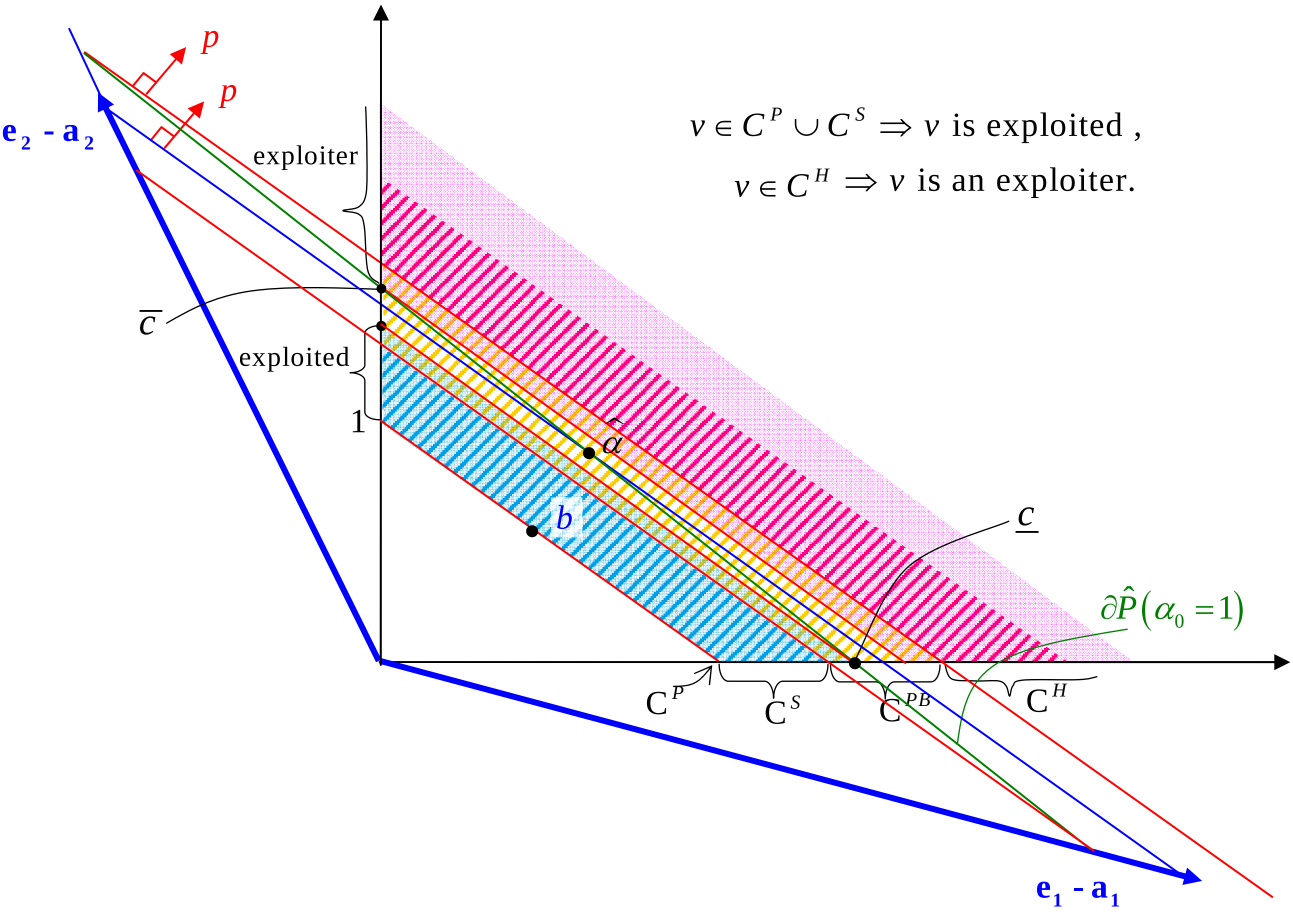
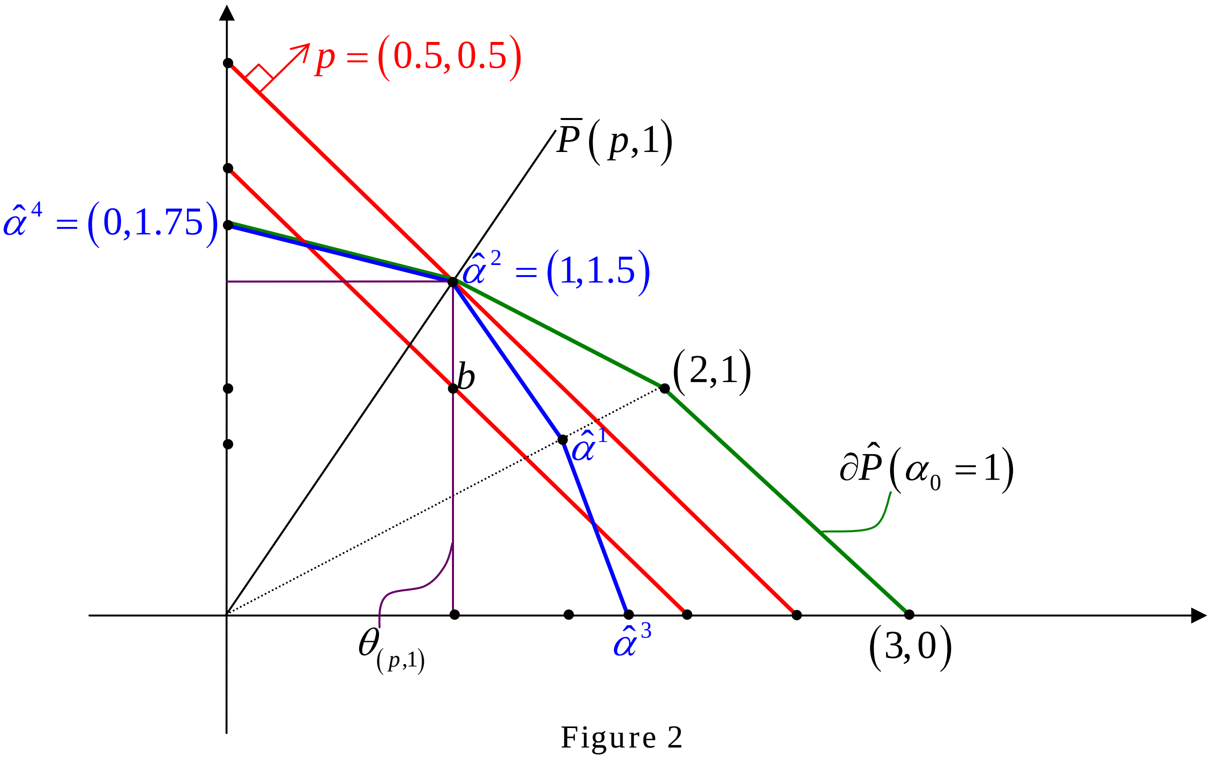


Figure1: Class-Exploitation Correspondence Principle holds under Definitions 3 and 5 in economies with Leontief technology



$p=(0.5,0.5)$
$\bar{P}(p,1)$
$\hat{\alpha}^4=(0,1.75)$
$\hat{\alpha}^2=(1,1.5)$
$(2,1)$
b
$\hat{\alpha}^1$
$\partial\hat{P}(\alpha_0=1)$
$\theta_{(p,1)}$
$\hat{\alpha}^3$
$(3,0)$

Figure 2

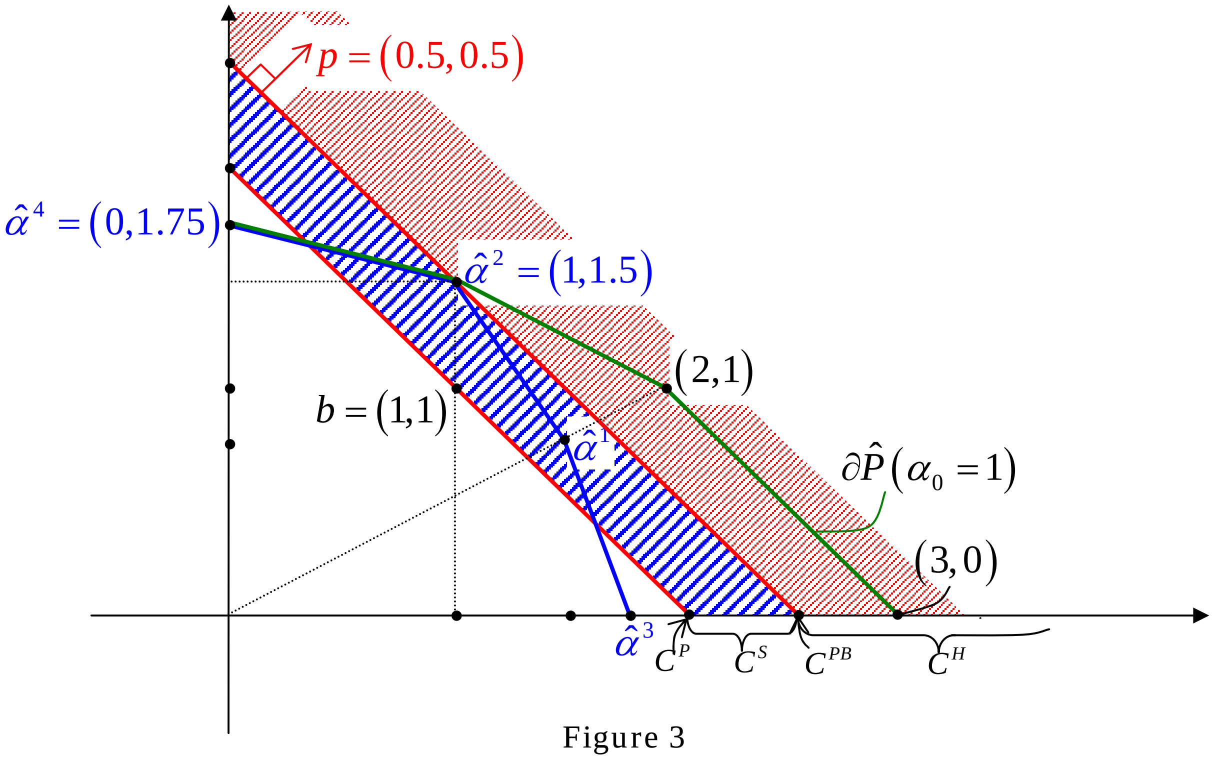

$p=(0.5,0.5)$
$\hat{\alpha}^4=(0,1.75)$
$\hat{\alpha}^2=(1,1.5)$
$(2,1)$
$b=(1,1)$
$\hat{\alpha}^1$
$\partial\hat{P}(\alpha_0=1)$
$(3,0)$
$\hat{\alpha}^3$
C^P
C^S
C^{PB}
C^H

Figure 3

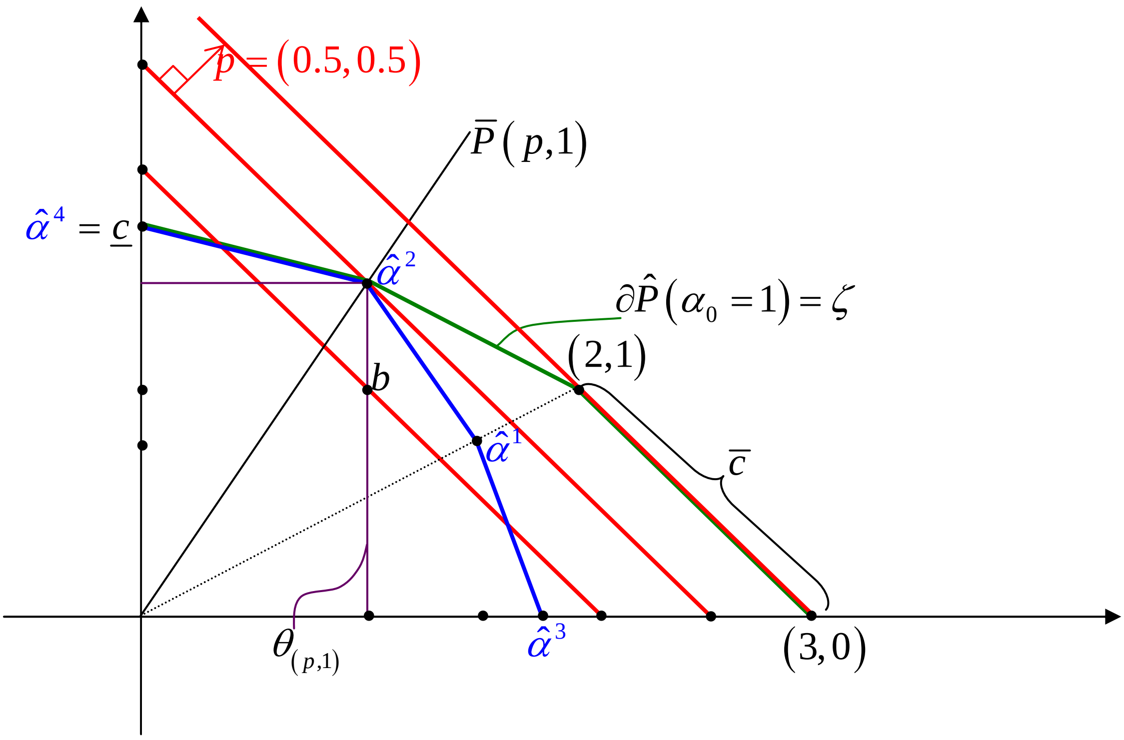


Figure 4: The Morishima (1974) definition for Marxian Labor Exploitation meets **LE** , but violates **CECP**.

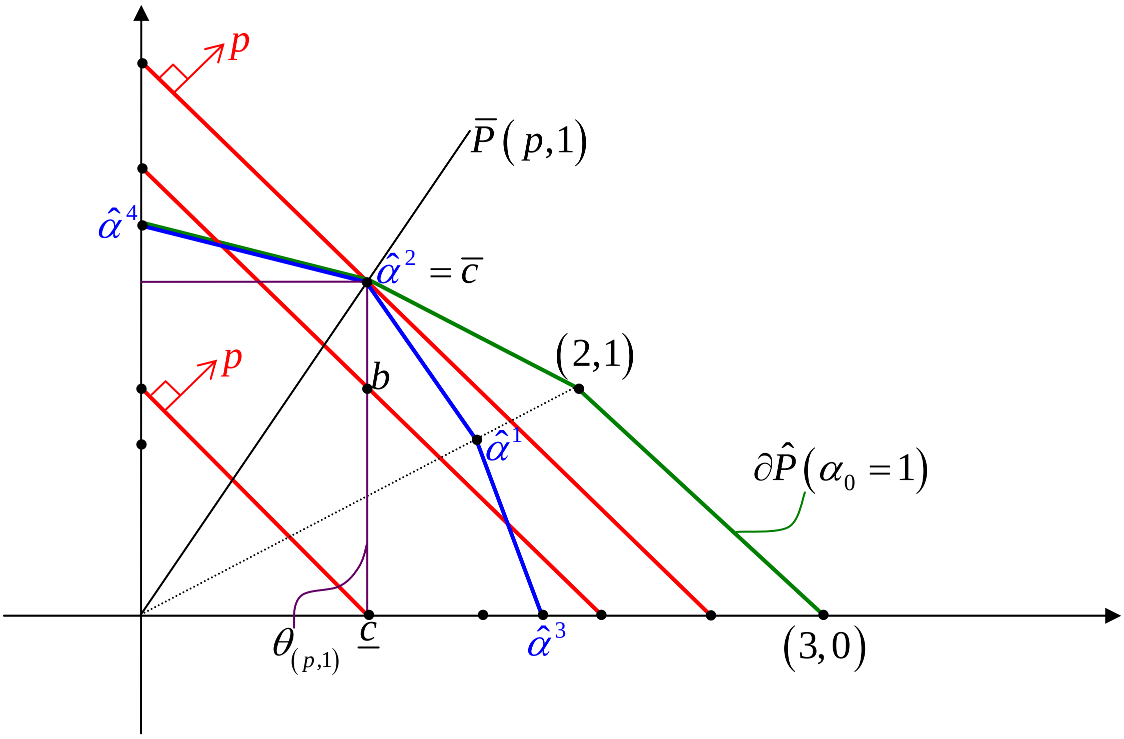


Figure 5: The Roemer (1982) definition for Marxian Labor Exploitation meets **LE**, but violates **CECP**.

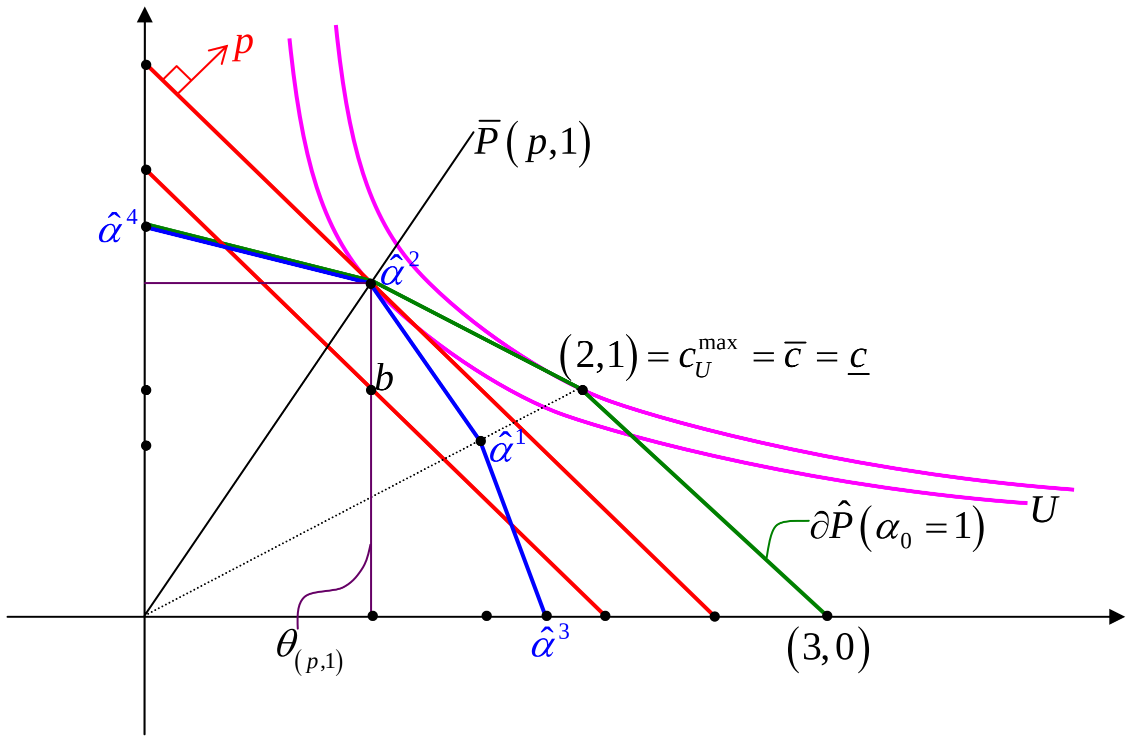


Figure 6: Definition 8 for Marxian Labor Exploitation meets **LE**, but violates **CECP**.

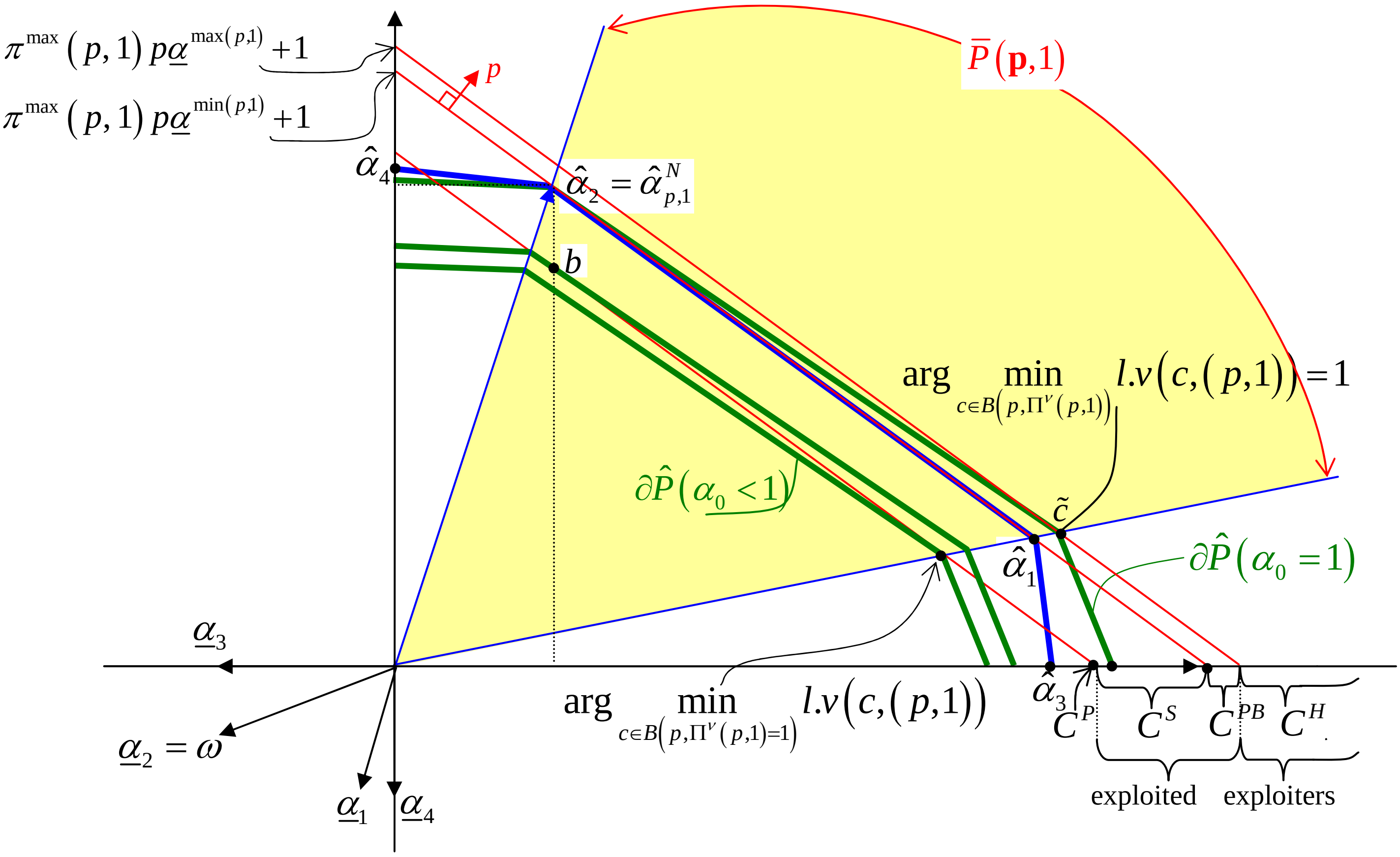


Figure 7: Class-Exploitation Correspondence Principle in a general convex cone economy when the formulation of exploitation is given by Definition 6

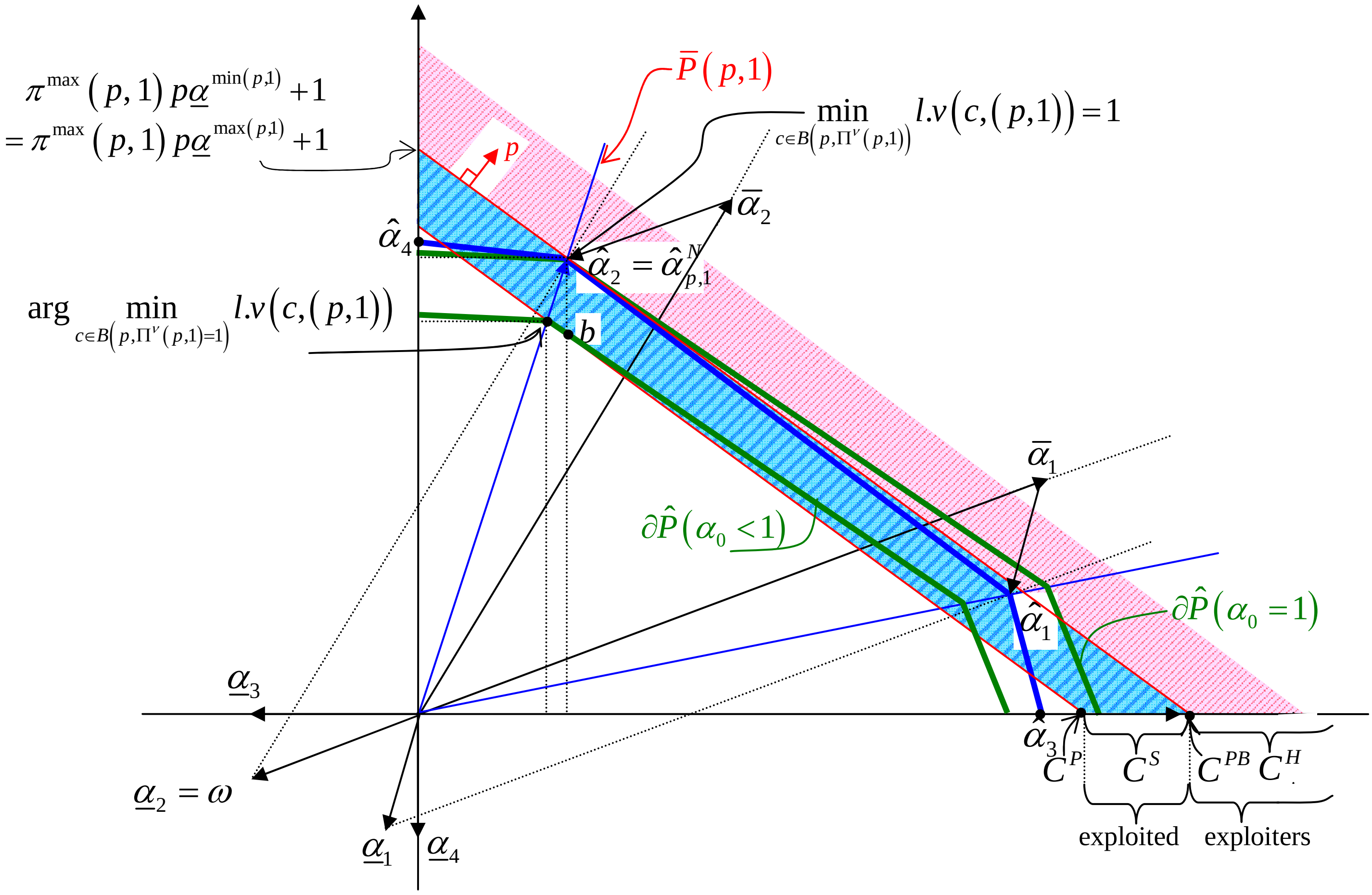


Figure 8: Class-Exploitation Correspondence Principle in a general convex cone economy when the formulation of exploitation is given by Definition 7.