Normative Economic Foundation for the Theory of Welfare State Policies*

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Abstract

In this paper, we propose an analytical foundation for normative economics of the welfare state policies. First, we argue that the so-called 'neo-liberal' reorganization of welfare states may be justified by the conventional, welfarist's welfare economics. Secondly, criticizing the welfaristic evaluation of social and economic policies, we propose a more comprehensive framework in which *extended social welfare functions* (**ESWF**s) are also introduced. In this framework, not only welfaristic values, but also non-welfaristic values can be treated appropriately. Then, we formulate a non-welfarist normative theory, *Real Libertarianism*, proposed by van Parijs (1995), as axioms of *Labor Sovereignty* and *Respect for Undominated Diversity*, and examine the possibility of consistent social evaluations based on the axioms of Real Libertarianism combined with the *Pareto principle.*

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1 Introduction

In the process of the 'globalization of the economy,' the Japanese state welfare system is undergoing a tangible reorganization. This is frequently viewed as being a part of the 'neo-liberal' movement. In other words, it is characterized as an approach that places much importance on advancing free economic competition by deregulation and a redefinition of competition policy. This creates a 'small government' by transferring what has been in the hands of the public sector such as social welfare and education to 'the vitality of the private sector,' in order to improve economic efficiency. One of the characteristics of such a policy, in terms of resource allocations, is tax reduction benefiting large scale enterprises and those in higher income bands,[1] as well as the change in the mechanism of income redistribution by cutting or reviewing social welfare benefits.[2] Such policy changes are causing a downward trend in total cash earnings,[3] while the proportion of non-regular employment is increasing[4] due to the deregulation of the labor market,[5] resulting in a larger income gap.[6] This income gap provides fewer opportunities for the children of

[1] The examples are the reduction of the corporation tax (From 37.5% in FY '97 to 34.5% in FY '98, 30% in FY '99) and the corporate enterprise tax (From 12% in FY '97 to 11% in FY '98, 9.6% in FY '99), etc, the reduction of the maximum rate of income tax (From 50% in FY '98 to 37.5% in FY '99) and the resident's tax (From 15% in FY '98 to 13% in FY '99), introduction of Research and Development Tax System (FY '03), the abolition of the securities transaction tax (FY '99), and the tax reduction for dividend income, and others.

[2] For example, in the review of Public Assistance System in Japan, introducing "Independence support services" was proposed, and the gradual cutting of the allowances for the special needs of elderly recipients and single-mother recipients was also discussed. As a whole, the reduction of fiscal expenditure for the Daily Life Protection, which will be implemented in 2007, and its shift to the local governments are discussed as a part of the "Trinity Reform" package.

[3] Since 1998, wage has decreased by 19.2 trillion yen in five years.

[4] This trend may result in solidifying the bargaining power of employers in the labor market. In conjunction with this, regular employees are tending to work longer hours.

[5] The examples are the introduction of the discretionary labor system in the Revision of the Labor Standards Law (promulgated in July 2003, came into effect in January 2004), and the extension of the maximum duration of employment on terminal contracts from one year to three years and the permission of temp workers in the manufacturing sector in the Revision of the Worker Dispatching Law enacted in June 2003.

[6] The "larger income gap" is currently a controversial issue in Japan. For instance, Ohtake (2005) argues that the large income gap since the 80's is mainly due to the increase in the aging population, since in general, the income inequality within older generations

lower-income parents which in turn reduces their earning power, thus further reducing their children's opportunities and earning power. This is destined to continue from generation to generation.

The 'neo-liberal' reform of the welfare state system is often a target of criticism in discussions[7] of political science, sociology, Marxian economics, and others. However, the 'neo-liberal' reform can also be evaluated from the point of view that it is concerned with the necessity of maintaining and reinforcing the international competitiveness of the national economy under economic 'globalization.' More precisely, it is often argued that the series of deregulations and the redefinition of competition policy may reform the competitive mechanism of markets so as to enhance the efficiency of economic resource allocations, which may contribute to improvement of social welfare in the long run. For instance, reforms such as lowering the maximum tax rate for corporate tax and income tax, abolishing securities transaction tax, reducing tax on dividend income, introduction of the Research and Development Tax System, and the deregulation of the labor market might steer the budget distribution of the government toward measures to strengthen the international competitiveness of the major domestic industries, rather than develop welfare-related policies for the socially underprivileged. This might eventually result in the maintenance and improvement of productivity and the international competitiveness of the national economy under the leadership of major large scale enterprises. As a result, this will also induce an improvement of employment conditions for the socially underprivileged, as well as secure and expand the welfare-related budget for the socially underprivileged in the long run. Such an argument could be justified to some extent by the microeconomic analysis of mainstream economics. In fact, the *hypothetical compensation principle* in the "new welfare economics" may provide the theoretical justification of the 'neo-liberal' policy scheme.

However, the hypothetical compensation principle is not necessarily able to evaluate the policy scheme consistently,[8] and it does not take into consid-

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is higher than within younger generations. Nevertheless, as Otake (2005) points out, the widening income gap and consumption gap among the youth have been recognized since the 90's, and also the growing gap of financial assets among people at the same age has been found.

[7]For instance, Saito (2004), Shionoya, Suzumura, and Gotoh (2004), Ikegami and Ninomiya (2005).

[8]The conventional criteria of the hypothetical compensation principle, although easily confirmed, have the possibility of producing cyclic binary relations.

eration the fairness of resource allocations. Thus, a new original criterion will need to be thought of in order to provide judgments on the problem: whether it is appropriate to neglect dealing with the inequality of opportunities, or to what extent corrective measures need to be established.

Traditional welfare economics basically recognizes that political intervention in the market is indispensable to solve the problems of market failure or the fairness of resource allocations, although it also recognizes that Pareto efficiency of economic resource allocations is realized in the market with perfect competition. Bearing this in mind, when suggesting further methods or criteria for appropriate policy evaluation or a policy mechanism that serves for such evaluations, such suggestions should take into account the values that individuals in society might instinctively have regarding how welfare should be. As long as this is the case, one might not be able to help but critically evaluate the welfarist viewpoint and the methodology that traditional welfare economics considers as its premise. This is a fundamental issue taken into account throughout the paper.

For instance, the review of the Public Assistance System in Japan, which is characterized as the introduction of Independence support services in order to promote labor market participation, can be placed in the 'workfare' policy.[9] [10] In fact, although the review of Public Assistance System has many positive points including Independence support services which try to reflect the diverse needs of the recipients and organize the assistance menus, this proposal also includes the suggestion to consider the gradual cutting of allowances for the special needs of elderly recipients and single-mother recipients, faced with the fact that there is an increasing number as well as proportion of long-term recipients as a result of the recent expanding economic gap. Furthermore, it also incorporates a sort of incentive mechanism where the recipients are required to participate in Independence support services in order to continue receiving their welfare benefits.

The idea of social welfare behind the workfare system is motivated by concerns mainly with the 'principle of market' and economic efficiency, and it can

[9]Note that the workfare type of social welfare system requires recipients, as the condition for continuing their welfare benefits, to participate in activities that help to improve their job prospects (such as training, rehabilitation and work experience).

[10]A clear-cut survey on the social and economic context and history of the crisis of the modern welfare states is given in Shinkawa (2004, 2004a). Also, for a survey on the development of workfare, the active labor market policy, etc., which characterize the "New Welfare State" movement, Miyamoto (2004) is beneficial.

be described as the introduction of 'liberalization of service provisions in the welfare field.' Regarding this phenomenon, many criticisms have been raised from the viewpoint of *fundamental human rights* (FHRs):[11] it is regarded as a system that threatens FHRs which need to be secured for any citizen. For instance, Independence support services may be valuable in the sense that it aims to give individuals independence by helping them acquire a job, but it could also create a situation where the mechanism forces individuals to get a job while ignoring the fact that there are single mothers or sick persons who are not able to work. Thus, there should be demands for some alternatives to such a policy.[12] However, within the welfarist policy evaluations such as the cost benefit analysis or the hypothetical compensation principle developed in traditional welfare economics, these types of concerns are put aside from the list of considerations as 'a field that concerns social sciences other than economics.' This is because the above mentioned evaluations are based on the welfare gauge, mentioned later, that measures the consumption preference satisfaction, which is convertible to monetary value. In fact, it is difficult to adopt the FHRs concerns as a criterion of reference for the social evaluation of economic policies, unless the notion of welfare and well-being which cannot be grasped merely by the level of consumption of goods and services is clarified.

Nonetheless, in general discussions on the validity of economic policies, not limited to the problems of the evaluation of the 'workfare' welfare policy, conflicts between the traditional economic viewpoint, in other words a welfarist criterion, and the traditionally 'non-economic' viewpoint, as in the above example which is still a valuable viewpoint or criterion to refer to, are among the problems that exist universally in the process of social policy decision-making. As a consequence, a more comprehensive system of social evaluation, enabling reference to both criteria will be needed. With this, it will be possible to present social evaluations and alternative suggestions with a more appropriate insight into the 'neo-liberal' way of reorganizing the welfare state.

In Section 2, a further discussion is held to clarify the limitations of a welfarist approach that traditional welfare economics considers as its premise. Furthermore in Section 3, a basic theoretical work is developed for the pur-

[11]The Constitution of Japan, Article 1, Section 1; "All people shall have the right to maintain the minimum standards of wholesome and cultured living."

[12]The *basic income* policy is positioned as one of such alternatives. For the basic income policy, see van Parijs (1992, 1995).

pose of establishing a more comprehensive social evaluation that overcomes its limitations. Finally, Section 4 is given for some concluding remarks.

2 What are the problems of the welfarist approach?

Welfarism is defined as a position or a methodology that evaluates individuals' welfare in a society according to their level of satisfaction with their subjective preferences. For this methodology of social welfare evaluation, the criticisms by Amartya Sen [Sen (1979; 1980)], Ronald Dworkin [Dworkin (1981a; 2000)], and others are well known.[13] On the other hand, problems with welfarist methodology can emerge in a more acute form within the domain of welfare economics.

Let us consider the policy evaluation based on the hypothetical compensation principle of the "new welfare economics." Suppose a public policy which causes the transition of the social situation from $\mathbf{x}$ to $\mathbf{x}'$. Generally, Pareto superior relation does not necessarily occur between $\mathbf{x}'$ and $\mathbf{x}$. In other words, during the transition from $\mathbf{x}$ to $\mathbf{x}'$, it is universally observed that some individuals (the beneficiaries) enjoy a utility improvement, while others (the disadvantaged) receive a utility loss. In this situation, however, after (hypothetically) compensating for the utility loss suffered by the public policy via transferring some of the beneficiary's benefits, if the beneficiary's situation compared to $\mathbf{x}$ is still the same or better, then the transition from $\mathbf{x}$ to $\mathbf{x}'$ brought about by the policy would be taken as a social improvement, regardless of whether or not such a compensation is implemented in reality. This is the meaning of the *Kaldor principle*.[14]

[13] These were the critiques against the limitation of the informational basis that evaluates individual's welfare from the viewpoint of the satisfaction of the subjective preference. Moreover, in relation to the concept of welfare as a satisfaction of preferences, they criticized the welfarist neutral attitude vis-à-vis the problem of what type of preference is satisfied. That is to say, there are types of preference satisfactions such as the utility satisfaction of individual offensive tastes, that of expensive tastes, that of formation of the adaptive preference, or that of cheaper tastes such as in the case of "termed housewife," all of which should be carefully and discriminatory treated in evaluation of social welfare from an ethical point of view. Thus, it is the point of the critiques against welfarism that the welfarist evaluation of social welfare has no conern for such difference of preferences.

[14] For the argument of the hypothetical compensation principle, there are also the Hicks principle, the Scitovsky principle, the Samuelson principle, etc., but here I would skip

The Kaldor principle is welfarist in nature, since it has an informational basis for policy evaluation only on a utility level that represents the level of individuals' satisfaction with their subjective preferences over goods and services, and it is also an extension of the Pareto principle as is clear from its definition. It makes clear judgments on policy changes that are unavoidably exempt from being judged by the normal Pareto principle, based on whether or not there is a possibility of potential Pareto improvement of said changes. Moreover, it is not even a welfarist theory of distributive justice, but rather utilitarian in the sense that it is related to the potential efficiency of resource allocations.

Let us discuss further the notion of welfare in the welfarist normative theory such as the hypothetical compensation principle. For that, let us remember the notion of *aggregate compensating variation* and *aggregate equivalent variation* defined in cost benefit analysis, and its logical relation to the hypothetical compensation principle. In standard micro economic theory, it is argued that the necessary condition for a change in a social situation by adopting a policy to be approved by the Kaldor compensation principle is that the aggregate equivalent variation (aggregate compensating variation) takes a positive value. Unfortunately, these variations do not provide sufficient conditions for the hypothetical compensation principle to be satisfied. Nonetheless, if the change in resource allocation caused by the policy is not radical, then its sufficiency can be confirmed.

Thus, the validity of executing a policy according to the hypothetical compensation principle is possibly confirmed by means of such monetary measures as the aggregate equivalent variation and the aggregate compensating variation, whenever the change in resource allocation caused by the policy is not radical. What may be of more interest is the corollary of this proposition. That is, the validity of executing a policy according to the hypothetical compensation principle is possibly confirmed by means of such monetary measures as gross national income, as long as the change in resource allocation caused by the policy is not radical. Thus, the hypothetical compensation principle and the *gross national income test* are equivalent as a test for the validity of a policy that is not expected to cause much change in resource allocations.

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the explanations of their differences in detail. It is because the critique on the concept of economic welfare that the Kaldor principle takes is valid for the other versions of the hypothetical compensation principle.

According to the above argument, the notion of social welfare that the hypothetical compensation principle considers as its premise is no less than the notion of a sum of different satisfaction levels of the subjective preferences obtained from the consumption of 'marketable' goods and services, whose degree can be evaluated by monetary measures. However, the notion of social welfare is significantly broad enough to encompass a wide range of ethical viewpoints. The social welfare that welfarist's welfare economics, represented by the hypothetical compensation principle, refers to is not that of this broad sense, but it is the one limited to that of (market) economic aspects related to monetary measures directly or indirectly.[15]

Such recognition of the limitation of social welfare in welfarist's welfare economics existed as early as in the time of A. C. Pigou [Pigou (1932)] who was the founder of welfare economics. For example, Pigou said in "Chapter I, Welfare and Economic Welfare" in his *The Economics of Welfare*, "the range of our inquiry becomes restricted to that part of social welfare that can be brought directly or indirectly into relation with the measuring-rod of money. This part of welfare may be called economic welfare." He distinguishes the notion of 'economic welfare' from that of welfare in the broad sense, limiting his study at that time to "certain important groups of causes that affect economic welfare in actual modern societies." In other words, the target

[15] Against such an argument, the following objection may arise from the welfarist position. 'It is true that the social welfare analysis utilized in conventional applied economics concerns only the social welfare as the total sum of satisfaction of the preferences over goods and services, which is convertible to monetary value. However, the concept of social welfare can be extended so as to consider the 'utility' from an outcome other than the private consumption of goods and services, extending the domain of the individual utility function if necessary.' Such an approach is one way to expand the limited informational basis of conventional welfarist's welfare economics. However, this approach would not be able to avoid the welfarist criticisms of Sen and Dworkin, which are mentioned at the beginning of this paper, because it takes the position of treating and evaluating everything on the sole utility function at the same dimension as the subjective preference of the private consumption of goods and services. In this sense, the concept of social welfare in this approach is still corresponding solely to the 'satisfaction of individual subjective preferences.' However, the concept of welfare and well-being cannot necessarily be grasped solely from the aspect of satisfaction of subjective preferences or tastes. The viewpoint of "respect for liberal rights" presented in Sen's Liberal Paradox [Sen (1970a,b)] and his theory of "functioning and capability" [Sen (1985)] were to offer the concept of welfare and well-being that are not grasped by such 'satisfaction of preferences.' This paper takes the position that the social welfare should be evaluated, not only from the satisfaction of subjective preferences or tastes, but also from the aspects of welfare and well-being that are not grasped by such 'satisfaction of individual preferences.'

of economics was limited to *economic welfare* where the monetary measure could be applied. It was Pigou's modest, reserved methodology, with its lack of pretension, that determined the provisions of "welfare economics" of later days, but he himself was well aware of the potential problems lying in such approaches. He wrote that "an economic cause may affect non-economic welfare in ways that cancel its effect on economic welfare," listing various examples of its regressive effects on the ethical attributes of individuals, an explicit one being, who concentrate on their pursuit of economical satisfaction. As he says in his writing, "the attention of the German people was so concentrated on the idea of learning to *do* that they did not care, as in former times, for learning to *be*."

Pigou also mentions the alienated relationship between capital and labor. He argues that the root of this hostile relationship cannot be merely viewed as an economic welfare problem such as a "dissatisfaction with rates of wages" but it also derives from dissatisfaction with "the general *status* of wage-labour" which "deprives the workpeople of the liberties and responsibilities proper to free men, and renders them mere tools to be used or dispensed with at the convenience of others" which defines such problems as those of non-economic welfare. Having said that, he also argues that changes "in industrial organisation that tend to give greater control over their own lives to workpeople, whether through workmen's councils to overlook matters of discipline and workshop organisation in conjunction with the employer, or through a democratically elected Parliament directly responsible for nationalised industries, ... might increase welfare as a whole, even though they were to leave unchanged, or actually to damage, economic welfare."

Pigou's comments mentioned above are important considerations in the discussion of contemporary welfare state policies. In fact, what brings the level of individuals' welfare higher in their lives is not merely their satisfaction from economic consumption; for instance, good health from a naturally rich environment and positive social relationships both within and outside family play a vital role. It is often in this way that individuals evaluate their lives, based on the extent to which their *basic needs* are met. It is not entirely clear whether or not aspects of the non-market life of a society, that affect the level of social welfare in a broad sense, are improved as a result of policies based on the hypothetical compensation principle which tries to achieve a (potential) Pareto improvement of the economic aspects of social welfare.

For instance, suppose that there was an improvement in the national income of the domestic economy as a whole due to the improvement of the corporate cost performance by public policies promoting industrial restructuring or technological innovation. The improvement of corporate cost performance has consequently caused unemployment, but at the same time, even the unemployed might be able to maintain a consumption close to the level they had prior to losing their jobs owing to the national unemployment benefit, which is based on increased tax revenue from companies that are now more profitable. Implementation of such a policy would be approved by the strong Kaldor principle because it means a (potential) Pareto improvement in the economic aspects of social welfare. Nevertheless, such evaluations are not necessarily concerned with individuals' failure to realize their aspirations due to unemployment and their loss of opportunity for their self-realization. Moreover, an extended period of unemployment will not only result in a partial loss of income but also a decline or drain in the individual labor force, which could lead to a loss of the basis of autonomous living. It is not entirely evident whether the above-mentioned policies for the improvement of national income should be encouraged immediately or not if they are based on a notion of social welfare in which individual autonomous living is viewed as a valuable element.

The above arguments highlight the limitations of welfarist's welfare economics, but Pigou himself intentionally restricted his theory within such a framework of welfarism.[16] In contrast, John Hicks [Hicks (1959)], who took an important role in new welfare economics, came to the conclusion that it was necessary to avoid such a restricted framework. He says in the introduction of *Essays in World Economics*, "I cannot therefore now feel that it is enough to admit, ..., that 'the economist must be prepared to see some suggested course of action which he thinks would promote economic wel-

[16] In fact, Pigou justifies the position to develop the argument limited to the concept of economic welfare. That is, "Nevertheless, I submit that, in the absence of special knowledge, there is room for a judgement of probability. When we have ascertained the effect of any cause on economic welfare, we may, unless, of course, there is specific evidence to the contrary, regarding this effect as probably equivalent in direction, though not in magnitude, to the effect of one cause is more favourable than that of another cause to economic welfare, we may, on the same terms, conclude that the effect of this cause on total welfare is probably more favorable. In short, there is a presumption ... that qualitative conclusions about the effect of an economic cause upon economic welfare will hold good also of the effect on total welfare. This presumption is especially strong where experience suggests that the non-economic effects produced are likely to be small. But in all circumstances the burden of proof lies upon those who hold that the presumption should be overruled." [Pigou (1932)]

fare turned down—his own judgement perhaps consenting, perhaps not—for over-riding reasons'." He also claims that, "This is still no more than an admission that there are 'parts' of welfare which are not included in Economic Welfare, and that the two sorts of ends may conflict. The economist, as such, is still allowed and even encouraged, to keep within his 'own' frontiers; if he has shown that a particular course of action is to be recommended , for economic reasons, he has done his job." Furthermore, he affirms, "I would now say that if he limits his function in that manner, he does not rise to his responsibility. It is impossible to make 'economic' proposals that do not have 'non-economic aspects,' as the Welfarist would call them; when the economist makes a recommendation, he is responsible for ... all aspects of that recommendation, whether he chooses to label them economic or not, are his concern." That is to say, it is impossible in reality to give advice on policies limited to the (market) economic aspects of social welfare. Thus, he says that economists are totally responsible for any aspect of the policy—whether it is an economic aspect or not—when they advise on public policies.

Herbert Gintis [Gintis (1972)], known as a radical economist, criticizes the welfare model of the neoclassical economic theory, and presents a radical theory of welfare as an alternative to the neoclassical theory. According to his discussion, individual preferences are exogenously given in neoclassical theory. Individual choice behaviors are taken as to maximize their preferences under some option sets constrained by available resources and knowledge of technologies, which constitute social outcomes as Pareto optimal. Thus, the social outcomes are taken as a reflection of individual preferences. Moreover, individuals' sovereignty as consumers, workers, and citizens are taken to be established in the capitalist society, which should be incorporated in the so-called social welfare function.

On the other hand, the radical theory, according to his discussion, argues that worker sovereignty does not hold in capitalist society because of the dominant relationship between capital and labor in the capitalist production process. Moreover, citizen sovereignty fails to hold because of the correspondence of political and economic power, which establishes the hegemony of capital in political decision making. Finally, compared to the above two types of sovereignties, the failure of consumer sovereignty is not taken as a major social problem. Rather, Gintis is concerned with the mechanism of generating "consumption-biased behavior" of individuals in the capitalist society, who prefer to fulfill their personal consumption of goods and services rather than maintain and improve the community through social development.

Then, Gintis presents the welfare model of the radical theory, which posits that well-being flows from the individual *activities* undertaken in social life of each person in the society. The contribution of an activity to individual well-being depends on: (a) The personal capacities developed by the individual in order to carry out and appreciate the activity; (b) The social contexts (work, community, family, environment, educational institutions, etc.) within which the activity takes place; and (c) The commodities available to the individual as *instruments* in the performance of the activity.

Gintis also points out that, in the capitalist society, performance of welfare-relevant social activity becomes a means toward the maximization of the instruments of performance (commodities), and a worker's access to the most positive social activity-contexts is dependent on his income-earning capacity. Consequently, Gintis says that the individual tends to use his money to gain personal access to those positive social activity-contexts *which exist*, rather than slightly increase their total supply. His discussion mentioned above has a content common to that of Hicks's self-criticism, and his welfare model can be said to have been followed by the non-welfarist's welfare theories such as Amartya Sen's theory of "functioning and capability."

3 Beyond the welfarist limitation

The criticism of welfarism mentioned above becomes relevant not only in the discussion of the criteria of policy evaluation based on the hypothetical compensation principle, but also in the general discussion of social welfare functions. A social welfare function associates a ranking over social alternatives with each social choice environment, especially taking economic resource allocations and economic environments into consideration, respectively. According to the ranking derived from such a function, the society can identify what are the most desired policies which would realize the most desired social alternatives.

This conceptual device is itself indispensable when considering the contemporary issues regarding the policies of a 'welfare state.' The basic problem in this context is what type of social welfare function should be composed, and it is in the course of such discussions that the conventional Bergson-Samuelson (B-S) social welfare functions are perceived as problematic. The very reason for this is that the B-S social welfare functions take the welfarist position that only the level of individual satisfaction with their subjective preferences is the sole basis of information. The rankings over social alternatives given by social welfare functions should sufficiently reflect an adequate indicator of individuals' well-being. The criticism of welfarism mentioned so far indicates that individual satisfaction with their subjective preferences is only one aspect of welfare and therefore a more pluralistic viewpoint is necessary.

The need for a pluralistic approach is argued by John Rawls [Rawls (1971; 1993; 2001)], Amartya Sen [Sen (1980); 1985]], and Philippe Van Parijs [Van Parijs (1992; 1993; 1995)]. Based on the theories of the three non-welfarists above, I would like to propose the following three basic criteria.

The first criterion is that individual autonomy in contemporary society should be guaranteed. It reflects a liberal value that contemporary civil societies respect, but I believe the guarantee of individual autonomy has become an important aspect for evaluating individual well-being. In fact, as opposed to the feudal society and the centralized socialist society where individual autonomy is suppressed, the modern civil society might be characterized as establishing a certain level of political liberalism in legal systems, a certain level of freedom of choice both in political and economic decision-makings, and a certain level of decentralized decision-making mechanisms such as markets, all of which constitute a necessary condition for the guarantee of individual autonomy. Such a viewpoint would suggest a certain constraint over the class of 'desired' social welfare functions. That is to say, if the social economic system cannot guarantee the decentralization and the freedom of choice in decision-making, the welfare that individuals receive under such social situations will not be highly valued by 'desired' social welfare functions, even if the system may support a sufficient level of individual consumption. Thus, this criterion represents a non-consequential value in nature.

The second criterion is that each and every individual should have as much opportunity to do whatever he might want to do as is feasibly possible. This criterion represents a non-welfaristic consequential value in the sense of the following two points. First, although this criterion concerns about social outcomes in terms of individual well-being, it hinges on an objective notion of individual well-being as opposed to welfarist criteria. Second, this criterion does not concern about the realization of individual well-being itself, but rather it is interested in the opportunity to pursue or realize individual well-being. Given these points, theories of distributive justice are relevant in the discussion of what concept of individual well-being is appropriate, and of what types of *equity* notions should be applied to the assignment problem of individual opportunity sets.

The third criterion represents a well-known welfarist consequential value such as the Pareto principle. It is worth noting that the standpoint of non-welfarism does not imply that welfarist notion of well-being is unnecessary. I believe that satisfaction of individual subjective preferences is still an important component of informational basis in order to constitute an overall notion of individual well-being. Thus, the Pareto principle is also taken into consideration as a condition to characterize what are 'desired' social welfare functions.

With the above discussion in mind, the question arises whether it is possible to construct a social welfare function consistent with the different pluralistic criteria mentioned above. These three criteria are all valid, and the validity of these three criteria means their realization as policies is desirable. The problem is whether such policies exist or not, and if such policies can be implemented or not. Moreover, if the social welfare function consistent with the three criteria above does not exist at all, realization of such policies will become unviable.

3.1 A Framework of Extended Social Welfare Functions

On the basis of the problems propounded in the previous section, in the following subsection, the notion of *extended social welfare function*[17] is introduced, which is based on the proposal of Gotoh, Suzumura, and Yoshihara (2005). There are two goods, one of which is an input (labor time) $x \in \mathbb{R}_+$ to be used to produce the other good $y \in \mathbb{R}_+$.[18] The population is given by the set $N = \{1, \ldots, n\}$, where $2 \leq n < +\infty$. Each agent i's consumption is denoted by $z_i = (x_i, y_i)$, where x_i denotes his labor time, and y_i the amount of his output. All agents face a common upper bound of labor time $\bar{x}$, where $0 < \bar{x} < +\infty$, and so have the same consumption set $[0, \bar{x}] \times \mathbb{R}_+$.

Each agent i's preference is defined on $Z \equiv [0, \bar{x}] \times \mathbb{R}_+$ and represented by a utility function $u_i : Z \to \mathbb{R}$, which is continuous and quasi-concave on Z, strictly monotonic (decreasing in labor time and increasing in the share

[17] It is called "extended social ordering function" in Gotoh, Suzumura, and Yoshihara (2005).

[18] The symbol $\mathbb{R}_+$ denotes the set of non-negative real numbers.

of output) on $\overset{\circ}{Z} \equiv [0, \bar{x}) \times \mathbb{R}_{++}$,[19] and $u_i(x, 0) = 0$ for any $x \in [0, \bar{x}]$. We use $\mathcal{U}$ to denote the class of such utility functions.

In addition, each agent i has a *labor skill*, $s_i \in \mathbb{R}_+$. The universal set of skills for all agents is denoted by $\mathcal{S} = \mathbb{R}_+$.[20] The labor skill $s_i \in \mathcal{S}$ is i's *labor supply* per hour measured in efficiency units. It can also be interpreted as i's *labor intensity* exercised in production. Thus, if the agent's *labor time* is $x_i \in [0, \bar{x}]$ and his labor skill is $s_i \in \mathcal{S}$, then $s_i x_i \in \mathbb{R}_+$ denotes the agent's *labor contribution* to production measured in efficiency units. The production technology is a function $f : \mathbb{R}_+ \to \mathbb{R}_+$, that is continuous, strictly increasing, concave, and $f(0) = 0$. For simplicity, we fix this f. Thus, an economy is a pair of profiles $\mathbf{e} \equiv (\mathbf{u}, \mathbf{s}^t)$ with $\mathbf{u} = (u_i)_{i \in N} \in \mathcal{U}^n$ and $\mathbf{s} = (s_i)_{i \in N} \in \mathcal{S}^n$. Denote the class of such economies by $\mathcal{E} \equiv \mathcal{U}^n \times \mathcal{S}^n$.

Given $\mathbf{s} = (s_i)_{i \in N} \in \mathcal{S}^n$, an allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in Z^n$ is *feasible for* $\mathbf{s}$ if $\sum y_i \leq f\left(\sum s_i x_i\right)$. We denote by $Z(\mathbf{s})$ the set of feasible allocations for $\mathbf{s} \in \mathcal{S}^n$. An allocation $\mathbf{z} = (z_i)_{i \in N} \in Z^n$ is *Pareto efficient for* $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$ if $\mathbf{z} \in Z(\mathbf{s})$ and there does not exist $\mathbf{z}' = (z'_i)_{i \in N} \in Z(\mathbf{s})$ such that for all $i \in N$, $u_i(z'_i) \geq u_i(z_i)$, and for some $i \in N$, $u_i(z'_i) > u_i(z_i)$. We use $P(\mathbf{e})$ to denote the set of Pareto efficient allocations for $\mathbf{e} \in \mathcal{E}$.

To complete the description of how our economy functions, what remains is to specify an *allocation rule* which assigns, to each $i \in N$, how many hours he/she works, and how much share of output he/she receives in return. In this paper, an allocation rule is modelled as a *game form* which is a pair $\gamma = (M, g)$, where $M = M_1 \times \cdots \times M_n$ is the set of admissible profiles of individual *strategies*, and g is the *outcome function* which maps each strategy profile $\mathbf{m} \in M$ into a unique outcome $g(\mathbf{m}) \in Z^n$. For each $\mathbf{m} \in M$, $g(\mathbf{m}) = (g_i(\mathbf{m}))_{i \in N}$, where $g_i(\mathbf{m}) = (g_{i1}(\mathbf{m}), g_{i2}(\mathbf{m}))$ and $g_{i1}(\mathbf{m}) \in [0, \bar{x}]$ and $g_{i2}(\mathbf{m}) \in \mathbb{R}_+$ for each $i \in N$. Let Γ be the set of all game forms representing allocation rules of our economy. Given an allocation rule $\gamma = (M, g) \in \Gamma$ and an economy $\mathbf{e} \in \mathcal{E}$, we obtain a non-cooperative game $(\gamma, \mathbf{e}) \in \Gamma \times \mathcal{E}$ without ambiguity.

Throughout this paper, we will focus on the Nash equilibrium concept in our analysis of the performance of game forms as allocation rules. Given a game form $\gamma = (M, g)$, let $\mathbf{m}_{-i} = (m_1, \ldots, m_{i-1}, m_{i+1}, \ldots, m_n) \in M_{-i} \equiv \times_{j \in N \setminus \{i\}} M_j$ for each $\mathbf{m} \in M$ and $i \in N$. Given an $\mathbf{m}_{-i} \in M_{-i}$ and an $m'_i \in$

[19] The symbol $\mathbb{R}_{++}$ denotes the set of positive real numbers.

[20] For any two sets X and Y, $X \subseteq Y$ whenever any $x \in X$ also belongs to Y, and $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$.

M_i, $(m_i'; \mathbf{m}_{-i})$ may be construed as an admissible strategy profile obtained from $\mathbf{m}$ by replacing m_i with m_i'. Given a game $(\gamma, \mathbf{e}) \in \Gamma \times \mathcal{E}$, an admissible strategy profile $\mathbf{m}^* \in M$ is a *pure strategy Nash equilibrium* if $u_i(g_i(\mathbf{m}^*)) \geq u_i(g_i(m_i, \mathbf{m}_{-i}^*))$ holds for all $i \in N$ and all $m_i \in M_i$. The set of all pure strategy Nash equilibria of the game $(\gamma, \mathbf{e})$ is denoted by $NE(\gamma, \mathbf{e})$. A feasible allocation $\mathbf{z}^* \in Z(\mathbf{s}^t)$ is a *pure strategy Nash equilibrium allocation* of the game $(\gamma, \mathbf{e})$ if $\mathbf{z}^* = g(\mathbf{m}^*)$ holds for some $\mathbf{m}^* \in NE(\gamma, \mathbf{u})$. The set of all pure strategy Nash equilibrium allocations of the game $(\gamma, \mathbf{e})$ is denoted by $\tau_{NE}(\gamma, \mathbf{e})$.

In this paper, the domain of social preference relations is given by pairs of allocations and allocation rules as game forms, which we call *extended social alternatives*. The intended interpretation of an extended social alternative, viz., a pair $(\mathbf{z}, \gamma) \in Z \times \Gamma$, is that an allocation $\mathbf{z}$ is attained through an allocation rule γ.[21] Moreover, given $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$, an extended social alternative $(\mathbf{z}, \gamma) \in Z \times \Gamma$ is *realizable* if $\mathbf{z} \in Z(\mathbf{s}) \cap \tau_{NE}(\gamma, \mathbf{e})$. Let $\mathcal{R}(\mathbf{e})$ denote the set of realizable extended social alternatives under $\mathbf{e} \in \mathcal{E}$.

What we call an *extended social welfare function* (**ESWF**) is a mapping $Q : \mathcal{E} \twoheadrightarrow (Z^n \times \Gamma)^2$ such that $Q(\mathbf{e})$ is an ordering on $\mathcal{R}(\mathbf{e})$ for every $\mathbf{e} \in \mathcal{E}$.[22] The intended interpretation of $Q(\mathbf{e})$ is that, for any extended social alternatives $(\mathbf{z}^1, \gamma^1)$, $(\mathbf{z}^2, \gamma^2) \in \mathcal{R}(\mathbf{e})$, $((\mathbf{z}^1, \gamma^1), (\mathbf{z}^2, \gamma^2)) \in Q(\mathbf{e})$ holds if and only if realizing a feasible allocation $\mathbf{z}^1$ through an allocation rule γ^1 is at least as good as realizing a feasible allocation $\mathbf{z}^2$ through an allocation rule γ^2 according to the social judgments embodied in $Q(\mathbf{e})$. The asymmetric part and the symmetric part of $Q(\mathbf{e})$ will be denoted by $P(Q(\mathbf{e}))$ and $I(Q(\mathbf{e}))$, respectively. The set of all **ESWF**s will be denoted by $\mathcal{Q}$.

The notion of extended social welfare functions enables us to treat the criteria of individual autonomy, equitable assignment of opportunities in terms of objective well-being, and the Pareto principle in a unified framework. Within the domain $Z^n \times \Gamma$ of social preference orderings derived from **ESWFs**, the component of game forms is a necessary informational basis for making orderings based on the criterion of individual autonomy, whereas the information of feasible allocations is relevant to the remaining two criteria.

In the following part, the above mentioned three criteria are formalized

[21] The concept of an extended social alternative was introduced by Pattanaik and Suzumura (1994; 1996), capitalizing on the thought-provoking suggestion by Arrow (1951, pp.89-91).

[22] A binary relation R on a universal set X is a *quasi-ordering* if it satisfies *reflexivity* and *transitivity*. An *ordering* is a quasi-ordering satisfying *completeness* as well.

as axioms of **ESWFs**.

(I) Individual Autonomy in terms of Choice of Labor Hours

According to the theory of individual liberty that John Stuart Mill proposed [Mill (1859)]; there ought to exist in human life a certain *minimal sphere of personal liberty* that should not be interfered with by anybody other than the person in question. Such a sphere should be socially respected and protected as part of individual rights in a liberal society. The question where exactly to draw the boundary between the sphere of personal liberty and that of social authority is a matter of great dispute, and, indeed, how much of a sphere each individual should be entitled to as his rights is a controversial issue. Nevertheless, a claim for inviolability of a minimal sphere of individual libertarian rights seems to be deeply rooted in our social and political goals.

Thus, a resource allocation policy would rarely be accepted, if its goal or its implementation is incompatible with even such minimal guarantee of individual liberty. Such a viewpoint is relevant to our first axiom of extended social welfare functions. We will discuss what constitutes the minimal guarantee of individual rights in the context of resource allocations that this paper considers.

In cases of resource allocation problems, the components of political freedom and the non-economic parts of individual rights might be assumed to be already established. However, there still remain non-established economic parts of individual rights, which might be either treated as *parameters* or as *variables* for relevant resource allocation problems. For instance, we may view *self-ownership* as such a part of individual rights that guarantee individual autonomy. The notion of self-ownership originates from the argument of the *Lockean proviso* of John Locke and was used by Robert Nozick (1974) as the principle to justify private ownership in capitalist societies. Nevertheless, the notion of self-ownership can be connected with two versions of *entitlement principles,* that is, the entitlement principles in the weak sense and in the strong sense, as van Parijs (1995) proposed.

The entitlement principle in the weaker sense regards self-ownership as a variable for society. Thus, according to this weaker sense of the principle, self-ownership can be seen as freedom or respect for the decision-making of individuals and identified with political freedom and freedom of choice of occupations, etc.[23] In this version, the notion of self-ownership is entirely

[23] In fact, van Parijs (1995) insists, "Though not strictly equivalent to 'basic liberties'

consistent with redistribution policies which may induce reconstruction of rights structure in order to achieve a given distributional goal. This is actually the position that van Parijs (1995) takes. On the other hand, *the entitlement principle in the stronger sense* no longer views self-ownership as a control variable, but as a parameter which society consists in respecting. This stronger sense of the principle can be identified with the arguments made by John Locke. This principle also made a solid basis for the *original appropriation of unowned external resources*, which was proposed by Libertarians including Locke and Nozick.

We also take the same position as van Parijs (1995) regarding the notion of self-ownership, and identify the contents of individual liberal rights within the context of resource allocation problems. First, individual liberal rights guarantee freedom of choice in terms of personal consumption. That is, other than the individual in question, no one else has the right to decide the way to dispose of private goods and leisure time available to him/her. W. l.o.g., we should assume that the right of the freedom of choice in consumption, in the context of passive freedom, is presumed to be guaranteed in the standard economic model of resource allocation problems such as this paper adopts.

Secondly, individual liberal rights contain *the right to freedom from forced labor.* This is to guarantee the right to choose one's own labor including the freedom of employment contract. That is to say, at least *the right to choose labor* that is secured in the labor market of contemporary civil society should be guaranteed. However, the simple economic model considered here presumes that there is no difference in professions, and all individuals engage in homogeneous labor. In this restricted world, it may be permissible to assume that the right to choose labor can be reduced to *the right to choose labor hours.*

The right to choose labor hours is defined as follows:

Definition 1 [Kranich (1994)]: *An allocation rule* $\gamma = (M, g) \in \Gamma$ *is* labor-sovereign *if, for all* $i \in N$ *and all* $x_i \in [0, \overline{x}]$, *there exists* $m_i \in M_i$ *such that, for all* $\mathbf{m}_{-i} \in M_{-i}$, $g_{i1}(m_i, \mathbf{m}_{-i}) = x_i$[24].

Let Γ_{LS} denote the subclass of Γ which consists solely of allocation rules

or 'human rights' as expressed, for example, in Rawls's first principle of justice or in the constitutions of liberal democracies, self-ownership is closely associated with most of them." [van Parijs (1995; p. 235; NOTES Chapter 1, 8.)]

[24] Here, that $g_{i1}(m_i, m_{-i}) = x_i$ is describing the work-hour supply of an individual $i \in N$ that the outcome function designates corresponding to the strategy $(m_i, m_{-i}) \in M$.

satisfying labor sovereignty. Then:

Labor Sovereignty (LS): *For any* $\mathbf{e} \in \mathcal{E}$ *and any* $(\mathbf{z}, \gamma), (\mathbf{z}', \gamma') \in \mathcal{R}(\mathbf{e})$, *if* $\gamma \in \Gamma_{LS}$ *and* $\gamma' \in \Gamma \setminus \Gamma_{LS}$, *then* $((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) \in P(Q(\mathbf{e}))$.

The axiom **LS** manifests that the extended social alternative with labor sovereign allocation rule should be given a higher priority than the alternative without it. This manifestation should be implemented regardless of what resource allocations the labor sovereign rule or the non-labor sovereign rule realizes as Nash equilibrium outcomes. In other words, the axiom **LS** claims that society should always be committed to implementing an economic institution (an allocation rule) that guarantees labor sovereignty, regardless of what outcome is expected to be realized. That is to say, this expresses an extremely non-consequential value.

Note that if a society executes a non-labor sovereign rule, then such a society might allow the policy-maker to execute some sort of forced labor. The axiom **LS** rejects such a society and an economic institution. According to this axiom, even an egalitarian redistribution policy would not possibly be accepted if it was carried out only with a process that involves forced labor. I believe that the principle of self-ownership based on the weak sense of entitlement principle, and also, even Rawls's *first principle of justice* [Rawls (1971)] should have the form of **LS** within the world of this economic model.

(II) Evaluation based on a criterion of Distributive Justice

Our next criterion is meant to capture an aspect of non-welfaristic egalitarianism. It hinges on what theories of distributive justice we take, which requires an instrument that incorporates various criteria of distributive justice.

Such an instrument is given by a mapping $J : \mathcal{E} \twoheadrightarrow Z^n \times Z^n$ which associates a binary relation $J(\mathbf{e}) \subseteq Z(\mathbf{e}) \times Z(\mathbf{e})$ with each economy $\mathbf{e} \in \mathcal{E}$. The interpretation of this is that such a binary relation $J(\mathbf{e})$ represents a criterion based on a certain theory of distributive justice and alternative feasible allocations are ranked according to this criterion. For instance, if the mapping J represents Sen's theory of equality of capability, then $J(\mathbf{e})$ provides a ranking over alternative capability assignments available to each economy $\mathbf{e} \in \mathcal{E}$, and the rational choice set derived from this $J(\mathbf{e})$ is regarded as consisting of the most 'equitable' capability assignments under $\mathbf{e} \in \mathcal{E}$.[25]

[25] Gotoh, Suzumura, and Yoshihara (2005) argues this case.

In this case, the rankings made by J should be invariant with respect to the change in the profile of utility functions: that is, $J(\mathbf{e}) = J(\mathbf{e}')$ holds whenever $\mathbf{s} = \mathbf{s}'$ holds. In contrast, if J represents Dworkinian theory of "equality of resources" [Dworkin, (1981b; 2000)], J might not have such an invariance property: that is, $J(\mathbf{e}) \neq J(\mathbf{e}')$ may hold even if $\mathbf{s} = \mathbf{s}'$. Moreover, if J represents the theory of "equality of welfare," then $J(\mathbf{e}) = J(\mathbf{e}')$ should hold for any $\mathbf{e}, \mathbf{e}' \in \mathcal{E}$ with $\mathbf{u} = \mathbf{u}'$. In such a way, this mapping can be universally applicable.

In this paper, by means of this instrument, we will formulate the criterion of leximin assignment of opportunity sets suggested by van Parijs (1995). Note that van Parijs (1995) defined the opportunity set as the *budget set* and also defined its leximin assignment by the condition called *undominated diversity*, which is defined in the following way:

Definition 2 [Parijs (1995)]: *Given any economy* $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$, *a feasible allocation* $\mathbf{z} \in Z(\mathbf{s})$ *meets* undominated diversity *if for any* $i, j \in N$, *there exists at least one individual* $k \in N$ *such that* $u_k(z_i) \geq u_k(z_j)$.

Note that undominated diversity is a far weaker condition than the *no envy* criterion [Foley (1967)].

Now, we are ready to define a binary relation mapping J^{UD} based on the undominated diversity criterion. Given an economy $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$ and a feasible allocation $\mathbf{z} \in Z(\mathbf{s})$, let $H(\mathbf{z}; \mathbf{u}) \subseteq N \times N$ be defined as:

$$H(\mathbf{z}; \mathbf{u}) \equiv \{(i, j) \in N \times N \mid \forall k \in N : u_k(z_j) > u_k(z_i)\}.$$

By the definition, the set $H(\mathbf{z}, \mathbf{u})$ is the set of pairs under 'dominant relationship.' Note that the pair (i, j) is under 'dominant relationship' in the sense that i is dominated by j if every individual judges that j's consumption bundle is strictly better than i's consumption bundle. The smaller this set is in terms of set-inclusion, the higher the degree of undominated diversity under this allocation $\mathbf{z}$ is considered to be. The binary relation mapping J^{UD} is now defined as follows: for any economy $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$ and any feasible allocations $\mathbf{z}, \mathbf{z}' \in Z(\mathbf{s})$,

$$\begin{aligned} I\left(J^{UD}(\mathbf{e})\right) &\equiv \{(\mathbf{z}, \mathbf{z}') \in Z(\mathbf{s}) \times Z(\mathbf{s}) \mid \mathbf{z} = \mathbf{z}'\}; \\ P\left(J^{UD}(\mathbf{e})\right) &\equiv \{(\mathbf{z}, \mathbf{z}') \in Z(\mathbf{s}) \times Z(\mathbf{s}) \mid H(\mathbf{z}; \mathbf{u}) \subsetneq H(\mathbf{z}'; \mathbf{u})\}. \end{aligned}$$

Note that this mapping J^{UD} guarantees that for every economy $\mathbf{e} \in \mathcal{E}$, $P\left(J^{UD}(\mathbf{e})\right)$ is a continuous strict partial ordering on $Z(\mathbf{s})$.

Now, our second axiom on extended social welfare function is given by means of the binary relation mapping J, as follows:

Respect for J-based fairness (J-RF): *For any* $\mathbf{e} \in \mathcal{E}$ *and any* $(\mathbf{z}, \gamma), (\mathbf{z}', \gamma') \in \mathcal{R}(\mathbf{e})$*, if* $\mathbf{z} = (\mathbf{x}, \mathbf{y})$, $\mathbf{z}' = (\mathbf{x}', \mathbf{y}')$ *and* $\mathbf{x} = \mathbf{x}'$*, then*:

$$\begin{aligned} ((\mathbf{z}, \gamma), (\mathbf{z}, \gamma)) &\in Q(\mathbf{e}) \Leftrightarrow (\mathbf{z}, \mathbf{z}') \in J(\mathbf{e}); \\ ((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) &\in P(Q(\mathbf{e})) \Leftrightarrow (\mathbf{z}, \mathbf{z}') \in P(J(\mathbf{e})). \end{aligned}$$

The axiom **J-RF** focuses only on the feasible allocation in evaluating relative wellness of any two extended alternatives, and under a certain constraint, it claims that the evaluation by the extended social welfare function over extended alternatives should be consistent with the evaluation by J over feasible allocations. Here, the "certain constraint" is given by "$\mathbf{z} = (\mathbf{x}, \mathbf{y})$, $\mathbf{z}' = (\mathbf{x}', \mathbf{y}')$ and $\mathbf{x} = \mathbf{x}'$."

As such, **J-RF** evaluates the desirability of extended social alternatives only from the viewpoint of J-fairness on resource allocations. Moreover, unlike the axiom **LS**, **J-RF** stands in the position of consequentialism. This is because this axiom evaluates the extended alternatives based solely on the evaluation of their corresponding resource allocations.

(III) Evaluation based on the welfarist value

Finally, let us introduce the axiom of the extended social welfare function based on the welfarist value. It is defined as an extension of the standard Pareto principle:

Pareto in Allocations (PA): *For any* $\mathbf{e} \in \mathcal{E}$ *and any* $(\mathbf{z}, \gamma), (\mathbf{z}', \gamma') \in \mathcal{R}(\mathbf{e})$*, if* $u_i(z_i) > u_i(z_i')$ *for all* $i \in N$*, then* $((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) \in P(Q(\mathbf{e}))$*, and if* $u_i(z_i) = u_i(z_i')$ *for all* $i \in N$*, then* $((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) \in I(Q(\mathbf{e}))$.

The axiom **PA** also focuses only on the feasible allocation in evaluating relative wellness of any two extended alternatives, and it claims that the evaluation by the extended social welfare function over extended alternatives should be consistent with the Pareto superiority relation or the Pareto indifference relation over feasible allocations. Moreover, by the almost same reason as the case of **J-RF**, **PA** also stands in the position of consequentialism.

3.2 Impossibility of ESWFs satisfying LS, J-RF, and PA

Now, we are ready to discuss the existence of **ESWF**s which simultaneously satisfy the axioms **LS**, **J-RF**, and **PA**. According to the technique introduced in **Appendix 1** of this paper, we can see this problem by examining the properties of binary relations, each of which respectively represents one of the above mentioned axioms.

Let us define three binary relation functions, each of which respectively represents one of the axioms **LS**, **J-RF**, and **PA**. Let $Q^{LS} : \mathcal{E} \twoheadrightarrow (Z^n \times \Gamma)^2$ be a binary relation function such that for any $\mathbf{e} \in \mathcal{E}$ and any $(\mathbf{z}, \gamma), (\mathbf{z}', \gamma') \in \mathcal{R}(\mathbf{e})$, the following holds:

$$\begin{aligned} ((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) &\in P(Q^{LS}(\mathbf{e})) \Leftrightarrow [\gamma \in \Gamma_{IS} \,\&\, \gamma' \notin \Gamma_{IS}]\,; \\ ((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) &\in I(Q^{LS}(\mathbf{e})) \Leftrightarrow (\mathbf{z}, \gamma) = (\mathbf{z}', \gamma'). \end{aligned}$$

Let $Q^{JRF} : \mathcal{E} \twoheadrightarrow (Z^n \times \Gamma)^2$ be a binary relation function such that for any $\mathbf{e} \in \mathcal{E}$ and any $(\mathbf{z}, \gamma), (\mathbf{z}', \gamma') \in \mathcal{R}(\mathbf{e})$ with $\mathbf{z} = (\mathbf{x}, \mathbf{y})$ and $\mathbf{z}' = (\mathbf{x}', \mathbf{y}')$, the following holds:

$$\begin{aligned} ((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) &\in I\left(Q^{JRF}(\mathbf{e})\right) \Leftrightarrow \mathbf{x} = \mathbf{x}' \text{ and } (\mathbf{z}, \mathbf{z}') \in I\left(J\left(\mathbf{e}\right)\right); \\ ((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) &\in P\left(Q^{JRF}(\mathbf{e})\right) \Leftrightarrow \mathbf{x} = \mathbf{x}' \text{ and } (\mathbf{z}, \mathbf{z}') \in P\left(J\left(\mathbf{e}\right)\right). \end{aligned}$$

Let $Q^{PA} : \mathcal{E} \twoheadrightarrow (Z^n \times \Gamma)^2$ be a binary relation function such that for any $\mathbf{e} \in \mathcal{E}$ and any $(\mathbf{z}, \gamma), (\mathbf{z}', \gamma') \in \mathcal{R}(\mathbf{e})$, the following holds:

$$\begin{aligned} ((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) &\in P\left(Q^{PA}(\mathbf{e})\right) \Leftrightarrow u_i(z_i) > u_i(z_i') \quad (\forall i \in N)\,; \\ ((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) &\in I\left(Q^{PA}(\mathbf{e})\right) \Leftrightarrow u_i(z_i) = u_i(z_i') \quad (\forall i \in N)\,. \end{aligned}$$

Thus, we can define a union $Q^{LJP}(\mathbf{e}) \equiv Q^{LS}(\mathbf{e}) \cup Q^{JRF}(\mathbf{e}) \cup Q^{PA}(\mathbf{e})$ for each $\mathbf{e} \in \mathcal{E}$. According to **Proposition 1** in **Appendix 1**, the existence of **ESWF**s which simultaneously satisfy the axioms **LS**, **J-RF**, and **PA** can confirm if and only if the binary relation $Q^{LJP}(\mathbf{e})$ is *consistent* for each $\mathbf{e} \in \mathcal{E}$.

Unfortunately, it is easily confirmed that $Q^{LJP}(\mathbf{e})$ is not consistent for some $\mathbf{e} \in \mathcal{E}$. Thus, the answer to the above existence issue is negative. The inconsistency of $Q^{LJP}(\mathbf{e})$ is due to the fact that the union of any two of the three binary relations $Q^{LS}(\mathbf{e})$, $Q^{JRF}(\mathbf{e})$, and $Q^{PA}(\mathbf{e})$ cannot constitute a consistent relation. To begin with, the inconsistency of $Q^{LS}(\mathbf{e}) \cup Q^{JRF}(\mathbf{e})$ is easily confirmed by the fact that $Q^{LS}(\mathbf{e})$ is interested solely in wellness of allocation rules, whereas $Q^{JRF}(\mathbf{e})$ represents the criterion which judges the wellness of extended alternatives, completely ignoring the wellness of allocation rules. A similar argument can be applied to the case of $Q^{LS}(\mathbf{e}) \cup Q^{PA}(\mathbf{e})$.

In contrast, how about the binary relation $Q^{JRF}(\mathbf{e}) \cup Q^{PA}(\mathbf{e})$? This is related to the issue known as the problem of compatibility between fairness and efficiency in resource allocations, and its answer exclusively depends on the properties of the criteria of distributive justice J.

If J represents a non-welfarist theory of distributive justice, then the binary relation made by J might totally ignore the information of individual utility functions. Thus, in such a case, it is easy to image that such $Q^{JRF}(\mathbf{e})$ becomes incompatible with $Q^{PA}(\mathbf{e})$ under some $\mathbf{e} \in \mathcal{E}$.[26] How about the case where the criterion of distributive justice is composed of the profile of utility functions as (a part of) the informational basis ? For instance, it is well known that $Q^{JRF}(\mathbf{e})$ and $Q^{PA}(\mathbf{e})$ are still incompatible with each other if the binary relation J represents the no envy criterion.[27] So, how about the case that J represents undominated diversity ? It is still impossible to avoid such a contradiction with the axiom **PA**, as the following example shows.

Example 1: Let $N = \{1, 2, 3\}$ and $\overline{x} = 3$. The production function is given by $f(x) = x$ for all $x \in \mathbb{R}_+$. Define an economic environment $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$ as follows: Let $s_i = 1$ for any $i \in N$. Now, the utility function of the individual 1 is given by: for any $(x, y) \in Z$, $u_1(x, y) = (1 - \theta) \cdot (\overline{x} - x) + y$ where $\theta \in (0, 1)$. Also, the utility function of the individual 2 is given by: for any $(x, y) \in Z$,

$$u_2(x, y) = \begin{cases} (1 - \theta) \cdot (\overline{x} - x) + y & \text{if } x \in [0, 1) \\ (1 + \theta) \cdot (\overline{x} - x) + y & \text{if } x \in [1, \overline{x}] \end{cases}.$$

Finally, let $u_3 = u_1$be the utility function of the individual 3.

Consider the following four feasible allocations: $\mathbf{z}^* = ((1, 1), (1, 1), (1, 1))$, $\mathbf{z}^{**} = ((2, 2), (2, 2), (1, 1))$, $\mathbf{z}^*(\theta) = ((1, 1 + \theta), (1, 1 - \theta), (1, 1))$, and $\mathbf{z}^{**}(\theta) = ((2, 2 + \theta), (2, 2 - \theta), (1, 1))$. Let γ^*, γ^{**}, $\gamma^*(\theta)$, and $\gamma^{**}(\theta)$ be the allocation rules respectively in which $\mathbf{z}^*$, $\mathbf{z}^{**}$, $\mathbf{z}^*(\theta)$, and $\mathbf{z}^{**}(\theta)$ become respectively Nash equilibrium outcomes under $\mathbf{e} \in \mathcal{E}$. Then, by the definition of

[26] In fact, Gotoh, Suzumura, and Yoshihara (2005) showed that if J represents Sen's theory of "equality of capability," then **J-RF** is incompatible with **PA**.

[27] The example of the latest successful research is Tadenuma (2002). Also, see Yoshihara (2005).

$Q^{J^{UD}RF}(\mathbf{e})$, we have:

$$((\mathbf{z}^*,\gamma^*),(\mathbf{z}^*(\theta),\gamma^*(\theta))) \in P\left(Q^{J^{UD}RF}(\mathbf{e})\right),\ ((\mathbf{z}^{**},\gamma^{**}),(\mathbf{z}^{**}(\theta),\gamma^{**}(\theta))) \in P\left(Q^{J^{UD}RF}(\mathbf{e})\right).$$

On the other hand, by the definition of $Q^{PA}(\mathbf{e})$, we have:

$$((\mathbf{z}^*(\theta),\gamma^*(\theta)),(\mathbf{z}^{**},\gamma^{**})) \in I\left(Q^{PA}(\mathbf{e})\right),\ ((\mathbf{z}^{**}(\theta),\gamma^{**}(\theta)),(\mathbf{z}^*,\gamma^*)) \in I\left(Q^{PA}(\mathbf{e})\right).$$

Thus, we can confirm that the binary relation $Q^{J^{UD}RF}(\mathbf{e}) \cup Q^{PA}(\mathbf{e})$ is not consistent. ■

3.3 On Possibility of Second-Best Extended Social Welfare Functions

So far, the previous section showed that there is no **ESWF** which satisfies the three basic axioms, **LS, J-RF, PA**, simultaneously. Then, the next step to argue is to examine the possibility of the second-best **ESWF**s which satisfy some weaker requirements of the three basic axioms. There are at least two types of methods to solve this problem. The first method is based on the *pluralistic application of axioms* proposed by Sen and Williams (1982). The second method is based on the *lexicographic application of axioms*. The formal definitions of these approaches are given in **Appendix 2**.

Here, we focus on the lexicographic application, which was sometimes practiced in the literature of normative theories such as Rawls (1971) and van Parijs (1995). To see the possibility of the second-best **ESWF**s based on the lexicographic application, let us first examine **J-RF** *first*-**PA** *second priority rule*, which is to consider the binary relation function $Q_{lex}^{J^{UD}RF \vdash PA} : \mathcal{E} \twoheadrightarrow (Z^n \times \Gamma)^2$ defined as follows. For any $\mathbf{e} \in \mathcal{E}$ and any binary relation $Q(\mathbf{e}) \subseteq (Z^n \times \Gamma)^2$, let $N(Q(\mathbf{e})) \subseteq (Z^n \times \Gamma)^2$ be defined by: for any $(\mathbf{z},\gamma), (\mathbf{z}',\gamma') \in \mathcal{R}(\mathbf{e})$,

$$((\mathbf{z},\gamma),(\mathbf{z}',\gamma')) \in N(Q(\mathbf{e})) \Leftrightarrow ((\mathbf{z},\gamma),(\mathbf{z}',\gamma')) \notin Q(\mathbf{e})\ \&\ ((\mathbf{z}',\gamma'),(\mathbf{z},\gamma)) \notin Q(\mathbf{e}).$$

Then, for any $\mathbf{e} \in \mathcal{E}$,

$$\begin{aligned} Q_{lex}^{J^{UD}RF \vdash PA}(\mathbf{e}) &\equiv Q^{J^{UD}RF}(\mathbf{e}) \cup \left[N\left(Q^{J^{UD}RF}(\mathbf{e})\right) \cap Q^{PA}(\mathbf{e})\right]; \text{ and} \\ P\left(Q_{lex}^{J^{UD}RF \vdash PA}(\mathbf{e})\right) &\equiv P\left(Q^{J^{UD}RF}(\mathbf{e})\right) \cup \left[N\left(Q^{J^{UD}RF}(\mathbf{e})\right) \cap P\left(Q^{PA}(\mathbf{e})\right)\right]. \end{aligned}$$

In a similar way, we can also consider **PA** *first*-**J-RF** *second priority rule*, and define the binary relation function $Q_{lex}^{PA \vdash J^{UD}RF} : \mathcal{E} \twoheadrightarrow (Z^n \times \Gamma)^2$.

Unfortunately, we can see both $Q_{lex}^{J^{UD}RF \vdash PA}(\mathbf{e})$ and $Q_{lex}^{PA \vdash J^{UD}RF}(\mathbf{e})$ are not consistent for some $\mathbf{e} \in \mathcal{E}$, which is checked by using the same four feasible allocations and the same economic environment as in **Example 1**. In fact, we can see in **Example 1** that

$$((\mathbf{z}^*, \gamma^*), (\mathbf{z}^*(\theta), \gamma^*(\theta))) \in N\left(Q^{PA}(\mathbf{e})\right) \ \& \ ((\mathbf{z}^{**}, \gamma^{**}), (\mathbf{z}^{**}(\theta), \gamma^{**}(\theta))) \in N\left(Q^{PA}(\mathbf{e})\right),$$

which implies the discussion of inconsistency in **Example 1** can be applied to $Q_{lex}^{PA \vdash J^{UD}RF}(\mathbf{e})$. The same discussion is applied to the binary relation $Q_{lex}^{J^{UD}RF \vdash PA}(\mathbf{e})$.

As the discussion above indicates, we cannot construct any second-best **ESWF** based on the lexicographic application, because neither $Q_{lex}^{J^{UD}RF \vdash PA}$ nor $Q_{lex}^{PA \vdash J^{UD}RF}$ is a consistent binary relation function. Thus, to secure the existence of a compatible lexicographic combination of our basic axioms, a further concession seems to be indispensable.

As such a further concession, let us consider, for each $\mathbf{e} \in \mathcal{E}$, to choose appropriately a subset $N^*\left(Q^{J^{UD}RF}(\mathbf{e})\right)$ from $N\left(Q^{J^{UD}RF}(\mathbf{e})\right)$ such that $Q^{J^{UD}RF}(\mathbf{e}) \cup \left[N^*\left(Q^{J^{UD}RF}(\mathbf{e})\right) \cap Q^{PA}(\mathbf{e})\right]$ becomes consistent. For any $\mathbf{e} \in \mathcal{E}$ and any $(\mathbf{z}, \gamma), (\mathbf{z}', \gamma') \in \mathcal{R}(\mathbf{e})$, let $N^*\left(Q^{J^{ND}RF}(\mathbf{e})\right) \subsetneq N\left(Q^{J^{ND}RF}(\mathbf{e})\right)$ be given as follows:

$$((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) \in N^*\left(Q^{J^{ND}RF}(\mathbf{e})\right) \Leftrightarrow \mathbf{x} \neq \mathbf{x}' \text{ or } H(\mathbf{z}; \mathbf{u}) = H(\mathbf{z}'; \mathbf{u}) = \varnothing.$$

Then, for any $\mathbf{e} \in \mathcal{E}$, let:

$$\begin{aligned} Q_{lex}^{*J^{UD}RF \vdash PA}(\mathbf{e}) &\equiv Q^{J^{UD}RF}(\mathbf{e}) \cup \left[N^*\left(Q^{J^{UD}RF}(\mathbf{e})\right) \cap Q^{PA}(\mathbf{e})\right]; \\ P\left(Q_{lex}^{*J^{UD}RF \vdash PA}(\mathbf{e})\right) &\equiv P\left(Q^{J^{UD}RF}(\mathbf{e})\right) \cup \left[N^*\left(Q^{J^{UD}RF}(\mathbf{e})\right) \cap P\left(Q^{PA}(\mathbf{e})\right)\right]. \end{aligned}$$

Also, for any $\mathbf{e} \in \mathcal{E}$, let $N^*\left(Q^{PA}(\mathbf{e})\right)$ be a subset of $N\left(Q^{PA}(\mathbf{e})\right)$ such that

$$((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) \in N^*\left(Q^{PA}(\mathbf{e})\right) \Leftrightarrow \mathbf{z}, \mathbf{z}' \in P(\mathbf{e}) \text{ and } \mathbf{u}(\mathbf{z}) \neq \mathbf{u}(\mathbf{z}').$$

Then, for any $\mathbf{e} \in \mathcal{E}$, let:

$$\begin{aligned} Q_{lex}^{*PA \vdash J^{UD}RF}(\mathbf{e}) &\equiv Q^{PA}(\mathbf{e}) \cup \left[N^*\left(Q^{PA}(\mathbf{e})\right) \cap Q^{J^{UD}RF}(\mathbf{e})\right]; \\ P\left(Q_{lex}^{*PA \vdash J^{UD}RF}(\mathbf{e})\right) &\equiv P\left(Q^{PA}(\mathbf{e})\right) \cup \left[N^*\left(Q^{PA}(\mathbf{e})\right) \cap P\left(Q^{J^{UD}RF}(\mathbf{e})\right)\right]. \end{aligned}$$

We can see that for any $\mathbf{e} \in \mathcal{E}$, both $N^*\left(Q^{J^{UD}RF}(\mathbf{e})\right)$ and $N^*\left(Q^{PA}(\mathbf{e})\right)$ are *connected* and *transitive*, where the definition of connectedness is given in **Definition 6** of **Appendix 2**. Hence, by **Proposition 2** of **Appendix 2**, we can see that both $Q_{lex}^{*J^{UD}RF \vdash PA}$ and $Q_{lex}^{*PA \vdash J^{UD}RF}$ are consistent binary relation functions. Thus, we obtain:

Theorem 1: *There exist at least four* ***ESWFs*** *such that each of which contains either of the following binary relation functions as subrelation mappings*:
(i) $Q_{lex}^{LS \vdash \left(*PA \vdash J^{UD}RF\right)}$; (ii) $Q_{lex}^{\left(*PA \vdash J^{UD}RF\right) \vdash LS}$; (iii) $Q_{lex}^{LS \vdash \left(*J^{UD}RF \vdash PA\right)}$; and
(iv) $Q_{lex}^{\left(*J^{UD}RF \vdash PA\right) \vdash LS}$.

Let $Q^{LS \vdash \left(*PA \vdash J^{UD}RF\right)}$ (resp. $Q^{LS \vdash \left(*J^{UD}RF \vdash PA\right)}$) be an **ESWF** which is obtained as an ordering extension of $Q_{lex}^{LS \vdash \left(*PA \vdash J^{UD}RF\right)}$ (resp. $Q_{lex}^{LS \vdash \left(*J^{UD}RF \vdash PA\right)}$). Note that both of $Q^{LS \vdash \left(*PA \vdash J^{UD}RF\right)}$ and $Q^{LS \vdash \left(*J^{UD}RF \vdash PA\right)}$ are interesting from the viewpoint of non-welfaristic normative theories. Both of them are sought out by the *weaker* sense of lexicographic application as discussed above, and give the first priority to a non-consequential axiom **LS** rather than the other two consequentialist axioms. Thus, we can regard each of them as a formulation of van Parijs's Real Libertarianism plus the Pareto principle, since van Parijs's Real Libertarianism is able to be formalized as $Q^{LS \vdash J^{UD}RF}$ within this economic model. In other words, the above theorem shows the possibility of consistent social evaluations based on '*Real Libertarianism combined with the Pareto principle.*'

3.4 Characterizations of Rationally Chosen Allocation Rules via ESWFs

In this subsection, we discuss characterizations of allocation rules which are rationally chosen via $Q^{LS \vdash \left(*PA \vdash J^{UD}RF\right)}$ and/or $Q^{LS \vdash \left(*J^{UD}RF \vdash PA\right)}$. In the first place, let us define rational choice sets of allocation rules in this framework. Given any **ESWF** Q, the rational choice set $C(Q)$ of allocation rules associated with Q is defined by: for any $\mathbf{e} \in \mathcal{E}$,

$$\gamma \in C(Q) \Leftrightarrow \forall \mathbf{e} \in \mathcal{E},\ \exists \mathbf{z} \in \tau_{NE}(\gamma, \mathbf{e}) \text{ s.t. } \forall (\mathbf{z}', \gamma') \in \mathcal{R}(\mathbf{e}),\ ((\mathbf{z}, \gamma), (\mathbf{z}', \gamma')) \in Q(\mathbf{e}).$$

As discussed in the previous subsection, both $Q^{LS\vdash(*PA\vdash J^{UD}RF)}$ and $Q^{LS\vdash(*J^{UD}RF\vdash PA)}$ can be regarded as formulations of 'Real Libertarianism combined with the Pareto principle.' Then, the next interesting issue is what kinds of allocation rules can be rationally chosen via these **ESWF**s. That is, we are interested in the characterizations of $C\left(Q^{LS\vdash(*PA\vdash J^{UD}RF)}\right)$ and/or $C\left(Q^{LS\vdash(*J^{UD}RF\vdash PA)}\right)$. In particular, if $\gamma \in C\left(Q^{LS\vdash(*PA\vdash J^{UD}RF)}\right) \cup C\left(Q^{LS\vdash(*J^{UD}RF\vdash PA)}\right)$, then what kinds of feasible allocations this γ can implement in Nash equilibria ? For instance, if $\mathbf{z} \in \tau_{NE}(\gamma, \mathbf{e})$ at some $\mathbf{e} \in \mathcal{E}$, is $\mathbf{z}$ Pareto efficient for $\mathbf{e}$? Or, does $\mathbf{z}$ satisfy the undominated diversity condition ? To examine such questions, let a rationally chosen allocation rule $\gamma \in C\left(Q^{LS\vdash(*PA\vdash J^{UD}RF)}\right) \cup C\left(Q^{LS\vdash(*J^{UD}RF\vdash PA)}\right)$ be called the *first best allocation rule* if for any $\mathbf{e} \in \mathcal{E}$ and any $\mathbf{z} \in \tau_{NE}(\gamma, \mathbf{e})$, $\mathbf{z}$ is Pareto efficient and meets undominated diversity. Now, our particular interest is reduced to the existence issue of the first best allocation rule rationalized by $Q^{LS\vdash(*PA\vdash J^{UD}RF)}$ and/or $Q^{LS\vdash(*J^{UD}RF\vdash PA)}$.

First of all, we have to check compatibility between Pareto efficiency and undominated diversity in feasible allocations. Given any $\mathbf{e} \in \mathcal{E}$, denote the set of all feasible allocations satisfying undominated diversity by $UD(\mathbf{e})$. Let $PD(\mathbf{e}) \equiv P(\mathbf{e}) \cap UD(\mathbf{e})$ for any $\mathbf{e} \in \mathcal{E}$. Then, our first problem is to check whether $PD(\mathbf{e})$ is non-empty or not for any $\mathbf{e} \in \mathcal{E}$. Fortunately, we obtain the positive answer of this as shown below.

Given any $\mathbf{e} \in \mathcal{E}$, let $\mathbf{z} \in P(\mathbf{e})$. Denote the set of efficiency prices each of which supports $\mathbf{z}$ at $\mathbf{e}$ by $\triangle(\mathbf{z}, \mathbf{e})$. Denote the generic element of $\triangle(\mathbf{z}, \mathbf{e})$ by $p^{(\mathbf{z},\mathbf{e})}$. Then, for any $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$, any $\mathbf{z} \in P(\mathbf{e})$, any $p^{(\mathbf{z},\mathbf{e})} \in \triangle(\mathbf{z}, \mathbf{e})$, and any $i \in N$, i's *budget set* is defined by:

$$B\left(z_i, s_i, p^{(\mathbf{z},\mathbf{e})}\right) \equiv \left\{(x, y) \in Z \mid p_y^{(\mathbf{z},\mathbf{e})} y - p_x^{(\mathbf{z},\mathbf{e})} s_i x \leq p_y^{(\mathbf{z},\mathbf{e})} y_i - p_x^{(\mathbf{z},\mathbf{e})} s_i x_i\right\}.$$

Given a budget set $B\left(z_i, s_i, p^{(\mathbf{z},\mathbf{e})}\right)$ and a utility function $u \in \mathcal{U}$, denote the set of *optimal consumptions for u over* $B\left(z_i, s_i, p^{(\mathbf{z},\mathbf{e})}\right)$ by $d\left(u, B\left(z_i, s_i, p^{(\mathbf{z},\mathbf{e})}\right)\right)$. Needless to say, $z_i \in d\left(u_i, B\left(z_i, s_i, p^{(\mathbf{z},\mathbf{e})}\right)\right)$ because $\mathbf{z} \in P(\mathbf{e})$. Given $u \in \mathcal{U}$ and $z \in Z$, let $r_z^u \in \mathbb{R}_+$ be given by $u(z) = u(0, r_z^u)$. Given $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$ and $\mathbf{z} \in P(\mathbf{e})$, let $i_{\min}^Y \equiv \arg\min \left\{r_{z_j}^{u_j}\right\}_{j \in N}$. Then:

Definition 7 [Fleurbaey and Maniquet (1996)]**:** *Given* $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$, $\mathbf{z} \in P(\mathbf{e})$ *is a* φ^Y-allocation for $\mathbf{e}$ *if there exists* $p^{(\mathbf{z},\mathbf{e})} \in \triangle(\mathbf{z}, \mathbf{e})$ *such that* *for any* $i, j \in N$,

$$u_{i^Y_{\min}}\left(d\left(u_{i^Y_{\min}}, B\left(z_i, s_i, p^{(\mathbf{z},\mathbf{e})}\right)\right)\right) = u_{i^Y_{\min}}\left(d\left(u_{i^Y_{\min}}, B\left(z_j, s_j, p^{(\mathbf{z},\mathbf{e})}\right)\right)\right).$$

Denote the set of all φ^Y-allocations for $\mathbf{e}$ by $\varphi^Y(\mathbf{e})$.

Next, given $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$, let $i^S_{\min} \equiv \arg\min \{s_j\}_{j \in N}$. Then:

Definition 8 [Fleurbaey and Maniquet (1996)]**:** *Given* $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$, $\mathbf{z} \in P(\mathbf{e})$ *is a* φ^S-allocation for $\mathbf{e}$ *if there exists* $p^{(\mathbf{z},\mathbf{e})} \in \triangle(\mathbf{z}, \mathbf{e})$ *such that for any* $i, j \in N$,

$$u_{i^S_{\min}}\left(d\left(u_{i^S_{\min}}, B\left(z_i, s_i, p^{(\mathbf{z},\mathbf{e})}\right)\right)\right) = u_{i^S_{\min}}\left(d\left(u_{i^S_{\min}}, B\left(z_j, s_j, p^{(\mathbf{z},\mathbf{e})}\right)\right)\right).$$

Denote the set of all φ^S-allocations for $\mathbf{e}$ by $\varphi^S(\mathbf{e})$.

Note that both $\varphi^Y(\mathbf{e})$ and $\varphi^S(\mathbf{e})$ are non-empty for any $\mathbf{e} \in \mathcal{E}$, which was shown by Fleurbaey and Maniquet (1996). Moreover, by the above definitions, it can be shown that both $\varphi^Y(\mathbf{e})$ and $\varphi^S(\mathbf{e})$ satisfy undominated diversity. Thus, we can now see that $PD(\mathbf{e})$ is non-empty for any $\mathbf{e} \in \mathcal{E}$.

Now, we are ready to move to the next step of the question. This is to examine the existence of $\gamma \in \Gamma_L$ with the property that for any $\mathbf{e} \in \mathcal{E}$, $\tau_{NE}(\gamma, \mathbf{e}) \subseteq PD(\mathbf{e})$. If the answer of this problem is negative, we should conclude that there is no first best allocation rule rationalized by either $Q^{LS\vdash\left(*PA\vdash J^{UD}RF\right)}$ or $Q^{LS\vdash\left(*J^{UD}RF\vdash PA\right)}$. Unfortunately, by means of the results in Fleurbaey and Maniquet (1996), the answer is negative.

Proposition 3: *There is no allocation rule* $\gamma \in \Gamma$ *such that for any* $\mathbf{e} \in \mathcal{E}$, $\tau_{NE}(\gamma, \mathbf{e}) \subseteq PD(\mathbf{e})$.

Proof. Define any correspondence φ such that: for any $\mathbf{e} \in \mathcal{E}$, $\varnothing \neq \varphi(\mathbf{e}) \subseteq PD(\mathbf{e})$. Then, the problem is reduced to that of Nash implementability of φ. Suppose an economy $\mathbf{e} \in \mathcal{E}$, in which $u_i = u_j$ holds for any $i, j \in N$. In this case, if $\mathbf{z} \in \varphi(\mathbf{e})$, it implies that $u_i(z_i) = u_j(z_j)$ for any $i, j \in N$. This follows from the fact that $\mathbf{z}$ satisfies undominated diversity. Thus, φ satisfies *Equal Welfare for Uniform Preferences* (**EWUP**) which was proposed by Fleurbaey and Maniquet (1996). According to Fleurbaey and Maniquet (1996), we can see that if φ satisfies **EWUP** and assigns a subset of Pareto efficient allocations for any $\mathbf{e} \in \mathcal{E}$, then φ does not satisfy *Maskin Monotonicity* (**MM**) [Maskin (1977)]. Thus, by the Maskin theorem [Maskin (1977)] on Nash implementation, we conclude that φ is not Nash-implementable. ■

This proposition implies the non-existence of the first best allocation rule meeting Real Libertarianism combined with the Pareto principle. Thus, any allocation rule $\gamma \in \Gamma_L$, which is rationalized by either $Q^{LS \vdash \left(*PA \vdash J^{UD}RF\right)}$ or $Q^{LS \vdash \left(*J^{UD}RF \vdash PA\right)}$, only implements either *Pareto efficient, but non-undominated diversity allocations* or *non-efficient and undominated diversity allocations* in Nash equilibria. Hence, whenever a society implements the basic income policy of Real Libertarianism, which is characterized by **LS** and $\mathbf{J}^{UD}$**-RF**, this society should sacrifice the efficiency in resource allocations.

Given the above impossibility result, we may consider an alternative formulation of the 'leximin assignment of opportunity sets.' Instead of undominated diversity, we may reformulate the idea of 'leximin assignment of opportunity sets' by using the information of budget sets straightforwardly. Given $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$ and $x \in \mathbb{R}_+$, let

$$\triangle (\mathbf{s}, x) \equiv \left\{p \in \mathbb{R}^2_+ \mid p_y f(x) - p_x x \geq p_y f(x') - p_x x' \ \ (\forall x' \in \mathbb{R}_+)\right\}.$$

Then:

Definition 9 [Yamada and Yoshihara (2005)]: *Given any economy* $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$, *a feasible allocation* $\mathbf{z} \in Z(\mathbf{s})$ *meets* set-inclusion undomination *if there exists* $p \in \triangle \left(\mathbf{s}, \sum_{i \in N} s_i x_i\right)$ *such that for any* $i, j \in N$, *neither* $B(z_i, s_i, p) \subsetneq B(z_j, s_j, p)$ *nor* $B(z_i, s_i, p) \supsetneq B(z_j, s_j, p)$.

I believe that set-inclusion undomination is a necessary condition that should be satisfied by any feasible allocation associated with a 'leximin assignment of opportunity sets' in terms of budget sets. In other words, if a feasible allocation does not meet this condition, such an allocation is no longer said to guarantee any kind of 'leximin assignment of opportunity sets' in terms of budget sets. In fact, set-inclusion undomination is implied by undominated diversity.

Define a binary relation mapping J^{SU} based on the set-inclusion undomination criterion. Given an economy $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$ and a feasible allocation $\mathbf{z} \in Z(\mathbf{s})$, let $\widetilde{H}(\mathbf{z}; \mathbf{s}) \subseteq N \times N$ be defined as:

$$\widetilde{H}(\mathbf{z}; \mathbf{s}) \equiv \left\{(i,j) \in N \times N \mid \exists p \in \triangle \left(\mathbf{s}, \sum_{i \in N} s_i x_i\right) : B(z_i, s_i, p) \subsetneq B(z_j, s_j, p)\right\}.$$

The binary relation mapping J^{SU} is now defined as follows: for any economy $\mathbf{e} = (\mathbf{u}, \mathbf{s}) \in \mathcal{E}$ and any feasible allocations $\mathbf{z}, \mathbf{z}' \in Z(\mathbf{s})$,

$$I\left(J^{UD}(\mathbf{e})\right) \equiv \left\{(\mathbf{z}, \mathbf{z}') \in Z(\mathbf{s}) \times Z(\mathbf{s}) \mid \mathbf{z} = \mathbf{z}'\right\};$$

$$P\left(J^{UD}(\mathbf{e})\right) \equiv \left\{(\mathbf{z},\mathbf{z}') \in Z(\mathbf{s}) \times Z(\mathbf{s}) \mid \mathbf{x} = \mathbf{x}' \ \& \ \widetilde{H}(\mathbf{z};\mathbf{s}) \subsetneq \widetilde{H}(\mathbf{z}';\mathbf{s})\right\}.$$

Then, we can show that there exists an **ESWF** of $Q^{LS\vdash\left(*PA\vdash J^{SU}RF\right)}$-type, following the same argument as **Theorem 1**.

Given any $\mathbf{e} \in \mathcal{E}$, denote the set of all feasible allocations satisfying set-inclusion undomination by $SU(\mathbf{e})$. Moreover, let $PS(\mathbf{e}) \equiv P(\mathbf{e}) \cap SU(\mathbf{e})$ for any $\mathbf{e} \in \mathcal{E}$. Note that $PS(\mathbf{e})$ is non-empty for any $\mathbf{e} \in \mathcal{E}$. For instance, the following is such an allocation:

Definition 10 [Fleurbaey and Maniquet (1996)]**:** *Let* $\widetilde{u} \in \mathcal{U}$ *be a* socially reference utility function. *Given* $\mathbf{e} = (\mathbf{u},\mathbf{s}) \in \mathcal{E}$*,* $\mathbf{z} \in P(\mathbf{e})$ *is a* $\varphi^{\widetilde{u}RWEB}$-allocation for $\mathbf{e}$ *if there exists* $p^{(\mathbf{z},\mathbf{e})} \in \triangle(\mathbf{z},\mathbf{e})$ *such that for any* $i,j \in N$,

$$\widetilde{u}\left(d\left(\widetilde{u}, B\left(z_i, s_i, p^{(\mathbf{z},\mathbf{e})}\right)\right)\right) = \widetilde{u}\left(d\left(\widetilde{u}, B\left(z_j, s_j, p^{(\mathbf{z},\mathbf{e})}\right)\right)\right).$$

Denote the set of all $\varphi^{\widetilde{u}RWEB}$-allocations for $\mathbf{e}$ by $\varphi^{\widetilde{u}RWEB}(\mathbf{e})$.

We can find an allocation rule $\gamma \in \Gamma$ with the property that for any $\mathbf{e} \in \mathcal{E}$, $\tau_{NE}(\gamma,\mathbf{e}) \subseteq PS(\mathbf{e})$. In fact, since the correspondence $\varphi^{\widetilde{u}RWEB}$ satisfies **MM**, there exists $\gamma \in \Gamma$ such that for any $\mathbf{e} \in \mathcal{E}$, $\tau_{NE}(\gamma,\mathbf{e}) = \varphi^{\widetilde{u}RWEB}(\mathbf{e})$. However, such an allocation rule can never be labor sovereign, as the following proposition shows:

Proposition 4 [Yamada and Yoshihara (2005)]**:** *There is no allocation rule* $\gamma \in \Gamma_L$ *such that for any* $\mathbf{e} \in \mathcal{E}$, $\tau_{NE}(\gamma,\mathbf{e}) \subseteq PS(\mathbf{e})$.

Proof. Define any correspondence φ such that: for any $\mathbf{e} \in \mathcal{E}$, $\varnothing \neq \varphi(\mathbf{e}) \subseteq PS(\mathbf{e})$. Then, the problem is reduced to that of Nash implementability of φ by some $\gamma \in \Gamma_L$. According to Yamada and Yoshihara (2005), if φ is Nash-implementable by some $\gamma \in \Gamma_L$, then it should satisfy the following property:

Independence of Unused Skills (IUS): *For each* $\mathbf{e} = (\mathbf{u},\mathbf{s}) \in \mathcal{E}$ *and each* $\mathbf{z} = (x_i,y_i)_{i\in N} \in \varphi(\mathbf{e})$*, there exists* $p^{(\mathbf{z},\mathbf{e})} \in \triangle(\mathbf{z},\mathbf{e})$ *such that for each* $\mathbf{e}' = (\mathbf{u},\mathbf{s}') \in \mathcal{E}$ *where* $s'_i = s_i$ *for each* $i \in N$ *with* $x_i > 0$, *if* $p^{(\mathbf{z},\mathbf{e})} \in \triangle(\mathbf{z},\mathbf{e}')$, *then* $\mathbf{z} \in \varphi(\mathbf{e}')$.

However, Yamada and Yoshihara (2005) shows that if $\varphi(\mathbf{e}) \subseteq PS(\mathbf{e})$ for any $\mathbf{e} \in \mathcal{E}$, then φ does not satisfy **IUS**. Thus, the proposition holds. ■

This proposition implies that even if the condition of the 'leximin assignment of opportunity set' is reformulated by set-inclusion undomination, instead of undominated diversity, we can still find no first best allocation rule among Γ_L. Since set-inclusion undomination is a minimal condition for the 'leximin assignment of opportunity sets' in terms of budget sets, we may conclude that there is no longer possibility of the first best 'basic income' policy, which are characterized by the criteria of labor sovereignty, Pareto efficiency, and the 'leximin assignment of opportunity sets' in terms of budget sets. We may have to either sacrifice the efficiency in resource allocations or replace the principle of leximin opportunity with another type of fairness principle for basic income policies.

4 Concluding Remarks

In the previous sections, we discussed that the welfarist's framework developed in traditional welfare economics provided us with a rather limited perspective for social evaluation of the welfare state policies, so a more comprehensive framework would be necessary. As such, we proposed the extended framework within which not only welfarist consequential values, but also non-welfarist consequential values and non-consequential values can be taken into consideration. Moreover, we introduced extended social welfare functions and, as axioms of which, Labor Sovereignty, Respect for **J**-based Fairness based on non-welfaristic well-being, and the Pareto principle. Then, we showed a method of applying these axioms based on a weaker lexicographic approach, by which some consistent extended social welfare functions can be constructed compatible with the above three values. In particular, if the **J**-based Fairness represents the undominated diversity condition proposed by van Parijs (1995), then some of the constructed extended social welfare functions can be regarded as the formulation of van Parijs's Real Libertarianism combined with the Pareto principle.

From now, we will provide some concluding remarks on the relevant literature of extended social welfare functions discussed in this paper. There is some literature such as Kaplow and Shavell (2001) and Blackorby, Bossert, and Donaldson (2005) which also discussed a sort of 'extended' social welfare functions satisfying some pluralistic values. In their frameworks of social welfare functions, not only the profile of utility information, but also the profile of non-welfaristic information are taken into account. Then, both papers showed that even in such frameworks with non-welfaristic information, the feasible class of social welfare functions are reduced to that of the welfarist types only, whenever the Pareto principle is required.

For instance, Kaplow and Shavell (2001) defined any "non-welfarist" axiom as being incompatible with the Pareto Indifference principle. Then, they showed that if a social welfare function satisfies continuity and such a "non-welfarist" axiom, then it violates the weak Pareto principle. This is derived from the fact that the continuity of the social welfare function and the weak Pareto principle immediately imply the Pareto Indifference principle.[28] Blackorby et al. (2005) showed that if a social welfare function defined over the domain of *multi-profiles* satisfies Universal Domain, Pareto Indifference, and Independence of Irrelevant Alternatives, then it implies Strong Neutrality. Note that Strong Neutrality is regarded as the axiom of Welfarism. This is because this axiom claims that social orderings should be determined independently of the profile of any non-utility information.

It is well known that, even in the case of B-S social welfare functions with the domains of the utility profiles only, Sen (1977) and Roberts (1980) showed that the conventional Arrovian axioms of universal domain, Pareto indifference, and independence of irrelevant alternatives together imply strong neutrality. The crucial difference of Blackorby et al. (2005) from Sen (1977) and Roberts (1980) is that the former defines Independence of Irrelevant Alternatives as requiring the social ranking of any two alternatives to depend on not only the utility information but also the non-welfaristic information associated with those two alternatives only. Hence, the independence axiom of Blackorby et al. (2005) is non-welfarist in nature, and it is weaker than the Arrovian independence axiom. Nevertheless, Blackorby et al. (2005) concluded that even in such a framework having the possibility of non-welfaristic evaluation, the possible social welfare function is only welfarist in nature, if it is required to satisfy the other Arrovian axioms such as Universal Domain and Pareto Indifference. This seems to provides us with a strong justification of welfarism.

I would like to review the relationship between our approach and the above mentioned works briefly. First, the welfarist theorem of Blackorby et al. (2005) relies strongly on the axiom of Universal Domain. Such a domain condition cannot be applied to the case of resource allocation problems this

[28]Kaplow and Shavell (2001) consider the Pareto indifference principle as the definition of welfarism for social welfare functions.

paper considers here, since in this paper, all available utility functions are restricted to satisfy strong monotonicity and quasi-concavity.[29] Thus, our result on the possibility of the non-welfarist social welfare functions is completely compatible with the result of Blackorby et al. (2005). Moreover, I believe that the universal domain assumption of the non-welfaristic information is not sound from the ethical point of view. This is because an objective well-being indicator expressing a non-welfaristic information should be defined as a binary relation function characterized by a system of axioms,[30] so it should need formally different treatment from the welfarist indicator (individual utility functions) as the representation of capricious subjective preferences.

Secondly, the conclusion of Kaplow and Shavell (2001) is basically identical with that of **Example 1** in this paper. However, **Example 1** does not suppose the continuity of social ordering, contrary to the assumption of Kaplow and Shavell (2001), so **Example 1** presents a stronger proposition than theirs in this sense. It is also worth noting that this impossibility result does not imply a justification of welfarism at all. This is because, as Fleurbaey, Tungodden, and Chang (2003) pointed out, the Pareto indifference principle and the welfarist axiom are not equivalent. In fact, our extended social welfare function $Q^{\left(*PA\vdash J^{UD}RF\right)\vdash LS}$ satisfies the weak Pareto principle as well as the Pareto indifference principle, and it also has the properties of the two types of non-welfarism. However, this type of function does not meet the continuity axiom. This implies that the real factor inducing the impossibility is not the trade-off between welfarism and non-welfarism, but rather the requirement of continuity.

To summarize this, despite the conclusions of Kaplow and Shavell (2001) and Blackorby et al. (2005), it is sufficiently possible to construct a desirable social welfare function that has the properties of the welfarist Pareto principle and the non-welfarist criteria. **Theorem 1** in this paper suggests this conclusion.

[29] In fact, there exists an extended social welfare function of $Q^{LS\vdash\left(*PA\vdash J^{UD}RF\right)}$-type which satisfies the independence axiom of Blackorby et al. (2005). This is because our extended social welfare functions do not satisfy the universal domain axiom of Blackorby et al. (2005).

[30] The example of this is a series of works by Pattanaik and Xu (1990), where the ranking over opportunity sets is characterized by the system of axioms which reflect the viewpoint of "freedom of choice."

5 Appendix 1

In this Appendix 1, the elementary properties of binary relations are provided, which constitute an analytical technique useful to consider the existence issue of extended social welfare functions. Let X be the universal set of any alternatives and R be a binary relation defined over this set. We may suppose that this binary relation R represents a preference relation on X that society expresses. If R satisfies *completeness* and *transitivity* in particular, we shall call it *social preference ordering.* Also:

Definition 3: *An axiom a is* represented by a binary relation $R^a \subseteq X \times X$ *if the following condition holds: for any* $\mathbf{x}, \mathbf{x}' \in X$,

$$\begin{aligned}(\mathbf{x}, \mathbf{x}') &\in R^a \Leftrightarrow \text{according to the axiom } a\text{, } \mathbf{x} \text{ is at least desired as } \mathbf{x}'; \\ (\mathbf{x}, \mathbf{x}') &\in P(R^a) \Leftrightarrow \text{according to the axiom } a\text{, } \mathbf{x} \text{ is strictly desirable than } \mathbf{x}'.\end{aligned}$$

In general, the binary relation representing an axiom is not necessarily a complete ordering. In the following discussion, let us denote the representation of the axiom a by R^a. Then, let us see how an ordering $R \subseteq X \times X$ satisfies an axiom in general.

Definition 4: *A binary relation R satisfies a class of axioms* $\{a^\lambda\}_{\lambda \in \Lambda}$ *if the following condition holds*:

$$R \supseteq \left[\cup_{\lambda \in \Lambda} R^{a^\lambda}\right] \text{ and } P(R) \supseteq \left[\cup_{\lambda \in \Lambda} P\left(R^{a^\lambda}\right)\right].$$

As **Definition 4** suggests, a binary relation R satisfies axioms $a^1, \ldots, a^m$ if and only if it contains all of the axiom-representing relations $R^{a^1}, \ldots, R^{a^m}$ as its subrelations. Otherwise, this relation does not satisfy some of these axioms.

Given a class of axioms on ordering relations, one interesting problem is to examine whether there exists an ordering relation that satisfies all of these axioms. To discuss this question, the following notion is crucial:

Definition 5 [Suzumura (1976)]: *A binary relation $R \subseteq X \times X$ is* consistent *if , for any finite subset* $\{x^1, x^2, ..., x^t\}$ *of* X*, the following condition does not hold:*

$$[(x^1, x^2) \in P(R), \left(x^k, x^{k+1}\right) \in R \; (\forall k = 2, \ldots, t-1)] \Rightarrow (x^t, x^1) \in R.$$

Proposition 1: *There exists an ordering relation $R \subseteq X \times X$ which satisfies a class of axioms $\left\{a^{\lambda}\right\}_{\lambda \in \Lambda}$ if and only if $\left[\cup_{\lambda \in \Lambda} R^{a^{\lambda}}\right]$ is* consistent.

According to **Proposition 1**, it is sufficient to confirm whether or not the union of the axiom-representing relations $\left\{R^{a^{\lambda}}\right\}_{\lambda \in \Lambda}$ meets the consistency. This condition can be useful when we discuss the existence of extended social welfare functions satisfying some class of axioms.

6 Appendix 2

1. The *pluralistic application of axioms* [Sen and Williams (1982)]

Given any two axioms a and b which are mutually incompatible, the *pluralistic application of axioms* is to construct a binary relation $R^{a \cap b} \subseteq X \times X$ which is defined as: $R^{a \cap b} \equiv R^a \cap R^b$ and $P\left(R^{a \cap b}\right) \equiv \left[P\left(R^a\right) \cap R^b\right] \cup \left[R^a \cap P\left(R^b\right)\right]$. Then, $R^{a \cap b}$ becomes consistent whenever R^a and R^b are respectively consistent. Thus, this kind of second best resolution is to consider an ordering extension of $R^{a \cap b}$.

2. The *lexicographic application of axioms*

Given any binary relation R, let $N(R) \subseteq X \times X$ be defined as follows: for any $\mathbf{x}, \mathbf{x}' \in X$, $(\mathbf{x}, \mathbf{x}') \in N(R) \Leftrightarrow (\mathbf{x}, \mathbf{x}') \notin R$ and $(\mathbf{x}', \mathbf{x}) \notin R$.[31] Given any two axioms a and b which are mutually incompatible, the *lexicographic application of axioms* is to construct a binary relation $R_{lex}^{a \vdash b} \subseteq X \times X$ which is defined by: for any $x, x' \in X$,

$$\begin{aligned} (x, x') &\in R_{lex}^{a \vdash b} \Leftrightarrow (x, x') \in R^a \cup \left[N\left(R^a\right) \cap R^b\right]; \text{ and} \\ (x, x') &\in P\left(R_{lex}^{a \vdash b}\right) \Leftrightarrow (x, x') \in P\left(R^a\right) \cup \left[N\left(R^a\right) \cap P\left(R^b\right)\right]. \end{aligned}$$

That is, suppose that the society gives a priority to the axiom a rather than to the b. Then, for any two alternatives, axiom a is applied by $R_{lex}^{a \vdash b}$ in the first place to make a comparison between them, and axiom b is applied only if these two alternatives are incomparable by axiom a. This is called *axiom a first-axiom b second priority rule.*

According to **Proposition 1**, an ordering extension of $R_{lex}^{a \vdash b}$ is possible whenever $R_{lex}^{a \vdash b}$ is consistent. Unfortunately, however, the consistency of $R_{lex}^{a \vdash b}$

[31] The definition of this binary relation is based on Suzumura (2004).

is not guaranteed in general. Thus, we need an algorithm to see what properties of the axioms a and/or b can make $R_{lex}^{a \vdash b}$ consistent.

Suppose that $R_{lex}^{a \vdash b}$ is not consistent. Our strategy is to choose appropriately a subset $N^*(R^a)$ from $N(R^a)$ such that

$$R_{lex}^{*a \vdash b} \equiv R^a \cup \left[N^*(R^a) \cap R^b\right]$$

becomes consistent. Then, the problem is to identify what conditions this $N^*(R^a)$ should satisfy so as to make $R_{lex}^{*a \vdash b}$ consistent. A general solution of this problem is given by Yoshihara (2005), and here we introduce a corollary of this solution given in Yoshihara (2005).

Definition 6 [Yoshihara (2005)]**:** *Given a binary relation $R \subseteq X \times X$, a subset $N^*(R) \subseteq N(R)$ is said to* be connected *if for any $(x, x'), (y, y') \in N^*(R)$, there exists $\{z^1, \ldots, z^t\} \subseteq X$ such that $z^1 = x'$, $z^t = y$, and $(z^k, z^{k+1}) \in N^*(R)$ holds for any $k = 1, \ldots, t-1$.*

Proposition 2 [Yoshihara (2005)]**:** *Let R^a be a quasi-ordering over X. Then,* if *the relation $N^*(R^a) \subseteq N(R^a)$ is* transitive *and* connected, *then the relation $R_{lex}^{*a \vdash b} \subseteq X \times X$ is* consistent *for any quasi-ordering $R^b \subseteq X \times X$.*

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