A Class of Fair Distribution Rules à la Rawls and Sen*

Reiko Gotoh† and Naoki Yoshihara‡

May 31, 1999, This version: January, 2002

Abstract

This paper discusses and develops "non-welfaristic" arguments on distributive justice à la J. Rawls and A. K. Sen, and formalizes, in cooperative production economies, "non-welfaristic" distribution rules as game form types of resource allocation schemes. First, it conceptualizes Needs Principle which the distribution rule should satisfy if this takes the individuals' needs into account. Second, one class of distribution rules which satisfy Needs Principle, a class of **J**-based Capability Maximin Rules, is proposed. Third, axiomatic characterizations of the class of **J**-based Capability Maximin Rules are provided.

JEL Classification Numbers: D 63, D 71, I 31

Corresponding Author: Naoki Yoshihara

*We are grateful to an anonymous referee of this journal, Professors Marc Fleurbaey, Nicolas Gravel, Ryo-ichi Nagahisa, Prasanta Pattanaik, Kotaro Suzumura, Koich Tadenuma, and Yongsheng Xu for their fruitful comments. An earlier version of this paper was published with the title name, "A Game Form Approach to Theories of Distributive Justice: Formalizing Needs Principle" as the Discussion Paper No. 407 of the Institute of Social and Economic Research, Osaka University, and in the proceedings of the International Conference on Logic, Game, and Social Choice held at Oisterwijk in May 1999. This research was partially supported by the Japanese Ministry of Education and the Ministry of Health and Welfare.

†National Institute of Population and Social Security Research, 1-2-3 Kasumigaseki, Chiyoda-Ku, Tokyo 100, Japan. Phone: (81)-3-3503-1711 ext. 4412, Fax: (81)-3-3591-4912. e-mail: gotoh-r@so.ipss.go.jp

‡Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-8603, Japan. Phone: (81)-42-580-8354, Fax: (81)-42-580-8333 or (81)-42-577-6192. e-mail: yosihara@ier.hit-u.ac.jp

1 Introduction

In this paper, we discuss fair allocations in cooperative production economies, mainly basing our arguments upon those of Rawls and Sen.

There have been two representative criteria on distributive justice, one of which is "*distribution with regard to individual contributions*," and the other being "*distribution with regard to individual needs.*" While the former criterion justifies resource allocations through competitive mechanisms, the latter criterion may play an important role in judging what redistribution policies are justifiable to achieve social security. In discussing fair allocations based upon the latter criterion, it is important to give a precise concept of *individual needs* as an objective one, as done by Rawls (1971, 1993) and Sen (1980, 1983). While Rawls (1993) argued upon the *citizens' needs* as indexed by primary goods, Sen (1980, 1983) understood the needs in terms of his *capability* approach. Sen's *capability* is interpreted as the set of an individual's various states of well-being possibly attainable by her through various ways of utilization of her share of resources. Based upon Sen's arguments (1980, 1983), the needs of individuals should accordingly be conceptualized as a lack of a reference level of capability, which is given for a given social context, but which may vary among various social contexts. Our object in this paper is just to propose an allocation scheme which meets the latter criterion discussed above, called the **J**-*based capability maximin rule* which is based upon Sen's capability.[1]

While most literature on fair allocation problems discussed allocation schemes of social choice correspondence types, we, in this paper, are particularly interested in *game form* types, which we call *distribution rules.* These are similar to *cost sharing rules* discussed in Moulin and Schenker (1994). A distribution rule is a function from the set of profiles of all individuals' labor time to the set of profiles of all individuals' shares of output. One reason for our approach is to clarify the aspect of interaction among individuals in assigning capabilities through the application of the allocation schemes.[2] Note that Sen's argument (1980) on equality of basic capability leaves out this aspect of interaction. In contrast, we can explicitly capture this aspect

[1] Herrero (1996) also discussed recently designing allocation schemes of social choice correspondence types in pure exchange economies by using Sen's capability index.

[2] This aspect is also pointed out by Atkinson (1995), where it is argued that the activity of monopolistic commodity supplier is influential in each individual's capability level determined by her share of commodity.

in our model by adopting the game form approach of allocation schemes, so that the determination of each individual's capability is influenced not only by his own strategy (choice of his labor time), but also by that of any other individual under a given distribution rule.

Another reason for our approach is related to the Rawlsian first principle of justice [Rawls (1971)], which seems to require allocation schemes to guarantee every individual an equal right to choose her own labor time freely. Such a procedural aspect of allocation schemes can be formally specified in the game form types,[3] while social choice correspondences specify no procedural aspect of decision making.

In this paper, we discuss the fairness of resource allocations by arguing the fairness of capability assignments given through the determination of resource allocations. Although recent literature on ranking of equal opportunity sets like Kranich (1996, 1997), etc.[4], may give us various criteria for evaluating the fairness of capability assignments, in this paper, we are not interested in what criteria the ranking on "fair" capability assignments should satisfy. Instead, we focus upon the problem of how social agreements on the ranking of "fair" capability assignments would be arrived at, so that we simply suppose that in the society, there exists a social procedure for determining "fair" capability assignments. In particular, we are interested in the types of social decision procedures which evaluate the fairness of capability assignments through the evaluation of what is the desirable *common capability*, and call them *social welfare functions*. The common capability is given by the intersection of all individuals' capabilities under the profile of individual handicaps and feasible allocations. The **J**-based capability maximin rule is just defined as being rationally chosen through a *Paretian* social welfare function whenever this function always generates a social ordering over common capabilities, which contains the set-inclusion relation as its subrelation. In other words, the **J**-based capability maximin rule is defined to guarantee every individual a maximal common capability with respect to set-inclusion. So, we first discuss under what conditions there exists a Paretian social welfare function which always generates a social ordering containing the set-inclusion relation, so that it always rationally chooses the **J**-based capability maximin rule.

[3]Right structures of freedom of choice can be best captured by game form approach. See Gaertner, Pattanaik, and Suzumura (1992).

[4]The literature of equal opportunity on basis of ranking opportunity profiles also includes Herrero (1997), Kranich and Ok (1998), Herrero, Iturbe-Ormaetxe, and Nieto (1998), Bossert, Fleurbaey, and Van de Gaer (1999), and Arlegi and Nieto (1999).

Next, we axiomatically characterize the social ordering on capability assignments which rationalizes the **J**-based capability maximin rule. A characterization result of this ordering is obtained by adopting the axioms of *Pareto inclusion on capability assignments* and *anonymity*, and an *equity* axiom based upon a particular *inequality measure* of capability assignments. Note that Herrero, Iturbe-Ormaetxe, and Nieto (1998) and Bossert, Fleurbaey, and Van de Gaer (1999) discussed the axiomatic characterizations of ranking opportunity set profiles on the basis of *common opportunity sets.* As discussed below, these characterization results cannot be directly applied to our discussion of ranking capability assignments based upon common capabilities, because of the different mathematical structure in economic models assumed in this study. So, our results are independent of the above literature.

Third, discussing several axioms on fairness of distribution rules, we analyze desirable properties which the class of **J**-based capability maximin rules satisfies. Our first characterization result on the class of **J**-based capability maximin rules is that it is the unique class of rules which meets the *Needs Principle* defined below and satisfies two axioms of *solidarity* concerning changes in individual supplies of labor time and individual handicap levels respectively. Our second characterization result shows that a subclass of capability maximin rules meets both axioms of *responsibility* and *compensation.* Although previous literature on responsibility and compensation by Fleurbaey (1994, 1995a,b), etc.,[5] was mainly based upon evaluating individual welfares or incomes, we evaluate the state of individuals by using the capability.

For the rest of this paper, section 2 defines a basic model; Section 3 discusses axioms on distribution rules; Section 4 proposes the capability maximin rules and analyzes their characteristics.

2 The Basic Model

There are two goods in the economy, one of which is labor time, $x \in \mathbb{R}_+$, to be used to produce the other good, $y \in \mathbb{R}_+$, where $\mathbb{R}_+$ denotes the set of non-negative real numbers. The population in the economy is given by the

[5]The recent literature on the issue of *responsibility* and *compensation* initiated by Dworkin (1981) and followed by Arneson (1989) and Cohen (1989, 1993), includes Bossert (1995), Bossert and Fleurbaey (1996), Fleurbaey (1998), Fleurbaey and Maniquet (1996, 1999), Iturbe (1997), Iturbe and Nieto (1996), Maniquet (1996), and Roemer (1993, 1996).

set $N = \{1, \cdots, n\}$, where $2 \leq n < +\infty$. Individual i's consumption vector is denoted by $z_i = (l_i, y_i)$, where $l_i = \overline{x} - x_i$ denotes his/her leisure time, and y_i denotes his/her assigned share of output, and $\overline{x}$, $0 < \overline{x} < +\infty$, denotes the upper bound of labor time. It is assumed that all individuals have the same consumption set $[0, \overline{x}] \times \mathbb{R}_+$. Individual i is characterized by *utilization ability of resources*, a_i, and *production skill*, s_i, both of which are assumed to be representable by real numbers. The universal set of utilization abilities, which is common for all individuals, is denoted by $A \subseteq \mathbb{R}$.[6] The universal set of production skills for all individuals is denoted by $S \subseteq \mathbb{R}_{++}$, where $\mathbb{R}_{++}$ denotes the set of positive real numbers. Thus, individual i's characteristics are denoted by $(a_i, s_i) \in A \times S$.[7]

A production process is described by a production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ which is assumed to be continuous, increasing, and $f(0) \geq 0$. The set of such functions is denoted by F.

Let there be m types of relevant *functionings* for each individual, which are common for all individuals and are attainable by means of his/her leisure time and share of output. Let us assume that we can measure these functionings by means of adequate non-negative real indices. Thus, an achievement of functioning k by individual i is denoted by $b_{ik} \in \mathbb{R}_+$. Individual i's achievement of relevant functionings is given by listing b_{ik}: $\mathbf{b}_i = (b_{i1}, \cdots, b_{im}) \in \mathbb{R}^m_+$.[8]

Individual i's utilization ability, leisure time, and share of output determine the vector of functionings he/she can achieve. It is assumed that there is a functional relationship which relates a triple of ability, leisure time, and share of output to a set of m-dimensional functioning vectors viz. $C : A \times [0, \overline{x}] \times \mathbb{R}_+ \;{}^{3}\; \mathbb{R}^m_+$ such that $C(a_i, l, y) \subseteq \mathbb{R}^m_+$ for all $i \in N$. We call C a *capability correspondence*. The intended interpretation is that individual i with a_i is able to attain the vector of relevant functionings $\mathbf{b}_i \in C(a_i, l, y)$ by utilizing the consumption vector (l, y). For simplicity, we assume that each admissible capability correspondence has the following properties:

[6] For any two sets X and Y, $X \subseteq Y$ whenever any $x \in X$ also belongs to Y, and $X \subsetneq Y$ if and only if $X \subseteq Y$ and *not* $(Y \subseteq X)$.

[7] There may exist a functional relationship between production skill and utilization ability such as $s_i = s(a_i)$ and s_i is strictly monotonic with respect to a_i. All the main results in this paper are valid, *mutatis mutandis*, even when such a relationship holds between s_i and a_i.

[8] For any two vectors $\mathbf{a} = (a_1, \ldots, a_p)$ and $\mathbf{b} = (b_1, \ldots, b_p)$, $\mathbf{a} \geq \mathbf{b}$ if and only if $a_i \geq b_i$ $(i = 1, \ldots, p)$, $\mathbf{a} > \mathbf{b}$ if and only if $\mathbf{a} \geq \mathbf{b}$ and *not* $(\mathbf{b} \geq \mathbf{a})$, and $\mathbf{a} \gg \mathbf{b}$ if and only if $a_i > b_i$ $(i = 1, \ldots, p)$.

(1)-(α) For all $(a, l) \in A \times [0, \overline{x}]$, $C(a, l, 0) = \{0\}$.
(1)-(β) If $(a, z) \leq (a', z')$ (resp. $(a, z) < (a', z')$), then $C(a, z) \subseteq C(a', z')$ (resp. $C(a, z) \subsetneq int\ C(a', z')$, where $int\ C(a', z')$ is the *interior* of $C(a', z')$ in $\mathbb{R}^m_+$).[9]
(1)-(γ) For all $(a, z) \in A \times [0, \overline{x}] \times \mathbb{R}_+$, $C(a, z)$ is compact and comprehensive in $\mathbb{R}^m_+$.
(1)-(δ) Given $a \in A$, $C : \{a\} \times [0, \overline{x}] \times \mathbb{R}_+ \rightrightarrows \mathbb{R}^m_+$ is continuous.

Let $\mathcal{C}$ be the class of capability correspondences satisfying these properties.[10] The condition (1)-(α) implies that no one can survive without consumption of output (income). The condition (1)-(β) implies the (strict) monotonicity property of capability correspondences with respect to utilization ability a and resource z. The conditions (1)-(γ) and (1)-(δ) are technical requirements which are used in showing Lemma 1 discussed later.

The *objective characteristics of the economy* is defined by a list $\mathbf{e} = (\mathbf{a}, \mathbf{s}, C, f) = ((a_i)_{i \in N}, (s_i)_{i \in N}, C, f) \in E \equiv A^n \times S^n \times \mathcal{C} \times F$, where A^n and S^n stand, respectively, for the n-fold Cartesian product of A and that of S. *A feasible allocation for* $\mathbf{e}$ is a vector $\mathbf{z} = (z_i)_{i \in N} \in ([0, \overline{x}] \times \mathbb{R}_+)^n$ such that $f(\sum s_i x_i) \geq \sum y_i$. We denote by $Z(\mathbf{e})$ the set of feasible allocations for $\mathbf{e} \in E$. Given the objective characteristics of the economy $\mathbf{e} = (\mathbf{a}, \mathbf{s}, C, f) \in E$, the *feasible assignment of capabilities under* $\mathbf{e}$ is a list $(C(a_i, z_i))_{i \in N}$ satisfying $(z_i)_{i \in N} \in Z(\mathbf{e})$.

A distribution rule is a function $h : E \times [0, \overline{x}]^n \rightarrow \mathbb{R}^n_+$ satisfying the following property: for any $\mathbf{e} = (\mathbf{a}, \mathbf{s}, C, f) \in E$ and any $\mathbf{x} = (x_i)_{i \in N} \in [0, \overline{x}]^n$, $h(\mathbf{e}, \mathbf{x}) = \mathbf{y} = (y_i)_{i \in N}$ such that $(\overline{x} - x_i, y_i)_{i \in N} \in Z(\mathbf{e})$. Notice that given $\mathbf{e} \in E$ and $\mathbf{x} \in [0, \overline{x}]^n$, the distribution rule h specifies a feasible assignment of capabilities for $\mathbf{e}$. In this formula, the strategy space of every individual is represented by $[0, \overline{x}]$, and each individual can be guaranteed equal freedom in choosing her own labor time, and the rule remunerates her with leisure as she sees fit.

[9] Incidentally, $\partial C(a', z')$ appearing below is the *boundary* of $C(a', z')$ in $\mathbb{R}^m_+$.

[10] Following Sen (1985), we may define a value function $v_i : \mathbb{R}^m_+ \rightarrow \mathbb{R}$ for each individual $i \in N$, and define his utility function $\overline{u}_i$ by:

$$\overline{u}_i(z) \equiv v_i(\mathbf{b}(z, C(a_i, z), v_i)), \text{ where } \mathbf{b}(z, C(a_i, z), v_i) \equiv \arg \max_{\mathbf{b}_i \in C(a_i, z)} v_i(\mathbf{b}_i), \text{ for all } z \in [0, \overline{x}] \times \mathbb{R}_+.$$

If $C(a_i, \cdot, \cdot)$ is convex-valued, and v_i is continuous, strictly monotonic, and quasi-concave on $\mathbb{R}^m_+$, then $\overline{u}_i$ is continuous, strictly monotonic, and quasi-concave on $[0, \overline{x}] \times \mathbb{R}_+$.

3 Two Fundamental Principles of Distributive Justice

In this section, we consider two fundamental principles of distributive justice, *Contribution Principle* and *Needs Principle*, which the society should consult in determining distribution rules while it guarantees every individual equal freedom of choice of labor time.

First, we define *Contribution Principle*:

Contribution Principle (CP) : *For all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$*, all* $\mathsf{x} \in [0, \overline{x}]^n$*, and all* $i, j \in N$*,* $[s_i x_i < s_j x_j \Rightarrow h_i(\mathsf{e}, \mathsf{x}) < h_j(\mathsf{e}, \mathsf{x})$ and $s_i x_i = s_j x_j \Rightarrow h_i(\mathsf{e}, \mathsf{x}) = h_j(\mathsf{e}, \mathsf{x})]$.

The class of distribution rules which satisfy Contribution Principle is defined by $CR \equiv \{h \mid h$ satisfies **CP** $\}$. There are many rules which satisfy Contribution Principle. Moulin and Schenker (1994) discussed the axiomatic characterizations of this type of distribution rule. An example of such a rule is the *proportional sharing rule* PR, which distributes outputs in proportion to each individual's labor contribution: for all $i \in N$, $h_i^{PR}(e, \mathsf{x}) = \frac{s_i x_i}{\sum s_j x_j} f(\sum s_j x_j)$.

The next principle we discuss is *Needs Principle.* This should define a reference level of the *real opportunity of well-being* for each social context, a lack of which should be taken into account as *one's needs* and being a subject for compensation. Note that, in this paper, the real opportunity of well-being is Sen's *capability.* Then, Needs Principle is defined as follows: given $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ and $\mathsf{x} \in [0, \overline{x}]^n$ such that for all $i, j \in N$, $x_i = x_j$, let $\mathsf{y}^H(\mathsf{e}, \mathsf{x}) = (\frac{f(\sum s_i x_i)}{n}, \cdots, \frac{f(\sum s_i x_i)}{n}) \in \mathbb{R}_+^n$ be a *hypothetical distribution.*[11] Then, a capability profile under $(\mathsf{e}, \mathsf{x}, \mathsf{y}^H(\mathsf{e}, \mathsf{x}))$ is determined as: $(C(a_i, \overline{x} - x_i, y^H(\mathsf{e}, \mathsf{x})))_{i \in N}$. Notice that by definition of the capability correspondence C, in this case, there is one individual $i^* \in N$ such that for any other $j \neq i^*$, $C(a_j, \overline{x} - x_j, y^H(\mathsf{e}, \mathsf{x})) \supseteq C(a_{i^*}, \overline{x} - x_{i^*}, y^H(\mathsf{e}, \mathsf{x}))$. Then, let $C^H(\mathsf{e}, \mathsf{x}) \equiv C(a_{i^*}, \overline{x} - x_{i^*}, y^H(\mathsf{e}, \mathsf{x}))$, and call it a *reference capability under* (e, x).

Needs Principle (NP) : *For all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ *and all* $\mathsf{x} \in [0, \overline{x}]^n$ *such that for all* $i, j \in N$*,* $x_i = x_j$*, there is no* $i \in N$ *such that* $C^H(\mathsf{e}, \mathsf{x}) \supsetneq C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x}))$.

[11] This hypothetical distribution can be regarded as the derivation from the *equal-division-for-equal-work* (*EDEW*) proposed by Kranich (1994). Note that Needs Principle defined below does not necessarily require distribution rules to satisfy the *EDEW.*

The class of distribution rules which satisfy Needs Principle is denoted by $NR \equiv \{h \mid h$ satisfies **NP** $\}$.

The above definition of Needs Principle is explained as follows: if for some $j \neq i^*$, $C(a_j, \overline{x} - x_j, y^H(\mathsf{e}, \mathsf{x})) \supsetneq C(a_{i^*}, \overline{x} - x_{i^*}, y^H(\mathsf{e}, \mathsf{x})) = C^H(\mathsf{e}, \mathsf{x})$, this difference in capabilities between j and i^* under the hypothetical situation $(\mathsf{x}, \mathsf{y}^H(\mathsf{e}, \mathsf{x}))$ is based only upon the difference in their utilization abilities, because, under the hypothetical situation, every individual enjoys the same resource vector. Thus, individual i^*'s least favorable situation in capability under the hypothetical distribution is attributed to her least favorable position in utilization ability. Consequently, if individual i^*'s situation after the application of the distribution rule is even worse than her situation under the hypothetical distribution, it would be presumably reasonable to recognize such a rule as failing to meet her *needs*. Thus, Needs Principle requires the society to reject such a distribution rule.

It is easy to show that Contribution Principle and Needs Principle are incompatible. This incompatibility is obtained, irrelevant to whether the value of production skill is strictly monotonic with respect to the value of utilization ability or not.

4 A Distribution Rule according to Needs

In this section, we propose a class of desirable distribution rules which are relevant to Needs Principle discussed in the last section.

4.1 Common Capabilities and Formulation of Social Ordering over Distribution Rules

Given $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ and $\mathsf{x} \in [0, \overline{x}]^n$, let $Y(\mathsf{e}, \mathsf{x}) \equiv \{\mathsf{y} = (y_i)_{i \in N} \in \mathbb{R}^n_+ \mid f(\sum s_i x_i) \geq \sum y_i\}$ be a set of feasible distributions of produced goods for (e, x). To move from the space of goods to the space of capabilities, let

(2) $\mathcal{FC}(\mathsf{e}, \mathsf{x}) \equiv \{(C(a_i, \overline{x} - x_i, y_i))_{i \in N} \mid \mathsf{y} = (y_i)_{i \in N} \in Y(\mathsf{e}, \mathsf{x})\}$

be a set of feasible assignments of capabilities under (e, x). Let

(3) $\mathcal{FC}(\mathsf{e}) \equiv \underset{\mathsf{x} \in [0, \overline{x}]^n}{\cup} \mathcal{FC}(\mathsf{e}, \mathsf{x})$ and $\mathcal{FC} \equiv \underset{\mathsf{e} \in E}{\cup} \mathcal{FC}(\mathsf{e})$.

For each $(C(a_i, \overline{x} - x_i, y_i))_{i \in N} \in \mathcal{FC}(\mathsf{e}, \mathsf{x})$, let

(4) $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \equiv \underset{i\in N}{\cap} C(a_i, \overline{x} - x_i, y_i)$,

which is to be called the *common capability under* $(\mathsf{e},\mathsf{x},\mathsf{y})$.[12] Notice that for all (e,x), common capabilities are non-empty sets. Given $\mathsf{e} = (\mathsf{a},\mathsf{s},C,f) \in E$ and $\mathsf{x} \in [0,\overline{x}]^n$, let

(5) $\mathcal{CC}(\mathsf{e},\mathsf{x}) \equiv \{CC(\mathsf{e},\mathsf{x},\mathsf{y}) \mid \mathsf{y} \in Y(\mathsf{e},\mathsf{x})\}$ [13]

be the set of common capabilities under (e,x). Moreover, let

(6) $\mathcal{CC}(\mathsf{e}) \equiv \underset{\mathsf{x}\in[0,\overline{x}]^n}{\cup} \mathcal{CC}(\mathsf{e},\mathsf{x})$ and $\mathcal{CC} \equiv \underset{\mathsf{e}\in E}{\cup} \mathcal{CC}(\mathsf{e})$.

Complicated though it may look, the intuitive meaning of the common capability is in fact very simple, and it has some intuitive appeal to those who are interested in distributive fairness. Given $\mathsf{e} = (\mathsf{a},\mathsf{s},C,f) \in E$, when individuals decide to supply $\mathsf{x} \in [0,\overline{x}]^n$, they are minimally warranted of functioning vectors in the common capability, no matter how much they differ in their utilization abilities. Clearly, this warranted set of functioning vectors hinges squarely on the feasible distribution of the produced good, viz, $\mathsf{y} \in Y(\mathsf{e},\mathsf{x})$. By choosing a distribution rule appropriately, we are interested in warranting individuals the "most fair" common capability. It is itself a problem to determine what is the "most fair" common capability, which is purely of social decision to choose one distribution rule from the admissible set of distribution rules so as to guarantee every individual a desirable common capability.

In this section, we assume that each and every individual judges the desirability of distribution rules through evaluating the common capabilities which the distribution rules guarantee to all individuals. Moreover, there is an aggregation procedure of all individuals' judgements about what is the "most fair" common capability, which just formulates the social decision process for choosing one distribution rule. Let every individual i's value judgement about common capabilities be represented by an ordering relation $J_i \subseteq \mathcal{CC} \times \mathcal{CC}$. Let $J_i(\mathsf{e}) \equiv J_i \cap [\mathcal{CC}(\mathsf{e})\times\mathcal{CC}(\mathsf{e})]$ and $J_i(\mathsf{e},\mathsf{x}) \equiv J_i(\mathsf{e})\cap[\mathcal{CC}(\mathsf{e},\mathsf{x})\times \mathcal{CC}(\mathsf{e},\mathsf{x})]$, and the asymmetric and symmetric parts of $J_i(\mathsf{e})$

[12]Herrero, Iturbe-Ormaetxe, and Nieto (1998), Bossert, Fleurbaey, and Van de Gaer (1999), and Arlegi and Nieto (1999) also discussed, in the problem of ranking opportunity profiles, a "common opportunity" which is defined as the intersection of all individuals' opportunity sets.

[13]For the sake of notational convenience, we sometimes denote the feasible allocation by (x,y) instead of z when $\mathsf{z} = (\overline{\mathsf{x}}-\mathsf{x},\mathsf{y}) \in Z(\mathsf{e})$.

(resp. $J_i(\mathsf{e},\mathsf{x})$) will be denoted by $P(J_i(\mathsf{e}))$ (resp. $P(J_i(\mathsf{e},\mathsf{x}))$) and $I(J_i(\mathsf{e}))$ (resp. $I(J_i(\mathsf{e},\mathsf{x}))$), respectively. The universal class of value judgements will be denoted by $\mathcal{J}$. The social decision process for choosing one distribution rule is formulated as a social welfare function ψ which is defined as follows: for every value judgement profile $\mathsf{J} = (J_i)_{i\in N} \in \mathcal{J}^n$, $J = \psi(\mathsf{J})$ is an ordering on $\mathcal{CC}$. We assume that ψ meets *Pareto Principle*: for all $\mathsf{J} = (J_i)_{i\in N}$, all (e,x), and all $\mathsf{y},\mathsf{y}^*\in Y(\mathsf{e},\mathsf{x})$, if $(CC(\mathsf{e},\mathsf{x},\mathsf{y}), CC(\mathsf{e},\mathsf{x},\mathsf{y}^*)) \in P(J_i(\mathsf{e},\mathsf{x}))$ for all $i \in N$, then $(CC(\mathsf{e},\mathsf{x},\mathsf{y}), CC(\mathsf{e},\mathsf{x},\mathsf{y}^*)) \in P(J(\mathsf{e},\mathsf{x}))$ where $J = \psi(\mathsf{J}) \in \mathcal{J}$.

Although each of the value judgements in $\mathcal{J}$ is interpreted as representing one possible idea as to what constitutes the "most fair" common capability, we are particularly interested in value judgements which satisfy the following two conditions:

Set-Inclusion Subrelations: *For all* (e,x) *and all* $\mathsf{y},\mathsf{y}^*\in Y(\mathsf{e},\mathsf{x})$,
(α) $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \supseteq CC(\mathsf{e},\mathsf{x},\mathsf{y}^*) \Rightarrow (CC(\mathsf{e},\mathsf{x},\mathsf{y}), CC(\mathsf{e},\mathsf{x},\mathsf{y}^*)) \in J(\mathsf{e},\mathsf{x})$*; and*
(β) $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \supsetneq CC(\mathsf{e},\mathsf{x},\mathsf{y}^*) \Rightarrow (CC(\mathsf{e},\mathsf{x},\mathsf{y}), CC(\mathsf{e},\mathsf{x},\mathsf{y}^*)) \in P(J(\mathsf{e},\mathsf{x}))$.

Anonymity: *For all* $\mathsf{e} \in E$, *all* $i,j \in N$ *such that* $a_i = a_j$ *and* $s_i = s_j$, *all* $\mathsf{x} \in [0,\overline{x}]^n$, *and all* $\mathsf{y},\mathsf{y}^* \in Y(\mathsf{e},\mathsf{x})$,
$(CC(\mathsf{e},\mathsf{x},\mathsf{y}), CC(\mathsf{e},\mathsf{x},\mathsf{y}^*)) \in J(\mathsf{e},\mathsf{x}) \Leftrightarrow (CC(\mathsf{e},\rho_{ij}(\mathsf{x},\mathsf{y})), CC(\mathsf{e},\rho_{ij}(\mathsf{x},\mathsf{y}^*))) \in J(\mathsf{e},\rho_{ij}(\mathsf{x}))$
where $\rho_{ij}(\mathsf{x},\mathsf{y}) = ((x_i^\rho, y_i^\rho),(x_j^\rho, y_j^\rho),(\mathsf{x}_{N\setminus\{i,j\}}, \mathsf{y}_{N\setminus\{i,j\}}))$, $x_i^\rho = x_j$, *(resp.* $y_i^\rho = y_j$*) and* $x_j^\rho = x_i$ *(resp.* $y_j^\rho = y_i$*).*

Let $\mathcal{J}_{SIA}$ be the class of value judgements which contain *set-inclusion subrelations* and satisfy **Anonymity**. The condition of **Set-Inclusion Subrelations** is to require the monotonicity of orderings over common capabilities in terms of set-inclusion. It is based upon the idea that the larger the minimally warranted capability for all individuals is, the more fair it will be. The condition of **Anonymity** implies that the ranking of common capabilities should not be influenced by replacement of the names of any two individuals who have the same objective characteristics.

4.2 The J-Based Capability Maximin Rule and its Social Choice Process

Based upon the *social value judgement* $\psi(\mathsf{J})$ defined by ψ and J, let us introduce a best element set by $B(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J})) \equiv \{CC(\mathsf{e},\mathsf{x},\mathsf{y}) \mid \forall CC(\mathsf{e},\mathsf{x},\mathsf{y}') \in \mathcal{CC}(\mathsf{e},\mathsf{x}) : (CC(\mathsf{e},\mathsf{x},\mathsf{y}),\ CC(\mathsf{e},\mathsf{x},\mathsf{y}')) \in \psi(\mathsf{J})\}$. Then, we can define a rational choice function φ as follows: for every $\mathsf{J} = (J_i)_{i\in N}$, $\varphi(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J})) \in B(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J}))$ for each $\mathsf{e} \in E$ and $\mathsf{x} \in [0,\overline{x}]^n$. Let $C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x}) \equiv \varphi(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J}))$. We refer to this $C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x})$ as a reference capability under (e,x). Given the social welfare function ψ and the rational choice function φ, such a reference capability can vary, depending upon a profile of individuals' value judgements, J. Thus, we call it J-*based minimal capability under* (e,x).

Given $\mathsf{x} = (x_g)_{g\in N} \in [0,\overline{x}]^n$ and $i \in N$, let $\mathsf{x}_{-i} \equiv (x_g)_{g\in N\setminus\{i\}} \in [0,\overline{x}]^{n-1}$. Given $\mathsf{x} = (x_g)_{g\in N} \in [0,\overline{x}]^n$ and $i,j \in N$, let $\rho_{ij}(\mathsf{x}) \equiv (x_i^\rho, x_j^\rho, \mathsf{x}_{N\setminus\{i,j\}})$ where $x_i^\rho = x_j$ and $x_j^\rho = x_i$. Given $\mathsf{x} = (x_g)_{g\in N} \in [0,\overline{x}]^n$ and $i,j \in N$, let $(\rho_{ij}(\mathsf{x}))_{-i} \equiv \mathsf{x}'_{-i}$ where $\mathsf{x}' = \rho_{ij}(\mathsf{x})$. Given $\mathsf{e} \in E$, $\mathsf{x}_{-i} \in [0,\overline{x}]^{n-1}$, and a distribution rule h, let $h_i(\mathsf{e},[0,\overline{x}],\mathsf{x}_{-i}) \equiv \{y \in \mathbb{R}_+ \mid \exists x \in [0,\overline{x}] : y = h_i(\mathsf{e},x,\mathsf{x}_{-i})\}$. Then, we can introduce an anonymity requirement for distribution rules in the following:

Equal Attainable Sets for Equal Handicaps (EAEH):[14] *For all* $\mathsf{e} = (\mathsf{a},\mathsf{s},C,f)$, *all* $i,j \in N$ *such that* $a_i = a_j$ *and* $s_i = s_j$, *and all* $\mathsf{x} \in [0,\overline{x}]^n$, $h_i(\mathsf{e},[0,\overline{x}],\mathsf{x}_{-i}) = h_j(\mathsf{e},[0,\overline{x}],(\rho_{ij}(\mathsf{x}))_{-j})$ *and* $h_i(\mathsf{e},[0,\overline{x}],(\rho_{ij}(\mathsf{x}))_{-i}) = h_j(\mathsf{e},[0,\overline{x}],\mathsf{x}_{-j})$.

The society can determine a desirable distribution rule by choosing a reference capability called J-based minimal capability and by using an anonymous requirement in the sense of **EAEH**:

Definition 1: *Given* ψ, φ, *and* $\mathsf{J} \in \mathcal{J}^n$, *the* J-based capability maximin rule *(*$\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}$*) is a function* $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}} : E \times [0,\overline{x}]^n \to \mathbb{R}^n_+$ *satisfying* **EAEH** *such that for all* $\mathsf{e} \in E$ *and all* $\mathsf{x} = (x_i)_{i\in N} \in [0,\overline{x}]^n$, $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e},\mathsf{x}) = \mathsf{y} = (y_i)_{i\in N}$ *meets:*
(α) *for all* $i \in N$, $C(a_i, \overline{x} - x_i, y_i) \supseteq C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x})$,
(β) *there is no other* $\mathsf{y}^* \in Y(\mathsf{e},\mathsf{x})$ *such that* $CC(\mathsf{e},\mathsf{x},\mathsf{y}^*) \supsetneq C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x})$.

[14] A similar requirement was originally discussed by Fleurbaey (1995b, 1998).

The implication of the J-based capability maximin rule is that all individuals are always guaranteed a maximum of common capability under (e,x), where the maximum is determined on the basis of all individuals' judgements over common capabilities, $\mathsf{J} \in \mathcal{J}^n$.

Note that the J-based capability maximin rule is not always necessarily well-defined by the Paretian social welfare function ψ: First, if the social judgement $\psi(\mathsf{J})$ has no maximal element on $\mathcal{CC}(\mathsf{e},\mathsf{x})$ for some (e,x), then the J-based capability maximin rule is not well-defined. Second, if the social judgement $\psi(\mathsf{J})$ does not contain the set-inclusion subrelation in $\mathcal{CC}(\mathsf{e},\mathsf{x})$ for some (e,x), then this rule cannot be generated through ψ . So, the problem is to determine under what conditions this rule can be generated through the Paretian social welfare function.

As an auxiliary step to guarantee well-definedness of the J-based capability maximin rule, let us introduce an appropriate topology into the space of compact sets in $\mathbb{R}^m_+$ in terms of the Hausdorff metric.[15] Equipped with this topology and given $\mathsf{e} \in E$ and $\mathsf{x} \in [0,\overline{x}]^n$, let us say that the common capability $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \in \mathcal{CC}(\mathsf{e},\mathsf{x})$ is *undominated under* (e,x) if there is no $\mathsf{y}' \in Y(\mathsf{e},\mathsf{x})$ such that $CC(\mathsf{e},\mathsf{x},\mathsf{y}') \supsetneq CC(\mathsf{e},\mathsf{x},\mathsf{y})$. Let $\mathcal{UC}(\mathsf{e},\mathsf{x})$ denote the set of undominated common capabilities under (e,x). Moreover, given $\mathsf{e} \in E$, $CC(\mathsf{e},\mathsf{z}) \in \mathcal{CC}(\mathsf{e},\mathsf{x})$, and an ordering $J(\mathsf{e},\mathsf{x})$ on $\mathcal{CC}(\mathsf{e},\mathsf{x})$, let[16]

$$\mathcal{L}\left(CC(\mathsf{e},\mathsf{z}),J(\mathsf{e},\mathsf{x})\right) \equiv \{CC(\mathsf{e},\mathsf{z}') \in \mathcal{CC}(\mathsf{e},\mathsf{x}) \mid (CC(\mathsf{e},\mathsf{z}),\, CC(\mathsf{e},\mathsf{z}')) \in P(J(\mathsf{e},\mathsf{x}))\}.$$

The ordering relation $J(\mathsf{e},\mathsf{x})$ is *upper semi-continuous on* $\mathcal{UC}(\mathsf{e},\mathsf{x})$ if for all $CC(\mathsf{e},\mathsf{z}) \in \mathcal{UC}(\mathsf{e},\mathsf{x})$, $\mathcal{L}\left(CC(\mathsf{e},\mathsf{z}),J(\mathsf{e},\mathsf{x})\right) \cap \mathcal{UC}(\mathsf{e},\mathsf{x})$ is open in the Hausdorff relative topology on $\mathcal{UC}(\mathsf{e},\mathsf{x})$.

Let $N^{SIA}(\subseteq N)$ be the set of individuals whose proposed value judgements always belong to $\mathcal{J}_{SIA}$, and are always upper-semi continuous on $\mathcal{UC}(\mathsf{e},\mathsf{x})$ for all $(\mathsf{e},\mathsf{x}) \in E \times [0,\overline{x}]^n$.

Lemma 1: *Suppose that* $\# N^{SIA} \geq 1$. *Then, there exists a* Paretian social welfare function ψ *such that for each proposed* $\mathsf{J} \in \mathcal{J}^n$, $B(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J})) \neq \varnothing$ *and* $B(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J})) \subseteq \mathcal{UC}(\mathsf{e},\mathsf{x})$ *for all* $(\mathsf{e},\mathsf{x}) \in E \times [0,\overline{x}]^n$.

[15] For any compact sets $C, C' \subseteq \mathbb{R}^{\mathsf{m}}$, the Hausdorff metric is defined by
$d(C,C') \equiv \max\{\max\{\delta(\mathsf{b},C) \mid \mathsf{b} \in C'\}, \max\{\delta(\mathsf{b},C') \mid \mathsf{b} \in C\}\}$,
where $\delta(\mathsf{b},C) \equiv \min_{\mathsf{b}' \in C} \parallel \mathsf{b},\mathsf{b}' \parallel$, and $\parallel \mathsf{b},\mathsf{b}' \parallel$ denotes the Euclidean distance between b and b'.

[16] For the sake of notational convenience, we sometimes denote the common capability by $CC(\mathsf{e},\mathsf{z})$ instead of $CC(\mathsf{e},\mathsf{x},\mathsf{y})$ when $\mathsf{z} = (\overline{\mathsf{x}} - \mathsf{x},\mathsf{y}) \in Z(\mathsf{e})$.

We are now ready to put forward the result on the possibility of social choice of the J-based capability maximin rule.

Theorem 1: *Suppose that* $\# N^{SIA} \geq 1$. *Then, there exists a* Paretian social welfare function ψ *and the corresponding rational choice function* φ *by which the* J-based capability maximin rule $h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}$ *is always generated. In addition, the* J-based capability maximin rule *is uniquely determined.*

Theorem 1 shows the existence of a Paretian social welfare function which never fails to generate the J-based capability maximin rule whenever there exists a person whose proposed value judgement, which is always upper semi-continuous, always satisfies **Anonymity** and **Set-Inclusion Subrelation**. Let ψ be just such a function and φ its corresponding rational choice function, both of which are assumed to be fixed in the following discussion.

4.3 A Characterization of the Social Value Judgement

We now characterize the social value judgement $\psi(\mathsf{J})$ which rationalizes the J-based capability maximin rule. First, we define a ranking of the feasible assignments of capabilities by an ordering relation $R^{\mathcal{FC}} \subseteq \mathcal{FC} \times \mathcal{FC}$ such that $(\mathsf{C}(\mathsf{e},\mathsf{z}), \mathsf{C}(\mathsf{e}',\mathsf{z}')) \in R^{\mathcal{FC}}$, where $\mathsf{C}(\mathsf{e},\mathsf{z}) \equiv (C(a_i, z_i))_{i \in N}$ and $\mathsf{C}(\mathsf{e}',\mathsf{z}') \equiv (C(a_i', z_i'))_{i \in N}$, which implies that the feasible assignment of capabilities $\mathsf{C}(\mathsf{e},\mathsf{z})$ is at least as good as the feasible assignment of capabilities $\mathsf{C}(\mathsf{e}',\mathsf{z}')$. The asymmetric and symmetric parts of $R^{\mathcal{FC}}$ are denoted by $P(R^{\mathcal{FC}})$ and $I(R^{\mathcal{FC}})$ respectively. Given $\mathsf{e} \in E$ and $\mathsf{x} \in [0, \overline{x}]^n$, let $R^{\mathcal{FC}}(\mathsf{e},\mathsf{x}) \equiv R^{\mathcal{FC}} \cap (\mathcal{FC}(\mathsf{e},\mathsf{x}))^2$ and $B(\mathcal{FC}(\mathsf{e},\mathsf{x}), R^{\mathcal{FC}}) \equiv \{\mathsf{C}(\mathsf{e},\mathsf{z}) \in \mathcal{FC}(\mathsf{e},\mathsf{x}) \mid \forall \mathsf{C}(\mathsf{e},\mathsf{z}') \in \mathcal{FC}(\mathsf{e},\mathsf{x}) : (\mathsf{C}(\mathsf{e},\mathsf{z}), \mathsf{C}(\mathsf{e},\mathsf{z}')) \in R^{\mathcal{FC}}\}$. Among various orderings on $\mathcal{FC}$, we are particularly interested in an ordering $R^{\mathcal{FC}}_{\psi(\mathsf{J})} \subseteq \mathcal{FC} \times \mathcal{FC}$ defined as follows: for all $\mathsf{e}, \mathsf{e}' \in E$, all $\mathsf{z} \in Z(\mathsf{e})$, and all $\mathsf{z}' \in Z(\mathsf{e}')$, $(\mathsf{C}(\mathsf{e},\mathsf{z}), \mathsf{C}(\mathsf{e}',\mathsf{z}')) \in R^{\mathcal{FC}}_{\psi(\mathsf{J})}$ if and only if $(CC(\mathsf{e},\mathsf{z}), CC(\mathsf{e}',\mathsf{z}')) \in \psi(\mathsf{J})$.

Second, we introduce axioms concerning the rankings of the feasible capability assignments as follows:

Pareto Inclusion in Capability Assignments (PICA): *For all* $\mathsf{e} \in E$, *all* $\mathsf{x} \in [0, \overline{x}]^n$, *and all* $\mathsf{y}, \mathsf{y}' \in Y(\mathsf{e},\mathsf{x})$,
$[C(a_i, \overline{x} - x_i, y_i) \supseteq C(a_i, \overline{x} - x_i, y_i')$ $(\forall i \in N)$ and $C(a_j, \overline{x} - x_j, y_j) \supsetneq C(a_j, \overline{x} - x_j, y_j')$ $(\exists j \in N) \Rightarrow (\mathsf{C}(\mathsf{e},\mathsf{z}), \mathsf{C}(\mathsf{e},\mathsf{z}')) \in P(R^{\mathcal{FC}}(\mathsf{e},\mathsf{x}))]$.

Anonymity in Capability Assignments (ACA): *For all* $\mathsf{e} \in E$, *all* $\mathsf{x} \in [0, \overline{x}]^n$, *all* $\mathsf{y}, \mathsf{y}' \in Y(\mathsf{e}, \mathsf{x})$, *and all* $i, j \in N$ *such that* $a_i = a_j$ *and* $s_i = s_j$, $[(\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}), \mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}')) \in R^{\mathcal{FC}}(\mathsf{e}, \mathsf{x}) \Leftrightarrow (\mathsf{C}(\mathsf{e}, \rho_{ij}(\mathsf{x}, \mathsf{y})), \mathsf{C}(\mathsf{e}, \rho_{ij}(\mathsf{x}, \mathsf{y}'))) \in R^{\mathcal{FC}}(\mathsf{e}, \rho_{ij}(\mathsf{x})))]$.

Let us define a *relative advantage function* $\mu : \mathcal{FC} \times N \rightarrow \mathbb{R}_+$ such that for all $\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}) \in \mathcal{FC}$ and all $i \in N$:
(i) $[C(a_i, \overline{x} - x_i, y_i) = CC(\mathsf{e}, \mathsf{x}, \mathsf{y}) \Rightarrow \mu(\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}), i) = 0]$,
(ii) for all $j \in N$, $[C(a_i, \overline{x} - x_i, y_i) \supseteq C(a_j, \overline{x} - x_j, y_j) \Rightarrow \mu(\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}), i) \geq \mu(\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}), j)]$.

This function provides a measure that is intended to capture the extent of inequality in capability assignments: *the higher the real number of* $\max_{i \in N} \mu(\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}), i)$ *is, the higher the degree of inequality in capabilities will be.* Among possibly various relative advantage functions, we are particularly interested in the following one: $\mu^* : \mathcal{FC} \times N \rightarrow \mathbb{R}_+$ such that for all $\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}) \in \mathcal{FC}$ and all $i \in N$, $\mu^*(\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}), i) \equiv \min\{\eta - 1 \in \mathbb{R}_+ \mid \exists \mathsf{b} \in \partial C(a_i, \overline{x} - x_i, y_i)$ s.t. $\eta^{-1}\mathsf{b} \in \partial CC(\mathsf{e}, \mathsf{x}, \mathsf{y})\}$. We can easily check that the function μ^* meets the two conditions (i) and (ii) of relative advantage functions. By using this μ^*, we introduce the following:

μ^*-**Equity in Capability Assignments** (μ^***ECA**)**:** *For all* $\mathsf{e} \in E$ *and all* $\mathsf{x} \in [0, \overline{x}]^n$, *there exists* $\max_{i \in N} \mu^*(\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}^*), i) \equiv \min_{\mathsf{y} \in Y(\mathsf{e}, \mathsf{x})} \max_{i \in N} \mu^*(\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}), i)$ *such that* $\mathsf{C}(\mathsf{e}, \mathsf{x}, \mathsf{y}^*) \in B(\mathcal{FC}(\mathsf{e}, \mathsf{x}), R^{\mathcal{FC}})$.

The implication of μ^***ECA** will be explained as follows: Given $(\mathsf{e}, \mathsf{x}, \mathsf{y})$ and given $i \in N$, any functioning vectors $\mathsf{b}_i, \mathsf{b}_i' \in \partial C(a_i, \overline{x} - x_i, y_i)$ are incomparable from each other and maximal in $C(a_i, \overline{x} - x_i, y_i)$ with respect to vector inequalities. Note that for any $\mathsf{b}_i \in \partial C(a_i, \overline{x} - x_i, y_i)$, there is a functioning vector $\mathsf{b}(\mathsf{b}_i) \equiv \eta^{-1}\mathsf{b}_i$ in $\partial CC(\mathsf{e}, \mathsf{x}, \mathsf{y})$ where $\eta \in [1, +\infty)$ implies the degree of advantage of the functioning b_i over the minimally warranted functioning $\mathsf{b}(\mathsf{b}_i)$. Among such functioning vectors in $\partial C(a_i, \overline{x} - x_i, y_i)$, there is a functioning vector b_i^* such that its corresponding scalar η^* is the *minimum* value in $\partial C(a_i, \overline{x} - x_i, y_i)$, which implies that the advantage of b_i^* over $\mathsf{b}(\mathsf{b}_i^*)$ is less than the advantage of any vector in $\partial C(a_i, \overline{x} - x_i, y_i)$. In the function μ^*, individual i's relative advantage over other individuals in capability assignments under $(\mathsf{e}, \mathsf{x}, \mathsf{y})$ is measured by $\eta^* - 1$. The axiom μ^***ECA** says that the capability assignment under which the degree of inequality in capabilities in terms of μ^* is minimized should be most preferable.

Theorem 2: *Given* $\mathsf{J} \in \mathcal{J}^n$*, suppose that there exists at least one individual* $i \in N$ *such that* $J_i \in \mathcal{J}_{SIA}$ *and* $J_i(\mathsf{e}, \mathsf{x})$ *is* upper-semi continuous on $\mathcal{UC}(\mathsf{e}, \mathsf{x})$. *Then, the ordering* $R^{\mathcal{FC}}_{\psi(\mathsf{J})} \subseteq \mathcal{FC} \times \mathcal{FC}$ *satisfies* PICA, ACA, *and* μ^*ECA. *Conversely, if the ordering* $R^{\mathcal{FC}} \subseteq \mathcal{FC} \times \mathcal{FC}$ *satisfies* PICA, ACA, *and* μ^*ECA, *then* $R^{\mathcal{FC}} = R^{\mathcal{FC}}_{\psi(\mathsf{J})}$ *for some* $\mathsf{J} \in \mathcal{J}^n$.

We should mention the literature related to the characterization of the ordering $R^{\mathcal{FC}}_{\psi(\mathsf{J})}$. Herrero et al. (1998) and Bossert et al. (1999) respectively defined the ordering of opportunity profiles, which is similar to $R^{\mathcal{FC}}_{\psi(\mathsf{J})}$, in the abstract *rich* domains, and provided axiomatic characterizations under the assumption of *richness* in Herrero et al. (1998) and Bossert et al. (1999). Note that their characterization results cannot be directly applied to $R^{\mathcal{FC}}_{\psi(\mathsf{J})}$, since the domain of the ordering $R^{\mathcal{FC}}$, which is the Hausdorff space $\mathcal{FC}$, does not necessarily satisfy the *richness* condition. Thus, in contrast to their results, our axiomatic characterization on $R^{\mathcal{FC}}_{\psi(\mathsf{J})}$ is obtained independently of the *richness* condition. Moreover, it is obtained by adopting a specific equity axiom, μ^**ECA,* instead of the *Hammond's equity* axioms in Herrero et al. (1998) and Bossert et al. (1999).

5 Characterizations of the Class of J-Based Capability Maximin Rules

5.1 Axioms on Distribution Rules

In this subsection, we discuss the desirability of distribution rules from the viewpoints of fair opportunity to well-being. Here, individual opportunity to well-being is identified with individual capabilities attainable through the application of distribution rules. How can we define the fairness of opportunity to well-being in this context?

In support of these viewpoints, we consider the following axioms:

Minimal Equality of Capability (MEC): *For all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ *and all* $\mathsf{x} = (x_i)_{i \in N} \in [0, \overline{x}]^n$,
$[\ \forall i, j \in N,\ a_i = a_j \text{ and } x_i = x_j] \Rightarrow [\ \forall i, j \in N,\ C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x})) = C(a_j, \overline{x} - x_j, h_j(\mathsf{e}, \mathsf{x}))]$.

Note that NP implies MEC, and CP also implies MEC whenever $s_i = s_j \Leftrightarrow a_i = a_j$ for all $i, j \in N$.

The next three axioms are related to *responsibility* and *compensation* discussed in Bossert (1995), Fleurbaey (1994, 1995a, b, 1998), and Fleurbaey and Maniquet (1994, 1996), etc. The first axiom is motivated by the following discussion: if there is reason to compensate someone in their capabilities, then there should exist differences in individuals' utilization abilities. To formulate a similar argument in the context of compensating inequal utility distributions, Fleurbaey (1995a, 1998), etc., introduced the *equal resource for equal handicap* axiom and/or other similar ones. Since our distribution rules are game forms, the formulation of which differs from that of allocation rules by Fleurbaey (1995a, 1998), etc., we cannot directly apply the *equal resource for equal handicap* and/or other similar axioms to our context. However, the following axiom seems to be along the same line as the *equal resource for equal handicap* axioms:

No Domination of Resources among Equally Handicapped (NDEH): *For all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$, *all* $\mathsf{x} \in [0, \overline{x}]^n$, *and all* $i, j \in N$ *such that* $a_i = a_j$, $[x_i = x_j \Rightarrow h_i(\mathsf{e}, \mathsf{x}) = h_j(\mathsf{e}, \mathsf{x})]$ and $[x_i < x_j \Rightarrow h_i(\mathsf{e}, \mathsf{x}) < h_j(\mathsf{e}, \mathsf{x})]$.

It is easy to show that *NDEH* implies *MEC*. Note that *NDEH* and *EAEH* are logically independent of each other.

The following two axioms are relevant to "*principle of compensation in capabilities.*"

No Domination of Capabilities among Equal Efforts (NDEE): *For all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$, all $\mathsf{x} \in [0, \overline{x}]^n$, *and all* $i, j \in N$ *such that* $x_i = x_j$, [neither $C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x})) \subsetneq C(a_j, \overline{x} - x_j, h_j(\mathsf{e}, \mathsf{x}))$ nor $C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x})) \supsetneq C(a_j, \overline{x} - x_j, h_j(\mathsf{e}, \mathsf{x}))$].

No Strict Domination of Capabilities among Equal Efforts (NS-DEE): *For all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$, *all* $\mathsf{x} \in [0, \overline{x}]^n$, *and all* $i, j \in N$ *such that* $x_i = x_j$, [neither $C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x})) \subsetneq int\ C(a_j, \overline{x} - x_j, h_j(\mathsf{e}, \mathsf{x}))$ nor $int\ C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x})) \supsetneq C(a_j, \overline{x} - x_j, h_j(\mathsf{e}, \mathsf{x}))$].

These two axioms mean that, given that one chooses the same amount of labor time as another, then should the individual's utilization ability be worse than the other, this should not necessarily determine a worse situation in that person's capability. Note that it seems to be more appealing if the principle of compensation in capabilities requires equal capabilities for any two persons who respectively provide the same amount of labor time. However, such a requirement is generally empty. So, these kinds of "non-domination in terms of set-inclusion" are more appropriate in discussing compensation of capabilities. Note that *NSDEE* is implied by *NDEE*. Moreover, it is easy to show that *NSDEE* implies *MEC*. However, *NSDEE* and *NP* are independent of each other.

The next two axioms concern "solidarity" conditions of distribution rules. The first axiom is relevant to the change in individuals' supplies of labor time. If, as a result of a change in their supplies of labor time, someone's capability is worse relative to another's, then the first one should be described as suffering the negative effect of the change more than the second. The axiom, *Solidarity in Rank of the Weakest Functioning Vectors*, defined below precludes any individual from falling into such a situation.

As an auxiliary step in introducing the above axiom, let us discuss the following: Given $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$, $\mathsf{z} \in Z(\mathsf{e})$, and $\mathsf{b}_i \in \partial C(a_i, z_i)$ *of* $i \in N$, let $N(i, \mathsf{b}_i, \mathsf{e}, \mathsf{z}) \equiv \{j \in N \mid \exists \lambda_j \geq 1$, s.t. $\lambda_j \mathsf{b}_i \in C(a_j, z_j)\}$.

Definition 2: *Given* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ *and* $\mathsf{z} \in Z(\mathsf{e})$, *the functioning vector* $\mathsf{b}_i \in \partial C(a_i, z_i)$ *of* $i \in N$ *is* the weakest vector for $i \in N$ *if for any other* $\mathsf{b}_i^* \in \partial C(a_i, z_i)$, $\# N(i, \mathsf{b}_i, \mathsf{e}, \mathsf{z}) \geq \# N(i, \mathsf{b}_i^*, \mathsf{e}, \mathsf{z})$.

Given $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ and $\mathsf{z} \in Z(\mathsf{e})$, let us denote the set of the weakest functioning vectors for $i \in N$ by $B_i^w(\mathsf{e}, \mathsf{z})$, and denote an element of $B_i^w(\mathsf{e}, \mathsf{z})$ by $\mathsf{b}_i^w(\mathsf{e}, \mathsf{z})$.

Solidarity in Rank of the Weakest Functioning Vectors (SRWF): *For all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ *and all* $\mathsf{x} = (x_i)_{i \in N} (\neq 0)$, $\mathsf{x}' = (x_i')_{i \in N} (\neq 0) \in [0, \overline{x}]^n$ *such that for all* $i \neq j$, $x_i = x_i'$ *and* $x_j \leq x_j'$, *there is no* $i \in N$ *such that* $\# N(i, \mathsf{b}_i^w(\mathsf{e}, \mathsf{z}), \mathsf{e}, \mathsf{z}) > \# N(i, \mathsf{b}_i^w(\mathsf{e}, \mathsf{z}'), \mathsf{e}, \mathsf{z}')$ *where* $\mathsf{z} = (\overline{\mathsf{x}} - \mathsf{x}, h(\mathsf{e}, \mathsf{x}))$, $\mathsf{z}' = (\overline{\mathsf{x}} - \mathsf{x}', h(\mathsf{e}, \mathsf{x}'))$, *and* $\overline{\mathsf{x}} = (\overline{x}, \ldots, \overline{x}) \in \mathbb{R}_+^n$.

This axiom stipulates that no one's rank in the weakest functioning vector will shift up through additional labor time by an individual.

The next axiom is relevant to the change in individuals' ability endowments. Consider that someone suffers from a bad accident, and as a result, can no longer maintain her current capability. The following axiom requires compensating the unlucky person in such a situation for her loss of capability.

No Relative Advantage by Bad Accidents (NRABA): *For all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f)$, $\mathsf{e}' = (\mathsf{a}', \mathsf{s}, C, f) \in E$ *such that for some* $j \in N$, $a_j \geq a_j'$ *and* *for any other* $i \neq j$, $a_i = a'_i$, *and all* $\mathsf{x} \in [0, \overline{x}]^n$, *there is no* $i \in N$ *such that* $\# N(i, \mathsf{b}_i^w(\mathsf{e}, \mathsf{z}), \mathsf{e}, \mathsf{z}) > \# N(i, \mathsf{b}_i^w(\mathsf{e}', \mathsf{z}'), \mathsf{e}', \mathsf{z}')$ *where* $\mathsf{z} = (\overline{\mathsf{x}} - \mathsf{x}, h(\mathsf{e}, \mathsf{x}))$ *and* $\mathsf{z}' = (\overline{\mathsf{x}} - \mathsf{x}, h(\mathsf{e}', \mathsf{x}))$.

This axiom requires precluding every individual from making her rank of the weakest vector shift up through another's unfortunate accident. This implies that all individuals are required to share the effect of someone's brute luck.

The following axiom is a requirement for *non-wastefulness* of resource allocations in the sense that it only requires distributing total outputs exhaustively.

Pareto Efficiency with respect to Capabilities (PEC): *For all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$, *all* $\mathsf{x} \in [0, \overline{x}]^n$, *and all* $\mathsf{y}' \in \mathbb{R}^n_+$, $[C(a_i, \overline{x} - x_i, y'_i) \supsetneq C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x}))\ (\forall i \in N) \Rightarrow (\overline{x} - x_i, y'_i)_{i \in N} \notin Z(\mathsf{e})]$.

Note that PEC and $NSDEE$ together imply NP.

5.2 Characterizations of the Class of J-Based Capability Maximin Rules

Note, that if the profile of individual value judgements changes from J to J^*, then the distribution rule selected through ψ and φ would change from $h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}$ to $h^{\mathsf{CM}_{\mathsf{J}^*}^{\psi,\varphi}}$, where $h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}} \neq h^{\mathsf{CM}_{\mathsf{J}^*}^{\psi,\varphi}}$. This implies that there are many J-based capability maximin rules which are socially chosen according to the profile of individual value judgements $\mathsf{J} \in \mathcal{J}^n$. Based upon this property, we can introduce the *class of* J*-based capability maximin rules* which are socially chosen through individual value judgements, and denote this class by $UCM \equiv \{h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}} \mid \mathsf{J} \in \mathcal{J}^n\}$. One of the properties of the set UCM deserves attention. The set UCM is defined independently of ψ and φ, whenever ψ satisfies Pareto Principle. To understand this point, let us take a distribution rule $h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}$ from UCM. By definition, this distribution rule is socially chosen through ψ and φ under the profile of individual value judgements $\mathsf{J} \in \mathcal{J}^n$. Then, corresponding to $h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}$, $C^{\min}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x}) \in \mathcal{UC}(\mathsf{e}, \mathsf{x})$ is defined for each and every $(\mathsf{e}, \mathsf{x}) \in E \times [0, \overline{x}]^n$. Let us define another Paretian social welfare function ψ^* and the associated rational choice function φ^*. Let us also consider another profile of individual value judgements $\mathsf{J}^* \in \mathcal{J}^n$ such that for each and every $(\mathsf{e}, \mathsf{x}) \in E \times [0, \overline{x}]^n$, $(C^{\min}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x}), CC(\mathsf{e}, \mathsf{x}, \mathsf{y})) \in P(J^*_i(\mathsf{e}, \mathsf{x}))$ for all $\mathsf{y} \in Y(\mathsf{e}, \mathsf{x})$ and all $i \in N$.

Then, we can definitely obtain $\varphi^*(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi^*(\mathsf{J}^*)) = C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x})$ for each and every $(\mathsf{e},\mathsf{x}) \in E \times [0,\overline{x}]^n$, since ψ^* satisfies the Pareto principle. This implies that the set UCM is determined, regardless of the Paretian social welfare functions and the rational choice functions.

To characterize the class of J-based capability maximin rules by means of the axioms defined in the previous section, we first introduce the following concept concerning assignments of capabilities by distribution rules:

Definition 3: *The* distribution rule h *meets* undominated property *if for all* $\mathsf{e} = (\mathsf{a},\mathsf{s},C,f) \in E$ *and all* $\mathsf{x} \in [0,\overline{x}]^n$*, there is no* $i \in N$ *such that for all* $j \neq i$*, int* $C(a_i,\overline{x}-x_i,h_i(\mathsf{e},\mathsf{x})) \supsetneq \bigcap_{j\neq i} C(a_j,\overline{x}-x_j,h_j(\mathsf{e},\mathsf{x}))$.

Let us denote the class of undominated distribution rules by UD.

Lemma 3: $UCM = EAEH \cap UD \cap PEC$.

By using Lemma 3, we obtain the following characterizations of UCM:

Theorem 3: $UCM = EAEH \cap MEC \cap SRWF \cap NRABA \cap PEC$.

This characterization shows that UCM is the set of anonymous and non-wasteful distribution rules satisfying MEC and the two solidarity conditions. By this characterization, we may see that if we support *equality of capabilities* [Sen (1980)] as the answer to the problem on "equality of what," all rules in UCM would be egalitarian in the sense of Sen (1980).

We can show that the above five axioms in Theorem 3 are logically independent. To see this, consider the following examples:

Example 1: Let h be a distribution rule having the following property: there is $\mathsf{J} \in \mathcal{J}^n$ such that for all $\mathsf{e} \in E$ and all $\mathsf{x} \in [0,\overline{x}]^n$,
(1) if for all $i,j \in N$, $x_i = x_j \neq 0$, then $h(\mathsf{e},\mathsf{x}) = h^{CM_{\mathsf{J}}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})$,
(2) otherwise, $h(\mathsf{e},\mathsf{x}) = 0$.
Then, h satisfies $EAEH$, MEC, $SRWF$, and $NRABA$, but not PEC.

Example 2: Let h be a distribution rule having the following property: there is $\mathsf{J} \in \mathcal{J}^n$ such that for all $\mathsf{e} \in E$ and all $\mathsf{x} \in [0,\overline{x}]^n$,
(1) if there exists $j \in N$ such that $x_j > 0$, and for any $i \neq j$, $x_i = 0$, then

$$h_j(\mathsf{e},\mathsf{x}) = f(s_j x_j) \text{ and } h_i(\mathsf{e},\mathsf{x}) = 0,$$

(2) otherwise, $h(\mathsf{e},\mathsf{x}) = h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})$.
Then, h satisfies $EAEH$, MEC, PEC, and $NRABA$, but not $SRWF$.

Example 3: Let h be a distribution rule having the following property: there is $\mathsf{J} \in \mathcal{J}^n$ such that for all $\mathsf{e} \in E$ and all $\mathsf{x} \in [0,\overline{x}]^n$,
(1) if there exists a pair of individuals $\{i,j\} \subseteq N$ such that $a_i = a_j$, then $h(\mathsf{e},\mathsf{x}) = h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})$,
(2) otherwise, h has the property that $f(\sum s_g x_g) = \sum h_g(\mathsf{e},\mathsf{x})$, and there exists a partition of N, $\mathcal{N}(\mathsf{e},\mathsf{x}) \equiv \{N^1(\mathsf{e},\mathsf{x}), N^2(\mathsf{e},\mathsf{x}), \cdots, N^T(\mathsf{e},\mathsf{x})\}$ where $2 \leq T \leq n$, such that
(α) for all $N^t(\mathsf{e},\mathsf{x}) \in \mathcal{N}(\mathsf{e},\mathsf{x})$ and all $i,j \in N^t(\mathsf{e},\mathsf{x})$,

$$\#N\left(i, \mathsf{b}_i^w\left(\mathsf{e},(\mathsf{x},h(\mathsf{e},\mathsf{x}))\right), \mathsf{e},(\mathsf{x},h(\mathsf{e},\mathsf{x}))\right) = \#N\left(j, \mathsf{b}_j^w\left(\mathsf{e},(\mathsf{x},h(\mathsf{e},\mathsf{x}))\right), \mathsf{e},(\mathsf{x},h(\mathsf{e},\mathsf{x}))\right),$$

(β) there exists $N^t(\mathsf{e},\mathsf{x}) \in \mathcal{N}(\mathsf{e},\mathsf{x})$ such that for all $i \in N^t(\mathsf{e},\mathsf{x})$,

$$\#N\left(i, \mathsf{b}_i^w\left(\mathsf{e},(\mathsf{x},h(\mathsf{e},\mathsf{x}))\right), \mathsf{e},(\mathsf{x},h(\mathsf{e},\mathsf{x}))\right) < n, \text{ and}$$

(γ) for any other $\mathsf{x}' \in [0,\overline{x}]^n$, $\mathcal{N}(\mathsf{e},\mathsf{x}') = \mathcal{N}(\mathsf{e},\mathsf{x})$.
Then, h satisfies $EAEH$, MEC, PEC, and $SRWF$, but not $NRABA$.

Example 4: Let h be a distribution rule having the following property: there is $\mathsf{J} \in \mathcal{J}^n$ such that for all $\mathsf{e} = (\mathsf{a},\mathsf{s},C,f) \in E$ and all $\mathsf{x} \in [0,\overline{x}]^n$,
(1) if there exists a pair of individuals $\{i,j\} \subseteq N$ such that $s_i = s_j$, then $h(\mathsf{e},\mathsf{x}) = h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})$,
(2) otherwise, h has the property that $f(\sum s_g x_g) = \sum h_g(\mathsf{e},\mathsf{x})$, and
(α) if for all $i,j \in N$, $a_i = a_j$ and $x_i = x_j \neq 0$, then $h_1(\mathsf{e},\mathsf{x}) < h_2(\mathsf{e},\mathsf{x}) < \cdots < h_n(\mathsf{e},\mathsf{x})$,
(β) otherwise, $C(a_1,\overline{x}-x_1,h_1(\mathsf{e},\mathsf{x})) \subsetneq C(a_2,\overline{x}-x_2,h_2(\mathsf{e},\mathsf{x})) \subsetneq \cdots \subsetneq C(a_n,\overline{x}-x_n,h_n(\mathsf{e},\mathsf{x}))$ whenever $\mathsf{x} \neq 0$, and $h(\mathsf{e},\mathsf{x}) = 0$ if $\mathsf{x} = 0$.
Then, h satisfies $EAEH$, $SRWF$, $NRABA$, and PEC, but not MEC.

Example 5: Let h be a distribution rule having the following property:
(1) for all $\mathsf{e} \in E$ and all $\mathsf{x} \in [0,\overline{x}]^n$, there exists $\mathsf{J} \in \mathcal{J}^n$ such that $h(\mathsf{e},\mathsf{x}) = h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})$,
(2) there is an economy $\mathsf{e}' = (\mathsf{a}',\mathsf{s}',C,f) \in E$ and a pair of individuals $\{i,j\} \subseteq N$ such that $a'_i = a'_j$ and $s'_i = s'_j$, but

$$h_i(\mathsf{e}',\mathsf{x}') \neq h_j(\mathsf{e}',\rho_{ij}(\mathsf{x}')) \text{ and } h_j(\mathsf{e}',\mathsf{x}') \neq h_i(\mathsf{e}',\rho_{ij}(\mathsf{x}'))$$

for some $\mathsf{x}' \in [0, \overline{x}]^n$ such that $x'_i \neq x'_j$.
Then, h satisfies MEC, $SRWF$, $NRABA$, and PEC, but not $EAEH$.

The following characterization shows that UCM is the unique class of anonymous and non-wasteful distribution rules satisfying Needs Principle and the two solidarity conditions:

Theorem 4: $UCM = NP \cap EAEH \cap SRWF \cap NRABA \cap PEC$.[17]

Since the Needs Principle and the two solidarity conditions together may be important requirements in determining resource allocations from the viewpoints of social security, it might be plausible to adopt the distribution rules in UCM in order to implement social security.

We can check the independence of the five axioms in Theorem 4 if $\#N \geq 3$. The independence of PEC, $SRABA$, NP, and $EAEH$ is shown by the above **Examples 1**, **2**, **4**, and **5**, respectively. The independence of $NRABA$ is shown by the following example:

Example 3*: Let h be a distribution rule having the properties of **Example 3**-(1) and (2). Moreover, for all $\mathsf{e} \in E$ and all $\mathsf{x} \in [0, \overline{x}]^n$, there is no pair of individuals, $\{i, j\} \subsetneq N$, such that

$$int\ C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x})) \supsetneq C(a_j, \overline{x} - x_j, h_j(\mathsf{e}, \mathsf{x})).$$

Then, h satisfies $EAEH$, NP, PEC, and $SRWF$, but not $NRABA$.

Theorem 5: *There exists a distribution rule h such that $h \in EAEH \cap NDEH \cap NDEE$. Moreover, for some $\mathsf{J} \in \mathcal{J}^n$, $h = h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}$.*[18]

Among the above three characterizations on UCM, it is particularly worth noting regarding Theorem 5 that UCM contains a class of rules satisfying the axioms of responsibility and of compensation, when we evaluate individuals' well-being by Sen's capabilities. In contrast, previous works

[17] If we assume two-person economies, Theorem 4 is reduced to the following:

$[\#N = 2] \Rightarrow [UCM = NP \cap EAEH \cap SRWF \cap PEC]$.

[18] We also obtain the following results similar to Theorem 5:

$UCM = NDEH \cap EAEH \cap SRWF \cap NRABA \cap PEC$,
$UCM = NSDEE \cap EAEH \cap SRWF \cap NRABA \cap PEC$.

showed the incompatibility between the axioms of responsibility and of full compensation, when individual well-being is evaluated by the standard individual utility [Fleurbaey (1994, 1995a), etc.]. This difference between us and the previous works may come from the different evaluative basis of comparing individuals' well-being: The basis in the previous works is put on individuals' achieved satisfactions (utilities), while in ours it is on individuals' opportunities (capabilities) to achieve satisfactions. In fact, in our model, if we introduce individual utility functions and define the axiom which requires equal utility for any two individual with equal utility functions in the same way as *Equal Welfare for Equal Preference* (EWEP) in Fleurbaey (1994, 1995a), then it might lead to the incompatibility result similar to Fleurbaey (1994, 1995a). This implies that if only the objective states of individual well-being, like capability assignments, are the informational basis for evaluating individuals' outcomes in resource allocations, the conflict between the requirements of responsibility and of compensation is not so serious as the one when the subjective states of individual well-being are the informational basis.

5.3 Discussion

As was shown in the formulation of capabilities and in the proposal of J-based capability maximin rules, we focussed only on the relevant functionings of consumption aspects in human life and on the determination of the opportunity sets of such functionings through the application of distribution rules. So, in this paper, there is no discussion about determining, through the application of distribution rules, the opportunity sets of functionings of production aspects such as that of "being able to work as much as one wants." This is because of the simple, static setting of production economies and resource allocations where every individual is endowed with a fixed production skill and the same set of labor time, and engages in the same type of labor.[19] So, the application of distribution rules does not influence the sizes of the opportunity sets of labor. Although we believe that such a limited analysis can be drawn from the Rawlsian viewpoint of distributive justice, of course, there may well be an argument that the opportunity sets of some functionings of production aspects such as "being able to work as much as one wants" should

[19] By taking skills of individuals fixed, we are in effect excluding the possibility of improving skills through education. Likewise, by assuming only one type of labor, we are in effect excluding the possibility of searching for alternative job opportunities.

be variable, depending upon the results of resource allocations. This would be true even if we assume only one type of labor. For example, the Marxian theory on *reproduction of labor-power* seems to lead us to such a perspective on the opportunity sets of some functionings of production aspects.

The last argument may make us consider resource allocation schemes which fairly assign to every individual, opportunity sets of functionings not only for consumption activities, but also for production activities. We think that this problem can appropriately be treated in dynamic settings of resource allocation problems. In this case, the reproduction process of "labor-power" and/or the education and learning process for improving production skills are explicitly discussed, although this is far beyond the scope of this paper.

6 Concluding Remarks

In this paper, we formalize, in cooperative production economies, the distribution rules as game forms, and define Sen's capability set in the context of existence of interaction among individuals concerning assignment of capabilities. Moreover, we propose a class of distribution rules, the class of J-*based capability maximin rules*, and discuss axiomatic characterizations of these rules mainly from normative viewpoints. Strategic aspects of the J-based capability maximin rules are discussed by Gotoh and Yoshihara (1997).

Although this paper discusses the social decision procedure for selecting distribution rules, which chooses the J-based capability maximin rule in the primordial stage of rule selection, the object of this paper is not to formalize systematically the *two principles of justice* proposed by Rawls (1971). This formalization has been done by Gotoh, Suzumura, and Yoshihara (2001).

7 Proofs of Theorems

Proof of Lemma 1: We will complete the proof by showing, first, Step (1) and second, Step (2) as follows:

Step (1): *$\mathcal{UC}(\mathsf{e}, \mathsf{x})$ is non-empty and compact for each $\mathsf{e} \in E$ and each $\mathsf{x} \in [0, \overline{x}]^n$.*

We define a correspondence $\zeta_{\mathsf{e},\mathsf{x}}$ and show that it is continuous on the compact set $Y_{\mathsf{e}}(\mathsf{x})$ which is also defined below. Next, we define a set $\mathcal{UIC}(\mathsf{e}, \mathsf{x})$

and show that it is compact. Finally, we show that the compactness of $\mathcal{UC}(e, x)$ by noting that $\mathcal{UC}(e, x) = \mathcal{UIC}(e, x) \cap \zeta_{e,x}(\partial Y_e(x))$.

Given $e \in E$, the set $Y(e, x)$ is well-defined and compact for each $x \in [0, \overline{x}]^n$, which can be used to define a compact-valued and continuous correspondence $Y_e : [0, \overline{x}]^n \; ^3 \; \mathbb{R}^n_+$ as $Y_e(x) = Y(e, x)$ for each $x \in [0, \overline{x}]^n$. Let $\partial Y_e(x) \equiv \{y = (y_i)_{i \in N} \in \mathbb{R}^n_+ \mid f(\sum s_i x_i) = \sum y_i\}$.

Given $e \in E$ and $x \in [0, \overline{x}]^n$, let $\zeta_{e,x} : Y_e(x) \; ^3 \; \mathbb{R}^m_+$ be such that, for all $y \in Y_e(x)$, $\zeta_{e,x}(y) \equiv CC(e, x, y)$. Since, by (1)-($\delta$), $C(a_i, \cdot, \cdot)$ is upper hemi-continuous (*u.h.c.*) for each $i \in N$, $\zeta_{e,x}$ is *u.h.c.* We show that $\zeta_{e,x}$ is also lower hemi-continuous (*l.h.c.*). Let $\eta^i_{e,x} : Y_e(x) \; ^3 \; \mathbb{R}^m_+$ be such that for all $y \in Y_e(x)$, $\eta^i_{e,x}(y) \equiv int\ C(a_i, \overline{x} - x_i, y_i)$. Since $\eta^i_{e,x}$ is *l.h.c.* by (1)-(δ), and $\eta^j_{e,x}$ can be shown to have open graph, $\eta^i_{e,x} \cap \eta^j_{e,x}$ is *l.h.c.* Similarly, we can show that $\bigcap_{i \in N} \eta^i_{e,x}$ is *l.h.c.* Since the closure of $\bigcap_{i \in N} \eta^i_{e,x}(y)$ is $\zeta_{e,x}(y)$ for all $y \in Y_e(x)$, we can see that $\zeta_{e,x}$ is *l.h.c.* Thus, $\zeta_{e,x}$ is continuous. Since, by (1)-(γ), $\mathcal{CC}(e, x)$ is a family of compact sets, $\zeta_{e,x}$ is a continuous function from $Y_e(x)$ to $\mathcal{CC}(e, x)$. Since $Y_e(x)$ is compact, $\zeta_{e,x}(Y_e(x)) = \mathcal{CC}(e, x)$ is compact for each $x \in [0, \overline{x}]^n$.

Given $CC(e, x, y) \in \mathcal{CC}(e, x)$, let

$$\mathcal{L}^{int}_{e,x}(CC(e, x, y), \supsetneq) \equiv \{CC(e, x, y') \in \mathcal{CC}(e, x) \mid int\ CC(e, x, y) \supsetneq CC(e, x, y')\}.$$

For any $CC(e, x, y') \in \mathcal{L}^{int}_{e,x}(CC(e, x, y), \supsetneq)$, there exists an $\epsilon > 0$ such that $\epsilon = \min\{\delta(b, CC(e, x, y')) \mid b \in \partial CC(e, x, y)\}$, where $\partial CC(e, x, y)$ is the boundary of the set $CC(e, x, y)$. Define

$$\mathcal{O}(CC(e, x, y'), \frac{1}{2}\epsilon) \equiv \{CC(e, x, y'') \in \mathcal{CC}(e, x) \mid d(CC(e, x, y'), CC(e, x, y'')) < \frac{1}{2}\epsilon\}.$$

By definition of the Hausdorff metric d, $int\ CC(e, x, y) \supsetneq CC(e, x, y'')$ for all $CC(e, x, y'') \in \mathcal{O}(CC(e, x, y'), \frac{1}{2}\epsilon)$. Thus, $\mathcal{L}^{int}_{e,x}(CC(e, x, y), \supsetneq)$ is open in the Hausdorff topology on $\mathcal{CC}(e, x)$, so $\supsetneq$ is upper semi-continuous. Then, since the relation $\supsetneq$ is transitive, hence acyclic, there exists a $CC(e, x, y^*) \in \mathcal{CC}(e, x)$ such that $int\ CC(e, x, y') \not\supsetneq CC(e, x, y^*)$ for all $CC(e, x, y') \in \mathcal{CC}(e, x)$. Denote the set of such $CC(e, x, y^*)$ by $\mathcal{UIC}(e, x)$ for each $e \in E$ and $x \in [0, \overline{x}]^n$. Note that the non-empty set $\mathcal{UIC}(e, x)$ is closed in $\mathcal{CC}(e, x)$, and $\mathcal{UIC}(e, x) \supseteq \mathcal{UC}(e, x)$. Since $\mathcal{CC}(e, x)$ is compact, $\mathcal{UIC}(e, x)$ is compact.

For $CC(e, x, y) \in \mathcal{UIC}(e, x)$, if $y \in Y_e(x) \backslash \partial Y_e(x)$, then there exists $y^* \in \partial Y_e(x)$ such that $CC(e, x, y^*) \supsetneq CC(e, x, y)$. Moreover, it is shown that $CC(e, x, y^*) \in \mathcal{UC}(e, x)$. Since $CC(e, x, y^*) \in \mathcal{UIC}(e, x)$, there exists

$\mathsf{b}_i \in \partial C(a_i, \overline{x} - x_i, y_i^*)$ for all $i \in N$ such that $\mathsf{b}_i \in \partial CC(\mathsf{e}, \mathsf{x}, \mathsf{y}^*)$. Since $\mathsf{y}^* \in \partial Y_{\mathsf{e}}(\mathsf{x})$, for any $\mathsf{y}^{**}(\neq \mathsf{y}^*) \in Y_{\mathsf{e}}(\mathsf{x})$, there exists at least one individual $j \in N$ such that $int\, C(a_j, \overline{x} - x_i, y_i^*) \supsetneq C(a_j, \overline{x} - x_i, y_i^{**})$ by the property (1)-(β) of the capability correspondence. It follows that there is no $CC(\mathsf{e}, \mathsf{x}, \mathsf{y}^{**}) \in \mathcal{CC}(\mathsf{e}, \mathsf{x})$ such that $CC(\mathsf{e}, \mathsf{x}, \mathsf{y}^{**}) \supsetneq CC(\mathsf{e}, \mathsf{x}, \mathsf{y}^*)$. Thus, $\mathcal{UC}(\mathsf{e}, \mathsf{x}) \neq \varnothing$ and $\mathcal{UIC}(\mathsf{e}, \mathsf{x}) \cap \zeta_{\mathsf{e},\mathsf{x}}(\partial Y_{\mathsf{e}}(\mathsf{x})) = \mathcal{UC}(\mathsf{e}, \mathsf{x})$. Since $\zeta_{\mathsf{e},\mathsf{x}}(\partial Y_{\mathsf{e}}(\mathsf{x}))$ is compact, $\mathcal{UC}(\mathsf{e}, \mathsf{x})$ is compact.

Step (2): *Existence of ψ having the property stated in Lemma 1.*

Given $\mathsf{J} \in \mathcal{J}^n$, let $\bigcap_{i \in N^{SIA}} J_i$. Note that $\bigcap_{i \in N^{SIA}} J_i$ is a quasi-ordering, and $\bigcap_{i \in N^{SIA}} J_i(\mathsf{e}, \mathsf{x})$ is upper semi-continuous on $\mathcal{UC}(\mathsf{e}, \mathsf{x})$ for each $\mathsf{e} \in E$ and each $\mathsf{x} \in [0, \overline{x}]^n$. Let $J_{SIA}^{\mathcal{UC}}(\mathsf{e}, \mathsf{x}) \equiv \bigcap_{i \in N^{SIA}} J_i(\mathsf{e}, \mathsf{x}) \cap (\mathcal{UC}(\mathsf{e}, \mathsf{x}) \times \mathcal{CC}(\mathsf{e}, \mathsf{x}))$ and $J_{SIA}^{\mathcal{UC}} \equiv \bigcup_{(\mathsf{e},\mathsf{x}) \in E \times [0,\overline{x}]^n} J_{SIA}^{\mathcal{UC}}(\mathsf{e}, \mathsf{x})$. Since $J_{SIA}^{\mathcal{UC}}$ is quasi-ordering, by Jaffray's (1975) theorem, there exists an ordering extension J^{SIA} of $J_{SIA}^{\mathcal{UC}}$ such that for each $\mathsf{e} \in E$ and each $\mathsf{x} \in [0, \overline{x}]^n$, $J^{SIA}(\mathsf{e}, \mathsf{x}) \cap (\mathcal{UC}(\mathsf{e}, \mathsf{x}))^2$ is upper semi-continuous on $\mathcal{UC}(\mathsf{e}, \mathsf{x})$.

Next, given $\mathsf{J} \in \mathcal{J}^n$, let $J_N \equiv \bigcap_{i \in N} J_i$. Then, since we can easily show that the relation $J^{SIA} \cup J_N$ is *consistent* in the sense of Suzumura (1983, chapter 1),[20] there exists an ordering extension J of $J^{SIA} \cup J_N$ over $\mathcal{CC}$. Let $\psi(\mathsf{J}) \equiv J$. Since $\psi(\mathsf{J})$ contains J^{SIA}, and every individuals in N^{SIA} always proposes the value judgement containing the set-inclusion subrelation, $B(\mathcal{CC}(\mathsf{e}, \mathsf{x}), \psi(\mathsf{J})) \subseteq \mathcal{UC}(\mathsf{e}, \mathsf{x})$. Since $\mathcal{UC}(\mathsf{e}, \mathsf{x})$ is non-empty and compact, $B(\mathcal{CC}(\mathsf{e}, \mathsf{x}), \psi(\mathsf{J})) = B(\mathcal{UC}(\mathsf{e}, \mathsf{x}), \psi(\mathsf{J})) \neq \varnothing$. Finally, by the construction of $\psi(\mathsf{J})$, it is clear that ψ satisfies Pareto Principle. ■

Given $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ and $\mathsf{x} = (x_i)_{i \in N} \in [0, \overline{x}]^n$, let $Y(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi) \equiv \{\mathsf{y} = (y_i)_{i \in N} \in Y(\mathsf{e}, \mathsf{x}) \mid \text{for all } i \in N,\ C(a_i, \overline{x} - x_i, y_i) \supseteq C_{\varphi \cdot \psi(\mathsf{J})}^{\min}(\mathsf{e}, \mathsf{x})\}$.

Lemma 2: *Suppose that the* J-based capability maximin rule $h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}$ *is generated through the* Paretian social welfare function ψ. *Then, for all* $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ *and all* $\mathsf{x} = (x_i)_{i \in N} \in [0, \overline{x}]^n$, *the* J-based minimal capability $C_{\varphi \cdot \psi(\mathsf{J})}^{\min}(\mathsf{e}, \mathsf{x})$, *which corresponds to* $h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}$, *has the following property*: *for*

[20] A binary relation R on a set X is *consistent* if there exists no finite subset $\{x^1, \cdots, x^t\}$ of X, where $2 \leq t < +\infty$, such that $(x^1, x^2) \in P(R)$, $(x^2, x^3) \in R$, $\cdots$, $(x^t, x^1) \in R$ hold. A binary relation R^* on X is called an *extension* of R if and only if $R \subseteq R^*$ and $P(R) \subseteq P(R^*)$ hold. It is shown in Suzumura (1983, Theorem A(5)) that *there exists an ordering extension of R if and only if R is consistent.*

all $i \in N$*, there exists* $b_i \in \partial C(a_i, \overline{x} - x_i, y_i)$ *such that* $b_i \in \partial C^{\min}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x})$ *where* $\mathsf{y} = (y_i)_{i \in N} \in Y(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi)$.

Proof: Let $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ and $\mathsf{x} \in [0, \overline{x}]^n$. Suppose that for some $\mathsf{y} = (y_i)_{i \in N} \in Y(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi)$, there exists $j \in N$ such that for all $b_j \in \partial C(a_j, \overline{x} - x_j, y_j)$, $b_j \notin C^{\min}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x})$. Let us denote the set of such individuals by $N(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi, \mathsf{y})$. Then, we can consider another distribution as follows:

$$\mathsf{y}' \equiv \begin{cases} y'_j = y_j - \varepsilon_j > 0 \ (\exists \varepsilon_j > 0) \text{ for } j \in N(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi, \mathsf{y}) \\ y'_i = y_i + \frac{\sum_{j \in N(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi, \mathsf{y})} \varepsilon_j}{\#[N \backslash N(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi, \mathsf{y})]} \text{ for } i \in N \backslash N(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi, \mathsf{y}) \end{cases}$$

where $\bigcap_{i \in N \backslash N(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi, \mathsf{y})} C(a_i, \overline{x} - x_i, y'_i) \subseteq C(a_j, \overline{x} - x_j, y'_j)$ for any $j \in N(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi, \mathsf{y})$. The existence of such a distribution is guaranteed by (1)-(δ). By (1)-(β), $C(a_i, \overline{x} - x_i, y'_i) \supsetneq C(a_i, \overline{x} - x_i, y_i)$. Then, since $\bigcap_{i \in N \backslash N(\mathsf{e}, \mathsf{x}, \psi(\mathsf{J}), \varphi, \mathsf{y})}$ $C(a_i, \overline{x} - x_i, y'_i) = \bigcap_{i \in N} C(a_i, \overline{x} - x_i, y'_i)$,
$\bigcap_{i \in N} C(a_i, \overline{x} - x_i, y'_i) \supsetneq C^{\min}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x})$. Since $\mathsf{y}' \in Y(\mathsf{e}, \mathsf{x})$,
$\bigcap_{i \in N} C(a_i, \overline{x} - x_i, y'_i) = CC(\mathsf{e}, \mathsf{y}', \mathsf{x}) \in \mathcal{CC}(\mathsf{e}, \mathsf{x})$, that is a contradiction. ■

Proof of Theorem 1: Given $\mathsf{e} \in E$ and $\mathsf{x} \in [0, \overline{x}]^n$, let $i, j \in N$ be such that $a_i = a_j$ and $s_i = s_j$. Then, by the definition of $\mathcal{J}_{SIA}$, we obtain that $J^{\mathcal{UC}}_{SIA}(\mathsf{e}, \mathsf{x}) = J^{\mathcal{UC}}_{SIA}(\mathsf{e}, \rho_{ij}(\mathsf{x}))$, where the quasi-ordering $J^{\mathcal{UC}}_{SIA}(\mathsf{e}, \mathsf{x})$ is defined in the proof of Lemma 1. Thus, by way of the proof of Lemma 1, we can guarantee that there exists an upper semi-continuous ordering extension J^{SIA^*} of $J^{\mathcal{UC}}_{SIA}$ such that for all $\mathsf{e} \in E$, all $\mathsf{x} \in [0, \overline{x}]^n$, and all $i, j \in N$ such that $a_i = a_j$ and $s_i = s_j$, $J^{SIA^*}(\mathsf{e}, \mathsf{x}) = J^{SIA^*}(\mathsf{e}, \rho_{ij}(\mathsf{x}))$. Moreover, by the same way as in the proof of Lemma 1, there exists an ordering extension J^* of $J^{SIA^*} \cup J_N$ over $\mathcal{CC}$. Let $\psi(\mathsf{J}) \equiv J^*$. Then, by Lemma 1, ψ satisfies Pareto Principle, and $B(\mathcal{CC}(\mathsf{e}, \mathsf{x}), \psi(\mathsf{J})) \neq \varnothing$ and $B(\mathcal{CC}(\mathsf{e}, \mathsf{x}), \psi(\mathsf{J})) \subseteq \mathcal{UC}(\mathsf{e}, \mathsf{x})$ for each $\mathsf{e} \in E$ and $\mathsf{x} \in [0, \overline{x}]^n$.

Given $\mathsf{e} \in E$ and $\mathsf{x} \in [0, \overline{x}]^n$, let $i, j \in N$ be such that $a_i = a_j$ and $s_i = s_j$. We next show that $CC(\mathsf{e}, \mathsf{x}, \mathsf{y}) \in B(\mathcal{CC}(\mathsf{e}, \mathsf{x}), \psi(\mathsf{J}))$ if and only if $CC(\mathsf{e}, \rho_{ij}(\mathsf{x}, \mathsf{y})) \in B(\mathcal{CC}(\mathsf{e}, \rho_{ij}(\mathsf{x})), \psi(\mathsf{J}))$. First, note that $\mathcal{CC}(\mathsf{e}, \mathsf{x}) = \mathcal{CC}(\mathsf{e}, \rho_{ij}(\mathsf{x}))$. Second, by construction of $\psi(\mathsf{J})$, we obtain that

$$\begin{aligned} B(\mathcal{CC}(\mathsf{e}, \mathsf{x}), \psi(\mathsf{J})) &= B(\mathcal{UC}(\mathsf{e}, \mathsf{x}), J^{SIA^*}(\mathsf{e}, \mathsf{x})) \text{ and} \\ B(\mathcal{CC}(\mathsf{e}, \rho_{ij}(\mathsf{x})), \psi(\mathsf{J})) &= B(\mathcal{UC}(\mathsf{e}, \rho_{ij}(\mathsf{x})), J^{SIA^*}(\mathsf{e}, \rho_{ij}(\mathsf{x}))). \end{aligned}$$

Since $J^{SIA^*}(\mathsf{e},\mathsf{x}) = J^{SIA^*}(\mathsf{e},\rho_{ij}(\mathsf{x}))$, we obtain that $B(\mathcal{UC}(\mathsf{e},\mathsf{x}),J^{SIA^*}(\mathsf{e},\mathsf{x})) = B(\mathcal{UC}(\mathsf{e},\rho_{ij}(\mathsf{x})),J^{SIA^*}(\mathsf{e},\rho_{ij}(\mathsf{x})))$. Thus, $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \in B(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J}))$ if and only if $CC(\mathsf{e},\rho_{ij}(\mathsf{x},\mathsf{y})) \in B(\mathcal{CC}(\mathsf{e},\rho_{ij}(\mathsf{x})),\psi(\mathsf{J}))$.

Thus, let us define a rational choice function φ as follows: for all $\mathsf{e} \in E$, all $\mathsf{x} \in [0,\overline{x}]^n$, and all $i,j \in N$ such that $a_i = a_j$ and $s_i = s_j$, $\varphi(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J})) \in B(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J}))$ and $\varphi(\mathcal{CC}(\mathsf{e},\rho_{ij}(\mathsf{x})),\psi(\mathsf{J})) = \varphi(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J}))$. The former property of φ implies that for each $\mathsf{e} \in E$ and $\mathsf{x} \in [0,\overline{x}]^n$, $Y(\mathsf{e},\mathsf{x},\psi(\mathsf{J}),\varphi)$ is non-empty. Let us define a distribution rule $h : E \times [0,\overline{x}]^n \to \mathbb{R}^n_+$ such that for all $\mathsf{e} \in E$ and all $\mathsf{x} \in [0,\overline{x}]^n$, $h(\mathsf{e},\mathsf{x}) \in Y(\mathsf{e},\mathsf{x},\psi(\mathsf{J}),\varphi)$. Then by the definition of $\varphi(\mathcal{CC}(\mathsf{e},\mathsf{x}),\psi(\mathsf{J}))$, $CC(\mathsf{e},\mathsf{x},h(\mathsf{e},\mathsf{x})) \in \mathcal{UC}(\mathsf{e},\mathsf{x})$ for all $\mathsf{e} \in E$ and all $\mathsf{x} \in [0,\overline{x}]^n$. Moreover, the second property of φ implies that the distribution rule h satisfies the **EAEH**. Since $CC(\mathsf{e},\mathsf{x},h(\mathsf{e},\mathsf{x})) = C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x})$, this implies that h is a J-based capability maximin rule: $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}} \equiv h$.

Next, we show that given $\psi(\mathsf{J})$ and φ, the J-based capability maximin rule $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}$ is uniquely determined: there is no other distribution rule h' such that for all $\mathsf{e} \in E$ and all $\mathsf{x} \in [0,\overline{x}]^n$, $h'(\mathsf{e},\mathsf{x}) \in Y(\mathsf{e},\mathsf{x},\psi(\mathsf{J}),\varphi)$ and $CC(\mathsf{e},\mathsf{x},h'(\mathsf{e},\mathsf{x})) \in \mathcal{UC}(\mathsf{e},\mathsf{x})$, and for some $\mathsf{e} \in E$ and some $\mathsf{x} \in [0,\overline{x}]^n$, $h'(\mathsf{e},\mathsf{x}) \neq h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e},\mathsf{x})$. It is sufficient to show that for any $\mathsf{e} \in E$, any $\mathsf{x} \in [0,\overline{x}]^n$, any $\mathsf{J} \in \mathcal{J}^n$ and ψ, and any φ, $Y(\mathsf{e},\mathsf{x},\psi(\mathsf{J}),\varphi)$ is singleton. Given $\mathsf{e} \in E$, $\mathsf{x} \in [0,\overline{x}]^n$, $\psi(\mathsf{J})$, and φ, let $C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x})$ be the J-based minimal capability under (e,x), and $\mathsf{y} \in Y(\mathsf{e},\mathsf{x},\psi(\mathsf{J}),\varphi)$. Suppose that there exists another distribution $\mathsf{y}' \in Y(\mathsf{e},\mathsf{x},\psi(\mathsf{J}),\varphi)$ such that $\mathsf{y}' \neq \mathsf{y}$. By (1)-($\beta$) of C and lemma 2, y and y' in $Y(\mathsf{e},\mathsf{x},\psi(\mathsf{J}),\varphi)$ have the following property: $\sum y_i = \sum y_i' = f(\sum s_i x_i)$. This implies that there exist at least two individuals $i,j \in N$ such that $y_i' < y_i$ and $y_j' > y_j$. Then, by (1)-(β) of the correspondence C, $\partial C(a_i,\overline{x}-x_i,y_i') \subsetneq int\ C(a_i,\overline{x}-x_i,y_i)$. By lemma 2, there exists $b_i \in \partial C(a_i,\overline{x}-x_i,y_i)$ such that $b_i \in \partial C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x})$. Hence, $b_i \notin \partial C(a_i,\overline{x}-x_i,y_i')$, so that $C(a_i,\overline{x}-x_i,y_i') \not\supseteq C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x})$. This is a contradiction. ■

Proof of Theorem 2: First, we show that the ordering $R^{\mathcal{FC}}_{\psi(\mathsf{J})} \subseteq \mathcal{FC} \times \mathcal{FC}$ satisfies $PICA$, ACA, and $\mu^* ECA$. By definition of $R^{\mathcal{FC}}_{\psi(\mathsf{J})}$, it satisfies ACA. For any two capability assignments $\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}), \mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}') \in \mathcal{FC}(\mathsf{e},\mathsf{x})$, if $C(a_i,\overline{x}-x_i,y_i) \supseteq C(a_i,\overline{x}-x_i,y_i')$ for all $i \in N$ and $C(a_j,\overline{x}-x_j,y_j) \supsetneq C(a_j,\overline{x}-x_j,y_j')$ for some $j \in N$, then this implies that $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \supsetneq CC(\mathsf{e},\mathsf{x},\mathsf{y}')$. Since $\psi(\mathsf{J})$ contains the set-inclusion subrelation by Theorem 1, $(\mathsf{C}(\mathsf{e},\mathsf{z}),\mathsf{C}(\mathsf{e},\mathsf{z}')) \in P(R^{\mathcal{FC}}_{\psi(\mathsf{J})}(\mathsf{e},\mathsf{x}))$. Thus, $R^{\mathcal{FC}}_{\psi(\mathsf{J})}$ satisfies $PICA$. Next, we show that $R^{\mathcal{FC}}_{\psi(\mathsf{J})}$ satisfies $\mu^* ECA$. Suppose that given $\mathsf{e} \in E$ and $\mathsf{x} \in [0,\overline{x}]^n$, $\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}^*) \in$

$B(\mathcal{FC}(\mathsf{e},\mathsf{x}), R^{\mathcal{FC}}_{\psi(\mathsf{J})})$. This supposition is not empty, since there exists an undominated common capability $CC(\mathsf{e},\mathsf{x},\mathsf{y}^*) \equiv C^{\min}_{\varphi\cdot\psi(\mathsf{J})}(\mathsf{e},\mathsf{x}) \in \mathcal{CC}(\mathsf{e},\mathsf{x})$, and by the definition of $R^{\mathcal{FC}}_{\psi(\mathsf{J})}$ and Theorem 1, $(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}^*), \mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y})) \in R^{\mathcal{FC}}_{\psi(\mathsf{J})}(\mathsf{e},\mathsf{x})$ for all $\mathsf{y} \in Y(\mathsf{e},\mathsf{x})$. Then, by definition of μ^*, $\max_{i\in N} \mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}^*), i) = 0$. Since $\mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}), i) \geq 0$ for all $i \in N$,

$$\max_{i\in N} \mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}^*), i) = \min_{\mathsf{y}\in Y(\mathsf{e},\mathsf{x})} \max_{i\in N} \mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}), i).$$

Next, suppose that the ordering $R^{\mathcal{FC}} \subseteq \mathcal{FC} \times \mathcal{FC}$ satisfies PICA, ACA, and μ^*ECA. Note that for all $\mathsf{e} \in E$, all $\mathsf{x} \in [0,\overline{x}]^n$, and all $\mathsf{y} \in Y(\mathsf{e},\mathsf{x})$, if $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \in \mathcal{UC}(\mathsf{e},\mathsf{x})$, then $\mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}), i) = 1$ for all $i \in N$, while if $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \in \mathcal{CC}(\mathsf{e},\mathsf{x})\backslash\mathcal{UC}(\mathsf{e},\mathsf{x})$, then for some $i \in N$, $\mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}), i) > 0$. This implies that $\max_{i\in N} \mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}^*), i) = \min_{\mathsf{y}\in Y(\mathsf{e},\mathsf{x})} \max_{i\in N} \mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}), i)$ for all $CC(\mathsf{e},\mathsf{x},\mathsf{y}^*) \in \mathcal{UC}(\mathsf{e},\mathsf{x})$. By the property of $R^{\mathcal{FC}}$ which satisifies the above three axioms, for all $\mathsf{e} \in E$, all $i, j \in N$ such that $a_i = a_j$ and $s_i = s_j$, and all $\mathsf{x} \in [0,\overline{x}]^n$, there exists $\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}^*) \in B(\mathcal{FC}(\mathsf{e},\mathsf{x}), R^{\mathcal{FC}})$ such that

$$\max_{i\in N} \mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}^*), i) = \min_{\mathsf{y}\in Y(\mathsf{e},\mathsf{x})} \max_{i\in N} \mu^*(\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}), i),$$

$CC(\mathsf{e},\mathsf{x},\mathsf{y}^*) \in \mathcal{UC}(\mathsf{e},\mathsf{x})$, and $\mathsf{C}(\mathsf{e}, \rho_{ij}(\mathsf{x},\mathsf{y}^*)) \in B(\mathcal{FC}(\mathsf{e}, \rho_{ij}(\mathsf{x})), R^{\mathcal{FC}})$.

Let us consider a profile of individual value judgements $\mathsf{J}^* \in \mathcal{J}^n$ such that for all $i \in N$, $(CC(\mathsf{e},\mathsf{x},\mathsf{y}), CC(\mathsf{e}',\mathsf{x}',\mathsf{y}')) \in J_i^* \Leftrightarrow (\mathsf{C}(\mathsf{e},\mathsf{x},\mathsf{y}), \mathsf{C}(\mathsf{e}',\mathsf{x}',\mathsf{y}')) \in R^{\mathcal{FC}}$. This implies for all $i \in N$, $J_i^* \in \mathcal{J}_{SIA}$. Thus, by construction of ψ, $\psi(\mathsf{J}^*) = J_i^*$ for all $i \in N$. Then, for all $\mathsf{e} \in E$, all $\mathsf{x} \in [0,\overline{x}]^n$, and all $i, j \in N$ such that $a_i = a_j$ and $s_i = s_j$, $CC(\mathsf{e},\mathsf{x},\mathsf{y}^*) = C^{\min}_{\varphi\cdot\psi(\mathsf{J}^*)}(\mathsf{e},\mathsf{x})$ and $CC(\mathsf{e}, \rho_{ij}(\mathsf{x},\mathsf{y}^*)) = C^{\min}_{\varphi\cdot\psi(\mathsf{J}^*)}(\mathsf{e}, \rho_{ij}(\mathsf{x})))$. Since $\psi(\mathsf{J}^*)$ rationalizes the J^*-capability maxmin rule $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}^*}}$, $R^{\mathcal{FC}} = R^{\mathcal{FC}}_{\psi(\mathsf{J}^*)}$. ■

Proof of Lemma 3: By lemma 2, it is easy to see that for any $\mathsf{J} \in \mathcal{J}^n$, $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}$ meets the undominated property, and it also satisfies PEC. We show that for any $h \in EAEH \cap UD \cap PEC$, there exists $\mathsf{J} \in \mathcal{J}^n$ such that $h = h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}$.

First, we show that for any $h \in EAEH \cap UD \cap PEC$, all $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$, and all $\mathsf{x} \in [0,\overline{x}]^n$, there is no $\mathsf{y} \in Y(\mathsf{e},\mathsf{x})$ such that $\mathsf{y} \neq h(\mathsf{e},\mathsf{x})$ and $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \supsetneq CC(\mathsf{e},\mathsf{x},h(\mathsf{e},\mathsf{x}))$. Suppose that for some $\mathsf{y} \in Y(\mathsf{e},\mathsf{x})$ such that $\mathsf{y} \neq h(\mathsf{e},\mathsf{x})$, $CC(\mathsf{e},\mathsf{x},\mathsf{y}) \supsetneq CC(\mathsf{e},\mathsf{x},h(\mathsf{e},\mathsf{x}))$. By UD, for all $i \in N$, there is $b_i \in \partial C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x}))$ such that $b_i \in \partial CC(\mathsf{e}, \mathsf{x}, h(\mathsf{e}, \mathsf{x}))$. Hence, $CC(\mathsf{e}, \mathsf{x}, \mathsf{y}) \supsetneq CC(\mathsf{e}, \mathsf{x}, h(\mathsf{e}, \mathsf{x}))$ implies that for some $j \in N$,

$$int\ C(a_j, \overline{x} - x_j, y_j) \supsetneq C(a_j, \overline{x} - x_j, h_j(\mathsf{e}, \mathsf{x}))$$

by (1)-(β) of the correspondence C. Thus, $y_j > h_j(\mathsf{e}, \mathsf{x})$ and for any other $i \neq j$, $y_i \geq h_i(\mathsf{e}, \mathsf{x})$. However, such a distribution is infeasible, because $h \in PEC$.

Second, let us show that if $h \in EAEH \cap UD \cap PEC$, then for some $\mathsf{J} \in \mathcal{J}^n$, for all $\mathsf{e} \in E$ and all $\mathsf{x} \in [0, \overline{x}]^n$, $CC(\mathsf{e}, \mathsf{x}, h(\mathsf{e}, \mathsf{x})) = C^{\mathsf{min}}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x})$. By the above argument, for each $\mathsf{e} \in E$ and $\mathsf{x} \in [0, \overline{x}]^n$, $CC(\mathsf{e}, \mathsf{x}, h(\mathsf{e}, \mathsf{x})) \in \mathcal{UC}(\mathsf{e}, \mathsf{x})$. Let $\mathsf{J}^* \in \mathcal{J}^n$ be such that for each $\mathsf{e} \in E$ and each $\mathsf{x} \in [0, \overline{x}]^n$, $(CC(\mathsf{e}, \mathsf{x}, h(\mathsf{e}, \mathsf{x})), CC(\mathsf{e}, \mathsf{x}, \mathsf{y})) \in P(J_i^*(\mathsf{e}, \mathsf{x}))$ for all $\mathsf{y} \in Y(\mathsf{e}, \mathsf{x}) \backslash \{h(\mathsf{e}, \mathsf{x})\}$ and all $i \in N$. Then, by Pareto Principle of ψ, $CC(\mathsf{e}, \mathsf{x}, h(\mathsf{e}, \mathsf{x})) = C^{\mathsf{min}}_{\varphi \cdot \psi(\mathsf{J}^*)}(\mathsf{e}, \mathsf{x})$. This implies that $h = h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}^*}}$ for $\mathsf{J}^* \in \mathcal{J}^n$. ■

Proof of Theorem 3: Given $\mathsf{J} \in \mathcal{J}^n$, let $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}} \in UCM$. Then, by Lemma 3, $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}} \in EAEH \cap UD \cap PEC$. It is easy to show that $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}} \in MEC$. So, we show first $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}} \in SRWF$. Suppose that there exist $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$, $\mathsf{x} = (x_i)_{i \in N} \in [0, \overline{x}]^n$, and $\mathsf{x}' = (x'_i)_{i \in N} \in [0, \overline{x}]^n$ such that for all $g \neq j$, $x_g = x'_g$ and $x_j \leq x'_j$, and for some $i \in N$,

$$\begin{aligned} & \#N(i, b_i^w(\mathsf{e}, (\mathsf{x}, h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e}, \mathsf{x}))), \mathsf{e}, (\mathsf{x}, h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e}, \mathsf{x}))) \\ > \ & \#N(i, b_i^w(\mathsf{e}, (\mathsf{x}', h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e}, \mathsf{x}'))), \mathsf{e}, (\mathsf{x}', h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e}, \mathsf{x}'))). \end{aligned}$$

This implies that for all $b_i \in \partial C(a_i, \overline{x} - x_i, h_i^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e}, \mathsf{x}'))$, there is $g \in N$ and $\lambda_g < 1$ such that $\lambda_g b_i \in \partial C(a_g, \overline{x} - x_g, h_g^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e}, \mathsf{x}'))$, since $\#N(i, b_i^w(\mathsf{e}, (\mathsf{x}', h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e}, \mathsf{x}'))), \mathsf{e}, (\mathsf{x}', h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(\mathsf{e}, \mathsf{x}'))) < n$. This also implies that $int\ C(a_i, \overline{x} - x_i, h_i^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}}(e, \mathsf{x}')) \supsetneq C^{\mathsf{min}}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x}')$. It is a contradiction by Lemma 2. In the same way, we can show that $h^{\mathsf{CM}^{\psi,\varphi}_{\mathsf{J}}} \in NRABA$.

Next, we show that for any $h \in EAEH \cap PEC$, if $h \in MEC \cap SRWF \cap NRABA$, then $h \in UD$. Let $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$ and $\mathsf{x} = (x_i)_{i \in N} \in [0, \overline{x}]^n$ be such that for all $i, j \in N$, $a_i = a_j$ and $x_i = x_j$. Then, $h \in MEC \cap PEC$ implies that

$$h_i(\mathsf{e}, \mathsf{x}) = \frac{f(\sum s_i x_i)}{n} \text{ for all } i \in N.$$

Since for all $i, j \in N$,

$$C(a_i, \overline{x} - x_i, h_i(\mathsf{e}, \mathsf{x})) = C(a_j, \overline{x} - x_j, h_j(\mathsf{e}, \mathsf{x})),$$

we obtain

$$\#N(i, b_i^w(\mathsf{e}, (\mathsf{x}, h(\mathsf{e}, \mathsf{x}))), \mathsf{e}, (\mathsf{x}, h(\mathsf{e}, \mathsf{x}))) = n \text{ for all } i \in N.$$

Let $\mathsf{e}' = (\mathsf{a}', \mathsf{s}, C, f) \in E$ and $\mathsf{x}' = (x_i')_{i \in N} \in [0, \overline{x}]^n$ be given. Without loss of generality, we assume that $a_1' \leq \cdots \leq a_n'$ and $x_1' \leq \cdots \leq x_n'$. Let $\mathsf{e}'' = (\mathsf{a}'', \mathsf{s}, C, f) \in E$ such that $\mathsf{a}'' = (a_n', \cdots, a_n')$ and $\mathsf{x}'' = (x_1', \cdots, x_1') \in [0, \overline{x}]^n$. Then, by $h \in MEC$,

$$\#N(i, b_i^w(\mathsf{e}'', (\mathsf{x}'', h(\mathsf{e}'', \mathsf{x}''))), \mathsf{e}'', (\mathsf{x}'', h(\mathsf{e}'', \mathsf{x}''))) = n \text{ for all } i \in N.$$

By repeatedly applying the $SRWF$ and $NRABA$, we obtain

$$\#N(i, b_i^w(\mathsf{e}', (\mathsf{x}', h(\mathsf{e}', \mathsf{x}'))), \mathsf{e}', (\mathsf{x}', h(\mathsf{e}', \mathsf{x}'))) = n \text{ for all } i \in N.$$

This implies that $h \in UD$. By Lemma 3, for some $\mathsf{J} \in \mathcal{J}^n$, $h = h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}} \in UCM$. ■

Proof of Theorem 4: Since NP implies MEC, from Theorem 3, it is sufficient to show that UCM implies NP. Let $\mathsf{e} = (\mathsf{a}, \mathsf{s}, C, f) \in E$, and $\mathsf{x} \in [0, \overline{x}]^n$ be such that for all $i, j \in N$, $x_i = x_j$. Then, $C^H(\mathsf{e}, \mathsf{x}) \in \mathcal{CC}(\mathsf{e}, \mathsf{x})$. If a is such that for all $i, j \in N$, $a_i = a_j$, then $C^H(\mathsf{e}, \mathsf{x}) = C^{\min}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x})$ for any $\mathsf{J} \in \mathcal{J}^n$. Otherwise, there exists an individual $i^* \in N$ such that for any other $j \neq i^*$, $a_{i^*} < a_j$, and that $C^H(\mathsf{e}, \mathsf{x}) = C(a_{i^*}, \overline{x} - x_{i^*}, y^H(\mathsf{e}, \mathsf{x}))$. Then, consider other distribution y^* such that for some $\varepsilon > 0$, for some $j \neq i^*$, $y_j^* = y^H(\mathsf{e}, \mathsf{x}) - \varepsilon$, for i^*, $y_{i^*}^* = y^H(\mathsf{e}, \mathsf{x}) + \varepsilon$, and for any other $g \neq i^*, j$, $y_g^* = y^H(\mathsf{e}, \mathsf{x})$. Then, by continuity and strict monotonicity of C, $CC(\mathsf{e}, \mathsf{x}, \mathsf{y}^*) = C(a_{i^*}, \overline{x} - x_{i^*}, y_{i^*}^*) \supsetneq C^H(\mathsf{e}, \mathsf{x})$. This implies that for any $\mathsf{J} \in \mathcal{J}^n$, $C^{\min}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x}) \supsetneq C^H(\mathsf{e}, \mathsf{x})$. Since $C(a_i, \overline{x} - x_i, h_i^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}(\mathsf{e}, \mathsf{x})) \supseteq C^{\min}_{\varphi \cdot \psi(\mathsf{J})}(\mathsf{e}, \mathsf{x})$ for all $i \in N$, there is no $i \in N$ such that $C^H(\mathsf{e}, \mathsf{x}) \supsetneq C(a_i, \overline{x} - x_i, h_i^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}(\mathsf{e}, \mathsf{x}))$. ■

Proof of Theorem 5: First, we show that UCM implies $NDEH$ and $NSDEE$. Given $\mathsf{J} \in \mathcal{J}^n$, let $h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}} \in UCM$. Suppose that $h^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}$ violates $NDEH$. Then, for some $(\mathsf{e}, \mathsf{x}) \in E \times [0, \overline{x}]^n$ and some $i, j \in N$ such that $a_i = a_j$, we obtain either (I) $x_i = x_j$ and $h_i^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}(\mathsf{e}, \mathsf{x}) < h_j^{\mathsf{CM}_{\mathsf{J}}^{\psi,\varphi}}(\mathsf{e}, \mathsf{x})$

or (II) $x_i < x_j$ and $h_i^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x}) \geq h_j^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})$. In case (I), $C(a_i, \overline{x} - x_i, h_i^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})) \subsetneq int\ C(a_j, \overline{x} - x_j, h_j^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x}))$, and in case (II), $int\ C(a_i, \overline{x} - x_i, h_i^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})) \supsetneq C(a_j, \overline{x} - x_j, h_j^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x}))$. The both cases imply that $h^{\mathsf{CMJ}^{\psi,\varphi}} \notin UD$, which is a contradiction by Lemma 3. Suppose that $h^{\mathsf{CMJ}^{\psi,\varphi}}$ violates $NSDEE$. Then, for some $(\mathsf{e},\mathsf{x}) \in E \times [0,\overline{x}]^n$ and some $i, j \in N$ such that $x_i = x_j$, $int\ C(a_i, \overline{x} - x_i, h_i^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})) \supsetneq C(a_j, \overline{x} - x_j, h_j^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x}))$. This implies that $int\ C(a_i, \overline{x} - x_i, h_i^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})) \supsetneq \bigcap_{j \neq i} C(a_j, \overline{x} - x_j, h_j^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x}))$. Hence, $h^{\mathsf{CMJ}^{\psi,\varphi}} \notin UD$, which is a contradiction by Lemma 3.

Next, based upon $h^{\mathsf{CMJ}^{\psi,\varphi}} \in UCM$, we construct another distribution rule $h \in EAEH \cap PEC$ satisfying the undominated property as follows: for each $(\mathsf{e},\mathsf{x}) \in E \times [0,\overline{x}]^n$,

(III) if there is $i^* \in N$ such that $C(a_{i^*}, \overline{x} - x_{i^*}, h_{i^*}^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})) = C_{\varphi \cdot \psi(\mathsf{J})}^{\min}(\mathsf{e},\mathsf{x})$ and there is a group $N(i^*) \subseteq N \setminus \{i^*\}$ such that for all $j^* \in N(i^*)$,

$$C(a_{i^*}, \overline{x} - x_{i^*}, h_{i^*}^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})) \subsetneq C(a_{j^*}, \overline{x} - x_{j^*}, h_{j^*}^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x})),$$

then

$$\begin{aligned} h_{i^*}(\mathsf{e},\mathsf{x}) &= h_{i^*}^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x}) + \varepsilon \text{ for some } \varepsilon > 0, \\ h_{j^*}(\mathsf{e},\mathsf{x}) &= h_{j^*}^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x}) - \alpha_{j^*}\varepsilon \text{ for all } j^* \in N(i^*) \end{aligned}$$

$$\begin{aligned} \text{where } \alpha_{j^*} &\geq 0 \text{ and } \sum_{N(i^*)} \alpha_{j^*} = 1, \\ \text{and } h_g(\mathsf{e},\mathsf{x}) &= h_g^{\mathsf{CMJ}^{\psi,\varphi}}(\mathsf{e},\mathsf{x}) \text{ for all } g \in N \setminus (N(i^*) \cup \{i^*\}) \end{aligned}$$

such that there is no $i \in N$,

$$C(a_i, \overline{x} - x_i, h_i(\mathsf{e},\mathsf{x})) \supsetneq \bigcap_{j \neq i} C(a_j, \overline{x} - x_j, h_j(\mathsf{e},\mathsf{x})),$$

and moreover, if there exists a pair of individuals $\{i, j\} \subseteq N$ such that $a_i = a_j$ and $s_i = s_j$, then for $\mathsf{x}' = \rho_{ij}(\mathsf{x}) \in [0,\overline{x}]^n$,

$$\begin{aligned} h_i(\mathsf{e},\mathsf{x}') &= h_j(\mathsf{e},\mathsf{x}),\ h_j(\mathsf{e},\mathsf{x}') = h_i(\mathsf{e},\mathsf{x}), \\ \text{and } h_g(\mathsf{e},\mathsf{x}') &= h_g(\mathsf{e},\mathsf{x}) \text{ for all } g \in N \setminus \{i, j\}, \end{aligned}$$

(IV) otherwise, $h(\mathsf{e}, \mathsf{x}) = h^{\mathsf{CM}_{\mathsf{J}^*}^{\psi;\varphi}}(\mathsf{e}, \mathsf{x})$.

The case (III) is possible by the continuity of the capability correspondence C. Then, by definition, $h \in EAEH \cap PEC \cap UD$. By Lemma 3, for some $\mathsf{J}^* \in \mathcal{J}^n$, $h = h^{\mathsf{CM}_{\mathsf{J}^*}^{\psi;\varphi}}$. Thus, by the above arguments, $h \in NDEH \cap NSDEE$. Moreover, the construction of h implies that $h \in NDEE$. ■

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