Triple Implementation by Sharing Mechanisms in Production Economies with Unequal Labor Skills*

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Abstract

In production economies with unequal labor skills, we study axiomatic characterizations of Pareto subsolutions which are implementable by *sharing mechanisms* in Nash, strong Nash, and subgame perfect equilibria. The sharing mechanism allows agents to work freely and distributes the produced output to the agents, according to the profile of labor hours and the information on demands, prices, and labor skills. Based on the characterizations, we find that most fair allocation rules, which embody the ethical principles of *responsibility and compensation,* cannot be implemented when individuals' labor skills are private information.

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1 Introduction

We consider the implementation of allocation rules in production economies with possibly unequal labor skills among individuals. Varian (1994), Hurwicz et al. (1995), Hong (1995), Suh (1995), Tian (1999, 2000), Yoshihara (1999), and Kaplan and Wettstein (2000) have proposed simple or natural mechanisms (game forms) to implement particular rules such as the Walrasian solution and the proportional solution [Roemer and Silvestre (1993)]. By contrast, a few works such as Shin and Suh (1997) and Yoshihara (2000) have discussed characterizations of allocation rules implementable by such simple or natural mechanisms. However, in these works, two implicit assumptions are made about the basic information structure among individuals and the social planner (or mechanism coordinator).

The first one is that the coordinator knows individuals' skills, or alternatively that all individuals have the same skill. Thus, the problem of asymmetric information is reduced to the possibility of an individual misrepresenting his preference ordering,[1] and at most, the possibility of an individual understating his endowment of material goods.[2] However, if individual skills differ, it is more natural to consider an informational structure in which the planner does not know each individual's true skill, and an individual has an incentive to *overstate,* or to understate, his own skill. The possibility of overstating one's skill is an essential feature of production economies with asymmetric information, because the coordinator cannot require individuals to "place the claimed endowments on the table" [Hurwicz et al. (1995)] in advance of production. Our goal is to characterize the class of Pareto efficient allocation rules that are implementable by a *natural mechanism even when individuals' skills are unknown to the planner.*

What kind of mechanism is natural in this context? This issue is relevant to our discussion of the second implicit assumption in the existing literature on implementation in production economies. Although Shin and Suh (1997) and Yoshihara (2000) define conditions for characterizing "natural mechanisms" in production economies, the list of these conditions[3] is not

[1]Varian (1994), Suh (1995), Shin and Suh (1997), Yoshihara (1999, 2000), and Kaplan and Wettstein (2000) discussed this type of problem.

[2]Hurwicz et al. (1995), Hong (1995), and Tian (1999, 2000) discussed this type of problem.

[3]Those are *feasibility*, *forthrightness*, *best response property*, and *simple strategy spaces*, which were originally proposed by Dutta, Sen, and Vohra (1995) and Saijo, Tatamitani,

yet satisfactory, because they omit another important feature of production economies with asymmetric information. Usually, a mechanism consists of a pair of a list of strategy spaces and an outcome function, a function that assigns an allocation to each profile of individuals' strategies. This function implies that in production economies, the planner is authorized to force individuals to supply the labor assigned by the outcome function.[4] However, the planner may not have such authority.

To address this issue, we require of mechanisms that they satisfy *labor sovereignty* [Kranich (1994)]. This is the requirement that every individual should have the right to choose his own labor time. We call a *sharing mechanism* a game form in which each individual can freely supply his labor time, and is asked to state his skill and his demand for consumption goods. The outcome function simply distributes the output produced, according to the information they provided and the labor actually supplied.

Thus, the question this paper addresses is: what efficient rules are implementable by sharing mechanisms? We consider three equilibrium notions, *Nash*, *strong Nash*, and *subgame perfect Nash*, for the non-cooperative games defined by sharing mechanisms.[5] We identify two axioms that characterize rules *triply implementable by sharing mechanisms*. The two axioms are relevant to the ethical principles of *responsibility and compensation* [Fleurbaey (1998)] in fair allocation problems. Thus, our characterization provides insight into the implementability of fair allocation rules in terms of responsibility and compensation.

The model is defined in Section 2. Section 3 provides a characterization of triple implementation by sharing mechanisms. Section 4 gives some examples of implementable and non-implementable rules. Concluding remarks appear in Section 5. All proofs are relegated to the Appendix.

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and Yamato (1996) to characterize "natural mechanisms" in pure exchange economies.

[4] Roemer (1989) pointed out this implicit assumption explicitly.

[5] Yamada and Yoshihara (2002) proposed a sharing mechanism that triply implements the proportional solution in these three equilibria, when the production function is differentiable.

2 The Basic Model

There are two goods, one of which is an input (labor time) $x \in \mathbb{R}_+$ to be used to produce the other good $y \in \mathbb{R}_+$.[6] There is a set $N = \{1, \ldots, n\}$ of agents, where $2 \leq n < +\infty$. Each agent i's consumption is denoted by $z_i = (x_i, y_i)$, where x_i denotes his labor time, and y_i his share of the output. All agents face a common upper bound of labor time $\bar{x}$, where $0 < \bar{x} < +\infty$, and so have the same consumption set $Z \equiv [0, \bar{x}] \times \mathbb{R}_+$.

Each agent i's preference is defined on Z and represented by a utility function $u_i : Z \to \mathbb{R}$, which is continuous and quasi-concave, and strictly monotonic (decreasing in labor time and increasing in the share of output) on $\overset{\circ}{Z} \equiv [0, \bar{x}) \times \mathbb{R}_{++}$.[7] Let $\mathcal{U}$ denote the class of all such utility functions.

Each agent i has a **labor skill**, $s_i \in \mathbb{R}_{++}$. The universal set of skills for all agents is denoted by $\mathcal{S} = \mathbb{R}_{++}$.[8] The labor skill $s_i \in \mathcal{S}$ is agent i's **effective labor supply** per hour measured in efficiency units. It can also be interpreted as agent i's **labor intensity** exercised in production.[9] Thus, if agent i's **labor time** is $x_i \in [0, \bar{x}]$ and his labor skill is $s_i \in \mathcal{S}$, then $s_i x_i \in \mathbb{R}_+$ denotes the agent's **effective labor contribution** to production measured in efficiency units. The production technology is a function $f : \mathbb{R}_+ \to \mathbb{R}_+$, which is continuous, strictly increasing, concave, and such that $f(0) = 0$. For simplicity, we fix f. Thus, an economy is a pair of profiles $\boldsymbol{e} \equiv (\boldsymbol{u}, \boldsymbol{s})$ with $\boldsymbol{u} = (u_i)_{i \in N} \in \mathcal{U}^n$ and $\boldsymbol{s} = (s_i)_{i \in N} \in \mathcal{S}^n$. Let $\mathcal{E} \equiv \mathcal{U}^n \times \mathcal{S}^n$ denote the class of all economies.

Given $\boldsymbol{s} \in \mathcal{S}^n$, an allocation $\boldsymbol{z} = (x_i, y_i)_{i \in N} \in Z^n$ is **feasible for** $\boldsymbol{s}$ if $\sum y_i \leq f(\sum s_i x_i)$. Let $Z(\boldsymbol{s})$ denote the set of (feasible) allocations for $\boldsymbol{s} \in \mathcal{S}^n$. An allocation $\boldsymbol{z} = (z_i)_{i \in N} \in Z^n$ is **Pareto efficient for** $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ if $\boldsymbol{z} \in Z(\boldsymbol{s})$ and there does not exist $\boldsymbol{z}' = (z'_i)_{i \in N} \in Z(\boldsymbol{s})$ such that for each

[6] The symbol $\mathbb{R}_+$ denotes the set of non-negative real numbers.

[7] The symbol $\mathbb{R}_{++}$ denotes the set of positive real numbers.

[8] For any two sets X and Y, $X \subseteq Y$ if and only if any $x \in X$ also belongs to Y, and $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$.

[9] It might be more natural to define labor skill and labor intensity in a discriminative way: for example, let $\overline{s}_i \in \mathcal{S}$ be i's labor skill, and s_i be i's labor intensity such that $0 < s_i \leq \overline{s}_i$. In such a formulation, we may view the amount of s_i as being determined endogenously by the agent i. In spite of this more natural view, we will assume in the following discussion that the labor intensity is a constant value, $s_i = \overline{s}_i$, for the sake of simplicity. The main theorems in the following discussion would remain valid with few changes in the settings of the economic environments even if the labor intensity were assumed to be varied.

$i \in N$, $u_i(z_i') \geq u_i(z_i)$, and for some $i \in N$, $u_i(z_i') > u_i(z_i)$. Let $P(\boldsymbol{e})$ denote the set of Pareto efficient allocations for $\boldsymbol{e} \in \mathcal{E}$. A **solution** is a correspondence $\varphi : \mathcal{E} \twoheadrightarrow Z^n$ such that for each $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, $\varphi(\boldsymbol{e}) \subseteq Z(\boldsymbol{s})$. A solution φ is a **Pareto subsolution** if for each $\boldsymbol{e} \in \mathcal{E}$, $\varphi(\boldsymbol{e}) \subseteq P(\boldsymbol{e})$. Given φ, $\boldsymbol{z} \in Z^n$ is φ**-optimal for** $\boldsymbol{e} \in \mathcal{E}$ if $\boldsymbol{z} \in \varphi(\boldsymbol{e})$.

2.1 Sharing Mechanisms

We are interested in mechanisms having the property of **labor sovereignty**, and in particular, we focus on the following types of mechanisms. For each $i \in N$, let his strategy space be $A_i \equiv M_i \times [0, \bar{x}]$, with generic element (m_i, x_i). Note that M_i denotes agent i's message space in general. Let $M \equiv \times_{i \in N} M_i$ and $A \equiv \times_{i \in N} A_i$. A **sharing mechanism** is a function $g : A \to \mathbb{R}^n_+$ such that for each $(\boldsymbol{m}, \boldsymbol{x}) \in A$, there is $\boldsymbol{y} \in \mathbb{R}^n_+$ such that $g(\boldsymbol{m}, \boldsymbol{x}) = \boldsymbol{y}$. A sharing mechanism g is **feasible** if for each $\boldsymbol{s} \in \mathcal{S}^n$ and each $(\boldsymbol{m}, \boldsymbol{x}) \in A$, $(\boldsymbol{x}, g(\boldsymbol{m}, \boldsymbol{x})) \in Z(\boldsymbol{s})$. Note that a feasible sharing mechanism g needs not refer to $\boldsymbol{s}$ in dividing the total output $f\left(\sum s_j x_j\right)$. Let $\mathcal{G}$ denote the class of all (feasible sharing) mechanisms.

Given $g \in \mathcal{G}$, a (**feasible**) **sharing game** is defined for each $\boldsymbol{e} \in \mathcal{E}$ as a non-cooperative game $(N, A, g, \boldsymbol{e})$. Fixing N and A, let $(g, \boldsymbol{e})$ simply denote the sharing game $(N, A, g, \boldsymbol{e})$.

Given a profile $(\boldsymbol{m}, \boldsymbol{x}) \in A$, let $(m_i', \boldsymbol{m}_{-i}, x_i', \boldsymbol{x}_{-i}) \in A$ be the profile that is obtained by replacing the i-th component (m_i, x_i) of $(\boldsymbol{m}, \boldsymbol{x})$ with (m_i', x_i'). A profile $(\boldsymbol{m}^*, \boldsymbol{x}^*) \in A$ is a (**pure-strategy**) **Nash equilibrium of** $(g, \boldsymbol{e})$ if for each $i \in N$ and each $(m_i, x_i) \in A_i$, $u_i(x_i^*, g_i(\boldsymbol{m}^*, \boldsymbol{x}^*)) \geq u_i\left(x_i, g_i\left(m_i, \boldsymbol{m}_{-i}^*, x_i, \boldsymbol{x}_{-i}^*\right)\right)$. Let $NE(g, \boldsymbol{e})$ denote the set of Nash equilibria of $(g, \boldsymbol{e})$. An allocation $\boldsymbol{z} = (x_i, y_i)_{i \in N} \in Z^n$ is a **Nash equilibrium allocation of** $(g, \boldsymbol{e})$ if there exists $\boldsymbol{m} \in M$ such that $(\boldsymbol{m}, \boldsymbol{x}) \in NE(g, \boldsymbol{e})$ and $\boldsymbol{y} = g(\boldsymbol{m}, \boldsymbol{x})$, where $\boldsymbol{x} = (x_i)_{i \in N}$ and $\boldsymbol{y} = (y_i)_{i \in N}$. Let $NA(g, \boldsymbol{e})$ denote the set of these allocations. A mechanism $g \in \mathcal{G}$ **implements** φ **in Nash equilibria** if for each $\boldsymbol{e} \in \mathcal{E}$, $NA(g, \boldsymbol{e}) = \varphi(\boldsymbol{e})$.

A profile $(\boldsymbol{m}^*, \boldsymbol{x}^*) \in A$ is a **(pure-strategy) strong (Nash) equilibrium of** $(g, \boldsymbol{e})$ if for each $T \subseteq N$ and each $(m_i, x_i)_{i \in T} \in (A_i)_{i \in T}$, there exists $j \in T$ such that

$$u_j\left(x_j^*, g_j(\boldsymbol{m}^*, \boldsymbol{x}^*)\right) \geq u_j\left(x_j, g_j\left((m_i, x_i)_{i \in T}, (m_k^*, x_k^*)_{k \in T^c}\right)\right).$$[10]

[10]For each $T \subseteq N$, T^c denotes the complement of T in N.

Let $SNE(g, \boldsymbol{e})$ denote the set of strong equilibria of $(g, \boldsymbol{e})$. An allocation $\boldsymbol{z} \in Z^n$ is a **strong equilibrium allocation of** $(g, \boldsymbol{e})$ if there exists $\boldsymbol{m} \in M$ such that $(\boldsymbol{m}, \boldsymbol{x}) \in SNE(g, \boldsymbol{e})$ and $\boldsymbol{y} = g(\boldsymbol{m}, \boldsymbol{x})$. Let $SNA(g, \boldsymbol{e})$ denote the set of strong equilibrium allocations of $(g, \boldsymbol{e})$. A mechanism $g \in \mathcal{G}$ **implements** φ **in strong equilibria**, if for each $\boldsymbol{e} \in \mathcal{E}$, $SNA(g, \boldsymbol{e}) = \varphi(\boldsymbol{e})$.

2.2 The Timing Problem with Sharing Mechanisms

Note that $\boldsymbol{m}$ and $\boldsymbol{x}$ represent different kinds of strategic choices: $\boldsymbol{m}$ is the list of agents' announcements concerning their private information, whereas $\boldsymbol{x}$ indicates their supplies of labor time. Thus, there may be a difference between *the point in time when* $\boldsymbol{m}$ *is announced and the point in time when* $\boldsymbol{x}$ *is exercised*. *Two polar opposite time sequences* can be distinguished: the agents may announce $\boldsymbol{m}$ before they engage in production, or they may announce $\boldsymbol{m}$ after supplying $\boldsymbol{x}$. The former enables each i to choose his labor supply with knowledge of $\boldsymbol{m}$, whereas the latter enables each i to choose m_i with knowledge of $\boldsymbol{x}$.

Thus, we consider two types of two-stage game forms: Given $g \in \mathcal{G}$, the first game form derived from g is the feasible mechanism $\Gamma_g^{\boldsymbol{m} \circ \boldsymbol{x}}$ in which Stage 1 consists of selecting $\boldsymbol{m} \in M$, Stage 2 consists of selecting $\boldsymbol{x} \in [0, \bar{x}]^n$, and $(\boldsymbol{x}, g(\boldsymbol{m}, \boldsymbol{x}))$ is the outcome. The second game form is the feasible mechanism $\Gamma_g^{\boldsymbol{x} \circ \boldsymbol{m}}$ in which Stage 1 consists of selecting $\boldsymbol{x} \in [0, \bar{x}]^n$, Stage 2 consists of selecting $\boldsymbol{m} \in M$, and $(\boldsymbol{x}, g(\boldsymbol{m}, \boldsymbol{x}))$ is the outcome.

Given a two-stage game $\left(\Gamma_g^{\boldsymbol{m} \circ \boldsymbol{x}}, \boldsymbol{e}\right)$ and a strategy profile $\boldsymbol{m} \in M$ in Stage 1, let $\left(\Gamma_g^{\boldsymbol{m} \circ \boldsymbol{x}}(\boldsymbol{m}), \boldsymbol{e}\right)$ be the corresponding Stage 2 subgame. A **strategy mapping** is a function $\boldsymbol{\chi} : M \to [0, \bar{x}]^n$ such that for each $\boldsymbol{m} \in M$, $\boldsymbol{\chi}(\boldsymbol{m})$ is a strategy profile of the subgame $\left(\Gamma_g^{\boldsymbol{m} \circ \boldsymbol{x}}(\boldsymbol{m}), \boldsymbol{e}\right)$. Let $\mathcal{X}$ be the set of all such mappings. A profile $(\boldsymbol{m}^*, \boldsymbol{\chi}^*) \in M \times \mathcal{X}$ is a **(pure-strategy) subgame perfect (Nash) equilibrium of** $\left(\Gamma_g^{\boldsymbol{m} \circ \boldsymbol{x}}, \boldsymbol{e}\right)$ if for each $i \in N$, each $m_i \in M_i$, each $\boldsymbol{\chi} \in \mathcal{X}$ with $\boldsymbol{\chi} = \left(\chi_i, \boldsymbol{\chi}_{-i}^*\right)$, and each $\boldsymbol{m} \in M$,

$$\begin{aligned} u_i\left(\chi_i^*(\boldsymbol{m}^*), g_i(\boldsymbol{m}^*, \boldsymbol{\chi}^*(\boldsymbol{m}^*))\right) &\geq u_i\left(\chi_i^*(m_i, \boldsymbol{m}_{-i}^*), g_i\left(m_i, \boldsymbol{m}_{-i}^*, \boldsymbol{\chi}^*(m_i, \boldsymbol{m}_{-i}^*)\right)\right) \\ \text{and } u_i\left(\chi_i^*(\boldsymbol{m}), g_i(\boldsymbol{m}, \boldsymbol{\chi}^*(\boldsymbol{m}))\right) &\geq u_i\left(\chi_i(\boldsymbol{m}), g_i(\boldsymbol{m}, \boldsymbol{\chi}(\boldsymbol{m}))\right), \end{aligned}$$

where $\chi_i^*(\boldsymbol{m})$ (*resp.* $\chi_i(\boldsymbol{m})$) is the i-th component of $\boldsymbol{\chi}^*(\boldsymbol{m})$ (*resp.* $\boldsymbol{\chi}(\boldsymbol{m})$) in Stage 2 subgame induced by the choice $\boldsymbol{m}$ in Stage 1.

Given a two-stage game $\left(\Gamma_g^{\boldsymbol{x} \circ \boldsymbol{m}}, \boldsymbol{e}\right)$ and a strategy profile $\boldsymbol{x} \in [0, \bar{x}]^n$ in Stage 1, let $\left(\Gamma_g^{\boldsymbol{x} \circ \boldsymbol{m}}(\boldsymbol{x}), \boldsymbol{e}\right)$ be the corresponding Stage 2 subgame. A **strategy**

mapping is a function $\boldsymbol{\mu} : [0,\bar{x}]^n \to M$ such that for each $\boldsymbol{x} \in [0,\bar{x}]^n$, $\boldsymbol{\mu}(\boldsymbol{x})$ is a strategy profile of the subgame $\left(\Gamma_g^{\boldsymbol{x}\circ\boldsymbol{m}}(\boldsymbol{x}), \boldsymbol{e}\right)$. Let $\mathcal{M}$ be the set of all such mappings. A profile $(\boldsymbol{\mu}^*, \boldsymbol{x}^*) \in \mathcal{M} \times [0,\bar{x}]^n$ is a **(pure-strategy) subgame perfect (Nash) equilibrium of** $\left(\Gamma_g^{\boldsymbol{x}\circ\boldsymbol{m}}, \boldsymbol{e}\right)$ if for each $i \in N$, each $x_i \in [0,\bar{x}]$, each $\boldsymbol{\mu} \in \mathcal{M}$ with $\boldsymbol{\mu} = \left(\mu_i, \boldsymbol{\mu}_{-i}^*\right)$, and each $\boldsymbol{x} \in [0,\bar{x}]^n$,

$$\begin{aligned} u_i\left(x_i^*, g_i\left(\boldsymbol{\mu}^*\left(\boldsymbol{x}^*\right), \boldsymbol{x}^*\right)\right) &\geq u_i\left(x_i, g_i\left(\boldsymbol{\mu}^*\left(x_i, \boldsymbol{x}_{-i}^*\right), x_i, \boldsymbol{x}_{-i}^*\right)\right), \\ \text{and } u_i\left(x_i, g_i\left(\boldsymbol{\mu}^*\left(\boldsymbol{x}\right), \boldsymbol{x}\right)\right) &\geq u_i\left(x_i, g_i\left(\boldsymbol{\mu}\left(\boldsymbol{x}\right), \boldsymbol{x}\right)\right). \end{aligned}$$

Let $SPE\left(\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right)$ be the set of subgame perfect equilibria of $\left(\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right)$. An allocation $\boldsymbol{z} \in Z^n$ is a **subgame perfect equilibrium allocation of** $\left(\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right)$ if there exists $(\boldsymbol{m}, \boldsymbol{\chi}) \in SPE\left(\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right)$ such that $\boldsymbol{\chi}(\boldsymbol{m}) = \boldsymbol{x}$ and $\boldsymbol{y} = g(\boldsymbol{m}, \boldsymbol{\chi}(\boldsymbol{m}))$. Let $SPA\left(\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right)$ be the set of subgame perfect equilibrium allocations of $\left(\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right)$. Given $g \in \mathcal{G}$, $\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}$ **implements** φ **in subgame perfect equilibria** if for each $\boldsymbol{e} \in \mathcal{E}$, $SPA\left(\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right) = \varphi(\boldsymbol{e})$. Given $g \in \mathcal{G}$, $\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}$ **triply implements** φ **in Nash, strong, and subgame perfect equilibria** if for each $\boldsymbol{e} \in \mathcal{E}$, $NA(g,\boldsymbol{e}) = SNA(g,\boldsymbol{e}) = SPA\left(\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right) = \varphi(\boldsymbol{e})$. Parallel definitions apply to $\left(\Gamma_g^{\boldsymbol{x}\circ\boldsymbol{m}}, \boldsymbol{e}\right)$.

3 Implementation by Sharing Mechanisms

We assume throughout that each agent prefers interior consumption vectors to boundary consumption vectors.

Assumption 1 (boundary condition of utility functions): *For each* $i \in N$, *each* $z_i \in \overset{\circ}{Z}$, *and each* $z_i' \in \partial Z \equiv Z \setminus \overset{\circ}{Z}$, $u_i(z_i) > u_i(z_i')$.

Let $p_x \in \mathbb{R}_+$ represent the price of labor (measured in efficiency units) and $p_y \in \mathbb{R}_+$ the price of output. The set of price vectors is the unit simplex $\Delta \equiv \left\{p = (p_x, p_y) \in \mathbb{R}_+^2 \mid p_x + p_y = 1\right\}$. Let $\boldsymbol{\rho} = (\rho_i)_{i \in N} \in \Delta^n$ be a profile of price vector announcements, $\boldsymbol{\sigma} = (\sigma_i)_{i \in N} \in \mathcal{S}^n$ a profile of skill announcements, and $\boldsymbol{w} = (w_i)_{i \in N} \in \mathbb{R}_+^n$ a profile of demands for output. Let $M \equiv \Delta^n \times \mathcal{S}^n \times \mathbb{R}_+^n$, with generic element $(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w})$.

Definition 1: *A vector* $p \in \Delta$ *is an* efficiency price for $\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y}) \in Z^n$ at $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ *if*
(i) *for each* $x' \in \mathbb{R}_+$, $p_y f(x') - p_x x' \leq \sum (p_y y_i - p_x s_i x_i)$;

(ii) *for each* $i \in N$ *and each* $z'_i \in Z$, *if* $u_i(z'_i) \geq u_i(z_i)$, *then* $p_y y'_i - p_x s_i x'_i \geq p_y y_i - p_x s_i x_i$.

Let $\Delta^P(\boldsymbol{e}, \boldsymbol{z})$ be the set of efficiency prices for $\boldsymbol{z}$ at $\boldsymbol{e}$.

Definition 2: *A Pareto subsolution* φ *is* triply labor sovereign implementable, *if there exists a feasible sharing mechanism* $g \in \mathcal{G}$ *such that*:

(i) $\Gamma_g^{\boldsymbol{m}\circ\boldsymbol{x}}$ (resp. $\Gamma_g^{\boldsymbol{x}\circ\boldsymbol{m}}$) triply implements φ in Nash, strong, and subgame perfect equilibria;
(ii) g *is* forthright: *for each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ *and each* $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{e})$, *there exists* $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$ *such that* $(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e})$ *and* $g(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{y}$ *with* $\boldsymbol{\rho} = (\rho_i)_{i \in N} = (p, \ldots, p)$;
(iii) *for each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, *if* $(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e})$ *and* $g(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{y}$ *such that* $\boldsymbol{\rho} = (\rho_i)_{i \in N} = (p, \ldots, p) \in \left(\Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})\right)^n$, *then for each* $i \in N$ *and each* $(\rho'_i, \sigma'_i, x'_i, w'_i) \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$,

$$g_i\left(\rho'_i, \sigma'_i, x'_i, w'_i, \boldsymbol{\rho}_{-i}, \boldsymbol{s}_{-i}, \boldsymbol{x}_{-i}, \boldsymbol{w}_{-i}\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s_i\left(x'_i - x_i\right)\right\};$$

(iv) *for each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, *if* $(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e})$, *then* [*for each* $i \in N$ *with* $x_i > 0$, $s'_i = s_i$] *implies* [$(\boldsymbol{\rho}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{w}) \in NE(g, \boldsymbol{e})$ *and* $g(\boldsymbol{\rho}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{w}) = g(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{w})$].

Forthrightness requires that if a strategy profile is consistent with a φ-optimal allocation, then it is a Nash equilibrium and the outcome coincides with this allocation [Dutta et al. (1995); Saijo et al. (1996)]. That is, any φ-optimal allocation should be realizable as an equilibrium outcome in a forthright way.

Definition 2 (iii) is a kind of informational efficiency of the mechanism. It says that in equilibrium, each agent's attainable set is included in a half space that is included in the lower contour set of the agent's utility function when the equilibrium allocation is Pareto efficient. An important point is that this half space depends only on the production point and the production possibility set. Thus, the coordinator does not need to know the entire preference profile in order to obtain φ-optimal allocations in equilibria.

Definition 2 (iv) is another requirement of informational efficiency. It says that the distribution of output is independent of the skill parameters stated by "non-working" agents.

We introduce two axioms as necessary conditions for labor sovereign implementation. First, any φ-optimal allocation should remain φ-optimal if the profile of the utility functions changes, without the Pareto efficiency of this allocation being affected. It is a condition of informational efficiency.

Supporting Price Independence (SPI) [Yoshihara (1998); Gaspart (1998)]**:** *For each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ *and each* $\boldsymbol{z} \in \varphi(\boldsymbol{e})$*, there exists* $p \in \Delta^P(\boldsymbol{e}, \boldsymbol{z})$ *such that for each* $\boldsymbol{e}' = (\boldsymbol{u}', \boldsymbol{s}) \in \mathcal{E}$*, if* $p \in \Delta^P(\boldsymbol{e}', \boldsymbol{z})$*, then* $\boldsymbol{z} \in \varphi(\boldsymbol{e}')$*.*

Let $\Delta^{SPI}(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z}) \equiv \{p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z}) \mid \forall \boldsymbol{u}' \in \mathcal{U}^n \text{ s.t. } p \in \Delta^P(\boldsymbol{u}', \boldsymbol{s}, \boldsymbol{z}), \boldsymbol{z} \in \varphi(\boldsymbol{u}', \boldsymbol{s})\}$.

Secondly, any φ-optimal allocation should remain φ-optimal if the skills of non-working agents change, without the Pareto efficiency of this allocation being affected.

Independence of Unused Skills (IUS): *For each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ *and each* $\boldsymbol{z} \in \varphi(\boldsymbol{e})$*, there exists* $p \in \Delta^P(\boldsymbol{e}, \boldsymbol{z})$ *such that for each* $\boldsymbol{e}' = (\boldsymbol{u}, \boldsymbol{s}') \in \mathcal{E}$ *where* $s'_i = s_i$ *for each* $i \in N$ *with* $x_i > 0$*, if* $p \in \Delta^P(\boldsymbol{e}', \boldsymbol{z})$*, then* $\boldsymbol{z} \in \varphi(\boldsymbol{e}')$*.*

Let $\Delta^{IUS}(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z}) \equiv \{p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z}) \mid \forall \boldsymbol{s}' \in \mathcal{S}^n \text{ s.t. } s'_i = s_i$ for each $i \in N$ with $x_i > 0$ and $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}', \boldsymbol{z})$, $\boldsymbol{z} \in \varphi(\boldsymbol{u}, \boldsymbol{s}')\}$.

The two axioms can also be interpreted in terms of *responsibility* and *compensation* in fair allocation [Fleurbaey (1998)]. SPI represents a "stronger" condition of responsibility, because it requires independence of *particular* changes of individuals' utility functions which are interpreted as factors for which the individuals are responsible.[11] It is "stronger" because SPI is stronger than *Maskin Monotonicity* [Maskin (1999)], which is seen as a relatively strong axiom of responsibility [Fleurbaey and Maniquet (1996)]. In contrast, IUS can be interpreted as a weaker condition of compensation, because it requires independence of *particular* changes of individuals' skills, for which the individuals cannot be held responsible. It is "weaker" because IUS is weaker than *Independence of Skill Endowments* [Yoshihara (2003)], which is a relatively weak axiom of compensation.

Note that a Pareto subsolution φ satisfies SPI and IUS if and only if for each $\boldsymbol{e} \in \mathcal{E}$ and each $\boldsymbol{z} \in \varphi(\boldsymbol{e})$, $\Delta^{SPI}(\boldsymbol{e}, \boldsymbol{z}) \neq \emptyset$ and $\Delta^{IUS}(\boldsymbol{e}, \boldsymbol{z}) \neq \emptyset$. Moreover, the following lemma shows that each $p \in \Delta^{SPI}(\boldsymbol{e}, \boldsymbol{z})$ is contained in $\Delta^{IUS}(\boldsymbol{e}, \boldsymbol{z})$.

[11] There is another axiom closely related to SPI, *Local Independence*, although it is applied only to economies with differentiable utility functions [Nagahisa (1991)].

Lemma 0: *Let φ satisfy SPI and IUS. Then, for each $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ and each $\boldsymbol{z} \in \varphi(\boldsymbol{e})$, $\Delta^{SPI}(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z}) \subseteq \Delta^{IUS}(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z})$.*

Proof. Given $(\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, let $\boldsymbol{z} = (x_i, y_i)_{i \in N} \in \varphi(\boldsymbol{u}, \boldsymbol{s})$ and $p \in \Delta^{SPI}(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z})$. W. l. o. g., suppose that $x_1 = 0$ and $x_i > 0$ for each $i \neq 1$. Let $\boldsymbol{s}' = (s_1', \boldsymbol{s}_{-1})$ be such that $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}', \boldsymbol{z})$. If $\boldsymbol{z} \in \varphi(\boldsymbol{u}, \boldsymbol{s}')$, then by the definition of Δ^{IUS}, we have that $p \in \Delta^{IUS}(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z})$.

For (x_2, y_2) of $\boldsymbol{z} = (x_i, y_i)_{i \in N}$, let $\alpha \in (0,1)$ satisfy $-\frac{\partial(\overline{x}-x_2)^{1-\alpha} \cdot (y_2)^\alpha / \partial x}{\partial(\overline{x}-x_2)^{1-\alpha} \cdot (y_2)^\alpha / \partial y} = \frac{p_x s_2}{p_y}$. The existence of α can be shown by the intermediate value theorem, since $-\frac{\partial(\overline{x}-x_2)^{1-\alpha} \cdot (y_2)^\alpha / \partial x}{\partial(\overline{x}-x_2)^{1-\alpha} \cdot (y_2)^\alpha / \partial y} = \frac{y_2}{\overline{x}-x_2}\left(\frac{1}{\alpha} - 1\right) \to 0$ as $\alpha \to 1$, whereas $-\frac{\partial(\overline{x}-x_2)^{1-\alpha} \cdot (y_2)^\alpha / \partial x}{\partial(\overline{x}-x_2)^{1-\alpha} \cdot (y_2)^\alpha / \partial y} \to \infty$ as $\alpha \to 0$. Let $u_2^* \in \mathcal{U}$ be, for each $(x, y) \in Z$, $u_2^*(x, y) = (\overline{x} - x)^{1-\alpha} \cdot y^\alpha$. Thus, u_2^* is compatible with Assumption 1.

Let $\boldsymbol{u}^* = (u_2^*, \boldsymbol{u}_{-2})$. As $\Delta^P(\boldsymbol{u}^*, \boldsymbol{s}, \boldsymbol{z}) = \{p\}$ and φ satisfies SPI, $\boldsymbol{z} \in \varphi(\boldsymbol{u}^*, \boldsymbol{s})$ and $\Delta^{SPI}(\boldsymbol{u}^*, \boldsymbol{s}, \boldsymbol{z}) = \{p\}$. Consider moving from $(\boldsymbol{u}^*, \boldsymbol{s})$ to $(\boldsymbol{u}^*, \boldsymbol{s}')$. From the definition of $\boldsymbol{u}^*$, $\Delta^P(\boldsymbol{u}^*, \boldsymbol{s}', \boldsymbol{z}) = \{p\}$. As φ satisfies IUS, $\boldsymbol{z} \in \varphi(\boldsymbol{u}^*, \boldsymbol{s}')$ and $\Delta^{IUS}(\boldsymbol{u}^*, \boldsymbol{s}', \boldsymbol{z}) = \{p\}$. Consider moving from $(\boldsymbol{u}^*, \boldsymbol{s}')$ to $(\boldsymbol{u}, \boldsymbol{s}')$. As $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}', \boldsymbol{z})$ and φ satisfies SPI, $\boldsymbol{z} \in \varphi(\boldsymbol{u}, \boldsymbol{s}')$. ■

SPI and IUS are necessary conditions for labor sovereign triple implementation.

Theorem 1: *If a Pareto subsolution φ is* triply labor sovereign implementable, *then φ satisfies SPI and IUS.*

Next, we show that SPI and IUS together are sufficient for labor sovereign implementation. Given $\boldsymbol{x} \in [0, \bar{x}]^n$ and $i \in N$, let $\pi(\boldsymbol{x}_{-i}) \equiv \max\left\{\frac{x_j + \bar{x}}{2} \mid x_j < \bar{x} \text{ for } j \neq i\right\}$. We construct the following two auxiliary mechanisms:

- Let $g^{\boldsymbol{w}}$ be such that for each $\boldsymbol{s} \in \mathcal{S}^n$, each $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$, and each $i \in N$,
$$g_i^{\boldsymbol{w}}(\boldsymbol{\tau}) = \begin{cases} f\left(\sum s_k x_k\right) & \text{if } x_i = \pi(\boldsymbol{x}_{-i}) \text{ and} \\ & w_i > \max\left\{f\left(\sum \sigma_k \bar{x}\right), \ \max\{w_j \mid j \neq i\}\right\}, \\ 0 & \text{otherwise.} \end{cases}$$

- Let $g^{\boldsymbol{\sigma}}$ be such that for each $\boldsymbol{s} \in \mathcal{S}^n$, each $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$, and each $i \in N$,
$$g_i^{\boldsymbol{\sigma}}(\boldsymbol{\tau}) = \begin{cases} f\left(\sum s_k x_k\right) & \text{if } x_i = 0,\ w_i = 0, \text{ and } \sigma_i > \sigma_j \text{ for each } j \neq i, \\ 0 & \text{otherwise.} \end{cases}$$

The mechanism $g^{\boldsymbol{w}} \in \mathcal{G}$ assigns all of the produced output[12] to only one agent, the agent who provides the maximal *positive* amount, but less than $\bar{x}$, of labor time and reports a maximal demand for the output. The mechanism $g^{\boldsymbol{\sigma}} \in \mathcal{G}$ assigns all of the produced output to only one agent, the agent who demands no output, reports the highest skill, and does not work.

Given $p \in \Delta$ and $(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w}) \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$, let $\varphi(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})^{-1} \equiv \left\{\boldsymbol{u} \in \mathcal{U}^n \mid (\boldsymbol{x},\boldsymbol{w}) \in \varphi(\boldsymbol{u},\boldsymbol{\sigma}) \text{ and } p \in \Delta^{SPI}(\boldsymbol{u},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})\right\}$. Given $p \in \Delta$ and $(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w}) \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$, let $N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w}) \equiv \Big\{i \in N \mid \exists (x'_i, w'_i) \in \overset{\circ}{Z}$ s.t. $\varphi(p,\boldsymbol{\sigma},(x'_i,\boldsymbol{x}_{-i}),(w'_i,\boldsymbol{w}_{-i}))^{-1} \neq \varnothing\Big\}$.

A profile $\boldsymbol{\tau} = (\boldsymbol{\rho},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w}) \in \triangle^n \times \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$ is φ-**consistent** if for some $p \in \Delta$, $\rho_i = p$ for each $i \in N$ and $\varphi(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})^{-1} \neq \varnothing$. Given a profile $\boldsymbol{\tau}$ such that for each j, $\rho_j = p$, $i \in N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})$ is called a "*potential deviator,*" for the following reason. Suppose $\varphi(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})^{-1} = \varnothing$ and $N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w}) \neq \varnothing$. The first statement implies that $\boldsymbol{\tau}$ is not φ-consistent. The second statement implies that there is an agent i who can switch his strategy to another one $(\rho_i,\sigma_i,x'_i,w'_i)$ so that the new profile $(\boldsymbol{\rho},\boldsymbol{\sigma},(x'_i,\boldsymbol{x}_{-i}),(w'_i,\boldsymbol{w}_{-i}))$ is consistent with φ. That is, it may be the agent who makes the current profile $\boldsymbol{\tau}$ inconsistent with φ. This means that $i \in N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})$ is a "potential deviator."

We define $g^* \in \mathcal{G}$ as follows:

For each $\boldsymbol{s} \in \mathcal{S}^n$ and each $\boldsymbol{\tau} = (\boldsymbol{\rho},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w}) \in \triangle^n \times \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$,
Rule 1: if $f(\sum \sigma_k x_k) = f(\sum s_k x_k)$, and
1-1: there exists $p \in \Delta$ such that $\rho_i = p$ for each $i \in N$ and $\varphi(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})^{-1} \neq \varnothing$, then $g^*(\boldsymbol{\tau}) = \boldsymbol{w}$,
1-2: there exist $j \in N$ and $p \in \Delta$ such that $p = \rho_i$ for each $i \neq j$, $\varphi(\rho_j,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})^{-1} = \varnothing$, and $j \in N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})$, then $g^*_i(\boldsymbol{\tau}) = 0$ for each $i \neq j$, and $g^*_j(\boldsymbol{\tau}) =$

$$\begin{cases} \max\left\{0, \min\left\{w'_j + \frac{p_x}{p_y}\left(\sigma_j x_j - \sigma_j x'_j\right), f\left(\sum s_k x_k\right)\right\}\right\} & \text{if } w_j > f\left(\sum \sigma_k \bar{x}\right) \\ 0 & \text{otherwise,} \end{cases}$$

for (x'_j, w'_j) with $\varphi\left(p,\boldsymbol{\sigma},\left(x'_j,\boldsymbol{x}_{-j}\right),\left(w'_j,\boldsymbol{w}_{-j}\right)\right)^{-1} \neq \varnothing$,
1-3: in any other case, $g^*(\boldsymbol{\tau}) = g^{\boldsymbol{w}}(\boldsymbol{\tau})$,

Rule 2: if $f(\sum \sigma_k x_k) \neq f(\sum s_k x_k)$, then $g^*(\boldsymbol{\tau}) = g^{\boldsymbol{\sigma}}(\boldsymbol{\tau})$.

[12] We implicitly assume that the mechanism coordinator can hold all of the produced output after the production process, although he may not monitor that process perfectly.

It is easy to see that g^* is *forthright*, satisfies the *best response property* [Jackson et al. (1994)], and is *self-relevant* [Hurwicz (1960)]. Note that the total output $f\left(\sum s_k x_k\right)$ is observable at the end of production, even without true information on labor skills.

Given $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$, g^* works as follows: First, g^* computes the amount $f\left(\sum \sigma_k x_k\right)$ and compares this with $f\left(\sum s_k x_k\right)$. Suppose equality holds. Then, if $\boldsymbol{\tau}$ is φ-consistent, g^* distributes $f\left(\sum s_k x_k\right)$ in accordance with $\boldsymbol{w}$ under Rule 1-1. Otherwise, and if there is a unique potential deviator, then g^* punishes this deviator according to Rule 1-2. In any other case, g^* selects the same outcome as $g^{\boldsymbol{w}}$ under Rule 1-3. If $f\left(\sum \sigma_k x_k\right) \neq f\left(\sum s_k x_k\right)$, then g^* selects the same outcome as $g^{\boldsymbol{\sigma}}$ under Rule 2.

We explain below how g^* induces true revelation of skills at least for working agents (A), and how it attains desirable allocations (B):

(A) g^* distributes $f\left(\sum s_k x_k\right)$ according to $\boldsymbol{\tau}$. The problem is that true skills are not observable. To solve this, a scheme of reward and punishment is set up as follows. First, if $f\left(\sum \sigma_k x_k\right) \neq f\left(\sum s_k x_k\right)$, then clearly $\boldsymbol{\sigma} \neq \boldsymbol{s}$, and at least one agent, say $j \in N$ whose labor supply is positive, $x_j > 0$, has misrepresented his skill, $\sigma_j \neq s_j$. This agent is punished under Rule 2.

Second, suppose $f\left(\sum \sigma_k x_k\right) = f\left(\sum s_k x_k\right)$ but $\boldsymbol{\sigma} \neq \boldsymbol{s}$. Then, at least two agents have misrepresented their skills while supplying positive amounts of labor, or someone, say j, has chosen "non-working" while misrepresenting his skill. Let us put aside the latter case for the moment. In the former case, suppose that one such misrepresenting agent, say $j \in N$, switches from $x_j > 0$ to $x'_j = 0$, while announcing a higher number σ'_j than any other in $\boldsymbol{\sigma}_{-j}$, so as to induce $f\left(\sigma'_j x'_j + \sum_{i \neq j} \sigma_i x_i\right) \neq f\left(s_j x'_j + \sum_{i \neq j} s_i x_i\right)$. In this case, j may be better off under Rule 2. Thus, the case may not correspond to an equilibrium situation. The following lemma confirms this insight.

Lemma 1: *Let Assumption 1 hold. Let $g^* \in \mathcal{G}$ be as above. Given $(\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, let $(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$ be a Nash equilibrium of $(g^*, \boldsymbol{u}, \boldsymbol{s})$ such that $f\left(\sum \sigma_k x_k\right) = f\left(\sum s_k x_k\right)$. Then, for each $i \in N$ with $x_i > 0$, $\sigma_i = s_i$.*

(B) We still need to explain how g^* implements the Pareto subsolution φ when all agents report their true skills, $\boldsymbol{\sigma} = \boldsymbol{s}$. Then, the profile $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$ induces only Rule 1. Note that among the three subrules of Rule 1, only Rule 1-1 can realize a desirable allocation in equilibrium, while the other two are to punish agents who have deviated from the situation of Rule 1-1. Suppose that $\boldsymbol{\tau}$ is φ-consistent. Then, $\boldsymbol{\tau}$ induces Rule 1-1 and the corresponding outcome $g^*(\boldsymbol{\tau}) = (\boldsymbol{x}, \boldsymbol{w})$ is φ-optimal for some economy. However, this does not necessarily imply that $(\boldsymbol{x}, \boldsymbol{w})$ is Pareto efficient for the actual economy. If $\boldsymbol{\tau}$ induces Rule 1-1, but $(\boldsymbol{x}, \boldsymbol{w})$ is not Pareto efficient for the actual economy, there is an agent, say j, and a consumption bundle $\left(x'_j, w'_j\right)$ which is better than (x_j, w_j) for him, and is also available to him within the budget set determined by the supporting price at $(\boldsymbol{x}, \boldsymbol{w})$. Then, Rule 1-2 is applied to select such $\left(x'_j, w'_j\right)$. Therefore, if $(\boldsymbol{x}, \boldsymbol{w})$ is an equilibrium allocation, then $(\boldsymbol{x}, \boldsymbol{w})$ is Pareto efficient for this actual economy.

We are now ready to discuss the full characterizations of labor sovereignty triple implementation by examining the performance of g^*.

Theorem 2: *Let Assumption 1 hold. Then, if a Pareto subsolution φ satisfies SPI and IUS, then φ is* triply labor sovereign implementable by g^**.*

This result holds even in economies of *two agents.*

Corollary 1: *Let Assumption 1 hold. Then, a Pareto subsolution φ is* triply labor sovereign implementable *if and only if φ satisfies SPI and IUS.*

If each agent i can control his contribution by selecting $\widetilde{s}_i \in [0, s_i]$, Corollary 1 still applies. In such a situation, the observable total output is given not by $f\left(\sum s_k x_k\right)$, but by $f\left(\sum \widetilde{s}_k x_k\right)$, where $\widetilde{s}_k \in [0, s_k]$ for each $k \in N$. Then, the coordinator can still compare $f\left(\sum \sigma_k x_k\right)$ with $f\left(\sum \widetilde{s}_k x_k\right)$.

From Corollary 1, we gain two new insights. First, we can classify which solutions remain implementable if skills are unknown to the coordinator, compared to when they are known.[13] Second, Corollary 1 indicates that the implementable solutions should satisfy a rather strong axiom of responsibility such as SPI, and a rather weak axiom of compensation such as IUS.

4 Characterizations

By applying Corollary 1, let us examine which Pareto subsolutions are implementable. First, we discuss three variants of the *Walrasian solution*:

[13] It is easy to see that any Pareto subsolution is labor sovereign implementable if and only if it satisfies SPI, whenever skills are known to the coordinator.

Definition 3: *The* **Walrasian solution with profit shares** $\theta = (\theta_i)_{i\in N} \in [0,1]^n$ *satisfying* $\sum \theta_i = 1$, *denoted* W^{θ}, *assigns with each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, $\boldsymbol{z} \in Z(\boldsymbol{s})$ *such that: there exists* $p = (p_x, p_y) \in \Delta$ *such that:*
(i) *for each* $x' \in \mathbb{R}_+$, $p_y f(x') - p_x x' \leq \sum (p_y y_j - p_x s_j x_j)$;
(ii) *for each* $i \in N$ *and each* $(x, y) \in Z$, *if* $u_i(x, y) > u_i(z_i)$, *then* $p_y y - p_x s_i x > \theta_i \sum (p_y y_j - p_x s_j x_j)$.

Definition 4 [Roemer and Silvestre (1989, 1993)]**:** *The* **proportional solution**, *denoted* PR, *assigns with each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, $\boldsymbol{z} \in P(\boldsymbol{e})$ *such that for each* $i \in N$, $y_i = \frac{s_i x_i}{\sum s_j x_j} \sum y_j$.

Definition 5 [Roemer and Silvestre (1989)]**:** *The* **equal benefit solution**, *denoted* EB, *assigns with each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, $\boldsymbol{z} \in P(\boldsymbol{e})$ *such that: there exists* $p = (p_x, p_y) \in \Delta^P(\boldsymbol{e}, \boldsymbol{z})$ *such that for each* $i \in N$, $p_y y_i - p_x s_i x_i = \frac{1}{n} \sum (p_y y_j - p_x s_j x_j)$.

All three solutions above satisfy SPI [Yoshihara (2000)]. Thus, to confirm their implementability, it suffices to examine IUS. Then:

Lemma 6: W^{θ}, *PR, and EB satisfy IUS.*

Corollary 2: *Let Assumption 1 hold. Then,* W^{θ}, *PR, and EB are* triply labor sovereign implementable*.*

Is there any non-Walrasian type of allocation rule that is implementable? To discuss this, let us consider the following types of rules:

Definition 6 [Yoshihara (2000a)]**:** *The* λ**-effort-reward solution** *with parameter* $\lambda \in [0,1]$, *denoted* ER^{λ}, *assigns with each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, $\boldsymbol{z} \in P(\boldsymbol{e})$ *such that for each* $i \in N$, $y_i = \left\{\lambda \frac{x_i}{\sum x_j} + (1-\lambda)\frac{1}{n}\right\} f\left(\sum s_j x_j\right)$.

For each $\lambda \in [0,1]$, the solution ER^{λ} is well defined [Yoshihara (2000a)]. It also satisfies the *equal-reward-for-equal-labor-time* (**EREL**) principle [Kranich (1994)]. Moreover, it satisfies SPI and IUS, since it distributes output completely independently of skills. Thus:

Corollary 3: *Let Assumption 1 hold. Then, for any* $\lambda \in [0,1]$, ER^{λ} *is* triply labor sovereign implementable*.*

As ER^{λ} is not a Walrasian type, but satisfies the EREL principle, Corollary 3 indicates that implementable EREL-Pareto subsolutions exist.

Note that among Pareto subsolutions meeting the ethical principles of responsibility and compensation, proposed by Fleurbaey and Maniquet (1996), the *$\widetilde{u}$-reference welfare equivalent budget solution* can be implementable by sharing mechanisms if skills are known to the coordinator, because it satisfies SPI. However, even this solution cannot be implementable if skills are unknown, because it does not satisfy IUS. Thus, the private information of skills causes this non-implementability result.

5 Concluding Remarks

We characterized implementation by sharing mechanisms in production economies with unequal labor skills. The class of Pareto subsolutions implementable by sharing mechanisms is characterized by two axioms, Supporting Price Independence and Independence of Unused Skills. The Walrasian, proportional, equal benefit, and λ-effort-reward solutions are implementable, whereas the $\widetilde{u}$-reference welfare equivalent budget solution is not implementable if skills are unknown to the coordinator. This result indicates the impossibility of implementing a Pareto subsolution that meets the ethical principles of responsibility and compensation, if skills are private information.

The workability of our proposed mechanism depends on two implicit assumptions: First, although every individual i's effective labor contribution, $s_i x_i$, is imperfectly observable and unverifiable by the coordinator, his labor supply, x_i, is observable. Second, despite such imperfect observability, the coordinator can observe the total output, so that he can compare this amount with the expected output based on the announcements of the individuals. We believe that these assumptions are reasonable. However, it is an open question whether the implementation of Pareto subsolutions by natural mechanisms in production economies with unequal skills holds without these.

6 Appendix

6.1 Proof of Theorem 1

Let φ be a Pareto subsolution that is triply labor sovereign implementable. Then, there exists $g \in \mathcal{G}$ that satisfies conditions (i)-(iv) in Definition 2.

Given $\boldsymbol{e} = (u_i, s_i)_{i\in N}$, $\boldsymbol{e}' = (u_i', s_i)_{i\in N} \in \mathcal{E}$, suppose that $\boldsymbol{z} = (x_i, y_i)_{i\in N} \in \varphi(\boldsymbol{e})$ and that there exists $p \in \Delta^P(\boldsymbol{e},\boldsymbol{z}) \cap \Delta^P(\boldsymbol{e}',\boldsymbol{z})$. From (ii), for $\boldsymbol{\rho} = (\rho_i)_{i\in N}$ with $\rho_i = p$ for each $i \in N$, $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e})$ and $g(\boldsymbol{\tau}) = \boldsymbol{y}$. Therefore, from (iii), $g_i(\tau_i', \boldsymbol{\tau}_{-i}) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s_i (x_i' - x_i)\right\}$ for each $i \in N$ and each $\tau_i' \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$. As $p \in \Delta^P(\boldsymbol{e}',\boldsymbol{z})$, this implies $\boldsymbol{\tau} \in NE(g, \boldsymbol{e}')$ and $(\boldsymbol{x}, \boldsymbol{y}) \in NA(g, \boldsymbol{e}')$. Hence, $\boldsymbol{z} \in \varphi(\boldsymbol{e}')$ by (i). Thus, φ satisfies SPI.

Given $\boldsymbol{e} = (u_i, s_i)_{i\in N}$, $\boldsymbol{e}' = (u_i, s_i')_{i\in N} \in \mathcal{E}$, let $\boldsymbol{z} \in \varphi(\boldsymbol{e})$, where $s_i = s_i'$ for each $i \in N$ with $x_i > 0$. Suppose that there exists $p \in \Delta^P(\boldsymbol{e},\boldsymbol{z}) \cap \Delta^P(\boldsymbol{e}',\boldsymbol{z})$. From (ii), for $\boldsymbol{\rho} = (\rho_i)_{i\in N}$ with $\rho_i = p$ for each $i \in N$, $\boldsymbol{\tau} \in NE(g, \boldsymbol{e})$ and $g(\boldsymbol{\tau}) = \boldsymbol{y}$, which implies $(\boldsymbol{\rho}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e})$ and $g(\boldsymbol{\rho}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{y}$ by (iv). Then, from (iii), $g_i(\tau_i', \boldsymbol{\tau}_{-i}) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s_i (x_i' - x_i)\right\}$ for each $i \in N$ and each $\tau_i' \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$. As $p \in \Delta^P(\boldsymbol{e}',\boldsymbol{z})$, this implies that $(\boldsymbol{p}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e}')$ and $(\boldsymbol{x}, \boldsymbol{y}) \in NA(g, \boldsymbol{e}')$. Hence, $\boldsymbol{z} \in \varphi(\boldsymbol{e}')$ from (i). Thus, φ satisfies IUS. ■

6.2 Proof of Theorem 2

Proof of Lemma 1. Suppose that there exists $j \in N$ with $\sigma_j \neq s_j$ and $x_j > 0$. Let $N(\boldsymbol{\sigma}, \boldsymbol{x})$ be the set of all such j. As $f(\sum \sigma_i x_i) = f(\sum s_i x_i)$, $N(\boldsymbol{\sigma}, \boldsymbol{x})$ is not a singleton. Moreover, under Rule 2, each $j \in N(\boldsymbol{\sigma}, \boldsymbol{x})$ can obtain $y_j' = f\left(\sum_{i\neq j} s_i x_i\right) > 0$ by switching to $\sigma_j' > \max\{\sigma_i \mid i \neq j\}$, $x_j' = 0$, and $w_j' = 0$. Note that:

$$\begin{aligned}
\sum_{j\in N(\boldsymbol{\sigma},\boldsymbol{x})} y_j' &= \sum_{j\in N(\boldsymbol{\sigma},\boldsymbol{x})} f\left(\sum_{i\neq j} s_i x_i\right) = \sum_{j\in N(\boldsymbol{\sigma},\boldsymbol{x})} f\left(\sum_{i\in N(\boldsymbol{\sigma},\boldsymbol{x})\setminus\{j\}} s_i x_i + \sum_{k\notin N(\boldsymbol{\sigma},\boldsymbol{x})} s_k x_k\right) \\
&\geq \sum_{j\in N(\boldsymbol{\sigma},\boldsymbol{x})} f\left(s_j x_j + \sum_{k\notin N(\boldsymbol{\sigma},\boldsymbol{x})} s_k x_k\right) \quad \text{(as } N(\boldsymbol{\sigma},\boldsymbol{x}) \text{ is not a singleton)} \\
&\geq f\left(\sum_{j\in N(\boldsymbol{\sigma},\boldsymbol{x})} \left(s_j x_j + \sum_{k\notin N(\boldsymbol{\sigma},\boldsymbol{x})} s_k x_k\right)\right) \quad \text{(as } f \text{ is concave and } f(0) \geq 0\text{)} \\
&\geq f\left(\sum_{j\in N(\boldsymbol{\sigma},\boldsymbol{x})} s_j x_j + \sum_{k\notin N(\boldsymbol{\sigma},\boldsymbol{x})} s_k x_k\right)
\end{aligned}$$

$$= \quad f\left(\sum s_k x_k\right) \geq \sum_{j \in N(\boldsymbol{\sigma},\boldsymbol{x})} y_j \equiv \sum_{j \in N(\boldsymbol{\sigma},\boldsymbol{x})} g_j^*\left(\boldsymbol{\rho},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w}\right).$$

Hence, there is $j \in N(\boldsymbol{\sigma},\boldsymbol{x})$ with $y_j' \geq y_j$. Note that $u_j(0,y_j') \geq u_j(0,y_j) \geq u_j(x_j,y_j)$ and $u_j(0,y_j') > u_j(x_j,0)$ by Assumption 1, whereas $u_j(0,y_j) > u_j(x_j,y_j)$ if $y_j > 0$ from the strict monotonicity of u_j. Hence, agent j has an incentive to switch from x_j to $x_j' = 0$ to obtain y_j'. Thus, $(\boldsymbol{\rho},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w})$ does not constitute a Nash equilibrium. ■

Lemma 2: *Let Assumption 1 hold. Then,* g^* implements *any Pareto subsolution* φ *satisfying SPI and IUS* in Nash equilibria*.*

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\boldsymbol{e} = (\boldsymbol{u},\boldsymbol{s}) \in \mathcal{E}$.

(1) First, we show that $\varphi(\boldsymbol{e}) \subseteq NA(g^*,\boldsymbol{e})$. Let $\boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \in \varphi(\boldsymbol{e})$. Let $\boldsymbol{\tau} = (\boldsymbol{\rho},\boldsymbol{s},\boldsymbol{x},\boldsymbol{y}) \in (\Delta \times \mathcal{S} \times [0,\bar{x}] \times \mathbb{R}_+)^n$ be such that $\rho_i = p$ for each $i \in N$ and $p \in \Delta^{SPI}(\boldsymbol{u},\boldsymbol{s},\boldsymbol{z})$. Then, $g^*(\boldsymbol{\tau}) = \boldsymbol{y}$ from Rule 1-1. Suppose $j \in N$ deviates to $\tau_j' = \left(\rho_j', s_j', x_j', w_j'\right) \in \Delta \times \mathcal{S} \times [0,\bar{x}] \times \mathbb{R}_+$. From Assumption 1 and the continuity of utility functions, if $g_j^*\left(\tau_j',\boldsymbol{\tau}_{-j}\right) = 0$, then j has no incentive to switch to τ_j'.

If τ_j' induces Rule 2, then $x_j' > 0$ and $g_j^*\left(\tau_j',\boldsymbol{\tau}_{-j}\right) = 0$. If τ_j' induces Rule 1-3, then $\varphi\left(\rho_j',\left(s_j',\boldsymbol{s}_{-j}\right),\left(x_j',\boldsymbol{x}_{-j}\right),\left(w_j',\boldsymbol{y}_{-j}\right)\right)^{-1} \neq \varnothing$ or $N\left(p,\left(s_j',\boldsymbol{s}_{-j}\right),\left(x_j',\boldsymbol{x}_{-j}\right),\left(w_j',\boldsymbol{y}_{-j}\right)\right) = \varnothing$. The former implies $s_j' = s_j$ and $w_j' \leq f\left(\sum_{i \neq j} s_i\bar{x} + s_j'\bar{x}\right)$, whereas the latter implies $x_j' = 0$ and $s_j' \neq s_j$. Thus, in either case, $g_j^*\left(\tau_j',\boldsymbol{\tau}_{-j}\right) = 0$. If τ_j' induces Rule 1-2, then either $x_j' > 0$ and $s_j' = s_j$ or $x_j' = 0$. If the former holds, then $g_j^*\left(\tau_j',\boldsymbol{\tau}_{-j}\right) \leq \max\left\{0, y_j + \frac{p_x}{p_y}\left(s_jx_j' - s_jx_j\right)\right\}$. If the latter holds, then there exists $\left(x_j'', w_j''\right)$ such that $\varphi\left(p,\left(s_j',\boldsymbol{s}_{-j}\right),\left(x_j'',\boldsymbol{x}_{-j}\right),\left(w_j'',\boldsymbol{y}_{-j}\right)\right)^{-1} \neq \varnothing$. Since $w_j'' - \frac{p_x}{p_y}s_j'x_j'' \leq y_j - \frac{p_x}{p_y}s_jx_j$ by the concavity of f, $g_j^*\left(\rho_j', s_j', 0, w_j', \boldsymbol{\tau}_{-j}\right) \leq \max\left\{0, y_j - \frac{p_x}{p_y}s_jx_j\right\}$. Finally, if τ_j' induces Rule 1-1, then $g_j^*\left(\tau_j',\boldsymbol{\tau}_{-j}\right) = w_j' = f(\sum_{i \neq j} s_ix_i + s_jx_j') - \sum_{i \neq j} y_i$. Thus, since $\boldsymbol{z} \in P(\boldsymbol{e})$, $u_j\left(x_j', w_j'\right) \leq u_j\left(x_j, y_j\right)$. In summary, j has no incentive to switch to τ_j'.

(2) Second, we show that $NA(g^*,\boldsymbol{e}) \subseteq \varphi(\boldsymbol{e})$. Let $\boldsymbol{\tau} = (\boldsymbol{\rho},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{w}) \in NE(g^*,\boldsymbol{e})$.

Suppose that $\boldsymbol{\tau}$ induces Rule 2. Then, either $N^0(\boldsymbol{x}) \equiv \{i \in N \mid x_i = 0\} = \varnothing$ or $N^0(\boldsymbol{x}) \neq \varnothing$. If $N^0(\boldsymbol{x}) = \varnothing$, then for each $i \in N$, $g_i^*(\boldsymbol{\tau}) = 0$. Then, if for each $k \in N$, $\sum_{i \neq k} \sigma_i x_i = \sum_{i \neq k} s_i x_i$, then $(n-1) \cdot (\sum \sigma_i x_i) = (n-1) \cdot (\sum s_i x_i)$. This contradicts the fact that Rule 2 is induced. Thus, for some $j \in N$, $\sum_{i \neq j} \sigma_i x_i \neq \sum_{i \neq j} s_i x_i$. Then, if j switches to $\tau_j' = (\rho_j', \sigma_j', x_j', w_j')$ with $\sigma_j' > \max\{\sigma_i \mid i \neq j\}$, $x_j' = 0$, and $w_j' = 0$, then $g_j^*(\tau_j', \boldsymbol{\tau}_{-j}) > 0$ under Rule 2.

Let $N^0(\boldsymbol{x}) \neq \varnothing$ with $\# N^0(\boldsymbol{x}) \geq 2$. Then, for each $j \in N^0(\boldsymbol{x})$, if j's deviating strategy τ_j' is such that for each $i \neq j$, $\sigma_j' > \sigma_i$ and $(x_j', w_j') = (0, 0)$, then $g_j^*(\tau_j', \boldsymbol{\tau}_{-j}) = f(\sum s_k x_k)$ under Rule 2.

Let $\# N^0(\boldsymbol{x}) = 1$ and $\# N \backslash N^0(\boldsymbol{x}) \geq 2$. Then, there exists $j \in N \backslash N^0(\boldsymbol{x})$ such that $\sum_{i \in N \backslash (N^0(\boldsymbol{x}) \cup \{j\})} \sigma_i x_i \neq \sum_{i \in N \backslash (N^0(\boldsymbol{x}) \cup \{j\})} s_i x_i$. Thus, j can switch to τ_j' such that $g_j^*(\tau_j', \boldsymbol{\tau}_{-j}) > 0$ under Rule 2. This can be shown in a similar way to the case $N^0(\boldsymbol{x}) = \varnothing$.

Let $N^0(\boldsymbol{x}) = \{i\}$ and $N \backslash N^0(\boldsymbol{x}) = \{j\}$. If $w_i > 0$, then switching i's strategy to $\sigma_i' > \sigma_j$, $x_i' = 0$, and $w_i' = 0$ implies $g_i^*(\rho_i', \sigma_i', x_i', w_i', \rho_j, \sigma_j, x_j, y_j) = f(s_j x_j)$ under Rule 2. If $w_i = 0$, then switching j's strategy to $\rho_j' = \rho_i$, $\sigma_j' = s_j$, $x_j' = \frac{\overline{x}}{2}$, and $w_j' > f(s_j \overline{x} + \sigma_i \overline{x})$ implies $g_j^*(\rho_i, \sigma_i, x_i, w_i, \rho_j', \sigma_j', x_j', w_j') = f(s_j x_j')$ under Rule 1-3. In summary, $\boldsymbol{\tau}$ does not induce Rule 2.

Suppose that $\boldsymbol{\tau}$ induces Rule 1-2 or 1-3. Then, there exists $j \in N$ such that $g_j^*(\boldsymbol{\tau}) = 0$. By Lemma 1, $\sigma_j = s_j$ or $x_j = 0$. Suppose $\boldsymbol{\tau}$ induces Rule 1-2, and for each $k, l \neq j$, $\rho_k = \rho_l = p$. Then, $g_j^*(\boldsymbol{\tau}) = 0$ implies that either $j \in N(p, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$ and $w_j \leq f(\sum \sigma_k \overline{x})$ or $j \notin N(p, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$. If the former holds, then there exists τ_j' such that $g_j^*(\tau_j', \boldsymbol{\tau}_{-j}) > 0$ under Rule 1-2. If the latter holds, then there exists τ_j' such that $\rho_j' \neq p$, $\sigma_j' = s_j$, $x_j' = \pi(\boldsymbol{x}_{-j}) < \overline{x}$, and $w_j' > \max\left\{f\left(\sum_{i \neq j} \sigma_i \bar{x} + \sigma_j' \bar{x}\right),\ \max\{w_i \mid i \neq j\}\right\}$. Such a deviation induces either Rule 1-2 or 1-3, and results in $g_j^*(\tau_j', \boldsymbol{\tau}_{-j}) > 0$. Suppose $\boldsymbol{\tau}$ induces Rule 1-2, and there exist $k, l \neq j$ such that $\rho_k \neq \rho_l$. Then, if j's deviation is such that $\rho_k \neq \rho_j' \neq \rho_l$, $\sigma_j' = s_j$, $x_j' = \pi(\boldsymbol{x}_{-j}) < \overline{x}$, and $w_j' > \max\left\{f\left(\sum_{i \neq j} \sigma_i \bar{x} + \sigma_j' \bar{x}\right),\ \max\{w_i \mid i \neq j\}\right\}$, then $g_j^*(\tau_j', \boldsymbol{\tau}_{-j}) > 0$ under Rule 1-3. Suppose $\boldsymbol{\tau}$ induces Rule 1-3. Then, there exists τ_j' such that $g_j^*(\tau_j', \boldsymbol{\tau}_{-j}) > 0$ under Rule 1-3. In summary, $\boldsymbol{\tau}$ induces neither Rule 1-2 nor 1-3.

Thus, $\boldsymbol{\tau}$ induces Rule 1-1, and $g^*(\boldsymbol{\tau}) = \boldsymbol{w}$. Then, there exists $\boldsymbol{u}' \in \mathcal{U}^n$ such that for each $i \in N$, $\rho_i = p \in \Delta^{SPI}(\boldsymbol{u}', \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$. Moreover, $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{w})$ and $(\boldsymbol{x}, \boldsymbol{w}) \in P(\boldsymbol{u}, \boldsymbol{s})$, because otherwise, some j has an incentive to deviate to Rule 1-2. Let $(\boldsymbol{u}', \boldsymbol{s}') \in \mathcal{E}$ be such that for each $i \in N$ with $x_i = 0$, $s'_i = \min\{\sigma_i, s_i\}$, and for each $i \in N$ with $x_i > 0$, $s'_i = \sigma_i\,(= s_i$ by Lemma 1). First, from the definition of $\boldsymbol{s}'$, $(\boldsymbol{x}, \boldsymbol{w}) \in P(\boldsymbol{u}', \boldsymbol{\sigma})$ implies $(\boldsymbol{x}, \boldsymbol{w}) \in P(\boldsymbol{u}', \boldsymbol{s}')$. Hence, from IUS, $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}', \boldsymbol{\sigma})$ implies $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}', \boldsymbol{s}')$. Next, from the definition of $\boldsymbol{s}'$, $(\boldsymbol{x}, \boldsymbol{w}) \in P(\boldsymbol{u}, \boldsymbol{s})$ implies $(\boldsymbol{x}, \boldsymbol{w}) \in P(\boldsymbol{u}, \boldsymbol{s}')$. Note here $p \in \Delta^{SPI}(\boldsymbol{u}', \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \cap \Delta^{P}(\boldsymbol{u}', \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{w})$ and $p \in \Delta^{P}(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{w}) \cap \Delta^{P}(\boldsymbol{u}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{w})$. Thus, from SPI, $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}', \boldsymbol{s}')$ implies $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}, \boldsymbol{s}')$. Finally, since $(\boldsymbol{x}, \boldsymbol{w}) \in P(\boldsymbol{u}, \boldsymbol{s})$ and $s'_i = s_i$ for each $i \in N$ with $x_i > 0$, $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}, \boldsymbol{s}')$ implies $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}, \boldsymbol{s})$ from IUS. ■

Lemma 3: *Let Assumption 1 hold. Then,* g^* implements *any Pareto subsolution* φ *satisfying SPI and IUS* in strong equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ be given. As $NA(g^*, \boldsymbol{e}) = \varphi(\boldsymbol{e})$, we only have to show that $NA(g^*, \boldsymbol{e}) \subseteq SNA(g^*, \boldsymbol{e})$. Suppose that there exists $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \in NE(g^*, \boldsymbol{e})$ such that for some $T \subseteq N$ with $2 \leq \#T < n$,[14] some $\boldsymbol{\tau}'_T = (\rho'_i, \sigma'_i, x'_i, w'_i)_{i \in T} \in (\Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^{\#T}$, and each $j \in T$, $u_j\left(x_j, g^*_j(\boldsymbol{\tau})\right) < u_j\left(x'_j, g^*_j\left(\boldsymbol{\tau}'_T, \boldsymbol{\tau}_{N \setminus T}\right)\right)$. Note that $\boldsymbol{\tau}$ induces Rule 1-1, and $g^*(\boldsymbol{\tau}) = \boldsymbol{w}$, as is shown in the proof of Lemma 2. Moreover, $(\boldsymbol{x}, \boldsymbol{w}) \in P(\boldsymbol{u}, \boldsymbol{s})$.

From the construction of g^*, there is at most one agent who obtains a positive share of output under Rules 1-2, 1-3, and Rule 2. Thus, from Assumption 1, the deviation by T induces Rule 1-1. Then, $g^*\left(\boldsymbol{\tau}'_T, \boldsymbol{\tau}_{N \setminus T}\right) = \left((w'_i)_{i \in T}, (w_k)_{k \in T^c}\right)$. Hence, $(\boldsymbol{x}, \boldsymbol{w}) \notin P(\boldsymbol{u}, \boldsymbol{s})$, a contradiction. Thus, $NA(g^*, \boldsymbol{e}) \subseteq SNA(g^*, \boldsymbol{e})$. ■

Lemma 4: *Let Assumption 1 hold. Then,* $\Gamma^{\boldsymbol{x} \circ \boldsymbol{m}}_{g^*}$ implements *any Pareto subsolution* φ *satisfying SPI and IUS* in subgame perfect equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ be given. From Lemma 2, we only have to show that $\varphi(\boldsymbol{e}) \subseteq SPA\left(\Gamma^{\boldsymbol{x} \circ \boldsymbol{m}}_{g^*}, \boldsymbol{e}\right)$.

First, we show that in each Stage 2 subgame, there is a Nash equilibrium. Let $NE\left(\Gamma^{\boldsymbol{x} \circ \boldsymbol{m}}_{g^*}(\boldsymbol{x}), \boldsymbol{e}\right)$ denote the set of Nash equilibria of $\left(\Gamma^{\boldsymbol{x} \circ \boldsymbol{m}}_{g}(\boldsymbol{x}), \boldsymbol{e}\right)$. Let $\boldsymbol{\mu}^* : [0, \bar{x}]^n \to \Delta^n \times \mathcal{S}^n \times \mathbb{R}^n_+$ be such that for each $\left(\Gamma^{\boldsymbol{x} \circ \boldsymbol{m}}_{g^*}(\boldsymbol{x}), \boldsymbol{e}\right)$, $\boldsymbol{\mu}^*(\boldsymbol{x}) =$

[14] For each $T \subseteq N$, $\#T$ denotes the number of agents in T.

$(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w})$, where for each $i \in N$

$$(\rho_i, \sigma_i, w_i) = \begin{cases} ((0,1), s_i, f(\sum s_k \bar{x}) + 1) & \text{if } x_i \neq \pi(\boldsymbol{x}_{-i}) \\ ((0,1), s_i, f(\sum s_k \bar{x}) + 2) & \text{otherwise.} \end{cases}$$

Note that $g^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$ corresponds to Rule 1-3. As $\rho_i = (0,1)$ for each $i \in N$, we have for any $(\rho'_j, \sigma'_j, x'_j, w'_j)$, $\varphi((0,1), (\sigma'_j, \boldsymbol{s}_{-j}), (x'_j, \boldsymbol{x}_{-j}), (w'_j, \boldsymbol{w}_{-j}))^{-1} = \varnothing$. This implies that no individual can induce Rule 1-1 by deviating. If $(\rho'_j, \sigma'_j, w'_j)$ induces Rule 1-2, j receives no output, because

$$j \notin N((0,1), (\sigma'_j, \boldsymbol{s}_{-j}), (x'_j, \boldsymbol{x}_{-j}), (w'_j, \boldsymbol{w}_{-j})).$$

If $(\rho'_j, \sigma'_j, w'_j)$ induces Rule 1-3, then $g^*_j(\rho'_j, \sigma'_j, w'_j, \boldsymbol{\rho}_{-j}, \boldsymbol{\sigma}_{-j}, \boldsymbol{x}, \boldsymbol{w}_{-j}) \leq g^*_j(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$. This is because whether $x_j = \pi(\boldsymbol{x}_{-j})$ holds or not is already fixed in Stage 1 game. If $(\rho'_j, \sigma'_j, w'_j)$ induces Rule 2, then $x_j > 0$, so that j receives no output. Thus, for any $\boldsymbol{x} \in [0, \overline{x}]^n$, $\boldsymbol{\mu}^*(\boldsymbol{x}) \in NE(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}(\boldsymbol{x}), \boldsymbol{e})$.

Now, we show that if $\widehat{\boldsymbol{z}} = (\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{y}}) \in \varphi(\boldsymbol{e})$, then there exists $(\boldsymbol{\mu}, \widehat{\boldsymbol{x}}) \in SPE(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}, \boldsymbol{e})$ such that $g^*(\boldsymbol{\mu}(\widehat{\boldsymbol{x}}), \widehat{\boldsymbol{x}}) = \widehat{\boldsymbol{y}}$. Define the strategy profile in $(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}, \boldsymbol{e})$ as follows.
(1) In Stage 1, each $i \in N$ supplies $\widehat{x}_i$.
(2) In Stage 2, $\boldsymbol{\mu} : [0, \bar{x}]^n \to \Delta^n \times \mathcal{S}^n \times \mathbb{R}^n_+$ is such that $\boldsymbol{\mu}(\boldsymbol{x}) = (\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w})$, which is defined as follows:
(2-1): if $\boldsymbol{x} = \widehat{\boldsymbol{x}}$ in Stage 1, then for some $p \in \Delta^{SPI}(\boldsymbol{u}, \boldsymbol{s}, \widehat{\boldsymbol{x}}, \widehat{\boldsymbol{y}})$ and each $i \in N$, $\mu_i(\boldsymbol{x}) = (p, s_i, \widehat{y}_i)$;
(2-2): if $\boldsymbol{x} = (x'_j, \widehat{\boldsymbol{x}}_{-j})$, where $x'_j \neq \widehat{x}_j$, in Stage 1, then for $j \in N$, $\mu_j(\boldsymbol{x}) = ((0,1), s_j, f(\sum s_k \bar{x}) + 1)$, and for some $p \in \Delta^{SPI}(\boldsymbol{u}, \boldsymbol{s}, \widehat{\boldsymbol{x}}, \widehat{\boldsymbol{y}})$ and each $i \neq j$,

$$\mu_i(\boldsymbol{x}) = \begin{cases} (p, s_i, \widehat{y}_i) & \text{if } x_i \neq \pi(\boldsymbol{x}_{-i}), \\ ((1,0), s_i, f(\sum s_k \bar{x}) + 2) & \text{otherwise;} \end{cases}$$

(2-3): in any other case, $\boldsymbol{\mu}(\boldsymbol{x}) = \boldsymbol{\mu}^*(\boldsymbol{x})$.

If $(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}(\boldsymbol{x}), \boldsymbol{e})$ corresponds to (2-1), then $\boldsymbol{\mu}(\boldsymbol{x}) \in NE(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}(\boldsymbol{x}), \boldsymbol{e})$. This is because $(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{y}}) \in NA(g^*, \boldsymbol{e})$ by Lemma 2. Also, by the above argument, $\boldsymbol{\mu}(\boldsymbol{x}) \in NE(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}(\boldsymbol{x}), \boldsymbol{e})$ in the subgame (2-3) of Stage 2. Suppose that $(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}(\boldsymbol{x}), \boldsymbol{e})$ corresponds to (2-2). Then, $g^*(\boldsymbol{\mu}(\boldsymbol{x}), \boldsymbol{x})$ does not correspond to Rule 1-1. If for each $i \neq j$, $x_i \neq \pi(\boldsymbol{x}_{-i})$, then $g^*(\boldsymbol{\mu}(\boldsymbol{x}), \boldsymbol{x})$ corresponds to Rule 1-2. Then, $\{j\} = N(p, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$. Suppose an agent $h \in N$ switches to $(\rho'_h, \sigma'_h, w'_h)$. If $h \neq j$, then h induces only Rule 1-3 or Rule 2 and receives no output. This is because $x_h \neq \pi(\boldsymbol{x}_{-h})$ is already fixed in Stage 1, and inducing Rule 2 implies $x_h > 0$. If $h = j$, then j induces only Rule 1-1 or Rule 1-2. In either case, as in the proof (1) of Lemma 2, it is shown that j receives no more than $\widehat{y}_j + \frac{p_x}{p_y}\left(s_j x'_j - s_j \widehat{x}_j\right)$. Finally, suppose $g^*\left(\boldsymbol{\mu}\left(\boldsymbol{x}\right), \boldsymbol{x}\right)$ corresponds to Rule 1-3. Then, there exists $l \in N \setminus \{j\}$ such that $x_l = \pi\left(\boldsymbol{x}_{-l}\right)$. Thus, $g^*_l\left(\boldsymbol{\mu}\left(\boldsymbol{x}\right), \boldsymbol{x}\right) = f\left(\sum s_k x_k\right)$. Then, if an agent $h \neq l$ switches to any $(\rho'_h, \sigma'_h, w'_h)$, then h receives no output. Also if l deviates, he receives at most $f\left(\sum s_k x_k\right)$ by such a deviation. Since $\boldsymbol{x}$ is already fixed in Stage 1, the above arguments imply $\boldsymbol{\mu}\left(\boldsymbol{x}\right) \in NE\left(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}(\boldsymbol{x}), \boldsymbol{e}\right)$ in the subgame (2-2) of Stage 2.

We show that $(\boldsymbol{\mu}, \widehat{\boldsymbol{x}}) \in SPE\left(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}, \boldsymbol{e}\right)$. In accordance with (1)-(2-1) of $(\boldsymbol{\mu}, \widehat{\boldsymbol{x}})$, $g^*\left(\boldsymbol{\mu}\left(\widehat{\boldsymbol{x}}\right), \widehat{\boldsymbol{x}}\right) = \widehat{\boldsymbol{y}}$. Suppose that an agent j deviates from $\widehat{x}_j$ to $x'_j \neq \widehat{x}_j$ in Stage 1. Then, from (2-2), $g^*_j\left(\boldsymbol{\mu}\left(x'_j, \widehat{\boldsymbol{x}}_{-j}\right), \left(x'_j, \widehat{\boldsymbol{x}}_{-j}\right)\right) \leq \widehat{y}_j + \frac{p_x}{p_y}\left(s_j x'_j - s_j \widehat{x}_j\right)$. Thus, since $\widehat{\boldsymbol{z}} \in P\left(\boldsymbol{e}\right)$, $\widehat{\boldsymbol{z}} \in SPA\left(\Gamma^{\boldsymbol{x}\circ\boldsymbol{m}}_{g^*}, \boldsymbol{e}\right)$. ■

Lemma 5: *Let Assumption 1 hold. Then,* $\Gamma^{\boldsymbol{m}\circ\boldsymbol{x}}_{g^*}$ implements *any Pareto subsolution* φ *satisfying SPI and IUS* in subgame perfect equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ be given. From Lemma 2, we only show $\varphi\left(\boldsymbol{e}\right) \subseteq SPA\left(\Gamma^{\boldsymbol{m}\circ\boldsymbol{x}}_{g^*}, \boldsymbol{e}\right)$.

First, we show that in each Stage 2 subgame, there exists a Nash equilibrium. Let $NE\left(\Gamma^{\boldsymbol{m}\circ\boldsymbol{x}}_{g^*}\left(\boldsymbol{m}\right), \boldsymbol{e}\right)$ denote the set of Nash equilibria of $\left(\Gamma^{\boldsymbol{m}\circ\boldsymbol{x}}_{g^*}\left(\boldsymbol{m}\right), \boldsymbol{e}\right)$. Let $I\left(p, \boldsymbol{\sigma}, \boldsymbol{0}, \boldsymbol{w}\right) \equiv \left\{i \in N \mid \exists x'_i \text{ s.t. } \varphi\left(p, \boldsymbol{\sigma}, \left(x'_i, \boldsymbol{0}_{-i}\right), \boldsymbol{w}\right)^{-1} \neq \varnothing\right\}$. Let $\boldsymbol{\chi}^* : \Delta^n \times \mathcal{S}^n \times \mathbb{R}^n_+ \to [0, \bar{x}]^n$ be such that for each $\left(\Gamma^{\boldsymbol{m}\circ\boldsymbol{x}}_{g^*}\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right), \boldsymbol{e}\right)$, $\boldsymbol{\chi}^*\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) \in [0, \bar{x}]^n$ is: for each $i \in N$,

(i) if $\sigma_i = s_i$, and there exists p such that for each $j \in N$, $\rho_j = p$ and $i = \min I\left(p, \boldsymbol{\sigma}, \boldsymbol{0}, \boldsymbol{w}\right)$, then $\chi^*_i\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) = x'_i$ such that $\varphi\left(p, \boldsymbol{\sigma}, \left(x'_i, \boldsymbol{0}_{-i}\right), \boldsymbol{w}\right)^{-1} \neq \varnothing$;

(ii) if $\sigma_i = s_i$, $w_i > f\left(\sum \sigma_k \bar{x}\right)$, and there exists p such that for each $j \neq i$, $\rho_j = p$ and $i \in N\left(p, \boldsymbol{\sigma}, \boldsymbol{0}, \boldsymbol{w}\right)$, then for (x'_i, w'_i) with $\varphi\left(p, \boldsymbol{\sigma}, \left(x'_i, \boldsymbol{0}_{-i}\right), \left(w'_i, \boldsymbol{w}_{-i}\right)\right)^{-1} \neq \varnothing$,

$\chi^*_i\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) = \arg\max_{x_i} u_i\left(x_i, \max\left\{0, \min\left\{w'_i + \frac{p_x}{p_y}\sigma_i\left(x_i - x'_i\right), f\left(\sigma_i x_i\right)\right\}\right\}\right)$;

(iii) if $\sigma_i = s_i$, $w_i > \max\left\{f\left(\sum \sigma_k \bar{x}\right), \max\left\{w_j \mid j \neq i\right\}\right\}$, and $\left[\left\{\exists p \text{ s.t. } \rho_j = p \ \left(\forall j \neq i\right)\right\} \Rightarrow i \notin N\left(p, \boldsymbol{\sigma}, \boldsymbol{0}, \boldsymbol{w}\right)\right]$, then $\chi^*_i\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) = \frac{\bar{x}}{2}$;

(iv) otherwise, $\chi^*_i\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) = 0$.

To simplify the notation, let us use $\boldsymbol{x}^*$ to denote $\boldsymbol{\chi}^*\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right)$ in the following discussion. Since $x_i = 0$ for each i with $\sigma_i \neq s_i$, $g^*\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}^*, \boldsymbol{w}\right)$ corresponds to one of the subrules of Rule 1. Suppose an agent h switches to x'_h.

If x'_h induces Rule 2, then $x'_h > 0$, so $g^*_h\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \left(x'_h, \boldsymbol{x}^*_{-h}\right), \boldsymbol{w}\right) = 0$. Suppose that $g^*\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}^*, \boldsymbol{w}\right)$ corresponds to Rule 1-1. Then, as $(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w})$ is already fixed, no unilateral deviation from $\boldsymbol{x}^*$ can induce Rule 1-1. Moreover, $w_i \leq f\left(\sum \sigma_k \bar{x}\right)$ for any i. Thus, if x'_h induces Rule 1-2 or Rule 1-3, then h receives no output. Suppose that $g^*\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}^*, \boldsymbol{w}\right)$ corresponds to Rule 1-2. Then, there exists $j \in N\left(p, \boldsymbol{\sigma}, \boldsymbol{x}^*, \boldsymbol{w}\right)$ such that for each $i \neq j$, $\rho_i = p$. This implies $w_i \leq f\left(\sum \sigma_k \bar{x}\right)$ for each $i \neq j$. Thus, under Rule 1-2, for each $i \neq j$, χ^*_i corresponds to the case (iv) solely, whereas for $j \in N\left(p, \boldsymbol{\sigma}, \boldsymbol{x}^*, \boldsymbol{w}\right)$, χ^*_j corresponds to the case (ii). Hence, if $h \neq j$ and the deviation x'_h induces Rule 1-2, Rule 1-3, or Rule 2, then $g^*_h\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \left(x'_h, \boldsymbol{x}^*_{-h}\right), \boldsymbol{w}\right) = 0$. Moreover, as $w_j > f\left(\sum \sigma_k \bar{x}\right)$, for any $h \in N$, x'_h cannot induce Rule 1-1. If $h = j$, then $h \in N\left(p, \boldsymbol{\sigma}, \left(x'_h, \boldsymbol{x}^*_{-h}\right), \boldsymbol{w}\right)$. Thus, x'_h cannot induce Rule 1-3. Moreover, since (ii) of χ^*_j is the best response for j, if $h = j$ and x'_h induces Rule 1-2, then $g^*_h\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \left(x'_h, \boldsymbol{x}^*_{-h}\right), \boldsymbol{w}\right) \leq g^*_h\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}^*, \boldsymbol{w}\right)$. Suppose that $g^*\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}^*, \boldsymbol{w}\right)$ corresponds to Rule 1-3. Then, $\boldsymbol{x}^*$ is either $x^*_j = \frac{\overline{x}}{2}$ and $x^*_i = 0$ for each $i \neq j$, or $\boldsymbol{x}^* = \mathbf{0}$. If the latter holds, then for each $i \in N$, χ^*_i corresponds to the case (iv). Then, for any x'_h, $g^*_h\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \left(x'_h, \boldsymbol{x}^*_{-h}\right), \boldsymbol{w}\right) = 0$. If the former holds, then there exists $j \in N$ whose χ^*_j corresponds to the case (iii). Note that $x^*_j = \frac{\overline{x}}{2}$ is the best response for $j \in N$ to $\boldsymbol{x}^*_{-j} = \mathbf{0}_{-j}$. If $h \neq j$, then no x'_h induces Rule 1-1, since $w_j > f\left(\sum s_k \bar{x}\right)$. Moreover, if x'_h induces Rule 1-2, Rule 1-3, or Rule 2, then $g^*_h\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \left(x'_h, \boldsymbol{x}^*_{-h}\right), \boldsymbol{w}\right) = 0$. Thus, in summary, $\boldsymbol{\chi}^*\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) \in NE\left(\Gamma^{\boldsymbol{m}\circ\boldsymbol{x}}_{g^*}\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right), \boldsymbol{e}\right)$ holds.

Now, we show that for $\boldsymbol{e} \in \mathcal{E}$, if $\widehat{\boldsymbol{z}} = (\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{y}}) \in \varphi(\boldsymbol{e})$, then there exists $(\boldsymbol{\rho}, \boldsymbol{s}, \widehat{\boldsymbol{y}}, \boldsymbol{\chi}) \in SPE\left(\Gamma^{\boldsymbol{m}\circ\boldsymbol{x}}_{g^*}, \boldsymbol{e}\right)$ such that $g^*\left(\boldsymbol{\rho}, \boldsymbol{s}, \widehat{\boldsymbol{y}}, \boldsymbol{\chi}\left(\boldsymbol{\rho}, \boldsymbol{s}, \widehat{\boldsymbol{y}}\right)\right) = \widehat{\boldsymbol{y}}$. Define a strategy profile in $\left(\Gamma^{\boldsymbol{m}\circ\boldsymbol{x}}_{g^*}, \boldsymbol{e}\right)$ as follows.

(1) In Stage 1, for $p \in \Delta^{SPI}\left(\boldsymbol{u}, \boldsymbol{s}, \widehat{\boldsymbol{x}}, \widehat{\boldsymbol{y}}\right)$, each $i \in N$ announces $(\rho_i, \sigma_i, w_i) = (p, s_i, \widehat{y}_i)$.

(2) In Stage 2, $\boldsymbol{\chi} : \Delta^n \times \mathcal{S}^n \times \mathbb{R}^n_+ \to [0, \bar{x}]^n$ is given as follows:

(2-1): if $(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}) = (\boldsymbol{\rho}, \boldsymbol{s}, \widehat{\boldsymbol{y}})$ is such that for each $i \in N$, $\rho_i = p$ in Stage 1, then each $i \in N$ supplies $\chi_i\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) = \widehat{x}_i$;

(2-2): if $(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}) = \left(\left(\rho'_j, \boldsymbol{\rho}_{-j}\right), \boldsymbol{s}, \left(w'_j, \widehat{\boldsymbol{y}}_{-j}\right)\right)$ is such that for each $i \neq j$, $\rho_i = p$ and $w'_j > f\left(\sum s_k \bar{x}\right)$ in Stage 1, then for $j \in N$,

$$\chi_j\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) = \arg\max_{x'_j} u_j\left(x'_j, \min\left\{\widehat{y}_j + \tfrac{p_x}{p_y} s_j\left(x'_j - \widehat{x}_j\right), f\left(\textstyle\sum_{i \neq j} s_i \widehat{x}_i + s_j x'_j\right)\right\}\right),$$

and for each $i \neq j$, $\chi_i\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) = \widehat{x}_i$;

(2-3): if $(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}) = \left(\left(\rho'_j, \boldsymbol{\rho}_{-j}\right), \boldsymbol{s}, \left(w'_j, \widehat{\boldsymbol{y}}_{-j}\right)\right)$ is such that for each $i \neq j$, $\rho_i = p$, $\left(\rho'_j, w'_j\right) \neq (p, \widehat{y}_j)$, and $w'_j \leq f\left(\sum s_k \bar{x}\right)$ in Stage 1, then $\boldsymbol{\chi}\left(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}\right) = \overline{\boldsymbol{x}}$;

(2-4): if $(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}) = \left(\left(\rho_j', \boldsymbol{\rho}_{-j}\right), \left(s_j', \boldsymbol{s}_{-j}\right), \left(w_j', \widehat{\boldsymbol{y}}_{-j}\right)\right)$ is such that for each $i \neq j$, $\rho_i = p$ and $s_j' \neq s_j$ in Stage 1, then for $j \in N$, $\chi_j(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}) = \frac{\overline{x}}{2}$, and for each $i \neq j$, $\chi_i(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}) = 0$;
(2-5): in any other case, $\boldsymbol{\chi}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}) = \boldsymbol{\chi}^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w})$.

To simplify the notation, let us use $\boldsymbol{x}$ to denote $\boldsymbol{\chi}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w})$ in the following discussion. If $\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}), \boldsymbol{e}\right)$ corresponds to (2-1), then $\boldsymbol{x} \in NE\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}), \boldsymbol{e}\right)$. This is because $(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{y}}) \in NA(g^*, \boldsymbol{e})$ by Lemma 2. Also, by the above argument, $\boldsymbol{x} \in NE\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}), \boldsymbol{e}\right)$ in the subgame (2-5) of Stage 2.

Suppose in $\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}), \boldsymbol{e}\right)$, an agent $h \in N$ switches to x_h'. If $\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}), \boldsymbol{e}\right)$ corresponds to (2-2), then $g^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$ corresponds to Rule 1-2. Then, if $h \neq j$, then x_h' cannot induce Rule 1-1 or Rule 2. Moreover, by $w_h = \widehat{y}_h \leq f\left(\sum s_k \bar{x}\right)$, if x_h' induces Rule 1-2 or Rule 1-3, then $g_h^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, (x_h', \boldsymbol{x}_{-h}), \boldsymbol{w}) = 0$. If $h = j$, then x_h' cannot induce Rule 1-1, Rule 1-3, or Rule 2. Thus, $g_h^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, (x_h', \boldsymbol{x}_{-h}), \boldsymbol{w}) \leq g_h^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$. If $\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}), \boldsymbol{e}\right)$ corresponds to (2-3), then $g^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$ corresponds to Rule 1-3, because $\boldsymbol{x} = \overline{\boldsymbol{x}}$. Then, since for any $h \in N$, $w_h \leq f\left(\sum s_k \bar{x}\right)$, if x_h' induces Rule 1-2 or Rule 1-3, then $g_h^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, (x_h', \boldsymbol{x}_{-h}), \boldsymbol{w}) = 0$. Moreover, x_h' cannot induce Rule 1-1 or Rule 2. If $\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}), \boldsymbol{e}\right)$ corresponds to (2-4), then $g^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$ corresponds to Rule 2. If $h \neq j$, then x_h' can induce Rule 2 only, and $g_h^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, (x_h', \boldsymbol{x}_{-h}), \boldsymbol{w}) = 0$. If $h = j$, then $x_h' = 0$ can induce Rule 1-3. Moreover, $x_h' > 0$ induces Rule 2. In any case, $g_h^*(\boldsymbol{\rho}, \boldsymbol{\sigma}, (x_h', \boldsymbol{x}_{-h}), \boldsymbol{w}) = 0$. Thus, in summary, $\boldsymbol{x} \in NE\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{w}), \boldsymbol{e}\right)$.

We show that $(\boldsymbol{\rho}, \boldsymbol{s}, \widehat{\boldsymbol{y}}, \boldsymbol{\chi}) \in SPE\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right)$. In accordance with (1)-(2-1) of $(\boldsymbol{\rho}, \boldsymbol{s}, \widehat{\boldsymbol{y}}, \boldsymbol{\chi})$, $g^*(\boldsymbol{\rho}, \boldsymbol{s}, \widehat{\boldsymbol{y}}, \boldsymbol{\chi}) = \widehat{\boldsymbol{y}}$. Suppose that an agent j deviates from $(p, s_j, \widehat{y}_j)$ to $\left(\rho_j', s_j', w_j'\right)$ in Stage 1. If $s_j' = s_j$, then, from (2-2) and (2-3), $g_j^*\left(\left(\rho_j', s_j', w_j', \boldsymbol{\rho}_{-j}, \boldsymbol{s}_{-j}, \widehat{\boldsymbol{y}}_{-j}\right), \boldsymbol{\chi}\left(\rho_j', s_j', w_j', \boldsymbol{\rho}_{-j}, \boldsymbol{s}_{-j}, \widehat{\boldsymbol{y}}_{-j}\right)\right) \leq \widehat{y}_j + \frac{p_x}{p_y} s_j(x_j - \widehat{x}_j)$, where $x_j = \chi_j\left(\rho_j', s_j', w_j', \boldsymbol{\rho}_{-j}, \boldsymbol{s}_{-j}, \widehat{\boldsymbol{y}}_{-j}\right)$. If $s_j' \neq s_j$, then, from (2-4), j receives no output under Rule 2. Thus, $\widehat{\boldsymbol{z}} \in SPA\left(\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}, \boldsymbol{e}\right)$. ■

Proof of Theorem 2. Let φ be a Pareto subsolution satisfying SPI and IUS. From Lemmas 2, 3, 4, and 5, $\Gamma_{g^*}^{\boldsymbol{m}\circ\boldsymbol{x}}$ (resp. $\Gamma_{g^*}^{\boldsymbol{x}\circ\boldsymbol{m}}$) triply implements φ in Nash, strong, and subgame perfect equilibria. Moreover, g^* is forthright, as is shown in the proof of Lemma 2. Thus, it suffices to show that g^* meets (iii) and (iv) of Definition 2.

1. Definition 2 (iii). The proof of Lemma 2 shows that if $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \in NE(g^*, \boldsymbol{e})$, then Rule 1-1 applies and $g^*(\boldsymbol{\tau}) = \boldsymbol{w}$. Given $j \in N$ and

$\tau_j' = \left(\rho_j', \sigma_j', x_j', w_j'\right) \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$, if $\left(\tau_j', \boldsymbol{\tau}_{-j}\right)$ induces Rules 1-1 or 1-2, then $g_j^*\left(\tau_j', \boldsymbol{\tau}_{-j}\right) \leq y_j + \frac{p_x}{p_y} s_j\left(x_j' - x_j\right)$. If $\left(\tau_j', \boldsymbol{\tau}_{-j}\right)$ induces Rule 1-3, then either (i) $\varphi\left(\rho_j', \left(\sigma_j', \boldsymbol{\sigma}_{-j}\right), \left(x_j', \boldsymbol{x}_{-j}\right), \left(w_j', \boldsymbol{w}_{-j}\right)\right) \neq \varnothing$ and $j \in N\left(p, \left(\sigma_j', \boldsymbol{\sigma}_{-j}\right), \left(x_j', \boldsymbol{x}_{-j}\right), \left(w_j', \boldsymbol{w}_{-j}\right)\right)$; or (ii) $j \notin N\left(p, \left(\sigma_j', \boldsymbol{\sigma}_{-j}\right), \left(x_j', \boldsymbol{x}_{-j}\right), \left(w_j', \boldsymbol{w}_{-j}\right)\right)$, where $p = \rho_k$ for each $k \neq j$. Case (i) implies $g_j^*\left(\tau_j', \boldsymbol{\tau}_{-j}\right) = 0$ by $w_j' \leq f\left(\sum s_k \bar{x}\right)$. Consider case (ii). Since $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{e})$ and $p \in \Delta^{SPI}(\boldsymbol{e}, \boldsymbol{x}, \boldsymbol{w})$ from Lemma 2, if $\sigma_j' = s_j$, then $j \in N\left(p, \left(\sigma_j', \boldsymbol{\sigma}_{-j}\right), \left(x_j', \boldsymbol{x}_{-j}\right), \left(w_j', \boldsymbol{w}_{-j}\right)\right)$. Thus, $\sigma_j' \neq s_j$. This implies $x_j' = 0$, so that $g_j^*\left(\tau_j', \boldsymbol{\tau}_{-j}\right) = 0$. If $\left(\tau_j', \boldsymbol{\tau}_{-j}\right)$ induces Rule 2, then $x_j' > 0$ by Lemma 1 and $g_j^*\left(\tau_j', \boldsymbol{\tau}_{-j}\right) = 0$. Thus, g^* meets Definition 2 (iii).

2. Definition 2 (iv). Note again that if $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \in NE(g^*, \boldsymbol{e})$, then $\boldsymbol{\tau}$ induces Rule 1-1, or there exists $\boldsymbol{u} \in \mathcal{U}^n$ such that $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}, \boldsymbol{\sigma})$ and for each $i \in N$, $\rho_i = p \in \Delta^{SPI}(\boldsymbol{u}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$. Moreover, if $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}, \boldsymbol{\sigma})$ for some $\boldsymbol{u} \in \mathcal{U}^n$, then $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$ implies that for each $\boldsymbol{\sigma}' \in \mathcal{S}^n$ such that $\sigma_i' = \sigma_i$ for each $i \in N$ with $x_i > 0$, there exists some $\boldsymbol{u}' \in \mathcal{U}^n$ such that $p \in \Delta^P(\boldsymbol{u}', \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \cap \Delta^P(\boldsymbol{u}', \boldsymbol{\sigma}', \boldsymbol{x}, \boldsymbol{w})$. By SPI, $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}, \boldsymbol{\sigma})$ and $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \cap \Delta^P(\boldsymbol{u}', \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w})$ together imply $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}', \boldsymbol{\sigma})$. By IUS, $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}', \boldsymbol{\sigma})$ and $p \in \Delta^P(\boldsymbol{u}', \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{w}) \cap \Delta^P(\boldsymbol{u}', \boldsymbol{\sigma}', \boldsymbol{x}, \boldsymbol{w})$ together imply $(\boldsymbol{x}, \boldsymbol{w}) \in \varphi(\boldsymbol{u}', \boldsymbol{\sigma}')$. Thus, $(\boldsymbol{\rho}, \boldsymbol{\sigma}', \boldsymbol{x}, \boldsymbol{w})$ induces Rule 1-1. Hence, $g^*(\boldsymbol{\rho}, \boldsymbol{\sigma}', \boldsymbol{x}, \boldsymbol{w}) = g^*(\boldsymbol{\tau}) = \boldsymbol{w}$, and $(\boldsymbol{\rho}, \boldsymbol{\sigma}', \boldsymbol{x}, \boldsymbol{w}) \in NE(g^*, \boldsymbol{e})$. ■

6.3 Proofs of Lemma 6 in Section 4

Proof of Lemma 6. Let $(\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ be such that $(\boldsymbol{x}, \boldsymbol{y}) \in W^\theta(\boldsymbol{u}, \boldsymbol{s})$. Let $p \in \Delta$ be a competitive equilibrium price for $(\boldsymbol{x}, \boldsymbol{y})$ at $(\boldsymbol{u}, \boldsymbol{s})$. Let $(\boldsymbol{u}, \boldsymbol{s}') \in \mathcal{E}$ be such that for each $i \in N$ with $x_i > 0$, $s_i' = s_i$, and $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{y})$. Then, by the definition of $\Delta^P(\boldsymbol{u}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{y})$ and strict monotonicity of utility functions, **(i)** for each $\boldsymbol{z}' \in Z(\boldsymbol{s}')$, $\sum\left(p_y y_i' - p_x s_i' x_i'\right) \leq \sum\left(p_y y_i - p_x s_i' x_i\right)$; and **(ii)** for each $i \in N$ and each $(x, y) \in Z$, if $u_i(x, y) > u_i(z_i)$, then $p_y y - p_x s_i' x > \theta_i \sum\left(p_y y_k - p_x s_k' x_k\right)$. Therefore, $(\boldsymbol{x}, \boldsymbol{y}) \in W^\theta(\boldsymbol{u}, \boldsymbol{s}')$. Thus, W^θ satisfies IUS. Similarly, EB also satisfies IUS.

Let $(\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ be such that $(\boldsymbol{x}, \boldsymbol{y}) \in PR(\boldsymbol{u}, \boldsymbol{s})$. Let $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$. Let $(\boldsymbol{u}, \boldsymbol{s}') \in \mathcal{E}$ be such that for each $i \in N$ with $x_i > 0$, $s_i' = s_i$, and $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{y})$. Then, as $(\boldsymbol{x}, \boldsymbol{y}) \in PR(\boldsymbol{u}, \boldsymbol{s})$, for each $i \in N$, $y_i = \frac{s_i x_i}{\sum s_k x_k} \sum y_k = \frac{s_i x_i}{\sum_{j \in N, x_j > 0} s_j x_j} \sum y_j = \frac{s_i' x_i}{\sum_{j \in N, x_j > 0} s_j' x_j} \sum y_j = \frac{s_i' x_i}{\sum s_k' x_k} \sum y_k$. Therefore, $(\boldsymbol{x}, \boldsymbol{y}) \in PR(\boldsymbol{u}, \boldsymbol{s}')$. Thus, PR satisfies IUS. ■

7 References

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