Exploitation as the Unequal Exchange of Labour: An Axiomatic Approach*

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Abstract

In subsistence economies with general convex technology and rational optimising agents, a new, axiomatic approach is developed, which allows an explicit analysis of the core positive and normative intuitions behind the concept of exploitation. Three main new axioms, called **Labour Exploitation in Subsistence Economies**, **Relational Exploitation**, and **Feasibility of Non-Exploitation**, are presented and it is proved that they uniquely characterise a definition of exploitation conceptually related to the so-called "New-Interpretation" (Duménil, 1980; Foley, 1982; Duménil at el., 2009), which focuses on the unequal distribution of (and control over) social labour, and on

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individual well-being freedom and the self-realisation of men. Then, the main results of Roemer's (1982, 1988) classical approach and all the crucial insights of exploitation theory are generalised, proving that every agent's class and exploitation status emerges in the competitive equilibrium, that there is a correspondence between an agent's class and exploitation status, and that the existence of exploitation is inherently linked to the existence of positive profits.

JEL: D63 (Equity, Justice, Inequality, and Other Normative Criteria and Measurement); D51 (General Equilibrium: Exchange and Production Economies); C62 (Existence and Stability Conditions of Equilibrium); B51 (Socialist; Marxist, Sraffian).

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1 Introduction

The notion of exploitation is prominent in the social sciences and in political discourse. It is central in a number of debates, ranging from analyses of labour relations, especially focusing on the weakest segments of the labour force, such as children, women, and migrants (see, e.g., ILO, 2005; 2005a; 2006); to controversies on drug-testing and on the price of life-saving drugs, especially in developing countries;[1] to ethical issues arising in surrogate motherhood (see, e.g., Field, 1989; Wood, 1995). The concept of exploitation is also central in the politics of the Left. In the 2007 programme of the German Social Democratic Party, for example, the very first paragraph advocates a society 'free from poverty, exploitation, and fear' (SPD, 2007, p.3), and the fight against exploitation is repeatedly indicated as a priority for the biggest party of the European Left. The notion of exploitation is arguably the cornerstone of Marxist social theory, but it is also extensively discussed in normative theory and political philosophy (see, e.g., Wertheimer, 1996; Wolff, 1999; Bigwood, 2003; and Sample, 2003).[2] Yet, there is little agreement concerning even the most basic features of exploitative relations, and both the definition of exploitation and its normative content are highly controversial.[3]

In general, agent A exploits agent B if and only if A takes unfair advantage of B. Despite its intuitive appeal, this definition leaves two major issues in need of precise specification from a normative perspective, namely the source of the unfairness and the structure of the relationship between A and B that allows A to take advantage of B. There is considerable debate

[1] In a section devoted to 'Ethical Issues,' the Investigation Committee on the clinical trial of the drug 'Trovan' conducted by Pfizer in 1996 in Kano (Nigeria) argue that 'Compensations to the participants were minimal or non existent, as such a clear case of exploitation of the ignorant was established' (Federal Ministry of Health of Nigeria, 2001, p.88).

[2] The notion of exploitation is relevant, for example, in Lockean or Neo-Lockean political philosophy, whereby exploitation occurs if the principle of Just Acquisition of unowned land is violated by State intervention; or in Neoclassical economic philosophy, whereby exploitation occurs in non-competitive markets if the distributive principle of marginal productivity is violated.

[3] For a review of some of the debates in exploitation theory, see Nielsen and Ware (1997).

in the economic and philosophical literature concerning both issues. At one extreme, in his seminal theory of exploitation, John Roemer (1982, 1988) argues that exploitation is a purely distributive concept which makes no reference to the interaction between agents and identifies an injustice stemming from an unequal distribution of assets. At the other extreme, *contra* Roemer, various authors deny that exploitation involves a distributive injustice and claim that the moral force of exploitation derives *entirely* from the objectionable features of the interaction between agents (e.g., Wolff, 1999; Wood, 2004).

As shown by Veneziani (2008), and as acknowledged by Roemer himself in later contributions, Roemer's claim that exploitation is a purely distributive concept is not convincing.[4] Some notion of unequal power, or dominance, is arguably crucial in any theory of exploitation and the positive and normative analysis of exploitative relations involves some consideration of the way in which A and B interact.[5] It seems, however, equally reductive to assume that inequalities deriving from exploitative relations between agents are immaterial in the judgment of exploitation. As forcefully argued by Warren (1997, p.63), "exploitation involves inequality on both ends of exchange: inequality defining the context of the exchange (that is, [differential ownership of productive assets]) and inequality defining the outcome."[6] So, although power, force, or dominance, are arguably crucial elements of exploitation theory, the analytical focus on this paper is on the source of the unfairness of exploitative relations, and on the injustice involved in the concept of exploitation within economic relations.

More specifically, this paper analyses the theory of *exploitation as an unequal exchange* (hereafter, UE) *of labour*, according to which exploitative relations are characterised by a difference between the hours of labour that an individual provides and the hours of labour necessary to produce commodities that she can purchase with her income. There are at least two reasons

[4] For a thorough analysis of Roemer's distributive approach, see Skillman (1995). Skillman (1995) argues that Roemer's approach is consistent with Marx's own account.

[5] In fact, Yoshihara (1998) showed that exploitative status is linked with the degree of labour-discipline, which reflects power relations in capitalist economies.

[6] In this perspective, "it is not unequal power itself that is supposed objectionable, but rather the fact that one person gains unjustly through the exercise of power (whether coercive or uncoercive) over another" (Warren, 1997, p.62). According to Warren, the relevant outcome inequality concerns indeed the unequal performance of labor, as suggested also in this paper.

to focus on *labour* as the measure of the injustice of exploitative relations. First, in a number of crucial economic interactions, the notion of exploitation seems inextricably linked with some form of labour exchange.[7] Second, the UE definition of exploitation captures some inequalities in the distribution of material well-being and free hours that are - at least *prima facie* - of normative relevance. As shown in this paper, for example, it is possible to significantly generalise the so-called *Class-Exploitation Correspondence Principle* (hereafter, CECP; see Roemer, 1982, 1988), according to which in a private-ownership economy with positive profits, class and UE exploitation status are strictly related, and they accurately reflect an unequal distribution of assets. That is, in equilibrium, the wealthiest agents emerge as exploiters, and members of the capitalist class, whereas poor agents are exploited, and members of the working class. As in standard Marxist theory, then, exploitative relations are relevant in that they reflect unequal opportunities of life options, due to unequal access to productive assets.

Interestingly, however, the UE concept of exploitation can also be seen as capturing inequalities in the distribution of *well-being freedom*. In fact, as argued by Rawls (1971) and Sen (1985a, 1985b), among the others, an individual's well-being freedom captures her ability to pursue the life she values.[8] [9] There are two crucial factors that determine the degree of individual's well-being freedom, or self-realisation: one is the amount of income she can spend to purchase the commodities necessary to achieve her goals, and the other is the amount of time she has to sacrifice as labour supply in order to purchase such commodities.[10] Then, the rate of labour exploitation can represent the degree of well-being freedom, or indeed *un*freedom, of the

[7]Despite his initial criticism of the UE definition, Roemer has later acknowledged that the exchange of labour is essential in explotiative relations, and "the expenditure of effort is characteristically associated with exploitation" (Roemer, 1989b, fn.11). See also Roemer (1989a, pp. 258-260; 1989b, p. 96).

[8]Interestingly, this notion of freedom is conceptually related to the Marxian notion of *self-realisation*, and one can argue that both commodities and free time are essential for an individual's self-realisation.

[9]In the Rawls-Sen theory, inequalities in the distribution of well-being freedom are formulated as inequalities of capabilities. The resource allocation problem, in terms of equality of capability, is explicitly analysed in Gotoh and Yoshihara (2003).

[10]In this view, labour only yields disutility and it reduces the possibility to self-realise. To assume away completely the possibility that work itself may be a source of well-being freedom may be unrealistic, but it is appropriate at the level of analysis of this paper, and it is consistent with a Marxian analysis of capitalist relations of production.

agent, since it can be taken as an index of the relative attainment of these two goods by using labour time as the *numéraire*: if an agent gains from the unequal exchange of labour, then she is *exploiting* the free hours which some other agents sacrificed as labour supply for the production of the commodities she can purchase, whereas if she suffers from the unequal exchange of labour, she is *exploited* in the sense that some of the free hours she sacrificed as labour supply to purchase the commodities are appropriated by somebody else.

Granting the normative relevance of the unequal exchange of labour, there are many possible ways of rigorously specifying the concept of UE exploitation and a number of alternative definitions have indeed been proposed in the literature (for a thorough discussion, see Yoshihara, 2007). This paper provides the first rigorous axiomatic analysis of UE exploitation: this is a completely new approach to exploitation theory and it provides a fully general framework to compare the most important definitions of exploitation discussed in the literature. An axiomatic approach was long overdue in exploitation theory, where the proposal of alternative definitions have sometimes appeared as a painful process of adjustment of the theory to the various counterexamples and formal exceptions found in the literature. The definitions of exploitation thus constructed have progressively lost the intuitive appeal, the normative relevance, and even the connection with the actual, observed variables emerging from a competitive mechanism. By taking an axiomatic approach, this paper suggests to start from first principles, thus explicitly discussing the normative intuitions behind exploitation theory. Therefore, the approach proposed in this paper, and the analysis developed below should be interesting for all exploitation theorists, and indeed for all social scientists and political philosophers, even if the specific axioms proposed may be deemed unsatisfactory.

To be precise, this paper analyses exploitation theory in a *class* of *convex subsistence economies* which generalise Roemer (1982, 1988). In this class of economies, each producer can use a general convex technology and is assumed to minimize her labour supply under the constraint that she has to earn enough to purchase a subsistence vector. First of all, an axiom is introduced, called *Labour Exploitation in Subsistence Economies* (hereafter, **LES**), which restricts the way in which the sets of exploiters and exploited agents should be identified in equilibrium. This axiom is taken as the minimal necessary condition to stipulate the normative intuitions behind exploitation theory, and it is shown that all the main definitions of exploitation proposed in the literature (see, for example, Morishima, 1974; Roemer, 1982; Yoshihara, 2007) do satisfy **LES**. Then, Theorems 1 and 2 provide a significant generalisation of Roemer's (1982, 1988) celebrated results: they derive the equilibrium class and exploitation structures of a general convex cone subsistence economy with optimising agents for a whole *class* of definitions of exploitation satisfying **LES**. Further, Theorems 3 and 4 derive the necessary and sufficient conditions under which, for a whole class of definitions of exploitation satisfying **LES**, and for any convex subsistence economy, the **CECP** holds and the existence of exploitation is synonymous with the existence of positive profits, respectively. These results are theoretically relevant because, as argued by Roemer (1982), although they are formally derived as theorems, their epistemological status is as postulates: any definition of exploitation should preserve them.

Then, three additional axioms are introduced: the first states that any definition of UE exploitation should guarantee the *feasibility of non-exploitative allocations*: this is a desirable property if exploitation is not to be considered an evil that agents should learn to live with. The second axiom requires that the definition of the status of exploiter or exploited agent should be *independent* of the *distribution of productive endowments* in the economy. The third axiom captures the *relational nature* of exploitative relations by ruling out the possibility that there exists exploiters without any agent being exploited, and vice versa. Interestingly, although almost all the formulations of UE exploitation discussed in the literature satisfy **LES**, the feasibility of non-exploitation, and the independence axiom, Theorem 5 proves that a generalised version of the so-called "New Interpretation" (Duménil, 1980; Foley, 1982; Duménil at el., 2009) is the unique definition of exploitation that satisfies all axioms. As a corollary, it follows that the generalised "New Interpretation" is the unique formulation of UE exploitation that satisfies the four axioms *and* under which the **CECP** holds in the class of general convex cone subsistence economies.

There are two main reasons to focus on static subsistence economies. First of all, the analysis of subsistence economies with a labour market is theoretically crucial in that they provide the simplest institutional framework in which exploitation arises. In particular, in order to analyse the distributive issues related to exploitation, it is appropriate to abstract from the role that exploitation plays in the accumulation process.[11] The model of a subsistence

[11]It is also worth noting that the subsistence vector can also be interpreted as a social

economy may not be a realistic representation of 'actual economies,' but it is an abstract model suitable to illustrate some of the *essential* characteristics of market economies with private ownership of productive assets. Besides, from a theoretical viewpoint, one may argue that the distinctive characteristic of capitalist economies is the existence of a labour market, in which labour is exchanged as a commodity, rather than accumulation and growth. Second, from a formal viewpoint, subsistence economies provide a particularly neat framework for the analysis of exploitation, which allows one to derive stark results. However, the main conclusions can be generalised to accumulation economies (see Yoshihara, 2007) and to dynamic economies along the lines, e.g., of Veneziani (2007, 2008) and Veneziani and Yoshihara (2009), albeit at the cost of a substantial increase in unnecessary technicalities.

The rest of the paper is organised as follows. In section 2, the model of a general convex subsistence economy is set up. In section 3, the notions of exploitation and classes are defined and axiom **LES** is presented. It is then shown that all the most important definitions of exploitation presented in the literature satisfy **LES**. The complete class and exploitation structures of the economy for the class of definitions of exploitation satisfying **LES** are derived. In section 4, the necessary and sufficient conditions for the **CECP** to hold for a class of definitions of exploitation satisfying **LES** are derived. In Section 5 the three additional axioms, called Feasibility of Non-Exploitation, Independence of Endowment Structure, and Relational Exploitation are presented and the main characterisation result of the paper is derived. Section 6 concludes and the existence of a general equilibrium is proved in Appendix 1, whereas all the proofs of the formal results are in Appendix 2.

2 A Model of General Convex Subsistence Economies

Let P be the production set. P has elements of the form $\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha})$ where $\alpha_0 \in \mathbb{R}_+$, $\underline{\alpha} \in \mathbb{R}^m_+$, and $\overline{\alpha} \in \mathbb{R}^m_+$. Thus, elements of P are vectors in

reference bundle of commodities, which represents a *decent living standard*, rather than as a consumption bundle necessary for survival. In this case, once the decent living standard is reached, each agent is free to determine how to use her spare time, if any. Then, the existence of UE exploitation represents unequal allocations of free hours among agents, given that everyone reaches the decent living standard, which straightforwardly implies unequal opportunities for self-realisation and well-being freeedom.

$\mathbb{R}^{2m+1}$. The first component, $-\alpha_0$, is the direct labour input of the process α; the next m components, $-\underline{\alpha}$, are the inputs of goods used in the process; and the last m components, $\overline{\alpha}$, are the outputs of the m goods from the process. The net output vector arising from α is denoted as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. The set P is assumed to be a closed convex cone containing the origin in $\mathbb{R}^{2m+1}$. Moreover, let $\mathbf{0} \in \mathbb{R}^m$ be such that $\mathbf{0} = (0, ..., 0)'$: it is assumed that[12]

A1. $\forall \alpha \in P$ s.t. $\alpha_0 \geq 0$ and $\underline{\alpha} \geqq \mathbf{0}$, $[\overline{\alpha} \geq \mathbf{0} \Rightarrow \alpha_0 > 0]$.
A2. $\forall\, c \in \mathbb{R}^m_+$, $\exists \alpha \in P$ s.t. $\widehat{\alpha} \geqq c$.
A3. $\forall \alpha \in P, \forall\, (-\underline{\alpha}', \overline{\alpha}') \in \mathbb{R}^m_- \times \mathbb{R}^m_+$, $[(-\underline{\alpha}', \overline{\alpha}') \leqq (-\underline{\alpha}, \overline{\alpha}) \Rightarrow (-\alpha_0, -\underline{\alpha}', \overline{\alpha}') \in P]$.

A1 implies that labour is indispensable to produce any non-negative output vector; A2 states that any non-negative commodity vector is producible as a net output; and A3 is a *free disposal* condition, which states that, given any feasible production process α, any vector producing (weakly) less net output than α is also feasible using the same amount of labour as α itself.

Given P, it is possible to define the set of net output vectors that can be produced using exactly l units of labour, denoted as $\widehat{P}\,(\alpha_0 = l)$. Formally:

$$\widehat{P}\,(\alpha_0 = l) \equiv \{\widehat{\alpha} \in \mathbb{R}^m \mid \exists \alpha = (-l, -\underline{\alpha}, \overline{\alpha}) \in P \text{ s.t. } \overline{\alpha} - \underline{\alpha} \geq \widehat{\alpha}\}\,.$$

Finally, for any set $X \subseteq \mathbb{R}^m$, the boundary of X is defined as $\partial X \equiv \{x \in X \mid \nexists x' \in X \text{ s.t. } x' > x\}$, and coX is the *convex hull of the set* X.

Consider a generalisation of Roemer's (1982) subsistence economy. Let N be the set of agents, with generic element ν. All agents $\nu \in N$ have access to the same technology P, but they possess different endowments ω^ν, whose distribution in the economy is given by $(\omega^\nu)_{\nu \in N} \in \mathbb{R}^{Nm}_+$. An agent $\nu \in N$ endowed with ω^ν can engage in three types of economic activity: she can sell her labour power γ_0^ν, she can hire others to operate $\beta^\nu = \left(-\beta_0^\nu, -\underline{\beta}^\nu, \overline{\beta}^\nu\right) \in P$, or she can work on her own operating $\alpha^\nu = (-\alpha_0^\nu, -\underline{\alpha}^\nu, \overline{\alpha}^\nu) \in P$. Given a price vector $p \in \mathbb{R}^m_+$ and a nominal wage rate w, it is assumed that each agent chooses her activities, α^ν, β^ν, and γ_0^ν, in order to minimise the labour she expends subject to earning enough income to purchase a *subsistence vector* of commodities $b \in \mathbb{R}^m_+$. Moreover, she must be able to lay out in advance the operating costs for the activities she chooses to operate, either with her own labour or with hired labour, using her wealth, and she cannot work more than her labour endowment.

[12] For all vectors $x = (x_1, \ldots, x_m)$ and $y = (y_1, \ldots, y_m) \in \mathbb{R}^m$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, m)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, m)$.

Formally, given (p, w), every agent chooses $(\alpha^\nu, \beta^\nu, \gamma_0^\nu)$ in order to solve program MP^ν:

$$\min \alpha_0^\nu + \gamma_0^\nu$$

subject to

$$\begin{aligned} [p(\overline{\alpha}^\nu - \underline{\alpha}^\nu)] + \left[p\left(\overline{\beta}^\nu - \underline{\beta}^\nu\right) - w\beta_0^\nu\right] + [w\gamma_0^\nu] &\geqq pb, \\ p\underline{\alpha}^\nu + p\underline{\beta}^\nu &\leqq p\omega^\nu, \\ \alpha_0^\nu + \gamma_0^\nu &\leqq 1. \end{aligned}$$

Given (p, w), let $\mathcal{A}^\nu(p, w)$ be the set of actions $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu) \in P \times P \times [0, 1]$, which solve ν's minimisation problem MP^ν at prices (p, w).

Let a *convex cone subsistence economy* be given by a list $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N}\rangle$. Let $\mathcal{E}$ denote the set of all convex cone subsistence economies. Based on Roemer (1982), the equilibrium notion for an economy $E \in \mathcal{E}$ can be defined.

Definition 1: A *reproducible solution* (RS) for an economy $E \in \mathcal{E}$ is a pair $\left((p, w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}\right)$, where $p \in \mathbb{R}_+^m$ and $w \geqq 0$ such that:

(a) $\forall \nu \in N$, $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu) \in \mathcal{A}^\nu(p, w)$ (individual optimality);

(b) $\underline{\alpha} + \underline{\beta} \leqq \omega$ (social feasibility),
where $\underline{\alpha} \equiv \sum_{\nu \in N} \underline{\alpha}^\nu$, $\underline{\beta} \equiv \sum_{\nu \in N} \underline{\beta}^\nu$, and $\omega \equiv \sum_{\nu \in N} \omega^\nu$;

(c) $\beta_0 = \gamma_0$ (labour market equilibrium)
where $\beta_0 \equiv \sum_{\nu \in N} \beta_0^\nu$ and $\gamma_0 \equiv \sum_{\nu \in N} \gamma_0^\nu$; and

(d) $\widehat{\alpha} + \widehat{\beta} \geqq Nb$ (reproducibility),
where $\widehat{\alpha} \equiv \sum_{\nu \in N}(\overline{\alpha}^\nu - \underline{\alpha}^\nu)$, $\widehat{\beta} \equiv \sum_{\nu \in N}(\overline{\beta}^\nu - \underline{\beta}^\nu)$, and $\alpha_0 \equiv \sum_{\nu \in N} \alpha_0^\nu$.

In other words, at a RS (a) all agents optimise; (b) aggregate capital is sufficient for production plans; (c) the labour market is in equilibrium; and (d) net output is sufficient for subsistence. For the sake of brevity, in what follows, the notation (p, w) is used to represent the RS $\left((p, w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}\right)$.

In order to avoid an excess of uninteresting technicalities, it is assumed, as in Roemer (1982), that agents who are able to reproduce themselves without working use just the amount of wealth strictly necessary to obtain their subsistence bundle b: in a subsistence economy, wealthy agents have no reason to accumulate or to consume more than b; hence, by stating that they do not "waste" their capital, assumption NBC is consistent with capitalist behaviour.

Non Benevolent Capitalists (NBC): *If agent ν has a solution to MP^{ν} with $\alpha_0^{\nu}+\gamma_0^{\nu}=0$, then agent ν chooses $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})$ to minimise $p\underline{\alpha}^{\nu}+p\underline{\beta}^{\nu}$.*

It is now possible to prove some preliminary results. Lemma 1 proves that at a RS, the net revenue constraint binds for all agents.

Lemma 1: *Assume* NBC. *Let (p,w) be a* RS *for $E=\langle N;(P,b);(\omega^{\nu})_{\nu\in N}\rangle$. Then, $p\widehat{\alpha}^{\nu}+p\widehat{\beta}^{\nu}-w\beta_0^{\nu}+w\gamma_0^{\nu}=pb$ for all ν.*

The next Lemma proves that the wealth constraint binds for all agents who work at the solution to MP^{ν}.

Lemma 2: *Let (p,w) be a* RS *for $E=\langle N;(P,b);(\omega^{\nu})_{\nu\in N}\rangle$ such that $p\widehat{\alpha}-w\alpha_0>0$. For any $\nu\in N$, if $\alpha_0^{\nu}+\gamma_0^{\nu}>0$, then $p\underline{\alpha}^{\nu}+p\underline{\beta}^{\nu}=p\omega^{\nu}$.*

For any (p,w) and any $\alpha\in P$, define the profit rate $\pi=\frac{p\widehat{\alpha}-w\alpha_0}{p\underline{\alpha}}$, and let $\pi^{\max}=\max_{\alpha\in P}\frac{p\widehat{\alpha}-w\alpha_0}{p\underline{\alpha}}$. By optimality, it is immediate to prove that only production processes yielding the maximal profit rate will be activated. The next result proves an important property of the set of solutions of MP^{ν}.

Lemma 3: *Let (p,w) be a price vector such that $\pi^{\max}>0$. If $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})$ solves MP^{ν}, then $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})$ also solves MP^{ν} whenever $\underline{\alpha}^{\nu}+\underline{\beta}^{\nu}=\underline{\alpha}'^{\nu}+\underline{\beta}'^{\nu}$ and $\gamma_0'^{\nu}-\beta_0'^{\nu}=\gamma_0^{\nu}-\beta_0^{\nu}$.*

The equilibrium price vector can be characterised.

Proposition 1: *Let (p,w) be a* RS *for $E=\langle N;(P,b);(\omega^{\nu})_{\nu\in N}\rangle$. Then* (i) *$p\geq\mathbf{0}$ with $pb>0$*; (ii) $\pi^{\max}\geqq 0$; (iii) $w>0$.

In general convex cone economies it is not possible to prove that at a RS the price vector will be strictly positive. In fact, it is possible for some good to be produced as a joint product without being used as an input.[13]

[13] It is reasonable to conjecture that Roemer's *Independence of Production* assumption may yield a strictly positive vector of commodities prices in equilibrium, by requiring that for any $\alpha\in P$ with $\widehat{\alpha}>c\geqq\mathbf{0}$, there is another vector $\alpha'\in P$ such that $\widehat{\alpha}'\geqq c$ and $\alpha_0>\alpha_0'$ (see Roemer, 1981, Assumption 7, p.47). However, all the arguments in this paper hold in the more general case and therefore no restriction is needed to guarantee a strictly positive price vector.

Proposition 2 derives aggregate net output in equilibrium.

Proposition 2: *Let* (p, w) *be* a RS *for* $E = \langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$. *If* $p > \mathbf{0}$, *then* $\widehat{\alpha} + \widehat{\beta} = Nb$. *Conversely, if* $\widehat{\alpha}_i + \widehat{\beta}_i > Nb_i$ *for some good* i, *then* $p_i = 0$.

The next result derives the optimal amount of labour expended by every agent at a RS.

Proposition 3: *Let* (p, w) *be a* RS *for* $E = \langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$. *Then,* $\alpha_0^{\nu} + \gamma_0^{\nu} = \max\{0, \frac{pb - \pi^{\max} p\omega^{\nu}}{w}\}$ *for all* ν.

By Proposition 3, it follows that agent ν will not work at the optimal solution if and only if $p\omega^{\nu} \geqq \frac{pb}{\pi^{\max}}$, which implies $\pi^{\max} > 0$.

3 Exploitation and Class in Convex Subsistence Economies

The concept of exploitation in a general convex economy can now be introduced. First of all, if exploitation is conceived of as involving an unequal exchange of labour (or simply as labour exploitation), it is necessary to identify the normative benchmark, that is the normatively relevant amount of labour involved in the exchange, which is usually defined as the labour value of an agent's '*labour power*.' Agent ν is said to be *exploited* (resp. an *exploiter*) if she performs *more* (resp. *less*) labour than the labour value of her '*labour power*.' The amount of labour expended by the agent is unambiguously $\Lambda^{\nu} \equiv \alpha_0^{\nu} + \gamma_0^{\nu}$, but there are various ways of defining the *value of labour power*, which is related to some reference bundle of commodities (e.g., that the agent does or can purchase). For any bundle $c \in \mathbb{R}_+^m$, the labour value of c must be defined. Unlike in standard Leontief economies, the definition of the labour value of c is not obvious, and various definitions have, in fact, been proposed (see Yoshihara, 2007, for a thorough discussion). Following are the most relevant ones discussed in the literature.

Definition 2 has been proposed by Morishima (1974). It suggests that the labour value of a given bundle of goods corresponds to the minimum amount of labour necessary to produce that bundle as net output. Given $c \in \mathbb{R}_+^m$, let $\phi(c) \equiv \{\alpha \in P \mid \widehat{\alpha} \geqq c\}$.

Definition 2 [Morishima (1974)]: Let $c \in \mathbb{R}^m_+$ be a given nonnegative bundle of commodities. Then, the labour value of c is given by:

$$l.v.(c) = \min\{\alpha_0 \in \mathbb{R}_+ | \alpha \in \phi(c)\}.$$

The next Definition has been proposed by Roemer (1982). It suggests that the labour value of a given bundle of goods corresponds to the minimum amount of labour necessary to produce that bundle as net output using a profit-rate maximising technique. Given a price vector (p, w), let

$$\overline{P}(p, w) \equiv \left\{ \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \mid \frac{p\overline{\alpha} - (p\underline{\alpha} + w\alpha_0)}{p\underline{\alpha}} = \pi^{\max} \right\}.$$

Definition 3 [Roemer (1982)]: Let $c \in \mathbb{R}^m_+$ be a given nonnegative bundle of commodities. Then, the labour value of c is given by

$$l.v.(c; p, w) = \min\{\alpha_0 \in \mathbb{R}_+ | \alpha \in \overline{P}(p, w) \cap \phi(c)\}.$$

Instead of discussing the virtues and limitations of existing definitions of exploitation, and possibly introducing a new one, this paper adopts a novel approach and suggests starting from first principles, by defining the desirable properties that *any* definition of exploitation should satisfy. At the most general level, the UE notion of exploitation aims to describe a relational property of a given social structure by focusing on the distribution of labour associated to a given resource allocation. In principle, there are many possible alternative definitions of exploitation, but one might argue that there should be some common structure characterising all forms of exploitation as the UE of labour, which characterises an *admissible class* of definitions. The next axiom represents a *domain condition* which precisely identifies the admissible domain of all forms of exploitation, thus specifying the relevant framework for the discussion of the properties of UE exploitation in the rest of the paper. Let $B(p, b) \equiv \{c \in \mathbb{R}^m_+ \mid pc = pb\}$: $B(p, b)$ is the set of bundles that cost exactly as much as the subsistence vector at prices p. Then:

Labour Exploitation in Subsistence Economies (LES): *Let (p, w) be a RS for $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$. Two subsets $N^{ter} \subseteq N$ and $N^{ted} \subseteq N$, $N^{ter} \cap N^{ted} = \varnothing$, constitute* the set of exploiters *and* the set of exploited agents *if and only if there exist $\overline{c}, \underline{c} \in B(p, b)$ such that there exist $\alpha^{\overline{c}} \in \phi(\overline{c})$ with $\widehat{\alpha}^{\overline{c}} = \overline{c}$ and $\alpha^{\underline{c}} \in \phi(\underline{c})$ with $\widehat{\alpha}^{\underline{c}} = \underline{c}$ such that $\alpha_0^{\underline{c}} \geqq \alpha_0^{\overline{c}}$ and for any $\nu \in N$,*

$$\nu \in N^{ter} \Leftrightarrow \alpha_0^{\overline{c}} > \Lambda^\nu;$$
$$\nu \in N^{ted} \Leftrightarrow \alpha_0^{\underline{c}} < \Lambda^\nu.$$

Axiom **LES** requires that, at any RS, the sets N^{ter} and N^{ted} are characterised by identifying two (possibly identical) reference commodity vectors $\overline{c}, \underline{c} \in \mathbb{R}_{+}^{m}$. Both reference bundles $\overline{c}$ and $\underline{c}$ can be purchased by the consumer and they identify the value of labour power. Thus, if an agent $\nu \in N$ optimally works Λ^{ν} to earn the income necessary to purchase the subsistence bundle b, and Λ^{ν} is less (*resp.* more) than the labour expended to produce $\overline{c}$ (*resp.* $\underline{c}$), then she is regarded as expending less (*resp.* more) labour than the 'value of labour power.' According to **LES,** the set of such agents coincides with N^{ter} (*resp.* N^{ted}).

As the domain condition for the admissible class of exploitation-forms, **LES** captures the essential insights of the UE theory of exploitation in convex subsistence economies.[14] Given any definition of exploitation, the sets N^{ter} and N^{ted} are identified: in the UE approach, the two sets, and the exploitation status of each agent ν, are determined by the difference between the amount of labour that ν 'contributes' to the economy, in some relevant sense, and the amount she 'receives', in some relevant sense. In convex subsistence economies, the former quantity is unambiguously given by the amount of labour performed, Λ^{ν}, whereas there are many possible UE views concerning the amount of labour that each agent receives, which incorporate different normative and positive concerns. As a domain condition, **LES** incorporates the main features of UE theory that are shared by *all* the main approaches proposed in the literature.

First, according to **LES**, the amount of labour that each agent receives depends on their equilibrium income, or more precisely, it is determined in equilibrium by some reference consumption vectors that agents can purchase. In the standard approach, the reference vector is unique and it corresponds to the bundle actually chosen by the agent. **LES** is much weaker in that allows for more than one reference vector and it only requires that the reference vectors be *potentially* affordable, even if they are not actually purchased.

Second, **LES** captures another key tenet of the UE theory of exploitation by stipulating that the amount of labour associated with each reference bundle - and thus potentially 'received' an agent - is related to the production conditions of the economy. More precisely, **LES** states that the reference bundles be technologically feasible as net output, and it defines their labour

[14]It should be stressed that **LES** only applies to labour-based definitions of exploitation. It is not relevant, for example, for Roemer's property-relations definition of exploitation that emphasises inequalities in ownership of productive assets. Similar versions of **LES** can be defined in different economies; see Yoshihara (2007).

'content' as the amount of labour necessary to produce them. Thus, the amount of labour 'received' by each agent - or, in the standard terminology, the value of labour power - is a function of the amount of social labour that is allocated to agents. It is worth noting that **LES** requires that the amount of labour associated with each reference bundle be uniquely determined with reference to production conditions, but it does not specify how such amount should be chosen, and there may be in principle many alternative ways of producing $\overline{c}, \underline{c}$, and thus of determining $\alpha_0^{\underline{c}}, \alpha_0^{\overline{c}}$.

To be sure, one might argue that an even weaker version of **LES** should be imposed which allows for more than two reference bundles, and associated labour amounts, as well as for heterogeneous bundles across individuals. For example, one may argue that all affordable bundles should be considered. Although the objection may be important in principle, the restrictions on reference bundles are arguably mild and reasonable in the economies considered in this paper, and the axiom **LES** can be generalised. If individual exploitation status is monotonic in labour performed, ceteris paribus, then all the relevant information to determine exploitation status can be summarised in at most two reference bundles, and the associated amounts of labour. Similarly, although **LES** might be generalised to include agent-specific reference bundles, this is redundant in the context of convex subsistence economies with agents of identical preference.

Finally, it is worth noting that the vectors $\overline{c}$ and $\underline{c}$ in **LES** need not be uniquely fixed, and may be functions of (p, w) and b. Further, once $\overline{c}$ and $\underline{c}$ are identified, the existence of $\alpha^{\overline{c}}$ and $\alpha^{\underline{c}}$ is guaranteed by A3. The condition $\alpha_0^{\underline{c}} \geqq \alpha_0^{\overline{c}}$ is necessary for N^{ter} and N^{ted} to be disjoint in every possible economy.

Various definitions of labour exploitation proposed in the literature are discussed below, which satisfy **LES**. These definitions may be partitioned into two main approaches, depending on how the value of labour power is defined. In what may be defined as the *direct approach*, the value of labour power is defined as the labour value of the bundles that agents actually consume, whatever the definition of labour value is. In the *indirect approach*, instead, the value of labour power is defined as the labour value of some reference bundle that agents can afford with their subsistence income, even though they do not necessarily purchase it.

Two definitions that follow the first approach are considered. The first one is an application to subsistence economies (in which every agent consumes the bundle b) of Morishima's (1974) classical definition.

Definition 4: Let (p, w) be a RS for $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$. Agent ν is *exploited* if and only if $\Lambda^\nu > l.v.(b)$; she is an *exploiter* if and only if $\Lambda^\nu < l.v.(b)$; and she is *neither exploited nor an exploiter* if and only if $\Lambda^\nu = l.v.(b)$.

Definition 4 satisfies **LES** by choosing $\overline{c} = \underline{c} = b$, where $\alpha^{\overline{c}} = \alpha^{\underline{c}}$ is chosen to satisfy $\alpha_0^{\overline{c}} = \alpha_0^{\underline{c}} = l.v.(b)$. The second definition is a refinement of Morishima's and is due to Roemer (1982).

Definition 5: Let (p, w) be a RS for $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$. Agent ν is *exploited* if and only if $\Lambda^\nu > l.v.(b; p, w)$; she is an *exploiter* if and only if $\Lambda^\nu < l.v.(b; p, w)$; and she is *neither exploited nor an exploiter* if and only if $\Lambda^\nu = l.v.(b; p, w)$.

Denote the production vector $\alpha \in \overline{P}(p, w)$ with $\alpha_0 = l.v.(c; p, w)$ by $\alpha(c)$. Definition 5 satisfies **LES** by choosing $\overline{c} = \underline{c} = \widehat{\alpha}(b)$ with $\alpha^{\overline{c}} = \alpha^{\underline{c}} = \alpha(b)$, or $\overline{c} = \underline{c} = b$ with $\alpha^{\overline{c}} = \alpha^{\underline{c}} = (\alpha_0(b), \underline{\alpha}(b), \underline{\alpha}(b) + b)$.

As an illustration of the second approach, two definitions are discussed. The first has been proposed by Yoshihara (2007):

Definition 6: Let (p, w) be a RS for $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$. Agent $\nu \in N$ is *exploited* if and only if $\Lambda^\nu > \min_{c \in B(p,b)} l.v.(c; p, w)$; she is an *exploiter* if and only if $\Lambda^\nu < \min_{c \in B(p,b)} l.v.(c; p, w)$, and she is *neither exploited nor an exploiter* if and only if $\Lambda^\nu = \min_{c \in B(p,b)} l.v.(c; p, w)$.

Let $c^* = \arg\min_{c \in B(p,b)} l.v.(c; p, w)$. Definition 6 satisfies **LES** by choosing $\overline{c} = \underline{c} = \widehat{\alpha}(c^*)$ with $\alpha^{\overline{c}} = \alpha^{\underline{c}} = \alpha(c^*)$, or $\overline{c} = \underline{c} = c^*$ with $\alpha^{\overline{c}} = \alpha^{\underline{c}} = (\alpha_0(c^*), \underline{\alpha}(c^*), \underline{\alpha}(c^*) + c^*)$.

The second example of the indirect approach is an extension of the so called "New Interpretation," originally developed by Dumenil and Foley [Dumenil (1980); Foley (1982)][15] to convex cone economies, which has been proposed by Yoshihara (2007). Let (p, w) be a RS and let $\alpha + \beta$ be the corresponding aggregate production point. Let $t \in [0, 1]$ be such that $t\left(\widehat{\alpha} + \widehat{\beta}\right) \in B(p, b)$.

Definition 7 : Let (p, w) be a RS for $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$. Agent $\nu \in N$ is *exploited* if and only if $\Lambda^\nu > t(\alpha_0 + \beta_0)$; she is an *exploiter* if and

[15] See also Lipietz (1982); for a recent survey see Mohun (2004).

only if $\Lambda^{\nu} < t(\alpha_0 + \beta_0)$, and she is *neither exploited nor an exploiter* if and only if $\Lambda^{\nu} = t(\alpha_0 + \beta_0)$.

Definition 7 satisfies **LES** by choosing $\overline{c} = \underline{c} = \frac{1}{N}\left(\widehat{\alpha} + \widehat{\beta}\right)$ with $\alpha^{\overline{c}} = \alpha^{\underline{c}} = \frac{1}{N}(\alpha + \beta)$ and $t = \frac{1}{N}$.

Although the most important definitions of exploitation proposed in the literature satisfy **LES**, the axiom is by no means trivial. Consider for instance the definition of exploitation proposed by Matsuo (2008), according to which an agent who consumes c is exploited if there is a feasible bundle that is at least as good as c, in utility terms, that can be produced using less labour than is actually expended by the agent. This definition does not seem immediately relevant in the framework of this paper, because by the subsistence hypothesis, the agents' utility functions are not strictly increasing in consumption, thus contradicting one of Matsuo's assumptions. However, one may define labour value *à la* Matsuo counterfactually by asking what would be the minimum amount of labour necessary to reach a given level of utility, if agents were endowed with a continuous, strictly increasing utility function $u : \mathbb{R}^m_+ \to \mathbb{R}$ defined over consumption goods. In this case, it can be shown that Matsuo's definition of exploitation does not satisfy **LES**. To see this, consider the following example, which also illustrates the implications of **LES**.[16]

Example 1: Consider the following von Neumann technology

$$A = \begin{bmatrix} 2 & 4 & 2 & 2 \\ 4 & 4 & 2 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 2 & 8 & 6 & 6 \\ 12 & 12 & 6 & 0 \end{bmatrix}, \; L = (1, 1, 1, 1).$$

where, following usual notational conventions, A is the input matrix, B is the output matrix, and L is the direct labour vector. The corresponding production possibility set is $P_{(A,B,L)} \equiv \left\{\alpha \in \mathbb{R}_- \times \mathbb{R}^2_- \times \mathbb{R}^2_+ \mid \exists x \in \mathbb{R}^4_+ : \alpha \leqq (-Lx, -Ax, Bx)\right\}$. Then, consider a convex cone subsistence economy $E \in \mathcal{E}$ defined by $P_{(A,B,L)}$, $b = (2, 2)$, and $\omega = (N, N)$. Let $(\omega^{\nu})_{\nu \in N}$ be such that $\omega^{\nu} = (\delta^{\nu}, \delta^{\nu})$, where $\delta^{\nu} \leqq 2$ for all $\nu \in N$, and $\omega^{\nu'} = (2, 2)$ for some $\nu' \in N$.

[16] Alternatively, the function u may be interpreted as an objective measure of material welfare deriving from consumption, rather than a representation of workers' subjective preferences. This interpretation is consistent with the equilibrium notion adopted in this paper. A detailed analysis of Matsuo's approach, and a discussion of recent developments in exploitation theory is in Yoshihara and Veneziani (2009).

Let $\mathbf{e}_j \in \mathbb{R}^4_+$ be a unit column vector with 1 in the j-th component and 0 in every other component. Consider four reference production processes: $\alpha^1 \equiv (-L\mathbf{e}_1, -A\mathbf{e}_1, B\mathbf{e}_1)$, $\alpha^2 \equiv (-L\mathbf{e}_2, -A\mathbf{e}_2, B\mathbf{e}_2)$, $\alpha^3 \equiv (-L\mathbf{e}_3, -A\mathbf{e}_3, B\mathbf{e}_3)$, and $\alpha^4 \equiv (-L\mathbf{e}_4, -A\mathbf{e}_4, B\mathbf{e}_4)$, such that $\widehat{\alpha}^1 \equiv (0, 8)$, $\widehat{\alpha}^2 \equiv (4, 8)$, $\widehat{\alpha}^3 \equiv (4, 4)$, and $\widehat{\alpha}^4 \equiv (4, 0)$. Note that, $\widehat{P}(\alpha_0 = 1) = co\left\{\widehat{\alpha}^1, \widehat{\alpha}^2, \widehat{\alpha}^3, \widehat{\alpha}^4, \mathbf{0}\right\}$.

In this economy, $p = (1, 0)$ and $w = 2$ constitute a RS, with $\alpha^\nu = 0$, $\beta^\nu = \frac{\delta^\nu \alpha^3}{2}$, and $\gamma_0^\nu = 1 - \frac{\delta^\nu}{2}$ for all $\nu \in N$. The corresponding aggregate production is $\alpha + \beta = \frac{N\alpha^3}{2}$. Note that, for each agent, $B(p, b) = \left\{c \in \mathbb{R}^2_+ \mid \exists x \in \mathbb{R}_+ : c = (2, x)\right\}$.

According to Matsuo (2008), the commodity vector which serves to define the value of labour power is given by:

$$c^M \equiv \arg \min_{c \in \mathbb{R}^2_+ ,\ \alpha \in P} \alpha_0 \text{ subject to } \widehat{\alpha} \geqq c \ \&\ u(c) \geqq u(b).$$

But, by the strict monotonicity of u, $c^M \in \widehat{P}\left(\alpha_0 = \frac{1}{2}\right) \backslash \partial \widehat{P}\left(\alpha_0 = \frac{1}{2}\right)$, which implies $pc^M < pb$. Thus, $c^M \notin B(p, b)$, which implies that exploitation *à la* Matsuo does not satisfy **LES**. ■

Let $W^\nu \equiv p\omega^\nu$. Theorem 1 characterises the exploitation status of every agent, based on their initial wealth W^ν, for *all* definitions of exploitation satisfying axiom **LES**.

Theorem 1: *Let* (p, w) *be a* RS *for* $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N}\rangle$ *such that* $\pi^{\max} > 0$. *Then, for any formulation of labour exploitation satisfying* **LES**:
(i) *agent* ν *is an* exploiter *if and only if* $W^\nu > \frac{1}{\pi^{\max}}\left[pb - w\alpha_0^{\overline{c}}\right]$;
(ii) *agent* ν *is* exploited *if and only if* $W^\nu < \frac{1}{\pi^{\max}}\left[pb - w\alpha_0^{\underline{c}}\right]$; *and*
(iii) *agent* ν *is* neither *an* exploiter nor exploited *if and only if*
$\frac{1}{\pi^{\max}}\left[pb - w\alpha_0^{\underline{c}}\right] \leqq W^\nu \leqq \frac{1}{\pi^{\max}}\left[pb - w\alpha_0^{\overline{c}}\right]$.

Theorem 1 is considerably more general than similar results derived by Roemer (1982), in that it applies to a whole class of definitions of exploitation, rather than a specific approach. Thus, for any definition of exploitation satisfying axiom **LES**, Theorem 1 identifies the wealth cut-offs that partition the set of agents based on their exploitation status. Given the analysis in the previous section, one immediately notes that for each of the Definitions analysed there is a unique wealth cut-off. Thus, by Theorem 1, under Definition 4, exploitation status depends on whether wealth is higher than, lower than or equal to $W^* = \frac{1}{\pi^{\max}}[pb - w(l.v.(b))]$; similarly, under Definition 5, the wealth cut-off is $W^* = \frac{1}{\pi^{\max}}[pb - w(l.v.(b;p,w))]$; under Definition 6, the wealth cut-off is $W^* = \frac{1}{\pi^{\max}}\left[pb - w\left[\min_{c\in B(p,b)} l.v.(c;p,w)\right]\right]$; finally, under Definition 7, the wealth cut-off is $W^* = \frac{1}{\pi^{\max}}\left[pb - \frac{w(\alpha_0+\gamma_0)}{N}\right]$. Actually, $\min_{c\in B(p,b)} l.v.(c;p,w) \leqq l.v.(b;p,w) \leqq t(\alpha_0+\beta_0) = \frac{\alpha_0+\gamma_0}{N}$, and $l.v.(b) \leqq l.v.(b;p,w)$ for all $E \in \mathcal{E}$, and thus the wealth cut-offs can be ranked. Therefore, *ceteris paribus*, the set of exploited agents according to Definition 7 is (weakly) included in the set of exploited agents according to Definition 5, which is in turn (weakly) included in the set of exploited agents according to Definition 4.[17]

Following Roemer (1982), *classes* can also be defined in this economy, based on the way in which agents relate to the means of production. At a RS of the subsistence economy, an individually optimal solution for an agent ν consists of a vector $(\alpha^\nu;\beta^\nu;\gamma_0^\nu)$. Therefore, let (a_1,a_2,a_3) be a vector where $a_i = \{+,\mathbf{0}\}$, $i=1,2$, and $a_3 = \{+,0\}$, where "+" means a non-zero vector in the appropriate place. Agent ν is said to be a member of class (a_1,a_2,a_3), if there is an individually optimal $(\alpha^\nu;\beta^\nu;\gamma_0^\nu)$ which has the form (a_1,a_2,a_3). The notation $(+,+,0)$ implies, for instance, that an agent works in her own 'shop' and hires others to work for her; $(+,\mathbf{0},+)$ implies that an agent works both in her own 'shop' and for others, etc. Although there are seven possible classes in the subsistence economy, it can be proved that at a RS, the set of producers N can be partitioned into the following five, theoretically relevant classes.

$$\begin{aligned}
C^1 &= \{\nu\in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (\mathbf{0},+,0)\},\\
C^2 &= \{\nu\in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,+,0)\setminus(+,\mathbf{0},0)\},\\
C^3 &= \{\nu\in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,\mathbf{0},0)\},\\
C^4 &= \{\nu\in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,\mathbf{0},+)\setminus(+,\mathbf{0},0)\},\\
C^5 &= \{\nu\in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (\mathbf{0},\mathbf{0},+)\}.
\end{aligned}$$

The notation $(+,+,0)\setminus(+,\mathbf{0},0)$ means that agent ν is a member of class $(+,+,0)$ but not of class $(+,\mathbf{0},0)$, and likewise for the other classes. As a first step in the analysis of classes, Lemma 4 proves that $(+,+,+)$ and $(\mathbf{0},+,+)$ are indeed redundant.

[17]It is worth noting that it is not difficult to construct economies $E\in\mathcal{E}$, in which the wealth cut-offs are indeed different and the inequalities are strict.

Lemma 4: *Let (p, w) be a given price vector. Let ν be such that $W^{\nu} > 0$ and $\Lambda^{\nu} > 0$ at the solution of MP^{ν}. Let ν belong to either $(+,+,+)$ or $(\mathbf{0},+,+)$. Then, exactly one of the following statements holds*:

$$\begin{aligned} \text{if } \gamma_0^{\nu} &< \beta_0^{\nu} \text{ for all optimal } (\alpha^{\nu};\beta^{\nu};\gamma_0^{\nu}), \text{ then } \nu \in (+,+,0)\backslash(+,\mathbf{0},0); \\ \text{if } \gamma_0^{\nu} &= \beta_0^{\nu} \text{ for some optimal } (\alpha^{\nu};\beta^{\nu};\gamma_0^{\nu}), \text{ then } \nu \in (+,\mathbf{0},0); \\ \text{if } \gamma_0^{\nu} &> \beta_0^{\nu} \text{ for all optimal } (\alpha^{\nu};\beta^{\nu};\gamma_0^{\nu}), \text{ then } \nu \in (+,\mathbf{0},+)\backslash(+,\mathbf{0},0). \end{aligned}$$

Given Lemma 4, it is now possible to prove a generalisation of Roemer's (1982) core result concerning the correspondence between class status and wealth. Theorem 2 proves that classes C^1 to C^5 are pairwise disjoint and exhaustive, and wealthier agents belong to the upper classes.

Theorem 2: *Let (p, w) be a* RS *for $E = \langle N; (P, b); (\omega^{\nu})_{\nu \in N}\rangle$ such that $\pi^{\max} > 0$. Then*:
(i) *For any $i \neq j$, $C^i \cap C^j = \varnothing$ and $\cup_{i=1}^{5} C^i = N$*;
(ii) *For any $\nu, \mu \in N$, if $\nu \in C^i$ and $\mu \in C^j$ with $i < j$ then $W^{\nu} > W^{\mu}$.*

Theorems 1 and 2 identify two different partitions of the set of agents based, respectively, on their exploitation status and their relation to the means of production (more precisely, their position in the labour market). In both cases, an agent's wealth is the main determinant of her position in the social structure, and therefore it is legitimate to ask whether class and exploitation status are related, as predicted in Marxian theory. This is the main object of analysis in the next section.

4 The Class-Exploitation Correspondence Principle

In classical Marxian theory, exploitation and classes are related: capitalists are exploiters and proletarians are exploited. On the one hand, this correspondence of class and exploitation status gives a specific normative relevance to the concept of class; on the other hand, it emphasises the importance of relations of production in the generation of exploitation. More formally, the Class-Exploitation Correspondence Principle (CECP) can be defined as follows.

Class-Exploitation Correspondence Principle (CECP) [Roemer (1982)]**:** *Given any economy $E \in \mathcal{E}$, at any RS with $\pi^{\max} > 0$,*

(A) every member of $C^1 \cup C^2$ is an exploiter.

(B) every member of $C^4 \cup C^5$ is exploited.

A fundamental contribution of Roemer's theory is the proof of the **CECP** in the private-ownership economy, which is derived as a *result* of the analysis, rather than being assumed. Epistemologically, though, Roemer forcefully argues that the central relevance of the **CECP** in class and exploitation theory implies that it should be considered as a *postulate*, by requiring that any satisfactory definition of exploitation (and classes) satisfies the **CECP**. Consistently with this approach, this section analyses the **CECP** under different definitions of exploitation satisfying **LES**.

As a first step, it is useful to provide a characterisation of class status in general convex cone subsistence economies. Let (p, w) be a RS such that $\pi^{\max} > 0$. Let $\alpha_{\min} \in \overline{P}(p, w)$ be such that $\frac{p\underline{\alpha}_{\min}}{\alpha_{0\min}} = \min_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right]$ and $\pi^{\max} p\underline{\alpha}_{\min} + w\alpha_{0\min} = pb$, and let $\alpha_{\max} \in \overline{P}(p, w)$ be such that $\frac{p\underline{\alpha}_{\max}}{\alpha_{0\max}} = \max_{\alpha' \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right]$ and $\pi^{\max} p\underline{\alpha}_{\max} + w\alpha_{0\max} = pb$. Note that $p\underline{\alpha}_{\min} \leqq p\underline{\alpha}_{\max}$, and that $\alpha_{\min}, \alpha_{\max}$ are well-defined. The next proposition provides a precise characterisation of class status based on an agent's wealth.

Proposition 4: *Let (p, w) be a* RS *for $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$ such that $\pi^{\max} > 0$. Then,*

(i) $\nu \in C^1 \Leftrightarrow W^\nu \geqq \frac{pb}{\pi^{\max}}$;

(ii) $\nu \in C^2 \Leftrightarrow p\underline{\alpha}_{\max} < W^\nu < \frac{pb}{\pi^{\max}}$;

(iii) $\nu \in C^3 \Leftrightarrow p\underline{\alpha}_{\min} \leqq W^\nu \leqq p\underline{\alpha}_{\max}$; and

(iv) $\nu \in C^4 \Leftrightarrow 0 < W^\nu < p\underline{\alpha}_{\min}$;

(v) $\nu \in C^5 \Leftrightarrow W^\nu = 0$.

In other words, Proposition 4 derives four wealth cut-off levels that partition the set of agents into five classes. The richest agents are big capitalists, who can reproduce themselves without working, and the poorest, propertyless agents are proletarians, who can only sell their labour in order to survive. All other agents fall into the intermediate classes, based on their wealth. In principle, the wealth cut-offs identified in Proposition 4 may be different from those characterising exploitation status identified in Theorem 1, which depend on the actual definition of exploitation adopted. The next theorem provides the general condition for the **CECP** to hold under *any* definition of exploitation satisfying **LES**.

Theorem 3: *Let* (p, w) *be a* RS *for* $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$ *such that* $\pi^{\max} > 0$. *For any definition of labour exploitation satisfying* **LES**, *the* **CECP** *holds if and only if* $\overline{c}, \underline{c}$ *are such that*

$$p\underline{\alpha}_{\min} \leqq \frac{1}{\pi^{\max}} \left[pb - w\alpha_0^{\underline{c}}\right] \leqq \frac{1}{\pi^{\max}} \left[pb - w\alpha_0^{\overline{c}}\right] \leqq p\underline{\alpha}_{\max}.$$

Again, Theorem 3 is significantly more general than the analogous results proved by Roemer (1982), in that it provides general necessary and sufficient conditions for the **CECP** to hold for an entire class of definitions of exploitation, rather than for a specific approach. Actually, there is another way of stating Theorem 3, which may illustrate its significance in terms of a definition of exploitation satisfying **LES**. Let (p, w) be a RS with $\pi^{\max} > 0$, and let

$$\Gamma(p, w) \equiv \{\alpha \in P \mid p\widehat{\alpha} = pb \text{ and } p\widehat{\alpha} - w\alpha_0 \in [\pi^{\max} p\underline{\alpha}_{\min}, \pi^{\max} p\underline{\alpha}_{\max}]\}.$$

Note that $\Gamma(p, w)$ is the set of feasible production processes yielding a net output with the same monetary value as the subsistence vector and a profit revenue equal to the profit revenue under one of the maximal-profit-rate processes. Similarly, the set of net outputs associated with $\Gamma(p, w)$ is defined as follows: $\widehat{\Gamma}(p, w) \equiv \left\{\widehat{\alpha} \in \mathbb{R}_+^m \mid \alpha \in \Gamma(p, w)\right\}$. Theorem 3 can be re-stated as proving that at a RS (p, w) such that $\pi^{\max} > 0$, for any definition of labour exploitation satisfying **LES**, the **CECP** holds if and only if $\overline{c}, \underline{c} \in \widehat{\Gamma}(p, w)$.

In other words, for any definition of labour exploitation satisfying **LES**, the **CECP** holds if and only if the profit revenue associated with the production of the reference bundles $\overline{c}$ and $\underline{c}$ is equal to the profit revenue under one of the maximal-profit-rate processes. This characterisation holds regardless of whether the amount of labour necessary to produce $\overline{c}, \underline{c}$ is minimal or not, and regardless of whether the net output is actually produced at the RS. Note that this does not exclude the possibility that the **CECP** holds under a labour value formulation with no maximal-profit-rate production process.[18] This property is due to the assumption of subsistence economies. In fact, in the case of accumulation economies, as Yoshihara (2007) shows, if the

[18] For a proof of this claim see Example A.1 in Appendix 2 below.

CECP holds under a labour value formulation, then this formulation should be associated with some maximal-profit-rate production process.[19]

Let (p, w) be a RS and let $\overline{\Gamma}(p, w) \equiv \left\{\alpha \in \overline{P}(p, w) \mid p\widehat{\alpha} = pb\right\}$: $\overline{\Gamma}(p, w)$ is the set of feasible, maximal-profit-rate production processes yielding a net output with the same monetary value as the subsistence vector. Similarly, the set of net outputs associated with $\overline{\Gamma}(p, w)$ can be defined as follows: $\widehat{\overline{\Gamma}}(p, w) \equiv \left\{\widehat{\alpha} \in \mathbb{R}^m_+ \mid \alpha \in \overline{\Gamma}(p, w)\right\}$. Then, noting that $\widehat{\overline{\Gamma}}(p, w) \subseteq \widehat{\Gamma}(p, w)$, the next Corollary immediately follows from Theorem 3.

Corollary 1: *Let* (p, w) *be a* RS *for* $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N}\rangle$ *such that* $\pi^{\max} > 0$. *For any formulation of labour exploitation satisfying* **LES**, *if its corresponding* $\overline{c}, \underline{c}$ *are such that* $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p, w)$, *then the* **CECP** *holds under this formulation.*

Corollary 1 provides sufficient conditions for the **CECP** to hold for *any* definition of labour exploitation satisfying **LES**. In particular, it states that the **CECP** holds if this definition evaluates the amount of labour necessary to produce a net output, which has the same monetary value as the subsistence vector at the RS, by operating one of the maximal-profit-rate processes. Based on Theorem 3 and Corollary 1, it is possible to prove that the **CECP** holds under Definitions 5, 6, and 7.

Corollary 2: *Let* (p, w) *be a* RS *for* $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N}\rangle$ *such that* $\pi^{\max} > 0$. *Then the* **CECP** *holds under* Definitions 5, 6, *and* 7.

Instead, Definition 4 does not preserve the **CECP**, because the commodity bundle used to define the value of labour power under Definition 4 need not be produced with the same profit revenue as that of the maximal-profit-rate processes. This is shown formally in the following proposition.

Proposition 5: *Let exploitation be defined according to* Definition 4. *There is a convex cone, subsistence economy* $E \in \mathcal{E}$ *in which there exists a* RS *with* $\pi^{\max} > 0$, *such that the* **CECP** *does not hold.*

In the context of accumulating economies, Roemer (1982; Chapter 5, p. 148) had already suggested that in accumulation economies, if Morishima's

[19]Note that Roemer (1982; Chapter 5; p.164) also stated basically the same claim, though he only discussed the case of exploiters.

definition of exploitation is used, then "the **CECP** is false for the cone technology" - a claim rigorously proved by Yoshihara (2007). The roots of the failure of the **CECP**, however, are different in the cases of subsistence and accumulation economies. In fact, as Roemer (1982) and Yoshihara (2007) show, in accumulation economies the failure of the **CECP** is due to the existence of both non-exploiting capitalists and non-exploited workers. In contrast, in subsistence economies, as the proof of Proposition 5 in the Appendix shows, the failure of the **CECP** is due to the existence of exploited capitalists.

The results presented in this section are quite relevant for exploitation theory. Given the epistemological relevance of the **CECP** discussed above, Proposition 5 suggests that Morishima's classical definition of exploitation is inadequate to capture the central intuitions of Marxian theory.[20] Contrary to Roemer's claims, however, Corollary 2 proves that, even in the general setting analysed in this paper, the **CECP** does hold for various definitions of exploitation presented in the literature. More generally, Theorem 3 suggests that, for all possible convex subsistence economies and all equilibria, the **CECP** holds for a whole class of notions of exploitation as UE of labour satisfying the arguably weak and reasonable axiom **LES**.[21]

Finally, the following equivalence relation can be proved:

Theorem 4: *Consider any definition of exploitation which satisfies* **LES**. *Let* (p, w) *be a* RS *for* $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$ *and* $\overline{c}, \underline{c} \in \widehat{\Gamma}(p, w)$ *for this definition of exploitation. Then, the following statements are equivalent*:

(1) $\pi^{\max} > 0$;

(2) *the* **CECP** *holds*;

(3) *every producer in* C^5 *is* exploited.

Theorem 4 has two important implications for exploitation theory. Firstly, the equivalence between (1) and (3) represents a significant generalisation of the so-called "*Fundamental Marxian Theorem*" (**FMT**) [Roemer (1980, 1981)], according to which the equilibrium maximal profit rate is positive if and only if every worker is exploited. The **FMT** is proved to hold in general

[20] Actually, Morishima's definition has another arguably undesirable property. In fact, it is not difficult to construct an example of a subsistence economy in which all producers have the same endowment of capital goods, but they are all exploited in the sense of Definition 4, a rather counterintuitive result.

[21] As shown by Yoshihara (2007), a similar result holds in accumulation economies.

convex cone economies with a complex class structure and for a whole set of definitions of exploitation satisfying axiom **LES**. This is important because, as for the **CECP**, although it is proved as a result, the epistemological status of the **FMT** in exploitation theory is usually that of an axiom, and alternative definitions of exploitation are often compared in terms of their ability to preserve it. Actually, one might discuss which axiom - the **CECP** or the **FMT** - is more relevant and whether imposing both significantly decreases the set of available definitions of exploitation. The second, and somewhat more surprising, implication of Theorem 4 is that for the whole class of definitions of exploitation satifying **LES** with $\overline{c}, \underline{c} \in \widehat{\Gamma}(p, w)$, the **CECP** and the **FMT** are equivalent, and therefore one need not choose among them: any definition preserving one, also preserves the other.

5 An Axiomatic Characterisation of Exploitation

The results presented in the previous sections are encouraging: there exist a set of definitions of exploitation as the unequal exchange of labour that satisfy the reasonable condition imposed by **LES**, and preserve the **CECP** (and the **FMT**) as a result, in the equilibrium of the private-ownership economy. Furthermore, both **LES** and the requirement that the **CECP** holds in general convex economies are not trivial, and some of the definitions proposed in the literature do not satisfy them. In terms of identifying and defending one definition of exploitation, however, the analysis developed so far is not conclusive as it cannot discriminate between a number of competing notions. In line with the novel axiomatic approach adopted in this paper, the main purpose of this section is to propose and defend some additional theoretical conditions that any definition of labour exploitation should satisfy and then to provide a characterisation result.

Consider any economy $E \in \mathcal{E}$. For any definition of exploitation satisfying **LES**, recall that the corresponding reference bundles $\overline{c}, \underline{c} \in \mathbb{R}^m_+$ identifying the value of labour power depend on p. So, let $\left((p, w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}\right)$ be a RS for $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$ and let $\overline{c}, \underline{c}$ be the corresponding reference bundles. Non-exploitative allocations can be defined as follows.

Definition 8: Suppose that the definition of exploitation satisfies **LES**. Given a RS $\left((p, w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}\right)$ and the corresponding reference bun-

dles $\overline{c}, \underline{c}$, the allocation $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu \in N} \in (P \times P \times [0,1])^N$ is *non-exploitative at* (p, w) if
(i) $Nb \leqq \sum_{\nu \in N} \widehat{\alpha}'^{\nu} + \sum_{\nu \in N} \widehat{\beta}'^{\nu}$ and $\sum_{\nu \in N} \beta_0'^{\nu} = \sum_{\nu \in N} \gamma_0'^{\nu}$; and
(ii) $\alpha_0^{\overline{c}} \leqq \alpha_0'^{\nu} + \gamma_0'^{\nu} \leqq \alpha_0^{\underline{c}}$ for all $\nu \in N$.

Definition 8 is a rather weak definition of non-exploitative allocations, because it only requires that subsistence be guaranteed and that all labour required to implement the allocations be available. No constraint is imposed, instead, on aggregate capital endowments, or on the way in which such allocation can be reached. Arguably, though, non-exploitative allocations that are within the realm of *social feasibility* are of focal normative relevance. The next definition provides a precise notion of social feasibility.

Definition 9: Given a RS $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu \in N}\right)$, an allocation $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu \in N}$ is *feasible non-exploitative w.r.t* (p', w') *in* $E = \langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$ if:
(i) the allocation $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu \in N}$ is non-exploitative at (p, w) and
(ii) $\left((p', w'), (\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu \in N}\right)$ is a RS for $\langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$.

Clearly, the feasibility of a non-exploitative allocation depends on the actual values of $\overline{c}, \underline{c} \in \mathbb{R}_+^m$, that is, on the notion of labour exploitation adopted.

The next Definition outlines the concept of efficiency that is relevant in convex subsistence economies.

Definition 10: An allocation $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu \in N} \in (P \times P \times [0,1])^N$ is *efficient for* $E = \langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$ if it satisfies:
(i) $\sum_{\nu \in N} \underline{\alpha}^{\nu} + \sum_{\nu \in N} \underline{\beta}^{\nu} \leqq \sum_{\nu \in N} \omega^{\nu}$, $Nb \leqq \sum_{\nu \in N} \widehat{\alpha}^{\nu} + \sum_{\nu \in N} \widehat{\beta}^{\nu}$, and $\sum_{\nu \in N} \beta_0^{\nu} = \sum_{\nu \in N} \gamma_0^{\nu}$; and there is no other allocation $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu \in N} \in (P \times P \times [0,1])^N$ such that the above (i) is satisfied and
(ii) $\alpha_0'^{\nu} + \gamma_0'^{\nu} \leqq \alpha_0^{\nu} + \gamma_0^{\nu}$ for all $\nu \in N$, and $\alpha_0'^{\nu^*} + \gamma_0'^{\nu^*} < \alpha_0^{\nu^*} + \gamma_0^{\nu^*}$ for some $\nu^* \in N$.

In other words, if an allocation is efficient, it is impossible to reduce the labour time of some agent without increasing the amount of labour expended by someone else.[22] It is also worth noting that if an allocation is efficient according to Definition 10, then even a benevolent social planner cannot

[22] Thus, Definition 10 appropriately focuses on the socially relevant phenomenon, namely the minimisation of labour, but it gives no weight to the behavioural assumption encompassed in **NBC**, according to which big capitalists minimise capital outlay.

improve on it by choosing $(\alpha'^{\nu})_{\nu\in N} \in P^N$ such that $\sum_{\nu\in N} \underline{\alpha}'^{\nu} \leqq \sum_{\nu\in N} \omega^{\nu}$, $Nb \leqq \sum_{\nu\in N} \widehat{\alpha}'^{\nu}$; and $\alpha_0'^{\nu} \leqq \alpha_0^{\nu} + \gamma_0^{\nu}$ for all $\nu \in N$, and $\alpha_0'^{\nu^*} < \alpha_0^{\nu^*} + \gamma_0^{\nu^*}$ for some $\nu^* \in N$.

The next axiom requires that, whatever the notion of labour exploitation is, its corresponding non-exploitative allocation should be feasible for each economy.

Feasibility of Non-exploitation (FNE): C*onsider a definition of exploitation satisfying* **LES**. *Given any economy* $E \in \mathcal{E}$*, let* (p, w) *be an efficient RS and let* $\overline{c}, \underline{c} \in \mathbb{R}^m_+$ *be the corresponding reference bundles. If* $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ is non-exploitative at (p, w) such that $\sum_{\nu\in N} \underline{\alpha}'^{\nu} + \sum_{\nu\in N} \underline{\beta}'^{\nu} = \sum_{\nu\in N} \omega^{\nu}$*, then there exists* (p', w') *such that for some suitable redistribution* $(\omega'^{\nu})_{\nu\in N}$ *of* $(\omega^{\nu})_{\nu\in N}$ *with* $\sum_{\nu\in N} \omega'^{\nu} = \sum_{\nu\in N} \omega^{\nu}$, $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ *is* feasible non-exploitative with respect to (p', w') in $\langle N; (P, b); (\omega'^{\nu})_{\nu\in N}\rangle$.

FNE is a rather weak and desirable property for all concepts of exploitation. For, first, all of the formulations of exploitation discussed in this paper satisfy it, and second, it requires that *if* a non-exploitative allocation exists with the current social endowment, then it must be possible to implement such an allocation with some appropriate distribution of social endowments. Arguably, any normatively relevant concept of exploitation should allow for the existence of non-exploitative allocations. If exploitation could not be eliminated, then its relevance for the evaluation of capitalist societies would arguably be diminished, as an evil that agents must learn to live with. Then, given that an ideal (non-exploitative) allocation is *technologically feasible* in terms of Definitions 1(b), 1(c), and 1(d), **FNE** would require that it be *rational* in terms of Definitions 1(a). The appeal to markets as the mechanism to implement non-exploitative allocations may be controversial, but one may argue that in the context of this paper the usual equivalence between market allocations and the social planner problem holds. Furthermore, theoretically, **FNE** can be seen as requiring that non-exploitation is compatible with the rational choices of individuals. To be precise, since the **CECP** shows that the emergence of class and exploitation is the consequence of rational choice of individuals, the **FNE** guarantees that *the emergence of class and exploitation is not a necessary condition for rational choice of individuals*, because non-exploitation is also required to be compatible with rational choices. More pragmatically, thus far, markets have proved to be the only sustainable, decentralised decision-making allocation mechanism in the sense of Hurwicz

(1972), and, absent a viable alternative, this justifies what may seem an excessively narrow focus. Furthermore, **FNE** is interesting also in relation to recent proposals of market socialism as a mechanism to combine efficiency and social justice, including the elimination of exploitation (see, e.g., Roemer, 1994). Actually, as shown below, at the level of abstraction at which this analysis is conducted, the focus on markets allows one to introduce some important issues concerning the relation between exploitation and efficiency.

In addition to **FNE**, the following axiom is a natural requirement for any definition of exploitation:

Independence of Endowment Structure (**IES**): C*onsider a definition of exploitation satisfying* **LES**. *Given two economies* $E = \langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$ *and* $E' = \langle N; (P, b); (\omega'^{\nu})_{\nu \in N} \rangle$, *let* $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu \in N}\right)$ *and* $\left((p, w), (\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu \in N}\right)$ *be their corresponding* RS*s such that* $\sum_{\nu \in N} (\alpha^{\nu} + \beta^{\nu}) = \sum_{\nu \in N} (\alpha'^{\nu} + \beta'^{\nu})$. *Moreover, let* $\overline{c}, \underline{c} \in \mathbb{R}_+^m$ *and* $\overline{c}', \underline{c}' \in \mathbb{R}_+^m$ *be the corresponding reference bundles. Then,* $\overline{c} = \overline{c}'$ *and* $\underline{c} = \underline{c}'$.

Axiom **IES** implies that the reference commodity bundles may depend on the price vector (p, w) and the equilibrium social production point $\alpha^{p,w}$ of the given RS, but they should be independent of the allocation of production activity vectors, which are determined by the underlying endowment structure. This condition seems uncontroversial and indeed all the main definitions of exploitation discussed in the literature satisfy it.

Finally, the following axiom captures an arguably essential feature of any theory of exploitation.

Relational Exploitation (RE): *Consider a definition of exploitation satisfying* **LES**. *At any RS for* $\langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$, *the two subsets* N^{ter} *and* N^{ted} *are such that* $N^{ter} \neq \varnothing$ *if and only if* $N^{ted} \neq \varnothing$.

From a formal viewpoint, axiom **RE** imposes a rather weak restriction on **LES**. From a theoretical viewpoint, it captures the crucial relational aspect inherent in exploitative relations, such that if an agent is exploited, she must be exploited *by someone*, and viceversa if an exploiter exists, she must be exploiting *someone*.

Before proving the main Theorem of this section, which provides a complete axiomatic characterisation of exploitation, some intermediate results are derived, which are also interesting in their own right. First of all, the next Lemma proves that all RS's in which the wealth constraints bind, are efficient in the sense of Definition 10.

Lemma 5: *Let* $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ *be a* RS *for* $E=\langle N;(P,b);(\omega^{\nu})_{\nu\in N}\rangle$. *If* $\sum_{\nu\in N} p\left(\underline{\alpha}^{\nu}+\underline{\beta}^{\nu}\right)=\sum_{\nu\in N} p\omega^{\nu}$, *then* $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}$ *is* efficient.

Lemma 5 suggests that the scarcity of capital is a sufficient condition for efficiency in equilibrium. Yet, it does not rule out the possibility of inefficient RS's. In particular, if $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ is a RS with $\sum_{\nu\in N} p\left(\underline{\alpha}^{\nu}+\underline{\beta}^{\nu}\right)<\sum_{\nu\in N} p\omega^{\nu}$, then $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}$ is not necessarily efficient. Formally, it is not possible to prove a Marxist version of the First Welfare Theorem because local nonsatiation may be violated in this economy for all those agents who do not work at the optimum. In fact, a necessary condition for a RS to be inefficient is the existence of big capitalists who can reproduce themselves without working, and who maintain capital scarcity "artificially" by minimising capital outlay, consistently with the assumption **NBC**. An example of an inefficient RS in a convex subsistence economy is discussed in detail in Appendix 3 below and the role of big capitalists is shown. This is an interesting case, as it suggests a relation between inefficiencies and extreme forms of injustice, viz. exploitative social relations. From a normative perspective, however, this is the least challenging case since there is no trade-off, at least locally, between an improvement in efficiency and a reduction of social injustices, namely exploitation. Even from a Marxian viewpoint, it may be argued that exploitation theory should focus on the amount of labour time necessary to produce the subsistence bundle by adopting efficient production techniques.

Proposition 6 proves that the amount of labour performed is uniquely determined at all efficient equilibria of the subsistence economy.

Proposition 6: *Let* $E=\langle N;(P,b);(\omega^{\nu})_{\nu\in N}\rangle$ *and* $E'=\langle N;(P,b);(\omega'^{\nu})_{\nu\in N}\rangle$ *be two economies such that* $\sum_{\nu\in N}\omega^{\nu}=\sum_{\nu\in N}\omega'^{\nu}$. *Let* $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ *and* $\left((p',w'),(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}\right)$ *be two* efficient RSs *for* E *and* E', *respectively. Then* $\alpha_0+\gamma_0=\alpha_0'+\gamma_0'$.

According to Proposition 6, for a given vector of aggregate productive assets, the amount of labour performed is independent of the distribution of endowments and it is equalised across all efficient RS's. The next Lemma, instead, focuses on efficient and resource-egalitarian equilibria and proves that if an efficient RS exists, then the same price vector supports an egalitarian RS in which all individuals have the same vector of productive endowments and activate the same production process as self-employed agents.

Lemma 6: *Let* $\left((p,w),(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu\in N}\right)$ *be a* RS *for* $E=\langle N;(P,b);(\omega^\nu)_{\nu\in N}\rangle$ *such that* $\sum_{\nu\in N} p\left(\underline{\alpha}^\nu+\underline{\beta}^\nu\right)=\sum_{\nu\in N} p\omega^\nu$. *Then* $\left((p,w),(\alpha'^\nu;\ 0;\ 0)_{\nu\in N}\right)$ *is a* RS *for* $E'=\langle N;(P,b);(\omega'^\nu)_{\nu\in N}\rangle$, *where* $\alpha'^\nu=\frac{\sum_{\nu\in N}(\alpha^\nu+\beta^\nu)}{N}$ *and* $\omega'^\nu=\frac{\omega}{N}$ *for all* $\nu\in N$.

Given the new axioms, **FNE**, **RE**, and **IES**, the following characterisations can be derived.

Theorem 5: *A formulation of labour exploitation satisfies* **LES**, **FNE**, **RE**, *and* **IES** *if and only if for all* $E\in\mathcal{E}$ *and every* RS (p,w), $\alpha_0^{\underline{c}}=\alpha_0^{\overline{c}}=\frac{\alpha_0+\beta_0}{N}$.[23]

Theorem 5 can thus be taken as providing support to the extension of Dumenil-Foley's "New Interpretation" (Definition 7) as the appropriate definition of exploitation, which is uniquely characterised from a rather small set of arguably weak axioms.[24] Actually, Theorem 5 has a rather striking implication: because, as shown above, virtually all of the most relevant definitions of exploitation proposed in the literature such as Definitions 4, 5, 6, and 7 satisfy axioms **LES**, **FNE**, and **IES**, it is the rather mild axiom **RE** - which requires the presence of exploiters, whenever some agent is exploited - that rules out all alternative definitions except Definition 7. By Theorem 5, not all definitions of exploitation satisfy these reasonable properties, and indeed our axiomatic analysis allows us to precisely identify the limits of the received definitions of exploitation: if Morishima's celebrated definition is adopted, for example, it is not difficult to construct examples in which **RE** is violated and *all* agents are exploited.

Interestingly, however, even the adoption of a more traditional stance emphasising the importance of the notions of *surplus labour* and *socially necessary labour time* in exploitation theory, rather than the relational property incorporated in **RE**, does not rescue the main received definitions of exploitation. In particular, following classical approaches to exploitation theory, it

[23] Such a characterisation by using **RE** still holds if **RE** is weakened so that its requirement is imposed only on any RS *with a positive profit rate.*

[24] It is worth noting that under Definitions 4 and 5, there indeed exist some $E\in\mathcal{E}$ and some RS (p,w), such that $l.v.(b)\neq\frac{\alpha_0+\beta_0}{N}$ and $l.v.(\widehat{\alpha}(b);p,w)\neq\frac{\alpha_0+\beta_0}{N}$, so that Theorem 5 does capture a fundamental difference between alternative definitions.

may be argued that *Socially Necessary Labour* corresponds to the minimum amount of labour necessary to produce the subsistence consumption bundle and this quantity is an important theoretical benchmark, such that the reference labour expenditures $\alpha_0^{\overline{c}}, \alpha_0^{\underline{c}}$ that identify exploiters and exploited agents should be no lower than socially necessary labour. More formally:

Exploitation as Efficient Use of Labour (**EEUL**): C*onsider a definition of exploitation satisfying* **LES**. *Given any economy* $\langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$, *let* $\left((p, w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}\right)$ *be a* RS *such that* $\overline{c}, \underline{c} \in \mathbb{R}_+^m$ *are the corresponding reference bundles. Then,* $\alpha_0^{\overline{c}}, \alpha_0^{\underline{c}} \geqq l.v.(b)$.

In the canonical Marxian theory of exploitation in the Okishio-Leontief framework, socially necessary labour time is uniquely defined and, since $\alpha_0^{\overline{c}} = \alpha_0^{\underline{c}} = l.v.(b)$, it identifies the sets of exploiters and exploited agents. Axiom **EEUL** generalises such a theory to convex subsistence economies, and in fact, this axiom represents a condition which the classical Marxian theory of labour value and exploitation satisfies if a subsistence economy $\langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$ is characterised by a simple Leontief technology $P = P_{(A,I,L)}$, where A and I are square matrices. However, this is a rather weak requirement in the sense that most of the formulations of exploitation discussed in this paper satisfy it.

In order to derive the characterisation result, it is necessary to slightly modify the axiom **FNE**.

Feasibility of Non-exploitation* (FNE*): C*onsider a definition of exploitation satisfying* **LES**. *Given any economy* $E \in \mathcal{E}$, *let* (p, w) *be an efficient RS and let* $\overline{c}, \underline{c} \in \mathbb{R}_+^m$ *be the corresponding reference bundles. Then, for any* non-exploitative allocation $(\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu \in N}$ at (p, w), *there exists* (p', w') *such that for some suitable redistribution* $(\omega'^\nu)_{\nu \in N}$ *of* $(\omega^\nu)_{\nu \in N}$ *with* $\sum_{\nu \in N} \omega'^\nu = \sum_{\nu \in N} \omega^\nu$, $(\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu \in N}$ *is* feasible non-exploitative with respect to (p', w') in $\langle N; (P, b); (\omega'^\nu)_{\nu \in N} \rangle$.

FNE* is slightly stronger than **FNE**, because it does not focus on non-exploitative allocations at the givel level of social endowments: implementability is imposed regardless of whether the non-exploitative allocation is physically feasible in the current economy or not.

Given **FNE***, it is now possible to derive an alternative characterisation of Definition 7.

Theorem 6: *A formulation of labour exploitation satisfies* **LES, FNE***, **EEUL,** and **IES** *if and only if for all* $E \in \mathcal{E}$ *and every* RS (p, w), $\alpha_0^{c} = \alpha_0^{\bar{c}} = \frac{\alpha_0 + \beta_0}{N}$.

In other words, one may argue that there are two possible views on exploitation. One is that exploitation is a notion that characterises social relations in market economies, as described by axiom **RE**. Alternatively, one may insist that exploitation theory is based on a notion of 'surplus labour' and socially necessary labour. Quite surprisingly, Theorems 5 and 6 prove that the two approaches coincide in the rather general context analysed in this paper and our extension of Dumenil-Foley's "New Interpretation" (Definition 7) is the only definition incorporating both perspectives.

Finally, from Theorems 3 and 5, the next result follows:

Corollary 3: Definition 7 *is the sole formulation of labour exploitation satisfying* **LES**, **FNE**, **RE**, *and* **IES**, *and under which the* **CECP** *holds at every* RS (p, w) *with* $\pi^{\max} > 0$ *for all* $E \in \mathcal{E}$.

Thus, the extension of Dumenil-Foley's "New Interpretation" (Definition 7) is the sole formulation of exploitation which satisfies all axioms and which preserves an important property of exploitation theory, namely the **CECP** in general convex economies. Of course, a similar corollary can be derived using Theorem 6.

6 Concluding remarks

This paper provides a formal analysis of exploitation in convex cone subsistence economies with rational optimising agents. The distributive injustice associated with exploitative relations is rigorously explored and the relevance of a notion of exploitation that emphasises the unequal exchange of labour is defended. Firstly, a definition of exploitation is provided according to which exploitative relations involve an unequal distribution of (and control over) social labour, consistently with a normative approach that focuses on individual well-being freedom and the self-realisation of men. The main results of Roemer's classical theory are generalised and it is proved that this definition - actually, a whole *class* of definitions of exploitation - preserves all the classical Marxist insights on exploitation in a framework that is considerably more general than the standard Leontief, or von Neumann economies: every agent's class and exploitation status emerges in the competitive equilibrium; there is a correspondence between an agent's class and exploitation status; and the existence of exploitation is inherently linked to the existence of positive profits. Thus, many important properties of exploitation theory are shown to be considerably more robust than is generally thought.

Secondly, and perhaps more importantly, unlike in the previous literature, this paper develops a new, axiomatic approach to exploitation theory. The core intuitions behind the concept of exploitation are thus directly addressed, in order to derive the appropriate definition of exploitation from first principles, rather than from ad hoc attempts to eschew a number of technical or theoretical problems. The main positive and normative aspects of exploitation theory are explicitly formalised in a set of axioms that every labour-based definition of exploitation should arguably satisfy. The axioms seem rather weak and reasonable, but they uniquely characterise the definition of exploitation adopted in the paper, which focuses on the distribution of social labour and which, quite interestingly, is conceptually related to the so-called "New-Interpretation" (Dumenil, 1980; Foley, 1982).

To be sure, the main results presented above hold in the rather large class of convex cone, subsistence economies considered in the paper, but nothing is said, for example, concerning economies in which agents accumulate, or have richer, and possibly heterogeneous preferences. The extension of the analysis to different contexts, or under alternative assumptions concerning agents' behaviour, indeed represents an interesting line for further research, but two important points should be made concerning the contributions of this paper. Firstly, although the details of the analysis are likely to change if different economies are analysed, the main insights of the paper, concerning the normative and positive relevance of exploitation and the appropriate definition of exploitation, would not be affected: they seem independent of the specific (subsistence) framework and their relevance seems to extend beyond the latter. Thus, for example, even if agents are allowed to consume different bundles of goods, an appropriate definition may still be seen as identifying a normatively relevant reference bundle - such as the 'average consumption bundle' or the per-capita value of net domestic product - and requiring that the value of labour power be measured accordingly. Secondly, and perhaps more importantly, the rigorous axiomatic analysis developed in this paper suggests a fruitful methodological approach to exploitation that can be applied to different contexts, and that should frame any future debates in exploitation theory.

7 Appendix

7.1 Appendix 1: The existence of a RS

This appendix provides a complete characterisation of reproducible solutions. Without loss of generality, let $S \equiv \left\{p \in \mathbb{R}^m_+ \mid pb = 1\right\}$ be the set of normalised price vectors. Let $\alpha_0(\omega) \equiv \max\{\alpha_0 \mid \exists\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \text{ s.t. } \underline{\alpha} \leqq \omega\}$. Given the production set P, let $\widetilde{P} \equiv \left\{(-\alpha_0, \overline{\alpha}) \in \mathbb{R}_- \times \mathbb{R}^m \mid (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P\right\}$ and let

$$\mathbb{C} \equiv \left\{\omega \in \mathbb{R}^m_+ \mid \exists\alpha \in P : \underline{\alpha} \leqq \omega,\ \overline{\alpha} - Nb \geqq \underline{\alpha},\ \overline{\alpha} - Nb \not> \underline{\alpha}\ \&\ (-\alpha_0, Nb + \underline{\alpha}) \in \partial\widetilde{P}\right\}.$$

That is, $\omega \in \mathbb{C}$ implies that there exists a production vector that is feasible from ω and produces Nb as net output. Moreover, such a production point is *weakly efficient* in the sense that $(-\alpha_0, Nb + \underline{\alpha}) \in \partial\widetilde{P}$. Note that by A1∽A3, $\mathbb{C}$ is non-empty, closed, and convex.

In order to provide a full characterisation of RS's, two more sets must be defined. Given the production set P and the scalar $w \geqq 1$, let $\widetilde{P}(w) \equiv \left\{\overline{\alpha} - w\alpha_0 b \in \mathbb{R}^m_+ \mid (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P\ \&\ \widehat{\alpha} - w\alpha_0 b \in \mathbb{R}^m_+\right\}$ and let

$$\mathbb{C}^* \equiv \left\{\omega \in \mathbb{R}^m_+ \mid \exists\alpha \in P\ \&\ \exists w \geqq 1 : \underline{\alpha} \leqq \omega,\ \overline{\alpha} - Nb \geqq \underline{\alpha},\ \overline{\alpha} - Nb \not> \underline{\alpha},\right.$$
$$\left.\&\ Nb + \underline{\alpha} - w\alpha_0 b \in \partial\widetilde{P}(w)\right\}.$$

That is, $\omega \in \mathbb{C}^*$ implies that there exists a production vector that is feasible from ω and produces Nb as net output. Moreover, its associated 'surplus product' $Nb - w\alpha_0 b$ is a non-negative vector for some $w \geqq 1$. Finally, such a production point is also *weakly efficient* in the sense that $Nb + \underline{\alpha} - w\alpha_0 b \in \partial\widetilde{P}(w)$ for some $w \geqq 1$. Note that $\mathbb{C}^* \subseteq \mathbb{C}$ and by A1∽A3, $\mathbb{C}^*$ is also non-empty, closed, and convex. Theorem A.1 proves a necessary condition for a RS to exist.

Theorem A.1: *Let $b \in \mathbb{R}^m_{++}$, $\omega \in \mathbb{R}^m_+$. Under A1∽A3, if a* reproducible solution (RS) *exists for the economy $E = \langle N; (P, b); (\omega^\nu)_{\nu\in N}\rangle$ then $\omega \in \mathbb{C}$. Furthermore, if there is some agent ν with $p\omega^\nu = 0$, then $\omega \in \mathbb{C}^*$.*

Proof. **1.** Let (p, w) be a RS with its corresponding aggregate production vector $\alpha + \beta \in P$, and such that $p \in S$. First, $\underline{\alpha} + \underline{\beta} \leqq \omega$ and $\overline{\alpha} + \overline{\beta} - Nb \geqq \underline{\alpha} + \underline{\beta}$ immediately follow from Definition 1(b)-(c).

Next, Propositions 1(i) and 2 imply that $\overline{\alpha} + \overline{\beta} - Nb \not> \underline{\alpha} + \underline{\beta}$. Finally, at the RS, $\pi^{\max} = \frac{pNb - w(\alpha_0+\beta_0)}{p(\underline{\alpha}+\underline{\beta})}$, and by Proposition 2, it is possible to choose a positive number $\lambda \geqq 1$ such that $\lambda p \left(\underline{\alpha} + \underline{\beta}\right) = p\omega$, and therefore $1 + \pi^{\max} = \frac{pN\lambda b + p\lambda(\underline{\alpha}+\underline{\beta}) - w\lambda(\alpha_0+\beta_0)}{p\omega}$. This implies that $pN\lambda b + p\lambda \left(\underline{\alpha} + \underline{\beta}\right) - w\lambda \left(\alpha_0 + \beta_0\right) \geq p\widehat{\alpha}' + p\underline{\alpha}' - w\alpha_0'$ for all $\left(-\alpha_0', \widehat{\alpha}' + \underline{\alpha}'\right) \in \widetilde{P}$ such that $p\underline{\alpha}' = p\omega$. Therefore $\left(-\lambda \left(\alpha_0 + \beta_0\right), N\lambda b + \lambda \left(\underline{\alpha} + \underline{\beta}\right)\right) \in \partial\widetilde{P}$ and (p, w) is a supporting price of it. Since $\widetilde{P}$ is a convex cone, $\left(-\left(\alpha_0 + \beta_0\right), Nb + \left(\underline{\alpha} + \underline{\beta}\right)\right) \in \partial\widetilde{P}$ and (p, w) is a supporting price of it.

2. Suppose there is some agent ν with $p\omega^{\nu} = 0$. The first part of the proof is as in step 1. Then note that since $p \in S$ and the upper bound of labour supply is one, it follows that $w \geqq 1$, for otherwise agent ν with $p\omega^{\nu} = 0$ cannot survive even if she supplies the maximum amount of labour. Finally, choose a positive number $\lambda \geqq 1$ such that $\lambda p \left(\underline{\alpha} + \underline{\beta}\right) = p\omega$ holds. Then, $1 + \pi^{\max} = \frac{pN\lambda b + p\lambda(\underline{\alpha}+\underline{\beta}) - w\lambda(\alpha_0+\beta_0)}{p\omega}$. This implies that $pN\lambda b + p\lambda \left(\underline{\alpha} + \underline{\beta}\right) - w\lambda \left(\alpha_0 + \beta_0\right) \geqq p\widehat{\alpha}' + p\underline{\alpha}' - w\alpha_0'$ for all $\widehat{\alpha}' + \underline{\alpha}' - w\alpha_0' b \in \widetilde{P}(w)$ such that $p\underline{\alpha}' = p\omega$. Moreover, since $\pi^{\max} \geqq 0$ at a RS and Lemma 1 holds, it follows from $b \in \mathbb{R}^m_{++}$ that $Nb - w\left(\alpha_0 + \beta_0\right) b \geqq 0$, which implies that $N\lambda b + \lambda \left(\underline{\alpha} + \underline{\beta}\right) - \lambda w \left(\alpha_0 + \beta_0\right) b \in \partial\widetilde{P}(w)$ and $p \in S$ is a supporting price of it. Since $\widetilde{P}(w)$ is a convex cone, $Nb + \left(\underline{\alpha} + \underline{\beta}\right) - w\left(\alpha_0 + \beta_0\right) b \in \partial\widetilde{P}(w)$ and $p \in S$ is a supporting price of it. ■

Define a domain of social endowments slightly smaller than $\mathbb{C}^*$ as follows:

$$\mathbb{C}^{**} \equiv \Big\{\omega \in \mathbb{R}^m_+ \mid \exists\alpha \in P \;\&\; \exists w \geqq 1 : \underline{\alpha} = \omega, \overline{\alpha} - Nb \geqq \underline{\alpha}, \overline{\alpha} - Nb \not> \underline{\alpha}, \\ \& \; Nb + \underline{\alpha} - w\alpha_0 b \in \partial\widetilde{P}(w)\Big\}.$$

Within this domain, the existence of a RS can be proved. For any given $\omega \in \mathbb{C}^{**}$, define the set

$$A(\omega) \equiv \\ \Big\{\alpha \in P \& \; w \geqq 1 \mid \underline{\alpha} = \omega, \overline{\alpha} - Nb \geqq \underline{\alpha}, \overline{\alpha} - Nb \not> \underline{\alpha}, \& \; Nb + \underline{\alpha} - w\alpha_0 b \in \partial\widetilde{P}(w)\Big\}.$$

Theorem A.2: *Let $b \in \mathbb{R}^m_{++}$, $\omega \in \mathbb{R}^m_+$ and $\alpha_0(\omega) \leqq N$. Under A1∽A3, if $\omega \in \mathbb{C}^{**}$ then there exists $(\alpha^*, w) \in A(\omega)$ such that there exists $p \in S$ that supports $Nb + \underline{\alpha}^* - w\alpha_0^* b \in \partial\widetilde{P}(w)$ in $\widetilde{P}(w)$. Furthermore, if $(\omega^{\nu})_{\nu \in N}$*

is such that $\sum_{\nu \in N} \omega^\nu = \omega$ *and* $pb - \pi^{\max} p\omega^\nu \geqq 0$ *for all* $\nu \in N$, *then a* reproducible solution (RS) *exists for the economy* $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$ at (p, w).

Proof. **1.** By definition, if $\omega \in \mathbb{C}^{**}$, there exists $\alpha^* = (-\alpha_0^*, -\underline{\alpha}^*, \overline{\alpha}^*) \in P$ such that $\underline{\alpha}^* = \omega$, $\overline{\alpha}^* - Nb \geqq \underline{\alpha}^*$ with $\overline{\alpha}^* - Nb \not> \underline{\alpha}^*$, and $Nb + \underline{\alpha}^* - w\alpha_0^* b \in \partial \widetilde{P}(w)$ for some $w \geqq 1$. Therefore the aggregate production vector α^* satisfies Definition 1, parts (b) and (d). It must be shown that there is a price vector such that α^* emerges from individually optimal choices and that the labour market clears. Note that since it is assumed that $\alpha_0(\omega) \leqq N$, it follows that $\overline{\alpha}^* - \alpha_0^* b \geqq \overline{\alpha}^* - Nb$. Since $Nb + \underline{\alpha}^* - w\alpha_0^* b \in \partial \widetilde{P}(w)$, by the supporting hyperplane theorem, there exists $p \in S$ that supports $Nb + \underline{\alpha}^* - w\alpha_0^* b \in \partial \widetilde{P}(w)$ in $\widetilde{P}(w)$. That is, $pNb + p\underline{\alpha}^* - w\alpha_0^* \geqq p\widehat{\alpha} + p\underline{\alpha} - w\alpha_0$ for all $\widehat{\alpha} + \underline{\alpha} - w\alpha_0 b \in \widetilde{P}(w)$. Hence, given that $\underline{\alpha}^* = \omega$, $pNb + p\underline{\alpha}^* - w\alpha_0^* \geqq p\widehat{\alpha} + p\underline{\alpha} - w\alpha_0$ holds for all $\widehat{\alpha} + \underline{\alpha} - w\alpha_0 b \in \widetilde{P}(w)$ such that $p\underline{\alpha} = p\omega$. Therefore, since $\widetilde{P}(w)$ is a convex cone, the previous argument shows that $(-\alpha_0^*, -\underline{\alpha}^*, Nb + \underline{\alpha}^*)$ realises the maximal profit rate at (p, w). However, since $\overline{\alpha}^* - Nb \geqq \underline{\alpha}^*$, α^* also realises the maximal profit rate at (p, w), and $p\widehat{\alpha}^* = pNb$. Furthermore, since $\widehat{\alpha}^* + \underline{\alpha}^* - w\alpha_0^* b \in \widetilde{P}(w)$, it follows that $pNb - w\alpha_0^* \geqq 0$, which implies $\pi^{\max} \geqq 0$.

2. Let $(\omega^\nu)_{\nu \in N}$ be a distribution of initial endowments such that $\sum_{\nu \in N} \omega^\nu = \omega$. Consider the price vector (p, w) derived in step 1. Note that $\alpha^* \in \overline{P}(p, w)$ and $\underline{\alpha}^* = \omega$. Let $\pi^{\max} \equiv \frac{p\widehat{\alpha}^* - w\alpha_0^*}{p\underline{\alpha}^*}$. Suppose that $(\omega^\nu)_{\nu \in N}$ satisfies the condition in the second part of the statement, so that $pb - \pi^{\max} p\omega^\nu \geqq 0$, for all $\nu \in N$. Then, let $\theta^\nu \equiv \frac{p\omega^\nu}{p\underline{\alpha}^*}$ for all $\nu \in N$, and let $\beta^\nu \equiv \theta^\nu \underline{\alpha}^*$ for all $\nu \in N$. Then, by definition, $p\underline{\beta}^\nu = p\omega^\nu$ and $\beta^\nu \in \overline{P}(p, w)$ for all $\nu \in N$. Moreover, let $\gamma^\nu \equiv \frac{pb - \pi^{\max} p\underline{\beta}^\nu}{w}$ for all $\nu \in N$. Then, by definition, $\frac{pb - \pi^{\max} p\underline{\beta}^\nu}{w} \geqq 0$ for all $\nu \in N$. Since $w \geqq 1$ and $pb = 1$, it follows that $1 \geqq \gamma_0^\nu \geqq 0$, all $\nu \in N$. Therefore by Lemma 3, $(\mathbf{0}; \beta^\nu; \gamma_0^\nu)_{\nu \in N} \in \times_{\nu \in N} \mathcal{A}^\nu(p, w)$. Furthermore

$$\sum_{\nu \in N} \gamma_0^\nu = \frac{\sum_{\nu \in N} \left(pb - \pi^{\max} p\underline{\beta}^\nu\right)}{w} = \frac{\left(Npb - \pi^{\max} \sum_{\nu \in N} p\underline{\beta}^\nu\right)}{w} = \frac{\left(Npb - \pi^{\max} p\underline{\alpha}^*\right)}{w}.$$

This implies $\sum_{\nu \in N} \gamma_0^\nu = \alpha_0^* = \sum_{\nu \in N} \beta_0^\nu$, which completes the proof. ■

Given Theorem A.2, it is possible to state a simpler sufficient condition for the existence of a RS. Consider the following domain of social endowments:

$$\mathbb{C}^{***} \equiv \left\{ \omega \in \mathbb{R}_{+}^{m} \mid \exists \alpha \in P \ \& \ \exists w \geqq 1 : \underline{\alpha} = \omega, \overline{\alpha} - Nb \geqq \underline{\alpha}, \overline{\alpha} - Nb \not> \underline{\alpha}, \right.$$
$$\left. \alpha_0 \geqq \tfrac{N-1}{w} \ \& \ Nb + \underline{\alpha} - w\alpha_0 b \in \partial \widetilde{P}(w) \right\}.$$

Corollary A.1: *Let* $b \in \mathbb{R}_{++}^{m}$, $\omega \in \mathbb{R}_{+}^{m}$ *and* $\alpha_0(\omega) \leqq N$. *Under A1∽A3, if* $\omega \in \mathbb{C}^{***}$ *then a* reproducible solution (RS) *exists for the economy* $E = \langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$.

7.2 Appendix 2: Proofs

Proof of Lemma 1: Suppose, contrary to the statement, that $p\widehat{\alpha}^{\nu} + p\widehat{\beta}^{\nu} - w\beta_0^{\nu} + w\gamma_0^{\nu} > pb$ for some ν. If $\alpha_0^{\nu} + \gamma_0^{\nu} > 0$, then by the convex cone property of the production set P, agent ν can reduce either γ_0^{ν} or α_0^{ν} without violating feasibility, which contradicts optimality. If $\alpha_0^{\nu} + \gamma_0^{\nu} = 0$, then by the convex cone property of the production set P, agent ν can reduce $\underline{\beta}^{\nu}$ without violating feasibility, which contradicts **NBC**. ■

Proof of Lemma 2: Suppose, contrary to the statement, that $p\underline{\alpha}^{\nu} + p\underline{\beta}^{\nu} < p\omega^{\nu}$. Then, by increasing the investment of capital and hiring other agents, ν can increase her profits and reduce her labour expended, while reaching subsistence, which contradicts optimality. ■

Proof of Lemma 3: By the convexity of MP^{ν}, it follows that $(\alpha'^{\nu}; \ \beta'^{\nu})$ is technically feasible with $\beta_0'^{\nu} + \alpha_0'^{\nu} = \beta_0^{\nu} + \alpha_0^{\nu}$. This implies that labour expenditure is the same in both production plans, since $(\gamma_0'^{\nu} + \alpha_0'^{\nu}) - (\beta_0'^{\nu} + \alpha_0'^{\nu}) = (\gamma_0^{\nu} + \alpha_0^{\nu}) - (\beta_0^{\nu} + \alpha_0^{\nu})$. By Lemma 1, noting that only processes that yield the maximal profit rate are operated, it is possible to write $\pi^{\max}(p\underline{\alpha}^{\nu} + p\underline{\beta}^{\nu}) + w\alpha_0^{\nu} + w\gamma_0^{\nu} = pb$. But then, it is immediate to check that $(\alpha'^{\nu}; \ \beta'^{\nu}; \ \gamma_0'^{\nu})$ yields the same amount of net revenue and capital outlay. ■

Proof of Proposition 1: Part (i). Suppose, contrary to the statement, that $p = 0$, which implies $pb = 0$. Then, at the solution to MP^{ν}, it will be $\alpha_0^{\nu} + \gamma_0^{\nu} = 0$, all ν, and thus $\alpha_0 + \gamma_0 = 0$. However, by A1 $\alpha_0 + \gamma_0 = 0$ implies that $\overline{\alpha} + \overline{\beta} = \mathbf{0}$, which contradicts part (d) of the Definition of RS.

Part (ii). Suppose, contrary to the statement, that $\pi^{\max} < 0$. By Lemma 1, for all ν, $(p\widehat{\alpha}^{\nu} - w\alpha_0^{\nu}) + \left(p\widehat{\beta}^{\nu} - w\beta_0^{\nu}\right) + w\alpha_0^{\nu} + w\gamma_0^{\nu} = pb$ where $(p\widehat{\alpha}^{\nu} - w\alpha_0^{\nu}) < 0$ and $\left(p\widehat{\beta}^{\nu} - w\beta_0^{\nu}\right) < 0$. Hence, at the solution to MP^{ν} it must be $\alpha^\nu = \beta^\nu = \mathbf{0}$, all ν, which contradicts part (d) of the Definition of RS.

Part (iii). Suppose, contrary to the statement, that $w \leqq 0$. Then, at the solution to MP^ν, it will be $\gamma_0^\nu = 0$ for all ν. Further, $\pi^{\max} \leqq 0$ can be ruled out, because by part (i), $pb > 0$. However, if $\pi^{\max} > 0$, then at the solution to MP^ν it will be $\beta_0^\nu > 0$ for all ν with $p\omega^\nu > 0$, contradicting part (c) of the Definition of RS. ■

Proof of Proposition 2: By Lemma 1, the net revenue constraint of every agent holds as an equality. Summing over ν, one obtains $p\widehat{\alpha}+p\widehat{\beta}-w\beta_0+w\gamma_0 = pNb$ and using part (c) of the Definition of RS, $p\widehat{\alpha} + p\widehat{\beta} = pNb$. The result then follows by part (d) of the Definition of RS. ■

Proof of Proposition 3: By Lemma 1, $p\widehat{\alpha}^\nu+p\widehat{\beta}^\nu-w\beta_0^\nu-w\alpha_0^\nu+w(\gamma_0^\nu+\alpha_0^\nu) = pb$ holds for all ν. Noting that only processes yielding the maximal profit rate will be used, the latter expression can be written as $\pi^{\max}(p\underline{\alpha}^\nu + p\underline{\beta}^\nu) + w\alpha_0^\nu + w\gamma_0^\nu = pb$ for all ν. The result then follows by Proposition 1(iii) and Lemma 2. ■

Proof of Theorem 1: Let $\overline{c}, \underline{c} \in B(p,b)$ be the reference consumption bundles of a given definition satisfying **LES**. For this definition, $\nu \in N^{ter}$ if and only if there is a $\alpha^{\overline{c}} \in \phi(\overline{c})$ such that $\widehat{\alpha}^{\overline{c}} = \overline{c}$ and $\alpha_0^{\overline{c}} > \alpha_0^\nu + \gamma_0^\nu$, $\nu \in N^{ted}$ if and only if there is a $\alpha^{\underline{c}} \in \phi(\underline{c})$ such that $\widehat{\alpha}^{\underline{c}} = \underline{c}$ and $\alpha_0^{\underline{c}} < \alpha_0^\nu + \gamma_0^\nu$, and $\nu \in N\backslash\left(N^{ter} \cup N^{ted}\right)$ if and only if $\alpha_0^{\overline{c}} \leqq \alpha_0^\nu + \gamma_0^\nu \leqq \alpha_0^{\underline{c}}$. Note that by A1, $\alpha_0^{\overline{c}} > 0$ and $\alpha_0^{\underline{c}} > 0$. Moreover, by Proposition 1(iii), it follows that $w\alpha_0^{\underline{c}} \geqq w\alpha_0^{\overline{c}} > 0$.

Consider agent ν with $\alpha_0^\nu + \gamma_0^\nu = 0$ at the solution to MP^ν. Since $\alpha_0^{\overline{c}} > 0$, such an agent is an exploiter by the above characterisation of N^{ter}. By Proposition 3, it follows that $W^\nu \geqq \frac{pb}{\pi^{\max}} > \frac{1}{\pi^{\max}}\left[pb - w\alpha_0^{\overline{c}}\right]$.

Next, consider any agent ν with $\alpha_0^\nu + \gamma_0^\nu > 0$ at the solution to MP^ν. By Proposition 3, $\alpha_0^\nu + \gamma_0^\nu = \frac{pb-\pi^{\max}W^\nu}{w}$ and therefore, by **LES**, ν will be an exploiter if and only if $\frac{pb-\pi^{\max}W^\nu}{w} < \alpha_0^{\overline{c}}$, which holds if and only if $W^\nu > \frac{1}{\pi^{\max}}\left[pb - w\alpha_0^{\overline{c}}\right]$. The other two conditions follow in like manner. ■

Proof of Lemma 4: 1. First, note that by the convexity of MP^ν, it follows that if $\gamma_0^\nu < \beta_0^\nu$ for some optimal $(\alpha^\nu;\beta^\nu;\gamma_0^\nu)$ and $\gamma_0^{\prime\nu} > \beta_0^{\prime\nu}$ for some other optimal $(\alpha^{\prime\nu};\beta^{\prime\nu};\gamma_0^{\prime\nu})$, then there is a solution to MP^ν such that $\gamma_0^{\prime\prime\nu} = \beta_0^{\prime\prime\nu}$. Therefore, the three cases in the statement are mutually exclusive and they decompose agents with $W^\nu > 0$ and $\Lambda^\nu > 0$ at the solution of MP^ν into disjoint sets.

2. Suppose $\gamma_0^\nu < \beta_0^\nu$ for all optimal $(\alpha^\nu; \beta^\nu; \gamma_0^\nu)$. If $\gamma_0^\nu = 0$, then clearly $\nu \in (+,+,0)$ because by assumption $\Lambda^\nu > 0$ and $0 = \gamma_0^\nu < \beta_0^\nu$. If $\gamma_0^\nu > 0$, then construct $(\alpha'^\nu; \beta'^\nu; \gamma_0'^\nu)$ such that $\gamma_0'^\nu = 0$, $\beta_0'^\nu = \beta_0^\nu - \gamma_0^\nu$ and $\alpha_0'^\nu = \alpha_0^\nu + \gamma_0^\nu$. By Lemma 3, $(\alpha'^\nu; \beta'^\nu; \gamma_0'^\nu)$ is also optimal (it yields the same amount of net revenue, labour expenditure, and capital outlay). It is sufficient to show that $\nu \notin (+,0,0)$. Suppose, contrary to the latter statement, that ν has an optimal solution of the form $(\alpha^\nu; 0; 0)$. As in Lemma 3, it is possible to show that ν also has a solution $(\alpha'^\nu; \beta'^\nu; \gamma_0'^\nu)$ such that $\alpha'^\nu = 0$, $\beta'^\nu = \alpha^\nu$ and $\gamma_0'^\nu = \alpha_0^\nu$. But this contradicts the assumption that all optimal solutions for ν are such that $\gamma_0^\nu < \beta_0^\nu$.

3. The other two cases are proved similarly. ■

Proof of Theorem 2: Part (i). First, $\nu \in C^5$ if and only if $W^\nu = 0$. Next, by Proposition 3, $\nu \in C^1$ if and only if $W^\nu \geqq \frac{pb}{\pi^{\max}}$. The result then follows from Lemma 4.

Part (ii). It is sufficient to prove that the wealth ordering in the statement is true for agents ν, $\mu \in N$ with $0 < W^\nu < \frac{pb}{\pi^{\max}}$ and $0 < W^\mu < \frac{pb}{\pi^{\max}}$. By Lemma 4, it follows that $\nu, \mu \in \cup_{i=2,3,4} C^i$. Suppose $\nu \in C^4$ and $\mu \in C^3$ but $W^\nu > W^\mu$. ($W^\nu = W^\mu$ is ruled out by the disjointedness of classes.) Since $\mu \in C^3$, by reversing the reasoning in Lemma 4, it is possible to show that μ has an optimal solution such that $\alpha'^\mu = \mathbf{0}$, $\beta'^\mu = \alpha^\mu$ and $\gamma_0'^\mu = \alpha_0^\mu$, with $\beta_0'^\mu = \gamma_0'^\mu$. Next, by Lemma 3, consider ν's solutions of the form $(\mathbf{0}; \beta^\nu; \gamma_0^\nu)$. Since working time is strictly decreasing in wealth, $W^\nu > W^\mu$ implies $\Lambda^\nu = \gamma_0^\nu < \Lambda^\mu = \gamma_0'^\mu$. However, $\beta_0'^\mu = \gamma_0'^\mu$ and $\beta_0^\nu < \gamma_0^\nu$ implies $\beta_0^\nu < \beta_0'^\mu$ which is impossible given that $W^\nu > W^\mu$ and thus agent ν can hire more labour than μ by investing in sectors with the maximal profit rate $\pi^{\max} > 0$. A similar argument proves that if $\nu \in C^2$ and $\mu \in C^3$ then $W^\nu > W^\mu$. ■

Proof of Proposition 4: 1. By Lemma 4, Proposition 3, and Theorem 2(ii), it is sufficient to prove part (iii) of the statement.

2. Suppose $p\underline{\alpha}_{\min} \leqq W^\nu \leqq p\underline{\alpha}_{\max}$. By Lemma 3, it is possible to consider ν's solutions of the form $(\mathbf{0}; \beta^\nu; \gamma_0^\nu)$, without loss of generality. By optimality, at the solution to MP^ν, $p\widehat{\beta}^\nu - w\beta_0^\nu + w\gamma_0^\nu = pb$, or equivalently $\pi^{\max} p\underline{\beta}^\nu + w\Lambda^\nu = pb$, with $p\underline{\beta} = W^\nu$. But then, since $p\underline{\alpha}_{\min} \leqq W^\nu \leqq p\underline{\alpha}_{\max}$, by the convexity of P, it follows that there exists some $\alpha \in P$, such that $\pi^{\max} p\underline{\alpha} +$

$w\alpha_0 = pb$, with $p\underline{\alpha} = W^\nu$. The latter equation implies that $\alpha_0 = \Lambda^\nu$, and thus Γ^ν has a solution of the form $(+, \mathbf{0}, 0)$.

3. Conversely, suppose that $\nu \in C^3$, so that Γ^ν has a solution of the form $(+, 0, \mathbf{0})$. This implies that there exists $\alpha \in P$ such that $\pi^{\max} p\underline{\alpha} + w\alpha_0 = pb$, with $p\underline{\alpha} = W^\nu$, which implies $p\underline{\alpha}_{\min} \leqq W^\nu \leqq p\underline{\alpha}_{\max}$. ■

Proof of Theorem 3: The result follows immediately from Theorem 1 and Proposition 4. ■

Proof of Corollary 2: 1. First, consider Definition 7. By Theorem 3, what needs to be shown is that $p\underline{\alpha}_{\min} \leqq \frac{1}{\pi^{\max}}\left[pb - w\left(\frac{(\alpha_0+\beta_0)}{N}\right)\right] \leqq p\underline{\alpha}_{\max}$ holds. Let $\overline{W} \equiv \frac{1}{\pi^{\max}}\left[pb - \frac{w(\alpha_0+\beta_0)}{N}\right]$. Then, $\pi^{\max}\overline{W} + \frac{w(\alpha_0+\beta_0)}{N} = pb$. By definition, $\overline{W} = \frac{p(\underline{\alpha}+\underline{\beta})}{N}$ holds, since by Lemma 1, $p\left(\widehat{\alpha}+\widehat{\beta}\right) = Npb$ holds, and at a RS, only processes yielding the maximal profit rate are operated. Then, $\frac{\alpha_{0\,\max}}{p\underline{\alpha}_{\max}} \leqq \frac{\alpha_0+\beta_0}{p(\underline{\alpha}+\underline{\beta})} \leqq \frac{\alpha_{0\,\min}}{p\underline{\alpha}_{\min}}$ by the optimality of production plans. Note that $\frac{\pi^{\max}p(\underline{\alpha}+\underline{\beta})}{\alpha_0+\beta_0} = \frac{pb}{(\alpha_0+\beta_0)/N} - w$, $\frac{\pi^{\max}p\underline{\alpha}_{\min}}{\alpha_{0\,\min}} = \frac{pb}{\alpha_{0\,\min}} - w$, and $\frac{\pi^{\max}p\underline{\alpha}_{\max}}{\alpha_{0\,\max}} = \frac{pb}{\alpha_{0\,\max}} - w$, which imply that $\frac{pb}{\alpha_{0\,\min}} - w \leqq \frac{pb}{(\alpha_0+\beta_0)/N} - w \leqq \frac{pb}{\alpha_{0\,\max}} - w$. Thus, $\alpha_{0\,\max} \leqq \frac{(\alpha_0+\beta_0)}{N} \leqq \alpha_{0\,\min}$ holds. Given that $\pi^{\max}p\underline{\alpha}_{\min} + w\alpha_{0\,\min} = \pi^{\max}\overline{W} + \frac{w(\alpha_0+\beta_0)}{N} = \pi^{\max}p\underline{\alpha}_{\max} + w\alpha_{0\,\max} = pb$, the last inequality implies that $p\underline{\alpha}_{\min} \leqq \overline{W} \leqq p\underline{\alpha}_{\max}$.

2. Consider next Definitions 5 and 6. As noted above, $\min_{c\in B(p,b)} l.v.(c;p,w) \leqq l.v.(b;p,w) \leqq t(\alpha_0+\beta_0) = \frac{\alpha_0+\gamma_0}{N}$. Note that $l.v.(b;p,w) \leqq \frac{\alpha_0+\gamma_0}{N}$ follows from $pb = \frac{p(\widehat{\alpha}+\widehat{\beta})}{N}$. Hence, by Theorem 3 and step 1 of the proof, it follows that $p\underline{\alpha}_{\min} \leqq \frac{1}{\pi^{\max}}\left[pb - w\frac{\alpha_0+\gamma_0}{N}\right] \leqq \frac{1}{\pi^{\max}}\left[pb - w\left(\min_{c\in B(p,b)} l.v.(c;p,w)\right)\right]$. Hence, what needs to be shown is $\frac{1}{\pi^{\max}}\left[pb - w\left(\min_{c\in B(p,b)} l.v.(c;p,w)\right)\right] \leqq p\underline{\alpha}_{\max}$.

Let $c^* \equiv \arg\min_{c\in B(p,b)} l.v.(c;p,w)$. By the definition of $l.v.(c^*;p,w)$, there exists $\alpha(c^*) \in \overline{P}(p,w)$ such that $\alpha_0(c^*) = l.v.(c^*;p,w)$. In this case, $pc^* = p\widehat{\alpha}(c^*)$ holds. Let us show this. Suppose $pc^* < p\widehat{\alpha}(c^*)$ if $c^* \leq \widehat{\alpha}(c^*)$. Since $p\widehat{\alpha}(c^*) = \pi^{\max}p\underline{\alpha}(c^*) + w\alpha_0(c^*)$, it follows that $pc^* < \pi^{\max}p\underline{\alpha}(c^*) + w\alpha_0(c^*)$. In this case, for some $0 < t < 1$, it is true that $pt\widehat{\alpha}(c^*) = pc^*$ and $t\alpha_0(c^*) = l.v.(t\widehat{\alpha}(c^*);p,w) < \alpha_0(c^*) = l.v.(c^*;p,w)$. This is a contradiction, since $t\widehat{\alpha}(c^*) \in B(p,b)$ and $\alpha_0(c^*) = \min_{c\in B(p,b)} l.v.(c;p,w)$. Thus, $pc^* = p\widehat{\alpha}(c^*)$. The last equation implies that $\alpha(c^*) \in \overline{\Gamma}(p,w)$ and $\widehat{\alpha}(c^*) \in \widehat{\overline{\Gamma}}(p,w)$. Note that , since $c^* \leqq \widehat{\alpha}(c^*)$ by definition, $\alpha(c^*) \in P$

implies $(-\alpha_0(c^*), -\underline{\alpha}(c^*), c^* + \underline{\alpha}(c^*)) \in P$ by A3. Since $pc^* = p\widehat{\alpha}(c^*)$, $\widehat{\alpha}(c^*) \in \widehat{\overline{\Gamma}}(p,w)$ implies that $(-\alpha_0(c^*), -\underline{\alpha}(c^*), c^* + \underline{\alpha}(c^*)) \in \overline{\Gamma}(p,w)$ and $c^* \in \widehat{\overline{\Gamma}}(p,w)$. Thus, since $c^* \in \widehat{\overline{\Gamma}}(p,w)$ implies $pc^* - w\alpha_0(c^*) \leqq \pi^{\max} p\underline{\alpha}_{\max}$, $\frac{1}{\pi^{\max}}\left[pb - w\left(\min_{c \in B(p,b)} l.v.(c;p,w)\right)\right] \leqq p\underline{\alpha}_{\max}$ holds from $pc^* = pb$, which completes the proof. ■

Proof of Proposition 5: Consider the following von Neumann technology:

$$A = \begin{bmatrix} 2 & 2 & 10 \\ 2 & 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 6 & 16 \\ 10 & 10 & 0 \end{bmatrix}, L = \left(1, 1, \frac{4}{3}\right).$$

Given this data, the production possibility set $P_{(A,B,L)}$ can be defined:

$$P_{(A,B,L)} \equiv \left\{\alpha \in \mathbb{R}_- \times \mathbb{R}^2_- \times \mathbb{R}^2_+ \mid \exists x \in \mathbb{R}^3_+ : \alpha \leqq (-Lx, -Ax, Bx)\right\}.$$

Then, consider a convex cone subsistence economy defined by $P_{(A,B,L)}$, $b = (2,2)$, and $\omega = (N,N)$. Let $(\omega^\nu)_{\nu \in N}$ be such that $\omega^\nu = (\delta^\nu, \delta^\nu)$, where $\delta^\nu \leqq 2$, all $\nu \in N$, and $\omega^i = (2,2)$, some $i \in N$.

Let $\mathbf{e}_j \in \mathbb{R}^3_+$ and α^j, $j = 1,2,3$, be defined as in Example 1 above. Then, $\widehat{\alpha}^1 \equiv (0,8)$, $\widehat{\alpha}^2 \equiv (4,8)$, and $\widehat{\alpha}^3 \equiv (6,0)$. Moreover, $\widehat{P}(\alpha_0 = 1) = co\left\{\widehat{\alpha}^1, \widehat{\alpha}^2, \frac{3}{4}\widehat{\alpha}^3, \mathbf{0}\right\}$. In this economy, $p = (1,0)$ and $w = 2$ constitute a *RS* with $\alpha^\nu = 0$, $\beta^\nu = \frac{\delta^\nu \alpha^2}{2}$, and $\gamma_0^\nu = 1 - \frac{\delta^\nu}{2}$ for all $\nu \in N$. The corresponding aggregate production is $\alpha + \beta = \frac{N\alpha^2}{2}$. In such a case, $\pi(\alpha + \beta; p, w) \equiv \frac{p\widehat{\alpha}^2 - w\alpha_0^2}{p\underline{\alpha}^2} = 1$, whereas $\pi(\alpha^1; p, w) \equiv \frac{p\widehat{\alpha}^1 - w\alpha_0^1}{p\underline{\alpha}^1} < 0$ and $\pi(\alpha^3; p, w) \equiv \frac{p\widehat{\alpha}^3 - w\alpha_0^3}{p\underline{\alpha}^3} = \frac{1}{3} = \pi\left(\frac{3}{4}\alpha^3; p, w\right)$. Thus, $\overline{P}(p,w) = \{\alpha \in P \mid \exists \lambda > 0 : \alpha = \lambda\alpha^2\}$ and $\beta^\nu \in \overline{P}(p,w)$ for all $\nu \in N$.

Hence, $p\underline{\alpha}_{\min} = p\underline{\alpha}_{\max} = p\frac{\underline{\alpha}^2}{2} = 1$. In contrast, $l.v.(b) = \frac{17}{36} = \frac{1}{4}\alpha_0^2 + \frac{1}{6}\alpha_0^3$, so that $\frac{1}{\pi^{\max}}[pb - w(l.v.(b))] = 2 - \frac{17}{18} = \frac{19}{18} > 1$. Therefore, every agent $\nu \in C^1 \cup C^2$ with $1 < W^\nu < \frac{19}{18}$ is exploited, and the **CECP** does not hold in this economy if Definition 4 is adopted. ■

Proof of Theorem 4: From Theorem 3, there only remains to prove that (3)⇒(1). Suppose that $\pi^{\max} = 0$. Then, every producer earns the income pb solely from the wage. Thus, to earn the same income, every producer supplies the same amount of labour which is equal to the labour input $\frac{\alpha_0 + \beta_0}{N}$ corresponding to the social production activity per capita. Note that if $\pi^{\max} = 0$, then $\overline{c}, \underline{c} \in \widehat{\Gamma}(p,w)$ implies $w\alpha_0^{\overline{c}} = w\alpha_0^{\underline{c}} = pb = w\frac{\alpha_0 + \beta_0}{N}$. Thus,

$\alpha_0^{\bar{c}} = \frac{\alpha_0+\beta_0}{N} = \alpha_0^{\underline{c}}$, which implies that there is no exploiter nor exploited agent by **LES**. ■

Proof of Lemma 5: Suppose that $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu\in N}$ is not efficient. This implies that there exists another allocation $(\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu\in N}$ such that $\sum_{\nu\in N}\underline{\alpha}'^\nu + \sum_{\nu\in N}\underline{\beta}'^\nu \leqq \sum_{\nu\in N}\omega^\nu$, $Nb \leqq \sum_{\nu\in N}\widehat{\alpha}'^\nu + \sum_{\nu\in N}\widehat{\beta}'^\nu$, $\sum_{\nu\in N}\beta_0'^\nu = \sum_{\nu\in N}\gamma_0'^\nu$, and $\alpha_0'^\nu + \gamma_0'^\nu \leqq \alpha_0^\nu + \gamma_0^\nu$ for all $\nu \in N$ and $\alpha_0'^{\nu^*} + \gamma_0'^{\nu^*} < \alpha_0^{\nu^*} + \gamma_0^{\nu^*}$ for some $\nu^* \in N$. Then, premultiplying the first two inequalities by p, one obtains

$$\begin{aligned}
\sum_{\nu\in N} p\underline{\alpha}'^\nu + \sum_{\nu\in N} p\underline{\beta}'^\nu &\leqq \sum_{\nu\in N} p\omega^\nu,\\
\sum_{\nu\in N} p\widehat{\alpha}'^\nu + \sum_{\nu\in N} p\widehat{\beta}'^\nu &\geqq pNb = \sum_{\nu\in N} p\widehat{\alpha}^\nu + \sum_{\nu\in N} p\widehat{\beta}^\nu,\\
\sum_{\nu\in N} \alpha_0'^\nu + \sum_{\nu\in N} \gamma_0'^\nu &< \sum_{\nu\in N} \alpha_0^\nu + \sum_{\nu\in N} \gamma_0^\nu.
\end{aligned}$$

Let $\beta'' \equiv \sum_{\nu\in N}\alpha'^\nu + \sum_{\nu\in N}\beta'^\nu$ and $\gamma_0'' \equiv \sum_{\nu\in N}\alpha_0'^\nu + \sum_{\nu\in N}\gamma_0'^\nu$. By definition, $\beta_0'' = \gamma_0''$. Thus, since $p\underline{\beta}'' \leqq \sum_{\nu\in N} p\omega^\nu$, $p\widehat{\beta}'' \geqq pNb = \sum_{\nu\in N} p\widehat{\alpha}^\nu + \sum_{\nu\in N} p\widehat{\beta}^\nu$, and $\gamma_0'' < \sum_{\nu\in N}\alpha_0^\nu + \sum_{\nu\in N}\gamma_0^\nu$, it follows that

$$\frac{p\widehat{\beta}'' - w\beta_0''}{p\underline{\beta}''} = \frac{p\widehat{\beta}'' - w\gamma_0''}{p\underline{\beta}''} > \frac{pNb - w\left(\sum_{\nu\in N}\alpha_0^\nu + \sum_{\nu\in N}\gamma_0^\nu\right)}{p\underline{\beta}''}.$$

This implies $\sum_{\nu\in N} p\underline{\alpha}^\nu + \sum_{\nu\in N} p\underline{\beta}^\nu < p\underline{\beta}''$, since $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu\in N}$ is a RS and thus only processes yielding the maximum profit rate are operated. However, since $\sum_{\nu\in N} p\underline{\alpha}^\nu + \sum_{\nu\in N} p\underline{\beta}^\nu = \sum_{\nu\in N} p\omega^\nu$, this implies $p\underline{\beta}'' > \sum_{\nu\in N} p\omega^\nu$, a contradiction. ■

Proof of Proposition 6: The result follows immediately noting that if $\left((p,w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu\in N}\right)$ is efficient then $\alpha_0 + \beta_0 \leqq \sum_{\nu\in N}\alpha_0''^\nu + \sum_{\nu\in N}\gamma_0''^\nu$ for all feasible $(\alpha''^\nu;\ \beta''^\nu;\ \gamma_0''^\nu)_{\nu\in N}$ that satisfy Definition 10(i). Therefore $\alpha_0 + \beta_0 \leqq \alpha_0' + \gamma_0'$. A similar argument holds for $\left((p',w'), (\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu\in N}\right)$ proving $\alpha_0 + \beta_0 \geqq \alpha_0' + \gamma_0'$, which establishes the desired result. ■

Proof of Lemma 6: By the construction of $(\alpha'^\nu)_{\nu\in N}$, $\left((p,w), (\alpha'^\nu;\ 0;\ 0)_{\nu\in N}\right)$ satisfies Definition 1(b), (c), and (d). Therefore, only individual optimality needs to be proved. Since $\sum_{\nu\in N} p\left(\underline{\alpha}^\nu + \underline{\beta}^\nu\right) = \sum_{\nu\in N} p\omega^\nu$, it follows that $\sum_{\nu\in N} p\underline{\alpha}'^\nu = \sum_{\nu\in N} p\omega'^\nu$, so that $p\underline{\alpha}'^\nu = p\omega'^\nu$ holds for all $\nu \in N$. By

Lemma 1, $\sum_{\nu\in N} p\left(\widehat{\alpha}^{\nu}+\widehat{\beta}^{\nu}\right) = Npb$, and therefore $\sum_{\nu\in N} p\widehat{\alpha}'^{\nu} = Npb$, so that $p\widehat{\alpha}'^{\nu} = pb$ holds for all $\nu \in N$. Further, note that, by Lemma 5, the allocation $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}$ is efficient, which implies that $\sum_{\nu\in N}(\alpha_0^{\nu}+\gamma_0^{\nu})$ is the minimal amount of labour expenditure to produce Nb as a social net output under the capital constraint $\sum_{\nu\in N}\omega^{\nu}$. This implies that $\alpha_0'^{\nu}$ is the minimal labour expenditure to produce b under the constraint ω'^{ν} for each $\nu \in N$. Hence, $\alpha_0'^{\nu}$ is the solution of the problem MP^{ν} given the price vector (p,w) in the economy $\langle N; (P,b); (\omega'^{\nu})_{\nu\in N}\rangle$. Since $\sum_{\nu\in N}\alpha'^{\nu} = \sum_{\nu\in N}(\alpha^{\nu}+\beta^{\nu})$ by definition, (p,w) supports $\sum_{\nu\in N}\alpha'^{\nu}$ as a profit-rate maximizing production point. Therefore, $\left((p,w),(\alpha'^{\nu};\ 0;\ 0)_{\nu\in N}\right)$ is a RS for the economy $\langle N; (P,b); (\omega'^{\nu})_{\nu\in N}\rangle$. ■

Proof of Theorem 5:

($\Leftarrow$)**:** It is easy to see that Definition 7 meets **RE** and **IES**. Thus, it suffices to show that Definition 7 meets **FNE**. Let $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ be an efficient RS for the economy $\langle N; (P,b); (\omega^{\nu})_{\nu\in N}\rangle$. Then, by Definition 7, there exists a non-exploitative allocation such that $\alpha_0'^{\nu}+\gamma_0'^{\nu} = \frac{\alpha_0+\beta_0}{N}$ for all $\nu \in N$, and $\underline{\alpha}'+\underline{\beta}' = \underline{\alpha}+\underline{\beta}$ holds. Suppose that $p\left(\underline{\alpha}+\underline{\beta}\right) = p\omega$ holds. Then, by Lemma 6, the non-exploitative allocation is feasible with respect to (p,w) in $\langle N; (P,b); \left(\frac{\omega}{N},\dots,\frac{\omega}{N}\right)\rangle$ with $(\alpha'^{\nu})_{\nu\in N} \in P^N$ such that $\widehat{\alpha}'^{\nu} = \frac{\widehat{\alpha}+\widehat{\beta}}{N}$ and $\alpha_0'^{\nu} = \frac{\alpha_0+\gamma_0}{N}$ for each $\nu \in N$. Suppose that $p\left(\underline{\alpha}+\underline{\beta}\right) < p\omega$ holds. Choose $(\omega'^{\nu})_{\nu\in N}$ such that $p\left(\underline{\alpha}+\underline{\beta}\right) = p\omega'$ with $p\left(\underline{\alpha}^{\nu}+\underline{\beta}^{\nu}\right) = p\omega'^{\nu}$ for all $\nu \in N$. Then, $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ is still an efficient RS for the economy $\langle N; (P,b); (\omega'^{\nu})_{\nu\in N}\rangle$. Then, by Lemma 6, the non-exploitative allocation is feasible with respect to (p,w) in $\langle N; (P,b); \left(\frac{\omega}{N},\dots,\frac{\omega}{N}\right)\rangle$ with $(\alpha'^{\nu})_{\nu\in N} \in P^N$ such that $\widehat{\alpha}'^{\nu} = \frac{\widehat{\alpha}+\widehat{\beta}}{N}$ and $\alpha_0'^{\nu} = \frac{\alpha_0+\gamma_0}{N}$ for each $\nu \in N$. Thus, in any case, Definition 7 meets **FNE**.

($\Rightarrow$)**:** Consider any definition of labour exploitation satisfying **LES**, **FNE**, **RE**, and **IES**.

Case 1, $\pi^{\max} > 0$. Consider an economy $\langle N; (P,b); (\omega^{\nu})_{\nu\in N}\rangle$ with a RS $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ such that $\pi^{\max} > 0$ and $\underline{\alpha}+\underline{\beta} = \sum_{\nu\in N}\omega^{\nu} = \omega$. By Lemma 5, $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}$ is efficient. Suppose the condition $\alpha_0^{c} = \alpha_0^{\bar{c}} = \frac{\alpha_0+\beta_0}{N}$ does not hold.

1. Suppose $\alpha_0^{c} < \frac{\alpha_0+\beta_0}{N}$. Let us consider another economy $\langle N; (P,b); (\omega'^{\nu})_{\nu\in N}\rangle$ with $\omega'^{\nu} = \frac{\omega}{N}$ for any $\nu \in N$. Then, let us consider an allocation $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N} \in (P\times P\times[0,1])^N$ such that $\alpha'^{\nu} = \frac{\alpha+\beta}{N}$, $\beta'^{\nu} = \mathbf{0}$, and $\gamma_0'^{\nu} = 0$ for any $\nu \in N$. Then, by construction, $\left((p,w),(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}\right)$ constitutes a RS with

$\pi^{\max} > 0$ and $\alpha' + \beta' = \alpha + \beta$. Then, by **IES**, its corresponding reference bundles $\overline{c}', \underline{c}' \in \mathbb{R}^m_+$ meet $\overline{c}' = \overline{c}$ and $\underline{c}' = \underline{c}$. Thus, since $\frac{\alpha_0+\beta_0}{N}$ is the labour expenditure of every agent in $\left((p,w), (\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu\in N}\right)$, $\alpha_0^{\underline{c}'} < \frac{\alpha_0+\beta_0}{N}$ implies that every agent is exploited, which contradicts **RE**.

2. Suppose $\alpha_0^{\overline{c}} > \frac{\alpha_0+\beta_0}{N}$. Let us consider the same economy and the same allocation as in the above argument. Then, we can see that $\alpha_0^{\overline{c}'} > \frac{\alpha_0+\beta_0}{N}$ implies that every agent is an exploiter in the RS $\left((p,w), (\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu\in N}\right)$ for the economy $\langle N; (P,b); (\omega'^\nu)_{\nu\in N}\rangle$, which also contradicts **RE**.

3. Suppose $\alpha_0^{\overline{c}} < \frac{\alpha_0+\beta_0}{N} < \alpha_0^{\underline{c}}$. If $\frac{\alpha_0+\beta_0}{N} - \alpha_0^{\overline{c}} \neq \alpha_0^{\underline{c}} - \frac{\alpha_0+\beta_0}{N}$, then we can use the same argument as in steps 1 and 2 above to obtain a contradiction. For instance, if $\frac{\alpha_0+\beta_0}{N} - \alpha_0^{\overline{c}} > \alpha_0^{\underline{c}} - \frac{\alpha_0+\beta_0}{N}$, then we can construct an alternative economy $\langle N; (P,b); (\omega'^\nu)_{\nu\in N}\rangle$ and its RS $\left((p,w), (\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu\in N}\right)$ such that $\alpha' + \beta' = \alpha + \beta$, and there exists one agent $\nu' \in N$ such that $\alpha_0'^{\nu'} + \gamma_0'^{\nu'} > \alpha_0^{\underline{c}}$ and for any other $\nu \in N\backslash\{\nu'\}$, $\alpha_0'^\nu + \gamma_0'^\nu \geqq \alpha_0^{\overline{c}}$. Then, the desired contradiction follows from **IES** and **RE**.

If $\frac{\alpha_0+\beta_0}{N} - \alpha_0^{\overline{c}} = \alpha_0^{\underline{c}} - \frac{\alpha_0+\beta_0}{N}$, then there exists a non-exploitative allocation $(\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu\in N}$ such that $\alpha' + \beta' = \alpha + \beta$, $\alpha_0' + \gamma_0' = \alpha_0 + \gamma_0$, and $\alpha_0'^{\nu'} + \gamma_0'^{\nu'} < \frac{\alpha_0+\beta_0}{N}$ for some $\nu' \in N$. Then, consider $(\alpha''^\nu;\ \beta''^\nu;\ \gamma_0''^\nu)_{\nu\in N}$ such that for any $\nu \in N\backslash\{\nu'\}$, $(\alpha''^\nu;\ \beta''^\nu;\ \gamma_0''^\nu) = (\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)$ and for ν', $\alpha_0''^{\nu'} + \gamma_0''^{\nu'} = \frac{\alpha_0+\beta_0}{N}$, $\left(\beta''^{\nu'}; \gamma_0''^{\nu'}\right) = \left(\beta'^{\nu'}; \gamma_0'^{\nu'}\right)$, and $\left(\underline{\alpha}''^{\nu'}, \overline{\alpha}''^{\nu'}\right) = \left(\underline{\alpha}'^{\nu'}, \overline{\alpha}'^{\nu'}\right)$. By A3, $\left(\alpha_0''^{\nu'}, \underline{\alpha}''^{\nu'}, \overline{\alpha}''^{\nu'}\right) \in P$. Moreover, since $\sum_{\nu\in N} \underline{\alpha}''^\nu + \sum_{\nu\in N} \underline{\beta}''^\nu = \sum_{\nu\in N} \underline{\alpha}'^\nu + \sum_{\nu\in N} \underline{\beta}'^\nu$, $\sum_{\nu\in N} \overline{\alpha}''^\nu + \sum_{\nu\in N} \overline{\beta}''^\nu = \sum_{\nu\in N} \overline{\alpha}'^\nu + \sum_{\nu\in N} \overline{\beta}'^\nu$, $\sum_{\nu\in N} \beta_0''^\nu = \sum_{\nu\in N} \beta_0'^\nu$, and $\sum_{\nu\in N} \gamma_0''^\nu = \sum_{\nu\in N} \gamma_0'^\nu$, if $(\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu\in N}$ is a non-exploitative allocation at (p,w), then so is $(\alpha''^\nu;\ \beta''^\nu;\ \gamma_0''^\nu)_{\nu\in N}$ by Definition 8. Moreover, since $\sum_{\nu\in N} \underline{\alpha}''^\nu + \sum_{\nu\in N} \underline{\beta}''^\nu = \sum_{\nu\in N} \underline{\alpha}'^\nu + \sum_{\nu\in N} \underline{\beta}'^\nu = \omega$, **FNE** can be applied to $(\alpha''^\nu;\ \beta''^\nu;\ \gamma_0''^\nu)_{\nu\in N}$, so that there exist (p', w') and $(\omega'^\nu)_{\nu\in N}$ with $\sum_{\nu\in N} \omega'^\nu = \omega$ such that $\left((p', w'), (\alpha''^\nu;\ \beta''^\nu;\ \gamma_0''^\nu)_{\nu\in N}\right)$ is a RS for the economy $\langle N; (P,b); (\omega'^\nu)_{\nu\in N}\rangle$. In this case, since $\sum_{\nu\in N} \alpha_0''^\nu + \sum_{\nu\in N} \gamma_0''^\nu > \alpha_0 + \beta_0$ and $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu\in N}$ is efficient, Proposition 6 implies that $(\alpha''^\nu;\ \beta''^\nu;\ \gamma_0''^\nu)_{\nu\in N}$ is inefficient. The last statement implies that $p'\left(\widehat{\alpha}'' + \widehat{\beta}''\right) > w\left(\alpha_0'' + \gamma_0''\right)$ (that is, $\pi''^{\max} > 0$), since any RS with $\pi^{\max} = 0$ is efficient. Moreover, by Lemmas 2 and 5, there must be some agent ν such that $\alpha_0''^\nu + \gamma_0''^\nu = 0$. But then, by assumption A1 on P, this implies that $(\alpha''^\nu;\ \beta''^\nu;\ \gamma_0''^\nu)_{\nu\in N}$ would only be non-exploitative if $\overline{c} = \mathbf{0}$, which contradicts the assumption that $\overline{c} \in B(p,b)$, because $pb > 0$ by Proposition 1.

4. Suppose $\alpha_0^{\overline{c}} = \frac{\alpha_0+\beta_0}{N} < \alpha_0^{\underline{c}}$. Then we can construct an alternative economy $\langle N;(P,b);(\omega'^{\nu})_{\nu\in N}\rangle$ and its RS $\left((p,w),(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}\right)$ such that $\alpha'+\beta'=\alpha+\beta$, and there exists one agent $\nu'\in N$ such that $\alpha_0'^{\nu'}+\gamma_0'^{\nu'}<\alpha_0^{\overline{c}}$ and for any other $\nu\in N\setminus\{\nu'\}$, $\alpha_0^{\underline{c}}\geqq\alpha_0'^{\nu}+\gamma_0'^{\nu}$. Then, the desired contradiction follow from **IES** and **RE**. Next, suppose $\alpha_0^{\overline{c}}<\frac{\alpha_0+\beta_0}{N}=\alpha_0^{\underline{c}}$. Then, applying the same logic as in the case of $\frac{\alpha_0+\beta_0}{N}-\alpha_0^{\overline{c}}\neq\alpha_0^{\underline{c}}-\frac{\alpha_0+\beta_0}{N}$, we can obtain a contradiction by **IES** and **RE**.

Consider an economy $\langle N;(P,b);(\omega^{\nu})_{\nu\in N}\rangle$ with a RS $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ with $\pi^{\max}>0$, such that $\underline{\alpha}+\underline{\beta}\leq\sum_{\nu\in N}\omega^{\nu}=\omega$. If $p\left(\underline{\alpha}+\underline{\beta}\right)=p\omega$ holds, then the same argument as for $\underline{\alpha}+\underline{\beta}=\omega$ can be applied, because $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}$ is efficient. If $p\left(\underline{\alpha}+\underline{\beta}\right)<p\omega$ holds, then by Lemma 2, there is a subset $N^{-}\subset N$ such that for each $\nu\in N^{-}$, $p\widehat{\beta}^{\nu}-w\beta_0^{\nu}=pb<\pi^{\max}p\omega^{\nu}-w\beta_0^{\nu}$ holds. Thus, by selecting $\omega'^{\nu}\leq\omega^{\nu}$ appropriately from each $\nu\in N^{-}$, it is possible to find $\omega'\equiv\sum_{\nu\in N\setminus N^{-}}\omega^{\nu}+\sum_{\nu\in N^{-}}\omega'^{\nu}$ such that $p\left(\underline{\alpha}+\underline{\beta}\right)=p\omega'$ holds. Then, by Lemma 5, $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ constitutes an efficient RS for this economy $\langle N;(P,b);(\omega'^{\nu})_{\nu\in N}\rangle$. By applying the same argument as in the case of $\underline{\alpha}+\underline{\beta}=\omega$, one can check that any formulation of exploitation satisfying **LES**, **FNE**, and **RE** is identical to Definition 7 at $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ in $\langle N;(P,b);(\omega'^{\nu})_{\nu\in N}\rangle$. Then, by **IES**, the same statement also holds for $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ in $\langle N;(P,b);(\omega^{\nu})_{\nu\in N}\rangle$.

Case 2, $\pi^{\max}=0$. Consider an economy with a RS $\left((p,w),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ with $\pi^{\max}=0$. Then, Proposition 3 implies that $\alpha_0^{\nu}+\gamma_0^{\nu}=\frac{pb}{w}$ holds for each $\nu\in N$. This implies that $\alpha_0^{\nu}+\gamma_0^{\nu}=\frac{\alpha_0+\beta_0}{N}$ holds for each $\nu\in N$ and this RS is efficient, according to Definition 10.

Let $\underline{c},\overline{c}\in B(p,b)$ be the corresponding reference bundles. Since $\pi^{\max}=0$, then $p\overline{c}-w\alpha_0^{\overline{c}}\leqq pb-w\frac{\alpha_0+\beta_0}{N}=0$ and $p\underline{c}-w\alpha_0^{\underline{c}}\leqq pb-w\frac{\alpha_0+\beta_0}{N}=0$. Thus, $\alpha_0^{\underline{c}},\alpha_0^{\overline{c}}\geqq\frac{\alpha_0+\beta_0}{N}$ by $\underline{c},\overline{c}\in B(p,b)$. By **LES**, $\alpha_0^{\underline{c}}\geqq\alpha_0^{\overline{c}}$ holds, so that either (i) every agent is neither exploiter nor exploited, or (ii) every agent is exploiter. Note that (ii) corresponds to the case that $\alpha_0^{\underline{c}}\geqq\alpha_0^{\overline{c}}>\frac{\alpha_0+\beta_0}{N}$. Then, by assumption A3, there exists a non-exploitative allocation $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ at (p,w) such that $\sum_{\nu\in N}\alpha_0'^{\nu}+\sum_{\nu\in N}\gamma_0'^{\nu}>\alpha_0+\beta_0$ and $\sum_{\nu\in N}\underline{\alpha}'^{\nu}+\sum_{\nu\in N}\underline{\beta}'^{\nu}=\omega$, and the same argument can be applied as in step 3 above to prove that $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ would only be non-exploitative if $\overline{c}=\mathbf{0}$, which contradicts the assumption that $\overline{c}\in B(p,b)$. Thus, $\alpha_0^{\underline{c}}\geqq\alpha_0^{\overline{c}}=\frac{\alpha_0+\beta_0}{N}$.

Let $\alpha_0^{\underline{c}}>\alpha_0^{\overline{c}}=\frac{\alpha_0+\beta_0}{N}$. Then, by assumption A3, there exists another non-exploitative allocation $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ at (p,w) such that $\sum_{\nu\in N}\alpha_0'^{\nu}+$

$\sum_{\nu \in N} \gamma_0'^{\nu} > \alpha_0 + \beta_0$ and $\sum_{\nu \in N} \underline{\alpha}'^{\nu} + \sum_{\nu \in N} \underline{\beta}'^{\nu} = \omega$, and the same argument can be applied as in step 3 above to prove that $(\alpha'^{\nu}; \beta'^{\nu}; \gamma_0'^{\nu})_{\nu \in N}$ would only be non-exploitative if $\overline{c} = \mathbf{0}$, which contradicts the assumption that $\overline{c} \in B(p, b)$.

In sum, if a definition of labour exploitation satisfies **LES**, **RE**, **FNE**, and **IES**, then the condition $\alpha_0^{\underline{c}} = \alpha_0^{\overline{c}} = \frac{\alpha_0 + \beta_0}{N}$ must hold. ■

Proof of Theorem 6:

($\Leftarrow$)**:** It is easy to see that Definition 7 meets **EEUL** and **IES**. Therefore, it suffices to show that Definition 7 satisfies **FNE***. Let $\left((p, w), (\alpha^{\nu}; \beta^{\nu}; \gamma_0^{\nu})_{\nu \in N}\right)$ be an efficient RS for the economy $\langle N; (P, b); (\omega^{\nu})_{\nu \in N}\rangle$. Then, by Definition 7, at any non-exploitative allocation it must be $\alpha_0'^{\nu} + \gamma_0'^{\nu} = \frac{\alpha_0 + \beta_0}{N}$ for all $\nu \in N$.

Suppose that $p\left(\underline{\alpha} + \underline{\beta}\right) = p\omega$ holds. Then, by Lemma 6, the non-exploitative allocation is feasible with respect to (p, w) in $\langle N; (P, b); \left(\frac{\omega}{N}, \dots, \frac{\omega}{N}\right)\rangle$ with $(\alpha'^{\nu})_{\nu \in N} \in P^N$ such that $\widehat{\alpha}'^{\nu} = \frac{\widehat{\alpha} + \widehat{\beta}}{N}$ and $\alpha_0'^{\nu} = \frac{\alpha_0 + \gamma_0}{N}$ for each $\nu \in N$. Suppose that $p\left(\underline{\alpha} + \underline{\beta}\right) < p\omega$ holds. Choose $(\omega'^{\nu})_{\nu \in N}$ such that $p\left(\underline{\alpha} + \underline{\beta}\right) = p\omega'$ with $p\left(\underline{\alpha}^{\nu} + \underline{\beta}^{\nu}\right) = p\omega'^{\nu}$ for all $\nu \in N$. Then, $\left((p, w), (\alpha^{\nu}; \beta^{\nu}; \gamma_0^{\nu})_{\nu \in N}\right)$ is still an efficient RS for the economy $\langle N; (P, b); (\omega'^{\nu})_{\nu \in N}\rangle$ and by Lemma 6, the non-exploitative allocation is feasible with respect to (p, w) in $\langle N; (P, b); \left(\frac{\omega}{N}, \dots, \frac{\omega}{N}\right)\rangle$ with $(\alpha'^{\nu})_{\nu \in N} \in P^N$ such that $\widehat{\alpha}'^{\nu} = \frac{\widehat{\alpha} + \widehat{\beta}}{N}$ and $\alpha_0'^{\nu} = \frac{\alpha_0 + \gamma_0}{N}$ for each $\nu \in N$. Thus, in either case, Definition 7 meets **FNE***.

($\Rightarrow$)**:** Consider any definition of labour exploitation satisfying **LES**, **FNE***, **EEUL**, and **IES**.

Case 1, $\pi^{\max} > 0$. Consider an economy $\langle N; (P, b); (\omega^{\nu})_{\nu \in N}\rangle$ with a RS $\left((p, w), (\alpha^{\nu}; \beta^{\nu}; \gamma_0^{\nu})_{\nu \in N}\right)$ such that $\pi^{\max} > 0$ and $\underline{\alpha} + \underline{\beta} = \sum_{\nu \in N} \omega^{\nu} = \omega$. By Lemma 5, $(\alpha^{\nu}; \beta^{\nu}; \gamma_0^{\nu})_{\nu \in N}$ is efficient. Suppose the condition $\alpha_0^{\underline{c}} = \alpha_0^{\overline{c}} = \frac{\alpha_0 + \beta_0}{N}$ does not hold. First of all, By **LES** and **EEUL**, $\alpha_0^{\underline{c}} \geqq \alpha_0^{\overline{c}} \geqq l.v.(b)$. Then, there always exists a non-exploitative allocation at (p, w).

1. Suppose $\alpha_0^{\overline{c}} > \frac{\alpha_0 + \beta_0}{N}$. Let $(\alpha'^{\nu}; \beta'^{\nu}; \gamma_0'^{\nu})_{\nu \in N}$ be a non-exploitative allocation at (p, w). Then, by **FNE***, this allocation is feasible non-exploitative with respect to some (p', w') in $\langle N; (P, b); (\omega'^{\nu})_{\nu \in N}\rangle$. By definition it must be $\sum_{\nu \in N} \alpha_0'^{\nu} + \sum_{\nu \in N} \gamma_0'^{\nu} \equiv \alpha_0' + \gamma_0' \geqq N\alpha_0^{\overline{c}} > \alpha_0 + \beta_0$, thus implying that $(\alpha'^{\nu}; \beta'^{\nu}; \gamma_0'^{\nu})_{\nu \in N}$ is inefficient for $\langle N; (P, b); (\omega'^{\nu})_{\nu \in N}\rangle$.

Let us show the last statement. If $\alpha_0' + \gamma_0' > \alpha_0 + \beta_0$, then another allocation $(\alpha''^{\nu}; \beta''^{\nu}; \gamma_0''^{\nu})_{\nu \in N}$ can be constructed such that $\sum_{\nu \in N} \alpha''^{\nu} + \sum_{\nu \in N} \beta''^{\nu} = \sum_{\nu \in N} \alpha^{\nu} + \sum_{\nu \in N} \beta^{\nu}$ and $\sum_{\nu \in N} \alpha_0''^{\nu} + \sum_{\nu \in N} \gamma_0''^{\nu} = \sum_{\nu \in N} \alpha_0^{\nu} +$

$\sum_{\nu\in N}\gamma_0^{\nu}$, and moreover, $\alpha_0^{\prime\prime\nu}+\gamma_0^{\prime\prime\nu}\leqq\alpha_0^{\prime\nu}+\gamma_0^{\prime\nu}$ holds for all $\nu\in N$, and strictly for some $\nu\in N$. By the above two equations, $\sum_{\nu\in N}\beta_0^{\prime\prime\nu}=\sum_{\nu\in N}\gamma_0^{\prime\prime\nu}$ holds, since $\sum_{\nu\in N}\beta_0^{\nu}=\sum_{\nu\in N}\gamma_0^{\nu}$. This construction implies that $(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})_{\nu\in N}$ is inefficient.

The last statement implies that $p'\left(\widehat{\alpha}'+\widehat{\beta}'\right)>w'\left(\alpha_0'+\gamma_0'\right)$ (that is, $\pi'^{\max}>0$), since any RS with $\pi^{\max}=0$ is efficient. Moreover, by Lemmas 2 and 5, there must be some agent ν such that $\alpha_0^{\prime\nu}+\gamma_0^{\prime\nu}=0$. But then, by assumption A1 on P, this implies that $(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})_{\nu\in N}$ would only be non-exploitative if $\overline{c}=\mathbf{0}$ which contradicts the assumption that $\overline{c}\in B(p,b)$, because $pb>0$ by Proposition 1.

2. Suppose $\alpha_0^{\underline{c}}<\frac{\alpha_0+\beta_0}{N}$. Let $(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})_{\nu\in N}$ be a non-exploitative allocation at (p,w). Then, by **FNE***, this allocation is feasible non-exploitative with respect to some (p',w') in $\langle N;(P,b)\,;(\omega^{\prime\nu})_{\nu\in N}\rangle$. By definition it must be $\sum_{\nu\in N}\alpha_0^{\prime\nu}+\sum_{\nu\in N}\gamma_0^{\prime\nu}\equiv\alpha_0'+\gamma_0'\leqq N\alpha_0^{\overline{c}}<\alpha_0+\beta_0$, thus implying that the original allocation $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}$ was inefficient, a contradiction.

3. Suppose $\alpha_0^{\overline{c}}<\frac{\alpha_0+\beta_0}{N}<\alpha_0^{\underline{c}}$. Let $(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})_{\nu\in N}$ be a non-exploitative allocation at (p,w): by **FNE***, $(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})_{\nu\in N}$ is feasible with respect to some (p',w') in some economy $\langle N;(P,b)\,;(\omega^{\prime\nu})_{\nu\in N}\rangle$. If $\sum_{\nu\in N}\alpha_0^{\prime\nu}+\sum_{\nu\in N}\gamma_0^{\prime\nu}<\alpha_0+\beta_0$, then an argument similar to $\alpha_0^{\underline{c}}<\frac{\alpha_0+\beta_0}{N}$ is applied. Suppose $\sum_{\nu\in N}\alpha_0^{\prime\nu}+\sum_{\nu\in N}\gamma_0^{\prime\nu}>\alpha_0+\beta_0$. Since $\underline{\alpha}+\underline{\beta}=\omega$ by construction, the RS allocation $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}$ is efficient for $\langle N;(P,b)\,;(\omega^{\nu})_{\nu\in N}\rangle$. However, since $\sum_{\nu\in N}\alpha_0^{\prime\nu}+\sum_{\nu\in N}\gamma_0^{\prime\nu}>\alpha_0+\beta_0$, then $(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})_{\nu\in N}$ is inefficient, as shown in the case of $\alpha_0^{\overline{c}}>\frac{\alpha_0+\beta_0}{N}$. Thus, as shown in the case of $\alpha_0^{\overline{c}}>\frac{\alpha_0+\beta_0}{N}$, $(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})_{\nu\in N}$ would only be non-exploitative if $\overline{c}=\mathbf{0}$, which contradicts the assumption that $\overline{c}\in B(p,b)$. If $\sum_{\nu\in N}\alpha_0^{\prime\nu}+\sum_{\nu\in N}\gamma_0^{\prime\nu}=\alpha_0+\beta_0$, then consider $(\alpha^{\prime\prime\nu};\ \beta^{\prime\prime\nu};\ \gamma_0^{\prime\prime\nu})_{\nu\in N}$ such that for any $\nu\in N\setminus\{\nu'\}$, $(\alpha^{\prime\prime\nu};\ \beta^{\prime\prime\nu};\ \gamma_0^{\prime\prime\nu})=(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})$ and for ν', $\alpha_0^{\prime\prime\nu'}+\gamma_0^{\prime\prime\nu'}>\alpha_0^{\prime\nu'}+\gamma_0^{\prime\nu'}$, $\left(\beta^{\prime\prime\nu'};\gamma_0^{\prime\prime\nu'}\right)=\left(\beta^{\prime\nu'};\gamma_0^{\prime\nu'}\right)$, and $\left(\underline{\alpha}^{\prime\prime\nu'},\overline{\alpha}^{\prime\prime\nu'}\right)=\left(\underline{\alpha}^{\prime\nu'},\overline{\alpha}^{\prime\nu'}\right)$. By A3, $\left(\alpha_0^{\prime\prime\nu'},\underline{\alpha}^{\prime\prime\nu'},\overline{\alpha}^{\prime\prime\nu'}\right)\in P$. Moreover, since $\sum_{\nu\in N}\underline{\alpha}^{\prime\prime\nu}+\sum_{\nu\in N}\underline{\beta}^{\prime\prime\nu}=\sum_{\nu\in N}\underline{\alpha}^{\prime\nu}+\sum_{\nu\in N}\underline{\beta}^{\prime\nu}$, $\sum_{\nu\in N}\overline{\alpha}^{\prime\prime\nu}+\sum_{\nu\in N}\overline{\beta}^{\prime\prime\nu}=\sum_{\nu\in N}\overline{\alpha}^{\prime\nu}+\sum_{\nu\in N}\overline{\beta}^{\prime\nu}$, $\sum_{\nu\in N}\beta_0^{\prime\prime\nu}=\sum_{\nu\in N}\beta_0^{\prime\nu}$, and $\sum_{\nu\in N}\gamma_0^{\prime\prime\nu}=\sum_{\nu\in N}\gamma_0^{\prime\nu}$, if $(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})_{\nu\in N}$ is a non-exploitative allocation at (p,w), then so is $(\alpha^{\prime\prime\nu};\ \beta^{\prime\prime\nu};\ \gamma_0^{\prime\prime\nu})_{\nu\in N}$ by Definition 8. However, $\sum_{\nu\in N}\alpha_0^{\prime\prime\nu}+\sum_{\nu\in N}\gamma_0^{\prime\prime\nu}>\alpha_0+\beta_0$ induces a contradiction as discussed in the above.

4. Suppose $\alpha_0^{\overline{c}}=\frac{\alpha_0+\beta_0}{N}<\alpha_0^{\underline{c}}$, and let $(\alpha^{\prime\nu};\ \beta^{\prime\nu};\ \gamma_0^{\prime\nu})_{\nu\in N}$ be a non-exploitative allocation at (p,w) such that for some $\nu',\nu''\in N$, $\alpha_0^{\prime\nu'}+\gamma_0^{\prime\nu'}=\alpha_0^{\overline{c}}$ and $\alpha_0^{\prime\nu''}+\gamma_0^{\prime\nu''}=\alpha_0^{\underline{c}}$. Then, since $\sum_{\nu\in N}\alpha_0^{\prime\nu}+\sum_{\nu\in N}\gamma_0^{\prime\nu}>\alpha_0+\beta_0$, we apply the same argument as in the last paragraph. Suppose $\alpha_0^{\overline{c}} < \frac{\alpha_0+\beta_0}{N} = \alpha_0^{\underline{c}}$, and let $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ be a non-exploitative allocation at (p, w). By **EEUL**, it follows that $\alpha_0+\beta_0 > l.v.(Nb)$. But then $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ can be such that $\sum_{\nu\in N}\alpha_0'^{\nu} + \sum_{\nu\in N}\gamma_0'^{\nu} < \alpha_0 + \beta_0$, so that an argument similar to $\alpha_0^{\underline{c}} < \frac{\alpha_0+\beta_0}{N}$ is applied. Thus, by **EEUL** and **LES**, $\alpha_0^{\overline{c}} = \alpha_0^{\underline{c}} = \frac{\alpha_0+\beta_0}{N}$ should hold, which is a contradiction.

Consider an economy $\langle N; (P, b); (\omega^{\nu})_{\nu\in N}\rangle$ with a RS $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ with $\pi^{\max} > 0$, such that $\underline{\alpha} + \underline{\beta} \leq \sum_{\nu\in N}\omega^{\nu} = \omega$. If $p\left(\underline{\alpha} + \underline{\beta}\right) = p\omega$ holds, then the same argument as the case of $\underline{\alpha} + \underline{\beta} = \omega$ can be applied, since $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}$ is efficient. If $p\left(\underline{\alpha} + \underline{\beta}\right) < p\omega$ holds, then by Lemma 2, there is a subset $N^- \subset N$ such that for each $\nu \in N^-$, $p\widehat{\beta}^{\nu} - w\beta_0^{\nu} = pb < \pi^{\max}p\omega^{\nu} - w\beta_0^{\nu}$ holds. Thus, by selecting $\omega'^{\nu} \leq \omega^{\nu}$ appropriately from each $\nu \in N^-$, it is possible to find $\omega' \equiv \sum_{\nu\in N\setminus N^-}\omega^{\nu} + \sum_{\nu\in N^-}\omega'^{\nu}$ such that $p\left(\underline{\alpha} + \underline{\beta}\right) = p\omega'$ holds. Then, by Lemma 5, $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ constitutes an efficient RS for this economy $\langle N; (P, b); (\omega'^{\nu})_{\nu\in N}\rangle$. By applying the same argument as in the case of $\underline{\alpha} + \underline{\beta} = \omega$, one can check that $\alpha_0^{\underline{c}} = \alpha_0^{\overline{c}} = \frac{\alpha_0+\beta_0}{N}$ for $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ in $\langle N; (P, b); (\omega'^{\nu})_{\nu\in N}\rangle$. Then, by **IES**, $\alpha_0^{\underline{c}} = \alpha_0^{\overline{c}} = \frac{\alpha_0+\beta_0}{N}$ holds for $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ in $\langle N; (P, b); (\omega^{\nu})_{\nu\in N}\rangle$.

Case 2, $\pi^{\max} = 0$. Consider an economy with a RS $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ with $\pi^{\max} = 0$. Then, Proposition 3 implies that $\alpha_0^{\nu} + \gamma_0^{\nu} = \frac{pb}{w}$ holds for each $\nu \in N$. This implies that $\alpha_0^{\nu} + \gamma_0^{\nu} = \frac{\alpha_0+\beta_0}{N}$ holds for each $\nu \in N$, and this RS is efficient, according to Definition 10.

Let $\underline{c}, \overline{c} \in B(p, b)$ be the corresponding reference bundles. Since $\pi^{\max} = 0$, then $p\overline{c} - w\alpha_0^{\overline{c}} \leqq pb - w\frac{\alpha_0+\beta_0}{N} = 0$ and $p\underline{c} - w\alpha_0^{\underline{c}} \leqq pb - w\frac{\alpha_0+\beta_0}{N} = 0$. Thus, $\alpha_0^{\underline{c}}, \alpha_0^{\overline{c}} \geqq \frac{\alpha_0+\beta_0}{N}$ by $\underline{c}, \overline{c} \in B(p, b)$. By **LES**, $\alpha_0^{\underline{c}} \geqq \alpha_0^{\overline{c}}$ holds, so that either (i) every agent is neither an exploiter nor exploited, or (ii) every agent is an exploiter. Note that (ii) corresponds to the case that $\alpha_0^{\underline{c}} \geqq \alpha_0^{\overline{c}} > \frac{\alpha_0+\beta_0}{N}$. Then, by assumption A3, there exists a non-exploitative allocation $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ at (p, w) such that $\sum_{\nu\in N}\alpha_0'^{\nu} + \sum_{\nu\in N}\gamma_0'^{\nu} > \alpha_0 + \beta_0$, which implies that $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ is inefficient, as shown in the case of $\pi^{\max} > 0$. Thus, as shown in step 1 of the proof above, $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ would only be non-exploitative if $\overline{c} = \mathbf{0}$, which contradicts the assumption that $\overline{c} \in B(p, b)$. Thus, $\alpha_0^{\underline{c}} \geqq \alpha_0^{\overline{c}} = \frac{\alpha_0+\beta_0}{N}$.

Let $\alpha_0^{\underline{c}} > \alpha_0^{\overline{c}} = \frac{\alpha_0+\beta_0}{N}$. Then, by assumption A3, there exists another non-exploitative allocation $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}$ at (p, w) such that $\sum_{\nu\in N}\alpha_0'^{\nu} +$

$\sum_{\nu \in N} \gamma_0'^{\nu} > \alpha_0 + \beta_0$, to which the same argument can be applied as in the case of $\alpha_0^{\underline{c}} \geqq \alpha_0^{\overline{c}} > \frac{\alpha_0 + \beta_0}{N}$. Thus, $(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu \in N}$ would only be non-exploitative if $\overline{c} = \mathbf{0}$, which contradicts the assumption that $\overline{c} \in B(p, b)$.

In sum, if a definition of labour exploitation satisfies **LES**, **EEUL**, **FNE***, and **IES**, then the condition $\alpha_0^{\underline{c}} = \alpha_0^{\overline{c}} = \frac{\alpha_0 + \beta_0}{N}$ must hold. ■

7.3 Appendix 3: Additional Claims

In this Appendix, some additional claims made in the paper are rigorously proved. First, Example A.1 proves that, if the definition of exploitation satisfies **LES**, the **CECP** may hold if a definition of labour value is adopted which does not focus on profit-maximising processes.

Example A.1: Consider the following von Neumann technology:

$$A = \begin{bmatrix} 2 & 4 & 2 & 2 \\ 4 & 4 & 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 8 & 6 & 6 \\ 12 & 12 & 6 & 0 \end{bmatrix}, L = (1, 1, 1, 1),$$

where the notation is the same as in Example 1 above and the production possibility set is $P_{(A,B,L)} \equiv \left\{ \alpha \in \mathbb{R}_- \times \mathbb{R}^2_- \times \mathbb{R}^2_+ \mid \exists x \in \mathbb{R}^4_+ : \alpha \leqq (-Lx, -Ax, Bx) \right\}$. Then, consider a convex cone subsistence economy defined by $P_{(A,B,L)}$, $b = (2, 2)$, and $\omega = (N, N)$. Let $(\omega^{\nu})_{\nu \in N}$ be such that $\omega^{\nu} = (\delta^{\nu}, \delta^{\nu})$, where $\delta^{\nu} \leqq 2$, all $\nu \in N$, and $\omega^i = (2, 2)$, some $i \in N$.

Let $\mathbf{e}_j$ and α^j, $j = 1, 2, 3, 4$, be defined as in Example 1 discussed in Section 3 above. Then $\widehat{\alpha}^1 \equiv (0, 8)$, $\widehat{\alpha}^2 \equiv (4, 8)$, $\widehat{\alpha}^3 \equiv (4, 4)$, and $\widehat{\alpha}^4 \equiv (4, 0)$. Also, $\widehat{P}(\alpha_0 = 1) = co\left\{\widehat{\alpha}^1, \widehat{\alpha}^2, \widehat{\alpha}^3, \widehat{\alpha}^4, \mathbf{0}\right\}$. In this economy, $p = (1, 0)$, $w = 2$ constitute a RS, with $\alpha^{\nu} = 0, \beta^{\nu} = \frac{\delta^{\nu} \alpha^3}{2}$, and $\gamma_0^{\nu} = 1 - \frac{\delta^{\nu}}{2}$, all $\nu \in N$. The corresponding aggregate production is $\alpha + \beta = \frac{N\alpha^3}{2}$. At this RS, $\pi(\alpha + \beta; p, w) \equiv \frac{p\widehat{\alpha}^3 - w\alpha_0^3}{p\underline{\alpha}^3} = 1$, whereas $\pi(\alpha^1; p, w) \equiv \frac{p\widehat{\alpha}^1 - w\alpha_0^1}{p\underline{\alpha}^1} < 0$, $\pi(\alpha^2; p, w) \equiv \frac{p\widehat{\alpha}^2 - w\alpha_0^2}{p\underline{\alpha}^2} = \frac{1}{2}, \pi(\alpha^4; p, w) \equiv \frac{p\widehat{\alpha}^4 - w\alpha_0^4}{p\underline{\alpha}^4} = 1$. Thus, $\overline{P}(p, w) = \{\alpha \in P \mid \exists \lambda > 0 : \lambda\alpha \in co\{\alpha^3, \alpha^4\}\}$.

Choose $\overline{c} = \underline{c} = \frac{\widehat{\alpha}^2}{2}$, so that $\overline{c}, \underline{c} \in B(p, b)$. Since $\frac{p\widehat{\alpha}^2 - w\alpha_0^2}{2} = 1 = \pi^{\max} p\underline{\alpha}_{\min} = \pi^{\max} p\underline{\alpha}_{\max} = \pi^{\max} p\frac{\underline{\alpha}_3}{2}$, Theorem 3 states that the **CECP** holds if a definition of exploitation is adopted which satisfies **LES** with $\overline{c} = \underline{c} = \frac{\widehat{\alpha}^2}{2}$. However, since $\pi\left(\frac{\alpha^2}{2}; p, w\right) = \frac{1}{2}$, it follows that $\alpha^{\underline{c}} = \alpha^{\overline{c}} = \frac{\alpha^2}{2} \notin \overline{P}(p, w)$. ■

Example A.2 instead proves the existence of inefficient RS's, and it highlights their structure and the role of big capitalists in generating inefficiencies.

Example A.2: Consider the following von Neumann technology:

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix}, L = (1, 0.8),$$

where the notation is the same as in Example 1 above and the production possibility set is $P_{(A,B,L)} \equiv \left\{ \alpha \in \mathbb{R}_- \times \mathbb{R}^2_- \times \mathbb{R}^2_+ \mid \exists x \in \mathbb{R}^2_+ : \alpha \leqq (-Lx, -Ax, Bx) \right\}$. Consider a subsistence economy $E = \langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$ defined by $N = \{1, 2\}$, $P = P_{(A,B,L)}$, $b = (1, 1)$, and $(\omega^1, \omega^2) = ((2, 1), (0, 0))$.

To begin with, it is shown that the price vector $(p, w) = ((1, 0), 1)$, and the allocation $(\alpha^1;\ \beta^1;\ \gamma_0^1) = (0; (-1, -(1, 1), (3, 3)); 0)$, and $(\alpha^2;\ \beta^2;\ \gamma_0^2) = (0; 0; 1)$ constitute a RS for this economy. First, given this (p, w), the activity $\beta^1 = (-1, -(1, 1), (3, 3)) \in P_{(A,B,L)}$ is a maximal profit-rate production point. Note that $\pi^1(p, w) \equiv \frac{p\widehat{\beta}^1 - w\beta_0^1}{p\underline{\beta}^1} = 1$. Let $\beta' \equiv (-0.8, -(2, 1), (4, 4)) \in P_{(A,B,L)}$. Then, $\pi'(p, w) \equiv \frac{p\widehat{\beta}' - w\beta_0'}{p\underline{\beta}'} = 0.6$. Thus, $\pi^1(p, w) > \pi'(p, w)$. By the property of $P_{(A,B,L)}$, any other production point $\alpha \in P_{(A,B,L)}$ is represented as $\alpha \leqq t\beta^1 + t'\beta'$ for some suitable non-negative values $t, t' \geqq 0$. Thus, $\pi^1(p, w) > \pi'(p, w)$ implies that β^1 is a maximal profit-rate point. Second, since $pb = 1$ and $w = 1$, agent 2's optimal solution is $\gamma_0^2 = 1$, since $\omega^2 = (0, 0)$. Third, for agent 1, $(0;\ \beta^1;\ 0)$ is the optimal solution, since $p\widehat{\beta}^1 - w\beta_0^1 = pb$, $\pi^1(p, w) = \pi^{\max}$, and $p\underline{\beta}^1 < p\omega^1$. Note that $\widehat{\beta}^1 = 2b$ and $\beta_0^1 = \gamma_0^2 = 1$. Since $\underline{\beta}^1 \leq \omega^1$, which implies $\underline{\beta}^1 \leq \omega \equiv \omega^1 + \omega^2$, $\left((p, w), (\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}\right)$ is a RS with $p\underline{\beta} < p\omega$.

The allocation $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}$ is inefficient. Consider β' as the alternative social production point. Note that $\underline{\beta}' = \omega$, $\widehat{\beta}' = (2, 3) \geq 2b$, and $\beta_0' = 0.8 < \beta_0^1$. Construct an alternative allocation $(\alpha'^\nu;\ \beta'^\nu;\ \gamma_0'^\nu)_{\nu \in N}$ as $(\alpha'^1;\ \beta'^1;\ \gamma_0'^1) = (0; \beta'; 0)$ and $(\alpha'^2;\ \beta'^2;\ \gamma_0'^2) = (0; 0; 0.8)$. Since $\gamma_0'^2 < \gamma_0^2$, this implies that the RS allocation $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)_{\nu \in N}$ is not efficient. ■

As noted in Section 5 above, the source of the inefficiency is the violation of the assumption of local nonsatiation: an increase in wealth does not make agent 1 better off. Thus, it is individually rational for agent 1 to use the labour intensive activity β^1 because it yields the maximum rate of profit at $(p, w) = ((1, 0), 1)$, instead of the capital-intensive, and socially optimal technique β' which yields a lower profit rate.

8 References

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