On Initial Conferment of Individual Rights*

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Abstract

An extended social choice framework is proposed for the analysis of initial conferment of individual rights. It captures the intuitive conception of decision-making procedure as a carrier of intrinsic value along with the instrumental usefulness thereof in realizing valuable culmination outcomes. Our model of social decision-making consists of two stages. In the first stage, the society decides on the game form rights-system to be promulgated. In the second stage, the promulgated game form rights-system, coupled with the revealed profile of individual preference orderings over the set of culmination outcomes, determines a fully-fledged game, the play of which determines a culmination outcome at the Nash equilibrium. A set of sufficient conditions for the existence of a democratic social choice procedure, which chooses a game form in the first stage that is not only liberal, efficient and Nash solvable, but also uniformly workable for every revealed profile of individual preference orderings over the set of culmination outcomes, is identified.

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1. Introduction

1.1. Historical Background

Ever since Sen (1970, Chapter 6 & Chapter 6*; 1970a; 1976) acutely crystallized the logical conflict between the welfaristic outcome morality in the weak form of the Pareto principle and the non-welfaristic claim of libertarian rights into the *impossibility of a Paretian liberal*, a huge literature has evolved along several distinct avenues.[1] In the first place, some of the early literature either repudiated the importance of Sen's impossibility theorem, or tried to find an escape route from the logical impasse identified by Sen. In the second place, capitalizing on the seminal observation by Nozick (1974, pp.164-166), alternative articulations of libertarian rights, which are game-theoretic in nature, were proposed by Gärdenfors (1981), Sugden (1985), Gaertner, Pattanaik and Suzumura (1992), Deb (1990/2004; 1994), Hammond (1995; 1996) and Peleg (1998). Recollect that Sen's original articulation of libertarian rights was in terms of the preference-contingent constraints on social choice rules by means of individual decisiveness. In contrast, these game-theoretic articulations captured the essence of libertarian rights by means of individual freedom of choosing admissible strategies in the game-theoretic situations where individual liberties are at stake. Unlike the first class of work, these alternative articulations were meant to provide more legitimate methods of capturing the essence of what libertarian rights should mean.[2] In the third place, the crucial problem of initial

[1]Some of these literature are succinctly surveyed and evaluated by Suzumura (1996; 2007). See also Sen (2002, Part VI) for his recent evaluation of the issues of liberty and social choice.

[2]Note that these alternative articulations of libertarian rights do *not* claim to resolve the impossibility of a Paretian liberal. Quite to the contrary, Pattanaik (1996) and Deb,

conferment of libertarian rights was occasionally mentioned in the literature without providing a fully-fledged analytical framework.[3] Suffice it to cite just one salient example. In his rebuttal to the game form articulation proposed by Gaertner, Pattanaik and Suzumura (1992), Sen (1992, p.155) concluded as follows: "Gaertner *et al.* (1992) do, in fact, pose the question, 'How does the society decide which strategies should or should not be admissible for a specific player in a given context?' This, as they rightly note, is 'an important question'. ... [I]t is precisely on the answer to this further question that the relationship between the game-form formulations and social-choice formulations depend ... . We must not be too impressed by the 'form' of the 'game forms'. We have to examine its contents and its rationale. The correspondence with social-choice formulations becomes transparent precisely there." The purpose of this paper is to contribute to this less cultivated issue within the theory of libertarian rights.

1.2. Basic Problem

Pattanaik and Razzolini (1997) showed that there are several natural variants of the impossibility of a Paretian liberal even when libertarian rights are articulated in terms of game forms.

[3]Pattanaik and Suzumura (1994; 1996) and Suzumura (1996; 2007) identified three distinct issues in the analysis of libertarian rights. The first issue is the *formal structure* of rights. The second issue is the *realization* of conferred rights. The third issue is the *initial conferment* of rights. In Sen's theory of libertarian rights, the formal structure of rights was articulated in terms of the preference-contingent constraints on social choice rules, whereas the issue of the realization of conferred rights could be boiled down to the existence of a social choice rule which respects the preference-contingent constraints on social choice rules. However, Sen has never addressed himself to the issue of initial conferment of rights. This is presumably because his interest was focussed squarely on the conflict between the non-welfaristic claim of libertarian rights and the welfaristic claim of the Pareto principle.

To illustrate the nature of the problem of initial conferment of game form rights, consider the following example.

Example 1: There are two passengers 1 and 2 in the compartment of a train, where 1 is a smoker and 2 is a non-smoker. The train company is to decide either to respect the smoker′s desire to smoke freely, or to respect the non-smoker′s desire not to be imposed secondary smoking. The company′s problem is to choose from the set of various game forms, which includes the following two game forms.

The first game form is $\gamma = (M_1^\gamma \times M_2^\gamma, g^\gamma)$, where M_i^γ is the set of player i's *strategies* $(i = 1, 2)$ and g^γ is the *outcome function*, viz., $g^\gamma(m_1, m_2)$ is the *culmination outcome* corresponding to the *strategy profile* $(m_1, m_2) \in M_1^\gamma \times M_2^\gamma$. It is defined by $M_1^\gamma = \{s, ns\}$, where $s =$ "to smoke" and $ns =$ "not to smoke", $M_2^\gamma = \{(l|s, r|ns), r\}$, where $(l|s, r|ns) =$ "to leave the compartment if the smoker smokes, to remain in the compartment if the smoker does not smoke" and $r =$ "to remain in the compartment no matter what", and g^γ is defined by

1 \ 2	$(l\,|\,s,\ r\,|\,ns)$	r
s	(s, l)	(s, r)
ns	(ns, r)	(ns, r)

where (s, l) is the culmination outcome such that the smoker smokes and the non-smoker leaves the compartment. The consequential outcomes (ns, r) and (s, r) may be interpreted similarly.

The second game form $\gamma^* = (M_1^{\gamma^*} \times M_2^{\gamma^*}, g^{\gamma^*})$ is defined by $M_1^{\gamma^*} = \{(s|p, ns|np), ns\}$, where $(s|p, ns|np) =$ "to smoke if the non-smoker permits it, not to smoke if the non-smoker does not permit it" and ns = "not to smoke no matter what", $M_2^{\gamma^*} = \{p \cdot r, p \cdot l, np\}$, where $p \cdot r$ = "to permit the smoker to smoke and remain in the compartment", $p \cdot l$ = "to permit the smoker to smoke and leave the compartment if and only if the smoker indeed smokes" and np = "not to permit the smoker to smoke", and g^{γ^*} is defined by

1 \ 2	$p \cdot r$	$p \cdot l$	np
$(s \mid p,\ ns \mid np)$	(s, r)	(s, l)	(ns, r)
ns	(ns, r)	(ns, r)	(ns, r)

Note that the set of culmination outcomes is given by $A = \{(s, l), (ns, r), (s, r)\}$. Note also that the company confers on the smoker (*resp.* the non-smoker) the right for free smoking (*resp.* the right for clean air) if it chooses the game form γ (*resp.* γ^*). ■

The gist of this example is that *the social choice of a game form is tantamount to the initial conferment of individual rights.* This social choice issue should be solved by designing and implementing a democratic social decision procedure for initial conferment of individual rights. This analysis can be based on the conceptual framework developed by Pattanaik and Suzumura (1994; 1996), which proposed to capture the intuitive conception of decision-making procedure as a carrier of intrinsic value beyond the instrumental usefulness thereof in realizing valuable culmination outcomes. The model of social decision-making consists of two stages. In the first stage, the society decides on the game form rights-system to be promulgated. In the second stage, the promulgated game form rights-system, coupled with the revealed profile of individual preference orderings over the set of culmination outcomes, determine a fully-fledged game, and the play of this game determines a culmination outcome at the Nash equilibrium.[4] It may deserve emphasis that this two-stage social choice procedure has a sharply contrasting feature vis-à-vis the classical Arrow (1963) social choice framework. In the classical framework, it is the culmination outcome that is socially chosen, whereas our two-stage social choice framework visualizes a procedure where it is the game form rights-system that is socially chosen, the culmination outcome being determined through the decentralized play of the game.

How then, should we articulate the first stage social decision-making procedure? In this paper, each individual is assumed to have an *ordering function* Q_i, which assigns an extended ordering $Q_i(\mathbf{R})$ over the pairs of game forms and realized culmination outcomes to the profile $\mathbf{R}$.[5] Let Ψ be the social aggregator, to be called the *extended constitution function*, which maps each admissible profile of individual ordering functions into a social ordering function. It is this social ordering function that determines the game form rights-system

[4]We may illustrate this two-stage framework by means of **Example 1**. Suppose that the two passengers have their own preference orderings over the set of culmination outcomes A, together forming the following profile $\mathbf{R} = (R_1, R_2)$:

$$R_1 : (s,l) \succ_1 (s,r) \succ_1 (ns,r)\,;\; R_2 : (ns,r) \succ_2 (s,l) \succ_2 (s,r)\,,$$

where $a \succ_i b$ denotes that $i \in \{1,2\}$ prefers a to b. Given this profile $\mathbf{R}$, (s,l) is the unique pure strategy Nash equilibirum outcome of the game $(\gamma, \mathbf{R})$, whereas (ns,r) is the unique pure strategy Nash equilibirum outcome of the game $(\gamma^*, \mathbf{R})$. For the sake of further argument, let us assume that γ^* is the socially chosen game form. Since γ^* is chosen in the first stage before the profile $\mathbf{R}$ is revealed, the two individuals play the game $(\gamma^*, \mathbf{R})$ in the second stage, and the unique pure strategy Nash equilibrium outcome (ns,r) will emerge as a consequence.

[5]In **Example 1**, $((ns,r),\gamma^*)Q_i(\mathbf{R})((s,l),\gamma)$ means that the situation where (ns,r) is realized as a Nash equilibrium outcome of the game $(\gamma^*, \mathbf{R})$ is at least as desirable for i as the situation where (s,l) is realized as a Nash equilibrium outcome of the game $(\gamma, \mathbf{R})$.

to be socially chosen and promulgated as the rule of the game to be played in the second stage. Within this conceptual framework, the crucial task in the analysis of social choice of game form rights-system is to show the existence of a "reasonable" extended constitution function Ψ. In this paper, we will introduce several axioms on Ψ to identify the conditions which qualify an extended constitution function to be "reasonable." Also, we will propose several conditions which identify the class of liberal game forms. Since the concept of game forms itself has very little, if any, to do with liberal rights-systems, we should discuss what conditions are needed to characterize the liberal rights-systems. To sum up, the purpose of this paper is to investigate the possibility of reasonable extended constitution functions, in terms of which a liberal game form can be rationally chosen.

1.3. Other Related Literature

A scheme similar to ours is pursued by Koray (2000) in the sense that Koray also addresses himself to the social choice of social decision rules. One of the crucial differences between us is that the social decision rules envisaged by Koray are the conventional social choice functions, whereas we focus on the social decision rules as game forms. Another difference is that Koray was concerned only about the consequential values of social decision rules, whereas we are interested in consequential as well as non-consequential values of social decision rules as game forms. We should also note that the main result of Koray is an impossibility theorem, whereas our main results are possibility theorems. This contrast is mainly due to the social concern about non-consequential values of game forms which our framework may capture.

Apart from this introduction, the paper consists of four sections and an appendix. Section 2 explains our basic model of extended social alternatives and game form rights-systems. It also defines the extended constitution function. Section 3 introduces the basic Arrovian axioms which identify democratic extended constitution functions, and explains what we mean by game forms being liberal. Section 4 identifies a set of conditions that guarantees the existence of an extended constitution function, which enables the society to decide on the initial conferment of liberal game form rights-system when all individuals are self-interested. In contrast, Section 5 briefly disucusses the case where individuals are ethically motivated. Section 6 concludes, and Appendix gathers involved proofs.

2. Basic Model

2.1. Description of Social States

The society consists of n individuals, where $2 \leq n < +\infty$. N is the set of all individuals, viz., $N = \{1, \cdots, i, \cdots, n\}$. A is the set of *feasible social states*. In what follows, it is assumed that $3 \leq \#A < +\infty$. For each individual $i \in N$, $R_i \subseteq A \times A$ denotes i's (weak) preference ordering defined over A. For any $\mathbf{x}, \mathbf{y} \in A, (\mathbf{x}, \mathbf{y}) \in R_i$, or more briefly $\mathbf{x} R_i \mathbf{y}$, means that $\mathbf{x}$ is at least as good as $\mathbf{y}$ from i's viewpoint. $P(R_i)$ and $I(R_i)$ denote, respectively, the strict preference relation and the indifference relation corresponding to R_i. Thus, $\mathbf{x}P(R_i)\mathbf{y}$ if and only if $[\mathbf{x}R_i\mathbf{y}$ & *not* $\mathbf{y}R_i\mathbf{x}]$, and $\mathbf{x}I(R_i)\mathbf{y}$ if and only if $[\mathbf{x}R_i\mathbf{y}$ & $\mathbf{y}R_i\mathbf{x}]$. $\mathcal{R}$ denotes the universal set of preference orderings defined over A. An n-tuple $\mathbf{R} = (R_1, \cdots, R_i, \cdots, R_n)$ of individual preference orderings, one ordering for each individual, is a *profile* of individual preference orderings over A. $\mathcal{R}^n$ denotes the universal set of logically conceivable profiles.

To articulate individual rights in our framework, we introduce rights-systems as game forms. A *game form* is the pair $\gamma = (M, g)$, where $M \equiv \prod_{i \in N} M_i$ and M_i is the set of *permissible strategies* for individual $i \in N$, and $g : M \to A$ is the *outcome function* which specifies, for each strategy profile $\mathbf{m} \in M$, a feasible outcome $g(\mathbf{m}) \in A$. We assume that g is *surjective,* viz., $g(M) = A$. The universal set of game forms is Γ.

Given a profile $\mathbf{R} \in \mathcal{R}^n$ and a game form $\gamma = (M, g) \in \Gamma$, a pair $(\gamma, \mathbf{R})$ defines a *non-cooperative game*. In this paper, we adopt the Nash equilibrium concept: given a game $(\gamma, \mathbf{R})$, a strategy profile $\mathbf{m}^* \in M$ is a *Nash equilibrium in pure strategies*, Nash equilibrium for short, if $g(\mathbf{m}^*) R_i g(m_i, \mathbf{m}^*_{-i})$ holds for all $i \in N$ and all $m_i \in M_i$.[6] The set of all Nash equilibria of the game $(\gamma, \mathbf{R})$ is $\epsilon(\gamma, \mathbf{R})$. A conceivable social outcome $\mathbf{x}^* \in A$ is a *Nash equilibrium outcome* of the game $(\gamma, \mathbf{R})$ if there exists a Nash equilibrium $\mathbf{m}^* \in \epsilon(\gamma, \mathbf{R})$ satisfying $\mathbf{x}^* = g(\mathbf{m}^*)$. The set of all Nash equilibrium outcomes of the game $(\gamma, \mathbf{R})$ is $\tau(\gamma, \mathbf{R})$.

2.2. Social Decision Procedure for Rule Selection

Let us visualize the two-stage social decision procedure in the general setting. To begin with, every individual expresses his value judgements on the social desirability of alternative methods of conferring game form rights-system. Then, all individuals engage in debates about each other′s value judgements, providing justifications for their own values, and offering criticisms of values held by others. Sooner or later, there comes the stage where debate must stop and action must be taken by the society. In the *primordial stage of rule selection*, the social decision is made on the rights-system to be promulgated by aggregating

[6] For every $i \in N$ and every $\mathbf{m} \in M$, let $\mathbf{m}_{-i} \equiv (m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_n)$ and $M_{-i} \equiv \Pi_{j \neq i} M_j$. For every $i \in N$, every $m_i^0 \in M_i$ and every $\mathbf{m}_{-i} \in M_{-i}$, $(m_i^0, \mathbf{m}_{-i}) \equiv (m_1, \ldots, m_{i-1}, m_i^0, m_{i+1}, \ldots, m_n)$.

individuals′ value judgements regarding the initial rights-conferment through some democratic social decision procedure. After the rights-system as a game form $\gamma \in \Gamma$ is promulgated, and the profile of individual preference orderings $\mathbf{R} \in \mathcal{R}^n$ on the set of culmination outcomes is revealed, a fully-fledged game $(\gamma, \mathbf{R})$ is played in the *realization stage of the conferred game form rights-system*, which determines a Nash equilibrium social outcome $\mathbf{x}^* \in \tau(\gamma, \mathbf{R})$ if $\epsilon(\gamma, \mathbf{R}) \neq \varnothing$.

To make this scenario precise, we invoke the extended social choice framework by Pattanaik and Suzumura (1994; 1996).[7] For every $\mathbf{x} \in A$ and every $\gamma \in \Gamma$, a pair $(\mathbf{x}, \gamma) \in A \times \Gamma$ is an *extended (social) alternative.* Given $\mathbf{R} \in \mathcal{R}^n$, $(\mathbf{x}, \gamma)$ is said to be *realizable at* $\mathbf{R}$ if and only if $\mathbf{x} \in \tau(\gamma, \mathbf{R})$ holds. The intended interpretation is that the social outcome $\mathbf{x}$ is realized through the exercise of rights-system γ when the profile $\mathbf{R}$ prevails. In what follows, $\Lambda(\mathbf{R})$ denotes the set of all realizable extended alternatives at $\mathbf{R}$, viz.,

$$\Lambda(\mathbf{R}) = \{(\mathbf{x}, \gamma) \mid \mathbf{x} \in \tau(\gamma, \mathbf{R}) \ \& \ \gamma \in \Gamma\}.$$

The social decision procedure is formulated as follows. First, each individual i's value judgements on the desirability of rights-systems are assumed to be represented by an *ordering function* $Q_i : \mathcal{R}^n \twoheadrightarrow (A \times \Gamma)^2$ such that, for

[7] This framework due to Pattanaik and Suzumura capitalizes on the insightful observation by Arrow (1963, pp.89-90): "Up to now, no attempt has been made to find guidance by considering the components of the vector which defines the social state. One especially interesting analysis of this sort considers that, among the variables which taken together define the social state, one is the very process by which the society makes its choice. This is especially important if the mechanism of choice itself has a value to the individuals in the society. For example, an individual may have a positive preference for achieving a given distribution through the free market mechanism over achieving the same distribution through rationing by the government." See, also, Suzumura (1996; 1999; 2000; 2007).

each $\mathbf{R} \in \mathcal{R}^n$, $Q_i(\mathbf{R}) \subseteq \Lambda(\mathbf{R}) \times \Lambda(\mathbf{R})$ is a complete and transitive relation (ordering) defined over $\Lambda(\mathbf{R})$. $P(Q_i(\mathbf{R}))$ and $I(Q_i(\mathbf{R}))$ stand for the asymmetric part and the symmetric part of $Q_i(\mathbf{R})$, respectively. By definition, $(\mathbf{x}, \gamma)Q_i(\mathbf{R})(\mathbf{x}', \gamma')$ means that, according to i's value judgements, having a social outcome $\mathbf{x}$ through the play of the game $(\gamma, \mathbf{R})$ is at least as good for the society as having a social outcome $\mathbf{x}'$ through the play of the game $(\gamma', \mathbf{R})$. Let $\mathcal{Q}$ be the set of all logically possible ordering functions.

In the second place, the democratic procedure for aggregating individual value judgements is defined as follows.

Definition 1: *An* extended constitution function (**ECF**) *is a function* Ψ *which maps each and every profile of individual ordering functions* $\mathbf{Q} = (Q_i)_{i \in N}$ *in an appropriate domain* $\Delta_\Psi \subseteq \mathcal{Q}^n$ *into a social ordering function* Q*,* viz., $Q = \Psi(\mathbf{Q}) \in \mathcal{Q}$ for every $\mathbf{Q} \in \Delta_\Psi$.

This concept is due originally to Pattanaik and Suzumura (1996), which is a natural extension of the Arrovian *social welfare function* or *constitution function* [Arrow (1963)]. Note that, in the present framework as well as in that of Pattanaik and Suzumura (1996), there are two types of individual preference orderings. One is an individual′s preference ordering R_i over A, which represents i's *subjective tastes* over the set of culmination outcomes. The other is i's ordering function Q_i, which represents i's *value judgements* over the set of extended alternatives.[8] The latter preferences constitute the informational basis of the **ECF** to select a rights-system in the primordial stage

[8] Note that the individual ordering function does not have to be *ethical* in nature. It may generate a selfish extended preference ordering, where Q_i expresses i's *selfish judgements* if and only if, for every $\mathbf{R} \in \mathcal{R}^n$ and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $(\mathbf{x}, \gamma)Q_i(\mathbf{R})(\mathbf{x}', \gamma')$ (resp. $(\mathbf{x}, \gamma)P(Q_i(\mathbf{R}))(\mathbf{x}', \gamma')$) holds if and only if $\mathbf{x}R_i\mathbf{x}'$ (resp. $\mathbf{x}P(R_i)\mathbf{x}'$) holds.

of rule selection, whereas the former preferences serve as the informational basis for realizing a feasible social outcome in the realization stage of conferred game form rights-system.

Given an **ECF** Ψ, we define the associated *rational social choice function* as follows. For each profile of individual ordering functions $\mathbf{Q} \in \Delta_\Psi$, and for each profile of individual preference orderings $\mathbf{R} \in \mathcal{R}^n$, the set of game forms chosen through Ψ is given by

$$C(\Psi(\mathbf{Q}); \mathbf{R}) \equiv \{\gamma \in \Gamma \mid \exists\, \mathbf{x} \in A,\ \forall (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R}) : (\mathbf{x}, \gamma) Q(\mathbf{R}) (\mathbf{x}', \gamma'), \\ \text{where } Q = \Psi(\mathbf{Q})\}.$$

3. Basic Axioms

3.1. Democratic Extended Constitution Functions

What are the properties that qualify an **ECF** to be "reasonable"? Our first requirement is that Ψ is *minimally democratic* in the Paretian sense that the unanimous individual judgements are faithfully reflected in the social judgements in the following two senses.

Strong Pareto Principle (SP): *For every* $\mathbf{Q} \in \Delta_\Psi$, *every* $\mathbf{R} \in \mathcal{R}^n$, *and every* $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $(\mathbf{x}, \gamma) Q_i(\mathbf{R}) (\mathbf{x}', \gamma')$ *for all* $i \in N$ *and* $(\mathbf{x}, \gamma) P(Q_j(\mathbf{R}))(\mathbf{x}', \gamma')$ *for some* $j \in N$ *imply* $(\mathbf{x}, \gamma) P(Q(\mathbf{R}))(\mathbf{x}', \gamma')$, *where* $Q = \Psi(\mathbf{Q})$.

Pareto Indifference Principle (PI): *For every* $\mathbf{Q} \in \Delta_\Psi$, *every* $\mathbf{R} \in \mathcal{R}^n$, *and every* $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $(\mathbf{x}, \gamma) I(Q_i(\mathbf{R}))(\mathbf{x}', \gamma')$ *for all* $i \in N$ *implies* $(\mathbf{x}, \gamma) I(Q(\mathbf{R}))(\mathbf{x}', \gamma')$, *where* $Q = \Psi(\mathbf{Q})$.

The next requirement is a version of the Arrovian independence of irrelevant alternatives [Arrow (1963)] in our framework of extended alternatives.

Independence (I): *For every* $\mathbf{R} \in \mathcal{R}^n$*, every* $\mathbf{Q}$*,* $\mathbf{Q}' \in \Delta_\Psi$*, and every* $(\mathbf{x}, \gamma)$, $(\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, *if* $(\mathbf{x}, \gamma) Q_i(\mathbf{R})(\mathbf{x}', \gamma') \Leftrightarrow (\mathbf{x}, \gamma) Q'_i(\mathbf{R})(\mathbf{x}', \gamma')$ *holds for all* $i \in N$, *then* $(\mathbf{x}, \gamma) Q(\mathbf{R})(\mathbf{x}', \gamma') \Leftrightarrow (\mathbf{x}, \gamma) Q'(\mathbf{R})(\mathbf{x}', \gamma')$ *holds as well, where* $Q = \Psi(\mathbf{Q})$ *and* $Q' = \Psi(\mathbf{Q}')$.

For every $\mathbf{R} \in \mathcal{R}^n$ and given an **ECF** Ψ, an individual $d \in N$ is called an $\mathbf{R}$-*dictator under* Ψ if, for every $\mathbf{Q} \in \Delta_\Psi$ and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $(\mathbf{x}, \gamma) P(Q_d(\mathbf{R}))(\mathbf{x}', \gamma')$ implies $(\mathbf{x}, \gamma) P(Q(\mathbf{R}))(\mathbf{x}', \gamma')$, where $Q = \Psi(\mathbf{Q})$. The last democratic requirement on Ψ is as follows.

Non-Dictatorship (**ND**): *For every* $\mathbf{R} \in \mathcal{R}^n$, *there is no* $\mathbf{R}$-*dictator under* Ψ.

Note that these four requirements on the extended constitution function are natural extensions of Arrow's axioms on the standard Arrovian constitution function [Arrow (1963)], except for the parametric role played by $\mathbf{R}$ in the definition of a dictator under Ψ.

3.2. Uniform Rationality of Social Choice Functions

In the second place, we want Ψ to generate uniformly rational social choice functions in the sense that it satisfies the following property.

Uniformity of Rational Choice (URC): *For every* $\mathbf{Q} \in \Delta_\Psi$,

$$\cap_{\mathbf{R} \in \mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R}) \neq \varnothing.$$

If this condition is satisfied and γ^* is chosen from this intersection, γ^* applies *uniformly* to every future realization of $\mathbf{R} \in \mathcal{R}^n$. Since the game form is the formal method of specifying the initial conferment of rights-system in the society prior to the realization of the profile of individual preference orderings over culmination outcomes, it seems desirable to design the extended constitution function satisfying the condition **URC**. Indeed, if we implement a game form $\gamma^* \in \cap_{\mathbf{R}\in\mathcal{R}^n} C(Q;\mathbf{R})$, γ^* prevails as the rights-system no matter how frivolously $\mathbf{R}$ may change.[9,10]

3.3. Nash Solvability, Liberalism, and Efficiency

In the third place, we introduce three "reasonable" requirements on the class of game forms, which qualify these game forms to be relevant from the viewpoint of welfare and rights. They embody a requirement of *stability* in the social decision-making, a requirement of *liberalism*, and a requirement of *social efficiency*, respectively. The first property is due to van Hees (1999), which is well-known in game theory as the *Nash solvability* of a game form.[11]

Definition 2: *A game form* $\gamma \in \Gamma$ *is* Nash solvable *if* $\tau(\gamma, \mathbf{R})$ *is non-empty for every profile* $\mathbf{R} \in \mathcal{R}^n$*.*

[9] It is true that the condition **URC** is strong, as it requires that the promulgated rules of the game remain insensitive to the unforeseen changes in $\mathbf{R} \in \mathcal{R}^n$. It follows that the conditions which guarantee the satisfaction of **URC** cannot but be stringent and go beyond the consequentialist border of informational constraints.

[10] Another argument in favor of the condition **URC** may proceed as follows. Suppose that an individual $i \in N$, a profile $\mathbf{R} \in \mathcal{R}^n$, and an ordering R_i^* are such that $\{\gamma\} = C(Q;\mathbf{R}), \{\mathbf{x}\} = \tau(\gamma,\mathbf{R}), \{\gamma^*\} = C(Q;(R_i^*;\mathbf{R}_{-i}))$, and $\{\mathbf{x}^*\} = \tau(\gamma^*,(R_i^*;\mathbf{R}_{-i}))$ satisfy $\mathbf{x}^* P(R_i)\mathbf{x}$ or $\mathbf{x}P(R_i^*)\mathbf{x}^*$. In this case, either $C(Q;\mathbf{R})$ is manipulable by means of R_i^*, or $C(Q;\,(R_i^*;\mathbf{R}_{-i}))$ is manipulable by means of R_i. The condition **URC** excludes the possibility of such manipulability.

[11] The Nash solvability plays an important role in the game form formulation of libertarian rights. Indeed, Peleg (1998) formulated the Gibbard paradox [Gibbard (1974)] in the game form formulation by means of the fact that the game form is not Nash solvable. Furthermore, Peleg, Peters and Storchen (2002) identified a necessary and sufficient condition for the Nash solvability with the purpose of providing a resolution of the Gibbard paradox.

Let Γ_{NS} denote the subclass of Γ which consists solely of the Nash solvable game forms.

The second property is related to the intrinsic value of libertarian rights-system. As an auxiliary step, we introduce the α-*effectivity function* of a game form, which enables us to capture the (*veto*) power structure which a game form confers to individuals. Given a game form $\gamma = (M, g)$, the associated α-effectivity function E^γ can be defined by $E^\gamma(\varnothing) = \varnothing$ and, for every non-empty $S \subseteq N$,

$$E^\gamma(S) \equiv \left\{B \subseteq A \mid \exists \mathbf{m}_S \in M_S, \forall \mathbf{m}_{N\setminus S} \in M_{N\setminus S} : g(\mathbf{m}_S, \mathbf{m}_{N\setminus S}) \in B\right\},$$

where $M_S \equiv \Pi_{i\in S} M_i$ and $M_{N\setminus S} \equiv \Pi_{i\in N\setminus S} M_i$.

The concept of the α-effectivity function enables us to identify two types of game forms. Note that we are hereby using somewhat abusive expression such as $E^\gamma(i)$ instead of $E^\gamma(\{i\})$ for every $i \in N$.

Definition 3: *A game form $\gamma = (M, g) \in \Gamma$ is* dictatorial *if there exists a unique individual $i \in N$, to be called the* dictator *of γ, such that $E^\gamma(i) = 2^A \setminus \{\varnothing\}$ and $E^\gamma(j) = \{A\}$ for every $j \neq i$. A dictatorial game form in which $i \in N$ is the dictator is called the* i-dictatorial game form.

For each $i \in N$, $\Gamma(i)$ denotes the set of all i-dictatorial game forms.

Definition 4 [Peleg (1998)]**:** *A game form $\gamma = (M, g) \in \Gamma$ satisfies* minimal liberalism *if there exist at least two individuals $i, j \in N$ such that there are $B^i \in E^\gamma(i)$ and $B^j \in E^\gamma(j)$ with $B^i \neq A \neq B^j$.*

In fact, the property of minimal liberalism may not be attractive in many person society, as it is compatible with the possibility of duopolistic allocation of effective powers in the presence of many other individuals with no power whatsoever. To avoid such a duopolistic power structure in many person society, we introduce a slightly stronger version of minimal liberalism.

Definition 5: *A game form* $\gamma = (M, g) \in \Gamma$ *is* liberal *if every individual* $i \in N$ *has an effective power so that there exists* $B^i \in E^\gamma(i)$ *with* $B^i \neq A$.

Let Γ_L denote the subclass of Γ which consists solely of liberal game forms.

The third property is the consequentialist value of social efficiency.

Definition 6: *A game form* $\gamma \in \Gamma$ *is* efficient *if, for every profile* $\mathbf{R} \in \mathcal{R}^n$, *there exists a Pareto efficient Nash equilibrium outcome in* A *whenever* $\tau(\gamma, \mathbf{R})$ *is non-empty.*

We denote the set of efficient game forms by Γ_{PE}.

The condition of efficiency is particularly relevant in the context of *liberal paradox* in the game form formulation of individual rights. Recollect that Deb, Pattanaik, and Razzolini (1997) proposed two notions of liberal paradox: *strong paradox* and *weak paradox*. The former says that, for some preference profile, every Nash equilibrium outcome is Pareto inefficient, whereas the latter says that, for some preference profile, there is a Nash equilibrium outcome which is Pareto inefficient. According to this classification, the existence of an efficient game form defined above resolves the strong paradox, but not the weak paradox. Although the resolution of the weak paradox is preferable to that of the strong paradox, it is a desideratum which is impossible to attain, since any game form satisfying minimal liberalism should have a Pareto inefficient outcome for some preference profile, as Peleg (1998) has shown.

We can show that there exists a game form which satisfies all of the above three conditions.

Proposition 1: *There exists a game form* $\gamma^* \in \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$.

According to Peleg, Peters and Storchen (2002), the Nash solvability is equivalent to the claim that, for every preference profile, there exists a *weakly Pareto efficient* Nash equilibrium outcome. Since we are requiring not weak Pareto efficiency, but strong Pareto efficiency, we cannot simply invoke their equivalence theorem in the context of verifying the validity of **Proposition 1**.

4. On the Existence of Reasonable ECF

Under what domain restrictions on the acceptable class of profiles of individual ordering functions can we construct an **ECF** which is not only consistent with the four Arrovian axioms of **SP**, **PI**, **I**, and **ND**, but also is capable of choosing a liberal game form? What about the stringent, but highly desirable property of uniformly rational choice of game form rights-system?

In what follows, we define a subclass $\mathcal{S}$ of individual ordering functions which may be called the *self-interested* class, and ask about the existence of an **ECF** which is workable for every profile of individual ordering functions within this specified class.

The definition of $\mathcal{S}$ goes as follows: for every $i \in N$, $Q_i \in \mathcal{S}$ implies, for every $\mathbf{R} \in \mathcal{R}^n$ and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, that

(a) if $\gamma = \gamma'$, then $(\mathbf{x}, \gamma) Q_i(\mathbf{R}) (\mathbf{x}', \gamma')$ if and only if $\mathbf{x} R_i \mathbf{x}'$; and

(b) if $\mathbf{x} = \mathbf{x}'$, then $E^{\gamma}(i) \supseteq$ (resp. $\supsetneq$) $E^{\gamma'}(i)$ implies $(\mathbf{x}, \gamma) Q_i(\mathbf{R}) (\mathbf{x}', \gamma')$ (resp. $(\mathbf{x}, \gamma) P(Q_i(\mathbf{R})) (\mathbf{x}', \gamma')$).

The meaning of the first restriction (a) should be clear: whenever the two extended alternatives $(\mathbf{x}, \gamma)$ and $(\mathbf{x}', \gamma')$ share the same game form $\gamma = \gamma'$, then the evaluation by Q_i over the pair $(\mathbf{x}, \gamma)$ and $(\mathbf{x}', \gamma')$ is in accordance with his personal preferences R_i over the pair of culmination outcomes $\mathbf{x}$ and $\mathbf{x}'$. It means that this individual transcribes his selfish preferences over the set of culmination outcomes at least partly into his value judgements over the set of extended alternatives. The second restriction (b) says that whenever the two extended alternatives $(\mathbf{x}, \gamma)$ and $(\mathbf{x}', \gamma')$ share the same culmination outcome $\mathbf{x} = \mathbf{x}'$, Q_i prefers $(\mathbf{x}, \gamma)$ to $(\mathbf{x}', \gamma')$ at every $\mathbf{R} \in \mathcal{R}^n$ with $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$ as long as γ endows i with more effective power than γ' does. Hence Q_i deserves to the name of *self-interested preferences*.

Although $\mathcal{S}^n$ is highly restrictive vis-à-vis $\mathcal{Q}^n$, we cannot thereby exorcise the Arrovian phantom. Indeed, we may show that, for every Ψ with $\Delta_\Psi = \mathcal{S}^n$ which satisfies **SP**, **PI** and **I**, there exists $d \in N$ such that $C(\Psi(\mathbf{Q}); \mathbf{R}) = C(Q_d; \mathbf{R})$ for every $\mathbf{Q} \in \mathcal{S}^n$. Moreover, if Ψ satisfies **URC**, then the class of d-dictatorial game forms is uniformly chosen, viz., $\Gamma(d) \subseteq \cap_{\mathbf{R} \in \mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R})$ for every $\mathbf{Q} \in \mathcal{S}^n$, and $\cap_{\mathbf{Q} \in \mathcal{S}^n} \cap_{\mathbf{R} \in \mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R}) = \Gamma(d)$. See Suzumura and Yoshihara (2008) for details.

The message of this assertion is simple. It says that the Arrovian impossibility result cannot be avoided in the present context of social choice of game form rights-system even on the strongly restrictive self-interested domain.

How, then, can we enable a society with self-interested population to find a method for conferring a liberal rights-system through a non-dictatorial procedure? To answer this question in the affirmative, we introduce a further restriction on the self-interested domain $\mathcal{S}^n$.

As an auxiliary step, let us define, for every $j \in N$, a subset $\Gamma_j^0 \subseteq \Gamma$ by

$$\Gamma_j^0 \equiv \{\gamma \in \Gamma \mid E^\gamma(j) = \{A\}\}.$$

Intuitively speaking, Γ_j^0 consists of game forms in which j is *powerless* in terms of effectivity. Note in particular that the set of all i-dictatorial game forms satisfies the following set-inclusion:

$$\Gamma(i) \subseteq \cap_{j \in N \setminus \{i\}} \Gamma_j^0.$$

For each $i \in N$, let $\Gamma^u(i) \subseteq \Gamma$ and $\Gamma^p(i) \subseteq \Gamma$ be defined, respectively, by

$$\Gamma^u(i) \equiv \cup_{j \in N \setminus \{i\}} \Gamma_j^0$$

and

$$\Gamma^p(i) \equiv \Gamma \setminus \Gamma^u(i) = \cap_{j \in N \setminus \{i\}} (\Gamma \setminus \Gamma_j^0).$$

Intuitively speaking, $\Gamma^u(i)$ consists of game forms in which somebody other than $i \in N$ is *unprivileged* in the sense of being powerless in terms of effectivity, whereas $\Gamma^p(i)$ consists of game forms in which nobody other than $i \in N$ is unprivileged in the same sense.

By means of these auxiliary concepts, we define a class of coalitions $\mathcal{N}_i(\mathbf{Q}) \subseteq 2^{N \setminus \{i\}}$ for every $i \in N$ and $\mathbf{Q} \in \mathcal{S}^n$ as follows: for every $S \subseteq N \setminus \{i\}$, $S \in \mathcal{N}_i(\mathbf{Q})$ if and only if, for every $\gamma \in \Gamma^p(i)$, every $\gamma' \in \Gamma^u(i)$ with $E^{\gamma'}(S) = \{A\}$, every $\mathbf{R}, \mathbf{R}' \in \mathcal{R}^n$, every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, and every $(\mathbf{y}, \gamma), (\mathbf{y}', \gamma') \in \Lambda(\mathbf{R}')$, there exists at least one $j \in S$ such that the following condition is satisfied, where **(n-c)** is named after *non-consequentialism*:

$$(\mathbf{x}, \gamma) Q_j(\mathbf{R}) (\mathbf{x}', \gamma') \Leftrightarrow (\mathbf{y}, \gamma) Q_j(\mathbf{R}') (\mathbf{y}', \gamma'). \qquad \textbf{(n-c)}$$

Thus, $S \in \mathcal{N}_i(\mathbf{Q})$ means that if γ' deprives all members in S taken together of their effective power, and at least some member in S of his/her effective power, whereas γ does not deprive any member in S of his/her effective power, then there should be some member $j \in S$ who ranks at Q_j the game form γ at least as high as the game form γ', regardless of the culmination outcomes which γ

and γ' bring about at $\mathbf{R}$ and $\mathbf{R}'$, respectively. Taking the condition (b) of $\mathcal{S}$ into consideration, it follows that the set $\mathcal{N}_i(\mathbf{Q})$ is the class of coalitions in $N \setminus \{i\}$, each element of which contains at least one member who consistently values at $\mathbf{Q}$, regardless of the culmination outcomes which may emerge, the protection of rights of all members of S higher than i's potential dictatorship.

Then we may assert the following general possibility theorem.

Theorem 1: *For every* $i \in N$*, there exists an* **ECF** Ψ *with* $\Delta_\Psi \subseteq \mathcal{S}^n$ *that satisfies* **SP**, **PI**, **I**, **ND**, *and that* $\varnothing \neq C(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$ *holds for every* $\mathbf{Q} \in \Delta_\Psi$ *and every* $\mathbf{R} \in \mathcal{R}^n$ *if* $\cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N \setminus \{i\}} \setminus \{\varnothing\}$.

The gist of **Theorem 1** is easy to explain. If every individual other than i is resolute enough to insist that the complete deprivation of his/her rights should be rejected no matter what may subsequently happen in the realization stage of the conferred rights, then his/her rights in terms of effective power can be protected through the democratic social decision procedure. Since the game form $\gamma^* \in C(\Psi(\mathbf{Q}); \mathbf{R})$ belongs to Γ_L, γ^* is not only non-dictatorial, but also confers liberty on every individual including i himself. [12]

[12] The proof of **Theorem 1** is relegated to Appendix for the sake of simplifying the exposition of the main text. It may facilitate the understanding of **Theorem 1**, however, if we explain the structure of **ECF** constructed in its proof. It hinges on the two-tier social ordering function in the following sense: if a pair $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma')$ is such that $\gamma \in \Gamma^p(i)$ and $\gamma' \in \Gamma^u(i)$, then $Q(\mathbf{R})$ always ranks $(\mathbf{x}, \gamma)$ higher than $(\mathbf{x}', \gamma')$ regardless of what $\mathbf{R} \in \mathcal{R}^n$ may materialize, and if the pair is such that both $(\mathbf{x}, \gamma)$ and $(\mathbf{x}', \gamma')$ belong to $\Gamma^p(i)$, then $Q(\mathbf{R})$ should be consistent with the two Pareto conditions **SP** and **PI**.

Why does Ψ perform nicely as **Theorem 1** asserts? Suppose that individual i is a potential dictator. Take a pair $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma')$ such that $\gamma \in \Gamma^p(i)$ and $\gamma' \in \Gamma^u(i)$. Then, $\cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N \setminus \{i\}} \setminus \{\varnothing\}$ implies that somebody other than i, say j, who is powerless in γ', ranks $(\mathbf{x}, \gamma)$ higher than $(\mathbf{x}', \gamma')$ regardless of which $\mathbf{R} \in \mathcal{R}^n$ may materialize, and

According to **Theorem 1**, there exists a Ψ such that $C(\Psi(\mathbf{Q}); \mathbf{R})$ consists solely of Nash solvable, efficient, and liberal game forms. Note, however, that if $\#N \geq 3$, there exists a class of Nash solvable, efficient, and non-dictatorial game forms which are not satisfactory from the point of view of liberty. This is the class of *King-maker mechanisms* due to Hurwicz and Schmeidler (1978): for every $i \in N$, a game form $\gamma^{K_i} = (M^{K_i}, g^{K_i})$ is a King-maker mechanism if $M_i^{K_i} = N, M_h^{K_i} = A$ for every $h \in N \setminus \{i\}$, and $g^{K_i}(\mathbf{m}) = m_{m_i}$ for every $\mathbf{m} \in M^{K_i}$. In view of this fact, it may be of some interest that the following corollary of **Theorem 1** holds true.

Corollary 1: *For every* $i \in N, \gamma^{K_i} \notin \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$.

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ranks $(\mathbf{y}, \gamma)$ higher than $(\mathbf{y}', \gamma')$ whenever $(\mathbf{y}, \gamma)$ and $(\mathbf{y}', \gamma')$ are realizable at any other $\mathbf{R}' \in \mathcal{R}^n$. On the basis of the existence of such a protesting individual, the non-dictatorial Paretian Ψ may define a social ordering function $Q(\mathbf{R})$ with the two-tier structure described above. Then, the rational choice set $C(\Psi(\mathbf{Q}); \mathbf{R})$ becomes a subset of $\Gamma^p(i)$. However, $\Gamma^p(i)$ may contain game forms in which i is powerless. Let γ be such a game form and $(\mathbf{x}, \gamma)$ be realizable at $\mathbf{R} \in \mathcal{R}^n$. As is shown in Appendix, there exists a liberal game form γ^* such that $(\mathbf{x}, \gamma^*)$ is realizable at $\mathbf{R}$. Then, since $\Delta_\Psi \subseteq \mathcal{S}^n$, i ranks $(\mathbf{x}, \gamma^*)$ higher than $(\mathbf{x}, \gamma)$ by means of Q_i at $\mathbf{R}$, which enables the corresponding social ordering $Q(\mathbf{R})$ to rank $(\mathbf{x}, \gamma^*)$ higher than $(\mathbf{x}, \gamma)$ at $\mathbf{R}$, since Ψ is Paretian. Thus, the rational choice set $C(\Psi(\mathbf{Q}); \mathbf{R})$ becomes a subset of Γ_L. Moreover, any game form in $C(\Psi(\mathbf{Q}); \mathbf{R})$ represents a $maximal freedom$ [van Hees (1999)], as we discuss in **Appendix**, which makes any game form in $C(\Psi(\mathbf{Q}); \mathbf{R})$ Nash solvable and efficient.

Thus, given the society with self-interested individuals, we can define a social decision procedure that chooses liberal, Nash solvable, and efficient rights-system. This procedure consists of two components: one is the mechanism Ψ with the simple two-tier structure, and the other is the individuals' protest against the complete deprivation of their own rights in effective power. The importance of the second component is worth emphasizing, as the mechanism design of **ECF** *per se* may be incapable of securing nice properties in the absence of individuals' attitudes towards protecting their own rights.

Proof. By definition, the α-effectivity function $E^{\gamma^{K_i}}$ associated with γ^{K_i} has $E^{\gamma^{K_i}}(h) = \{A\}$ for every $h \in N$, which implies that γ^{K_i} is not a liberal game form. ■

Combined with **Theorem 1**, it follows that *any King-maker mechanism cannot be rationally chosen via* Ψ *even when* $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N\setminus\{i\}} \setminus \{\varnothing\}$.

Observe, however, that **Theorem 1** falls short of guaranteeing the uniform rational choice of a liberal game form. To secure the desirable property of uniformity, we must introduce another domain restriction. For every $\mathbf{Q} \in \mathcal{S}^n$ define a class of coalitions $\mathcal{M}_i(\mathbf{Q}) \subseteq 2^N$ as follows: for every $S \subseteq N$, $S \in \mathcal{M}_i(\mathbf{Q})$ if and only if, for every $\gamma, \gamma' \in \Gamma^p(i)$, every $\mathbf{R}$, $\mathbf{R}' \in \mathcal{R}^n$, every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, and every $(\mathbf{y}, \gamma), (\mathbf{y}', \gamma') \in \Lambda(\mathbf{R}')$, there exists at least one individual $j \in S$ such that Q_j satisfies the condition **(n-c)**. We may then assert the following:

Theorem 2: *For every* $i \in N$, *there exists an* **ECF** Ψ *with* $\Delta_\Psi \subseteq \mathcal{S}^n$ *such that* **SP**, **PI**, **I**, **ND** *and* $\varnothing \neq \cap_{\mathbf{R}\in\mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$ *holds for every* $\mathbf{Q} \in \Delta_\Psi$ *if* $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N\setminus\{i\}}\setminus \{\varnothing\}$ *and* $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{M}_i(\mathbf{Q}) \supseteq 2^N\setminus\{\varnothing\}$.

According to **Theorem 2**, if every individual has only the non-consequential preferences on rights-systems, it is not only possible to resolve the Arrovian impossibility impasse, but it is also possible to protect every individual′s liberty in terms of his/her effective power, and to choose uniformly rational, liberal, Nash solvable, and Pareto efficient game form as a rights-system to be promulgated.

Before concluding this section, it is worthwhile to examine the following augmented **Example 1**, which may facilitate the understanding of what **Theorem 1** asserts.

Example 2: Let us augment **Example 1** by introducing two dictatorial game forms γ^1 and γ^2 which are defined as follows. The 1-dictatorial game form $\gamma^1 = (M_1^{\gamma^1} \times M_2^{\gamma^1}, g^{\gamma^1})$ is such that $M_1^{\gamma^1} = \{m_{11}^{\gamma^1}, m_{12}^{\gamma^1}, m_{13}^{\gamma^1}\}$, and $M_2^{\gamma^1} = \{m_{2h}^{\gamma^1}\}_{h\in H}$, where $m_{11}^{\gamma^1}$= "to smoke in the presence of 2", $m_{12}^{\gamma^1}$="to smoke in the absence of 2", and $m_{13}^{\gamma^1}$="not to smoke in the presence of 2", and the outcome function g^{γ^1} is defined by

$g^{\gamma^1}(m_{11}^{\gamma^1}, m_{2h}^{\gamma^1}) = (s, r)$ for all $h \in H$

$g^{\gamma^1}(m_{12}^{\gamma^1}, m_{2h}^{\gamma^1}) = (s, l)$ for all $h \in H$

and

$g^{\gamma^1}(m_{13}^{\gamma^1}, m_{2h}^{\gamma^1}) = (ns, r)$ for all $h \in H$.

Likewise, the 2-dictatorial game form $\gamma^2 = (M_1^{\gamma^2} \times M_2^{\gamma^2}, g^{\gamma^2})$ is such that $M_1^{\gamma^2} = \{m_{1k}^{\gamma^2}\}_{k\in K}$ and $M_2^{\gamma^2} = \{m_{21}^{\gamma^2}, m_{22}^{\gamma^2}, m_{23}^{\gamma^2}\}$, where $m_{21}^{\gamma^2}$ = "to force 1 to smoke and leave the compartment", $m_{22}^{\gamma^2}$ = "to force 1 not to smoke and remain in the compartment" and $m_{23}^{\gamma^2}$ = " to force 1 to smoke and remain in the compartment", and the outcome function g^{γ^2} is defined by

$g^{\gamma^2}(m_{1k}^{\gamma^2}, m_{21}^{\gamma^2}) = (s, l)$ for all $k \in K$

$g^{\gamma^2}(m_{1k}^{\gamma^2}, m_{22}^{\gamma^2}) = (ns, r)$ for all $k \in K$

and

$g^{\gamma^2}(m_{1k}^{\gamma^2}, m_{23}^{\gamma^2}) = (s, r)$ for all $k \in K$.

Given the profile $\mathbf{R} = (R_1, R_2)$ that is defined in footnote 4, we may check that $\tau(\gamma^1, \mathbf{R}) = \{(s, l)\}$ and $\tau(\gamma^2, \mathbf{R}) = \{(ns, r)\}$.

Let us now check whether or not $\gamma, \gamma^*, \gamma^1$ and γ^2 are capable of being chosen by means of an **ECF** Ψ in **Theorem 1**. To begin with, it is clear that

$E^{\gamma}(1) = \Omega(\{(s, l), (s, r)\}) \cup \Omega(\{(ns, r)\})$;

$E^{\gamma}(2) = \Omega(\{(s, l), (ns, r)\}) \cup \Omega(\{(s, r), (ns, r)\})$;

$E^{\gamma^*}(1) = \Omega(\{(ns, r)\}), E^{\gamma^*}(2) = \Omega(\{(ns, r)\})$;

$$E^{\gamma^1}(1) = \Omega(\{(ns,r)\}) \cup \Omega(\{(s,l)\}) \cup \Omega(\{(s,l)\}), E^{\gamma^1}(2) = \{A\}; \text{ and}$$
$$E^{\gamma^2}(1) = \{A\}, E^{\gamma^2}(2) = \Omega(\{(ns,r)\}) \cup \Omega(\{(s,r)\}) \cup \Omega(\{(s,l)\}),$$

where $\Omega(B)$ denotes the family of sets consisting of B and all its supersets in A.

We may also verify that, for any game form $\delta \in \{\gamma, \gamma^*, \gamma^1, \gamma^2\}$, there exists no game form $\delta' \in \Gamma$ such that δ' *power-dominates* δ in the sense that $E^{\delta'}(i) \supseteq E^{\delta}(i)$ for $i = 1$ and 2, and $E^{\delta'}(j) \supsetneq E^{\delta}(j)$ for $j = 1$ or 2. Furthermore, γ and γ^* are liberal game forms, whereas γ^1 and γ^2 are not. By virtue of **Lemma 1** in Appendix, for any game form $\delta \in \{\gamma, \gamma^*\}$, there exists a Nash solvable, liberal and Pareto efficient game form δ^* such that $E^{\delta} = E^{\delta^*}$, which we will denote by $\delta^*(\gamma)$ and $\delta^*(\gamma^*)$. According to **Theorem 1**, $\delta^*(\gamma)$ and $\delta^*(\gamma^*)$, being in $\Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$, are eligible from Γ. However, whether or not any one of $\delta^*(\gamma)$ and $\delta^*(\gamma^*)$ is chosen, or a game form in $(\Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}) \backslash \{\delta^*(\gamma), \delta^*(\gamma^*)\}$ is chosen instead, hinges squarely on the profile $\mathbf{Q} = (Q_1, Q_2)$, and lies outside the reach of the design of the mechanism Ψ itself. ■

5. Ethical Domain: Alternative Avenue to Possibility Theorems

In the context of social choice theoretic articulation of libertarian rights, there are several conspicuous attempts in the literature to resolve the Pareto libertarian paradox by introducing some ethical constraints on the class of admissible profiles, or on the attitudes of people towards others within their personal spheres, or the combination of both.[13] It is interesting to see how the ethical constraints can be articulated within the game form formulation of libertarian rights, and how they affect the strenuous impossibility theorem in this

[13] For a survey of these social choice theoretic literature, see Suzumura (2008, Section 4.3).

arena. The purpose of this section is to describe our theoretical scenario for performing this plan without going into details.

To begin with, we define a non-consequentialist liberal ordering function by the combination of two requirements. The first requirement is that an ordering function Q is *liberal* in the sense that $\gamma \in \Gamma_L$ and $\gamma' \in \Gamma \setminus \Gamma_L$ necessarily imply $(\mathbf{x}, \gamma) P(Q(\mathbf{R}))(\mathbf{x}', \gamma')$ for every $\mathbf{R} \in \mathcal{R}^n$ and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$. The second requirement is that Q is *non-consequentialist* in the sense that it satisfies the condition (**n**-**c**) for every $\gamma, \gamma' \in \Gamma_L$, every $\mathbf{R}, \mathbf{R}' \in \mathcal{R}^n$, every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, and every $(\mathbf{y}, \gamma), (\mathbf{y}', \gamma') \in \Lambda(\mathbf{R}')$. If an ordering function Q is liberal as well as non-consequentialist, Q is said to be *non-consequentialist liberal*. The target of this analysis is to identify some conditions on the domain Δ_Ψ of an extended constitution function Ψ and on people's attitudes towards others, so that Ψ can aggregate every profile $\mathbf{Q}$ of individual ordering functions in the suitably circumscribed domain Δ_Ψ into a non-consequentialist liberal social ordering function $\Psi(\mathbf{Q})$ and that the socially chosen game forms satisfy the following:

$$\varnothing \neq C(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE} \text{ for every } \mathbf{R} \in \mathcal{R}^n.$$

In Suzumura and Yoshihara (2008), we have identified the necessary and sufficient condition for the above property to hold. We have also identified the sufficient condition for the stronger property of uniformity of rational choice:

$$\varnothing \neq \cap_{\mathbf{R} \in \mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$$

for every $\mathbf{Q} \in \Delta_\Psi$.

6. Concluding Remarks

Instead of summarizing the main contents of this paper, let us conclude it by making three brief observations on the general nature of our analysis.

In the first place, this paper represents a non-standard attempt to embody non-consequentialism within the extended framework of social choice theory à la Pattanaik and Suzumura (1996). In the context of the theory of welfare and rights, which was first explored by Sen (1970; 1970a) in terms of the *impossibility of a Paretian liberal*, we have shown how the seldom-discussed problem of initial conferment of libertarian rights-system can be neatly analyzed by means of the Pattanaik-Suzumura framework. In particular, it is shown that an extended constitution that satisfies Arrovian conditions of democratic social decision-making exists, which can choose a game form rights-system satisfying Nash solvability, efficiency, liberalism and uniform applicability, whenever individuals resolutely resist to the loss of their effective libertarian power in the initial conferment of individual rights-system even if doing so may result in their welfare loss in terms of culmination outcomes. In this sense, it may be understood as an attempt to pay non-consequentialist attention to the intrinsic value of *social choice procedures*. In this sense, it may be construed as the counterpart to an earlier attempt by Suzumura and Xu (2001; 2004) within the Arrovian social choice framework, which tried to pay non-consequentialist attention to the intrinsic value of *social choice opportunities*.

In the second place, the possibility theorem in this paper cannot be secured simply by means of the clever design of social choice mechanisms. The efficacy of our escape routes from the logical impasse of welfare and rights hinges squarely on the individual attitudes towards intrinsic value of liberty. In this sense, it has some family resemblance to the well-known resolution scheme within the context of social choice theoretic articulation of individual rights, which is due to Sen (1976) and Suzumura (1978) where the concept of a *liberal indivudual* plays a pivotal role. Recollect that "an individual is said to be liberal [in the Sen-Suzumura scheme] if and only if he claims only those parts of his preferences that are compatible with others' preferences over their respective protected spheres to count in social choice [Suzumura (1983, p.196)]." It is the existence of such an individual who pays attention to the social realization of individual libertarian rights that serves us as an ultimate guarantee for the successful working of a liberal society.

In the third and last place, the rights conferment is a repeated exercise for a society in view of the past performance of the previously conferred rights. The present analysis is confined to the initial segment of this larger problem. The fixed and finite set of individuals as well as the static analytical framework is chosen for the sake of analyzing this tractable initial segment. The fully-fledged analysis of the rights-conferment may well require us to go far beyond the present framework. Since the current state of the art falls much short of attemping such an ambitious analysis, the limitation of the present paper may be tolerated as our modest first attempt.

Appendix: Proofs

For any $\mathbf{R} \in \mathcal{R}^n$ and any $\mathbf{Q} \in \Delta_\Psi$, we define $Q_N(\mathbf{R})$ by $(\mathbf{x}, \gamma)Q_N(\mathbf{R})(\mathbf{y}, \gamma')$ for all $\mathbf{x}, \mathbf{y} \in A$ and $\gamma, \gamma' \in \Gamma$ if and only if $(\mathbf{x}, \gamma)Q_h(\mathbf{R})(\mathbf{y}, \gamma')$ for all $h \in N$. $I(Q_N(\mathbf{R}))$ and $P(Q_N(\mathbf{R}))$ are the symmetric part and the asymmetric part of $Q_N(\mathbf{R})$, respectively. Let a profile $\mathbf{R}^0 \in \mathcal{R}^n$ be such that every individual is universally indifferent over A. Given $\gamma = (M, g) \in \Gamma$, for any $h \in N$ and any $m_h \in M_h$, let $B^h_{m^h} \equiv g(m_h, M_h)$. Then, $E^\gamma(h) = \cup_{m_h \in M_h} \Omega(B^h_{m_h})$ for each and every $h \in N$, where and hereafter $\Omega(B)$ denotes the family of sets consisting of B and all its supersets in A. Given $\gamma = (M, g) \in \Gamma$ and for every $h \in N$, let Θ_h^γ be the set of *minimal subsets* in $E^\gamma(h)$ in terms of set-theoretic inclusion.

Let us say that a game form $\gamma \in \Gamma$ is *power-dominated* by another game form $\gamma' \in \Gamma$ if and only if $E^{\gamma'}(i) \supseteq E^\gamma(i)$ for all $i \in N$ and $E^{\gamma'}(j) \supsetneq E^\gamma(j)$ for some $j \in N$. A game form $\gamma^* \in \Gamma$ represents a *maximal power structure* if there is no other game form $\gamma \in \Gamma$ which power-dominates γ^*. Let us denote by $\mu(\Gamma)$ the set of game forms such that each member of $\mu(\Gamma)$ represents a maximal power structure in Γ. Given a game form $\gamma \in \Gamma$, E^γ satisfies *maximal freedom* in the sense of van Hees (1999) if $\gamma \in \mu(\Gamma)$ and, for any $\left(B^h\right)_{h \in N} \in \prod_{h \in N} \Theta_h^\gamma$, $\left|\cap_{h \in N} B^h\right| = 1$. Thus, E^γ satisfies maximal freedom if γ represents a maximal power structure in Γ, and the exercise of maximal power by each and every individual is enough to identify a unique culmination outcome.

Proof of Proposition 1: For every game form $\gamma = (M, g) \in \Gamma$ and culmination outcome $\mathbf{x} \in A$, define a new game form $\gamma' = (M', g') \in \Gamma$ as follows: for every $h \in N$, $M'_h = M_h \cup \{\mathbf{x}\}$, and let g' be such that, for every $\mathbf{m} \in M'$,

$$\begin{cases} g'(\mathbf{m}) = \mathbf{x} & \text{if } m_h = \mathbf{x} \in M'_h \text{ for some } h \in N; \\ g'(\mathbf{m}) = g(\mathbf{m}) & \text{otherwise.} \end{cases}$$

Then, $E^{\gamma'}(h) = \Omega(\{\mathbf{x}\})$ for each $h \in N$. Note that $\gamma' \in \mu(\Gamma)$ holds true. This is because, for any $\gamma'' \in \Gamma \backslash \{\gamma'\}$, if $E^{\gamma''}(k) \supseteq \Omega(\{\mathbf{x}\}) \cup \Omega(B)$ holds for some $B \subseteq A \backslash \{\mathbf{x}\}$ and some $k \in N$, then $\{\mathbf{x}\} \notin E^{\gamma''}(h)$ holds for any other $h \neq k$. Moreover, $\Theta_h^{\gamma'} = \{\{\mathbf{x}\}\}$ holds for any $h \in N$ in γ'. Thus, the associated $E^{\gamma'}$ satisfies maximal freedom. Then, by **Lemma 1** below, there exists a Nash solvable and efficient $\gamma^* \in \mu(\Gamma)$ such that $E^{\gamma^*} = E^{\gamma'}$. Finally, since $E^{\gamma^*}(h) = \Omega(\{\mathbf{x}\})$ for each $h \in N$, γ^* satisfies liberalism. ■

Lemma 1: *Assume that a game form* $\gamma \in \Gamma$ *represents a maximal power structure in* Γ. *Then, if the associated* E^{γ} *satisfies maximal freedom, there exists a Nash solvable and efficient* $\gamma^{*} \in \Gamma$ *such that* $E^{\gamma^{*}} = E^{\gamma}$.

Proof. By definition, $\gamma \in \mu(\Gamma)$ is such that E^{γ} satisfies maximal freedom. According to van Hees (1999, Theorem 1), there exists $\gamma^{*} \in \Gamma$ which is Nash solvable, efficient, and induced by E^{γ}. ■

Lemma 2: *Let a game form* $\gamma \in \Gamma$ *be such that* $\gamma \notin \mu(\Gamma)$. *Then, for any* $\mathbf{R} \in \mathcal{R}^{n}$ *and any* $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, *there exists* $\gamma^{*} \in \mu(\Gamma) \cap \Gamma_{L}$ *such that, for some* $j \in N$, $E^{\gamma^{*}}(j) \supsetneqq E^{\gamma}(j)$ *and* $(\mathbf{x}, \gamma^{*}) \in \Lambda(\mathbf{R})$.

Proof. Given $\gamma = (M, g) \notin \mu(\Gamma)$, there exists at least one $j \in N$ such that $E^{\gamma}(j) \not\supseteq \Omega(\{\mathbf{x}\})$ and $E^{\gamma}(j) \not\supseteq \Omega(\{\mathbf{y}\})$ for any $\mathbf{y} \in A \setminus \{\mathbf{x}\}$. This is because, if there exists $k \in N$ such that $E^{\gamma}(k) \supseteq \Omega(\{\mathbf{y}\})$ for some $\mathbf{y} \in A$, then for any other $h \neq k$ and any $B \in E^{\gamma}(h)$, $\mathbf{y} \in B$ holds. Since $\gamma \notin \mu(\Gamma)$, there exists $j \in N \setminus \{k\}$ such that $\{\mathbf{y}\} \notin E^{\gamma}(j)$. Thus, for this j and for any $B \in E^{\gamma}(j)$, $\mathbf{y} \in B$ holds, but $\{\mathbf{y}\} \notin E^{\gamma}(j)$, which implies that $E^{\gamma}(j) \not\supseteq \Omega(\{\mathbf{y}'\})$ for every $\mathbf{y}' \in A$.

For such $j \in N$, define $\gamma' = (M', g') \in \Gamma$ as follows: $M'_{j} = M_{j} \cup \{\mathbf{x}\}$ and $M'_{h} = M_{h}$ for $h \neq j$, and let g' be defined by

$$g'(\mathbf{m}) = \begin{cases} \mathbf{x} & \text{if } m_{j} = \mathbf{x} \in M'_{j}; \\ g(\mathbf{m}) & \text{otherwise} \end{cases}$$

for every $\mathbf{m} \in M'$. By construction, $E^{\gamma'}(j) = E^{\gamma}(j) \cup \Omega(\{\mathbf{x}\})$. Also, if $(m_{h}^{\mathbf{x}})_{h \in N} \in \epsilon(\mathbf{R}, \gamma)$ with $\mathbf{x} = g((m_{h}^{\mathbf{x}})_{h \in N})$, then $(m_{h}^{\mathbf{x}})_{h \in N} \in \epsilon(\mathbf{R}, \gamma')$ with $\mathbf{x} = g'((m_{h}^{\mathbf{x}})_{h \in N})$. If there is no $B \in \Theta_{j}^{\gamma'}$ such that $|B| > 1$ and $\mathbf{x} \notin B$, we define $\gamma^{*} = (M^{*}, g^{*}) \in \mu(\Gamma)$ as follows: $M_{j}^{*} = M'_{j}$ for $j \in N$ and $M_{h}^{*} = M'_{h} \cup \{\mathbf{x}\}$, for

$h \neq j$, and g^* is defined, for every $\mathbf{m} \in M^*$, by

$$g^*(\mathbf{m}) = \begin{cases} \mathbf{x} & \text{if } m_h = \mathbf{x} \in M_h^* \text{ for some } h \in N; \\ g'(\mathbf{m}) & \text{otherwise.} \end{cases}$$

Then, $E^{\gamma^*}(h) = \Omega(\{\mathbf{x}\})$ for every $h \in N$, and $E^{\gamma^*}(j) = E^{\gamma'}(j) \supsetneq E^{\gamma}(j)$. Moreover, $(m_h^{\mathbf{x}})_{h\in N} \in \epsilon(\mathbf{R}, \gamma^*)$ with $\mathbf{x} = g^*((m_h^{\mathbf{x}})_{h\in N})$.

If there exists $B^1 \in \Theta_j^{\gamma'}$ with $|B^1| > 1$ and $\mathbf{x} \notin B^1$, we choose an outcome $\mathbf{z}^1 \in B^1 \cap g'(M_j', \mathbf{m}_{-j}^{\mathbf{x}})$ and define $\gamma^{(1)} = \left(M^{(1)}, g^{(1)}\right) \in \Gamma$ as follows: $M_j^{(1)} = M_j' \cup \{\mathbf{z}^1\}$ for $j \in N$, $M_h^{(1)} = M_h'$ for $h \neq j$, and $g^{(1)}$ is defined, for every $\mathbf{m} \in M^{(1)}$, by

$$g^{(1)}(\mathbf{m}) = \begin{cases} \mathbf{z}^1 & \text{if } m_j = \mathbf{z}^1 \in M_j^{(1)}; \\ g'(\mathbf{m}) & \text{otherwise.} \end{cases}$$

By construction, $E^{\gamma^{(1)}}(j) \supsetneq E^{\gamma'}(j)$ and $(\mathbf{x}, \gamma^{(1)}) \in \Lambda(\mathbf{R})$. If there still exists $B^2 \in \Theta_j^{\gamma^{(1)}}$ such that $|B^2| > 1$ and $\mathbf{x}, \mathbf{z}^1 \notin B^2$, then we choose an outcome $\mathbf{z}^2 \in B^2 \cap g^{(1)}(M_j^{(1)}, \mathbf{m}_{-j}^{\mathbf{x}})$, and define $\gamma^{(2)} = \left(M^{(2)}, g^{(2)}\right) \in \Gamma$ as follows: $M_j^{(2)} = M_j^{(1)} \cup \{\mathbf{z}^2\}$ for $j \in N$ and $M_h^{(2)} = M_h^{(1)}$ for $h \neq j$, and $g^{(2)}$ is defined, for each and every $\mathbf{m} \in M^{(2)}$, by

$$g^{(2)}(\mathbf{m}) = \begin{cases} \mathbf{z}^2 & \text{if } m_j = \mathbf{z}^2 \in M_j^{(2)}; \\ g^{(1)}(\mathbf{m}) & \text{otherwise.} \end{cases}$$

In this way, we may arrive at the stage k where, in $\gamma^{(k)}$, there is no more $B^{k+1} \in \Theta_j^{\gamma^{(k)}}$ with $\left|B^{k+1}\right| > 1$ and $\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k \notin B^{k+1}$, since A is finite and $\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^l \notin B^{l+1} \subsetneq A$ holds for each $l = 1, 2, \ldots, k$. Note that $E^{\gamma^{(k)}}(j) = \Omega(\{\mathbf{x}\}) \cup \left[\cup_{l=1,\ldots,k} \Omega(\left\{\mathbf{z}^l\right\})\right]$, and, for all $h \neq j$ and all $B \in E^{\gamma^{(k)}}(h)$, $\left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\} \subseteq B$. Then, let $\gamma^* = (M^*, g^*) \in \mu(\Gamma) \cap \Gamma_L$ be such that $E^{\gamma^*}(j) = \Omega(\{\mathbf{x}\}) \cup \left[\cup_{l=1,\ldots,k} \Omega(\left\{\mathbf{z}^l\right\})\right]$ and $E^{\gamma^*}(h) = \Omega(\left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\})$ for all $h \neq j$. To show that $\gamma^* \in \mu(\Gamma)$, observe that if $\gamma'' \in \Gamma \backslash \{\gamma^*\}$ is such that $E^{\gamma''}(j) \supseteq E^{\gamma^*}(j) \cup \Omega(B)$ holds for some $B \subseteq A \backslash \{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\}$, then $\left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\} \notin E^{\gamma''}(h)$ holds for any other $h \neq j$. Second, if $\gamma'' \in \Gamma \backslash \{\gamma^*\}$ is such that $E^{\gamma''}(h) \supseteq E^{\gamma^*}(h) \cup \Omega(B)$ holds for some $B \not\supseteq \{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\}$ and some $h \neq j$, then there exists $\mathbf{y} \in \left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\}$ such that $\{\mathbf{y}\} \notin E^{\gamma''}(j)$ holds. Thus, $\gamma^* \in \mu(\Gamma)$. To see $\gamma^* \in \Gamma_L$, let us examine $\gamma^{(k-1)}$. Because of the definition of $\gamma^{(k)}$, there exists at least one $B^k \in \Theta_j^{\gamma^{(k-1)}}$ in $\gamma^{(k-1)}$ such that $\left|B^k\right| > 1$, $\mathbf{z}^k \in B^k$, and $\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^{k-1} \notin B^k$, whereas there is no more $B'^k \in \Theta_j^{\gamma^{(k-1)}}$ in $\gamma^{(k-1)}$ such that $\left|B'^k\right| > 1$, $\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^{k-1} \notin B'^k$ and $\mathbf{z}^k \notin B'^k$. Let $\mathbf{z}^{k+1} \in B^k \backslash \left\{\mathbf{z}^k\right\}$. Then, by the construction of $\gamma^{(k)}$, $\left\{\mathbf{z}^{k+1}\right\} \notin E^{\gamma^{(k)}}(j)$ and $\mathbf{z}^{k+1} \notin \left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\}$. This implies that $\gamma^* \in \Gamma_L$.

Such a game form γ^* can be defined as follows: $M_j^* = M_j^{(k)}$ for $j \in N$, $M_h^* = M_h^{(k)} \cup \left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\}$ for $h \neq j$, and g^* is defined, for every $\mathbf{m} \in M^*$, by

$$g^*(\mathbf{m}) = \begin{cases} \mathbf{z}^l & \text{if } m_j = \mathbf{z}^l \in M_j^{(k)}; \\ \mathbf{x} & \text{if } m_j = \mathbf{x} \in M_j'; \\ \mathbf{y} & \begin{array}{l} \text{if } \mathbf{y} = g^{(k)}(m_j, \mathbf{m}_{-j}^{\mathbf{x}}) \in \left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\} \text{ for } m_j \in M_j, \\ \text{ and } m_h = \left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\} \in M_h^* \text{ for some } h \neq j; \end{array} \\ \mathbf{x} & \begin{array}{l} \text{if } g^{(k)}(m_j, \mathbf{m}_{-j}^{\mathbf{x}}) \notin \left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\} \text{ for } m_j \in M_j, \\ \text{ and } m_h = \left\{\mathbf{x}, \mathbf{z}^1, \ldots, \mathbf{z}^k\right\} \in M_h^* \text{ for some } h \neq j; \end{array} \\ g^{(k)}(\mathbf{m}) & \text{otherwise.} \end{cases}$$

Then, $(m_h^{\mathbf{x}})_{h \in N} \in \epsilon(\mathbf{R}, \gamma^*)$ with $\mathbf{x} = g^*((m_h^{\mathbf{x}})_{h \in N})$, and $E^{\gamma^*}(j) \supsetneq E^{\gamma}(j)$. ■

Lemma 3: *Let a game form* $\gamma \in \mu(\Gamma)$ *be such that the associated* E^{γ} *does not satisfy* maximal freedom. *Then, for every* $\mathbf{R} \in \mathcal{R}^n$ *and every* $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, *there exists* $\gamma' \in \mu(\Gamma)$ *such that the associated* $E^{\gamma'}$ *satisfies* maximal freedom *with* $E^{\gamma'}(j) \supsetneq E^{\gamma}(j)$ *for some* $j \in N$ *and* $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$.

Proof. If $\gamma = (M, g) \in \mu(\Gamma)$ is such that the associated E^{γ} does not satisfy maximal freedom, there exists at least one $j \in N$ with $E^{\gamma}(j) \not\supseteq \Omega(\{\mathbf{x}\})$. Then, define $\gamma' = (M', g') \in \mu(\Gamma)$ as follows: for such $j \in N$, $M'_j = M_j \cup \{\mathbf{x}\}$ and $M'_h = M_h$ for $h \neq j$, and g' is defined, for every $\mathbf{m} \in M'$, by

$$g'(\mathbf{m}) = \begin{cases} \mathbf{x} & \text{if } m_j = \mathbf{x} \in M'_j; \\ g(\mathbf{m}) & \text{otherwise.} \end{cases}$$

Then, by construction, $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$ and $E^{\gamma'}(j) \supsetneq E^{\gamma}(j)$ holds. Note that if $E^{\gamma'}(h) \supseteq \Omega(\{\mathbf{x}\})$ for all $h \in N$, then $E^{\gamma'}$ satisfies maximal freedom. Otherwise, follow the instruction for constructing the game form $\gamma^* = (M^*, g^*) \in \mu(\Gamma)$ in the proof of **Lemma 2**. For this γ^*, the associated E^{γ^*} satisfies maximal freedom. ■

Lemma 4: *Suppose the game form* $\gamma \in \Gamma$ *has the* α*-effectivity function* E^{γ} *such that* $E^{\gamma}(i) = \{A\}$ *for some* $i \in N$. *Then, for every* $\mathbf{R} \in \mathcal{R}^n$ *and every* $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, *there exists* $\gamma' \in \Gamma$ *such that* $E^{\gamma'}(i) \neq \{A\}$ *and* $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$. *Moreover, if* $\{\mathbf{x}\} = \tau(\mathbf{R}, \gamma)$, *then* $\{\mathbf{x}\} = \tau(\mathbf{R}, \gamma')$.

Proof. We construct a new game form $\gamma' = (M', g') \in \Gamma$ as follows: for each $h \in N \setminus \{i\}$, $M'_h = M_h$ and $M'_i = M_i \cup \{\mathbf{x}\}$, and for each $\mathbf{m} \in M'$,

$$g'(\mathbf{m}) = \begin{cases} \mathbf{x} & \text{if } m_i = \mathbf{x}; \\ g(\mathbf{m}) & \text{otherwise.} \end{cases}$$

Then, $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$. If $\{\mathbf{x}\} = \tau(\mathbf{R}, \gamma)$, then $\{\mathbf{x}\} = \tau(\mathbf{R}, \gamma')$ holds by construction. Moreover, $E^{\gamma'}(h) = \cup_{m_h \in M'_h} \Omega(B^h_{m_h} \cup \{\mathbf{x}\})$ for each $h \in N \setminus \{i\}$, and $E^{\gamma'}(i) = \Omega(\{\mathbf{x}\})$. ■

Lemma 5: *Let $\gamma \in \Gamma$ be an i-dictatorial game form. Then, for every $\mathbf{R} \in \mathcal{R}^n$ and every $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, there exists $\gamma' \in \mu(\Gamma)$ such that $E^{\gamma'}(j) \neq \{A\}$ for some $j \in N \setminus \{i\}$ and $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$.*

Proof. By the similar method used in the proof of **Lemma 4**, we can construct a desired game form γ'. ■

Lemma 6: *Given $i \in N$ and $\mathbf{Q} \in \mathcal{S}^n \subseteq \mathcal{Q}^n$, let $\{j\} \in \mathcal{N}_i(\mathbf{Q})$. Then, for every $(\gamma, \gamma') \in \Gamma^p(i) \times \Gamma^u(i)$ with $E^{\gamma'}(j) = \{A\}$, every $\mathbf{R} \in \mathcal{R}^n$, and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $(\mathbf{x}, \gamma) P(Q_j(\mathbf{R}))(\mathbf{x}', \gamma')$ holds.*

Proof. Let a profile $\mathbf{R}^0 \in \mathcal{R}^n$ be such that every individual is universally indifferent over A. Then, for every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in A \times \Gamma$, $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R}^0)$. Moreover, if $\gamma \in \Gamma^p(i)$ and $\gamma' \in \Gamma^u(i)$ with $E^{\gamma'}(j) = \{A\}$, then $(\mathbf{x}, \gamma) P(Q_j(\mathbf{R}^0))(\mathbf{x}', \gamma')$. This follows from $(\mathbf{x}, \gamma) P(Q_j(\mathbf{R}^0))(\mathbf{x}, \gamma')$ as well as $(\mathbf{x}, \gamma') I(Q_j(\mathbf{R}^0))(\mathbf{x}', \gamma')$ by the property of $\mathcal{S}^n$, coupled with the transitivity of $Q_j(\mathbf{R}^0)$. Thus, by the condition **(n-c)**, for every $\mathbf{R} \in \mathcal{R}^n$, if $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, then $(\mathbf{x}, \gamma) P(Q_j(\mathbf{R}))(\mathbf{x}', \gamma')$. ■

Proof of Theorem 1:[14] Let $\cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N \setminus \{i\}} \setminus \{\varnothing\}$. For every $\mathbf{R} \in \mathcal{R}^n$, every $\mathbf{Q} \in \Delta_\Psi$, and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, we define Ψ as follows:
(i) if $\gamma, \gamma' \in \Gamma^p(i)$, then $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q(\mathbf{R})) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q_i(\mathbf{R}))$ or $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in I(Q_i(\mathbf{R})) \cap P(Q_1(\mathbf{R}))$ or $\ldots$ $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in$

[14]Throughout this proof, we make use of the set-theoretic representation of a binary relation, so that $(\mathbf{x}, \gamma) Q(\mathbf{R})(\mathbf{x}', \gamma')$ will be equivalently denoted by $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q(\mathbf{R})$.

$I\left(Q_i(\mathbf{R})\right) \cap \left(\bigcap_{h=1,\neq i}^{k} I\left(Q_h(\mathbf{R})\right) \right) \cap P\left(Q_{k+1}(\mathbf{R})\right)$ for any $k \in \{1, \ldots, n-1\} \setminus \{i\}$, and $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in I\left(Q(\mathbf{R})\right) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in I\left(Q_N(\mathbf{R})\right)$;

(ii) if $\gamma \in \Gamma^p(i)$ and $\gamma' \in \Gamma^u(i)$, then $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P\left(Q(\mathbf{R})\right)$; and

(iii) if $\gamma, \gamma' \in \Gamma^u(i)$, then $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P\left(Q(\mathbf{R})\right) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P\left(Q_j(\mathbf{R})\right) \cup P\left(Q_N(\mathbf{R})\right)$, and $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in I\left(Q(\mathbf{R})\right) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in I\left(Q_j(\mathbf{R})\right) \setminus P\left(Q_N(\mathbf{R})\right)$ for some $\{j\} \in \cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{N}_i(\mathbf{Q})$, where $Q = \Psi(\mathbf{Q})$.

Note that (i), (ii) and (iii) are mutually exclusive and jointly exhaustive. The above $Q(\mathbf{R})$ is complete, and has a two-tier structure. It is also an ordering. We must examine whether or not the part (ii) is consistent with the four Arrovian conditions and the domain restrictions. For every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, if $\gamma \in \Gamma^p(i)$ and $\gamma' \in \Gamma^u(i)$, then by the condition $\cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N \setminus \{i\}} \setminus \{\varnothing\}$ and **Lemma 6**, there exists at least one individual $j \in N \setminus \{i\}$ such that $E^{\gamma'}(j) = \{A\}$ and $(\mathbf{x}, \gamma) P\left(Q_j(\mathbf{R})\right)(\mathbf{x}', \gamma')$. Thus, the part (ii) is consistent with **SP** and **PI**.

By construction, we may verify that Ψ satisfies **SP**, **PI**, **I**, and **ND** as follows. First, by the part (ii) of Ψ, Ψ satisfies **ND**. Second, to show that Ψ satisfies **I**, assume that $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q_h(\mathbf{R}) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q'_h(\mathbf{R})$ for every $h \in N$. Note that the social preferences $Q(\mathbf{R})$ and $Q'(\mathbf{R})$ over $\{(\mathbf{x}, \gamma), (\mathbf{x}', \gamma')\}$ are completely determined by applying one of the parts (i), (ii), and (iii) of Ψ. Moreover, $Q(\mathbf{R})$ over $\{(\mathbf{x}, \gamma), (\mathbf{x}', \gamma')\}$ is determined by applying the part (i), for instance, if and only if $Q'(\mathbf{R})$ over $\{(\mathbf{x}, \gamma), (\mathbf{x}', \gamma')\}$ is determined by applying the part (i). The same argument applies to (ii) and (iii). This implies that Ψ satisfies **I**. Finally, by construction, the part (i) and (iii) are respectively consistent with **SP** and **PI**. Thus, Ψ satisfies **SP** and **PI**. Moreover, $\cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma^p(i)$ by the part (ii) of

Ψ. Note that $\Gamma^p(i)$ contains a game form γ with $E^{\gamma}(i) = \{A\}$. However, such a game form cannot be rationally chosen. This is because, for every $\mathbf{R} \in \mathcal{R}^n$, if $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, then there exists another game form $\gamma' \in \Gamma^p(i)$ with $E^{\gamma'}(i) \neq \{A\}$ and $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$, which is guaranteed by **Lemma 4** and **Lemma 5**. Thus, by the restriction (b) of $\mathcal{S}^n$, $(\mathbf{x}, \gamma')P(Q_i(\mathbf{R}))(\mathbf{x}, \gamma)$, which implies $(\mathbf{x}, \gamma')P(Q(\mathbf{R}))(\mathbf{x}, \gamma)$.

In summary, we have:

$$\gamma \in \cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R}) \Rightarrow \; \forall j \in N, \exists B^j \in E^{\gamma}(j) \text{ s.t. } B^j \neq A.$$

Thus, $\cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_L$.

Note that for every $\gamma \in \cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R})$, the associated E^{γ} satisfies maximal freedom, which can be verified as follows. First, to show that $\cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \mu(\Gamma)$, take a game form $\gamma \in \Gamma^p(i) \setminus \mu(\Gamma)$ with $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$. Then, by **Lemma 2**, we can see that there exists another game form $\gamma' \in \mu(\Gamma) \cap \Gamma^p(i)$ with $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$, and either $E^{\gamma'}(i) \supsetneq E^{\gamma}(i)$ or $[E^{\gamma'}(i) = E^{\gamma}(i)$, $E^{\gamma'}(h) = E^{\gamma}(h)$ for $h = 1, \ldots, k-1$, and $E^{\gamma'}(k) \supsetneq E^{\gamma}(k)$ for some $k \neq i]$. This is because, if $E^{\gamma}(i) \not\supseteq \Omega(\{\mathbf{x}\})$, then γ' can be constructed so as to satisfy $E^{\gamma'}(i) \supseteq \Omega(\{\mathbf{x}\})$ and $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$ by the proof of **Lemma 2**. If $E^{\gamma}(i) = \Omega(\{\mathbf{x}\})$, then γ' can be constructed so as to satisfy $E^{\gamma'}(h) = \Omega(\{\mathbf{x}\})$ for any $h \in N$, and $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$. Let $E^{\gamma}(i) = \Omega(\{\mathbf{x}\}) \cup \Omega(B)$, where $B \subseteq A \setminus \{\mathbf{x}\}$ is non-empty. If $|B| > 1$, then we can appropriately choose $\mathbf{y} \in B$ such that $E^{\gamma'}(i) = \Omega(\{\mathbf{x}\}) \cup \Omega(\{\mathbf{y}\})$ and $E^{\gamma'}(h) = \Omega(\{\mathbf{x}, \mathbf{y}\})$ for any $h \neq i$, and $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$. Then, $E^{\gamma'}(i) \supsetneq E^{\gamma}(i)$ and $\gamma' \in \mu(\Gamma) \cap \Gamma^p(i)$. If $B = \{\mathbf{y}\}$, then $\gamma \notin \mu(\Gamma)$ implies that there exists at least one individual $k \neq i$ such that $\{\mathbf{x}, \mathbf{y}\} \notin E^{\gamma}(k)$. By constructing γ' as $E^{\gamma'}(i) = \Omega(\{\mathbf{x}\}) \cup \Omega(\{\mathbf{y}\})$ and $E^{\gamma'}(h) = \Omega(\{\mathbf{x}, \mathbf{y}\})$ for any $h \neq i$, $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$ holds, and $[E^{\gamma'}(i) = E^{\gamma}(i)$, $E^{\gamma'}(h) = E^{\gamma}(h)$ for $h = 1, \ldots, k-1$, and $E^{\gamma'}(k) \supsetneq E^{\gamma}(k)$

for some $k \neq i$]. Moreover, if $E^{\gamma}(i) = \Omega(\{\mathbf{x}\}) \cup \Omega(B) \cup \Omega(B')$ such that $\varnothing \neq B' \subseteq A \backslash (B \cup \{\mathbf{x}\})$, then by repeating the same argument as in the proof of **Lemma 2**, we can construct a desired game form, $\gamma' \in \mu(\Gamma) \cap \Gamma^p(i)$. Thus, by the restriction (b) of $\mathcal{S}^n$ and (i) of Ψ, $(\mathbf{x}, \gamma') P(Q(\mathbf{R}))(\mathbf{x}, \gamma)$ holds, since either $(\mathbf{x}, \gamma') P(Q_i(\mathbf{R}))(\mathbf{x}, \gamma)$ or $[(\mathbf{x}, \gamma') I(Q_i(\mathbf{R}))(\mathbf{x}, \gamma)$, $(\mathbf{x}, \gamma') I(Q_h(\mathbf{R}))(\mathbf{x}, \gamma)$ for $h = 1, \ldots, k-1$, and $(\mathbf{x}, \gamma') P(Q_k(\mathbf{R}))(\mathbf{x}, \gamma)$ for some $k \neq i$]. Second, suppose that $\gamma \in \mu(\Gamma) \cap \Gamma^p(i)$ with $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, but the associated E^{γ} does not satisfy maximal freedom. Then, by **Lemma 3**, we can see that there exists another game form $\gamma' \in \mu(\Gamma) \cap \Gamma^p(i)$, whose associated E^{γ} satisfies maximal freedom, such that $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$ and either $E^{\gamma'}(i) \supsetneq E^{\gamma}(i)$ or $[E^{\gamma'}(i) = E^{\gamma}(i)$, $E^{\gamma'}(h) = E^{\gamma}(h)$ for $h = 1, \ldots, k-1$, and $E^{\gamma'}(k) \supsetneq E^{\gamma}(k)$ for some $k \neq i$]. Thus, by the restriction (b) of $\mathcal{S}^n$ and (i) of Ψ, $(\mathbf{x}, \gamma') P(Q(\mathbf{R}))(\mathbf{x}, \gamma)$ holds. By **Lemma 1**, there exists a liberal game form in $C(\Psi(\mathbf{Q}); \mathbf{R})$ for any $\mathbf{Q} \in \Delta_{\Psi}$ and any $\mathbf{R} \in \mathcal{R}^n$, which is Nash solvable and efficient.

Finally, we show that $C(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$ holds for every $\mathbf{Q} \in \Delta_{\Psi}$ and every $\mathbf{R} \in \mathcal{R}^n$. Suppose that $\gamma^*, \gamma^{*\prime} \in C(\Psi(\mathbf{Q}); \mathbf{R})$ such that $E^{\gamma^*} = E^{\gamma^{*\prime}}$ and $\gamma^* \in \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$, but $\gamma^{*\prime} \in (\Gamma_{NS} \cap \Gamma_L) \backslash \Gamma_{PE}$. Let $\mathbf{x} \in \tau(\gamma^{*\prime}, \mathbf{R})$ and $\mathbf{x}$ be Pareto inefficient. Then, there exists $\left(B^h_{m_h}\right)_{h \in N} \in \prod_{h \in N} E^{\gamma^{*\prime}}(h)$ such that $(m_h)_{h \in N} \in \epsilon(\gamma^{*\prime}, \mathbf{R})$ and $\mathbf{x} = g^{*\prime}\left((m_h)_{h \in N}\right)$. Since $E^{\gamma^*} = E^{\gamma^{*\prime}}$, we may assume that $\left(B^h_{m_h}\right)_{h \in N} \in \prod_{h \in N} E^{\gamma^*}(h)$ such that $(m_h)_{h \in N} \in \epsilon(\gamma^*, \mathbf{R})$ and $\mathbf{x} = g^*\left((m_h)_{h \in N}\right)$. Since $\mathbf{x}$ is Pareto inefficient, there exists $\mathbf{y} \in A$ that is Pareto efficient and Pareto dominates $\mathbf{x}$. By van Hees (1999, Theorem 1), we may assume that the efficient game form γ^* has the property that, for any $h \in N$ and any $B^h \in E^{\gamma^*}(h)$, there exists $m'_h \in M^*_h$ such that $g^*\left(m'_h, M^*_{-h}\right) = B^h$. Thus, since $B^h_{m_h} \cup \{\mathbf{y}\} \in E^{\gamma^*}(h)$ for every $h \in N$, there exists $(\overline{m}_h)_{h \in N} \in M^*$ such that $g^*\left(\overline{m}_h, M^*_{-h}\right) = B^h_{m_h} \cup \{\mathbf{y}\}$ for every $h \in N$, and $g^*\left((\overline{m}_h)_{h\in N}\right)=\mathbf{y}$. Then, $(\overline{m}_h)_{h\in N}\in\epsilon(\gamma^*,\mathbf{R})$, since $\mathbf{y}$ Pareto dominates $\mathbf{x}$ which is a best outcome within $g^*\left(m_{-h},M_h^*\right)\subseteq\cap_{j\neq h}B^j_{m_j}$ for every $h\in N$. Thus, $(\mathbf{y},\gamma^*)\in\Lambda(\mathbf{R})$. Note by $(\mathbf{x},\gamma^*),(\mathbf{y},\gamma^*)\in\Lambda(\mathbf{R})$ and the restriction (a) of $\mathcal{S}^n$, $(\mathbf{y},\gamma^*)P\left(Q_N(\mathbf{R})\right)(\mathbf{x},\gamma^*)$ holds, which implies $(\mathbf{y},\gamma^*)P\left(Q_N(\mathbf{R})\right)(\mathbf{x},\gamma^{*\prime})$ by the restriction (b) of $\mathcal{S}^n$ and $E^{\gamma^*}=E^{\gamma^{*\prime}}$. This implies $\gamma^{*\prime}\notin C(\Psi(\mathbf{Q});\mathbf{R})$, a desired contradiction. ■

Proof of Theorem 2: Let $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{N}_i(\mathbf{Q})\supseteq 2^{N\setminus\{i\}}\setminus\{\varnothing\}$ and $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{M}_i(\mathbf{Q})\supseteq 2^N\setminus\{\varnothing\}$. For every $\mathbf{R}\in\mathcal{R}^n$, every $\mathbf{Q}\in\Delta_\Psi$, and every $(\mathbf{x},\gamma),(\mathbf{x}',\gamma')\in\Lambda(\mathbf{R})$, define Ψ as in the proof of **Theorem 1**. We have only to examine the uniform rationalizability by means of $Q=\Psi(\mathbf{Q})$. By **Theorem 1**, it follows from $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{N}_i(\mathbf{Q})\supseteq 2^{N\setminus\{i\}}\setminus\{\varnothing\}$ that $C(Q;\mathbf{R})\subseteq\mu(\Gamma)\cap\Gamma_L$ holds for every $\mathbf{Q}\in\Delta_\Psi$ and every $\mathbf{R}\in\mathcal{R}^n$. Take $\mathbf{R}^0\in\mathcal{R}^n$ such that every individual is universally indifferent over A. Thus, for every $h\in N$, every $\mathbf{R}\in\mathcal{R}^n$ and every $(\mathbf{x},\gamma),(\mathbf{x}',\gamma')\in\Lambda(\mathbf{R})$, if $\gamma,\gamma'\in\mu(\Gamma)\cap\Gamma_L$ with $E^\gamma\neq E^{\gamma'}$, then $(\mathbf{x},\gamma)Q_h(\mathbf{R})(\mathbf{x}',\gamma')\Leftrightarrow(\mathbf{x},\gamma)Q_h(\mathbf{R}^0)(\mathbf{x}',\gamma')$, where use is made of $\{h\}\in\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{M}_i(\mathbf{Q})$. Thus, in this case, $(\mathbf{x},\gamma)Q(\mathbf{R})(\mathbf{x}',\gamma')\Leftrightarrow(\mathbf{x},\gamma)Q(\mathbf{R}^0)(\mathbf{x}',\gamma')$. Note that, for every $\gamma\in\mu(\Gamma)\cap\Gamma_L$, there exists $\gamma^*\in\Gamma_{NS}\cap\Gamma_{PE}\cap\Gamma_L$ such that $E^{\gamma^*}=E^\gamma$. Then, as shown in the proof of **Theorem 1**, there exists $\mathbf{x}^*\in\tau(\gamma^*,\mathbf{R})$ and $\mathbf{x}\in\tau(\gamma,\mathbf{R})$ for every $\mathbf{R}\in\mathcal{R}^n$ such that $\mathbf{x}^*$ Pareto dominates $\mathbf{x}$ at $\mathbf{R}$ and $(\mathbf{x}^*,\gamma^*)P\left(Q(\mathbf{R})\right)(\mathbf{x},\gamma)$. Thus, if $(\mathbf{x},\gamma)Q(\mathbf{R}^0)(\mathbf{x}',\gamma')$ holds for any $\gamma'\in\mu(\Gamma)\cap\Gamma_L$ with $E^{\gamma'}\neq E^\gamma$, then $(\mathbf{x}^*,\gamma^*)Q(\mathbf{R})(\mathbf{x}'',\gamma'')$ holds for any $\gamma''\in\mu(\Gamma)\cap\Gamma_L$ and for any $\mathbf{R}\in\mathcal{R}^n$, whenever $(\mathbf{x}'',\gamma'')\in\Lambda(\mathbf{R})$. This implies that $\gamma^*\in\cap_{\mathbf{R}\in\mathcal{R}^n}C(Q;\mathbf{R})$. ■

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