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新成長理論

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1 New Growth Theory

2 Introduction

Understanding of the economic growth theory was deepened during 1960's-early 70's. A lot of papers were written by famous theoretical economists such as Uzawa, Arrow, Koopmance, Kurz, Hahn, McKenzie, etc., etc.. As we have discussed in previous lectures, the neoclassical growth model started from a very simple one by Solow (1956). The motivation at that time was to build a model to explain the stylized facts raised by Kaldor. The growth theory in the mid of 60's became very mathematical and departed from empirical works completely. The main issues at that time were to find conditions to assure existence and stability of equilibria under very general assumptions. They knew that under some circumstances, steady states disappear and many kinds of complex equilibrium paths might appear. The researches at that time, however, didn't think that such characteristics of the dynamic economy important in real economies. Rather, they regarded such complex dynamics as anomalies that should be eliminated from the model. The studies of dynamic economy became unpopular in the mid of 70's.

In the mid of 80's, Paul Romer, as a part of his dissertation submitted to the University of Chicago, wrote a very influential paper on economic growth. That was a real resurrection! After that, tons of papers on economic growth appeared. I think there are two reasons for the resurrection. The first is the availability of historical international data sets complied by Madison (1982, 1991) and Summers and Heston (1991). Their data sets are easy to obtain and handle. As we discussed in our class, the neoclassical growth model, both Solow's and Ramsey's, have a strong implication on convergence. The poorer economy is, the faster the economy grows! We can check the proposition easily using the historical data. Take a look at the figures in the end of this lecture note. The figures are taken from a very comprehensive textbook on growth theory by Acemoglu (2009). If we look at the entire world, clearly there is no conversion. The relationship between income level and growth rate is zero! But if we look at OECD countries only, there is a strong convergence as the optimal growth model predicts. Obviously, the simple optimal growth model cannot explain such difference. This is a challenge to a neoclassical growth model because the model apparently lacks some very important features observed in the real economy. What kind of model is convincing theoretically? What kind of aspects are important or missing in the model? Education? Public infrastructure? Competition? Since we have the historical data, it is possible to evaluate a new model we came up.

The second reason is a historical background of macroeconomics during the mid 80's. Just before Romer submitted his dissertation, new Keynesian school had extended the scope of macroeconomics significantly. That is, a lot of models with imperfect competition, externality, public goods, etc., were written. Most papers in new Keynesian school do not have generality or robustness in terms of model structures that was very important in growth studies 40 years ago. However, the school got attention because of their rich economic implications. When Romer wrote his dissertation, many students of macroeconomics did not have difficulty to accept introduction of externality or many other things in a very simple way.

Analyses of so called, the new growth theory, can be classified into two categories, model building and growth regression. The former investigates the model structures that bring us endogenous growth. The latter tries to find out the determinants of economic growth rates.

3 Basic Idea

Convergence appears because the economy has a steady state. Without steady states, economy must keep moving. One of the crucial assumptions for the existence of steady states is Inada condition, that is,

$$\lim_{k \to \infty} f'(k) = 0.$$

In Solow's model, the steady state is defined by

$$sf(k^*) = k^*(g + n + \delta).$$

In the optimal growth model,

$$f'(k) = \rho + \theta g.$$

If the slope of the production function in intensive form, f', is always greater than some positive real number, we may not be able to obtain the steady state. The economy is always on the path on which both consumption and capital grow forever.

Under what circumstances can we get an economy without Inada condition? A simple answer is just dropping it! More than 30 years ago, Kurz (1968) analyzed a simple optimal growth model in which the marginal products of capital does not converge to zero. Recently, this approach is called AK model. Let's assume that the production function depends only on capital, and the relation is linear, that is,

$$Y = AK.$$

Ignore many factors such as labor, population growth, or technological progress. If A is greater than some values, such as $g + n + \delta$, or $\rho + \theta g$, the economy keeps growing.

3.1 Knowledge (Romer, Rebelo)

In the Ramsey's problem, technology is given by

$$Y_i = F(K_i, AL_i).$$

Assume that there are many identical producers. Now let's assume that the productivity of labor, A, is increasing with respect to the economy wide capital level, K(without subscript). In Cobb-Douglas case, the producer i has the following production function,

$$Y_i = K_i^\alpha (AL_i)^{1-\alpha},$$

$$\text{where } A = BK$$

$$K = \sum_i K_i.$$

Suppose there are so many producers that each producer regards the aggregate capital level K as given. Then, the marginal product of private capital is

$$\frac{\partial Y_i}{\partial K_i} = \alpha K_i^{\alpha-1} (L_i)^{1-\alpha} B^{1-\alpha} K^{1-\alpha} = a \left(\frac{K_i}{L_i} \right)^{a-1} B^{1-\alpha} K^{1-\alpha}.$$

If all the firms are identical, $\frac{K_i}{L_i} = \frac{K}{L}$, that is,

$$\frac{\partial Y_i}{\partial K_i} = a\left(\frac{K}{L}\right)^{a-1} B^{1-\alpha} K^{1-\alpha} = aB^{1-\alpha} L^{1-\alpha}.$$

If the total labor force, L, is constant, the above equation becomes

$$\frac{\partial Y_i}{\partial K_i} = aB^{1-\alpha} L^{1-\alpha} = const \equiv A.$$

Since A is the real interest rate, the consumption growth rate is

$$\frac{\dot{C}}{C} = \frac{(A-\rho)}{\theta}.$$

On the balanced growth path,

$$\frac{\dot{K}}{K} = A - \frac{C}{K}$$

3.2 Public Goods (Barro)

Suppose the infrastructure provided by the government is an important factor for production, that is,

$$Y = AK^{\alpha} L^{\beta} G^{1-\alpha}.$$

Suppose that the government finances its expenditure by the proportional income tax, that is,

$$G = \tau Y.$$

The marginal product of capital in the private sector, i.e., the real interest rate is

$$(1-\tau)\, r = (1-\tau)\, \frac{\partial Y}{\partial K} = \alpha\, (1-\tau)\, \frac{Y}{K}.$$

Considering the budget constraint of the government,

$$Y = AK^{\alpha} L^{\beta} G^{1-\alpha} = AK^{\alpha} L^{\beta} \tau^{1-\alpha} Y^{1-\alpha}$$

$$Y = \tau^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} K L^{\frac{\beta}{\alpha}}.$$

Therefore, the real interest rate becomes

$$(1-\tau)\, r_t = \alpha\, (1-\tau)\, \tau^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} L^{\frac{\beta}{\alpha}}.$$

Basically, the structure is the same as the former AK model.

Both endogenous growth models, Knowledge spillover and public goods share one very important feature. In both models, the economic growth model is an increasing function of the scale of the economy. If the number of labor inputs are larger, the capital returns become larger, which creates faster growth. Many people criticized the two endogenous growth models in this point. If this is correct, Indonesia should grow much faster than Korea. This feature comes from their reliance on the externality (or public goods). Many people are skeptical about this. But still, AK model is extremely popular because the model is very tractable. No capital dynamics is in here. The system is completely linear if we assume log utility. No complicated staff like linearization is involved. Now, AK model is used when a researcher would like to build a general equilibrium growth model with capital accumulation but also would like to keep the interest rate constant.

3.3 Human Capital (Lucas)

Another way to get endogenous growth is to introduce accumulable production factors other than physical capital. For example, let's assume that production level does not depend not only on the labor inputs but on the human capital, H as well. That is

$$Y = (H \times L_{pt})^{1-\alpha k} K^{\alpha k}.$$

Human capital H is accumulated by

$$\frac{\dot{H_t}}{H_t} = \psi_E \left(\frac{L_{Et}}{L_t} \right)$$

$$\psi_E : \text{ constant}$$

$$L_{Et} + L_{pt} = 1$$

So, the economy consists of one agent who divides his time to either learning L_{Et} , or producing, L_{pt}. Assume that the agent has CRRA utility function,

$$U = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\theta}}{1-\theta} dt.$$

The feasible condition for physical capital accumulation is

$$\dot{K_t} = (H \times L_{pt})^{1-\alpha} K_t^\alpha - \delta K_t - C_t.$$

The Hamiltonian is given by

$$H_t = \frac{C_t^{1-\theta}}{1-\theta} + \lambda_t \left((H \times L_{pt})^{1-\alpha} K_t^\alpha - \delta K_t - C_t \right) + \mu_t \psi_E (1 - L_{pt}) H_t.$$

The FOCs. are

$$c^{-\theta} = \lambda,$$

$$\dot{\lambda} = \rho\lambda - \lambda\left(\frac{\alpha Y}{K} - \delta\right),$$

$$\dot{\mu} = \rho\mu - \mu\psi_E\left(1 - L_p\right) - \lambda\frac{Y}{H}\left(1 - \alpha\right),$$

$$\lambda\frac{Y}{L_p}\left(1 - \alpha\right) - \mu\psi_E H = 0,$$

$$\lim_{t\to\infty} e^{-\rho t}K_t = \lim_{t\to\infty} e^{-\rho t}H_t = 0,$$

$$\dot{K}_t = \left(H \times L_{pt}\right)^{1-\alpha} K_t^{\alpha} - \delta K_t - C_t,$$

$$\frac{\dot{H}_t}{H_t} = \psi_E\left(1 - L_{pt}\right).$$

Arranging the equations a bit,

$$\frac{\dot{\lambda}}{\lambda} = \rho - \left(\frac{\alpha Y}{K} - \delta\right),$$

$$\frac{\dot{\mu}}{\mu} = \rho - \psi_E\left(1 - L_p\right) - \frac{\lambda}{\mu}\frac{Y}{H}\left(1 - \alpha\right) = \rho - \psi_E,$$

$$\frac{\dot{C}}{C} = \frac{-1}{\theta}\left(\rho - \left(\frac{\alpha Y}{K} - \delta\right)\right),$$

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \delta - \frac{C}{K},$$

$$\frac{\dot{H}}{H} = \psi_E\left(1 - L_{pt}\right).$$

Therefore, on the balanced growth path,

$$\frac{\dot{H}}{H} = \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \psi_E\left(1 - L_p\right) = \frac{Y}{K} - \delta - \frac{C}{K} = \frac{1}{\theta}\left(\rho - \left(\frac{\alpha Y}{K} - \delta\right)\right).$$

The above condition determines the $\frac{Y}{K}$, $\frac{C}{K}$, and L_p.

This human capital model is very different from Knowledge spillover model or public goods model. Basically, the equilibrium path is efficient. The only externality included in this model is that the human capital can move from parent to child, which is not so unrealistic. The paper by Lucas

(1988) is very influential and becomes one of the most cited papers in all economic fields. But it is also worth noting that there are three variables in his model, H, K, and C. So, the equilibrium other than the balanced growth path is very difficult to analyze. We can't use the Poincare-Bendixson theore. That is, it is difficult to show that the balanced growth path is stable. Even more difficult is to derive the transition dynamics if any.

3.4 Increasing Variety

Another strand of growth theory focuses on an increase in variety of goods. The basic idea behind it is simple. The more variety, the more productive firms become, or the happier people feel. In this section, a model with increasing variety of consumer products model is introduced. I chose this model because this model contains a lot of components adopted in other fields such as strategic trade theory, or New Keynesian macroeconomic theory.

3.4.1 Utility Function

Household utility depends on her consumption of "aggregates" defined by

$$C = \left[\sum_{j=1}^{M} (c_j)^{\varepsilon} \right]^{1/\varepsilon}.$$

This is sometimes called Dixit and Stiglitz type aggregator because in their famous work on monopolistic competition in 1977, Dixit and Stiglitz first introduced this type of aggregator. ε must be $0 < \varepsilon \leq 1$. Obviously, this aggregate consumption is an increasing function of the number of variety, M. The intertemporal utility function is

$$U_i = \int_0^{\infty} \frac{c^{1-\theta} - 1}{1-\theta} e^{-\pi t} dt.$$

Denote the price of goods c_{*j} as p_j.

The budget constraint of the consumer is

$$\dot{a} = ra + w - \sum_{j=1}^{M} p_j c_j.$$

The Hamiltonian becomes

$$H = \frac{C^{1-\theta} - 1}{1-\theta} + \lambda_t \left(ra + w - \sum_{j=1}^{M} p_j c_j \right).$$

This is a little bit complicated problem. First, let's think about the cost minimization problem,

$$Min \sum_{j=1}^{M} p_j c_j$$

$$s.t. \overline{C} = \left[\sum_{j=1}^{M} (c_j)^{\varepsilon} \right]^{1/\varepsilon}$$

The Lagrangian is

$$L = \sum_{j=1}^{M} p_j c_j - \alpha \left(\left[\sum_{j=1}^{M} (c_j)^{\varepsilon} \right]^{1/\varepsilon} - \overline{C} \right)$$

The first order conditions are

$$p_j = a \left[\sum_{j=1}^{M} (c_{ij})^{\varepsilon} \right]^{(1-\varepsilon)/\varepsilon} (c_j)^{\varepsilon - 1} \text{ for all j.}$$

Therefore,

$$\frac{p_j}{p_k} = \left(\frac{c_j}{c_i} \right)^{\varepsilon - 1}$$

and

$$p_j c_j = a \left[\sum_{j=1}^{M} (c_j)^{\varepsilon} \right]^{(1-\varepsilon)/\varepsilon} (c_j)^{\varepsilon} = a \overline{c}^{1-\varepsilon} (c_j)^{\varepsilon}.$$

By summing,

$$\sum_{j=1}^{M} p_j c_j = a \overline{C}^{1-\varepsilon} \sum_{j=1}^{M} (c_j)^{\varepsilon} = a \overline{C}^{1-\varepsilon} \overline{C}^{\varepsilon} = a \overline{C}.$$

That is,

$$a = \frac{\sum_{j=1}^{M} p_j c_j}{\overline{C}}.$$

Therefore, given the aggregate consumption goods $\overline{c}$, and the expenditure, $\sum_{j=1}^{M} p_j c_j$, the demand function for c_j is

$$(c_j)^{\varepsilon - 1} = \frac{p_j}{a \overline{C}^{1-\varepsilon}}$$

$$c_j = \left(\frac{p_j}{\left(\sum_{j=1}^{M} p_j c_j \right) \overline{C}^{-\varepsilon}} \right)^{1/(\varepsilon - 1)}.$$

Now, define

$$P=\left[\sum_{j=1}^{M}p_j^{\varepsilon/(\varepsilon-1)}\right]^{(\varepsilon-1)/\varepsilon}.$$

Then,

$$P\overline{C}=\left(\sum_{j=1}^{M}p_j^{\varepsilon/(\varepsilon-1)}\right)^{(\varepsilon-1)/\varepsilon}\left(\left[\sum_{j=1}^{M}(c_j)^{\varepsilon}\right]^{1/\varepsilon}\right).$$

BTW,

$$p_j=a\overline{c}^{(1-\varepsilon)}(c_j)^{\varepsilon-1}$$

$$\sum_{j=1}^{M}p_j^{\varepsilon/(\varepsilon-1)}=a^{\varepsilon/(\varepsilon-1)}\overline{c}^{-\varepsilon}\sum_{j=1}^{M}(c_j)^{\varepsilon}.$$

Therefore,

$$\begin{aligned}\left(\sum_{j=1}^{M}p_j^{\varepsilon/(\varepsilon-1)}\right)^{(\varepsilon-1)/\varepsilon}&=a\overline{c_i}^{1-\varepsilon}\left[\sum_{j=1}^{M}(c_{ij})^{\varepsilon}\right]^{(\varepsilon-1)/\varepsilon}\\&=a\overline{c_i}^{1-\varepsilon}\overline{c_i}^{\varepsilon-1}=a.\end{aligned}$$

That is,

$$P=a.$$

Therefore,

$$Pc_i=\left(\sum_{j=1}^{M}p_j^{\varepsilon/(\varepsilon-1)}\right)^{(\varepsilon-1)/\varepsilon}\left(\left[\sum_{j=1}^{M}(c_{ij})^{\varepsilon}\right]^{1/\varepsilon}\right)=\sum_{j=1}^{M}p_jc_{ij}.$$

So, P can be regarded as the aggregate price index corresponding to the aggregate consumption goods. The individual demand becomes

$$c_{ij}=\left(\frac{p_j}{P}\right)^{1/(\varepsilon-1)}\overline{C}. \tag{1}$$

The consumer's maximization problem can be written as the standard problem like

$$\text{Max } U_i=\int_0^{\infty}\frac{C^{1-\theta}-1}{1-\theta}e^{-\pi t}dt$$

$$\text{s.t. } \dot{a}=ra+w-PC.$$

Now, let's think about production. In this economy, there are a lot of different products. Suppose for producing some goods, say product j, somebody must invent the goods by paying fixed costs for invention, η. Assume that in order to sell a goods j in the market, one unit of the goods should be used. In other words, the firm that produces goods j's profit is $p_j - 1$.

Suppose at time t, a new product is invented. The discounted sum of future profit from the invention will be

$$V(t) = \int_{v=t}^{\infty} (p_{jt} - 1)\, c_{jt} e^{-r(v,t)(v-t)} dv.$$

Where C_{jt} is the sum of consumption of goods j. At each time, the firm sets the price facing the demand function,

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{1/(\varepsilon-1)} C_t.$$

. Assume that each firm regards the aggregate price index, P_t, as given. The maximized price is

$$p_{jt} = \frac{1}{\varepsilon}.$$

Notice that the individual price is fixed and identical among various goods. This is simply because the demand elasticity and marginal costs are constant. Therefore,

$$P = \left[\sum_{j=1}^{M} p_j^{\varepsilon/(\varepsilon-1)}\right]^{(\varepsilon-1)/\varepsilon} = \left[M\left(\frac{1}{\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}\right]^{(\varepsilon-1)/\varepsilon} = \left(\frac{1}{\varepsilon}\right) M^{(\varepsilon-1)/\varepsilon}.$$

Since all the prices are identical,

$$c_{jt} = c_{k,t} = c_j.$$

Calculating the discounted sum of profit, if interest rate is constant, we can get

$$V(t) = \int_{v=t}^{\infty} \left(\frac{1-\varepsilon}{\varepsilon}\right) c_j e^{-r(v-t)} dv = \left(\frac{1-\varepsilon}{\varepsilon r}\right) c_j.$$

Suppose there is free entry-exit for inventing the product. In this case, the discounted sum of future profit must not be larger than the cost for inventing the product, that is,

$$\eta \geq \left(\frac{1-\varepsilon}{\varepsilon r}\right) c_j.$$

If there are always some products invented, that is, if $\dot{M} > 0$, we get $\eta = \left(\frac{1-\varepsilon}{\varepsilon r}\right) c_j$.

From the aggregator, we can get

$$C = \left[\sum_{j=1}^{M} (c_j)^{\varepsilon}\right]^{1/\varepsilon} = \left[\sum_{j=1}^{M} (c_k)^{\varepsilon}\right]^{1/\varepsilon} = M^{1/\varepsilon}\left(\frac{\eta\varepsilon r}{1-\varepsilon}\right).$$

Now, let's go back to the consumer's problem.

Hamiltonian becomes

$$H = \frac{C_t^{1-\theta} - 1}{1-\theta} + \lambda_t \left(ra + w - P_t C_t \right)$$

Simple calculation gives us

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta} \left(r - \rho - \frac{\dot{P}_t}{P_t} \right).$$

Remember

$$C_t = M_t^{1/\varepsilon} \left(\frac{\eta \varepsilon r}{1-\varepsilon} \right),$$

And

$$\frac{\dot{P}_t}{P_t} = \frac{(\varepsilon - 1)}{\varepsilon} \frac{\dot{M}_t}{M_t},$$

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\varepsilon} \frac{\dot{M}_t}{M_t},$$

$$\frac{\dot{P}_t}{P_t} = (\varepsilon - 1) \frac{\dot{C}_t}{C_t}.$$

Therefore,

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta} \left(r - \rho - (\varepsilon - 1) \frac{\dot{C}_t}{C_t} \right),$$

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta + (\varepsilon - 1)} \left(r - \rho \right),$$

$$\frac{\dot{M}_t}{M_t} = \frac{\varepsilon}{\theta + (\varepsilon - 1)} \left(r - \rho \right). \tag{2}$$

In the balanced growth path, since the individual output is constant, the growth rate of total output is equal to the growth rate of the variety. Therefore, in the balanced growth path, the growth rate is given by (2) that depends on ε. Since $\theta, \varepsilon > 0$, the growth rate is positive if $r - \rho > 0$. ε is the inverse of the markup ratio. The greater ε implies smaller monopoly power, which leads to higher economic growth rate. However, ε in this model is derived from consumer's preference.

4 Other Models

As you can guess, there are lots of different ways to obtain endogenous growth. Some consider public stock rather than flow in Barro. Introduction of environmental stock, tax and trade policies, more complex schooling system, political system and inequality, foreign aid, learning from foreign competitiors, and macroeconomic policies, etc. In textbooks by Jones (1998), Aghion and Howitt (1998), and most recently by Acemoglu (2009) you will find many different kinds of growth models. It is astonishing to see so many papers have been published for these 20 years on this issue. Still, this is a hot field.

4.1 Growth Regression

Barro and Lee (1994):

$$\begin{aligned} g = &-0.0255\log(GDP)^{**} + 0.0138 Male\_\text{sec}^{**} - 0.0092 FEM\_\text{sec}^{**} \\ &+ 0.0801\log(life)^{**} + 0.0770(I/Y)^{**} - 0.1550(G/Y)^{**} \\ &- 0.0304\log(1+BMP)^{**} - 0.0178 REV^{**} \end{aligned}$$

Sample: 1965-1984, 138 countries. g: growth rate of GDP per capita. GDP : initial level of GDP, $Male\_\text{sec}$ and: FEM_sec : Male and Female secondly school attainment, respectively, LIFE: life expectancy, BMP: Black Market Premium, REV: the number of successful and unsuccessful revolutions per years. Means are subtracted from all the variables. I: investment, G: government consumption. All the variables are significant. Barro (1994) included inflation and some other policy variables. Other variables often considered important in growth regressions are (1) income inequality, (2) corruption, (3) macroeconomic stability, (4) legal system, (5) culture, etc.

How can we interpret the results? For example, the coefficient of the female secondly school attainment is negative. Does it mean by discouraging female from schooling, we can increase the economic growth rate? Probably not.

It is well known that the coefficients of many important variables in growth regressions change their signs so easily depending on the specifications. In other words, most results in growth regressions are not robust. I remember that when I was in grad school, one my friend said he could get ANY results by changing the sample and specifications in growth regressions.

Probably, the biggest problem in growth regressions are endogeneity of regressors. Obviously, the richer country allows more people to study. Lots variables such as life expectancy, investment-GDP ratios, are not exogenous. They are determined inside the economy depending on various factors. To interpret the results, we need to find good instrumental variables. After discussing the details of the variables and instruments Barro used, Sims (1996) wrote "The conclusion from all these considerations is that the coefficient on inflation in the equation Barro presents to us represents, at best, a small piece of the story of how policy-induced changes in inflation influence output growth and at worst an uninterpretbale hodgepodge."

Acemoglu(2009) Chapter 1

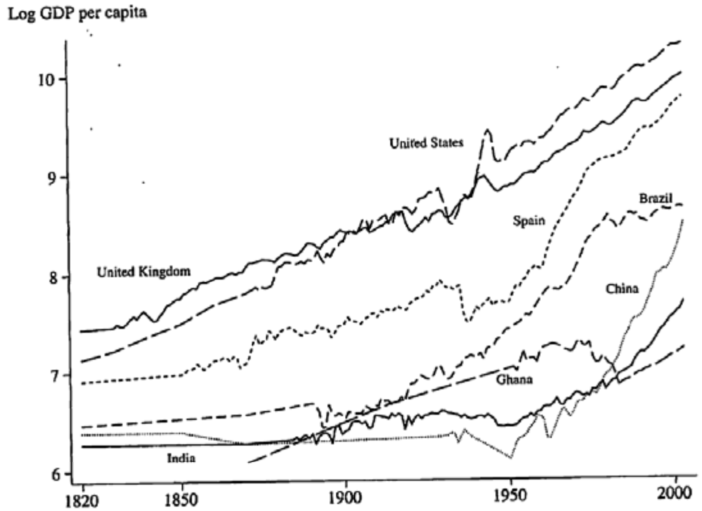

FIGURE 1.12 The evolution of income per capita in the United States, the United Kindgom, Spain, Brazil, China, India, and Ghana, 1820–2000.

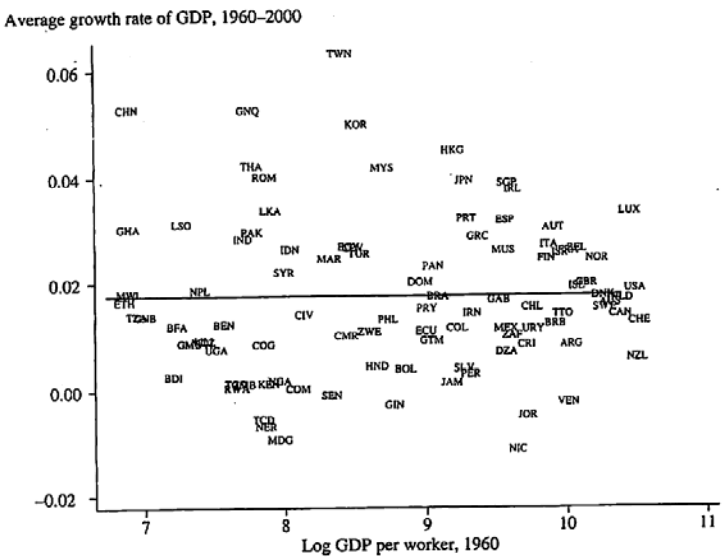

FIGURE 1.13 Annual growth rate of GDP per worker between 1960 and 2000 versus log GDP per worker in 1960 for the entire world.

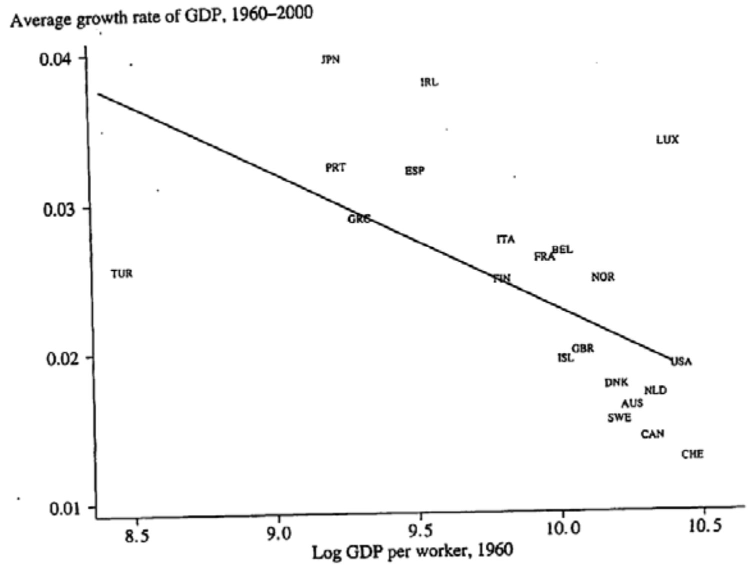

FIGURE 1.14 Annual growth rate of GDP per worker between 1960 and 2000 versus log GDP per worker in 1960 for core OECD countries.