AJAE Appendix for "Weather Risk, Wages in Kind, and the Off-Farm Labor Supply of Agricultural Households in a Developing Country"*

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Appendix to "Weather Risk, Wages in Kind, and the Off-Farm Labor Supply of Agricultural Households in a Developing Country"

Appendix I: Comparative Statics

This appendix provides a comparative-static analysis of $\ell_j(X_p, X_w, \Sigma)$, $j = a, b, c, d$ (the optimal labor supply shares). In the comparative-static analysis, the term v_y in the first order conditions (5) and (6) is the key. Applying a Taylor approximation to v_y and then totally differentiating Roy's identity, we obtain:

$$v_y \approx \bar{v}_y \left\{ 1 - \psi \frac{y - \bar{y}}{\bar{y}} + s(\psi - \eta) \frac{p - \bar{p}}{\bar{p}} \right\}, \tag{9}$$

where ψ ($\equiv -y v_{yy}/v_y$) is the Arrow-Pratt measure of relative risk aversion, s ($\equiv pq/y$, where q is the Marshallian demand for food) is the budget share of food, and η ($\equiv \partial \ln q / \partial \ln y$) is the income elasticity of food demand. ψ, s, and η are all evaluated at the means of y and p so that they are treated as constant in the following exposition. Note that the assumption of $v_{yp} > 0$ is equivalent to the assumption of $\psi > \eta$ in this approximation, which is likely to be satisfied for low-income households (Fafchamps 1992).

The assumptions in the theoretical section imply the following structure of Σ (the covariance matrix of θ_a, θ_b, θ_c, θ_d, and θ_p):

$$\Sigma = \begin{pmatrix} \sigma_a^2 & \sigma_a \sigma_b \rho_{ab} & \sigma_a \sigma_c \rho_{ac} & 0 & \sigma_a \sigma_p \rho_a \\ \sigma_a \rho_{ab} \sigma_b & \sigma_b^2 & \sigma_b \sigma_c \rho_{bc} & 0 & \sigma_b \sigma_p \rho_b \\ \sigma_a \sigma_c \rho_{ac} & \sigma_b \sigma_c \rho_{bc} & \sigma_c^2 & 0 & \sigma_c \sigma_p \rho_c \\ 0 & 0 & 0 & \sigma_d^2 & 0 \\ \sigma_a \sigma_p \rho_a & \sigma_b \sigma_p \rho_b & \sigma_c \sigma_p \rho_c & 0 & \sigma_p^2 \end{pmatrix}, \tag{10}$$

where σ_k is the coefficient of variation of θ_k (note that the mean of θ_k is one), ρ is the correlation coefficient, $0 < \rho_{ab} < \rho_{ac}$, and $0 < \rho_b < \rho_c$. We also assume that the magnitudes of σ_j ($j = a, b, c, d$) are not very different. By inserting (9) and (10) into the first order conditions (5) and (6), we obtain a system of equations, based on which we conduct the comparative-static analysis.

Since the system cannot be analyzed without additional restrictions, we investigate the simplest case for which it is possible to obtain analytical results and which is useful to understand the risk-aversion mechanism underlying the optimal labor choice. More concretely, we begin with the case when the total labor supply is fixed at $\bar{L}$, ignoring the labor-leisure choice, and $\partial f_j / \partial L_j = \partial f_j / \partial(\ell_j \bar{L}) = w$, i.e., labor returns are linear and their expected values are the same across sectors. With this specification, the household income becomes

$$y = y_0 + w\bar{L}\{\ell_a \theta_a + \ell_b \theta_b + \ell_c \theta_c + (1 - \ell_a - \ell_b - \ell_c)\theta_d\}. \tag{11}$$

Inserting (9) into (5) and re-arranging, we obtain

$$E\left[\overline{v}_y \left\{1 - \psi \frac{y - \overline{y}}{\overline{y}} + s(\psi - \eta)\frac{p - \overline{p}}{\overline{p}}\right\}(\theta_k - \theta_d)\right] = 0, \quad k = a, b, c. \tag{12}$$

We then insert (10) and (11) into the expression above. After re-arranging, we obtain three equations:

$$\begin{aligned}
\ell_a \overbrace{(\sigma_a^2 + \sigma_d^2)}^{=\Sigma_{aa}} + \ell_b \overbrace{(\sigma_a\sigma_b\rho_{ab} + \sigma_d^2)}^{=\Sigma_{ab}} + \ell_c \overbrace{(\sigma_a\sigma_c\rho_{ac} + \sigma_d^2)}^{=\Sigma_{ac}} - \sigma_d^2 &= \overbrace{\frac{\overline{y}s}{w\overline{L}}\left(1 - \frac{\eta}{\psi}\right)}^{s''} \sigma_a\sigma_p\rho_a, \\
\ell_a(\sigma_a\sigma_b\rho_{ab} + \sigma_d^2) + \ell_b \overbrace{(\sigma_b^2 + \sigma_d^2)}^{=\Sigma_{bb}} + \ell_c \overbrace{(\sigma_b\sigma_c\rho_{bc} + \sigma_d^2)}^{=\Sigma_{bc}} - \sigma_d^2 &= \frac{\overline{y}s}{w\overline{L}}\left(1 - \frac{\eta}{\psi}\right) \sigma_b\sigma_p\rho_b, \\
\ell_a(\sigma_a\sigma_c\rho_{ac} + \sigma_d^2) + \ell_b(\sigma_b\sigma_c\rho_{bc} + \sigma_d^2) + \ell_c \overbrace{(\sigma_c^2 + \sigma_d^2)}^{=\Sigma_{cc}} - \sigma_d^2 &= \frac{\overline{y}s}{w\overline{L}}\left(1 - \frac{\eta}{\psi}\right) \sigma_c\sigma_p\rho_c,
\end{aligned}$$

where $\bar{y} = y_0 + w\bar{L}$, which does not depend on the portfolio choice. For this reason, we treat it as a parameter and replace $\bar{y}s(1 - \eta/\psi)/(w\bar{L})$ by s''. Therefore, the above system can be expressed as

$$\begin{pmatrix} \Sigma_{aa} & \Sigma_{ab} & \Sigma_{ac} \\ \Sigma_{ab} & \Sigma_{bb} & \Sigma_{bc} \\ \Sigma_{ac} & \Sigma_{bc} & \Sigma_{cc} \end{pmatrix} \begin{pmatrix} \ell_a \\ \ell_b \\ \ell_c \end{pmatrix} = \begin{pmatrix} \sigma_d^2 + s''\sigma_a\sigma_p\rho_a \\ \sigma_d^2 + s''\sigma_b\sigma_p\rho_b \\ \sigma_d^2 + s''\sigma_c\sigma_p\rho_c \end{pmatrix},$$

which can be solved to obtain a closed-form solution. Letting D denote the determinant of the three-by-three matrix above, i.e., $D = \Sigma_{aa}\Sigma_{bb}\Sigma_{cc} + 2\Sigma_{ab}\Sigma_{bc}\Sigma_{ac} - \Sigma_{bc}^2\Sigma_{aa} - \Sigma_{ac}^2\Sigma_{bb} - \Sigma_{ab}^2\Sigma_{cc}$, we obtain the following closed-form solution:

$$\begin{aligned}
\ell_a &= \frac{1}{D}\Bigg[\sigma_d^2\{\overbrace{(\Sigma_{bb}\Sigma_{cc} - \Sigma_{bc}^2) + (\Sigma_{bc}\Sigma_{ac} - \Sigma_{ab}\Sigma_{cc}) + (\Sigma_{ab}\Sigma_{bc} - \Sigma_{bb}\Sigma_{ac})}^{=R_a}\} \\
&+ s''\sigma_p\{\underbrace{\sigma_a\rho_a(\Sigma_{bb}\Sigma_{cc} - \Sigma_{bc}^2) + \sigma_b\rho_b(\Sigma_{bc}\Sigma_{ac} - \Sigma_{ab}\Sigma_{cc}) + \sigma_c\rho_c(\Sigma_{ab}\Sigma_{bc} - \Sigma_{ac}\Sigma_{bb})}_{=Q_a}\}\Bigg],
\end{aligned} \tag{13}$$

$$\begin{aligned}
\ell_b &= \frac{1}{D}\Bigg[\sigma_d^2\{\overbrace{(\Sigma_{bc}\Sigma_{ac} - \Sigma_{ab}\Sigma_{cc}) + (\Sigma_{aa}\Sigma_{cc} - \Sigma_{ac}^2) + (\Sigma_{ab}\Sigma_{ac} - \Sigma_{aa}\Sigma_{bc})}^{=R_b}\} \\
&+ s''\sigma_p\{\underbrace{\sigma_a\rho_a(\Sigma_{ac}\Sigma_{bc} - \Sigma_{ab}\Sigma_{cc}) + \sigma_b\rho_b(\Sigma_{aa}\Sigma_{cc} - \Sigma_{ac}^2) + \sigma_c\rho_c(\Sigma_{ab}\Sigma_{ac} - \Sigma_{aa}\Sigma_{bc})}_{=Q_b}\}\Bigg],
\end{aligned} \tag{14}$$

$$\ell_c = \frac{1}{D}\Bigg[\sigma_d^2\{\overbrace{(\Sigma_{ab}\Sigma_{bc} - \Sigma_{ac}\Sigma_{bb}) + (\Sigma_{ac}\Sigma_{ab} - \Sigma_{aa}\Sigma_{bc}) + (\Sigma_{aa}\Sigma_{bb} - \Sigma_{ab}^2)}^{=R_c}\}$$

$$+s''\sigma_p\{\underbrace{\sigma_a\rho_a(\Sigma_{ab}\Sigma_{bc}-\Sigma_{ac}\Sigma_{bb})+\sigma_b\rho_b(\Sigma_{ab}\Sigma_{ac}-\Sigma_{aa}\Sigma_{bc})+\sigma_c\rho_c(\Sigma_{aa}\Sigma_{bb}-\Sigma_{ab}^2)}_{=Q_c}\}\Bigg], \quad (15)$$

$$\ell_d \;=\; 1-\sum_{i=a,b,c}\ell_i = 1-\frac{1}{D}\left[\sigma_d^2(R_a+R_b+R_c)+s''\sigma_p(Q_a+Q_b+Q_c)\right]. \quad (16)$$

Now we investigate the comparative statics with respect to σ_a. First, a numerical example is shown in Figure A.1, where we set s at 0.5, $\bar{y}/(w\bar{L})$ at 1/0.8, η at 0.4, ψ at 2.0, ρ_{ab} at 0.1, ρ_{ac} at 0.2, ρ_{bc} at 0.4, ρ_a at -0.05, ρ_b at 0.1, ρ_c at 0.2, σ_b, σ_c, σ_d and σ_p at 0.5. The figure clearly supports the three predictions in (8): As self-employed farming becomes riskier, the own-farm labor supply (ℓ_a) declines, the labor supply share to agricultural wage work paid in kind (ℓ_c) increases more rapidly than that to agricultural wage work paid in cash (ℓ_b), and the labor supply share to non-agricultural wage work (ℓ_d) increases more rapidly than that to agricultural wage work paid in cash (ℓ_b).

A.I.1 Impact of Farm Income Risk on the Farm Labor Share

Since the shape of Figure A.1 is contingent on our specific choice of parameters, we examine the robustness of this shape in the followings. For simplicity's sake, in what follows, we assume that all the variances of risk factors are equal in order to focus on the effect of the covariances between risk factors.

Regarding the impact of farm income risk on the farm labor share, we take the partial derivative of (13) and obtain

$$\frac{\partial \ell_a}{\partial \sigma_a}=\frac{1}{D}\left\{\sigma_d^2\frac{\partial R_a}{\partial \sigma_a}+s''\sigma_p\frac{\partial Q_a}{\partial \sigma_a}\right\}-\frac{\ell_a}{D}\frac{\partial D}{\partial \sigma_a}. \quad (17)$$

In general, the sign of the above expression is indeterminate. However, with some additional assumptions, we can show that $\partial\ell_a/\partial\sigma_a<0$. First,

$$\begin{aligned}\frac{\partial R_a}{\partial \sigma_a} &= \Sigma_{bc}\sigma_c\rho_{ac}-\Sigma_{cc}\sigma_b\rho_{ab}+\Sigma_{bc}\sigma_b\rho_{ab}-\Sigma_{bb}\sigma_c\rho_{ac}\\ &= \sigma_b\rho_{ab}(\Sigma_{bc}-\Sigma_{cc})+\sigma_c\rho_{ac}(\Sigma_{bc}-\Sigma_{bb})\underbrace{<}_{\text{since }\rho_{bc}<1\ \&\ \sigma_b\approx\sigma_c}0.\end{aligned}$$

Second,

$$\begin{aligned}\frac{\partial Q_a}{\partial \sigma_a} &= \rho_a(\Sigma_{bb}\Sigma_{cc}-\Sigma_{bc}^2)+\sigma_b\rho_b(\sigma_c\rho_{ac}\Sigma_{bc}-\sigma_b\rho_{ab}\Sigma_{cc})+\sigma_c\rho_c(\sigma_b\rho_{ab}\Sigma_{bc}-\sigma_c\rho_{ac}\Sigma_{bb})\\ &\underbrace{\approx}_{\text{since }\sigma_b\approx\sigma_c} \rho_a(\Sigma_{bb}^2-\Sigma_{bc}^2)+\sigma_b^2\{\rho_{ac}(\rho_b\Sigma_{bc}-\rho_c\Sigma_{bb})+\rho_{ab}(\rho_c\Sigma_{bc}-\rho_b\Sigma_{bb})\}\\ &\underbrace{<}_{\text{since }\rho_{bc}<1} \rho_a(\Sigma_{bb}^2-\Sigma_{bc}^2)+\sigma_b^2\Sigma_{bb}\{\rho_{ac}(\rho_b-\rho_c)+\rho_{ab}(\rho_c-\rho_b)\}\end{aligned}$$

$$
\begin{aligned}
&= \rho_a(\Sigma_{bb}^2 - \Sigma_{bc}^2) + \sigma_b^2\Sigma_{bb}\{(\rho_{ac} - \rho_{ab})(\rho_b - \rho_c)\} \\
&\underbrace{<}_{\text{since } \rho_{ac} > \rho_{ab} \ \& \ \rho_c > \rho_b} \rho_a(\Sigma_{bb}^2 - \Sigma_{bc}^2) \underbrace{<}_{\text{if } \rho_a < 0} 0.
\end{aligned}
$$

Note that $\partial Q_a/\partial\sigma_a$ is more likely to be negative when $\rho_a < 0$, i.e., when farmers enjoy a higher gross income from crops, the food price tends to be lower, which seems to fit the situations in rural India. The assumption of the negative correlation between farm income and food price, $\rho_a < 0$, is not necessary to show our predictions in (8), however. We can obtain a similar conclusion if ρ_a is positive but sufficiently small. And third,

$$
\begin{aligned}
\frac{\partial D}{\partial\sigma_a} &= 2\sigma_a\Sigma_{bb}\Sigma_{cc} + 2\sigma_b\rho_{ab}\Sigma_{ac}\Sigma_{bc} + 2\sigma_c\rho_{ac}\Sigma_{ab}\Sigma_{bc} - 2\sigma_a\Sigma_{bc}^2 - 2\sigma_c\rho_{ac}\Sigma_{ac}\Sigma_{bb} - 2\sigma_b\rho_{ab}\Sigma_{ab}\Sigma_{cc} \\
&= 2\sigma_a\Sigma_{bb}\Sigma_{cc}\left(1 - \frac{\Sigma_{bc}^2}{\Sigma_{bb}\Sigma_{cc}}\right) - 2\sigma_b\rho_{ab}\Sigma_{ab}\Sigma_{cc}\left(1 - \frac{\Sigma_{ac}\Sigma_{bc}}{\Sigma_{ab}\Sigma_{cc}}\right) - 2\sigma_c\rho_{ac}\Sigma_{ac}\Sigma_{bb}\left(1 - \frac{\Sigma_{ab}\Sigma_{bc}}{\Sigma_{ac}\Sigma_{bb}}\right) \\
&\underbrace{\approx}_{\text{since } \sigma_a \approx \sigma_b \approx \sigma_c} 2\sigma_b\Sigma_{bb}^2\left(1 - \frac{\Sigma_{bc}^2}{\Sigma_{bb}^2}\right) - 2\sigma_b\rho_{ab}\Sigma_{ab}\Sigma_{bb}\left(1 - \frac{\Sigma_{ac}\Sigma_{bc}}{\Sigma_{ab}\Sigma_{bb}}\right) - 2\sigma_b\rho_{ac}\Sigma_{ac}\Sigma_{bb}\left(1 - \frac{\Sigma_{ab}\Sigma_{bc}}{\Sigma_{ac}\Sigma_{bb}}\right) \\
&\underbrace{>}_{\text{since } \rho_{ac} > \rho_{ab}} 2\sigma_b\Sigma_{bb}\left\{\Sigma_{bb}\left(1 - \frac{\Sigma_{bc}^2}{\Sigma_{bb}^2}\right) - (\rho_{ab}\Sigma_{ab} + \rho_{ac}\Sigma_{ac})\left(1 - \frac{\Sigma_{ab}\Sigma_{bc}}{\Sigma_{ac}\Sigma_{bb}}\right)\right\} \\
&\underbrace{\geq}_{\text{if } \rho_{ac},\ \rho_{bc} \leq \frac{1}{2} \ \& \ \rho_{ac}\rho_{bc} \leq 2\rho_{ab}} 2\sigma_b\Sigma_{bb}(\Sigma_{bb} - \rho_{ab}\Sigma_{ab} - \rho_{ac}\Sigma_{ac})\left(1 - \frac{\Sigma_{bc}^2}{\Sigma_{bb}^2}\right) \underbrace{>}_{\text{if } \rho_{ab},\ \rho_{ac} < \frac{1}{2}} 0.
\end{aligned}
$$

Note that $\partial D/\partial\sigma_a$ is more likely to be positive when $\sigma_a > \sigma_b$ (σ_c), which seems to fit the situations in rural India, but as shown above, even in the case of $\sigma_a \approx \sigma_b$ (σ_c), it becomes positive if the correlation coefficients are sufficiently small to satisfy $\rho_{ac} < 1/2$, $\rho_{bc} \leq 1/2$ and $\rho_{ac}\rho_{bc}/2 \leq \rho_{ab} < 1/2$. Thus, we obtain the relation $\partial\ell_a/\partial\sigma_a < 0$, which predicts that the own-farm labor supply declines as production becomes riskier. A corollary of this prediction is $\partial(\ell_b + \ell_c + \ell_d)/\partial\sigma_a > 0$, which predicts that the sum of the off-farm labor supply shares increases as self-employed farming becomes riskier.

A.I.2 Impact of Farm Income Risk on Labor Supply to Off-Farm Sectors

Now we investigate which among the three off-farm sectors expands most rapidly when self-employed farming becomes riskier. First, we examine the choice between agricultural wage work paid in cash and agricultural wage work paid in kind. Taking the partial derivatives of (14) and (15), we obtain

$$
\frac{\partial\ell_c}{\partial\sigma_a} - \frac{\partial\ell_b}{\partial\sigma_a} = \frac{1}{D}\left\{\sigma_d^2\left(\frac{\partial R_c}{\partial\sigma_a} - \frac{\partial R_b}{\partial\sigma_a}\right) + s''\sigma_p\left(\frac{\partial Q_c}{\partial\sigma_a} - \frac{\partial Q_b}{\partial\sigma_a}\right)\right\} - \frac{(\ell_c - \ell_b)}{D}\frac{\partial D}{\partial\sigma_a}.
$$

The sign of the above expression depends on the signs of $\partial(R_c - R_b)/\partial\sigma_a$, $\partial(Q_c - Q_b)/\partial\sigma_a$, $\ell_c - \ell_b$, and $\partial D/\partial\sigma_a$. As shown for the case of $\partial\ell_a/\partial\sigma_a$, it is likely that $\partial D/\partial\sigma_a > 0$. Furthermore,

$$
\begin{aligned}
\frac{\partial Q_c}{\partial\sigma_a} - \frac{\partial Q_b}{\partial\sigma_a} &= -\rho_a(\Sigma_{ac}\Sigma_{bb} - \Sigma_{ab}\Sigma_{cc}) + (\Sigma_{bc}\Sigma_{ac} - \Sigma_{ab}\Sigma_{bc}) \\
&\quad +(\sigma_a\rho_{ab}\sigma_b\Sigma_{cc} - \sigma_a\rho_{ab}\sigma_b\Sigma_{bc}) + (\sigma_a\rho_{ac}\sigma_c\Sigma_{bb} - \sigma_a\rho_{ac}\sigma_c\Sigma_{bc})\} \\
&\quad +\rho_b\sigma_b\{2(-\sigma_a\Sigma_{cc} + \sigma_c\rho_{ac}\Sigma_{ac}) \\
&\quad +2\sigma_a\sigma_b\sigma_c(-\rho_{bc} + \rho_{ab}\rho_{ac}) + \sigma_d^2(-2\sigma_a + \sigma_b\rho_{ab} + \sigma_c\rho_{ac})\} \\
&\quad +\rho_c\sigma_c\{2(\sigma_a\Sigma_{bb} - \sigma_b\rho_{ab}\Sigma_{ab}) \\
&\quad +2\sigma_a\sigma_b\sigma_c(\rho_{bc} - \rho_{ab}\rho_{ac}) + \sigma_d^2(2\sigma_a - \sigma_b\rho_{ab} - \sigma_c\rho_{ac})\} \\
&\underbrace{>}_{\text{since } \rho_{ab} < \rho_{ac}} -\rho_a\{\underbrace{(\Sigma_{ac}\Sigma_{bb} - \Sigma_{ab}\Sigma_{cc})}_{>0} + \underbrace{(\Sigma_{bc}\Sigma_{ac} - \Sigma_{ab}\Sigma_{bc})}_{>0} \\
&\quad + \underbrace{(\sigma_a\rho_{ab}\sigma_b\Sigma_{cc} - \sigma_a\rho_{ab}\sigma_b\Sigma_{bc})}_{>0} + \underbrace{(\sigma_a\rho_{ac}\sigma_c\Sigma_{bb} - \sigma_a\rho_{ac}\sigma_c\Sigma_{bc})}_{>0}\} \\
&\quad + \underbrace{(\rho_c\sigma_c - \rho_b\sigma_b)}_{>0}\{2\underbrace{(\sigma_a\Sigma_{bb} - \sigma_b\rho_{ab}\Sigma_{ab})}_{>0} \\
&\quad +2\sigma_a\sigma_b\sigma_c(\rho_{bc} - \rho_{ab}\rho_{ac}) + \sigma_d^2\underbrace{(2\sigma_a - \sigma_b\rho_{ab} - \sigma_c\rho_{ac})}_{>0}\}.
\end{aligned}
$$

Therefore, if we additionally assume that $\rho_a < 0$ and the correlation between cash and in-kind wages in agricultural labor market is moderately high so that $\rho_{bc} > \rho_{ab}\rho_{ac}$, which seems plausible in the context of rural India, we can assign the sign of $\partial(Q_c - Q_b)/\partial\sigma_a$ as positive. Thus, when $\ell_c \leq \ell_b$ and $\partial(R_c - R_b)/\partial\sigma_a \geq 0$, we obtain the relation $\partial(\ell_c - \ell_b)/\partial\sigma_a > 0$, which predicts that the labor supply share to wage work paid in kind increases more rapidly than that to wage work paid in cash, as self-employed farming becomes riskier. When $\ell_c > \ell_b$ or $\partial(R_c - R_b)/\partial\sigma_a < 0$, the sign of $\partial(\ell_c - \ell_b)/\partial\sigma_a$ is indeterminate, although it is more likely to be positive when s'' is large, i.e., the household's food budget share is high, the household is highly risk averse, and the household's food demand is inelastic. In the numerical simulation, the positive effect of $\partial(Q_c - Q_b)/\partial\sigma_a$ is dominant, although $(\ell_c - \ell_b)$ is positive and $\partial(R_c - R_b)/\partial\sigma_a$ is negative.

Finally, we investigate the choice between agricultural and non-agricultural wage work. From (14) and (16), we obtain

$$
\begin{aligned}
\frac{\partial\ell_d}{\partial\sigma_a} - \frac{\partial\ell_b}{\partial\sigma_a} &= \frac{1}{D}\left\{\sigma_d^2\left(-\frac{\partial R_a}{\partial\sigma_a} - 2\frac{\partial R_b}{\partial\sigma_a} - \frac{\partial R_c}{\partial\sigma_a}\right) + s''\sigma_p\left(-\frac{\partial Q_a}{\partial\sigma_a} - 2\frac{\partial Q_b}{\partial\sigma_a} - \frac{\partial Q_c}{\partial\sigma_a}\right)\right\} \\
&+ \frac{\ell_a + 2\ell_b + \ell_c}{D}\frac{\partial D}{\partial\sigma_a}.
\end{aligned}
$$

We already showed that the combination of $\partial R_a/\partial\sigma_a < 0$, $\partial Q_a/\partial\sigma_a < 0$, and $\partial D/\partial\sigma_a > 0$ is likely. Therefore, when the absolute values of $\partial R_b/\partial\sigma_a \approx \partial R_c/\partial\sigma_a$ are small and the absolute values of $\partial Q_b/\partial\sigma_a$ and $\partial Q_c/\partial\sigma_a$ are small, we expect the relation $\partial(\ell_d - \ell_b)/\partial\sigma_a > 0$, which predicts that the labor supply share to non-agricultural wage work increases more rapidly than that to agricultural wage work, as self-employed farming becomes riskier. This relation also holds in cases where σ_d^2 and s'' are sufficiently small. Regarding Figure A.1, we observe the relation $\partial(\ell_d - \ell_b)/\partial\sigma_a > 0$ because the absolute values of $\partial R_b/\partial\sigma_a$, $\partial R_c/\partial\sigma_a$, $\partial Q_b/\partial\sigma_a$, and $\partial Q_c/\partial\sigma_a$ are small. Note that in typical situations in developing countries, s'' is not very small, because the household's food budget share is high, the household is highly risk averse, and the household's food demand is inelastic.

A.I.3 Allowing for the Labor-Leisure Choice

We briefly sketch the case when we allow for the labor-leisure choice. By solving the first order conditions (5) and (6) and applying the implicit function theorem, we obtain the reduced-form solution for the total labor supply, $L^* = L(X_p, X_w, \Sigma)$. Meanwhile, we can solve the first order condition (5) only and derive the partially reduced-form solution, $\ell_j^* = \ell_j(L^*, X_p, X_w, \Sigma)$, $j = a, b, c, d$, which is conditional on the value of L^*. Then,

$$\frac{\partial \ell_j}{\partial \sigma_a} = \frac{\partial \ell_j(L^*, X_p, X_w, \Sigma)}{\partial \sigma_a} + \frac{\partial \ell_j(L^*, X_p, X_w, \Sigma)}{\partial L^*}\frac{\partial L(X_p, X_w, \Sigma)}{\partial \sigma_a}. \tag{18}$$

The first term of (18) is what we discussed in the comparative analysis above. The second term of (18) is the additional impact attributable to the endogeneity of the labor-leisure choice. This decomposition thus shows that the previous argument is valid even when we allow for the labor-leisure choice if we re-interpret the above exercise as the comparative-static analysis of $\ell_j(L^*, X_p, X_w, \Sigma)$, $j = a, b, c, d$, conditional on L^*.

Furthermore, in the specific cases investigated in this appendix, $\partial \ell_j(L^*, X_p, X_w, \Sigma)/\partial L^* = 0$ when $y_0 = 0$, because s'' does not depend on L^* when $y_0 = 0$ (remember that we approximate risk and consumption preference parameters at the expected values of y and p, and we assume the production technology to be constant-returns-to-scale). We also obtained the results that $\partial \ell_j(L^*, X_p, X_w, \Sigma)/\partial\sigma_a \approx \partial \ell_j(X_p, X_w, \Sigma)/\partial\sigma_a$ from numerical exercises that are extended from the one leading to Figure A.1 to allow for the labor-lesire choice. In other words, in empirical situations corresponding to rural India, where the dominant share of income of the poor comes from their hard labor, the theoretical predictions on the optimal labor supply shares derived under the assumption that the total labor supply is fixed are reasonable approximations for those on the optimal labor supply shares allowing for the labor-leisure choice.

Appendix II: Robustness Checks

In this appendix, we conduct several robustness checks of our main result shown in table 4, regarding the impact of rainfall risk (*CV of rainfall*) on labor supply. First, we re-estimate the same model under alternative specification with no adjustment for the possible correlation between errors. The estimation results are reported in the last row of table 6. Although the null hypothesis that the restricted model is true is rejected at the 1% level, the magnitudes of the coefficients on *CV of rainfall* are very similar to those reported in table 4.

Second, we examine the robustness with respect to the specification of the rainfall variables. In the default specification, district-level rainfall data are calculated as the weighted average of four nearest neighbor grid points ([two nearest neighbor longitudes] times [two nearest neighbor latitudes]), using the distance as weights. Instead, we could simply use rainfall data at the nearest grid point from each district. The results are very similar to those reported in the November 2007 version of this manuscript (not reported here).

In the default specification, district-level rainfall variables of *CV of rainfall* and *Rainfall shock* are calculated using fifteen-year rainfall data from 1985 to 1999. Considering the possibility that rainfall in the earlier years is weighted lighter in farmers' subjective assessment of weather risk, *CV of rainfall* and *Rainfall shock* are re-calculated using ten-year rainfall data from 1990 to 1999. The results are reported in table A.1, showing that coefficients on *CV of rainfall* show the same relations as predicted theoretically in (8). Statistical significance levels are improved in equations for ℓ_c and ℓ_d.

In the default specification, various variables at the district level are included, to control for the demand factors and other local conditions that affect labor allocations by farmers. Nevertheless, a suspicion of omitted variable bias cannot be ruled out. For instance, it is possible that the districts are different in terms of labor market conditions, this heterogeneity is not controlled adequately in our main result, and the heterogeneity is correlated with the variable *CV of rainfall*. In order to examine whether the bias due to this heterogeneity is substantial, we estimate the labor supply model with district dummies included, instead of district characteristics and rainfall variables. If the coefficients on household-level and village-level variables change substantially from our main result, a suspicion of omitted variable bias could be raised. By using a Wald test, we test the null hypothesis that the coefficient estimates in our main result and those in the regression with district dummies are equal. The test results indicate that the difference in the estimates is not statistically significant. To show that the omitted variable bias is likely to be small from a different angle, we re-estimate the model without district-level variables except for *CV of rainfall* and *Rainfall shock* variables. The estimation results regarding *CV of rainfall* are reported in table A.1. We find that the sign and statistical significance of coefficients on *CV of rainfall* are essentially unchanged, but the absolute values of the coefficients become larger as we include more district-level control variables. This seems to suggest that the impacts of risk factors are likely to be underestimated when heterogeneity across villages or districts is ignored. Thus, we expect the omitted variable bias to be rather small and not affect our qualitative finding that $\partial\ell_c^*/\partial\sigma_a > \partial\ell_b^*/\partial\sigma_a$ and $\partial\ell_d^*/\partial\sigma_a > \partial\ell_b^*/\partial\sigma_a$, even if unobserved heterogeneity exists across districts.

Appendix III: Simulation Procedure

In this appendix, we explain the simulation procedure used to obtain the results reported in table 5. We follow the procedure outlined by Cornick et al. (1994).

First, we simulate T runs of a (4×1) vector of error terms u using Cholesky factorization of the covariance matrix $\widehat{\Sigma}$ estimated by the multivariate tobit model:

$$\begin{aligned} \widehat{u}_t &= LS_t, & (19)\\ E[\widehat{u}_t] &= LE[S_t] = 0, & (20)\\ V[\widehat{u}_t] &= LV[S_t]L' = LIL' = \widehat{\Sigma}, & (21) \end{aligned}$$

where S_t is a (4×1) vector of random numbers obtained from a univariate standard normal distribution in the t-th trial, and L is a lower triangular matrix defined in the last equation of (21). Then for each run, we assign each observation (household) to a pattern of labor allocation shown in table 1, and obtain the following two pattern vectors, both of which are 4×1 (N: *interior* and C: *corner* solution outcome):

$$\begin{aligned} N_t &= \begin{pmatrix} 1[100 - X\widehat{\beta}_a > \widehat{u}_{a,t} > -X\widehat{\beta}_a] \\ \vdots \\ 1[100 - X\widehat{\beta}_d > \widehat{u}_{d,t} > -X\widehat{\beta}_d] \end{pmatrix} = \begin{pmatrix} N_{a,t} \\ \vdots \\ N_{d,t} \end{pmatrix}, \\ C_t &= \begin{pmatrix} 1[\widehat{u}_{a,t} \geq 100 - X\widehat{\beta}_a] \\ \vdots \\ 1[\widehat{u}_{d,t} \geq 100 - X\widehat{\beta}_d] \end{pmatrix} = \begin{pmatrix} C_{a,t} \\ \vdots \\ C_{d,t} \end{pmatrix}, \end{aligned}$$

where $1[\cdot]$ is an indicator function that takes unity if the condition in the bracket is true and zero otherwise, X is the vector of explanatory variables, and $\widehat{\beta}_k$ is the vector of estimated coefficients in the equation k ($k = a$: self-employment in agriculture, b: wage work in agriculture paid in cash, c: wage work in agriculture paid in kind, d: wage work in non-agriculture).

Using these pattern vectors and letting $\tilde{\ell}_k = X\widehat{\beta}_k + \widehat{u}_k$, we approximate the probabilities that a household allocates labor to each type of work by the followings.

$$\begin{pmatrix} \widehat{\Pr}(\ell_a > 0) \\ \vdots \\ \widehat{\Pr}(\ell_d > 0) \end{pmatrix} = \begin{pmatrix} \widehat{\Pr}(100 > \tilde{\ell}_a > 0) + \widehat{\Pr}(\tilde{\ell}_a \geq 100) \\ \vdots \\ \widehat{\Pr}(100 > \tilde{\ell}_d > 0) + \widehat{\Pr}(\tilde{\ell}_d \geq 100) \end{pmatrix} = \frac{\sum_{t=1}^{T} N_t + \sum_{t=1}^{T} C_t}{T}. \tag{22}$$

In addition, the expected labor supply share is given by

$$\begin{aligned} E[\ell_k] &= 0 \times \Pr(\tilde{\ell}_k \leq 0) + E[\tilde{\ell}_k | 100 > \tilde{\ell}_k > 0] \times \Pr(100 > \tilde{\ell}_k > 0) + 100 \times \Pr(\tilde{\ell}_k \geq 100) \\ &= \{X\beta_k + E[u_k | 100 > \tilde{\ell}_k > 0]\} \times \Pr(100 > \tilde{\ell}_k > 0) + 100 \times \Pr(\tilde{\ell}_k \geq 100), \quad k = a, b, c, d. \end{aligned}$$

Therefore, $E[\ell_k]$ can be estimated by using the predicted probabilities, $\widehat{\Pr}(100 > \tilde{\ell}_k > 0)$ and $\widehat{\Pr}(\tilde{\ell}_k \geq 100)$ in equation (22), and the expected value of error terms conditional on $(100 > \tilde{\ell}_k > 0)$ defined by

$$\widehat{E}[u_k | 100 > \tilde{\ell}_k > 0] = \frac{\sum_{t=1}^{T} \widehat{u}_k N_{k,t}}{\sum_{t=1}^{T} N_{k,t}}.$$

Note that the reported figures in table 6 are the mean predicted values when T is set to 50. The simulation results are not sensitive to marginal changes in T around 50.

Table A.1: Robustness Checks for Rainfall Risk (*CV of Rainfall*)

	(a) Self-emp., Agriculture		(b) Wage work, agriculture (cash payment)		(c) Wage work, agriculture (in-kind payment)		(d) Wage work, non-agriculture	
Base specification [Rainfall data: 15 years from 1985 to 99] (Table 4)								
CV of rainfall $\times 10^2$	-2.55	(2.74)***	0.19	(0.19)	1.40	(1.70)*	2.32	(2.35)**
Log-likelihood = -15229.483.								
Base specification [Rainfall data: 10 years from 1990 to 99]								
CV of rainfall $\times 10^2$	-3.07	(6.85)***	0.53	(0.89)	1.01	(2.31)**	2.66	(3.71)***
Log-likelihood = -15208.579.								
Without district variables [Rainfall data: 15 years from 1985 to 99]								
CV of rainfall $\times 10^2$	-2.26	(2.30)**	-0.58	(0.59)	0.40	(0.46)	1.74	(1.83)*
Log-likelihood = -15257.523.								
Without district variables [Rainfall data: 10 years from 1990 to 99]								
CV of rainfall $\times 10^2$	-2.32	(5.10)***	0.72	(1.32)	1.62	(2.50)**	2.10	(3.08)***
Log-likelihood = -15242.773.								

Note: (1) All regressions are implemented with other variables included, such as household characteristics, district average rainfall and UP state dummy. Coefficient estimates on these variables have been dropped for brevity but are available on request.
(2) Numbers in parentheses are z-values based on clustering robust standard errors using districts as clusters.

Figure A.1: An Example of the Optimal Labor Supply