4.

Russian Growth Path in Light of Production Function Estimation Using Quarterly Data

Masaaki Kuboniwa

Introduction

In this paper we characterize Russian growth in 1995Q1–2010Q2 by estimating a Cobb-Douglas production function using quarterly data. Most of the studies dealing with the Russian production function have contributed to a better understanding of Russian economic performance, but it seems to me that the results are not totally satisfactory, in part because annual data were used. Usually production function estimation is highly sensitive to the data employed. The use of quarterly data could reduce such sensitivity. Most of these studies also employed a conventional method of capital distribution ratio (capital share) of 30 percent. However, this conventional assumption should be verified by estimating the Russian production function itself. Here we present some results of our estimation attempts, which are at least statistically meaningful. The lack of appropriate data, e.g. on capital stock, has stood in the way of further progress in researching the Russian economy on the basis of a production function. Needless to say, our attempts are not free from this constraint. What we present here is only a step toward further investigation. However, we are able to show that the productivity of the Russian economy has increased rapidly in the last decade. Russia’s dependence on oil and energy products in its foreign trade has not prevented this development.
Estimation of Cobb-Douglas Production Function

We employ a Cobb-Douglas production function with steady exogenous technological progress:

\[ Y = A \exp(\lambda t) K^\alpha L^{(1-\alpha)}, \]

where \( Y \) = real GDP, \( K \) = real capital stock. \( L \) = employment, \( \alpha \) = elasticity of GDP with respect to capital (capital distribution ratio), \( \lambda \) = the constant technical progress rate or total factor productivity (TFP), and \( A \) is a positive constant.

Since this function is homogeneous of degree 1 with respect to \( K \) and \( L \), we can write it as

\[ y = A \exp(\lambda t) k^\alpha, \]

where \( y = Y/L \) and \( k = K/L \). It follows that

\[ \log y = \alpha \log k + \lambda t + \log A. \]

We estimate coefficients \( \alpha, \lambda \) and a constant \( \log A \) using ordinary least squares regression based on Russian quarterly data.

From the production function, we obtain the following well known growth accounting equation:

\[ g(Y) = \alpha g(K) + (1 - \alpha) g(L) + \lambda \]

Since we denote the growth rate of a variable \( X \) as \( g(X) = dX/X \), \( g(Y) \) denotes the growth rate of real GDP. \( g(K) \) and \( g(L) \) denote the increment rates of capital and labor respectively. The first two terms of the right-hand side of this equation give the capital and labor contributions to GDP growth respectively. The last term is the TFP contribution to GDP growth, which measures the effects of resource reallocations, modernization, technical progress, and catch-up efforts in the Russian economy.

We are now in a position to compile the data on GDP, capital and labor in a well defined manner. Real quarterly GDP data at 2003 prices can be estimated, based on the official non-seasonally-adjusted (nsa) values for 2003Q1–2010Q2 at the reference year 2003 (Rosstat as of October 1, 2010) and the official non-seasonally-adjusted growth rates for 1996Q1–2003Q4 (Archives of Rosstat as of December
31, 2009). We convert these estimated data into seasonally adjusted (sa) data via the census X-12 method – a fairly easy task.

One of the most difficult tasks in estimating a production function may be to estimate the data on capital stock. We estimate the capital stock time series at 2003 prices according to

\[ K_t = K_{t-1}(1-\delta) + I_t, \]

where \( K_t \) is real fixed capital stock measured in constant 2003 prices at the end of period \( t \), \( I_t \) is real investment (fixed capital formation) measured in 2003 prices at period \( t \), and \( \delta \) is the constant depreciation rate.

In estimating the capital stock, the official SNA investment data were employed. As in GDP, using the official values (nsa) for 2003Q1–2010Q2 and the official growth rates (nsa) for 1996Q1–2003Q4, our data (sa) on investment are derived via X-12. Figure 4.1 shows data (nsa and sa) on investment at 2003 prices.

![Figure 4.1 Movement of Investment in Russia, constant 2003 roubles bn. Source: Rosstat and own calculations.](image)

We regard the initial value of the capital stock at the end of 2002Q4 (i.e. at the start of 2003) as 52,904 billion roubles at current prices, which may be 1.7 times higher than the official value of overall capital stock of 30,329 billion roubles at book prices. Our initial value is based on preliminary data that Rosstat compiled in accord with 2003 input-output tables. Although the data employed are preliminary, we feel
that they reflect the current values for reproduction of corresponding capital stocks better than the official values at mixed book prices.

Our baseline assumption of the annual depreciation rate is 1.8 percent (quarterly rate 0.45 percent), which corresponds to the ratio of capital depreciation (consumption of capital stock) to the capital stock for 2003. The value of capital depreciation in 2003, derived from the official integrated SNA table, amounts to 959 billion roubles at current prices (Rosstat). Hence we obtain an annual depreciation rate of 1.8 percent ($959/52,904=0.018$).

Thus we have our time series (levels) of Russian capital stock. Figure 4.2 presents a comparison of annual growth rates of data aggregated from our series with the official annual rates of capital stock. As can be seen from this figure, our annual growth rates of Russian capital stock are much higher than the official growth rates at 2000 constant prices. Assuming a higher depreciation rate would result in a lower real growth rate of capital stock. This suggests that a higher depreciation rate (capital replacement) would result in a larger TFP contribution with a smaller capital contribution in the growth accounting. This result reminds us of the importance of meticulous accounting for variation in capital stock, as small differences in these assumptions change our interpretation of productivity developments also for the Russian economy.

![Figure 4.2 Annual Percent Changes in the Estimates of Capital Stock. Source: Rosstat and own calculations.](image-url)

---

Russian Growth Path in Light of Production Function Estimation Using Quarterly Data
We use our employment (in number of persons) estimate for 1999Q1–2010Q2, based on the official labor survey (average employment data for February, May, August and November of each year) adjusted so that the aggregated annual data of our quarterly employment estimates correspond with the official annual employment data. For 1995Q1–1998Q4 we estimate quarterly employment, based on the official annual data for 1995–1998 and quarterly changes in employment for 1999. These data (nsa) are seasonally adjusted via X-12. Thus we arrive at the employment estimate shown in Figure 4.3.

![Figure 4.3](image-url)  
Figure 4.3 Movements of Employment in Russia, thousands. Source: Rosstat and own calculations.

Based on the estimated quarterly data for the favorable economic growth period (1999Q1–2008Q2), we obtain the following results for coefficients, which are significant at the 1 percent level (regression details are given in the Appendix):

\[
\log y = 0.640 \cdot \log k + 0.0113;
\]

(1) \[ g(Y) = 0.640 \cdot g(K) + 0.360 \cdot g(L) + 0.011 \text{ (TFP's annualized rate of 4.6 percent)} \]

These variables are not spurriously related but are cointegrated (all regressions are shown in the Appendix).
Equation (1) shows that the capital distribution ratio for the favorable period is 64 percent, which is approximately double the conventional a priori ratio of 30 percent. This equation also shows that the average TFP is 4.6 percent for the favorable growth period.

The average annual growth rates for Russian GDP, capital stock and employment for 1999Q1–2008Q2 were 7.5 percent, 3.2 percent and 0.8 percent, respectively. The capital contribution to the growth $(0.640 \cdot 3.2 = 2.0$ percentage points) explains 27 percent of the overall growth rate, and the labor contribution to the growth $(0.360 \cdot 0.8 = 0.3$ percentage point) explains 4 percent of the overall growth rate. The residual 69 percent consists of the TFP of 62 percent and statistical error of 7 percent. More than half of the growth can be explained by the TFP impact and one-fourth by the capital contribution.

Figure 4.4 shows actual and fitted GDPs at 2003 prices using Equation (1). The fitted values are just within the neighborhood of the actual ones.

![Graph](image)

**Figure 4.4** Actual and Fitted Values of GDP for 1999Q1–2008Q2, 2003 bn roubles. Source: Rosstat and own calculations.

Figure 4.5 shows the changes in the actual capital distribution ratio (capital share) using the official national accounts (Rosstat). As is shown, the capital distribution ratio posted marked increases after the 1998 financial crisis and during the oil price rises in 2000. In contrast, it suffered much more than the labor side after the Lehman shock and oil price declines. The official data on labor income (compensation for employees) include the 'hidden wages'. In the Russian
national accounts these hidden wages are recorded as the transfer from gross profits (including mixed income), as is shown in Table 1. Before the transfer, the average capital distribution ratio is approximately 66 percent; afterwards, it is about 54 percent. Whether this kind of transfer is proper is debatable. The issue requires further investigation. So the actual capital distribution ratio may be between 54 percent and 66 percent. Interestingly, our estimation of the capital distribution ratio (62 percent) is within this range.

Figure 4.5 Actual Capital Distribution Ratio (percent). Source: Rosstat and own calculations.
Table 1. Capital Distribution Ratios before and after Transfer of “Hidden” Income (percent share in GDP)

<table>
<thead>
<tr>
<th></th>
<th>Before transfer</th>
<th></th>
<th></th>
<th>After transfer</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GDP</td>
<td>Capital</td>
<td>Labor</td>
<td>“Hidden” labor income</td>
<td>Capital</td>
</tr>
<tr>
<td></td>
<td>distribution ratio</td>
<td>distribution ratio</td>
<td>income</td>
<td>distribution ratio</td>
<td>distribution ratio</td>
</tr>
<tr>
<td>1995</td>
<td>100</td>
<td>65.8</td>
<td>34.2</td>
<td>11.2</td>
<td>54.6</td>
</tr>
<tr>
<td>1996</td>
<td>100</td>
<td>61.5</td>
<td>38.5</td>
<td>12.5</td>
<td>49.1</td>
</tr>
<tr>
<td>1997</td>
<td>100</td>
<td>61.0</td>
<td>39.0</td>
<td>12.4</td>
<td>48.6</td>
</tr>
<tr>
<td>1998</td>
<td>100</td>
<td>62.5</td>
<td>37.5</td>
<td>10.5</td>
<td>52.0</td>
</tr>
<tr>
<td>1999</td>
<td>100</td>
<td>70.8</td>
<td>29.2</td>
<td>10.9</td>
<td>59.9</td>
</tr>
<tr>
<td>2000</td>
<td>100</td>
<td>70.9</td>
<td>29.1</td>
<td>11.1</td>
<td>59.8</td>
</tr>
<tr>
<td>2001</td>
<td>100</td>
<td>68.2</td>
<td>31.8</td>
<td>11.2</td>
<td>57.0</td>
</tr>
<tr>
<td>2002</td>
<td>100</td>
<td>64.7</td>
<td>35.3</td>
<td>11.5</td>
<td>53.2</td>
</tr>
<tr>
<td>2003</td>
<td>100</td>
<td>64.2</td>
<td>35.8</td>
<td>11.3</td>
<td>52.8</td>
</tr>
<tr>
<td>2004</td>
<td>100</td>
<td>65.6</td>
<td>34.4</td>
<td>11.7</td>
<td>53.9</td>
</tr>
<tr>
<td>2005</td>
<td>100</td>
<td>68.0</td>
<td>32.0</td>
<td>11.8</td>
<td>56.2</td>
</tr>
<tr>
<td>2006</td>
<td>100</td>
<td>68.3</td>
<td>31.7</td>
<td>12.8</td>
<td>55.5</td>
</tr>
<tr>
<td>2007</td>
<td>100</td>
<td>66.7</td>
<td>33.3</td>
<td>13.4</td>
<td>53.3</td>
</tr>
<tr>
<td>2008</td>
<td>100</td>
<td>65.4</td>
<td>34.6</td>
<td>12.6</td>
<td>52.8</td>
</tr>
<tr>
<td>2009</td>
<td>100</td>
<td>62.0</td>
<td>38.0</td>
<td>13.8</td>
<td>48.3</td>
</tr>
<tr>
<td>2009</td>
<td>100</td>
<td>62.0</td>
<td>38.0</td>
<td>13.8</td>
<td>48.3</td>
</tr>
<tr>
<td>Average (1995–2009)</td>
<td>100</td>
<td>65.7</td>
<td>34.3</td>
<td>11.9</td>
<td>53.8</td>
</tr>
<tr>
<td>Average (1999–2008)</td>
<td>100</td>
<td>67.3</td>
<td>32.7</td>
<td>11.8</td>
<td>55.4</td>
</tr>
</tbody>
</table>

Sources: Rosstat and Annual National Accounts Report (NSR) various issues.

Notes: 1. Capital income consist of the gross profits and net taxes on production and imports.
2. “Hidden” income is estimated as the difference between reported income and estimated income from household survey.

Estimation of Production Function using Capital Utilization Rate

Our estimation of Equation (1) is indeed statistically satisfactory, but it covers only the favorable growth period. In order to include periods of the 1998 crisis and Lehman shock coupled with the adverse oil price shock, we have to consider utilization rates of capital as well as labor. However, we do not have much data on these utilization rates for the overall economy of Russia. As suggested by Figures 4.2 and 4.3, the movement of labor is more sensitive to business cycles than are capital movements.

As a proxy for movement of the capital utilization rate we would like to use an index of terms-of-trade effects. We may assume that

---

5 See Oomes and Dynnikova (2006) for details of Russian statistics of utilization rates of capital and labor.
Russian managers of enterprises and factories can decide on the capital utilization rate in response to terms-of-trade effects, which are truly indicative of business conditions in Russia.

Our index \( u \) is defined as the ratio of real GDI (gross domestic income) to real GDP, that is to say, \( u = \frac{\text{GDI}}{\text{GDP}} \).

Here according to the SNA 2008 and the U.S. Bureau of Economic Analysis, the real GDI (\( Z \)) or the command-basis GDP is defined as the real GDP (\( Y \)) plus the real trading gain.

\[ Z = Y + T. \]

The trading gain (\( T \)) from changes in the terms-of-trade can be defined as the nominal net exports deflated by the import price index minus the conventional real net exports.

\[ T = \frac{(E_n - M_n)}{P^m} - \frac{(E_r - M_r)}{P^m}. \]

It follows from this equation that

\[ T = \frac{E_n}{P^m} - \frac{E_r}{P^m} = E_R \left( \frac{P^e}{P^m} - 1 \right). \]

Therefore, \( T > = < 0 \) if \( \frac{P^e}{P^m} > = < 1 \). If the terms-of-trade \( P^e/P^m \) improve (worsen), the trading gain should increase (decrease). At the base period, \( P^e = P^m = 1 \) and hence \( T \) must be zero.

Using seasonally adjusted exports and imports at current and 2003 prices, we can derive the series \{GDI\}, and hence \{u\} or \{GDI/GDP\} at 2003 prices. (For details on this index and terms-of-trade effects, see Kubonichi, 2010). The adjusted capital stock series \{uK\} and non-adjusted series \{K\} are shown in Figure 4.6 as normalized data with mean 0 and variance 1. As can be seen from this figure, the capital utilization rate decreases during recessions and increases in the period 2005–2008Q2, where the price of oil increases.
Figure 4.6 Normalized Capital Stock Utilization ratio. Source: Rosstat and own calculations.

We consider here a modified production function: in place of $K$, we use $uK$.

$$Y = A \exp(\lambda t) \ (uK)^\alpha L^{(1-\alpha)}.$$  

It follows from this equation that

$$y = A \exp(\lambda t) \ (uk)^\alpha,$$

$$\log y = \log A + \alpha \log uk + \lambda t.$$  

Thus the growth accounting equation is written as

$$g(Y) = \alpha \ g(uK) + (1 - \alpha) \ g(L) + \lambda.$$  

For 1995Q1–2010Q2 we obtain the following regression result with coefficients significant at the 1 percent level.

$$\log y = 0.629 \cdot \log uk + 0.0045;$$

(2)  

$$g(Y) = 0.629 \cdot g(K) + 0.371 \cdot g(L) + 0.0145 \ (TFP's \ annualized \ rate \ of \ 1.8 \ percent)$$

These variables are not spuriousely related but cointegrated (see Appendix).
Equation (2) shows that the capital distribution ratio for the favorable period is 63 percent, which is similar to the case of non-adjusted capital stock. This equation also shows that the average TFP is 1.8 percent for the period including two crises, which is much smaller than the result for the favorable period.

The average annual growth rates of Russian GDP, adjusted capital stock (uK) and employment for 1995Q1–2010Q2 were 3.6 percent, 4.4 percent and 0.08 percent, respectively. The capital contribution to the growth (0.63 · 4.4 = 2.8 percentage points) explains 77 percent of the overall growth rate, and the labor contribution (0.37 · 0.08 = 0.03 percentage point) explains 1 percent. The residual 21 percent consists of the TFP of 51 percent and statistical error of –29 percent. More than three-fourths of the growth can be explained by the capital contribution impact; and one-fifth by the residuals, including the TFP contribution share of more than 50 percent. The residuals include rather large statistical errors. The fitted GDP explains the recessions well, but it does not reflect well the actual growth after 2009Q2.

For the favorable period (1999Q1–2008Q2) we obtain the following regression result with coefficients significant at the 1 percent level.

\[
\log y = 0.239 \cdot \log uK + 0.0116;
\]

(3) \[ g(Y) = 0.239 \cdot g(K) + 0.761 \cdot g(L) + 0.0116 \text{ (TFP's annualized rate of 4.8 percent)} \]

Equation (3) shows that the capital distribution ratio for the favorable period is a rather low 24 percent. This equation also shows that the average TFP is 4.8 percent for the favorable period.

The average annual growth rates of Russian GDP, adjusted capital stock (uK) and employment for 1999Q1–2008Q2 were 7.5 percent, 7.6 percent and 0.8 percent, respectively. The capital contribution to the growth (0.24 · 7.6 = 1.8 percentage points) explains 24 percent of the overall growth rate, and the labor contribution (0.76 · 0.8 = 0.6 percentage point) explains 8 percent. The residual, 68 percent, consists of the TFP of 63.4 percent and statistical error of 4.3 percent. The growth rate of adjusted capital stock is much higher than that of non-adjusted stock, and hence the capital distribution ratio in Equation (3) is much less than that in Equation (1). After all, the capital contribution in Equation (3) is similar to that in Equation (1). It is also noteworthy that the TFP contribution in Equation (3) is very close to that in Equation (1).
Concluding Remarks

We presented explicit estimations of the Cobb-Douglas production function using quarterly data. This attempt suggests that it is better to use quarterly data in estimating the Russian production function. The bulk of the literature on the Russian production function seems not to be so successful partly because annual data are used. Furthermore, we showed that for the favorable period the key growth driver from the supply-side was the TFP, followed by the capital contribution. It is noteworthy that the favorable growth rate of 7.5 percent was induced by a TFP contribution of more than 4.5 percent. This suggests a significant improvement in the efficiency of the Russian economy, contrary to some commentators’ views. However, we should note that this favorable TFP development corresponds to a period of rising oil and energy prices.

Furthermore, we also offered a method of adjusting capital stock data to capture the recessions, based on the production function, albeit further studies on the adjustments are needed. In our analysis for the whole period, including two recessions, the key driver of Russian growth may be the capital contribution, followed by TFP. This may be due to the large contraction of the TFP base during the recessions. These results caution against overemphasizing structural changes in the Russian economy.

Growth accounting analyses based on alternative assumptions for capital depreciation rates as well as the initial capital stock also remain unresolved. We would like to briefly sketch the results based on alternative assumptions. When we use a depreciation rate of 3 percent per annum, instead of 1.8 percent, for the favorable period we obtain a lower capital increment rate (annual rate of 1 percent) and thus a higher rate of TFP (annual rate of 6 percent) with a slightly lower capital distribution ratio (49 percent). If we take the official capital stock at book values as the initial value, with a depreciation rate of 3 percent per annum, we obtain a slightly higher capital increment rate (annual rate of 4.5 percent) and a reasonable rate of TFP (annual rate of 4.6 percent) with a lower capital distribution rate (42 percent).

Therefore, one should be careful in interpreting any one set of results. Nevertheless, it appears that productivity of the Russian economy has risen in the recent years. If this positive trend continues, one can be cautiously optimistic about Russia’s future growth prospects as well.
Appendix

Table A1 below gives the details of the regressions in this paper. The order of integration of the series is important for the selected regressions. We tested for unit roots by the commonly used Augmented Dickey-Fuller (ADF) tests and found all variables nonstationary. Performing the tests for the first differences of variables, we reject the null of nonstationarity. Since all variables must be differenced once to obtain stationarity, they are integrated of order 1, I(1).

To test whether the nonstationary variables in our regressions are cointegrated or spuriously related, we examined the properties of the regression residuals via the ADF test. For all regressions we can reject the null of no cointegration. In other words, the nonstationary variables in all of our regressions are cointegrated.  

Table A1. Results of Regressions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
<th>Adj. R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Equation (1) for 1990Q1–2008Q2 (38 observations))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9950</td>
</tr>
<tr>
<td>constant</td>
<td>-3.2438</td>
<td>0.0550</td>
<td>-58.9769</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>log uk</td>
<td>0.6398</td>
<td>0.1430</td>
<td>4.4736</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>TFP;</td>
<td>0.0113</td>
<td>0.0008</td>
<td>13.4939</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>(Equation (2) for 1995Q1–2010Q2 (62 observations))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9692</td>
</tr>
<tr>
<td>constant</td>
<td>-3.0550</td>
<td>0.0423</td>
<td>-72.2285</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>log uk</td>
<td>0.6288</td>
<td>0.0841</td>
<td>7.4790</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>TFP;</td>
<td>0.0045</td>
<td>0.0010</td>
<td>4.7347</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>(Equation (3) for 1999Q1–2008Q2 (38 observations))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9963</td>
</tr>
<tr>
<td>constant</td>
<td>-3.3409</td>
<td>0.0242</td>
<td>-138.0904</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>log uk</td>
<td>0.2392</td>
<td>0.0383</td>
<td>6.2478</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>TFP;</td>
<td>0.0116</td>
<td>0.0006</td>
<td>20.7742</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>

Notes: y = Y/L, k = K/L, u = GDI/GDP.

6 Results are available from the author upon request.
References


NSR, Natsional’nye scheta Rossii, various issues. Moscow: Rosstat.
