Discussion Paper Series A No.601

Bilateral Equivalence between Trade in Value Added and Value Added Content of Trade

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February, 2014

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Abstract

This paper demonstrates that the bilateral equivalence between trade in value added (TiVA) and value added content of trade. TiVA, based on value added exports, which is proposed by Johnson-Noguera and OECD-WTO, measures origin country's value added exports induced by each destination country's final demand, excluding intermediates, for the world. Value added content of trade or "value added in trade (VAiT)," which is proposed by Trefler and followed by Stehrer, measures value added induced by appropriately arranged gross trade, including intermediates. At a glance, these two measures may look quite different. This paper shows that in the world with two countries and many countries these two measures of TiVA and value added content of trade are bilaterally equivalent.

JEL classification codes: F1, F15, F19, R15

Key words: trade in value added, value added content of trade, gross trade, input-output

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1. Introduction

In light of the development of intermediate goods trade, Johnson and Noguera (2012) as well as WTO and IDE JETRO (2011) addressed the new concept of trade in value added (TiVA) in place of conventional trade in gross terms. OECD and WTO (2012) also provided empirical results based on the OECD international input-output tables. The global trade network captured and generated by TiVA is called global value chains (GVC). The new concept of value added exports from an origin country to a destination country is defined as the origin county's value added induced by the destination country's final demand, excluding exports and imports of intermediate goods, for the world.

On the other hand, Stehrer (2012, 2013) and Foster-McGregor and Stehrer (2013) presented another approach to value added content of gross trade or value added in trade (VAiT) through a direct application of Trefler and Zhu (2010) to Stehrer's preferred world with three countries and many sectors. In this paper, first, we show that in the world with two countries these two measures are doubtlessly equivalent. Second, we theoretically demonstrate that in the world with three and more countries these two measures are also equivalent through presenting the logically consistent definition of bilateral value added content of gross trade in place of Stehrer's bilateral setting. We also numerically and empirically verify our theorem on the bilateral equivalence between TiVA and value added contents of trade.

2. Model and definition of TiVA

2.1. Model

Following Isard (1951), WTO and IDE (2011), and Johnson and Noriega (2012), we reproduce an inter-country multi-sector model in a general framework, shown by Table 1.

Table 1

Data structure of an international input-output table.

	Intermediate demand/input								Final demand (destination)								
	Country 1	Country 2	$\cdots$	Country r	$\cdots$	Country s	$\cdots$	ROW R	Country 1	Country 2	$\cdots$	Country r	$\cdots$	Country s	$\cdots$	ROW R	Output
									$\boldsymbol{F}_1$	$\boldsymbol{F}_2$	$\cdots$	$\boldsymbol{F}_r$	$\cdots$	$\boldsymbol{F}_s$	$\cdots$	$\boldsymbol{F}_R$	$\boldsymbol{X}$
Country 1	$\underline{\boldsymbol{X}}_{11}$	$\underline{\boldsymbol{X}}_{12}$	$\cdots$	$\underline{\boldsymbol{X}}_{1r}$	$\cdots$	$\underline{\boldsymbol{X}}_{1s}$	$\cdots$	$\underline{\boldsymbol{X}}_{1R}$	$\boldsymbol{Y}_{11}$	$\boldsymbol{Y}_{12}$	$\cdots$	$\boldsymbol{Y}_{1r}$	$\cdots$	$\boldsymbol{Y}_{1s}$	$\cdots$	$\boldsymbol{Y}_{1R}$	$\boldsymbol{X}_1$
Country 2	$\underline{\boldsymbol{X}}_{21}$	$\underline{\boldsymbol{X}}_{22}$	$\cdots$	$\underline{\boldsymbol{X}}_{2r}$	$\cdots$	$\underline{\boldsymbol{X}}_{2s}$	$\cdots$	$\underline{\boldsymbol{X}}_{2R}$	$\boldsymbol{Y}_{21}$	$\boldsymbol{Y}_{22}$	$\cdots$	$\boldsymbol{Y}_{2r}$	$\cdots$	$\boldsymbol{Y}_{2s}$	$\cdots$	$\boldsymbol{Y}_{2R}$	$\boldsymbol{X}_2$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
Country r	$\underline{\boldsymbol{X}}_{r1}$	$\underline{\boldsymbol{X}}_{r2}$	$\cdots$	$\underline{\boldsymbol{X}}_{rr}$	$\cdots$	$\underline{\boldsymbol{X}}_{rs}$	$\cdots$	$\underline{\boldsymbol{X}}_{rR}$	$\mathbf{Y}_{r1}$	$\mathbf{Y}_{r2}$	$\cdots$	$\boldsymbol{Y}_{rr}$	$\cdots$	$\boldsymbol{Y}_{rs}$	$\cdots$	$\boldsymbol{Y}_{rR}$	$\boldsymbol{X}_r$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
Country s	$\underline{\boldsymbol{X}}_{s1}$	$\underline{\boldsymbol{X}}_{s2}$	$\cdots$	$\underline{\boldsymbol{X}}_{sr}$	$\cdots$	$\underline{\boldsymbol{X}}_{ss}$	$\cdots$	$\underline{\boldsymbol{X}}_{sR}$	$\boldsymbol{Y}_{s1}$	$\boldsymbol{Y}_{s2}$	$\cdots$	$\boldsymbol{Y}_{sr}$	$\cdots$	$\boldsymbol{Y}_{ss}$	$\cdots$	$\boldsymbol{Y}_{sR}$	$\boldsymbol{X}_s$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
ROW R	$\underline{\boldsymbol{X}}_{R1}$	$\underline{\boldsymbol{X}}_{R2}$	$\cdots$	$\underline{\boldsymbol{X}}_{Rr}$	$\cdots$	$\underline{\boldsymbol{X}}_{Rs}$	$\cdots$	$\underline{\boldsymbol{X}}_{RR}$	$\mathbf{Y}_{R1}$	$\mathbf{Y}_{R2}$	$\cdots$	$\boldsymbol{Y}_{Rr}$	$\cdots$	$\boldsymbol{Y}_{Rs}$	$\cdots$	$\boldsymbol{Y}_{RR}$	$\boldsymbol{X}_R$
Value added	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	$\cdots$	$\boldsymbol{v}_r$	$\cdots$	$\boldsymbol{v}_s$	$\cdots$	$\boldsymbol{v}_R$									
Output	$\boldsymbol{X}_1$	$\boldsymbol{X}_2$	$\cdots$	$\mathbf{X}_r$	$\cdots$	$\boldsymbol{X}_s$	$\cdots$	$\boldsymbol{X}_R$									

$\underline{\boldsymbol{X}}rs$ ($s\neq r$): country r's gross export matrix of intermediate goods to country s or country s's gross import matrix of intermediate goods from country r. $\underline{\boldsymbol{X}}rr$: country r's input matrix of intermediate goods domestically produced.

We assume there are r, s =1, 2, …, R countries (areas or regions) each of which produces and inputs $r(i)$, $s(j)$ =1, 2, ..., n products. We assume the classical Leontief open input-output model with fixed input coefficients and final demand for each country. In this model each sector produces a single commodity without joint production. We regard the last country R as the rest of the world (ROW). We consider an international input-output system not in physical terms but in value terms.

We denote: $\boldsymbol{A}_{rs} = (a_{r(i)s(j)})$ $(n\times n)$: country r's export coefficient matrix to country s or country s's import coefficient matrix from country r if $r\neq s$, and country r's input coefficient matrix of domestically produced intermediate goods if s=r; $\boldsymbol{Y}_r=[Y_{r(i)}]$ $(n\times 1)$: country r's final demand vector in an international input-output table; $\widetilde{\boldsymbol{Y}}_r = [\widetilde{\boldsymbol{Y}}_{r(i)}](n\times 1)$: country r's final demand vector, including exports of intermediate goods, in each country's input-output system; $\boldsymbol{Y}_{rs}= [Y_{r(i)s}]$ $(n\times 1)$: country s's final demand vector for country r $(n\times 1)$ or country r's final goods export vector to country s if $r\neq s$; $\boldsymbol{F}_s = [\boldsymbol{Y}_{rs}]$ $((n\times R)\times 1)$: country s's final demand vector for all countries; $\boldsymbol{X}_r= [X_{r(i)}]$ $(n\times 1)$: country r's output vector ; $\boldsymbol{X}= [\boldsymbol{X}_r]$ $((n\times R)\times 1)$: an overall output vector ; $\boldsymbol{I}$: an $(n\times R)$ dimensional identity matrix; $\boldsymbol{I}_n$: an n dimensional identity matrix. We assume that non-negative matrixes $\boldsymbol{A}$ and $\boldsymbol{A}_{rr}$ are productive.

Denoting $\boldsymbol{X}^*$ as the equilibrium output vector, the global equilibrium (market clearing) condition for an Isard type of non-competitive inter-country multi-sector input-output table in value terms can be written as:

$$\boldsymbol{X}^* = \boldsymbol{A}\boldsymbol{X}^* + \boldsymbol{Y};\ \boldsymbol{X}^* = \boldsymbol{B}\boldsymbol{Y}, \text{where } \boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{A})^{-1}, \tag{1}$$

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} & \dots & \boldsymbol{A}_{1s} \dots & \boldsymbol{A}_{1R} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{A}_{r1} & \boldsymbol{A}_{r1} & \dots & \boldsymbol{A}_{rs} \dots & \boldsymbol{A}_{rR} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{A}_{R1} & \boldsymbol{A}_{R2} & \dots & \boldsymbol{A}_{Rs} \dots & \boldsymbol{A}_{RR} \end{pmatrix},$$

$$\boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{pmatrix} \boldsymbol{B}_{11} & \boldsymbol{B}_{12} & \dots & \boldsymbol{B}_{1s} & \boldsymbol{B}_{1R} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{B}_{r1} & \boldsymbol{B}_{r1} & \dots & \boldsymbol{B}_{rs} & \boldsymbol{B}_{rR} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{B}_{R1} & \boldsymbol{B}_{R2} & \dots & \boldsymbol{B}_{Rs} & \boldsymbol{B}_{RR} \end{pmatrix},$$

$$\boldsymbol{Y} = \begin{pmatrix} \boldsymbol{Y}_1 \\ \dots \\ \boldsymbol{Y}_r \\ \dots \\ \boldsymbol{Y}_R \end{pmatrix} = \begin{pmatrix} \boldsymbol{Y}_{11} \\ \dots \\ \boldsymbol{Y}_{r1} \\ \dots \\ \boldsymbol{Y}_{R1} \end{pmatrix} + \cdots + \begin{pmatrix} \boldsymbol{Y}_{1s} \\ \dots \\ \boldsymbol{Y}_{rs} \\ \dots \\ \boldsymbol{Y}_{Rs} \end{pmatrix} + \cdots + \begin{pmatrix} \boldsymbol{Y}_{1R} \\ \dots \\ \boldsymbol{Y}_{rR} \\ \dots \\ \boldsymbol{Y}_{RR} \end{pmatrix}$$

$$= \boldsymbol{F}_1 + \cdots + \boldsymbol{F}_s + \cdots + \boldsymbol{F}_R; \boldsymbol{X} = \begin{pmatrix} X_1 \\ \dots \\ X_r \\ \dots \\ X_R \end{pmatrix}; \ \boldsymbol{F}_s = \begin{pmatrix} \boldsymbol{Y}_{1s} \\ \dots \\ \boldsymbol{Y}_{rs} \\ \dots \\ \boldsymbol{Y}_{Rs} \end{pmatrix}.$$

Overall output $\boldsymbol{X}^*_{*s}$ and country *r*'s output $\boldsymbol{X}^*_{r*s}$, induced by a fixed destination country **s*'s final demand $\boldsymbol{F}_{*s}$, are given by

$$\boldsymbol{X}^*_{*s} = \boldsymbol{A}\boldsymbol{X}^*_{*s} + \boldsymbol{F}_{*s} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{F}_{*s}; \ \boldsymbol{X}^*_{r*s} = \Sigma_k \boldsymbol{A}_{rk}\boldsymbol{X}^*_{k*s} + \boldsymbol{Y}_{r*s}\,. \tag{2}$$

This equation is essential for the definition of trade in value added. For productive semi-positive matrix $\boldsymbol{A}$ and semi-positive $\boldsymbol{F}_{*s}$**,** each of $\boldsymbol{B}$ and the solution $\boldsymbol{X}^*_{*s}$ is semi-positive and unique.

By definitions of $\boldsymbol{F}_s$ and $\boldsymbol{Y}_{rs}$ we have

$$\boldsymbol{X}^* = \Sigma_s \boldsymbol{X}^*_{*s}; \ \boldsymbol{X}^*_r = \Sigma_s \boldsymbol{X}^*_{r*s}\,. \tag{3}$$

Country *r*'s gross exports to country *s*, $\boldsymbol{E}_{rs}$ are given by

$$\boldsymbol{E}_{rs} = \boldsymbol{A}_{rs}\boldsymbol{X}^*{}_s + \boldsymbol{Y}_{rs}\ (s \neq r)\textbf{.} \tag{4}$$

2.2. General definition of TiVA

Let us define country *r*'s *i*-th value added ratio as $v_{r(i)} = V_{r(i)}/X_{r(i)}$ where $V_{r(i)}$ is country *r*'s *i*-th value added. Country *r*'s value added ratio vector and the overall value added vector are $\boldsymbol{v}_r = (v_{r(i)})(1 \times n)$ and $\boldsymbol{v} = (\boldsymbol{v}_r)(1 \times (n \times R))$ respectively.

The new concept of value added trade is defined as follows:

Definition 1. The new concept of bilateral value added exports: Johnson and Noguera (2012), and WTO and IDE (2011)

Country r's value added exports to country s are defined as $\widehat{\boldsymbol{V}}_r \boldsymbol{X}^*_{rs}$ where $\widehat{\boldsymbol{V}}_r = diag\left\{v_{r(1)}, \ldots, v_{r(n)}\right\}$ (n×n). The total value added exports of origin country r to destination country s amounts to $\boldsymbol{u}_n \widehat{\boldsymbol{V}}_r \boldsymbol{X}^*_{rs} = \boldsymbol{v}_r \boldsymbol{X}^*_{rs}$ where $\boldsymbol{u}_n = (1,1, \ldots, 1)$ (1×n) is an aggregation vector of unities.

Based on Definition 1, Johnson and Noguera (2012), WTO and OECD tried to demonstrate empirical results of the relationship between value added exports and gross exports.

3. Bilateral equivalence between TiVA and value added content of trade: two-country case

3.1. Definition of value added content of gross trade: two-country case

In discussing bilateral relationships between TiVA and value added content in trade, we start with the model with two countries and many sectors. This is rather important because a country's bilateral trade with its partners can always be summarized by that with one aggregate partner, or all partners including the rest of the world.

In the world with two countries (r, s =1, 2), Definition 1 can be written as follows:

Definition 1a. Value added exports in the case with two countries and many sectors. We consider the following output transfer equations for destination country *s=1

$$\begin{pmatrix} \boldsymbol{X}^*_{11} \\ \boldsymbol{X}^*_{21} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \boldsymbol{X}^*_{11} \\ \boldsymbol{X}^*_{21} \end{pmatrix} + \begin{pmatrix} \boldsymbol{Y}_{11} \\ \boldsymbol{Y}_{21} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} \boldsymbol{Y}_{11} \\ \boldsymbol{Y}_{21} \end{pmatrix} \tag{5}$$

and for destination country *s=2

$$\begin{pmatrix} \boldsymbol{X}_{12}^{*} \\ \boldsymbol{X}_{22}^{*} \end{pmatrix} = \boldsymbol{A}\begin{pmatrix} \boldsymbol{X}_{21}^{*} \\ \boldsymbol{X}_{12}^{*} \end{pmatrix} + \begin{pmatrix} \boldsymbol{Y}_{12} \\ \boldsymbol{Y}_{22} \end{pmatrix} = \boldsymbol{B}\begin{pmatrix} \boldsymbol{Y}_{12} \\ \boldsymbol{Y}_{22} \end{pmatrix} \tag{6}$$

where $\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{pmatrix}$, $\boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{A})^{-\mathbf{1}} = \begin{pmatrix} \boldsymbol{B}_{11} & \boldsymbol{B}_{12} \\ \boldsymbol{B}_{21} & \boldsymbol{B}_{22} \end{pmatrix}$.

Then value added exports from origin country 1 to destination country 2 are defined as $\boldsymbol{v}_1\boldsymbol{X}_{12}^{*}$, whereas value added exports from origin country 2 to destination country 1 are defined by $\boldsymbol{v}_2\boldsymbol{X}_{21}^{*}$. $\{\boldsymbol{X}_{12}^{*}, \boldsymbol{X}_{21}^{*}\}$ is uniquely determined. We have to solve two equations (5) and (6) to complete the Definition 1 with two countries.

In the world with two countries and many sectors, value added content of gross trade or "value added in trade," which is proposed by Trefler and Zhu (2010) and followed up by Stehrer (2012, 2013) and Foster-McGregor and Stehrer (2013), can be reformulated as follows:

Definition 2a. Value added contents of gross exports with two countries and many sectors

When we consider the following output transfer equation

$$\begin{pmatrix} \boldsymbol{X}_{12}^{**} \\ -\boldsymbol{X}_{21}^{**} \end{pmatrix} = \boldsymbol{A}\begin{pmatrix} \boldsymbol{X}_{12}^{**} \\ -\boldsymbol{X}_{21}^{**} \end{pmatrix} + \begin{pmatrix} \boldsymbol{E}_{12} \\ -\boldsymbol{E}_{21} \end{pmatrix} = \boldsymbol{B}\begin{pmatrix} \boldsymbol{E}_{12} \\ -\boldsymbol{E}_{21} \end{pmatrix}, \tag{7}$$

then value added content of gross exports from country 1 to 2, $\boldsymbol{E}_{12}$, is defined as $\boldsymbol{v}_1\boldsymbol{X}_{12}^{**}$. Value added content of gross exports from country 2 to 1, $\boldsymbol{E}_{21}$, is defined as $\boldsymbol{v}_2\boldsymbol{X}_{21}^{**}$. $\boldsymbol{X}_{12}^{**}$ is output transfer from country 1 to 2 induced by gross exports from country 1 to 2, $\boldsymbol{E}_{12}$, through the international Leontief inverse $\boldsymbol{B}$, whereas $\boldsymbol{X}_{21}^{**}$ or $-\boldsymbol{X}_{21}^{**}$ is output transfer from country 2 to 1 induced by gross exports from country 2 to 1, $\boldsymbol{E}_{12}$, or country 1's gross imports from country 2, $-\boldsymbol{E}_{21}$ through the international Leontief inverse $\boldsymbol{B}$.

For a productive semi-positive (non-negative and non-zero) matrix $\boldsymbol{A}$, $\boldsymbol{B}$ is unique and semi-positive. Hence, the solution $(\boldsymbol{X}_{12}^{**}, -\boldsymbol{X}_{21}^{**})'$ is also unique for a given non-zero vector $(\boldsymbol{E}_{12}, -\boldsymbol{E}_{21})'$.

3.2. Bilateral equivalence between TiVA and value added content of trade

Equations (5) and (6) can be written as

for destination 1

$$\boldsymbol{X}_{11}^{*} = \boldsymbol{A}_{11}\boldsymbol{X}_{11}^{*} + \boldsymbol{A}_{12}\boldsymbol{X}_{21}^{*} + \boldsymbol{Y}_{11}, \tag{5a}$$

$$\boldsymbol{X}_{21}^{*} = \boldsymbol{A}_{21}\boldsymbol{X}_{11}^{*} + \boldsymbol{A}_{22}\boldsymbol{X}_{21}^{*} + \boldsymbol{Y}_{21}, \tag{5b}$$

and for destination 2

$$\boldsymbol{X}_{12}^{*} = \boldsymbol{A}_{11}\boldsymbol{X}_{12}^{*} + \boldsymbol{A}_{12}\boldsymbol{X}_{22}^{*} + \boldsymbol{Y}_{12}, \tag{6a}$$

$$\boldsymbol{X}_{22}^{*} = \boldsymbol{A}_{21}\boldsymbol{X}_{12}^{*} + \boldsymbol{A}_{22}\boldsymbol{X}_{22}^{*} + \boldsymbol{Y}_{22}. \tag{6b}$$

Note that by definition

$$\boldsymbol{X}_{1}^{*} = \boldsymbol{X}_{11}^{*} + \boldsymbol{X}_{12}^{*}, \ \boldsymbol{X}_{2}^{*} = \boldsymbol{X}_{21}^{*} + \boldsymbol{X}_{22}^{*}.$$

On the other hand, equation (7) can be written as

$$\boldsymbol{X}_{12}^{**} = \boldsymbol{A}_{11}\boldsymbol{X}_{12}^{**} - \boldsymbol{A}_{12}\boldsymbol{X}_{21}^{**} + \boldsymbol{E}_{12} \tag{7a}$$

$$-\boldsymbol{X}_{21}^{**} = \boldsymbol{A}_{21}\boldsymbol{X}_{12}^{**} - \boldsymbol{A}_{22}\boldsymbol{X}_{21}^{**} - \boldsymbol{E}_{21}; \ \boldsymbol{X}_{21}^{**} = -\boldsymbol{A}_{21}\boldsymbol{X}_{12}^{**} + \boldsymbol{A}_{22}\boldsymbol{X}_{21}^{**} + \boldsymbol{E}_{21} \tag{7b}$$

From the definition of gross exports from country r to country s, $\boldsymbol{E}_{rs} = \boldsymbol{A}_{rs}\boldsymbol{X}_{s}^{*} + \boldsymbol{Y}_{rs}\ (s \neq r)$, we have

$$\boldsymbol{E}_{12} = \boldsymbol{A}_{12}\boldsymbol{X}_{2}^{*} + \boldsymbol{Y}_{12} = \boldsymbol{A}_{12}(\boldsymbol{X}_{21}^{*} + \boldsymbol{X}_{22}^{*}) + \boldsymbol{Y}_{12},$$

$$\boldsymbol{E}_{21} = \boldsymbol{A}_{21}\boldsymbol{X}_{1}^{*} + \boldsymbol{Y}_{21} = \boldsymbol{A}_{21}(\boldsymbol{X}_{11}^{*} + \boldsymbol{X}_{12}^{*}) + \boldsymbol{Y}_{21}.$$

Substituting these in equations (7), we have

$$\boldsymbol{X}_{12}^{**} = \boldsymbol{A}_{11}\boldsymbol{X}_{12}^{**} + \boldsymbol{A}_{12}(\boldsymbol{X}_{21}^{*} + \boldsymbol{X}_{22}^{*} - \boldsymbol{X}_{21}^{**}) + \boldsymbol{Y}_{12}, \tag{7a'}$$

$$\boldsymbol{X}_{21}^{**} = \boldsymbol{A}_{21}(\boldsymbol{X}_{11}^{*} + \boldsymbol{X}_{12}^{*} - \boldsymbol{X}_{12}^{**}) + \boldsymbol{A}_{22}\boldsymbol{X}_{21}^{**} + \boldsymbol{Y}_{21}. \tag{7b'}$$

If $\boldsymbol{X}_{22}^{*} = \boldsymbol{X}_{21}^{*} + \boldsymbol{X}_{22}^{*} - \boldsymbol{X}_{21}^{**}$ and $\boldsymbol{X}_{11}^{*} = \boldsymbol{X}_{11}^{*} + \boldsymbol{X}_{12}^{*} - \boldsymbol{X}_{12}^{**}$, that is to say, $\boldsymbol{X}_{12}^{**} = \boldsymbol{X}_{12}^{*}$ and $\boldsymbol{X}_{21}^{**} = \boldsymbol{X}_{21}^{*}$, the solution set $\{\boldsymbol{X}_{12}^{**}, \boldsymbol{X}_{21}^{**}\}$ satisfy both of the equation system of (#6a) and (#6b) and another system of (#7a) and (#7b). Each solution set for two systems is unique. Therefore, in general, $\boldsymbol{X}_{12}^{**} = \boldsymbol{X}_{12}^{*}$ and $\boldsymbol{X}_{21}^{**} = \boldsymbol{X}_{21}^{*}$. This implies the following theorem.

Theorem 1a in the case with two countries and many sectors

Definition 1a is equivalent to Definition 2b.

In fact, Definition 2a is a special case of Stehrer (2012, 2013), starting with, and focusing on, the world with three countries. However, he did not find the theorem at all. Instead, rather surprisingly, he gave different empirical results from ours in Stehrer (2012, Table 1). This may be due to his inconsistent treatment of bilateral trade relations as shown below.

3.3. A numerical example for a two-country case

To make our proposition legible, we would like to provide a numerical example. Table 2 displays an international input-output table with two countries each of which produces a differentiated aggregate product.

Table 2.

A numerical example with two countries and two sectors. (in billion US$)

	Intermediate demand		Final demand			
	country 1	country 2	country 1	country 2	final demand	Output
Country 1	2	6	8	4	12	20
Country 2	8	18	7	7	14	40
Value added	10	16				
Output	20	40				

It follows from Table 2 that

$$\boldsymbol{A} = \begin{pmatrix} 0.1 & 0.15 \\ 0.4 & 0.45 \end{pmatrix},\ (\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{B} = \begin{pmatrix} 1.264 & 0.345 \\ 0.920 & 2.069 \end{pmatrix},\ \boldsymbol{F}_1 = \begin{pmatrix} 8 \\ 7 \end{pmatrix},\ \boldsymbol{F}_2 = \begin{pmatrix} 4 \\ 7 \end{pmatrix}\ .$$

$$\boldsymbol{v} = (0.5 \quad 0.4)\ .$$

Using equations (5) and (6), we immediately reach the result for TiVA:

$$\begin{pmatrix} X_{12}^* \\ X_{22}^* \end{pmatrix} = \boldsymbol{B}\boldsymbol{F}_2 = \begin{pmatrix} 7.471 \\ 18.161 \end{pmatrix};\ \begin{pmatrix} X_{11}^* \\ X_{21}^* \end{pmatrix} = \boldsymbol{B}F_1 = \begin{pmatrix} 12.529 \\ 21.839 \end{pmatrix}.$$

$v_1 X_{12}^* = 0.5 \times 7.471 = 3.736$; $v_2 X_{21}^* = 0.4 \times 21.839 = 8.736$.

On the other hand, for value added content of trade we have:

$$\begin{pmatrix} X_{12}^{**} \\ -X_{21}^{**} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} X_{12}^{**} \\ -X_{21}^{**} \end{pmatrix} + \begin{pmatrix} E_{12} \\ -E_{21} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} E_{12} \\ -E_{21} \end{pmatrix}.$$

From Table 2, $\begin{pmatrix} E_{12} \\ -E_{21} \end{pmatrix} = \begin{pmatrix} 6+4 \\ -(8+7) \end{pmatrix} = \begin{pmatrix} 10 \\ -15 \end{pmatrix}$. Using the above equation, we can easily have the result for value added content of gross trade:

$$\begin{pmatrix} X_{12}^{**} \\ -X_{21}^{**} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} E_{12} \\ -E_{21} \end{pmatrix} = \begin{pmatrix} 7.471 \\ -21.839 \end{pmatrix}.$$

It follows from this that $v_1 X_{12}^{**} = 0.5 \times 7.471 = 3.736$; $v_2 X_{21}^{**} = 0.4 \times 21.839 = 8.736$.

Thus, we can confirm Theorem 1a.

4. Bilateral equivalence between TiVA and value added content of trade: three country-case

4.1. Definition of value added content of gross trade: three-country case

Definition 1b. Value added exports in the case with three countries and many sectors.

We have the following output transfer equations

for destination country *s=1

$$\begin{pmatrix} \boldsymbol{X}_{11}^* \\ \boldsymbol{X}_{21}^* \\ \boldsymbol{X}_{31}^* \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \boldsymbol{X}_{11}^* \\ \boldsymbol{X}_{21}^* \\ \boldsymbol{X}_{31}^* \end{pmatrix} + \begin{pmatrix} \boldsymbol{Y}_{11} \\ \boldsymbol{Y}_{21} \\ \boldsymbol{Y}_{31} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} \boldsymbol{Y}_{11} \\ \boldsymbol{Y}_{21} \\ \boldsymbol{Y}_{31} \end{pmatrix}, \tag{8}$$

for destination country *s=2

$$\begin{pmatrix} \boldsymbol{X}_{12}^* \\ \boldsymbol{X}_{22}^* \\ \boldsymbol{X}_{32}^* \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \boldsymbol{X}_{12}^* \\ \boldsymbol{X}_{22}^* \\ \boldsymbol{X}_{32}^* \end{pmatrix} + \begin{pmatrix} \boldsymbol{Y}_{12} \\ \boldsymbol{Y}_{22} \\ \boldsymbol{Y}_{32} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} \boldsymbol{Y}_{12} \\ \boldsymbol{Y}_{22} \\ \boldsymbol{Y}_{32} \end{pmatrix}, \tag{9}$$

for destination country *s=3

$$\begin{pmatrix} \boldsymbol{X}_{13}^{*} \\ \boldsymbol{X}_{23}^{*} \\ \boldsymbol{X}_{33}^{*} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \boldsymbol{X}_{13}^{*} \\ \boldsymbol{X}_{23}^{*} \\ \boldsymbol{X}_{33}^{*} \end{pmatrix} + \begin{pmatrix} \boldsymbol{Y}_{13} \\ \boldsymbol{Y}_{23} \\ \boldsymbol{Y}_{33} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} \boldsymbol{Y}_{13} \\ \boldsymbol{Y}_{23} \\ \boldsymbol{Y}_{33} \end{pmatrix}, \tag{10}$$

where $\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} & \boldsymbol{A}_{13} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} & \boldsymbol{A}_{23} \\ \boldsymbol{A}_{31} & \boldsymbol{A}_{23} & \boldsymbol{A}_{33} \end{pmatrix}$, $\boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{A})^{-\mathbf{1}} = \begin{pmatrix} \boldsymbol{B}_{11} & \boldsymbol{B}_{12} & \boldsymbol{B}_{13} \\ \boldsymbol{B}_{21} & \boldsymbol{B}_{22} & \boldsymbol{B}_{23} \\ \boldsymbol{B}_{31} & \boldsymbol{B}_{23} & \boldsymbol{B}_{33} \end{pmatrix}$.

Then value added exports from origin country 1 to destination countries 2 and 3 are defined as $\boldsymbol{v}_1\boldsymbol{X}_{12}^{*}$ and $\boldsymbol{v}_1\boldsymbol{X}_{13}^{*}$ respectively. Value added exports from origin country 2 to destination countries 1 and 3 are defined by $\boldsymbol{v}_2\boldsymbol{X}_{21}^{*}$and $\boldsymbol{v}_2\boldsymbol{X}_{23}^{*}$**.** **V**alue added exports from origin country 3 to destination countries 1 and 2 are defined as $\boldsymbol{v}_3\boldsymbol{X}_{31}^{*}$ and $\boldsymbol{v}_3\boldsymbol{X}_{32}^{*}$**.**

Definition 2b. Value added contents of gross trade in the case with three countries and many sectors

We consider following output transfer equations:

for the case that country 1 exports to, and imports from, countries 2 and 3

$$\begin{pmatrix} \boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**} \\ -\boldsymbol{X}_{21}^{**} \\ -\boldsymbol{X}_{31}^{**} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**} \\ -\boldsymbol{X}_{21}^{**} \\ -\boldsymbol{X}_{31}^{**} \end{pmatrix} + \begin{pmatrix} \boldsymbol{E}_{12} + \boldsymbol{E}_{13} \\ -\boldsymbol{E}_{21} \\ -\boldsymbol{E}_{31} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} \boldsymbol{E}_{12} + \boldsymbol{E}_{13} \\ -\boldsymbol{E}_{21} \\ -\boldsymbol{E}_{31} \end{pmatrix}, \tag{11}$$

for the case that country 2 exports to, and imports from, countries 1 and 3

$$\begin{pmatrix} -\boldsymbol{X}_{12}^{**} \\ \boldsymbol{X}_{21}^{**} + \boldsymbol{X}_{23}^{**} \\ -\boldsymbol{X}_{32}^{**} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} -\boldsymbol{X}_{12}^{**} \\ \boldsymbol{X}_{21}^{**} + \boldsymbol{X}_{23}^{**} \\ -\boldsymbol{X}_{32}^{**} \end{pmatrix} + \begin{pmatrix} -\boldsymbol{E}_{12} \\ \boldsymbol{E}_{21} + \boldsymbol{E}_{23} \\ -\boldsymbol{E}_{32} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -\boldsymbol{E}_{12} \\ \boldsymbol{E}_{21} + \boldsymbol{E}_{23} \\ -\boldsymbol{E}_{32} \end{pmatrix}, \tag{12}$$

and for the case that country 3 exports to, and imports from, countries 1 and 2

$$\begin{pmatrix} -\boldsymbol{X}_{13}^{**} \\ -\boldsymbol{X}_{23}^{**} \\ \boldsymbol{X}_{31}^{**} + \boldsymbol{X}_{32}^{**} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} -\boldsymbol{X}_{13}^{**} \\ -\boldsymbol{X}_{23}^{**} \\ \boldsymbol{X}_{31}^{**} + \boldsymbol{X}_{32}^{**} \end{pmatrix} + \begin{pmatrix} -\boldsymbol{E}_{13} \\ -\boldsymbol{E}_{23} \\ \boldsymbol{E}_{31} + \boldsymbol{E}_{32} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -\boldsymbol{E}_{13} \\ -\boldsymbol{E}_{23} \\ \boldsymbol{E}_{31} + \boldsymbol{E}_{32} \end{pmatrix}. \tag{13}$$

Then value added contents of gross exports from origin country 1 to destination countries 2 and 3 are defined as $\boldsymbol{v}_1\boldsymbol{X}_{12}^{**}$ and $\boldsymbol{v}_1\boldsymbol{X}_{13}^{**}$ respectively. Value added contents of gross exports from origin country 2 to destination countries 1 and 3 are defined by $\boldsymbol{v}_2\boldsymbol{X}_{21}^{**}$ and $\boldsymbol{v}_2\boldsymbol{X}_{23}^{**}$respectively. **V**alue added contents of gross exports from origin country 3 to destination countries 1 and 2 are defined as $\boldsymbol{v}_3\boldsymbol{X}_{31}^{**}$ and $\boldsymbol{v}_3\boldsymbol{X}_{32}^{**}$

respectively. It should be noted that for the computation of all bilateral value added contents of gross exports or imports, we need only two of equations (11)-(13).

4.2. Bilateral equivalence between TiVA and value added content of trade

Equations (8)-(10) can be written as

for destination 1

$$\boldsymbol{X}_{11}^{*} = \boldsymbol{A}_{11}\boldsymbol{X}_{11}^{*} + \boldsymbol{A}_{12}\boldsymbol{X}_{21}^{*} + \boldsymbol{A}_{13}\boldsymbol{X}_{31}^{*} + \boldsymbol{Y}_{11}, \tag{8a}$$

$$\boldsymbol{X}_{21}^{*} = \boldsymbol{A}_{21}\boldsymbol{X}_{11}^{*} + \boldsymbol{A}_{22}\boldsymbol{X}_{21}^{*} + \boldsymbol{A}_{23}\boldsymbol{X}_{31}^{*} + \boldsymbol{Y}_{21}, \tag{8b}$$

$$\boldsymbol{X}_{31}^{*} = \boldsymbol{A}_{31}\boldsymbol{X}_{11}^{*} + \boldsymbol{A}_{32}\boldsymbol{X}_{21}^{*} + \boldsymbol{A}_{33}\boldsymbol{X}_{31}^{*} + \boldsymbol{Y}_{31}, \tag{8c}$$

for destination 2

$$\boldsymbol{X}_{12}^{*} = \boldsymbol{A}_{11}\boldsymbol{X}_{12}^{*} + \boldsymbol{A}_{12}\boldsymbol{X}_{22}^{*} + \boldsymbol{A}_{13}\boldsymbol{X}_{32}^{*} + \boldsymbol{Y}_{12}, \tag{9a}$$

$$\boldsymbol{X}_{22}^{*} = \boldsymbol{A}_{21}\boldsymbol{X}_{12}^{*} + \boldsymbol{A}_{22}\boldsymbol{X}_{22}^{*} + \boldsymbol{A}_{23}\boldsymbol{X}_{32}^{*} + \boldsymbol{Y}_{22}, \tag{9b}$$

$$\boldsymbol{X}_{32}^{*} = \boldsymbol{A}_{31}\boldsymbol{X}_{12}^{*} + \boldsymbol{A}_{32}\boldsymbol{X}_{22}^{*} + \boldsymbol{A}_{33}\boldsymbol{X}_{32}^{*} + \boldsymbol{Y}_{31}, \tag{9c}$$

and for destination 3

$$\boldsymbol{X}_{13}^{*} = \boldsymbol{A}_{11}\boldsymbol{X}_{13}^{*} + \boldsymbol{A}_{12}\boldsymbol{X}_{23}^{*} + \boldsymbol{A}_{13}\boldsymbol{X}_{33}^{*} + \boldsymbol{Y}_{13}, \tag{10a}$$

$$\boldsymbol{X}_{23}^{*} = \boldsymbol{A}_{21}\boldsymbol{X}_{13}^{*} + \boldsymbol{A}_{22}\boldsymbol{X}_{23}^{*} + \boldsymbol{A}_{23}\boldsymbol{X}_{33}^{*} + \boldsymbol{Y}_{23}, \tag{10b}$$

$$\boldsymbol{X}_{33}^{*} = \boldsymbol{A}_{31}\boldsymbol{X}_{13}^{*} + \boldsymbol{A}_{32}\boldsymbol{X}_{23}^{*} + \boldsymbol{A}_{33}\boldsymbol{X}_{33}^{*} + \boldsymbol{Y}_{33}. \tag{10c}$$

It follows from (9a) and (10a) that

$$\begin{aligned}\boldsymbol{X}_{12}^{*} + \boldsymbol{X}_{13}^{*} = {} & \boldsymbol{A}_{11}(\boldsymbol{X}_{12}^{*} + \boldsymbol{X}_{13}^{*}) + \boldsymbol{A}_{12}(\boldsymbol{X}_{22}^{*} + \boldsymbol{X}_{23}^{*}) + \boldsymbol{A}_{13}(\boldsymbol{X}_{32}^{*} + \boldsymbol{X}_{33}^{*}) \\ & + \boldsymbol{Y}_{12} + \boldsymbol{Y}_{13}.\end{aligned} \tag{9a'}$$

On the other hand, equation (11) can be written as

$$\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**} = \boldsymbol{A}_{11}(\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**}) - \boldsymbol{A}_{12}\boldsymbol{X}_{21}^{**} - \boldsymbol{A}_{13}\boldsymbol{X}_{31}^{**} + (\boldsymbol{E}_{12} + \boldsymbol{E}_{13}), \tag{11a}$$

$$\boldsymbol{X}_{21}^{**} = -\boldsymbol{A}_{21}(\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**}) + \boldsymbol{A}_{22}\boldsymbol{X}_{21}^{**} + \boldsymbol{A}_{23}\boldsymbol{X}_{31}^{**} + \boldsymbol{E}_{21}, \tag{11b}$$

$$\boldsymbol{X}_{31}^{**} = -\boldsymbol{A}_{31}(\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**}) + \boldsymbol{A}_{32}\boldsymbol{X}_{21}^{**} + \boldsymbol{A}_{33}\boldsymbol{X}_{31}^{**} + \boldsymbol{E}_{31}. \tag{11c}$$

Gross exports are given by

$$\boldsymbol{E}_{12} = \boldsymbol{A}_{12}\boldsymbol{X}_{2}^{*} + \boldsymbol{Y}_{12} = \boldsymbol{A}_{12}(\boldsymbol{X}_{21}^{*} + \boldsymbol{X}_{22}^{*} + \boldsymbol{X}_{23}^{*}) + \boldsymbol{Y}_{12},$$

$$\boldsymbol{E}_{13} = \boldsymbol{A}_{13}\boldsymbol{X}_{3}^{*} + \boldsymbol{Y}_{13} = \boldsymbol{A}_{13}(\boldsymbol{X}_{31}^{*} + \boldsymbol{X}_{32}^{*} + \boldsymbol{X}_{33}^{*}) + \boldsymbol{Y}_{13},$$

$$\boldsymbol{E}_{21} = \boldsymbol{A}_{21}\boldsymbol{X}_1^* + \boldsymbol{Y}_{21} = \boldsymbol{A}_{21}(\boldsymbol{X}_{11}^* + \boldsymbol{X}_{12}^* + \boldsymbol{X}_{13}^*) + \boldsymbol{Y}_{21},$$
$$\boldsymbol{E}_{31} = \boldsymbol{A}_{31}\boldsymbol{X}_1^* + \boldsymbol{Y}_{31} = \boldsymbol{A}_{31}(\boldsymbol{X}_{11}^* + \boldsymbol{X}_{12}^* + \boldsymbol{X}_{13}^*) + \boldsymbol{Y}_{31}.$$

Substituting these equations in equation (11a)-(11c), we have

$$\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**} = \boldsymbol{A}_{11}(\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**}) + \boldsymbol{A}_{12}(\boldsymbol{X}_{21}^* + \boldsymbol{X}_{22}^* + \boldsymbol{X}_{23}^* - \boldsymbol{X}_{21}^{**})$$
$$+ \boldsymbol{A}_{13}(\boldsymbol{X}_{31}^* + \boldsymbol{X}_{32}^* + \boldsymbol{X}_{33}^* - \boldsymbol{X}_{31}^{**}) + \boldsymbol{Y}_{12} + \boldsymbol{Y}_{13}, \tag{11a'}$$
$$\boldsymbol{X}_{21}^{**} = \boldsymbol{A}_{21}\{(\boldsymbol{X}_{11}^* + \boldsymbol{X}_{12}^* + \boldsymbol{X}_{13}^*) - (\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**})\} + \boldsymbol{A}_{22}\boldsymbol{X}_{21}^{**}$$
$$+ \boldsymbol{A}_{23}\boldsymbol{X}_{31}^{**} + \boldsymbol{Y}_{21}, \tag{11b'}$$
$$\boldsymbol{X}_{31}^{**} = \boldsymbol{A}_{31}\{(\boldsymbol{X}_{11}^* + \boldsymbol{X}_{12}^* + \boldsymbol{X}_{13}^*) - (\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**})\} + \boldsymbol{A}_{32}\boldsymbol{X}_{21}^{**}$$
$$+ \boldsymbol{A}_{33}\boldsymbol{X}_{31}^{**} + \boldsymbol{Y}_{31}. \tag{11c'}$$

Let us compare these equations for value added contents of gross exports with those for value added exports.

If $(\boldsymbol{X}_{11}^* + \boldsymbol{X}_{12}^* + \boldsymbol{X}_{13}^*) - (\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**}) = \boldsymbol{X}_{11}^*$, $(\boldsymbol{X}_{11}^* + \boldsymbol{X}_{12}^* + \boldsymbol{X}_{13}^*) - (\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**}) = \boldsymbol{X}_{11}^*$, $\boldsymbol{X}_{21}^* + \boldsymbol{X}_{22}^* + \boldsymbol{X}_{23}^* - \boldsymbol{X}_{21}^{**} = \boldsymbol{X}_{22}^* + \boldsymbol{X}_{23}^*$ and $\boldsymbol{X}_{31}^* + \boldsymbol{X}_{32}^* + \boldsymbol{X}_{33}^* - \boldsymbol{X}_{31}^{**} = \boldsymbol{X}_{32}^* + \boldsymbol{X}_{33}^*$, that is to say, $\boldsymbol{X}_{12}^* + \boldsymbol{X}_{13}^* = \boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**}$, $\boldsymbol{X}_{21}^* = \boldsymbol{X}_{21}^{**}$ and $\boldsymbol{X}_{31}^* = \boldsymbol{X}_{31}^{**}$, the solution set $\{\boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**}, \boldsymbol{X}_{21}^{**}, \boldsymbol{X}_{31}^{**}\}$ satisfy both of the equation system of (8b), (8c) and (9a'), and another one of (11a)-(11c). Each solution set for the two systems is unique. Therefore, $\boldsymbol{X}_{12}^* + \boldsymbol{X}_{13}^* = \boldsymbol{X}_{12}^{**} + \boldsymbol{X}_{13}^{**}$, $\boldsymbol{X}_{21}^* = \boldsymbol{X}_{21}^{**}$ and $\boldsymbol{X}_{31}^* = \boldsymbol{X}_{31}^{**}$. $\boldsymbol{X}_{21}^{**}$ and $\boldsymbol{X}_{31}^{**}$ are uniquely determined while neither of $\boldsymbol{X}_{12}^{**}$ and $\boldsymbol{X}_{13}^{**}$ is yet determined in the above expansion. However, similarly, from equations (12) and (8)-(10), we can easily obtain $\boldsymbol{X}_{21}^* + \boldsymbol{X}_{23}^* = \boldsymbol{X}_{21}^{**} + \boldsymbol{X}_{23}^{**}$, $\boldsymbol{X}_{12}^* = \boldsymbol{X}_{12}^{**}$ and $\boldsymbol{X}_{32}^* = \boldsymbol{X}_{32}^{**}$**.** It also follows from the above that $\boldsymbol{X}_{13}^* = \boldsymbol{X}_{13}^{**}$ and $\boldsymbol{X}_{23}^* = \boldsymbol{X}_{23}^{**}$. $\boldsymbol{X}_{13}^* = \boldsymbol{X}_{13}^{**}$ and $\boldsymbol{X}_{23}^* = \boldsymbol{X}_{23}^{**}$ can also be derived from equations (13) and (8)-(10) because we can directly reach $\boldsymbol{X}_{31}^* + \boldsymbol{X}_{32}^* = \boldsymbol{X}_{31}^{**} + \boldsymbol{X}_{32}^{**}$, $\boldsymbol{X}_{13}^* = \boldsymbol{X}_{13}^{**}$ and $\boldsymbol{X}_{23}^* = \boldsymbol{X}_{23}^{**}$**.** In all, $\boldsymbol{X}_{12}^* = \boldsymbol{X}_{12}^{**}$, $\boldsymbol{X}_{13}^* = \boldsymbol{X}_{13}^{**}$, $\boldsymbol{X}_{21}^* = \boldsymbol{X}_{21}^{**}$, $\boldsymbol{X}_{23}^* = \boldsymbol{X}_{23}^{**}$, $\boldsymbol{X}_{31}^* = \boldsymbol{X}_{31}^{**}$ and $\boldsymbol{X}_{32}^* = \boldsymbol{X}_{32}^{**}$**.** Therefore we can conclude the following theorem.

Theorem 1b in the case with three countries and many sectors
Definition 1b is equivalent to Definition 2b.

4.3. Discussion

Stehrer (2012, 2013) initiated the discussion of value added content of trade based on Trefler and Zhu (2010) whereas he looked at only equation (11) in the world with three countries. Without considering equations (12) and/or (13), he introduced the following definition of bilateral output transfer on which bilateral value added content of trade between country 1 and country 2 is calculated in the world with three countries:

$$\begin{pmatrix} \overline{\boldsymbol{X}}_{12}^{**} \\ -\overline{\boldsymbol{X}}_{21}^{**} \\ \boldsymbol{\Gamma}_{(3,12)}^{**} \end{pmatrix} = \boldsymbol{A}\begin{pmatrix} \overline{\boldsymbol{X}}_{12}^{**} \\ -\overline{\boldsymbol{X}}_{21}^{**} \\ \boldsymbol{\Gamma}_{(3,12)}^{**} \end{pmatrix} + \begin{pmatrix} \boldsymbol{E}_{12} \\ -\boldsymbol{E}_{21} \\ \boldsymbol{O} \end{pmatrix} = \boldsymbol{B}\begin{pmatrix} \boldsymbol{E}_{12} \\ -\boldsymbol{E}_{21} \\ \boldsymbol{O} \end{pmatrix}. \tag{14}$$

$\overline{\boldsymbol{X}}_{12}^{**}$ might denote country 1’s output induced by its exports to country 2. $\overline{\boldsymbol{X}}_{21}^{**}$ might denote country 2’s output induced by its exports to country 1 in the absence of country 1’s output induced by its exports to country 3. $\boldsymbol{\Gamma}_{(3,12)}^{**}$ might be an adjustment term which is not explained in a well-defined-manner. Equation (14) can be written as:

$$\overline{\boldsymbol{X}}_{12}^{**} = \boldsymbol{A}_{11}\overline{\boldsymbol{X}}_{12}^{**} - \boldsymbol{A}_{12}\overline{\boldsymbol{X}}_{21}^{**} + \boldsymbol{A}_{13}\boldsymbol{\Gamma}_{(3,12)}^{**} + \boldsymbol{E}_{12}\,, \tag{14a}$$

$$\overline{\boldsymbol{X}}_{21}^{**} = -\boldsymbol{A}_{21}\overline{\boldsymbol{X}}_{12}^{**} + \boldsymbol{A}_{22}\overline{\boldsymbol{X}}_{21}^{**} - \boldsymbol{A}_{23}\boldsymbol{\Gamma}_{(3,12)}^{**} + \boldsymbol{E}_{21}, \tag{14b}$$

$$\boldsymbol{\Gamma}_{(3,12)}^{**} = \boldsymbol{A}_{31}\overline{\boldsymbol{X}}_{12}^{**} - \boldsymbol{A}_{32}\overline{\boldsymbol{X}}_{21}^{**} + \boldsymbol{A}_{33}\boldsymbol{\Gamma}_{(3,12)}^{**}. \tag{14c}$$

Comparing equations (14a)-(14c) with equations (11a)-(11c), clearly in general $\overline{\boldsymbol{X}}_{21}^{**} \neq \boldsymbol{X}_{21}^{**}$ and hence $\boldsymbol{v}_2\overline{\boldsymbol{X}}_{21}^{**} \neq \boldsymbol{v}_2\boldsymbol{X}_{21}^{**}$ in the world with three countries. That is to say, he gave two different definitions for the concept of value added contents of country 2’s gross exports to country 1 or country 1’s gross imports from country 2 in the world with three countries. His double definitions are prohibited when considering a system with a unique solution set. Definition (11) is always followed by equations (12) and (13). We must consider (12) and/or (13) at the same time whenever we consider equation (11). He might have introduced prohibited double definitions due to the non-determination of $\boldsymbol{X}_{12}^{**}$ and $\boldsymbol{X}_{13}^{**}$ when we look at only equation (11). However, as shown, this problem can be solved by considering at least two of equations (11)-(13) for the definition of bilateral value added contents of gross exports in the system with three countries. His definition (14) of bilateral trade may be accepted only for the world with two countries although he did not examine the case with two countries at all. We only complete

Stehrer's pioneering work by slightly amending his definition of value added content of trade.

4.4. A numerical example for three-country case

Table 3 shows an international input-output table with three countries each of which produces a differentiated aggregate product.

Table 3.

A numerical example with three countries. (in billion US$)

	intermediate demand			final demand				
	country 1	country 2	country 3	country 1	country 2	country 3	final demand	Output
country 1	2	6	3	8	4	7	19	30
country 2	8	18	4	7	7	10	24	54
country 3	5	12	53	5	10	15	30	100
Value added	15	18	40					
Output	30	54	100					

It follows from Table 3 that

$$\boldsymbol{A}=\begin{pmatrix}0.067 & 0.111 & 0.03\\ 0.267 & 0.333 & 0.04\\ 0.167 & 0.222 & 0.53\end{pmatrix},\ \boldsymbol{B}=(\boldsymbol{I}-\boldsymbol{A})^{-1}=\begin{pmatrix}1.152 & 0.223 & 0.092\\ 0.499 & 1.640 & 0.171\\ 0.644 & 0.855 & 2.242\end{pmatrix}.$$

$$\boldsymbol{F}_1=\begin{pmatrix}8\\7\\5\end{pmatrix},\ \boldsymbol{F}_2=\begin{pmatrix}4\\7\\10\end{pmatrix},\ \boldsymbol{F}_3=\begin{pmatrix}7\\10\\15\end{pmatrix}\ \text{and}\ \ \boldsymbol{v}=(0.5\ \ 0.333\ \ 0.4).$$

Using equations (8)-(10), we have

$$\begin{pmatrix}X_{12}^*\\X_{22}^*\\X_{32}^*\end{pmatrix}=\boldsymbol{BF}_2=\begin{pmatrix}7.090\\15.195\\30.975\end{pmatrix};\ \begin{pmatrix}X_{11}^*\\X_{21}^*\\X_{31}^*\end{pmatrix}=\boldsymbol{BF}_1=\begin{pmatrix}11.234\\16.334\\22.345\end{pmatrix};$$

$$\begin{pmatrix}X_{13}^*\\X_{23}^*\\X_{33}^*\end{pmatrix}=\boldsymbol{BF}_3=\begin{pmatrix}11.676\\22.471\\46.680\end{pmatrix}.$$

$v_1X_{12}^*$=0.5×7.090=3.545; $v_2X_{21}^*=0.333\times16.334=5.445$;

$v_1X_{13}^*$=0.5×11.676=5.838; $v_3X_{31}^*=0.4\times22.345$ =8.938;

$v_2X_{23}^{*}$=0.333×22.471=7.490; $v_3X_{32}^{*} = 0.4 \times 30.975 = 12.390$.

On the other hand, from Table 3 and equations (11)-(13) we have the following results of bilateral value added contents of gross trade:

$$\begin{pmatrix} X_{12}^{**} + X_{13}^{**} \\ -X_{21}^{**} \\ -X_{31}^{**} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} E_{12} + E_{13} \\ -E_{21} \\ -E_{31} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} 20 \\ -15 \\ -10 \end{pmatrix} = \begin{pmatrix} 18.766 \\ -16.334 \\ -22.345 \end{pmatrix};$$

$$\begin{pmatrix} -X_{12}^{**} \\ X_{21}^{**} + X_{23}^{**} \\ -X_{32}^{**} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -E_{12} \\ E_{21} + E_{23} \\ -E_{32} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -10 \\ 29 \\ -22 \end{pmatrix} = \begin{pmatrix} -7.090 \\ 38.805 \\ -30.975 \end{pmatrix};$$

$$\begin{pmatrix} -X_{13}^{**} \\ -X_{23}^{**} \\ X_{31}^{**} + X_{32}^{**} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -E_{13} \\ -E_{23} \\ E_{31} + E_{32} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} -10 \\ -14 \\ 32 \end{pmatrix} = \begin{pmatrix} -11.676 \\ -22.471 \\ 53.320 \end{pmatrix}.$$

Clearly,

$$X_{12}^{*} = X_{12}^{**},\ X_{13}^{*} = X_{13}^{**},\ X_{21}^{*} = X_{21}^{**},\ X_{23}^{*} = X_{23}^{**},\ X_{31}^{*} = X_{31}^{**},\ X_{32}^{*} = X_{32}^{**}.$$

Hence

$$v_1\boldsymbol{X}_{12}^{*} = v_1\boldsymbol{X}_{12}^{**},\ v_1\boldsymbol{X}_{13}^{*} = v_1\boldsymbol{X}_{13}^{**},\ v_2\boldsymbol{X}_{21}^{*} = v_2\boldsymbol{X}_{21}^{**},\ v_2\boldsymbol{X}_{23}^{*} = v_2\boldsymbol{X}_{23}^{**},$$

$$v_3\boldsymbol{X}_{31}^{*} = v_3\boldsymbol{X}_{31}^{**},\ v_3\boldsymbol{X}_{32}^{*} = v_3\boldsymbol{X}_{32}^{**}.$$

Theorem 1b is verified.

Stehrer's definition of bilateral output transfers between country 1 and country 2 can be calculated as follows:

$$\begin{pmatrix} \bar{X}_{12}^{**} \\ -\bar{X}_{21}^{**} \\ \Gamma_{(3,12)}^{**} \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} E_{12} \\ -E_{21} \\ O \end{pmatrix} = \boldsymbol{B} \begin{pmatrix} 10 \\ -15 \\ 0 \end{pmatrix} = \begin{pmatrix} 8.175 \\ -19.613 \\ -6.374 \end{pmatrix}.$$

$\bar{X}_{21}^{**} = 19.613 > X_{21}^{**}$=16.334 and $\bar{X}_{12}^{**}$= 8.175 > X_{12}^{**}=7.090. The definition of bilateral value added content of trade based on the above output transfers contradicts his initial definition of value added content of trade, shown by equation (11).

5. A generalization

A general definition of TiVA was given as Definition 1 in section 2.2 of this paper. Corresponding to Definition 1, we can provide a general definition of value added content of trade in the world with many countries and many sectors as follows:

Definition 2. Value added contents of gross trade in the case with many countries and many sectors

We consider following *R* output transfer equations:

for the case that country 1 exports to, and imports from, countries 2, 3, …, *R*

$$\begin{pmatrix} \boldsymbol{X}_{12}^{**}+\cdots+\boldsymbol{X}_{1R}^{**} \\ -\boldsymbol{X}_{21}^{**} \\ \dots \\ -\boldsymbol{X}_{R1}^{**} \end{pmatrix} = \boldsymbol{A}\begin{pmatrix} \boldsymbol{X}_{12}^{**}+\cdots+\boldsymbol{X}_{1R}^{**} \\ -\boldsymbol{X}_{21}^{**} \\ \dots \\ -\boldsymbol{X}_{R1}^{**} \end{pmatrix} + \begin{pmatrix} \boldsymbol{E}_{12}+\cdots+\boldsymbol{E}_{1R} \\ -\boldsymbol{E}_{21} \\ \dots \\ -\boldsymbol{E}_{R1} \end{pmatrix}$$

$$= \boldsymbol{B}\begin{pmatrix} \boldsymbol{E}_{12}+\cdots+\boldsymbol{E}_{1R} \\ -\boldsymbol{E}_{21} \\ \dots \\ -\boldsymbol{E}_{R1} \end{pmatrix},$$

………………………………………………………………….

for the case that country *r* exports to, and imports from, countries 1, …, *s*,…, *R* (*s*≠*r*)

$$\begin{pmatrix} -\boldsymbol{X}_{1r}^{**} \\ \dots \\ \boldsymbol{X}_{r1}^{**}+\cdots+\boldsymbol{X}_{rR}^{**} \\ \dots \\ -\boldsymbol{X}_{Rr}^{**} \end{pmatrix} = \boldsymbol{A}\begin{pmatrix} -\boldsymbol{X}_{1r}^{**} \\ \dots \\ \boldsymbol{X}_{r1}^{**}+\cdots+\boldsymbol{X}_{rR}^{**} \\ \dots \\ -\boldsymbol{X}_{Rr}^{**} \end{pmatrix} + \begin{pmatrix} -\boldsymbol{E}_{1r} \\ \dots \\ \boldsymbol{E}_{r1}+\cdots+\boldsymbol{E}_{rR} \\ \dots \\ -\boldsymbol{E}_{Rr} \end{pmatrix}$$

$$= \boldsymbol{B}\begin{pmatrix} -\boldsymbol{E}_{1r} \\ \dots \\ \boldsymbol{E}_{r1}+\cdots+\boldsymbol{E}_{rR} \\ \dots \\ -\boldsymbol{E}_{Rr} \end{pmatrix},$$

………………………………………………………………….

and finally for the case that country R exports to, and imports from, countries 1,…, r, …, R-1 ($r≠R$)

$$\begin{pmatrix} -\boldsymbol{X}_{1R}^{**} \\ \cdots \\ -\boldsymbol{X}_{rR}^{**} \\ \cdots \\ \boldsymbol{X}_{R1}^{**}+\cdots+\boldsymbol{X}_{R,R-1}^{**} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} -\boldsymbol{X}_{1R}^{**} \\ \cdots \\ -\boldsymbol{X}_{rR}^{**} \\ \cdots \\ \boldsymbol{X}_{R1}^{**}+\cdots+\boldsymbol{X}_{R,R-1}^{**} \end{pmatrix} + \begin{pmatrix} -\boldsymbol{E}_{1R} \\ \cdots \\ -\boldsymbol{E}_{rR} \\ \cdots \\ \boldsymbol{E}_{R1}+\cdots+\boldsymbol{E}_{R,R-1} \end{pmatrix}$$

$$= \boldsymbol{B} \begin{pmatrix} -\boldsymbol{E}_{1R} \\ \cdots \\ -\boldsymbol{E}_{rR} \\ \cdots \\ \boldsymbol{E}_{R1}+\cdots+\boldsymbol{E}_{R,R-1} \end{pmatrix}.$$

Then value added content of gross exports from origin country r to destination country s is defined as $\boldsymbol{v}_r\boldsymbol{X}_{rs}^{**}$.

Using the same method as for the case with three countries, we can easily prove that $\boldsymbol{X}_{rs}^{**}=\boldsymbol{X}_{rs}^{*}$ and $\boldsymbol{v}_r\boldsymbol{X}_{rs}^{**}=\boldsymbol{v}_r\boldsymbol{X}_{rs}^{*}$ for r, s=1, 2,…., R ($s≠r$). Therefore, we can arrive at the following equivalence theorem:

Theorem 1.
Definition 1 is equivalent to Definition 2.

6. Empirical results

Applying some aggregated versions of WIOD of Groningen University (Timmer *et al*. (2012)) to equations for definitions of value added content of gross trade, we could have the completely same results as in Kuboniwa (2014a, 2014b) for trade in value added. The results are totally identical and computation processes using matrix-vector algebra with more than 70×70 and 70×1 dimensions are too large to display here. As upon request details of calculation processes are available from the author, here, for comparison with Stehrer (2012), we would like to show only Table 4 of China's trade with ROW (aggregating other 39 countries and the rest of the world in the original data) for 2005. Based on Table 4 we have Table 5. Table 5 shows that our result for value added content or VAiT is completely different from that in Stehrer (2012).

Table 4.
China's trade with the rest of the world in 2005

(in billion US$)

		Gross trade			Trade in value added TiVA			Value added content of trade "Value added in trade		
		exports	imports	balance	exports	imports	balance	exports	imports	balance
1	Agriculture	8.2	20.1	-12.0	48.0	14.1	33.9	48.0	14.1	33.9
2	Mining	10.0	72.3	-62.3	38.3	53.2	-15.0	38.3	53.2	-15.0
3	Food	20.1	11.9	8.1	15.2	5.0	10.2	15.2	5.0	10.2
4	Textiles	108.9	14.4	94.5	42.0	3.5	38.5	42.0	3.5	38.5
5	Leather	22.7	4.5	18.3	7.2	1.0	6.2	7.2	1.0	6.2
6	Wood	5.7	2.5	3.2	5.2	1.7	3.5	5.2	1.7	3.5
7	Pulp and paper	4.3	8.7	-4.4	10.4	7.3	3.1	10.4	7.3	3.1
8	Coke and oil products	7.9	14.8	-6.9	9.0	5.8	3.2	9.0	5.8	3.2
9	Chemicals	36.0	75.8	-39.7	31.3	25.3	5.9	31.3	25.3	5.9
10	Rubber and plastics	25.4	8.8	16.6	15.5	6.4	9.2	15.5	6.4	9.2
11	Other non-metallic mine	10.4	4.4	6.0	10.0	3.6	6.4	10.0	3.6	6.4
12	Basic metal products	53.1	54.1	-1.0	43.3	32.5	10.8	43.3	32.5	10.8
13	Machinery, nec	43.6	63.0	-19.4	20.8	23.1	-2.4	20.8	23.1	-2.4
14	Electrical equipment.	296.9	222.4	74.5	70.4	62.0	8.5	70.4	62.0	8.5
15	Transport Equipment	24.6	24.4	0.3	12.0	9.2	2.8	12.0	9.2	2.8
16	Manufacturing nec	22.7	6.2	16.5	9.4	2.9	6.5	9.4	2.9	6.5
17	Electricity, gas & water	1.2	0.9	0.2	25.0	8.3	16.7	25.0	8.3	16.7
18	Construction	3.2	2.5	0.7	1.2	3.4	-2.2	1.2	3.4	-2.2
19	Sale & repair of motor vehicles	0.0	0.2	-0.2	0.0	3.1	-3.1	0.0	3.1	-3.1
20	Wholesale trade	37.3	4.7	32.5	37.0	23.6	13.4	37.0	23.6	13.4
21	Retail trade	7.7	0.8	6.9	16.8	11.6	5.2	16.8	11.6	5.2
22	Hotels & restaurants	7.8	1.2	6.6	9.4	3.3	6.1	9.4	3.3	6.1
23	Inland transport	7.6	6.1	1.6	20.3	10.7	9.6	20.3	10.7	9.6
24	Water transport	16.2	0.8	15.3	13.1	1.6	11.5	13.1	1.6	11.5
25	Air transport	11.5	5.0	6.5	3.9	2.3	1.6	3.9	2.3	1.6
26	Other transport activitie:	2.6	0.3	2.3	3.4	4.5	-1.1	3.4	4.5	-1.1
27	Communications	4.5	4.2	0.3	10.5	7.3	3.2	10.5	7.3	3.2
28	Financial intermediation	0.6	2.5	-1.9	17.5	16.2	1.3	17.5	16.2	1.3
29	Real estate	0.0	0.1	-0.1	6.9	8.7	-1.8	6.9	8.7	-1.8
30	Renting of M&Eq	25.5	23.0	2.5	27.2	47.6	-20.5	27.2	47.6	-20.5
31	Public administration	0.5	3.3	-2.8	0.4	3.4	-3.0	0.4	3.4	-3.0
32	Education	0.3	0.8	-0.5	1.5	1.4	0.0	1.5	1.4	0.0
33	Health and social work	0.0	0.1	-0.1	1.2	0.4	0.8	1.2	0.4	0.8
34	Other services	9.9	5.3	4.6	10.3	7.9	2.4	10.3	7.9	2.4
35	Private house employ	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Total excluding dummy	836.7	670.1	166.7	593.7	422.1	171.6	593.7	422.1	171.6
	Dummy sector				13.7	18.6	-4.9	13.7	18.6	-4.9
	Total	836.7	670.1	166.7	607.4	440.7	166.7	607.4	440.7	166.7

Sources: Author's calculation using WIOD for 2005.

For dummy sector, see kuboniwa (2014a, 2014b).

Table 5.

Totals of China's gross trade, trade in value added and value added content of trade (value added in trade) in 2005

(in billion US$)

	Gross trade			Trade in value added TiVA excluding dummy			Value added content of trade "Value added in trade (VAiT)" excluding dummy		
	exports	imports	net	exports	imports	net	exports	imports	net
Stehrer (2012)	836.7	669.8	166.9	592.0	421.1	170.9	813.9	643.0	170.9
Kuboniwa	836.7	670.1	166.7	593.7	422.1	171.6	593.7	422.1	171.6

Sources: Stehrer (2012, Table 1) and Table 4.

7. Concluding remarks

We developed a definition of bilateral value added content of gross trade, based on Trefler and Zhu (2010) and Stehrer (2012, 2013), along with Johnson and Noguera (2012)'s definition of TiVA. Two definitions look different. However, we proved that these two definitions are equivalent. This implies that we provided an alternative definition and equation system to compute TiVA. We can use final demand vector by destination, excluding intermediates, as in Johnson and Noguera (2012), and WTO and OECD (website for TiVA). We can also employ the gross exports-imports vector by destination, including intermediates, if the trade vector is appropriately arranged in Trefler and Zhu (2010)'s manner. An international Leontief inverse is the common multiplier device for the two definitions.

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