

# Chapter 3 Individual Income Taxation

## 3.1 Introduction

The individual income tax is the most important single tax in many countries. The basic principle of the individual income tax is that the taxpayer's income from all sources should be combined into a single or global measure of income. Total income is then reduced by certain exemptions and deductions to arrive at income subject to tax. This is the base to which tax rates are applied when computing tax.

A degree and coverage of exemptions and deductions vary from country to country. A degree of progressivity of tax rates also varies.

Nevertheless the underlying principles of the tax system are common among countries and are worth reviewing.

## 3.2 The Income-Based Principle

Economists have argued that a *comprehensive* definition of income must be used that includes not only cash income but capital gains. A number of other adjustments have to be made to convert your "cash" income into the "comprehensive" income that, in principle, should form the basis of taxation.

This comprehensive definition of income is referred to as the Hicksian concept or the Haig-Simons concept. This concept measures most accurately reflects "*ability to pay*".

(1) *Cash basis*: In practice, only *cash-basis* market transactions are taxed. The tax is thus levied on a notion of income that is somewhat narrower than that which most economists would argue. Certain non-marketed (non-cash) economic activities are excluded, though identical activities in the market are subject to taxation (e.g. housewife's work at home (*vis-à-vis* a maid's work), and own house (*vis-à-vis* rented house)).

Some non-cash transactions are listed in the tax code but are difficult to enforce. *Barter arrangements* are subject to tax.

*Unrealized capital gains* is also not included in the income tax bases. Capital gains are taxed only when the asset is sold (not on an *accrual* basis).

(2) *Equity-based adjustments*: Individuals who have large medical expenses or casualty losses are allowed to deduct a portion of those expenses from their income, presumably

on the grounds that they are not in as good a position for paying taxes as someone with the same income without those expenses.

- (3) *Incentive-based adjustments*: The tax code is used to encourage certain activities by allowing tax credits or deductions for those expenditures. Incentives are provided for energy conservation, for investment, and for charitable contributions.
- (4) *Special Treatment of Capital Income*: The tax laws treat capital and wage income separately. The difficulty of assessing the magnitude of the returns to capital plays some role, while attempts to encourage savings as a source of domestic investment and growth.

## The Progressivity Principle<sup>1</sup>

Even the simplification of tax schedule prevails among countries, the premise remains that those with higher incomes not only should pay more but should pay a larger fraction of their income in taxes. In other words, progressivity is reflected in an increase not only in average rates but in marginal rates.

Defining progressive and regressive is not easy and, unfortunately, ambiguities in definition sometimes confuse public debate. A natural way to define these words is in terms of the **average tax rate**, the ratio of taxes paid to income. If the average tax rate increases with income, the system is **progressive**; if it falls, the tax is **regressive**.

Confusion arises because some people think of progressiveness in terms of the **marginal tax rate** – the *change* in taxes paid with respect to a change in income. To illustrate the distinction, consider the following very simple income tax structure. Each individual computes her tax bill by subtracting \$3,000 from income and paying an amount equal to 20 percent of the remainder. (If the difference is negative, the individual gets a subsidy equal to 20 percent of the figure.)

**Table 1 Tax Liabilities under a Hypothetical Tax System**

Income	Tax Liability	Average Tax Rate	Marginal Tax Rate
2,000	-200	-0.10	0.2
3,000	0	0	0.2
5,000	400	0.08	0.2
10,000	1,400	0.14	0.2
30,000	5,400	0.18	0.2

Table 1 shows the amount of tax paid, the average tax rate, and the marginal tax rate for each of several income levels. The average rates increase with income. However, the marginal tax rate is constant at 0.2 because for each additional dollar earned, the individual pays an additional 20 cents, regardless of income level. People could disagree about the

<sup>1</sup> This Part draws heavily from Rosen (1999), pp.258-260.

progressiveness of this tax system and each be right according to their own definitions. It is therefore very important to make the definition clear when using the terms *regressive* and *progressive*. In the remainder of this section, we assume they are defined in terms of average tax rates.

### The degree of progression

Progression in the income tax schedule introduces disproportionality into the distribution of the tax burden and exerts a redistributive effect on the distribution of income. In order to explore these properties further, we need to be able to measure the degree of income tax progression along the income scale. Such measures are called measures of *structural progression* (sometimes, measures of *local progression*). There is more than one possibility, as we will see. Each such measure will induce a partial ordering on the set of all possible income tax schedules. We could not expect always to be able to rank a schedule  $t_2(x)$  unambiguously more, or less, structurally progressive than another schedule  $t_1(x)$ : we must allow that a schedule could display more progression in one income range and less in another.

Nevertheless, the policymaker and tax practitioner, and indeed the man in the street, would like to be able to say which of any two alternative income tax systems is the more progressive *in its effects*. Is the federal income tax in the USA more redistributive than the personal income tax in Germany? This sort of question will take us from measures of structural progression to measures of *effective progression*. Measuring effective progression is a matter of reducing a tax schedule and income distribution pair to a scalar index number. The same schedule  $t(x)$  could be more progressive *in effect* when applied to distribution A than to distribution B. Trends in effective progression for a given country over time, as well as differences between the income taxes of different countries, can be examined using such index numbers.

Let us begin by defining as  $m(x)$  and  $a(x)$  respectively the marginal and average rates of tax experienced by an income  $x$ :

$$a(x) = \frac{t(x)}{x} \quad \text{and} \quad m(x) = t'(x) \tag{1}$$

Since

$$\frac{d[t(x)]/x}{dx} = \frac{xt'(x) - t(x)}{x^2} = \frac{m(x) - a(x)}{x} \tag{2}$$

for strict progression it is necessary and sufficient that

$$m(x) > a(x) \text{ for all } x \quad (3)$$

The strict inequality rules out the possibility that the tax could be proportional to income in any interval: we may relax it if we wish. Measures of structural progression quantify, in various ways, the excess of the marginal rate  $m(x)$  over the average rate  $a(x)$  at income level  $x$ . We introduce two particularly important measures here.

First, *liability progression*  $LP(x)$  is defined at any income level  $x$  for which  $t(x) > 0$  as the elasticity of tax liability to pre-tax income:

$$LP(x) = e^{t(x),x} = \frac{xt'(x)}{t(x)} = \frac{m(x)}{a(x)} > 1 \quad (4)$$

As we have already noted, for a strictly progressive income tax a 1 percent increase in pre-tax income  $x$  leads to an increase of more than 1 percent in tax liability.  $LP(x)$  measures the actual percentage increase experienced. A change of tax schedule which, for some  $x_0$ , causes an increase in  $LP(x_0)$  connotes, in an obvious sense, an increase in progression at that income level  $x_0$ . If the change in a strictly positive income tax involves an upward shift of the entire function  $LP(x)$ , then the tax has become everywhere more liability progressive.

Second, *residual progression*  $RP(x)$  is defined at all income levels  $x$  as the elasticity of post-tax income to pre-tax income:

$$RP(x) = e^{x-t(x),x} = \frac{x[1-t'(x)]}{x-t(x)} = \frac{1-m(x)}{1-a(x)} < 1 \quad (5)$$

The counterpart to the observation above is that a 1 percent increase in pretax income  $x$  leads to an increase of less than 1 percent in post-tax income.  $RP(x)$  measures the actual percentage increase in post-tax income. A *reduction* in  $RP(x)$  must clearly be interpreted as an *increase in progression*, according to this measure.

These measures were first proposed by Musgrave and Thin (1948). It is inconvenient that  $RP(x)$  should decrease when the tax becomes more progressive. We therefore make a minor change of definition in this book, which is not standard in all of the literature, replacing the measure  $RP(x)$  by its reciprocal:

$$RP^*(x) = \frac{1}{RP(x)} = \frac{1-a(x)}{1-m(x)} > 1 \quad (5a)$$

$RP^*(x)$  can be interpreted as the elasticity of pre-tax income to post-tax income. An increase in  $RP^*(x)$  makes the tax more residually progressive at  $x$ .

Each of these measures quantifies in its own way the excess of the marginal rate of tax over the average rate of tax at the income level  $x$ , and accordingly induces a different partial ordering on the set of all tax schedules. Equipped with these measures, we shall, in the next two sections of the chapter, be able to demonstrate that if the income tax becomes structurally more progressive, and the pre-tax income distribution does not change, then this implies enhanced deviation from proportionality (in the case of liability progression) and enhanced redistributive effect (in the case of residual progression).

### 3.3 The Individual or Family-Based Principle: Choice of Unit<sup>2</sup>

The basic unit of taxation can be either the individual or the family. Many countries adopt the individual-based principle while some adopt the family-based principle.

#### *Background*

To begin, it is useful to consider the following three principles:

1. The income tax should embody increasing marginal tax rates.
2. Families with equal incomes should, other things being the same, pay equal taxes.
3. Two individuals' tax burdens should not change when they marry; the tax system should be **marriage neutral**.

**Table 2 Tax liabilities under a hypothetical tax system**

	Individual Income	Individual Tax	Family Tax with Individual Filing	Joint Income	Joint Tax
Lucy	1,000	100	12,200	30,000	12,600
Ricky	29,000	12,100			
Ethel	15,000	5,100	10,200	30,000	12,600
Fred	15,000	5,100			

Although a certain amount of controversy surrounds the second and third principles, it is probably fair to say they reflect a broad consensus as to desirable features of a tax system. While agreement on the first principle is weaker, increasing marginal tax rates seem to have wide political support.

Despite the appeal of these principles, a problem arises when it comes to implementing them: In general, no tax system can adhere to all three simultaneously. This point is made most

<sup>2</sup> This Part draws heavily on Rosen (1999, Chap.16, pp.363-5).

easily with an arithmetic example. Consider the following simple progressive tax schedule: a taxable unit pays in tax 10 percent of all income up to \$6,000, and 50 percent of all income in excess of \$6,000. The first two columns of Table 2 show the incomes and tax liabilities of four individuals, Lucy, Ricky, Fred, and Ethel. (For example, Ricky's tax liability is \$12,100 [ $.10 \times \$6,000 + .50 \times \$23,000$ ].) Now assume that romances develop – Lucy marries Ricky, and Ethel marries Fred. In the absence of joint filing, the tax liability of each individual is unchanged. However, two families with the same income (\$30,000) will be paying different amounts of tax. (The Lucy-Rickys pay \$12,200 while the Ethel-Freds pay on \$10,200, as noted in the third column.) Suppose instead that the law views the family as the taxable unit, so that the tax schedule applies to joint income. In this case, the two families pay equal amounts of tax, but now tax burdens have been changed by marriage. Of course, the actual change in the tax burden depends on the difference between the tax schedules applied to individual and joint returns. This example has assumed for simplicity that the schedule remains unchanged. But it does make the main point: given increasing marginal tax rates, we cannot have both principles 2 and 3.

What choice has the United States made? Over time, the choice has changed. Before 1948, the taxable unit was the individual, and principle 2 was violated. In 1948, the family became the taxable unit, and simultaneously income splitting was introduced. Under income splitting, a family with an income of \$50,000 is taxed as if it were two individuals with incomes of \$25,000. Clearly, with increasing marginal tax rates, this can be a major advantage. Note also that under such a regime, an unmarried person with a given income finds his or her tax liability reduced substantially if he or she marries a person with little or no income. Indeed, under the 1948 law, it was possible for an individual's tax liability to fall drastically when the person married – a violation of principle 3.

The differential between a single person's tax liability and that of a married couple with the same income was so large that Congress created a new schedule for unmarried people in 1969. Under this schedule, a single person's tax liability could never be more than 20 percent higher than the tax liability of a married couple with the same taxable income. (Under the old regime, differentials of up to 40 percent were possible.)

Unfortunately, this decrease in the single/married differential was purchased at the price of a violation of principle 3 in the opposite direction: it was now possible for person's tax liabilities to increase when they married. In effect, the personal income tax levied a tax on marriage. In 1981, congress attempted to reduce the "marriage tax" by introducing a new deduction for two-earner married couples. Two-earner families received a deduction equal to 10 percent of the lower earning spouse's wage income, but no more than \$3,000. However, the two-earner deduction was eliminated by The Tax Reform Act of 1986 (TRA86). It was deemed to be

unnecessary because lower marginal tax rates reduced the importance of the “marriage tax.”

Nevertheless, a substantial penalty still exists, and it tends to be highest when both spouses have similar earnings. Under certain conditions, for example, when two individuals with \$25,000 adjusted gross income (AGIs) marry, their joint tax liability can increase by more than \$700. On the other hand, when there are considerable differences in individuals’ earnings, the tax code provides a bonus for marriage. If two people with \$10,000 and \$50,000 AGIs marry, their joint tax liability can decrease by \$1,100. In cases like these, the law provides a “tax dowry.”

### 3.4 The Annual Measure of Income Principle

Income tax is usually based on *annual income*, not *lifetime income*. Economic theory is usually based on the lifetime utility maximization (e.g. life-cycle hypothesis), with the current annual income taxation, consumption smoothing does not avoid tax distortion. Because of the progressive nature of our tax system, the individual with the fluctuating income has to pay more taxes over his lifetime than the individual with a steady income.

## The Basic Framework

### 1) The Criteria for Optimality

Musgrave in his classic text book (*The Theory of Public Finance* (1959), McGraw-Hill) provides three criteria for appropriate taxation;

- (a) the need for taxes to be *fair*;
- (b) the need to *minimize administrative costs*; and
- (c) the need to *minimize disincentive effects*.

The approach of the optimal taxation literature is to use economic analysis to combine the criteria into one, implicitly deriving the relative weights that should be applied to each criterion. This is done by using the concepts of individual utility and social welfare.

Economists found it very difficult to model the relationship between tax rates and administrative costs. They usually ignored administrative costs in their analysis and concentrated on criteria (a) and (c)<sup>3</sup>. Effectively, they tried to determine the tax system that will provide the best compromise between *equality (fairness)* and *efficiency (incentives)*.

### 2) The Specification of Social Welfare

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<sup>3</sup> It is possible to include administrative costs in the tax analysis both in theory and in empirical investigations. It is rather important to take administration into account in tax analysis.

As we discussed before, social welfare function can take many forms as policy makers have different policy objectives and welfare criteria.

If our idea of a fair tax system is one that reduces inequality of utility, our social welfare function must place more weight on utility gains of poor people than those of rich people.

This is achieved by using the following formulae,

$$\begin{aligned}
 w &= \frac{1}{1-\varepsilon} \sum_h (u^h)^{1-\varepsilon} && \text{for } \varepsilon \neq 1 \\
 &= \sum_h \log(u^h) && \text{for } \varepsilon = 1
 \end{aligned} \tag{6}$$

where  $\varepsilon$  is a degree of inequality aversion.

### 3) The Modeling of Disincentives

In case of optimal income taxation in a model where labor supply response is the only disincentive problem, the utility function for each individual is used both to predict how that person will alter his/her labor supply when taxes are changed and to evaluate the resulting level of individual utility. The changes in labor supply will then be used to calculate the change in tax revenue, while the changes in utility will be used to calculate the change in social welfare. The optimal tax system will be the one where it is impossible to increase social welfare without reducing overall tax revenue.

The requirement to raise a specific amount of tax revenue is obviously fundamental. It has two important implications. First, it means that the solution to the optimal tax problem depends on the size of the revenue requirement. Second, it means that the tax changes that are considered should be revenue-neutral.

Why does it matter that a higher tax rate with higher personal allowances will reduce labor supply? After all, the objective is to maximize social welfare, not the size of the national income. The answer is that, by choosing to work less on average, workers will have lower incomes and thus will pay less taxes. Thus a change that would have been revenue-neutral for a fixed level of labor supply will, as a result of the reduction in work, produce a revenue loss. It is this revenue loss that represents the 'excess burden' of taxation. It requires an increase in tax rates to offset it – an increase that will reduce social welfare and counteract, at least in part, the gain is social welfare from the reduction in inequality that is produced by the increase in tax progressivity.

### 4) Problems of Application

The usefulness of the optimal tax results depends on the realism of the economic models. This is not to say that the presence of any unrealistic assumption invalidates the results. Rather,



any practical application of theoretical analysis requires an evaluation of whether any violation of the assumption can be expected to alter the results significantly.

Examples of unrealistic assumptions:

- (a) neglect of administrative and compliance costs of tax collection.
- (b) Assumption of perfect competition
- (c) neglect of heterogeneity of households in terms of their composition and preferences

What, then, the modern theory of income taxation ought to be concerned?

1. It must capture the efficiency/equity trade off involved in income taxation.
2. The structure of the income tax must be compatible with the revelation of the ability of households.

In a simpler term, the fundamental policy issue is whether it would be a good idea to increase the rate of income tax and use the proceeds to fund an increase in tax allowances, thus reducing after-tax income inequality.

### 3.5 Optimal Income taxation<sup>4</sup>

A linear income tax schedule is one possibility for choosing an income tax. More generally, however, the structure of personal income taxation need not be linear with a constant rate of taxation, but rather marginal tax rates can vary with different levels of income. This is the case in practice.

Suppose that we set out to determine the general optimal income tax structure that maximizes a social welfare function. We are then looking for a relationship between the tax rate and earned income that could in principle be progressive or regressive. If the rate of taxation,  $t$ , were to change with every change in income, we would be looking for an optimal income schedule

$$t = f(Y) \tag{7}$$

If a linear income tax schedule were to maximize social welfare, this would be revealed as the solution to the general problem of optimal personal income taxation.

We can expect the quest to identify a general structure of optimal income taxation as expressed by (7) to be complicated. In the case of the linear income tax, we had to find values for income subsidy  $S$  and  $t$  related through the government budget. In the case of the tax schedule (7), we are looking for a relationship between the tax rate and income that maximizes social welfare. That is, the solution to the general optimal taxation problem is a function that

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<sup>4</sup> This part quotes Hillman (2003, Chap.7, pp.479-83.)

tells us how to set the tax rate  $t$  for all values of income  $Y$ . We have seen that there are cases for both progressive and regressive taxes, and we therefore do not know beforehand whether the relationship expressed in (7) will indicate progressive or regressive taxation. We do know that finding an optimal income tax schedule requires a compromise between the progressive taxation sought for reasons of social justice (through the equal-sacrifice principle) and the efficiency and tax-base benefits of regressive taxation.

Through the substitution effect between work (and effort) and leisure, progressive taxation increases the leaks in the redistributive bucket. We can return to our example of the dentist who responds to increasingly greater marginal tax rates by stopping work and heading for the golf course. The substitution effect places an excess burden of taxation on the dentist, but also we saw that the dentist is led by progressive taxes to take personal utility in the form of leisure rather than in the form of earned income. Because leisure cannot be taxed, the beneficiaries of the income transfers financed by the dentist's taxes have reason to want the dentist to keep working and earning taxable income. In deciding on a tax schedule, an important question is therefore, how do taxpayers respond in their work and leisure decisions to the degree of progressivity or regressivity in the income tax schedule? The answer to this question determines the efficiency losses (through the excess burden of taxation) that are required to be incurred for the sake of social justice defined as a more equal post-tax income distribution. The answer tells us how far inside the efficient frontier a society has to go to approach greater post-tax income equality.

In choosing a social welfare function, society can stress efficiency or social justice (expressed as a preference for post-tax equality). We have seen that society's choice of social welfare function correspondingly expresses the society's aversion to risk and determines whether social insurance is complete (with Rawls) or incomplete (with Bentham and other formulations of social welfare).

The extent of inefficiency, or the leak in the bucket of redistribution through the response of taxpayers to progressivity or regressivity in the income tax schedule, is an empirical matter. We need to be able to observe labor-supply behavior to determine how people respond to taxes. The choice of the social welfare function to be maximized is an ideological issue. Some economists and political decision makers stress the desirability of social justice with little concern for efficiency (they are followers of Rawls) and want highly progressive income taxes. Others (who are closer to Bentham) stress the desirability of efficiency and want low marginal income tax rates or flat tax rates.

Although labor-supply behavior is empirically determined, different people often have different views or priorities about how labor-supply decisions respond to taxes. For economists and political decision makers who take the view that people more or less "contribute according to their ability", work and effort substitution responses to taxes are low, and

efficiency losses through excess burdens of taxation are not a deterrent to highly progressive income taxes. Such economists and political decision makers might then see their way free to choose a social welfare function close to Rawls, with resulting high tax rates and high progressivity in the tax schedule. Economists and political decision makers who interpret the evidence as an indication that incentives to work and exert effort are important stress the efficiency losses from taxation and recommend income tax structures with low tax rates and low levels of progressivity. In particular, the latter group of economists and political decision makers often recommends a linear income tax schedule, or a schedule with a small number of tax brackets with low rates of taxation and low progressivity.

### 3.6 The Mirrleesian Economy

#### Assumptions on the economy

1. The economy is *competitive*.
2. Households differ only in *the level of skill* in employment. A household's level of skill determines their hourly wage and hence income.
3. The skill level is *private information* which is not known to the government.
4. The only tax instrument of the state is *an income tax*. An income tax is employed both because lump-sum taxes are infeasible and because it is assumed that it is not possible for the state to observe separately hours worked and income per hour. Therefore, since only total income is observed, it has to be the basis for the tax system.

#### The Basic Structure of the Economy<sup>5</sup>

1. Two commodities: a consumption good  $x$  ( $x \geq 0$ ) and a single labor service,  $l$  ( $0 \leq l \leq 1$ ).
2. Each household is characterized by their skill level,  $s$ . The value of  $s$  gives the relative effectiveness of the labor supplied per unit of time. If a household of ability  $s$  supplies  $l$  hours of labor, they provide a quantity  $sl$  of effective labor. For simplicity, the marginal product of labor is equal to a worker's ability  $s$ . The total productivity of a worker during the  $l$  hours at work is equal to  $sl$ .
3. Denote the supply of effective labor of a household with ability  $s$  by  $z(s) \equiv sl(s)$ .
4. The price of the consumption good is normalized at 1.
5.  $z(s)$  is the household's pre-tax income in units of consumption. Denoting *the tax function* by  $T(z)$  and *the consumption function* by  $c(z)$ , a household that earns  $z(s)$  units of income

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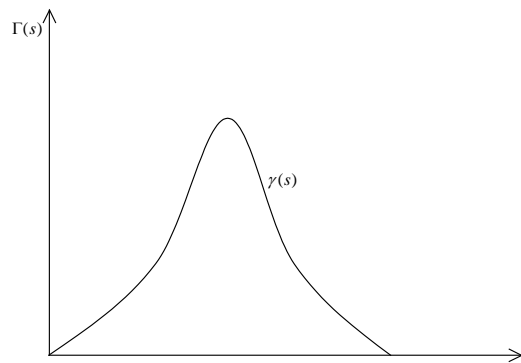
<sup>5</sup> This part draws heavily from Myles (1995) Chap 2, pp.133-55.

can consume

$$x(s) \leq c(z(s)) = z(s) - T(z(s)) \quad (8)$$

6. The ability parameter  $s$  is continuously distributed throughout the population with support  $s$  ( $s$  can be finite with  $s=[s_1, s_2]$  or infinite with  $s=[0, \infty)$ ). The cumulative distribution of  $s$  is given by  $\Gamma(s)$ , so there are  $\Gamma(s)$  households with ability  $s$ . The corresponding density function is denoted  $\gamma(s)$ .

**Figure 1 Distribution of ability  $s$**



7. All households have the same strictly concave utility function

$$U = U(x, l) \quad (9)$$

Each household makes the choice of labor supply and consumption demand to maximize utility subject to the budget constraint.

$$\text{Max } U(x, l) \text{ subject to } x(s) \leq c(sl(s)) \quad (10)$$

In the absence of income taxation, a household of ability  $s$  would face the budget constraint

$$x \leq sl \quad (11)$$

From (11), it is obvious that the budget constraint differs with ability.

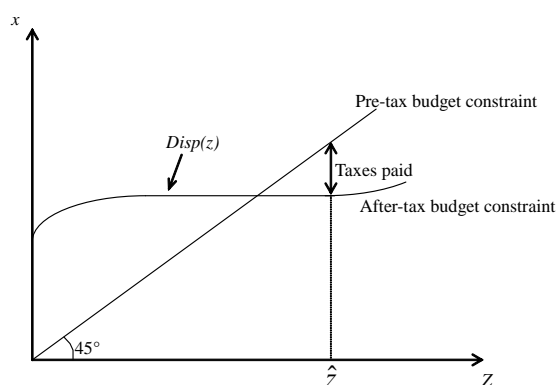
For simplicity, all households face the same budget constraint. This can be achieved by setting the analysis in  $(z,x)$  space. In this space, the pre-tax budget constraint is given by the 45° line for households of all abilities.

Figure 2 presents one case for the general shape of the pattern of pre-tax and post-tax income,  $z$  and  $Disp(z)$  respectively. The curve  $Disp(z)$  describes the case where marginal tax rates defined by  $T'(z)$  are first high for lower income, then rather low for middle-range income, and finally high again for high income. The shape of  $Disp(z)$  is very close to the marginal tax rate structure commonly observed in many countries.

$Disp(z)$  is sometimes referred to as a tax structure which sets the floor for poverty and the ceiling for wealth. At the same time it is very favorable for middle income groups. It can, however, be argued that when the possible disincentive effects of high marginal tax rates are taken into account, the pattern described above is no longer desirable. The problem is how to seek a compromise between the disincentive effects of marginal tax rates and their effects in achieving a more equal distribution of economic welfare.

Let  $c(z(s))$  denote consumption, and  $l(s)$  hours worked by an individual whose ability (productivity) is  $s$  and whose gross income is  $z = sl(s)$ . Let  $c(z(s))$  be the consumption schedule imposed by the government, with respect to which each individual must make his or her choice. Further, it is assumed that the indifference curves between  $c(z)$  and  $l(s)$  for all individuals are the same. By expanding each  $(c,l)$  curve horizontally by the factor  $s$ , indifference curves between  $c$  and  $sl$  can be drawn. If  $c$  is non-inferior, these curves are strictly flatter at each point the greater the value of  $s$ . This means that an individual with a greater  $s$  is more able to substitute labor for consumption. Under this assumption it is clear that  $c$  and  $z$  must be increasing functions of  $s$ .

**Figure 2 After-tax budget constraints**



## The Structure of Utility

The households have identical preferences over consumption and leisure. The utility function is continuously differentiable, strictly increasing in consumption and strictly decreasing in leisure. In addition, it satisfies

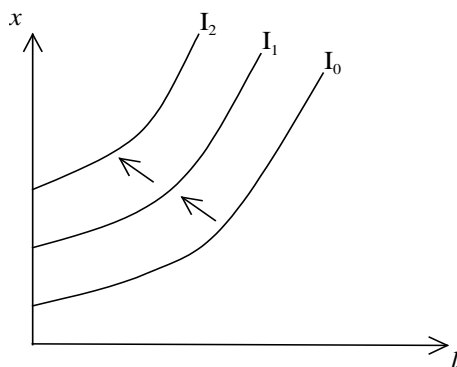
$$U_x > 0, \quad U_l < 0, \quad U_{xx} < 0 \quad (12)$$

and

$$U_l(x, l) \rightarrow -\infty \text{ as } l \rightarrow 1 \quad (13)$$

(15) implies that each household will endeavour to avoid corner solutions with  $l=1$  (no one wants to work all day long!!). The indifference curves of the utility function are illustrated bellows in which utility increases to the north west.

**Figure 3 Preference**



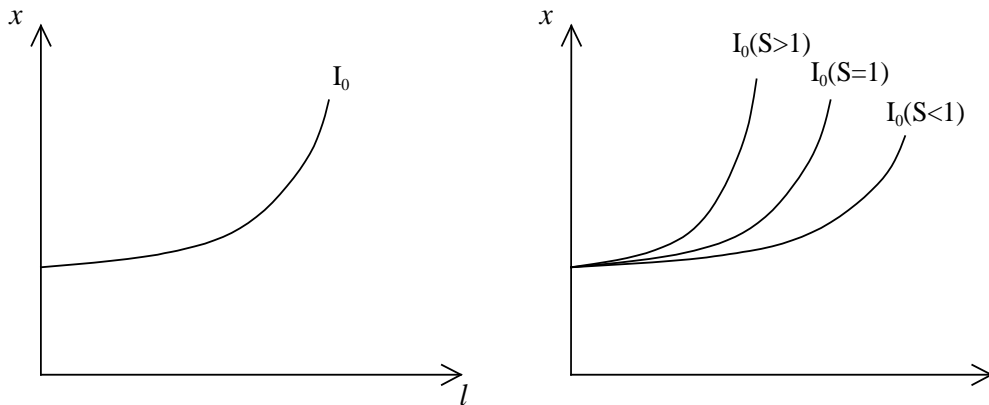
To allow preferences and the budget constraint to be depicted on the same diagram, the utility function can be written

$$U = U(x, l) = U(x, z/s) = u(x, z, s) \quad (14)$$

The indifference curves of  $u(x, z, s)$ , drawn  $(z, x)$ -space are dependent upon the ability level of the household since it takes a high-ability household less labor time to achieve any given level of income.

In fact, the indifference curves are constructed from those in  $(l,x)$ -space by multiplying by the relevant value of  $s$ . This construction for the single indifference curve  $I_0$  and households of three different ability levels.

**Figure 4 Translation of indifference curves.**



## Agent Monotonicity

The utility function (14) satisfies agent monotonicity if  $-u_z/u_x$ , is a decreasing function of  $s$ . Note that  $\Phi \equiv \frac{-u_z}{u_x}$  is the marginal rate of substitution between consumption and pre-tax

income and that agent monotonicity requires  $\Phi_s \equiv \frac{\partial \Phi}{\partial s} < 0$ .

An equivalent definition of agent monotonicity is that  $-l \frac{U_l}{U_x}$  is an increasing function of  $l$  as  $-\frac{u_z}{u_x} = -\frac{U_l(x, z/s)}{s U_x(x, z/s)}$ . Calculating  $\partial \left[ -l \frac{U_l}{U_x} \right] / \partial l$  and  $\Phi_s$  shows

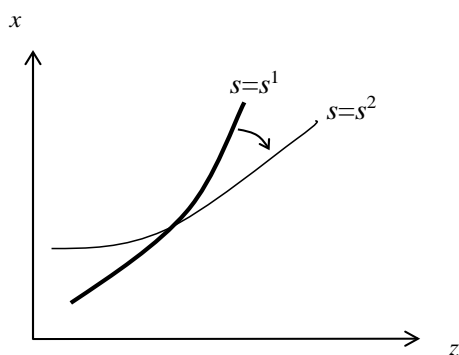
$$\Phi_s = -\frac{1}{s^2} \partial \left[ -l \frac{U_l}{U_x} \right] / \partial l \quad (15)$$

Agent monotonicity is equivalent to the condition that, in the absence of taxation, consumption will increase as the wage rate increases. A sufficient condition for agent monotonicity is that consumption is *not inferior*, i.e. it does not decrease as lump-sum income increases.

The marginal rate of substitution is the gradient of the indifference curve, agent monotonicity

implies that at any point in  $(z,x)$ -space the indifference curve of a household of ability  $s^1$  passing through that point is steeper than the curve of a household of ability  $s^2$  if  $s^2 > s^1$ . Agent monotonicity implies that any two indifference curves of households of different abilities only cross once. In other words, the indifference curve of an  $s$ -ability individual through the point  $(x,z)$  in consumption-labor space rotates strictly clockwise as  $s$  increases.

**Figure 5**



Mirrlees proved a theorem which shows, when the consumption function is a differentiable function of labor supply, *agent monotonicity implies that gross income is an increasing function of ability* (in other words, if agent monotonicity holds and the implemented tax function has pre-tax income increasing with ability, then the second-order condition for utility maximization must hold). This is important as to identify one's ability by watching gross income.

## Self-selection

Let  $x(s)$  and  $z(s)$  represent the consumption and income levels that the government intends a household of ability  $s$  to choose. The household of ability  $s$  will choose  $(x(s), z(s))$  provided that this pair generates at least as much utility as any other choice. This condition must apply to all consumption-income pairs and to all households. Formally we can write,

The self-selection constraint is satisfied if  $u(x(s), z(s), s) \geq u(x(s'), z(s'), s')$  for all  $s$  and  $s'$ .

In case of linear taxation, it does *not need to consider the self-selection constraints* since the behavior of the household can be determined as a function of the two parameters that describe the tax function; the lump-sum payment and the marginal rate of tax.

In case of non-linear taxation, *the self-selection constraints must be included*. This is achieved by noting that the satisfaction of the self-selection constraint is equivalent to achieving



the minimum of a certain minimization problem. If the sufficient conditions for the minimization are satisfied by the allocation resulting from the tax function, then the self-selection constraint is satisfied. The idea is to induce *the more able group to 'reveal' that they have a high income, not the reverse.*

To derive the required minimization problem, let  $u(s)=u(x(s), z(s), s)$  represent the maximized level of utility for a consumer of ability  $s$  resulting from (10).

$$0= u(s)-u(x(s), z(s), s) \leq u(s')-u(x(s), z(s), s') \quad (16)$$

so that  $s' = s$  minimizes  $u(s')-u(x(s), z(s), s')$ . Hence

$$u'(s)=u_s(x(s), z(s), s). \quad (17)$$

From the definition of  $u(s)$  it follows that

$$u_x x'(s) + u_z z'(s) = 0 \quad (18)$$

is equivalent to (17).

Condition (17) or (18) is the necessary (the first order) condition for the self-selection constraint to be satisfied.

The second-order condition for the self-selection constraint is found from the second derivative of  $u(s')-u(x, z, s')$  with respect to  $s'$  to be

$$u''(s) - u_{ss}(x(s), z(s), s) \geq 0 \quad (19)$$

Again using the definition of  $u(s)$ ,

$$u''(s) - u_{sx}x'(s) + u_{sz}z'(s) + u_{ss} \quad (20)$$

which gives, by using (19).

$$u_{sx}x'(s) + u_{sz}z'(s) \geq 0 \quad (21)$$

Eliminating  $x'(s)$  using (18) provides the final condition

$$\left[ u_{sz} - u_{sx} \frac{u_z}{u_x} \right] z'(s) = - \frac{\Phi_s}{u_x} z'(s) \geq 0 \quad (22)$$

where  $\Phi_s$  is the marginal rate of substitution introduced in the discussion of agent monotonicity. With agent monotonicity  $\Phi_s < 0$ , so that satisfaction of the second-order condition for self-selection is equivalent to  $z'(s) \geq 0$ . Any tax function that leads to an outcome satisfying (14) and  $z'(s) \geq 0$  will therefore satisfy the self-selection constraint.

## Characterization of Optimal Tax Function

It will clearly not be possible to calculate the function without precisely stating the functional forms of utility, production and skill distribution. What will be achieved is the derivation of a set of restrictions that the optimal function must satisfy.

### 3.7 The General Problem

Using the individual demand and supply functions and integrating over the population, it is possible to define total effective labor supply  $Z$ , by

$$Z = \int_0^{\infty} z(s) \gamma(s) ds \quad (23)$$

and aggregate demand,  $X$ , where

$$X = \int_0^{\infty} x(s) \gamma(s) ds \quad (24)$$

The optimal tax function is then chosen to maximize social welfare, where social welfare is given by the Bergson-Samuelson function.

$$W = \int_0^{\infty} w(u(s)) \gamma(s) ds \quad (25)$$

with  $W'' \leq 0$ .

There are two constraints upon the maximization of (25). The first is that the chosen allocation must be productively feasible such that,

$$X \leq F(Z) \tag{26}$$

where  $F$  is the production function for the economy.

This definition of productive feasibility can incorporate the government revenue requirement, expressed as a quantity of labor consumed by the government  $Z^G$ , by noting that (26) can be written  $X \leq \hat{F}(Z - Z^G) = F(Z)$ .

Denoting the level of revenue required by  $R(\equiv Z^G)$ , the revenue constraint can be written

$$R \geq \int_0^\infty [z(s) - x(s)]\gamma(s)ds \tag{27}$$

The second constraint is that it must satisfy the self-selection constraint which has already been discussed.

### 3.8 Optimal Linear Taxation<sup>6</sup>

#### Basic Model

Linear labor income taxation simplifies considerably the exposition but captures the key equity-efficiency trade off. Sheshinski (1972) offered the first modern treatment of optimal linear income taxation following the nonlinear income tax analysis of Mirrlees (1971). Both the derivation and the optimal formulas are also closely related to the more complex nonlinear case. It is therefore pedagogically useful to start with the linear case where the government uses a linear tax at rate  $\tau$  to fund a demogrant  $R$  (and additional non-transfer spending  $E$  taken as exogenous)<sup>7</sup>.

Summing the Marshallian individual earnings functions  $z_u^i(1 - \tau, R)$ , we obtain aggregate earnings which depend upon  $1 - \tau$  and  $R$  and can be denoted by  $Z_u(1 - \tau, R)$ . The government's budget constraint is  $R + E = \tau Z_u(1 - \tau, R)$ , which defines implicitly  $R$  as a function of  $\tau$  only (as we assume that  $E$  is fixed exogenously). Hence, we can express aggregate earnings as a sole function of  $1 - \tau$ :  $Z(1 - \tau) = Z_u(1 - \tau, R(\tau))$ . The tax revenue

<sup>6</sup> This part draws from Piketty, T and E. Saez (2013) Chap 5, pp.410-13.

<sup>7</sup> In terms of informational constraints, the government would be constrained to use linear taxation (instead of the more general nonlinear taxation) if it can only observe the amount of each earnings transaction but cannot observe the identity of individual earners. This could happen for example if the government can only observe the total payroll paid by each employer but cannot observe individual earnings perhaps because there is no identity number system for individuals.

function  $\tau \rightarrow \tau Z(1 - \tau)$  has an inverted U-shape. It is equal to zero both when  $\tau = \mathbf{0}$  (no taxation) and when  $\tau = \mathbf{1}$  (complete taxation) as 100% taxation entirely discourages labor supply. This curve is popularly called the Laffer curve although the concept of the revenue curve has been known since at least Dupuit (1844). Let us denote by  $e = \frac{1-\tau}{Z} \frac{dZ}{d(1-\tau)}$  the elasticity of aggregate earnings with respect to the net-of-tax rate. The tax rate  $\tau^*$  maximizing tax revenue is such that  $(1 - \tau) - \tau \frac{dZ}{d(1-\tau)} = \mathbf{0}$ , i.e.,  $\frac{\tau}{1-\tau} e = \mathbf{1}$ . Hence, we can express  $\tau^*$  as a sole function of  $e$ :

$$\text{Revenue maximizing linear tax rate: } \frac{\tau^*}{1-\tau^*} = \frac{1}{e} \text{ or } \tau^* = \frac{1}{1+e} \quad (28)$$

Let us now consider the maximization of a general social welfare function. The demogrant  $R$  evenly distributed to everybody is equal to  $\tau Z(1 - \tau) - E$  and hence disposable income for individual  $i$  is  $c^i = (1 - \tau)z^i + \tau Z(1 - \tau) - E$  (recall that population size is normalized to one). Therefore, the government chooses  $\tau$  to maximize

$$SWF = \int_i \omega^i G[u^i((1 - \tau)z^i + \tau Z(1 - \tau) - E, z^i)] dv(i).$$

Using the envelope theorem from the choice of  $z^i$  in the utility maximization problem of individual  $i$ , the first order condition for the government is simply

$$\mathbf{0} = \frac{dSWF}{d\tau} = \int_i \omega^i G'(u^i) u_c^i \cdot \left[ Z - z^i - \tau \frac{dZ}{d(1-\tau)} \right] dv(i),$$

The first term in the square brackets  $Z - z^i$  reflects the mechanical effect of increasing taxes (and the demogrant) absent any behavioral response. This effect is positive when individual income  $z^i$  is less than average income  $Z$ . The second term  $-\tau \frac{dZ}{d(1-\tau)}$  reflects the efficiency cost of increasing taxes due to the aggregate behavioral response. This is an efficiency cost because such behavioral responses have no first order positive welfare effect on individuals but have a first order negative effect on tax revenue.

Introducing the aggregate elasticity  $e$  and the “normalized” social marginal welfare weight  $g^i = \omega^i G'(u^i) u_c^i / \int \omega^i G'(u^i) u_c^i dv(j)$ , we can rewrite the first order condition as:

$$Z \cdot \left[ 1 - \frac{\tau}{1-\tau} e \right] = \int_i g_i z_i dv(i).$$

Hence, we have the following optimal linear income tax formula

$$\text{Optimal linear tax rate: } \tau = \frac{1-\bar{g}}{1-\bar{g}+e} \text{ with } \bar{g} = \frac{\int g_i z_i dv(i)}{Z} \quad (29)$$

$\bar{g}$  is the average “normalized” social marginal welfare weight weighted by pre-tax incomes  $z^i$ .  $\bar{g}$  is also the ratio of the average income weighted by individual social welfare weights  $g_i$  to the actual average income  $Z$ . Hence,  $\bar{g}$  measures where social welfare weights are concentrated on average over the distribution of earnings. An alternative form for formula (29) often presented in the literature takes the form  $\tau = -\text{cov}(g_i, z_i/Z) / [-\text{cov}(g_i, z_i/Z) + e]$  where  $\text{cov}(g_i, z_i/Z)$  is the covariance between social marginal welfare weights  $g_i$  and normalized earnings  $z_i/Z$ . As long as the correlation between  $g_i$  and  $z_i$  is negative, i.e., those with higher incomes have lower social marginal welfare weights, the optimum  $\tau$  is positive. Five points are worth noting about formula (29).

- 1) The optimal tax rate decreases with the aggregate elasticity  $e$ . This elasticity is a mix of substitution and income effects as an increase in the tax rate  $\tau$  is associated with an increase in the demogrant  $R = \tau Z(1 - \tau) - E$ . Formally, one can show that  $e = [\bar{e}_u - \bar{\eta}] / [1 - \bar{\eta}\tau / (1 - \tau)]$  where  $\bar{e}_u = \frac{1-\tau}{Z_u} \frac{\partial z}{\partial(1-\tau)}$  is the average of the individual uncompensated elasticities  $e_u^i$  weighted by income  $z^i$  and  $\bar{\eta} = (1 - \tau) \frac{\partial Z_u}{\partial R}$  is the unweighted average of individual income effects  $\eta^i$ .<sup>8</sup> This allows us to rewrite the optimal tax formula (29) in a slightly more structural form as  $\tau = (1 - \bar{g}) / (1 - \bar{g} - \bar{g} \cdot \bar{\eta} + \bar{e}_u)$ .  
When the tax rate maximizes tax revenue, we have  $\tau = 1 / (1 + e)$  and then  $e = \bar{e}_u$  is a pure uncompensated elasticity (as the tax rate does not raise any extra revenue at the margin). When the tax rate is zero,  $e$  is conceptually close to a compensated elasticity as taxes raised are fully rebated with no efficiency loss<sup>9</sup>.
- 2) The optimal tax rate naturally decreases with  $\bar{g}$  which measures the redistributive tastes of the government. In the extreme case where the government does not value redistribution

<sup>8</sup> To see this, recall that  $Z(1 - \tau) = Z_u(1 - \tau, \tau Z(1 - \tau) - E)$  so that  $\frac{dZ}{d(1-\tau)} \left[ 1 - \tau \frac{\partial Z_u}{\partial R} \right] = \frac{\partial Z_u}{\partial(1-\tau)} - Z \frac{\partial Z_u}{\partial R}$ .

<sup>9</sup> It is not exactly a compensated elasticity as  $\bar{e}_u$  is income weighted while  $\bar{\eta}$  is not.

at all,  $g_i \equiv 1$  and hence  $\bar{g} = 1$  and  $\tau = 0$  is optimal<sup>10</sup>. In the polar opposite case where the government is Rawlsian and maximizes the lump sum demogrant (assuming the worst-off individual has zero earnings), then  $\bar{g} = 0$  and  $\tau = 1/(1 + e)$ , which is the revenue maximizing tax rate from Eq.(28). As mentioned above, in that case  $e = \bar{e}_u$  is an uncompensated elasticity.

- 3) For a given profile of social welfare weights (or for a given degree of concavity of the utility function in the homogeneous utilitarian case), the higher the pre-tax inequality at a given  $\tau$ , the lower  $\bar{g}$ , and hence the higher the optimal tax rate. If there is no inequality, then  $\bar{g} = 1$  and  $\tau = 0$  with a lump-sum tax  $-R = E$  is optimal. If inequality is maximal, i.e., nobody earns anything except for a single person who earns everything and has a social marginal welfare weight of zero, then  $\tau = 1/(1 + e)$ , again equal to the revenue maximizing tax rate.
- 4) It is important to note that, as is usual in optimal tax theory, formula (29) is an implicit formula for  $\tau$  as both  $e$  and especially  $\bar{g}$  vary with  $\tau$ . Under a standard utilitarian social welfare criterion with concave utility of consumption,  $\bar{g}$  increases with  $\tau$  as the need for redistribution (i.e., the variation of the  $g_i$  with  $z_i$ ) decreases with the level of taxation  $\tau$ . This ensures that formula (29) generates a unique equilibrium for  $\tau$ .
- 5) Formula (29) can also be used to assess tax reform. Starting from the current  $\tau$ , the current estimated elasticity  $e$ , and the current welfare weight parameter  $\bar{g}$ , if  $\tau < (1 - \bar{g})/(1 - \bar{g} + e)$  then increasing  $\tau$  increases social welfare (and conversely). The tax reform approach has the advantage that it does not require knowing how  $e$  and  $\bar{g}$  change with  $\tau$ , since it only considers local variations.

**Generality of the Formula.** The optimal linear tax formula is very general as it applies to many alternative models for the income generating process. All that matters is the aggregate elasticity  $e$  and how the government sets normalized marginal welfare weights  $g_i$ . First, if the population is discrete, the same derivation and formula obviously apply. Second, if labor supply responses are (partly or fully) along the extensive margin, the same formula applies. Third, the same formula also applies in the long run when educational and human capital decisions are potentially affected by the tax rate as those responses are reflected in the long-run aggregate elasticity  $e$  (see e.g., Best & Kleven, 2012).<sup>11</sup>

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<sup>10</sup> This assumes that a lump sum tax  $E$  is feasible to fund government spending. If lump sum taxes are not feasible, for example because it is impossible to set taxes higher than earnings at the bottom, then the optimal tax in that case is the smallest  $\tau$  such that  $\tau Z(1 - \tau) = E$ , i.e., the level of tax required to fund government spending  $E$ .

<sup>11</sup> Naturally, such long-run responses are challenging to estimate empirically as short-term comparisons around a tax reform cannot capture them.

### 3.9 Optimal Nonlinear Taxation<sup>12</sup>

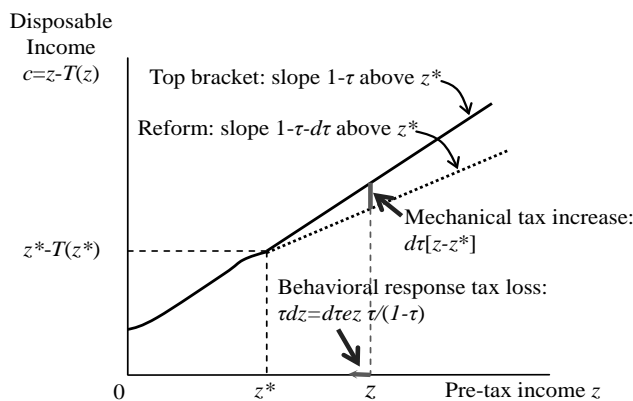
Formally, the optimal nonlinear tax problem is easy to pose. It is the same as the linear tax problem except that the government can now choose any nonlinear tax schedule  $T(\mathbf{z})$  instead of a single linear tax rate  $\tau$  with a demogrant  $R$ . Therefore, the government chooses  $T(\mathbf{z})$  to maximize.

$$SWF = \int_i \omega_i G(\mathbf{u}^i(\mathbf{z}^i - T(\mathbf{z}^i), \mathbf{z}^i)) d\nu(\mathbf{i}). \quad \text{subject to} \quad \int_i T(\mathbf{z}^i) d\nu(\mathbf{i}) \geq E(\mathbf{p}),$$

and the fact that  $\mathbf{z}^i$  is chosen by individual  $\mathbf{i}$  to maximize her utility  $\mathbf{u}^i(\mathbf{z}^i - T(\mathbf{z}^i), \mathbf{z}^i)$ . Note that transfers and taxes are fully integrated. Those with no earnings receive a transfer  $-T(\mathbf{0})$ . We start the analysis with the optimal top tax rate. Next, we derive the optimal marginal tax rate at any income level  $\mathbf{z}$ . Finally, we focus on the bottom of the income distribution to discuss the optimal profile of transfers.

In this chapter, we purposefully focus on intuitive derivations using small reforms around the optimum. This allows us to understand the key economic mechanisms and obtain formulas directly expressed in terms of estimable “sufficient statistics” (Chetty, 2009a; Saez, 2001). Hence, we will omit discussions of technical issues about regularity conditions needed for the optimal tax formulas<sup>13</sup>.

**Figure 6 Optimal Top Tax Rate Derivation**



The figure adapted from Diamond and Saez (2011), depicts the derivation of the optimal top tax rate  $\tau = 1/(1 + ae)$  by considering a small reform around the optimum which increases the

<sup>12</sup> This part draws from Piketty, T and E. Saez (2013) Chap 5, pp.421-25.

<sup>13</sup> The optimal income tax theory following Mirrlees (1971) has devoted substantial effort studying those issues thoroughly (see e.g., Mirrlees (1976, 1986, chap. 24) for extensive surveys). The formal derivations are gathered in the appendix.

top marginal tax rate  $\tau$  by  $d\tau$  above  $z^*$ . A taxpayer with income  $z$  mechanically pays  $d\tau[z - z^*]$  extra taxes but, by definition of the elasticity  $e$  of earnings with respect to the net-of-tax rate  $1 - \tau$ , also reduces his income by  $dz = -d\tau ez(1 - \tau)$  leading to a loss in tax revenue equal to  $d\tau ez\tau(1 - \tau)$ . Summing across all top bracket taxpayers and denoting by  $z$  the average income above  $z^*$  and  $a = z/(z - z^*)$ , we obtain the revenue maximizing tax rate  $\tau = 1/(1 + ae)$ . This is the optimum tax rate when the government sets zero marginal welfare weights on top income earners.

### 3.9.1 Optimal Top Tax Rate

The taxation of high income earners is a very important aspect of the tax policy debate. Initial progressive income tax systems were typically limited to the top of the distribution. Today, because of large increases in income concentration in a number of countries and particularly the United States (Piketty and Saez, 2003), the level of taxation of top incomes (e.g., the top 1%) matters not only for symbolic equity reasons but also for quantitatively for revenue raising needs.

#### Standard Model

Let us assume that the top tax rate above a fixed income level  $z^*$  is constant and equal to  $\tau$  as illustrated on Figure 6. Let us assume that a fraction  $q$  of individuals are in the top bracket. To obtain the optimal  $\tau$ , we consider a small variation  $d\tau$  as depicted on Figure 6. Individual  $i$  earning  $z^i$  above  $z^*$ , mechanically pays  $[z^i - z^*]d\tau$  extra in taxes. This extra tax payment creates a social welfare loss (expressed in terms of government public funds) equal to  $-g^i \cdot [z^i - z^*]d\tau$  where  $g^i = \omega_i G'(u^i) u_c^i / p$  is the social marginal welfare weight on individual  $i$ <sup>14</sup>. Finally, the tax change triggers a behavioral response  $dz^i$  leading to an additional change in tax  $\tau dz^i$ . Using the elasticity of reported income  $z^i$  with respect to the net-of tax rate  $1 - \tau$ , we have  $dz^i = -e^i z^i d\tau / (1 - \tau)$ . Hence, the net effect of the small reform on individual  $i$  is:

$$\left[ (1 - g^i)(z^i - z^*) - e^i z^i \frac{\tau}{1 - \tau} \right] d\tau.$$

To obtain the total effect on social welfare, we simply aggregate the welfare effects across all top bracket taxpayers so that we have:

<sup>14</sup> Because the individual chooses  $z^i$  to maximize utility, the money-metric welfare effect of the reform on individual  $i$  is given by  $[z^i - z^*]d\tau$  using the standard envelope theorem argument.



$$dSWF \left[ (1 - g)(z - z^*) - e z \frac{\tau}{1 - \tau} \right] q d\tau,$$

where  $q$  is the fraction of individuals in the top bracket,  $z$  is average income in the top bracket,  $g$  is the average social marginal welfare weight (weighted by income in the top bracket  $z^i - z^*$ ) of top bracket individuals, and  $e$  is the average elasticity (weighted by income  $z^i$ ) of top bracket individuals. We can introduce the tail-parameter  $a = z/(z - z^*)$  to rewrite  $dSWF$  as

$$dSWF \left[ 1 - g - a \cdot e \frac{\tau}{1 - \tau} \right] (z - z^*) q d\tau.$$

At the optimum,  $dSWF = 0$ , leading to the following optimal top rate formula.

$$\text{Optimal top tax rate: } \tau = \frac{1 - g}{1 - g + a e}. \quad (30)$$

Formula (30) expresses the optimal top tax rate in terms of three parameters: a parameter  $g$  for social preferences, a parameter  $e$  for behavioral responses to taxes, and a parameter  $a$  for the shape of the income distribution<sup>15</sup>. Five points are worth noting about formula (30).

First, the optimal tax rate decreases with  $g$ , the social marginal welfare weight on top bracket earners. In the limit case where society does not put any value on the marginal consumption of top earners, the formula simplifies to  $\tau = 1/(1 + a \cdot e)$  which is the revenue maximizing top tax rate. A utilitarian social welfare criterion with marginal utility of consumption declining to zero, the most commonly used specification in optimal tax models following Mirrlees (1971), has the implication that  $g$  converges to zero when  $z^*$  grows to infinity.

Second, the optimal tax rate decreases with the elasticity  $e$  as a higher elasticity leads to larger efficiency costs. Note that this elasticity is a mixture of substitution and income effects as an increase in the top tax rate generates both substitution and income effects<sup>16</sup>.

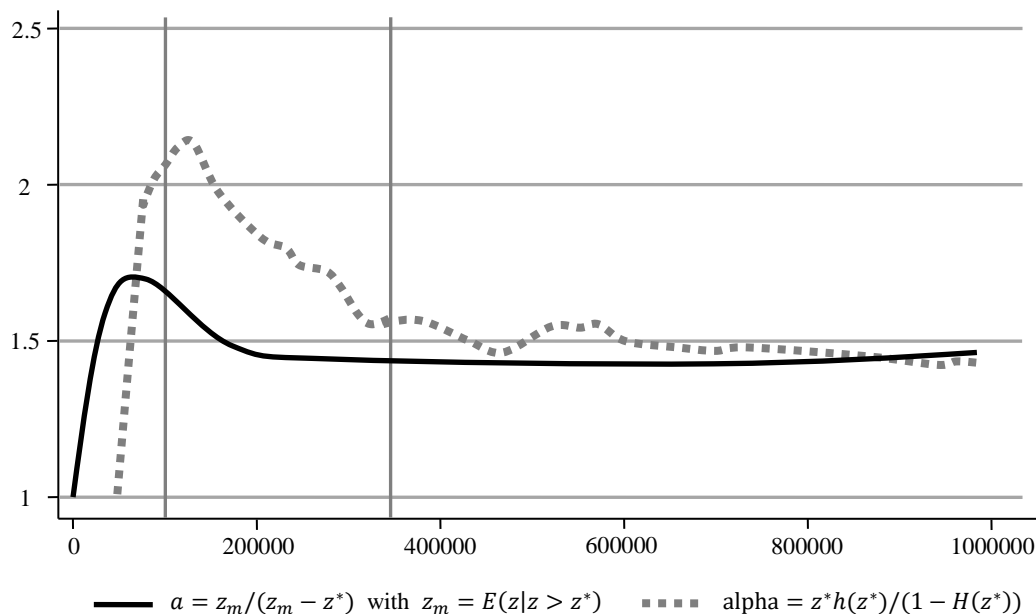
Importantly, for a given compensated elasticity, the presence of income effects *increases* the optimal top tax rate as raising the tax rate reduces disposable income and hence increases labor

<sup>15</sup> Note that the derivation and formula are virtually the same as for the optimal linear rate by simply multiplying  $e$  by the factor  $a > 1$ . Indeed, when  $z^* = 0$ ,  $a = z/(z - z^*) = 1$  and the problem boils down to the optimal linear tax problem.

<sup>16</sup> Saez (2001) provides a decomposition and shows that  $e = \bar{e}^u + \bar{\eta} \cdot (a - 1)/a$  with  $\bar{e}^u$  the average (income weighted) uncompensated elasticity and  $\bar{\eta}$  the (unweighted) average income effect.

supply.

**Figure 7**



Empirical Pareto coefficients in the United States, 2005. The figure, from Diamond and Saez (2011), depicts in solid line the ratio  $\alpha = z_m / (z_m - z^*)$  with  $z^*$  ranging from \$0 to \$1,000,000 annual income and  $z_m$  the average income above  $z^*$  using US tax return micro data for 2005. Income is defined as Adjusted Gross Income reported on tax returns and is expressed in current 2005 dollars. Vertical lines depict the 90<sup>th</sup> percentile (\$99,200) and 99<sup>th</sup> percentile (\$350,500) nominal thresholds as of 2005. The ratio  $\alpha$  is equal to one at  $z^* = 0$ , and is almost constant above the 99<sup>th</sup> percentile and slightly below 1.5, showing that the top of the distribution is extremely well approximated by a Pareto distribution for purposes of implementing the optimal top tax rate formula  $\tau = 1 / (1 + \alpha e)$ . Denoting by  $h(z)$  the density and by  $H(z)$  the cdf of the income distribution, the figure also displays in dotted line the ratio  $\alpha(z^*) = z^* h(z^*) / (1 - H(z^*))$  which is also approximately constant, around 1.5, above the top percentile. A decreasing (or constant)  $\alpha(z)$  combined with a decreasing  $g^+(z)$  and a constant  $e(z)$  implies that the optimal marginal tax rate  $T'(z) = [1 - g^+(z)] / [1 - g^+(z) + \alpha(z)e(z)]$  increases with  $z$ .

Third, the optimal tax rate decreases with the parameter  $\alpha \geq 1$  which measures the thinness of the top tail of the income distribution. Empirically,  $\alpha = z / (z - z^*)$  is almost constant as  $z^*$  varies in the top tail of the earnings distribution. Figure 7 depicts  $\alpha$  (as a function of  $z^*$ ) for the case of the US pre-tax income distribution and shows that it is extremely stable above  $z^* = \$400,000$ , approximately the top 1% threshold<sup>17</sup>. This is due to the well-known fact – since at least Pareto (1896) – that the top tail is very closely approximated by a Pareto distribution<sup>18</sup>.

Forth, and related, the formula shows the limited relevance of the zero-top tax rate result. Formally,  $z/z^*$  reaches 1 when  $z^*$  reaches the level of income of the single highest income

<sup>17</sup> This graph is taken from Diamond and Saez (2011) who use the 2005 distribution of total pre-tax family income (including capital income and realized capital gains) based on tax return data.

<sup>18</sup> A Pareto distribution with parameter  $a$  has a distribution of the form  $H(z) = 1 - k/z^a$  and density  $h(z) = ka/z^{1+a}$  (with  $k$  a constant parameter). For any  $z^*$ , the average income above  $z^*$  is equal to  $z^* \cdot a / (a - 1)$ .

earner, in which case  $\mathbf{a} = \mathbf{z}/(\mathbf{z} - \mathbf{z}^*)$  is infinite and indeed  $\boldsymbol{\tau} = \mathbf{0}$ , which is the famous zero-top rate result first demonstrated by Sadka (1976) and Seade (1977). However, notice that this result applies only to the very top income earner. Its lack of wider applicability can be verified empirically using distributional income tax statistics as we did in Figure 7 (see Saez, 2001 for an extensive analysis). Furthermore, under the reasonable assumption that the level of top earnings is not known in advance and where potential earnings are drawn randomly from an underlying Pareto distribution then, with the budget constraint satisfied in expectation, formula (30) remains the natural optimum tax rate (Diamond & Saez, 2011). This finding implies that the zero-top rate result and its corollary that marginal tax rates should decline at the top have no policy relevance.

Fifth, the optimal top tax rate formula is fairly general and applies equally to populations with heterogeneous preferences discrete populations, or continuous populations. Although the optimal formula does not require the strong homogeneity assumptions of the Mirrlees (1971) problem, it is also the asymptotic limit of the optimal marginal tax rate of the fully nonlinear tax problem of Mirrlees (1971).

### 3.10 Numerical Results<sup>19</sup>

To generate numerical results, Mirrlees (1971) assumed that the social welfare function took the form

$$\begin{aligned} w &= \int_0^\infty \frac{1}{v} e^{-vU} \gamma(s) ds, & v > 0 \\ &= \int_0^\infty U \gamma(s) ds, & v = 0 \end{aligned} \tag{31}$$

Higher values of  $v$  represent greater concern for equity, with  $v=0$  representing the utilitarian case.

The individual utility function was the Cobb-Douglas,

$$U = \log x + \log(1 - l) \tag{32}$$

and the skill distribution is log-normal,

---

<sup>19</sup> This part draws from Myles (1955) Chap 2, pp.156-9.

$$\gamma(s) = \frac{1}{s} \exp \left[ -\frac{(\log(s+1))^2}{2} \right] \quad (33)$$

With a standard deviation  $\sigma$  (=0.39 from Lydall (1968)). An implicit assumption is that the skill distribution can be inferred directly from an observed income distribution.

**Table 3 Optimal Tax Schedule**

Income	Consumption	Average tax (%)	Marginal tax (%)
(a) $z^G=0.013, \nu=0, \sigma=0.39$			
0.00	0.03	---	23
0.05	0.07	-34	26
0.10	0.10	-5	24
0.20	0.18	9	21
0.30	0.26	13	19
0.40	0.34	14	18
0.50	0.43	15	16
(b) $z^G=0.003, \nu=1, \sigma=0.39$			
0.00	0.05	---	30
0.05	0.08	-66	34
0.10	0.12	-34	32
0.20	0.19	7	28
0.30	0.26	13	25
0.40	0.34	16	22
0.50	0.41	17	20
(c) $z^G=0.013, \nu=1, \sigma=1$			
0.00	0.10	---	50
0.10	0.15	-50	58
0.25	0.20	20	60
0.50	0.30	40	59
1.00	0.52	48	57
1.50	0.73	51	54
2.00	0.97	51	52
3.00	1.47	51	49

Source: Mirrlees (1971)

The most important feature of the first two panels (a) and (b) in Table 3 is the low marginal rates of tax, with the maximal rate being only 34%. There is also limited deviation in these rates. The marginal rates become lower at high incomes but do not reach 0 because the skill distribution is unbounded. The average rate of tax is negative for low incomes so that low-income consumers are receiving an income supplement from the government.

The panel (c) of Table 3 show the effect of increasing the dispersion of skills (changing standard deviation from 0.39 to 1.00). This raises the marginal tax rates but there remain fairly constant across the income range. Kanbur and Tuomala (1994) find that an increased dispersion of skills raises the marginal tax rate at each income level and that it also has the effect of moving the maximum tax rate up the income range, so that the marginal tax rate is increasing over the majority of households.

Atkinson (1975) considered the effect of changing the social welfare function to the extreme maxi-min form,

$$w = \min\{U\} \tag{34}$$

From the above table, it can be seen that increased concern for equity,  $v$  going from 0 to 1, increased the optimal marginal tax rates. The natural question is “can strong equity considerations such as maxi-min SWF lead to high marginal rates?”. The result is given the below table. The maxi-min criterion leads to generally higher taxes. However they are again highest at low incomes and then decline. Absolute rate is lower than expected in all cases.

**Table 4 Optimal tax Schedule: Utilitarian vs. Maxi-min**

Level of $s$	Utilitarian		Maxi-min	
	Average rate (%)	Marginal rate (%)	Average rate (%)	Marginal rate (%)
Median (50%)	6	21	10	52
Top decile (10%)	14	20	28	34
Top percentile (1%)	16	17	28	26

Source: Atkinson and Stiglitz (1980, Table 13-3, p.421)

### 3.12 Voting over a Flat Tax<sup>20</sup>

Having identified the properties of the optimal tax structure, we now consider the tax system that emerges from the political process. To do this, we consider people voting over tax schedules that have some degree of redistribution. Because it is difficult to model voting on nonlinear tax schemes given the high dimensionality of the problem, we will restrict attention to a linear tax structure as originally proposed by Romer (1975). We specify the model further with quasi-linear preferences to avoid unnecessary complications and to simplify the analysis of the voting equilibrium.

Assume, as before, that individuals differ only in their level of skill. We assume that skills are distributed in the population according to a cumulative distribution function  $F(s)$  that is known to everyone, with mean skill  $\bar{s}$  and median  $s_m$ . Individuals work and consume. They also vote on a linear tax scheme that pays a lumpsum benefit  $b$  to each individual financed by a proportional income tax at rate  $t$ . The individual utility function has the quasi-linear form

$$u\left(x, \frac{z}{s}\right) = x - \frac{1}{2} \left[\frac{z}{s}\right]^2, \tag{35}$$

<sup>20</sup> This part is drawn from Hindriks and Myles (2006) Chapter 15, pp.503-5.

and the individual budget constraint is

$$x = [1 - t]z + b. \quad (36)$$

It is easy to verify that in this simple model the optimal income choice of a consumer with skill level  $s$  is

$$z(s) = [1 - t]s^2. \quad (37)$$

The quasi-linear preferences imply that there is no income effect on labor supply (i.e.,  $z(s)$  is independent of the lump-sum benefit  $b$ ). This simplifies the expression of the tax distortion and makes the analysis of the voting equilibrium easier. Less surprisingly a higher tax rate induces taxpayers to work less and earn less income.

The lump-sum transfer  $b$  is constrained by the government budget balance condition

$$b = tE(z(s)) = t[1 - t]E(s^2), \quad (38)$$

where  $E(\cdot)$  is the mathematical expectation, and we used the optimal income choice to derive the second equality. This constraint says that the lump-sum benefit paid to each individual must be equal to the expected tax payment  $tE(z(s))$ . This expression is termed the *Dupuit-Laffer curve* and describes tax revenue as a function of the tax rate. In this simple model the Dupuit-Laffer curve is bell-shaped with a peak at  $t = 1/2$  and no tax collected at the ends  $t = 0$  and  $t = 1$ . We can now derive individual preferences over tax schedules by substituting (34) and (35) into (33). After re-arrangement (indirect) utility can be written

$$v(t, b, s) = b + \frac{1}{2}[1 - t]^2 s^2. \quad (39)$$

Taking the total differential of (36) gives

$$dv = db - [1 - t]s^2 dt. \quad (40)$$

so that along an indifference curve where  $dv = 0$ ,

$$\frac{db}{dt} = [1 - t]s^2. \quad (41)$$

It can be seen from this that for given  $t$ , the indifference curve becomes steeper in  $(t, b)$  space as  $s$  increases. This monotonicity is a consequence of the single-crossing property of the indifference curves. The single-crossing property is a sufficient condition for the Median Voter Theorem to apply. It follows that there is only one tax policy that can result from majority voting: it is the policy preferred by the median voter (half the voters are poorer than the median and prefer higher tax rates, and the other half are richer and prefer lower tax rates). Letting  $t_m$  be the tax rate preferred by the median voter, then we have  $t_m$  implicitly defined by the solution to the first-order condition for maximizing the median voter's utility. We differentiate (37) with respect to  $t$ , taking into account the government budget constraint (36) to obtain

$$\frac{\partial v}{\partial t} = [1 - 2t]E(s^2) - [1 - t]s^2. \quad (42)$$

Setting this expression equal to zero for the median skill level  $s_m$  yields the tax rate preferred by the median voter

$$t_m = \frac{E(s^2) - s_m^2}{2E(s^2) - s_m^2}, \quad (43)$$

or, using the optimal income choice (35),

$$t_m = \frac{E(z) - z_m}{2E(z) - z_m}. \quad (44)$$

This simple model predicts that the political equilibrium tax rate is determined by the position of the median voter in the income distribution. The greater is income inequality as measured by the distance between median and mean income, the higher the tax rate. If the median voter is relatively worse off, with income well below the mean income, then equilibrium redistribution is large. In practice, the income distribution has a median income below the mean income, so a majority of voters would favor redistribution through proportional income

taxation. More general utility functions would also predict that the extent of this redistribution decreases with the elasticity of labor supply.



## Empirical Fact in Japan

**Table 5 Individual Inhabitants' and Income Tax**

	Inhabitants' Tax	Income Tax																																																																													
Tax Recipient	Municipal governments on 1 <sup>st</sup> January	National government																																																																													
Tax Payer	Individual lives in a municipal government	Individual lives in Japan																																																																													
Tax Method	With holding	Assessment																																																																													
Tax Base	Last year's income	This year's income																																																																													
Income Deduction	<ul style="list-style-type: none"> <li>• Basic deduction ¥330,000</li> <li>• Spouse deduction ¥330,000</li> <li>• Special spouse deduction max ¥330,000</li> <li>• Family deduction ¥330,000</li> </ul>	<ul style="list-style-type: none"> <li>• Basic deduction ¥380,000</li> <li>• Spouse deduction ¥380,000</li> <li>• Special spouse deduction max ¥380,000</li> <li>• Family deduction ¥380,000</li> </ul>																																																																													
Minimum Taxable Income (1998)	A couple with 2 children Before a special tax deduction ¥3,031,000 After special tax deduction ¥4,373,000	A couple with 2 children Before a special tax deduction ¥3,616,000 After special tax deduction ¥4,917,000																																																																													
Tax Rate	<table border="0" style="width: 100%;"> <thead> <tr> <th rowspan="2"></th> <th colspan="3">Tax Rates</th> </tr> <tr> <th>Prefectural</th> <th>Municipal</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>Taxable Income</td> <td></td> <td></td> <td></td> </tr> <tr> <td>Below ¥2,000,000</td> <td>2%</td> <td>3%</td> <td>5%</td> </tr> <tr> <td>Below ¥7,000,000</td> <td>2%</td> <td>7%</td> <td>10%</td> </tr> <tr> <td>Above ¥7,000,000</td> <td>3%</td> <td>12%</td> <td>15%</td> </tr> <tr> <td>Capital Gain Income</td> <td></td> <td></td> <td></td> </tr> <tr> <td>Below ¥40,000,000</td> <td>2%</td> <td>4%</td> <td>6%</td> </tr> <tr> <td>Below ¥80,000,000</td> <td>2%</td> <td>5.5%</td> <td>7.5%</td> </tr> <tr> <td>Above ¥80,000,000</td> <td>3%</td> <td>6%</td> <td>9%</td> </tr> <tr> <td>In 1999-2001</td> <td></td> <td></td> <td></td> </tr> <tr> <td>Below ¥60,000,000</td> <td>2%</td> <td>4%</td> <td>6%</td> </tr> <tr> <td>Above ¥60,000,000</td> <td>2%</td> <td>5.5%</td> <td>7.5%</td> </tr> </tbody> </table>		Tax Rates			Prefectural	Municipal	Total	Taxable Income				Below ¥2,000,000	2%	3%	5%	Below ¥7,000,000	2%	7%	10%	Above ¥7,000,000	3%	12%	15%	Capital Gain Income				Below ¥40,000,000	2%	4%	6%	Below ¥80,000,000	2%	5.5%	7.5%	Above ¥80,000,000	3%	6%	9%	In 1999-2001				Below ¥60,000,000	2%	4%	6%	Above ¥60,000,000	2%	5.5%	7.5%	<table border="0" style="width: 100%;"> <thead> <tr> <th>Taxable Income</th> <th>Tax Rates</th> </tr> </thead> <tbody> <tr> <td>Below ¥3,300,000</td> <td>10%</td> </tr> <tr> <td>Below ¥9,000,000</td> <td>20%</td> </tr> <tr> <td>Below ¥18,000,000</td> <td>30%</td> </tr> <tr> <td>Below ¥30,000,000</td> <td>40%</td> </tr> <tr> <td>Above ¥30,000,000</td> <td>50%</td> </tr> <tr> <td>Capital Gain Income</td> <td></td> </tr> <tr> <td>Below ¥40,000,000</td> <td>20%</td> </tr> <tr> <td>Below ¥80,000,000</td> <td>25%</td> </tr> <tr> <td>Above ¥80,000,000</td> <td>30%</td> </tr> <tr> <td>In 1998-2000</td> <td></td> </tr> <tr> <td>Below ¥60,000,000</td> <td>20%</td> </tr> <tr> <td>Above ¥60,000,000</td> <td>25%</td> </tr> </tbody> </table>	Taxable Income	Tax Rates	Below ¥3,300,000	10%	Below ¥9,000,000	20%	Below ¥18,000,000	30%	Below ¥30,000,000	40%	Above ¥30,000,000	50%	Capital Gain Income		Below ¥40,000,000	20%	Below ¥80,000,000	25%	Above ¥80,000,000	30%	In 1998-2000		Below ¥60,000,000	20%	Above ¥60,000,000	25%
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Tax Exemption	<ul style="list-style-type: none"> <li>- Adjustment for Double Taxation</li> <li>• Dividends</li> <li>• Foreign Tax</li> </ul>	<ul style="list-style-type: none"> <li>- Adjustment for Double Taxation</li> <li>• Dividends</li> <li>• Foreign Tax</li> <li>- Tax Incentives</li> <li>• Housing Acquisition</li> <li>• Research &amp; Development</li> </ul>																																																																													
Tax Revenue (1996)	90,17 trillion yen	189,649 trillion yen																																																																													

**Table 6 Historical Changes in Income Tax Rates Schedule**

	1974	1984	1987	1988	1989	1995	1999
	%	%	%	%	%	million	million
	10	10.5	10.5	10	10	(~3)	10
	12	12	12	20	20	(~6)	20
	14	14	16	30	30	(~1)	30
	16	17	20	40	40	(~2)	40
	18	21	25	50	50	(2~)	50
	21	25	30	60			
	24	30	35				
	27	35	40				
	30	40	45				
Tax Rate	34	45	50				
	38	50	55				
	42	55	60				
	46	60					
	50	65					
	55	70					
	60						
	65						
	70						
	75						
Maximum Inhabitant's Tax Rate	18%	18%	18%	16%	15%	15%	13%
Combined Maximum Tax Rate	93%	88%	78%	76%	65%	65%	50%
Number of Tax Brackets	19 (13)	15 (14)	12 (14)	6 (7)	5 (3)	5 (3)	4 (3)
Minimum Taxable Income	¥1,707,000	¥2,357,000	¥2,615,000	¥2,619,000	¥3,198,000	¥3,539,000 (¥3,616,000 after the 1998 amendment)	¥3,821,000

**Table 7 Change in Income Tax Payers**

	of Declared Earners		Total	of Declared Earners				Applicants for Refund
	Salary Earners	Declared Earners		Business	Agriculture	Small Business	Others	
1985	3,665	737	4,402	228	31	67	411	599
1986	3,728	770	4,498	231	32	70	437	654
1987	3,767	771	4,538	235	25	70	441	699
1988	3,909	780	4,689	245	24	70	441	696
1989	3,961	796	4,757	242	23	67	464	659
1990	4,158	855	5,013	250	25	67	513	663
1991	4,333	856	5,189	252	24	68	512	699
1992	4,403	858	5,261	247	24	69	518	735
1993	4,473	843	5,316	230	22	67	524	781
1994	4,478	822	5,300	223	27	62	510	867
1995	4,484	802	5,286	213	19	60	510	864
1996	4,537	824	5,361	213	20	60	531	883
1997	4,627	827	5,454	206	16	60	545	909
1998	4,703	848	5,551	210	20	61	557	---

Source: Ministry of Finance.

**Table 8 The 1996 Declared Income Tax Burden Rate by Income Class**

(million yen)

Income class	Average Income ①	Average Income Deduction ②	Average Taxable Income ①-②	Average Calculated Tax ③	Average Tax Deduction ④	Average Tax Payment ⑤	Gross Income Tax Rate ⑤/①	Effective Income Tax Rate ⑤/(①-②)
- 100	748	546	202	20	0	18	2.4%	8.9%
100 - 200	1,540	995	545	56	0	48	3.1%	8.8%
200 - 300	2,476	1,459	1,016	103	1	88	3.5%	8.7%
300 - 500	3,870	1,796	2,074	233	6	197	5.1%	9.5%
500 - 1000	6,899	2,020	4,879	700	14	635	9.2%	13.0%
1000+	23,005	1,893	21,112	4,789	32	4,707	20.5%	22.3%
Total Average	5,864	1,572	4,292	795	8	759	12.9%	17.7%

Source: *The 1996 Sample Survey of Declared Income Tax* (Tax Bureau).**Table 9 Number of Salary Earners, Total Salary and Tax in 1996**

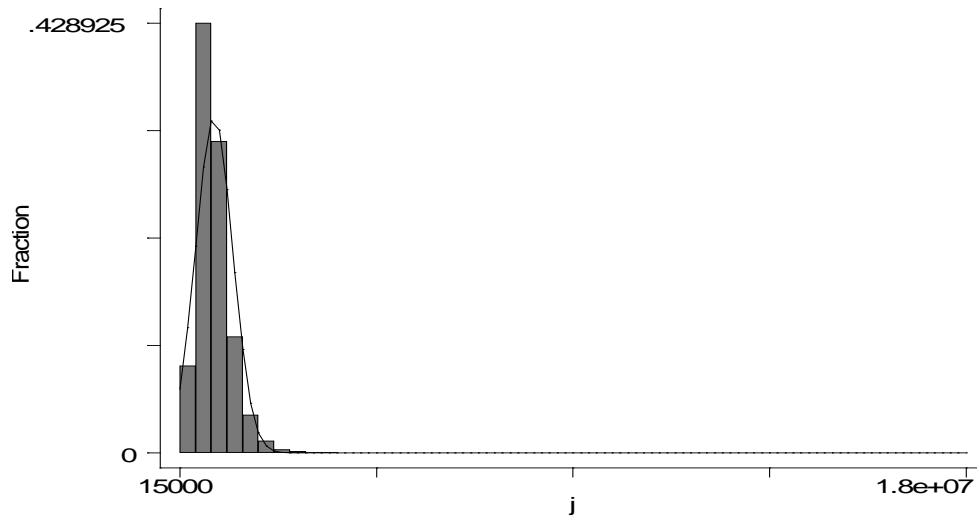
Income Class	Number of Salary Earners (thousands)				Total Salary (100 million yen)				Tax paid (100million yen)		Effective Tax Rate (%)
	of Tax Payers		of Tax Payers		of Tax Payers		of Tax Payers		Share	Share	
	Share	Share	Share	Share	Share	Share					
- 100	3,228	7.2	479	1.2	23,993	1.2	2,789	0.1	213	0.2	7.6
100 - 200	4,818	10.7	3,419	8.7	72,423	3.5	54,310	2.8	1,345	1.3	2.5
200 - 300	6,818	15.2	6,230	15.9	173,522	8.4	158,778	8.0	5,021	4.9	3.2
300 - 400	7,780	17.3	7,328	18.7	272,122	13.2	256,351	13.0	8,471	8.2	3.3
400 - 500	6,530	14.5	6,244	15.9	292,908	14.2	280,175	14.2	9,222	9.0	3.3
500 - 600	4,964	11.1	4,802	12.3	272,721	13.2	263,861	13.4	8,898	8.7	3.4
600 - 700	3,273	7.3	3,215	8.2	211,900	10.2	208,179	10.6	7,429	7.2	3.6
700 - 800	2,384	5.3	2,372	6.1	178,002	8.6	177,094	9.0	7,707	7.5	4.4
800 - 900	1,604	3.6	1,604	4.1	135,837	6.6	135,837	6.9	7,168	7.0	5.3
900 - 1000	1,004	2.2	1,004	2.6	95,168	4.6	95,168	4.8	6,137	6.0	6.4
1000 - 1500	1,963	4.4	1,963	5.0	232,610	11.2	232,610	11.8	21,461	20.9	9.2
1500 - 2000	378	0.8	378	1.0	64,247	3.1	64,247	3.3	9,475	9.2	14.7
2000 - 2500	87	0.2	87	0.2	20,098	1.0	20,098	1.0	3,956	3.8	19.7
2500+	64	0.1	64	0.2	23,254	1.1	23,254	1.2	6,294	6.1	27.1
Total	44,896	100.0	39,189	100.0	2,068,805	100.0	1,972,750	100.0	102,797	100.0	5.2

Note: Employees on December 31, 1996.

Source: *The 1996 Tax Bureau Private Sector Income Survey* (Tax Bureau).**Table 10 Tax Elasticity**

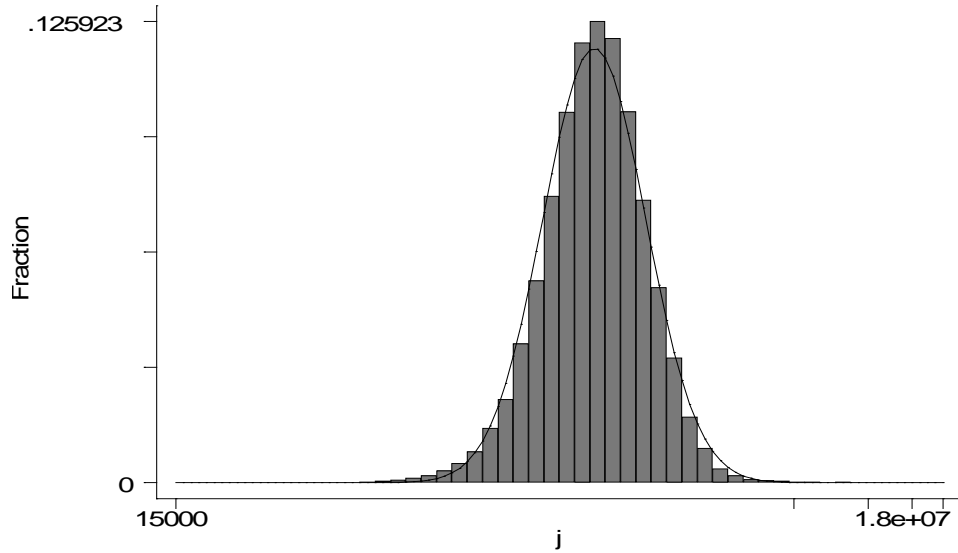
	Direct Tax (Income Tax)			Indirect Tax	Total Tax Revenue
	Individual	Corporate			
1985	1.00	1.31	0.64	0.42	0.86
1986	1.89	1.74	1.66	1.83	1.87
1987	3.64	2.58	4.78	1.94	3.20
1988	2.25	1.57	2.88	1.04	1.92
1989	1.66	2.73	0.58	0.77	1.44
1990	1.36	2.69	0.16	0.06	1.06
1991	0.02	0.06	▲1.43	0.10	0.03
1992	▲6.18	▲6.09	▲7.77	1.09	▲4.50
1993	▲4.14	2.14	▲17.71	4.71	▲1.86
1994	4.75	5.50	8.25	8.25	5.57
1995	1.10	▲2.00	6.50	1.55	1.20

**Figure 8 Before Tax Household Income Distribution in Japan**



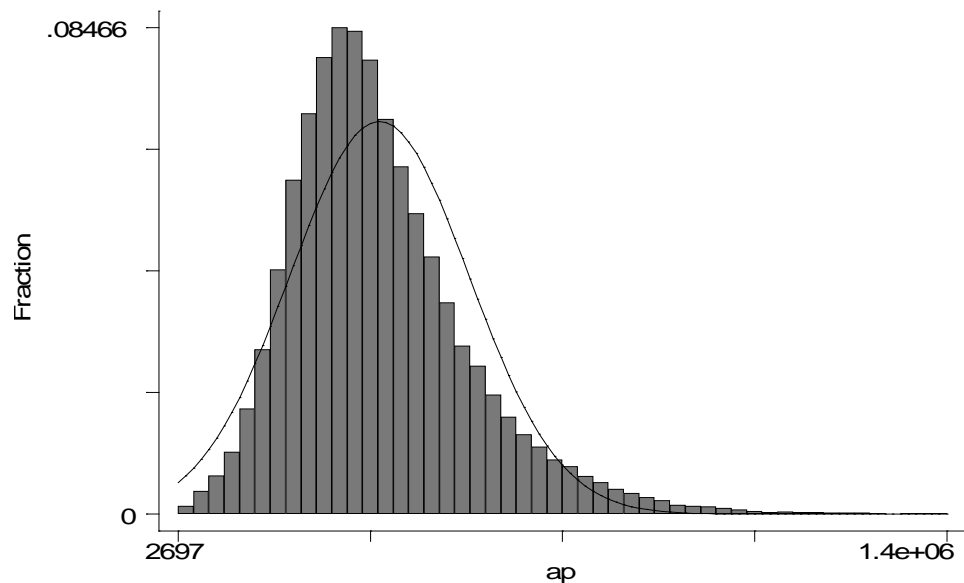
Source : The National Survey of Family Income and Expenditure 1984, 1989 and 1994 (pooled)  
Note: Monthly average income during September through November.

**Figure 9 Before Tax Household Income Distribution in Japan (log normal transformation)**



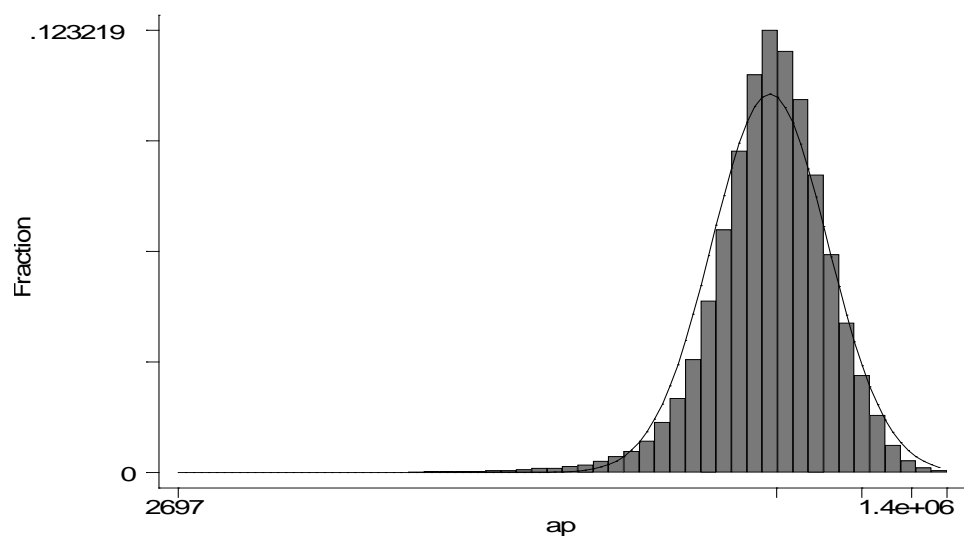
Source : The National Survey of Family Income and Expenditure 1984, 1989 and 1994 (pooled)  
Note: Monthly average income during September through November.

**Figure 10 After Tax Household Income Distribution in Japan**



Source : The National Survey of Family Income and Expenditure 1984, 1989 and 1994 (pooled)  
Note: Monthly average income during September through November.

**Figure 11 After Tax Household Income Distribution in Japan (log normal transformation)**



Source : The National Survey of Family Income and Expenditure 1984, 1989 and 1994 (pooled)  
Note: Monthly average income during September through November.

## Exercises

1. [Atkinson and Stiglitz (1980), p.376]

For the utility function  $U = \sum_{i=1}^n A_i \frac{X_i^{1-1/\varepsilon_i}}{1-1/\varepsilon_i} - vL$ , where  $L$  units of labor, wage is  $w$  thus the budget constraint for the household is  $\sum q_i X_i = wL = y$ . Show that the income terms ( $\partial x_i / \partial y$ ) and cross price terms are zero. Derive the optimal tax structure where  $\varepsilon_i$  are (positive) constants.

2. Recently many countries adopt indirect taxation (e.g. VAT) and shift its weight from direct taxation (e.g. individual income). Could you justify this shift of tax reform?
3. Stiglitz once argued that “it can be shown, that if one has a well-designed income tax, adding differential commodity taxation is likely to add little, if anything.” Would you agree with him? Or in what circumstances does the use of commodity taxation allow a higher level of social welfare to be achieved  $x = b + [1-t]w\ell$  in the presence of income taxation?
4. [Hindriks and Myles (2006) Chapter 15, Exercises 15.1]  
Consider the budget constraint  $x = b + [1-t]w\ell$ . Provide an interpretation of  $b$ . How does the average rate of tax change with income? Let utility be given by  $U = x - \ell^2$ . How is the choice of  $\ell$  affected by increases in  $b$  and  $t$ ? Explain these effects.
5. [Hindriks and Myles (2006) Chapter 15, Exercises 15.2]  
Assume that a consumer has preferences over consumption and leisure described by  $U = x[1-\ell]$ , where  $x$  is consumption and  $\ell$  is labor. For a given wage rate  $w$ , which leads to a higher labor supply: an income tax at constant rate  $t$  or a lump-sum tax  $T$  that raises the same revenue as the income tax?
6. [Hindriks and Myles (2006) Chapter 15, Exercises 15.3]  
Let the utility function be  $U = \log(x) - \ell$ . Find the level of labor supply if the wage rate,  $w$ , is equal to 10. What is the effect of the introduction of an overtime premium that raises  $w$  to 12 for hours in excess of that worked at the wage of 10?
7. [Hindriks and Myles (2006) Chapter 15, Exercises 15.4]  
Assume that utility is  $U = \log(x) - \log(1-\ell)$ . Calculate the labor supply function. Explain the form of this function by calculating the income and substitution effects of a wage increase.
8. [Hindriks and Myles (2006) Chapter 15, Exercises 15.6]  
Show that a tax function is average-rate progressive (the average rate of tax rises with income) if  $MRT > ART$ .
9. [Hindriks and Myles (2006) Chapter 15, Exercises 15.7]

Which is better: a uniform tax on consumption or a uniform tax on income?

10. [Hindriks and Myles (2006) Chapter 15, Exercises 15.8]

Consider the utility function  $U = x - \ell^2$ .

- For  $U = 10$ , plot the indifference curve with  $\ell$  on the horizontal axis and  $x$  on the vertical axis.
- Now define  $z = s\ell$ . For  $s = 0.5, 1, \text{ and } 2$  plot the indifference curves for  $U = 10$  with  $z$  on the horizontal axis and  $x$  on the vertical.
- Plot the indifference curves for  $s = 0.5, 1, \text{ and } 2$  through the point  $x = 20, z = 2$ .
- Prove that at any point  $(x, z)$  the indifference curve of a high-skill consumer is flatter than that of a low-skill.

11. [Hindriks and Myles (2006) Chapter 15, Exercises 15.9]

Consider an economy with two consumers who have skill levels  $s_1 = 1$  and  $s_2 = 2$  and utility function  $U = 10x^{1/2} - \ell^2$ . Let the government employ an income tax function that leads to the allocation  $x = 4, z = 5$  for the consumer of skill  $s = 1$  and  $x = 9, z = 8$  for the consumer of skill  $s = 2$ .

- Show that this allocation satisfies the incentive compatibility constraint that each consumer must prefer his allocation to that of the other.
- Keeping incomes fixed, consider a transfer of 0.01 units of consumption from the high-skill to the low-skill consumer.
  - Calculate the effect on each consumer's utility.
  - Show that the sum of utilities increases.
  - Show that the incentive compatibility constraint is still satisfied.
  - Use parts i through iii to prove that the initial allocation is not optimal for a utilitarian social welfare function.

12. [Hindriks and Myles (2006) Chapter 15, Exercises 15.11]

Assume that skill is uniformly distributed between 0 and 1 and that total population size is normalized at 1. If utility is given by  $U = \log(x)\log(1 - \ell)$  and the budget constraint is  $x = b + (1 - t)s\ell$ , find the optimal values of  $b$  and  $t$  when zero revenue is to be raised. Is the optimal tax system progressive?

13. [Hindriks and Myles (2006) Chapter 15, Exercises 15.13]

Consider an economy with two consumers of skill levels  $s_1$  and  $s_2, s_2 > s_1$ . Denote the allocation to the low-skill consumer by  $x_1, z_1$  and that to the high-skill consumer by  $x_2, z_2$ .

- For the utility function  $U = u(x) - z/s$  show that incentive compatibility requires that  $z_2 = z_1 + [u(x_2) - u(x_1)]$ .

- b. For the utilitarian social welfare function  $W = u(x_1) - \frac{z_1}{s_1} + u(x_2) - \frac{z_2}{s_2}$ , express  $W$  as a function of  $x_1$  and  $x_2$  alone.
- c. Assuming  $u(x_h) = \log(x_h)$ , derive the optimal values of  $x_1$  and  $x_2$  and hence of  $z_1$  and  $z_2$ .
- d. Calculate the marginal rate of substitution for the two consumers at the optimal allocation. Comment on your results.
14. [Hindriks and Myles (2006) Chapter 15, Exercises 15.15]  
 Suppose two types of consumers with skill levels 10 and 20. There is an equal number of consumers of both types. If the social welfare function is utilitarian and no revenue is to be raised, find the optimal allocation under a nonlinear income tax for the utility function  $U = \log(x) - \ell$ . Contrast this to the optimal allocation if skill was publicly observable.
15. [Hindriks and Myles (2006) Chapter 15, Exercises 15.16]  
 Tax revenue is given by  $R(t) = tB(t)$ , where  $t \in [0,1]$  is the tax rate and  $B(t)$  is the tax base. Suppose that the tax elasticity of the tax base is  $\varepsilon = -\frac{\gamma}{1-\gamma}$  with  $\gamma \in \left[\frac{1}{2}, 1\right]$ .
- a. What is the revenue-maximizing tax rate?
- b. Graph tax revenue as a function of the tax rate both for  $\gamma = 1/2$  and  $\gamma = 1$ . Discuss the implications of this Dupuit-Laffer curve.
16. [Hindriks and Myles (2006) Chapter 15, Exercises 15.17]  
 Consider an economy populated by a large number of workers with utility function  $U = x^\alpha [1 - \ell]^{1-\alpha}$ , where  $x$  is disposable income,  $\ell$  is the fraction of time worked ( $0 \leq \ell \leq 1$ ), and  $\alpha$  is a preference parameter (with  $0 < \alpha < 1$ ). Each worker's disposable income depends on his fixed "skill" as represented by wage  $w$  and a tax-transfer scheme  $(t, B)$  such that  $x = B + [1 - t]w\ell$ , where  $t \in (0,1)$  is the marginal tax rate and  $B > 0$  is the unconditional benefit payment.
- a. Find the optimal labor supply for someone with ability  $w$ . Will the high-skill person work more than the low-skill person? Will the high-skill person have higher disposable income than the low-skill person? Show that the condition for job market participation is  $w > \frac{[(1-\alpha)\alpha]B}{1-t}$ .
- b. If tax proceeds are only used to finance the benefit  $B$ , what is the government's budget constraint?
- c. Suppose that the mean skill in the population is  $\bar{w}$  and that the lowest skill is a fraction  $\gamma < 1$  of the mean skill. If the government wants to redistribute all tax



proceeds to finance the cash benefit  $B$ , what condition should the tax-transfer scheme  $(t, B)$  satisfy?

- d. Find the optimal tax rate if the government seeks to maximize the disposable income of the lowest skill worker subject to everyone working.

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