Chapter 10 Social Security[1]

10.1 Introduction

A typical social security system provides income during periods of unemployment, ill-health or disability, and financial support, in the form of pensions, to the retired. Although the generosity of systems varies among countries, these elements are present in all developed economies. The focus of this chapter is the economic implications of financial assistance to the retired. The overlapping generations economy provies to be ideal for this purpose.

In economic terms, the analysis of the aprt of the social security system that provides assistance during umemployment or ill-health is concerned with issues of uncertainty and insurance. Specifically, unemployment and ill-health can be viewed as events that are fundamentally uncertain, and the provision of social security is insurance cover against bad outcomes. In contrast, retirement is an inevitable outcome, or at least an option, once the retirement age has been reached. Insurance is therefore not the main issue (except for the problem of living for longer than accumulated wealth can finance). Instead, the issues that are raised with pensions are the potential transfers of resources between generations and the effect on savings behavior in the economy. Both of these issues require a treatment that is set within an explicitly intertemporal framework.

The pensions systems in many developed econmes are coing under pressure in a process that has become known as the “pensions crisis”. The roots of this crisis can be found in the design of the systems and the process of change in population structure. The potential extent of this crisis provides strong gound for holding the view that reform of the pension system is currently one of the most pressing economic policy challenges.

After describing alternative forms of pension systems, the nature of the pensions crisis is described. This introduces the concept of the dependency ratio and how this ratio links pensions and pension contributions. The economic analysis of social security begins with a study of their effect on the equilibrium of the economy. We will introduce the overlapping generations economy and showed how its competitive equilibrium may be inefficient. The potential for inefficiency opens up the possibility of efficiency-enhancing policy interventions. From this perspective we consider whether social security can be used to secure a gain in

[1] This chapter draws from Hindriks and Myles (2006, Chapter 20).

efficiency. The fact that a social security program may enhance efficiency can be understood from the effect of social security on the level of the capital stock. If a social security program has the form of forced saving, so that consumers are provided with greater second-period income than they would naturally choose, then the program will raise the capital stock through the increased savings it generates. This will be beneficial in an undercapitalized economy. Conversely, if the program simply transfers earnings from those who are working to those who are retired, savings will fall and hence the level of capital. These observations motivate the search for a social security program that can guide the economy to the Golden Rule.

The fall in the birth rate is one of the cuases of the pensions crisis. It is an interesting question to consider how a change in the birth rate affects the level of welfare at the steady state of an overlapping generations economy. We pursue this issue by considering how the birth rate affects the structure of the consumption possibility frontier, both in the absence and in the presence of a social security program. Social security may be beneficial for the economy, but there are issues of political economy connected with the continuation of a program. The introduction of a program with the structure observed in practice results in a transfer of resources toward the first generation of retired (they receive but do not contribute) and away from some of the generations that follow. This raises the question of how such a program is ever sustained, since each generation has an incentive to receive but not to contribute. The final analytical issue is to review the concept of Ricardian equivalence and its implications for social security. Ricardian equivalence is the observation that by changing their behavior, consumers are able to offset the actions of the government. We show the consequences this can have for social security and address the limitations for the argument. Finally, after having completed the analytical material, we return to address some of the proposals that have been made for the reform of social security programs.

10.2 Types of System

One defining characteristic of a social security system is whether pensions are paid from an accumulated fund or from current tax contributions. This feature forms the distinction between fully funded and pay-as-you-go social security systems. The economic effects, both in terms of efficiency and distribution, between these two polar forms of system are markedly different.

In a *pay-as-you-go* social security program the current contributions through taxation of those in employment provide the pensions of those who are retired. At any point in time the contributions to the system must match the pension payments made by the system. The social security systems presently in operation in the United States, the United Kingdom, and numerous other countries are broadly of this form. The qualifier "broadly" is used because, for example, although the US system owns some assets and could afford a short-term deficit, the assets would fund only a very short period of payments. At each point in time a pay-as-you-go system satisfies the equality

Benefits received by retired = Contributions of workers (1)

This equality can be expressed in terms of the number of workers and pensioners by

$$\beta R = \tau E \tag{2}$$

where τ is the average social security contribution of each worker, β is the average pension received, E the number of workers in employment, and R the number of retired. If there is a constant rate of growth of population, so that the workforce is a constant multiple of the retired population, then $E = [1+n]R$. Using this in (2) yields $\beta R = \tau[1+n]R$ or

$$\beta = [1+n]\tau \tag{3}$$

This relationship implies that the tax paid when young earns interest at rate n before being returned as a pension when old. Hence in a pay-as-you-go pension system the return on contributions is determined by the growth rate of population.

In a *fully funded system* each worker makes contributions toward social security via the social security tax, and the contributions are invested by the social security program. The program therefore builds up a pension fund for each worker. The total pension benefits received by the worker when retired are then equal to their contribution to the program plus the return received on the investment. Such a program satisfies the equalities

Pensions = Social security tax plus interest = Investment plus return (4)

The implication of this constraint is that the fund earns interest at rate r, so the pension and the tax are related by

$$\beta = [1 + r]\tau \qquad (5)$$

A fully funded social security system forces each worker to save an amount at least equal to the tax they pay. It remains possible for workers to save more if they choose to do so. If, in the absence of social security, all workers chose to save an amount in excess of the taxed levied by the program then, holding all else constant, a fully funded system will simply replace some of the private saving by an equivalent amount of public saving. In this case a fully funded sytem will have no effect on the equilibrium outcome. We explore this observation further when we discuss Ricardian equivalence in section 10.8. In more general settings with a variety of investment opportunities, the possibility must be considered that the rate of return on private savings may differ from that on public savings. When it does a fully funded system may affect the equilibrium. This point arises again in the analysis of pension reform.

Contrasting these two forms of system, it can be observed that a pay-as-you-go system leads to an intergenerational transfer of resources, from current workers to current retired, whereas a fully funded system can at most cause an intertemporal reallocation for each generation. This observation suggests that the two systems will have rather different welfare implications; these will be investigated in the following sections. In addition the pay-as-you-go system has a return of n on contributions and the fully funded system has a return of r. These returns will differ unless the economy is at the Golden Rule allocation.

Systems that fall between these two extremes are termed *non-fully funded*. Such systems make some investments, but the payments made in any given period may be greater than or less than the revenue, composed of tax payments plus return on investment, received in that period. The difference between payments and revenue will comprise investment, or disinvestment, in the pension fund.

10.3 The Pensions Crisis

Many countries face a pensions crisis that will require that their pensions systems be significantly reformed. This section identifies the nature and consequences of this crisis. Once the analysis of social security is completed, we return in section 10.9 to review a range of proposals for reform of the system in the light of this crisis.

The basis of the pensions crisis is threefold. First, the birth rate has fallen in most developed economies. Although immigration has partially offset the effect of this in some countries, there has still been a net effect of a steady reduction in the addition of new workers. The second effect is that longevity is increasing, since people are one average living longer. For any given retirement age, this is increasing the number of retired. Third, there is also a tendency for the retirement age to fall.

Table 1 Dependency Ratio (population over 65 as a proportion of population 15-64)

	1980	1990	2000	2010	2020	2030	2040
Australia	14.7	16.7	18.2	19.9	25.9	32.3	36.1
France	21.9	21.3	24.5	25.4	32.7	39.8	45.4
Japan	13.4	17.2	25.2	34.8	46.9	51.7	63.6
United Kingdom	23.5	24.1	24.1	25.3	31.1	40.4	47.2
United States	16.9	18.9	18.6	19.0	25.0	32.9	34.6

Source: OECD (http://www.oecd.org/dataoecd/40/27/2492139.xls).

The net effect of these three factors is that the proportion of retired in the population is growing, and it is this increase that is problematic. In general terms, as the proportion of the population that is retired rises, the output of each worker must support an ever larger number of people. Output per capita must rise just to keep consumption per capita constant. If output does not rise quickly enough, then productivity gains will be diluted and output per capita will fall. Furthermore, supporting the retired at a given standard of living will impose an increasing burden on the economy.

The size of this effect can be seen by looking at forecasts for the *dependency raito*. The dependency ratio measures the relative size of the retired population and is defined as the size of the retired population relative to the size of the working population. Table 1 reports the dependency ratio for a range of countries over the recent past and forecast for its development into the future. The countries in the table are typical with the dependency ratio forecast to increase substantially – in all cases the ratio more than doubles from 1980 to 2040. This

means that those working have to support an increasing proportion of retired. In some cases, for instance, Japan, the forecast increase in the dependency ratio is dramatic.

The consequence of the increase in the dependency ratio can be expressed in more precise terms by looking at the relationship between the contributions to pay for social security and the resulting level of social security. Using the identity (2) for a pay-as-you-go system and dividing through by E, the relationship between social security tax, pension, and dependency ratio is given by

$$\tau = \beta D \tag{6}$$

where D is the dependency ratio, R/E. Hence as D rises, τ must increase if the level of the pension β is to be maintained. Alternatively, the pension decreases as D increases if the tax rate is held constant. If some combination of such changes is not made, then the social security system will go into deficit if the dependency ratio increases. Neither a deficit, a falling pension, or an increasing tax are attractive options for governments to present to their electors.

To avoid such deficits, what these facts imply is that governments face a choice between maintaining the value of pension payments but with an ever-increasing tax rate, or they must allow the value of pensions to erode so as to keep the tax rate broadly constant. For example, the UK government has reacted to this situation by allowing the real value of the state pension to steadily erode. As shown in Table 2 the value of the pension has fallen from almost 40 percent of average earnings in 1975 to 26 percent in 2000, and it is expected to continue to fall, especially since the pension is now indexed to prices rather than earnings. These reductions have taken the value of the pension well below the subsistence level of income. Consequently pensioners with no other source of income receive supplementary state benefits to take them to the subsistence level. This reduction in the state pension has been accompanied by government encouragement of the use of private pensions.

Table 2 Forecasts for UK basic state pension

Date	Rate as percentage of average earnings
1975	39.3
1980	39.4
1985	35.8
1990	29.1
1995	28.3
2000	25.7

Source: UK, Department of Work and Pensions (http://www.dwp.gov.uk/asd/asd1/abstract/Abstrat2003.pdf).

In conclusion, the basis of the pensions crisis has been identified, and it has been shown haw this impacts on the state pensions that will be paid in the future. The depth of this crisis showns why social security reform is such an important policy issue. The chapter now proceeds to look at the economic effects of social security as a basis for understanding more about the arguments behind the alternative reforms that have been proposed.

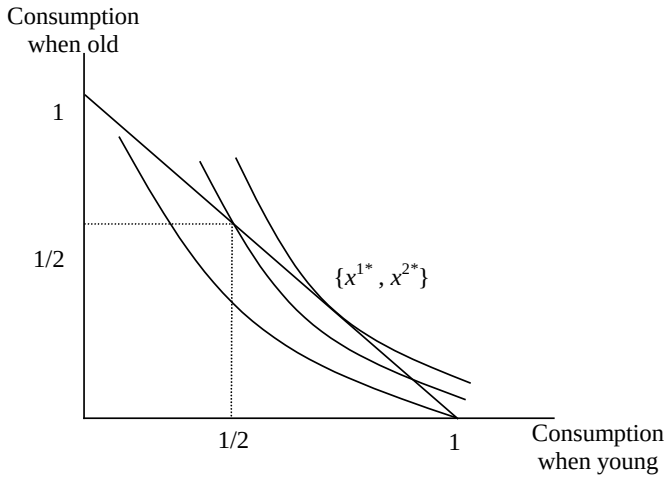
10.4 The Simplest Program

Having set out the issues connected with social security programs, the focus is now placed on their economic effects. The fundamental insight into the effect of social security upon the economy can be obtained using the simple model. In this economy there is no production but only the exchange of endowments. Although simple, this economy is still capable of supporting a role for social security.

In the economy under analysis, each consumer receives an endowment of one unit of the single consumption good in the first period of their life but receives no endowment in the second period. To simplify, the population is assumed to be constant. The equilibrium of this economy without any government intervention has the endowment entirely consumed when young so that there is no consumption when old. This has to be the equilibrium, since the old have nothing to offer the young in trade. This autarkic equilibrium is not Pareto-efficient, since all consumers would prefer a more even distribution of consumption over the two periods of life.

How can a social security program improve on the autarkic equilibrium? Consider a pay-as-you-go program that taxes each young consumer half a unit of consumption and transfers this to an old consumer. The lifetime consumption plan for every consumer then changes from the autarkic equilibrium consumption plan of $\{1,0\}$ to the new consumption plan of $\left\{\frac{1}{2},\frac{1}{2}\right\}$. Provided that the preferences of the consumers are convex, the new allocation is preferred to the original allocation. Since this applies to all generations, the social security system has achieved a Pareto improvement. This argument is illustrated in Figure 2. The Pareto improvement from the social security system is represented by the move from the lowest indifference curve to the central indifference curve.

Figure 2 Pareto Improvement and Social Security



In fact a far stronger conclusion can be obtained than just the ability of social security to achieve a Pareto improvement. To see this, note that the assumption of a constant population means that the per capita consumption possibilities for the economy lie on the line joining $\{1,0\}$ to $\{0,1\}$. In the same way that the Golden Rule was defined for the economy with production, the Golden Rule allocation can be defined for this economy as that which maximizes utility subject to the first- and second-period consumption levels summing to 1. Denote this allocation by $\{x^{1*}, x^{2*}\}$. The Golden Rule allocation can then be achieved by a pay-as-you-go social security program that transfers x^{2*} units of the consumption good from the young consumer to the old consumer.

These arguments show how social security can achieve a Pareto improvement and, for the simple exchange economy described, even achieve the Golden Rule allocation. The social security program is effective because of the intergenerational transfer that it engineers and the consequent revision in the consumption plans. The optimality result was built upon the use of a pay-as-you-go program. In contrast, a fully funded program cannot be employed, since there is no commodity that can be used as an investment vehicle. The form in which these conclusions extend to the more general overlapping generations economy with production is now discussed.

10.5 Social Security and Production

It has already been shown haw social security can obtain a Pareto improvement in an overlapping generations economy with no production. When there is production, a wider range of effects can arise, since social security affects the level of savings and hence capital accumulation. These additional features have to be accounted for in the analysis of social security.

The concept of the Golden Rule and its associated capital – labor ratio is well known. This showed that the optimal capital stock is the level which equates the rate of interest to the rate of population growth. If the capital stock is larger than this, the economy is dynamically inefficient and a Pareto improvement can be made by reducing it. When it is smaller, the economy is dynamically efficient, so no Pareto improvement can be made, but the economy is not in an optimal position. These observations then raise the questions: How does social security affect capital accumulation? Can it be used to move a nonoptimal economy closer to the Golden Rule?

To answer these questions, consider a social security program that taxes each worker an amount τ and pays each retired person a pension β. The program also owns a quantity K_t^s of capital at time t. Equivalently, it can be said to own k_t^s, $k_t^s = \frac{K_t^s}{L_t}$, of capital per unit of labor. A social security program will be optimal if the combination of τ, β, and k_t^s is feasible for the program and ensures the economy achieves the Golden Rule.

A feasible social security program must satisfy the budget identity

$$\beta L_{t-1} = \tau L_t + r_t k_t^s L_t - [k_{t+1}^s L_{t+1} - k_t^s L_t] \qquad (7)$$

which states that pension payments must be equal to tax revenue plus the return on capital holdings less investment in new capital. Since the population grows at rate n, in a steady state the identities $L_{t-1} = \frac{L_t}{1+n}$, $L_{t+1} = [1+n]L_t$ and $k_{t+1}^s = k_t^s \equiv k^s$ can be used in (7) to generate the steady-state budget identity

$$\frac{\beta}{1+n} = \tau + [r-n]k^s \qquad (8)$$

Nothing that the pension, β, which is received in the second period of life, is discounted in a consumer's budget constraint (since $x^1 + s = w - \tau$ and $[1+r]s + \beta = x^2$, it follows that $s = \frac{x^2 - \beta}{1+r}$), the budget constraint under the program can be written

$$x^1 = \frac{x^2}{1+r} = w - \tau + \frac{\beta}{1+r} \tag{9}$$

The condition describing consumer choice remains

$$\frac{U_1(x^1, x^2)}{U_2(x^1, x^2)} = 1 + r \tag{10}$$

Equilibrium on the capital market requires that private savings are equal to total capital less the capital owned by the social security program. This condition can be expressed as

$$w - x^1 - \tau = [1+n][k - k^s] \tag{11}$$

The choices of the representative firm do not change, so the conditions relating factor prices to capital still apply with

$$f'(k) = r \tag{12}$$

$$f(k) - kf'(k) = w \tag{13}$$

The steady-state equilibrium with the pension is the solution to equations (8) to (13).

The aim now is to investigate the effect that the social security policy can have on the equilibrium. To see why it may be possible to design a program that can achieve the Golden Rule, it should be noted that the failure of the competitive equilibrium without intervention to achieve efficiency results from the savings behavior of individuals leading to over- or under-accumulation of capital. With the correct choice of social security program the government can effectively force-save for individuals. This alters the steady-state level of the capital stock and hence the growth path of output.

In equations (8) to (13) there are five private-sector choice variables (k, x^1, x^2, w, and r) that are treated as endogenous, plus the three variables (β, τ and k^s) that describe the social security program. Given that there are six equilibrium conditions, the pension system can choose any two of the variables describing the program with the third determined alongside the endogenous variables. To analyze the system, it is simplest to treat β as endogenous and τ and k^s as exogenous.

The method of analysis is to assume that the Golden Rule is achieved and then to work back to the implications of this assumption. Consequently let $r = n$. From the firm's choice of capital, the Golden Rule is consistent with a capital stock that solves $f'(k^*) = n$ and hence a wage rate that satisfies $w = f(k^*) - k^* f'(k^*)$. The important observation is that with $r = n$, the budget constraint for the social security program collapses to

$$\frac{\beta}{1+n} = \tau + [r - n]k^s = \tau \qquad (14)$$

so a program attaining the Golden Rule must have the form of a pay-as-you-go system with $\beta = [1+n]\tau$. It is important to observe that any value of k^s is consistent with (14) when $r = n$, including positive values. This observation seems to conflict with the definition of a pay-as-you-go-system ,it does not add to or subtract from this level of capital. Instead, the return on the capital it owns is just sufficient to maintain it at a constant level. It remains true that along any growth path, including the steady state, a pay-as-you-go system cannot increase its capital holdings.

The value of the tax and capital stock of the program required to support the Golden Rule can now be found by using the fact that the program is pay-as-you-go to reduce the consumer's budget constraint to

$$x^1 + \frac{x^2}{1+r} = w \qquad (15)$$

Combining this constraint with the condition describing consumer choice indicates that the demand for first-period consumption must depend only on the wage rate and the interest rate, so $x^1 = x^1(w, r)$. Using the conditions for the choice of the firm, we have that the wage rate and

interest rate depend on the level of capital, so demand for first-period consumption can be written as

$$x^1 = x^1(w,r) = x^1(f(k) - kf'(k),\ f'(k)) = x^1(k) \tag{16}$$

The capital market-clearing condition can then be written as

$$w - x^1(k) - \tau = [1+n][k - k^s] \tag{17}$$

Using the conditions for the choice of the firm and evaluating at the Golden Rule level generates

$$\tau = [f(k^*) - k^* f'(k^*) - x^1(k^*) - [1+n]k^*] + [1+n]k^s \tag{18}$$

Condition (18) determines pairs of values $\{\tau, k^s\}$ that will achieve the Golden Rule.

Any pair $\{\tau, k^s\}$ that satisfies (18) will generate the Golden Rule, provided that the capital stock held by the program is not negative. For instance, if the program holds no capital, so that $k^s = 0$, then the value of the social security tax will be

$$\tau = f(k^*) - k^* f'(k^*) - x^1(k^*) - [1+n]k^* \tag{19}$$

Although the discussion to this point has implicitly been based on the tax, τ, being positive, it is possible that the optimal program may require it to be negative. If it is negative, the social security program will generate a transfer from the old to the young.

As an example, if $x^1(w,r) = \frac{w}{2}$ and $f(k) = k^\alpha$, then $k^* = \left[\frac{\alpha}{n}\right]^{1/[1-\alpha]}$ (see exercise (3) for the details of this derivation). Substituting these values into (19) gives

$$\tau = \left[\frac{\alpha}{n}\right]^{1/[1-\alpha]}\left[\frac{[1-\alpha]n}{2\alpha} - [1+n]\right] \tag{20}$$

If the rate of population growth is 5 percent, then the tax will be negative whenever

$$\frac{1}{43} < \alpha \tag{21}$$

For this example the tax rate is positive only for very small values of α.

The results have shown that attainment of the Golden Rule requires a pay-as-you-go social security system. By implication, a fully funded program will fail to attain the Golden Rule. In fact an even stronger result can be shown: a fully funded program will have no effect on the equilibrium. To demonstrate this result, observe that a fully funded program must satisfy the identity that the value of pension paid must equal the value of tax contributions plus interest, or

$$\beta L_{t-1} = \tau L_{t-1}[1 + r_t] = k^s L_t[1 + r_t] \tag{22}$$

Evaluated at a steady state,

$$\beta = \tau[1 + r] = k^s[1 + n][1 + r] \tag{23}$$

The substitution of (23) into the equilibrium conditions (8) to (13) shows that they reduce to the original market equilibrium conditions. The fully funded system therefore replaces private saving by public saving and does not affect the consumption choices of individual consumers. It therefore has no real effect on the equilibrium and, if the initial steady state were not at the Golden Rule, the fully funded social security program would not restore efficiency.

This analysis has demonstrated how a correctly designed social security program can generate the Golden Rule equilibrium, provided that it is not of the fully funded kind. A fully funded system does not affect the growth path. In contrast, a pay-as-you-go system can affect the aggregate levels of savings and hence the steady-state capital-labor ratio. This allows it to achieve the Golden Rule.

10.6 Population Growth

The fall in the rate of population growth is an important factor in the pensions crisis. While operating a simple pay-as-you-go program, a decreasing population size makes it harder to sustain any given level of pension. Observing this fact raises the general question of how the

level of welfare is related to the rate of population growth. This section addresses this issue both with and without a social security program.

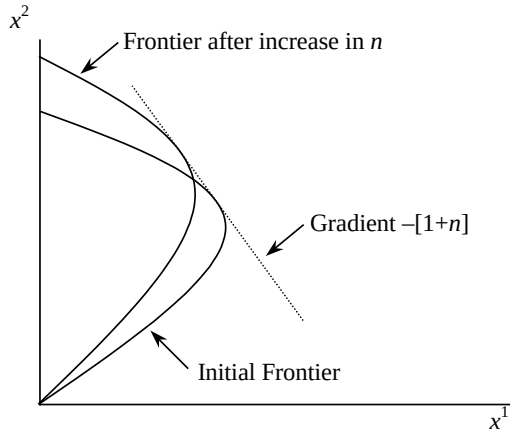
Assume first that there is no social security program in operation. Recall that the consumption possibility frontier is defined by a pair of consumption levels x^1 and x^2 that satisfy the conditions

$$x^1 = f(k) - kf'(k) - [1+n]k \tag{24}$$

and

$$x^2 = [1+n]k[1+f'(k)] \tag{25}$$

Figure 3 Population growth and Consumption Possibilities



The effect of a change in the population growth rate can be determined by calculating how it modifies this consumption possibility frontier. For a given value of *k*, it follows that $\frac{\partial x^1}{\partial n} = -k$ and $\frac{\partial x^2}{\partial n} = -k[1+f'(k)]$. Consequently, holding k fixed, an increase in the growth rate of population reduces the level of first-period consumption but raises the second-period level. This moves each point on the consumption possibility frontier inward and upward. Furthermore, when evaluated at the Golden Rule capital-labor ratio, these changes in the consumption levels satisfy

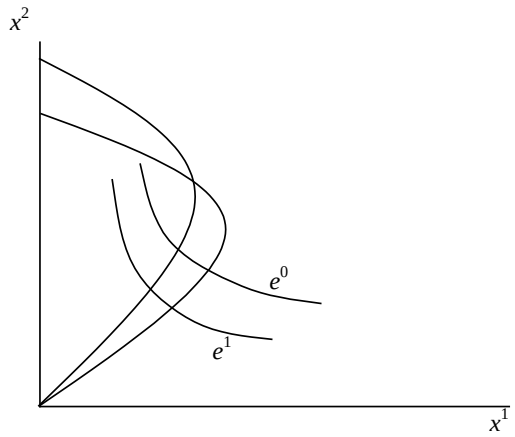
$$\frac{\partial x^2 / \partial n}{\partial x^1 / \partial n} = -[1+f'(k^*)] = -[1+n] \tag{26}$$

Hence, for a small increase in n, the point on the frontier corresponding to the Golden Rule equilibrium must shift upward along a line with gradient $-[1+n]$. The consequence of these calculations is that the shift of the consumption possibility must be as illustrated in Figure 3.

How the level of welfare generated by the economy is affected by an increase in n then depends on whether the initial equilibrium level of capital is above or below the Golden Rule level. If it is below, then welfare is reduced by an increase in the population growth rate – the capital stock moves further from the Golden Rule level. The converse occurs if the initial equilibrium is above the Golden Rule. This is illustrated in Figure 4 where the initial equilibrium is at e^0 with a capital – labor ratio below the Golden Rule. The equilibrium moves to e^1 following an increase in n. It can also be seen in the figure that if the initial equilibrium had been at a point on the frontier above the Golden Rule, then the upward shift in the frontier would imply that the new equilibrium moves onto a higher indifference curve.

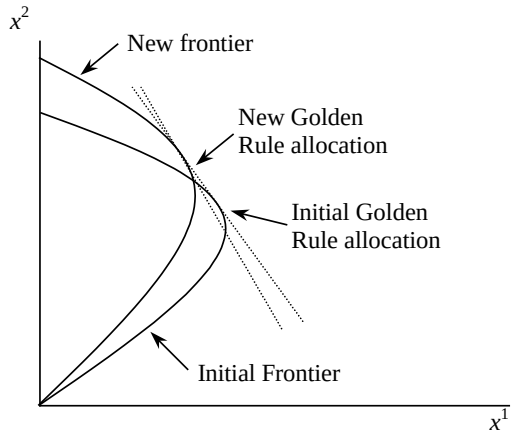
Now introduce a social security system and assume that this is adjusted as population growth changes to ensure that the Golden Rule is satisfied for all values of n. For a small change in n, the Golden Rule allocation moves along the line with gradient $-[1+n]$, as noted above. However, for large increases in n, the gradient of this line becomes steeper. This moves the Golden Rule equilibrium as shown in Figure 5 to a point below the original tangent line. As a consequence the increase in population growth must reduce the per capita level of consumption $x^1 + \frac{x^2}{1+n}$. Therefore, even with an optimal social security scheme in operation, an increase in population growth will reduce per capita consumption.

Figure 4 Population growth and Consumption Possibilities



The effect of changes in the rate of population growth are not as clear as the simple equilibrium identity for a pay-as-you-go program suggests. As well as the mechanics of the dependency ratio, a change in population growth also affects the shape of the consumption possibility frontier. How welfare changes depends on whether a social security program is in operation and on the location of the initial equilibrium relative to the Golden Rule. If an optimal program is in operation, then an increase in population growth must necessarily reduce the level of per capita consumption.

Figure 5 Population growth and Social Security



10.7 Sustaining a Program

In the simple economy without production, a social security program involving the transfer of resources between generations achieves a Pareto improvement. This raises the obvious question of why such a program will not always be introduced.

The basic nature of the pay-as-you go pension program described above is that the young make a transfer to the old without receiving anything directly from those old in return. Instead, they must wait until their own old age before receiving the compensating payment. Although these transfers do give rise to a Pareto improvement, it can be argued that it is not in the young consumer's private interest to make the transfer provided they expect to receive a transfer (Think of the generations playing a game. Giving a transfer cannot be a Nash equilibrium strategy). If the young consumers do not give their transfer but still expect to receive their pensions, then their consumption level will be increased. Clearly, this makes them better off,

so they will not wish to make the transfer. Since the social security system is not individually rational, how can the young be persuaded to consent to the imposition of the social security program?

Two different answers to this question will be considered. The first answer is based on altruism on the part of the young – they are willing to provide the transfer because they care about the old. This rationalizes the existence of a social security program but only by making an assumption that moves outside the standard economic framework of individual self-interest. The second answer works with the standard neoclassical model of self-interest but shows how the program can be sustained by the use of "punishment strategies" in an intertemporal game. It should be stressed that the fact that participation in a social security program is mandatory is not by itself a valid explanation of the existence of the program. All programs have to have willing participants to initiate them (so they must be individually rational at their introduction) and need continuing support to sustain them.

Altruism refers to feelings of concern for others beside oneself. It is natural to think that altruism applies to close family members, but it may also apply to concern for people generally.

Although the existence of altruism takes us outside the standard perspective of behavior driven by narrow self-interest, it need not affect the tools we employ to analyze behavior. What is meant by this is that altruism alters the nature of preferences but does not affect the fact that a consumer will want to achieve the highest level of preference possible. Consequently, given a set of altruistic preferences, the consumer will still choose the action that best satisfies those preferences subject to the constraint placed on their choices. The standard tools remain valid but operate on different preferences.

There are numerous ways to represent altruism, but one of the simplest is to view it as a consumption externality. Writing the utility of a consumer in generation t in the form

$$U_t = U(x_t^t, x_t^{t+1}, x_{t-1}^t) \tag{27}$$

gives an interpretation of altruism as concern for the consumption level, x_{t-1}^t, achieved by a member of the earlier generation (which is usually interpreted as the parent of the consumer). A very similar alternative would be to assume that

$$U_t = U(x_t^t, x_t^{t+1}, U_{t-1}) \tag{28}$$

so that altruism is reflected in a concern for the utility of the member of the earlier generation.

Both of these forms of altruism provide a motive for a social security program that transfers resources from the young to the old. Consider (27). A consumer with this utility function can be thought of as choosing their personal consumption levels x_t^t, x_t^{t+1}, and a transfer , τ , to the old consumer. The effect of the transfer is to arise the consumption level x_{t-1}^t, since the budget constraint of the old consumer is

$$x_{t-1}^t = [1 + r_t]s_{t-1} + \tau \qquad (29)$$

Provided that the marginal utility generated by an increase in x_{t-1}^t is sufficiently high, the consumer will willingly choose to make a positive transfer. In this sense the provision of social security has become individually rational because of altruism.

The second reason why transfers may be sustained is now considered. A rational explanation for participating in a social security program can be found in the fact that each young person expects a similar transfer when he is old. Young persons can then be threatened with having this removed if they do not themselves act in the appropriate manner. This punishment can sometimes (but not always) be sufficient to ensure that compliance with the social security program is maintained.

To give substance to these observations, it is best to express the argument using the language of game theory. The analysis so far has shown that the strategy to provide a transfer is not a Nash equilibrium. Recall that in the determination of a Nash equilibrium each individual holds the strategies of all others constant as they consider their own choice. So, if all others are providing transfers, it will be a better strategy not to do so but to still receive. If others are not transferring, then it is also best not to do so. Therefore not providing a transfer is a dominant strategy, and the individually rational Nash equilibrium must be for no transfers to take place.

These simple Nash strategies are not only ones that can be played. To motivate what else can be done, it is best to think about repeated games and the more sophisticated strategies that can be played in them. A repeated game is one where the same "stage" game is played once each period for an endless number of periods by the same players. The Prisoner's Dilemma given in the matrix in Figure 6 has the general features of the social security model. It is not exactly the same, since the social security model has many generations of consumers and not just the two given in the game.

Figure 6 Social Security Game

		Player 1	
		Contribute	Don't Contribute
Player 2	Contribute	5, 5	0, 10
	Don't Contribute	10, 0	2, 2

If both players contribute to social security, then a payoff of 5 is attained. If neither contributes, the payoff is only 2. This reflects the fact that the social security equilibrium is Pareto-preferred to the equilibrium without. However, the highest payoff is obtained if a player chooses not to contribute but the other does. When played a single time, the unique Nash equilibrium is for both players to choose Don't contribute – if the other contributes, then it pays not to. This reasoning applies to both players and hence the equilibrium. This equilibrium is inefficient and is Pareto-dominated by {Contribute, Contribute}.

The situation is completely changed if the game is repeated indefinitely. Doing so allows the efficient equilibrium {Contribute, Contribute}.

The situation is completely changed if the game is repeated indefinitely. Doing so allows the efficient equilibrium {Contribute, Contribute} to be sustained. The strategy that supports this is for each player to choose Contribute until their opponent chooses Don't contribute. Once this has happened, they should continue to play Don't contribute from that point on.

To evaluate the payoffs from this strategy, assume that the discount rate between periods is δ. The payoff from always playing Contribute is then

$$5 + 5\delta + 5\delta^2 + 5\delta^3 + \cdots = 5\left[\frac{1}{1-\delta}\right] \tag{30}$$

Alternatively, if Don't contribute is played unilaterally a temporary gain will be obtained but the payoff will then fall back to that at the Nash equilibrium of the single-period game once the other player switches to Don't contribute. This gives the payoff

$$10 + 2\delta + 2\delta^2 + 2\delta^3 + \cdots = 10 + 2\left[\frac{\delta}{1-\delta}\right] \tag{31}$$

Contrasting these, playing Contribute in every period will give a higher payoff if

$$5\left[\frac{1}{1-\delta}\right] = 10 + 2\left[\frac{\delta}{1-\delta}\right] \tag{32}$$

or

$$\delta > \frac{5}{8} \tag{33}$$

That is, {Contribute, Contribute} will be an equilibrium if the players are sufficiently patient. The reason behind this is that a patient player will put a high value on payoffs well into the future. Therefore the reduction to a payoff of 2 after the first period will be very painful. For a very impatient player, only the payoff of 10 will really matter and they are driven to Don't contribute.

The strategy just described is known as a "punishment strategy": the deviation from Contribute is punished by reversion to the inefficient Nash equilibrium. Although the punishment will hurt both players, the point is that it will not happen in equilibrium, since the optimal play with these strategies is always to choose Contribute when players are patient. In summary, in an infinitely repeated game, punishment strategies can be used to support efficient equilibria.

The same line of reasoning can be applied to the analysis of social security. What is different in this context is that the same players do not interact every period. Instead, it is a different pair of old and young consumers that meet in each period. However, the punishment strategy can still be employed in the following way: Each consumer when young will provide a transfer of size x to the old consumer that overlaps with them only if that old person alive at the same time provided a transfer in the previous period; otherwise no transfer is provided. If all generations of consumers play according to this strategy, then the transfers can be made self-supporting.

There remains one important limitation to this use of punishment strategies in the social security environment. To implement the strategy, each young consumer must know whether

the transfer was made in the period before they were alive. This issue does not arise in the standard application of punishment strategies, since the players are alive in all periods – they need only remember what happened in the previous period. Consequently some form of verification device is necessary to support the punishment strategy. Without the verification the only equilibrium is for there to be no transfers which is a Pareto-inferior outcome.

This discussion of pay-as-you-go social security has shown how such a system can be sustained even when there is a short-run incentive for consumers not to make the required transfers. The basis for this claim is that social security in an overlapping generations economy has the nature of a repeated game so that strategies that punish the failure to provide a transfer can be employed. What this analysis shows is that an apparent act of generosity – the gift of a transfer to the older generation – can be made to be rational for each individual. So the provision of social security may occur not through altruism but through rationality.

10.8 Ricardian Equivalence

Ricardian equivalence refers to the proposition that the government can alter an economic policy and yet the equilibrium of the economy can remain unchanged. This occurs if consumers can respond to the policy by making off-setting changes in their behavior that neutralize the effect of the policy change. In terms of the present chapter, Richardian equivalence holds when the government introduces, or changes, a social security system and yet the changes in individual behavior render the policy change ineffectual.

Such equivalence results have already featured twice in the text. On the first occasion, in the analysis of the private purchase of public goods, it was shown that by changing their purchases, the individuals could offset the effect of income redistribution. Furthermore it was also retional for the individuals to make the off-setting changes. The second case of equivalence arose in the derivation of the optimal social security program where it was noted that a fully funded system would not affect the capital – labor ratio. The explanation for this equivalence was that consumers react to a fully funded social security program by making a reduction in their private saving that ensures that total savings is unchanged.

The common feature of these examples is that the effect of the policy change and the off-setting reaction involves the same individuals. It is this that provides them with a direct incentive to modify their behavior. Clearly, this is true only of a social security system that is fully funded with a return equal to that on private savings. If social security is anything but

fully funded, a change in the system will affect a number of generations, since the system must be redistributive over time. In the case of pay-as-you-go, social security involves purely intertemporal redistribution. A change in a program can therefore affect consumers in different generations who need not be alive at the time the program is changed nor even be alive at the same time. At first sight, this would seem to mean that it cannot be possible for equivalence to hold. This argument is in fact correct given the assumptions made so far.

To obtain a basis for eliminating the effect of policy, it is necessary to link the generations across time so that something that affects one generation directly somehow affects all generations indirectly. The way that this can be done is to return to the idea of altruism and intergenerational concern. Intuitively we can think of each consumer as having familial forebears and descendents (or parents and children in simple language). This time we assume that each parent is concerned with the welfare of their children, and that their children are concerned with the welfare of the grandchildren. Indirectly, although they are not alive at the same time in the model, this makes the parents concerned about the grandchildren. What effect does this have? It makes each family act as if it was a dynasty stretching through time, and its decisions at any one moment take into account all later consequences. A change in a social security program then causes a reaction right through the decision process of the dynasty.

To provide some details, let the utility of the generation born at time t be

$$U_t = U(x_t^t, x_t^{t+1}, \tilde{U}_{t+1}) \tag{34}$$

It is the term $\tilde{U}_{t+1}$ that represents the concern for the next generation. Here $\tilde{U}_{t+1}$ is defined as the maximum utility that will be obtained by the children, who are born at $t+1$, of the parent born in t. The fact that the family will act as adynasty can then be seen by substituting for $\tilde{U}_{t+1}$ to give

$$U_t = U(x_t^t, x_t^{t+1}, \tilde{U}(x_{t+1}^{t+1}, x_{t+1}^{t+2}, \tilde{U}_{t+2})) \tag{35}$$

If this substitution is continually repeated, then the single parent born at t ultimately cares about consumption levels in all future time periods.

By this fact it is now possible to demonstrate that Ricardian equivalence applies to social security in these circumstances. Consider an initial position with no social security program and no population growth (so $n = 0$). The consumer at t reflects his concern for the

descendent by making a bequest of value b^t. Hence the consumption level in the second period of life is

$$x_t^{t+1} = s_t[1 + r_{t+1}] - b_t \tag{36}$$

and that of his descendent is

$$x_{t+1}^{t+1} = w_{t+1} + b_t - s_{t+1} \tag{37}$$

Assume that a social security program is now introduced and that each consumer has one descendent. Under the terms of the program, young consumers are taxed an amount τ to pay a pension of equal value to old consumers. Then the consumption level of each parent satisfies

$$x_t^{t+1} = s_t[1 + r_{t+1}] + \tau - \hat{b}_t \tag{38}$$

and that of his descendent

$$x_{t+1}^{t+1} = w_{t+1} + \hat{b}_t - \tau - s_{t+1} \tag{39}$$

But note that if the bequest is changed so that $\hat{b}_t = b_t + \tau$, the same consumption levels can be achieved for both the parent and the child as for the case with no pension. Furthermore, since these consumptions levels were the optimal choice initially, they will still be the optimal choice. So the old consumer will make this change to their bequest, and the social security scheme will have no effect.

The conclusion of this analysis is that the change in the bequest can offset the intertemporal transfer caused by a social security system. Although this was only a two-period system, it can easily be seen that the same logic can be applied to any series of transfers. All that the dynasty has to do is adjust each bequest to offset the effect of the social security system between any two generations. The outcome is that the policy has no effect. This is the basic point of Ricardian equivalence.

It must be noted that there are limitations to this argument. First, it is necessary that there be active intergenerational altruism. Without this there is no dynastic structure, and the offsetting changes in bequests will not occur. In addition the argument only works if the initial bequest is sufficiently large that it can be changed to offset the policy without becoming negative. Does it apply in practice? We clearly observe bequests but many of these may be unintentional and occur due to premature death.

The concept of Ricardian equivalence can be extended into other areas of policy. Closely related to social security is the issue of government debt, which is also an intergenerational transfer (but from children to parents), and its effects on the economy. This was the initial area of application for Ricardian equivalence, with changes in bequests offsetting changes in government debt policy. Furthermore, if links are made across households, it becomes possible for changes in household choices to offset a policy that causes transfers between households. This has lead to the question of whether "everything is neutral". The answer depends on the extent of the links.

10.9 Social Security Reform

The basic nature of the pensions crisis facing a range of economies was identified in section 3: increasing longevity and the decline in the birth rate are combining to increase the dependency ratio. Without major reform or an unacceptably high increase in tax rates, the pension programs will either go into deficit or pay a much reduced pension. A variety of reforms have been proposed in response to this crisis. Some of these are now briefly reviewed.

Underlying the crisis is the fact that the pension systems are essentially of the pay-as-you-go form. With such a structure an increase in the dependency ratio will always put pressure on the pension system. The reform most often discussed in the United States is for the social security system to move toward a fully funded structure. Once the system reaches the point of being fully funded, pensions are paid from the pension fund accumulated by each worker. This breaks the identity relating pensins to the dependency ratio. A fully funded system can operate either as a government-run scheme or on the basis of private pensions. We comment on this choice below. For now, we note that as well as reducing the real value of the pension, the UK government has moved in the direction of a fully funded program by encouraging the use of private pensions. The difficulty with this approach is that it relies on workers making adequate provision for their retirement – and there is much evidence that this is not the case.

If an economy were to reform its pension system, it would take some time to transit from the pay-as-you-go system to the fully funded system. The reform requires that a capital fund be established that takes a period of investment. Furthermore the pay-as-you-go system cannot be terminated abruptly. Those already retired will still require the provision of their pensions, and those close to retirement will have too little time to invest in a pension fund and so will require the continuation of the pay-as-you-go element. These facts imply that those who are in work during the transition process will have to both pay the pensions of the retired and pay to finance their own pension fund. In simple terms, they are paying for two sets of pensions and fare badly during the reform process. At the very least, this suggests that there could be significant political pressure against the proposed reform.

It is interesting to consider the extent to which social security provision is determined by political considerations. Evidence on this is provided by Mulligan, Gil, and Sala-i-Martin in their analysis of social security and democracy. Their key finding is that social security has little to do with the voting process because countries without voting still supply public insurance in the same way. They even observe for Chile that most of the growth in social security spending occurred under nondemocratic regimes, and payroll taxes reached extremely high levels under General Pinochet. In fact they report on nine dynamic case studies – Greece, Portugal, Spain, Italy, Argentina, Brazil, Chile, Peru, and Uruguay – for the period 1960 to 1990. The countries were selected on the basis of their extreme political changes over this period. With the exception of Greece, it is found that formerly nondemocratic countries do not, relative to their democratic neighbors, change their social security programs after experiencing democracy (in terms of the amount of public insurance spending, and the design of tax and benefit formulas). Similarly formerly democratic countries do not change their program when they become nondemocratic. Furthermore multiple regression studies fo the determinants of public insurance spending, controlling for population age and per capita income, find neither a significant partial correlation between democracy and social insurance spending (relative to GDP), nor a significant interaction between democracy and the other variables in a spending regression. These results suggest that the role of political constraints on social security may sometimes be overstated.

It is useful to stress a classical error that often accompanies discussion of switching to a fully funded system. The error arises from comparing the likely rates of return on personal accounts with those paid under the current pay-as-you-go system. The proposition that suggests switching to the fully funded system to benefit from the opportunity for higher rates of return is a fallacy. Compare first the real rate of return delivered by the existing social

security over the last decades (about 2 percent per year) with the risk-free rate of return of 3 to 4 percent that personal accounts could guarantee by holding inflation-indexed US Treasury securities. The return in the existing system is only 2 percent because of the arithmetic of the pay-as-you-go system.

Suppose that all workers contribute a fixed fraction of their income to social security. The key point is that today's contributions cover the pension benefits of today's retirees, who were the previous generation of workers that contributed. The total return corresponds to the growth of overall wage income (population plus productivity growth rate). Thus the real rate of return in an ongoing system is 2 percent if the economy grows at that rate in the long run.

There is a fallacy to the argument that 3 to 4 percent yield on personal accounts is better. The fallacy is that the return on the existing system is low because workers start with a liability to provide for the retirees of the previous generation. If the workers could defect from their liability to the current elderly, they could earn a rate much higher than 2 percent, even if no personal accounts were introduced. But, of course, no one wants to cut the benefits of the elderly who contributed to the system throughout all their working lives. To put it differently, the opportunity of a higher rate of return with personal accounts comes from the misleading feature that they come with no obligation to raise the pensions of the current elderly. This is the feature that accounts for the differences in returns. Moreover the higher expected return is offset by at least the perception of greater risk. This is not to say that the returns in the existing system are risk-free. The major risk in the present system is probably that pension benefits paid in the future are subject to the political whims of future governments.

The distributional effects of a reform from a pay-as-you-go system to a fully funded system are illustrated by the simulation reported in Table 3. This simulation determines the growth path of an economic model for a reference case in which the state pension is held constant. Applied to the United Kingdom, the model assumes that the value of the pension is 20 percent. A reform is then considered where an announcement is made in 1997 (the year the research was conducted) that the state pensin will be steadily reduced from the year 2020 until being phased out in 2040. The aim of the long period between announcement and reduction is to allow for adjustment in private behavior. The removal of the state pension implies that private savings will have to increase to compensate.

The negative ages in the first column of Table 3 refer to consumers who had not yet been born in 1997, so a consumer with age – 10 in 1997 will be born in 2007. The numbers in the second and third columns shows the percentage by which the lifetime wage of that age group would need to be changed in the reference case to give the same level of welfare as in the

reform case. Hence the value of –1.1 for the age group 40 to 50 in the United Kingdom shows that this group is worse off with the reform – a reduction of 1.1 percent of their wage in the base case would give then the same walfare level as in the reform case.

Table 3 Gains and losses in transition

Age in 1997	United Kingdom	Europe
>57	0	0
50-57	-0.09	-0.6
40-50	-1.1	-2.3
30-40	-3.0	-5.7
20-30	-3.8	-7.2
10-20	-2.3	-4.2
0-10	0.7	1.7
-10-0	3.95	9.2
-20- -10	6.5	15.7
-40- -30	7.4	18.7
<-40	7.2	18.9

The values in Table 3 show that the pension reform hurts those early in life who must pay the pensions of the retired and pay into their own retirement fund. Ultimately the reform benefits consumers in the long run. The long-run gain comes from the fact that the reduction in the pension leads to an increase In private saving. Private saving has to be invested, so there is also an increase in the capital stock. The consequence of this capital stock increase depends on the initial level of capital compared to the Golden Rule level. In the simulations, capital is initially below the Golden Rule level and remains so throughout the transition. But since this is moving the economy closer to the Golden Rule, there is ultimately a gain in welfare for later generations. The structure of the gains and losses also illustrates the political problem involved in implementing the reform: those who must vote in favor of its implementation are those who lose the most. This political problem will be exacerbated by the aging of the electorate that is expected over the next 50 years. Estimates of the age of the median voter are given in Table 4. These estimates reveal that the age of the median voter is likely to rise form the midforties to the midfifties. So the electorate will become dominated by the age group that will lose most if the pension system reform is undertaken.

Table 4 Age of the Median Voter

Country	Year	Age of median voter
France	2000	43
	2050	53
Germany	2000	46
	2050	55
Italy	1992	44
	2050	57
Spain	2000	44
	2050	57
United Kingdom	2000	45
	2050	53
United States	2000	47
	2050	53

Source: Galasso and Profeta (2004)

It has already been noted that a fully funded scheme run by the government is equivalent to a system of private pension provision. This is only strictly true in an economy, like the overlapping generations model we have studies, that has a single capital good. In a more practical setting with a range of investment assets, the equivalence will only hold if the same portfolio choices are made. Moving from a pay-as-you-go system to a fully funded system run by the government raises the issue of the portfolio of investments made by the pension fund. In the United States the assets of the fund are invested entirely in long-term Treasury debt. Such debt is very low risk, but as a consequence it also has a low return. This is not a portfolio that any private sector institution would choose, except one that is especially risk-averse. Nor is it one that many private investors would choose. Permitting the social security fund to invest in a wider portfolio opens the possibility for a higher return to be obtained but introduces questions about the degree of investment risk that the pension fund could accept. In addition changing the portfolio structure of the social security fund could have significant macroeconomic consequences because of its potential size.

A further issue in the design of a pensions system is the choice between a *defined contributions* system and a *defined benefits* system. In a defined contribution scheme, social security contributions are paid into an investment fund, and at the time of retirement the accumulated fund is annuitized. What annuitized means is that the fund purchases an annuity that is a financial instrument paying a constant income to the purchaser until his date of death. In a defined benefits scheme, contributions are made at a constant proportion of income and the benefit is a known fraction of income at retirement (or some average over income levels in years close to retirement).

The consequences of these differences are most apparent in the apportionment of risk under the two types of system. With a defined contributions system, the level of payment into the

pension fund is certain for the worker. What is not certain is the maturity value of the pension fund, since this depends on the return earned on the fund, or the pension that will be received, since this depends on the rate offered on annuities at retirement. All risk therefore falls upon the worker. With a defined benefits system, the risk is placed entirely on the pension fund, since it must meet the promises that have been made. The pension fund receives contributions that it can invest, but it runs the risk that the returns on these investments may not meet pension commitments. This is currently the situation of the US fund where the forecast deficit is a consequence of the defined benefits it has promised.

Assuming that a defined contributions scheme is chosen, there is a further reform that can be made. In the discussion of the simulation it was noted that the reform involved a move from a state pension scheme to private pension schemes. In a defined contribution system there is no real distinction between state and private schemes in principal. When put into practice, distinctions will arise in the choice of investment portfolio, the returns earned on the portfolio and the transactions costs incurred in running the scheme. If moving to a fully funded system pensions, the choice between state and private become a real issue. One option is to use a public fund, either directly administered or run privately after a competitive tendering process. Alternatively, a limited range of approved private funds could be made available. Both choices would lead to a problem of monitoring the performance of the schemes given the fundamentally uncertain nature of financial markets. In addition seeking low transactions costs could prove detrimental to other areas of performance. A final option is to make use of an open selection of private investment funds. Doing so relies on investors making informed choices between the providers and between the funds on offer to ensure that therisk characteristics of the fund match their preferences. Such a scheme will not work with poorly informed investors and may run foul of high transactions costs. Both of these have been significant problems in the United Kingdom where “misselling” – the selling of pensions plans with inappropriate risk characteristics for the purchasers – and high costs have accompanied the move toward the private financing of pensions.

The reform of pensions systems is an issue with much current policy relevance. A range of reforms have been suggested to cope with the forecast change in the dependency ratio. Some of these represent adjustments to the structure of pension schemes, whereas others seek a major reorganization of pension provision.

10.10 Conclusions

Social security in the form of pensions is important both in policy relevance and for its effect on the economy. The generosity of a pension scheme has implications for individual's savings behavior and, in the aggregate, for capital accumulation. Since an economy may reach an inefficient steady state, the designs of pension schemes have an impact on economic efficiency.

Demographic changes and changes in employment behavior are currently putting existing state pension schemes under pressure because of their fundamentally pay-as-you-go nature. Reform proposals have focused on a move to a fully funded system, but such a reform can be detrimental to the welfare level of consumers living during the transition period.

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Exercises

1) If you work for 30 years and wish to retire for 15 years on 50 percent of your working income, how much of your income must be saved when working? (Assume that the interest rate and income when working are constant, and that there are no taxes.)
2) Assume that all consumers have preferences represented by $U = x_t^t x_t^{t+1}$. If the budget constraint is $x_t^t + \frac{x_t^{t+1}}{[1+r_{t+1}]} = w_t - \tau + \frac{\beta}{[1+r_{t+1}]}$, determine the relationship between the level of savings and the parameters τ and β of the social security program. Assuming that $y_t = k_t^\alpha$, find the steady-state level of the capital-labor ratio. Solve for the social security programs that lead to the Golden Rule. Show that none of these programs is fully funded. What is the form of the pay-as-you-go system that achieves the Golden Rule?
3) For the economy described in exercise 2), relate the structure of social security programs achieving the Golden Rule to dynamic efficiency and inefficiency.
4) A common policy is to make pension contributions tax deductible and to insist that the pension fund be annuitized on retirement. Explain the logic behind this policy.
5) Consider a consumer with true preferences $U = \left[x_t^t\right]^\alpha \left[x_t^{t+1}\right]^{1-\alpha}$. Rather than acting on the basis of these preferences, the consumer is myopic and does not realize the true value of second-period consumption. The myopic preferences are given by $U = \left[x_t^t\right]^\alpha \left[\rho x_t^{t+1}\right]^{1-\alpha}$, $\rho < 1$.
   a. determine how the level of saving depends on ρ.
   b. How does the level of welfare measured by true preferences depend on ρ?
   c. Ssume that there is a population of *H* consumers who act according to these myopic preferences and that the equilibrium interest rate is $r_{t+1} = a - bs_t$, where s_t is the total level of savings in the economy. Can myopia ever increase the consumers' true utilities?
   d. Does this form of myopia provide a justification for social security?
6) For the myopia model, assume a pay-as-you-go pension system. The consumers over estimate the generosity of the pension scheme and believe that the pension, β, and the social security tax, τ, are related by $\beta = (1+\phi)\tau$, where $\phi > 0$. There is no population growth, so the true value of the pension is $\beta = \tau$. What effect does an increase in ϕ

have on savings? Does welfare increase or decrease in ϕ? Should we have the social security program when consumers have this from of myopia?

7) Consider an economy where individuals live for two periods only. Their utility function over consumption in periods 1 and 2 is given by $U = 2\log(C_1) + 2\log(C_2)$, where C_1 and C_2 are period 1 and period 2 consumption levels respectively. They have labor income of \$100 in period 1 and labor income of \$50 in period 2. They can save as much of their income in period 1 as they like in bank accounts, earning interest rate of 5 percent per period 2.
   a. What is each individual's lifetime budget constraint? If they choose consumption in each period so as to maximize their lifetime utility subject to their lifetime budget constraint, what is the optimal consumption in each period? How much do the consumers save in the first period?
   b. Suppose that the government introduces a social security system that will take \$10 from each individual in period 1, put it in a bank account, and transfer it back to them with interest in period 2. What is the new lifetime budget constraint? What is the effect of this social security system on private savings? How does the system affect total savings in society?

8) Consider the previous exercise and suppose that the introduction of social security induces the individuals to retire in period 2. So they receive no labor income in period 2.
   a. What is the new optimal consumption in each period? How much do the consumers save? How does it compare with previous exercise? Explain.
   b. Now building on this example, should the actual social security system lead to early retirement? Why or why not? What is the evidence on the impact of social security on the retirement decision in the United States and elsewhere?

9) Consider an individual who lives for two periods and has utility of lifetime consumption $U = \log(C_1) + \frac{1}{1+\delta}\log(C_2)$, where C_1 and C_2 are the consumption levels in the first and second period respectively, and δ, $0 < \delta < 1$, denotes the per period discount rate. Suppose that the individual has an income of $Y_1 > 0$ in the first period and no income in the second period, so $Y_2 = 0$. He can transfer some income to the second period at a before-tax rate of return of r, so saving $\$S$ in the first period gives $\$[1+r]S$ in the second period. The government levies a capital tax at rate τ on capital income received

in the second period. The tax proceeds are paid as a lump-sum transfer to the following generation. The present generation does not care about the next one.

a. What is the lifetime consumption profile of this individual? What is his lifetime indirect utility function expressed as a function of Y_1 and $[1-\tau]r$?

b. Evaluate the change in initial income Y_1 that is required to compensate the individual for the welfare loss doe to the capital income tax τ .

c. What is the impact of a tax rate change on consumption level in the first period? And in the second period? What conclusion about the welfare cost of capital income taxation can you draw from your analysis?

10) Consider an economy where individuals live for two periods only. They have the utility function over consumption in period 1 (C_1) and period 2 (C_2) given by $U = 2\log(C_1) + 2\log(C_2)$. The labor income of each individual in period 1 is fixed at \$10, and there is no labor income in period 2. They can save as much of their income in period 1 as they like in bank accounts, earning interest rates of 200 percent per period (recall, a period is the entire active life). The income tax rate is 50 percent, which is used to pay back the public debt inherited from the past generation.

a. Derive the optimal lifetime consumption profile of this consumer. What would be the consumption profile without income tax?

b. Suppose that a "retirement saving program" is introduced allowing each consumer to save up to 20 percent in the first period in a tax-free account. Compare the lifetime budget constraints with and without the retirement savings program.

c. Derive the optimal lifetime consumption profile with the retirement savings program. Explain the impact of this program on private savings.

d. Now suppose that the retirement savings program in part b is replaced by a new savings program taxing investment income on the first 50 percent of savings and exempting any savings In excess of 50 percent from taxation. Draw the budget set associated with this program, and find the optimal lifetime consumption profile. Explain the difference with the program in part b.

e. If the threshold for tax-exempt savings in part b is increased from 50 to 51 percent, how would this affect private savings? How does this affect total savings in society?

11) What are the advantages and problems related to a reform of social security that consists of switching to individual annuitized accounts?