Optimal Timing for Banks’ Write-off Decisions  
under the Possible Implementation of a Subsidy Scheme: 
A Stochastic Analysis

Naohiko Baba*

Key Words: Write-off, Non-Performing Loans, Dynamic Programming, 
Real Options, Reputation, Forbearance Policy

JEL Classification: G21, G28

Abstract

This paper provides a formal model that investigates the optimal timing for 
banks to write-off their non-performing loans. The motivation comes from 
the recent episodes of Japanese banks, which have been slow to clean up 
their non-performing loans in the wake of the collapse of the “bubble 
economy” in the early 1990s. A real options approach is employed to 
measure the value of the rationality of the “forbearance policy”. Uncertainty 
is assumed to arise from the following sources: (i) the re-investment return 
from freeing up funds by write-offs, (ii) the liquidation loss, (iii) the 
possible implementation of a subsidy scheme, and (iv) the reputational 
repercussions from not writing off their non-performing loans immediately. 
This paper attaches particular importance to the uncertainty from the 
possible implementation of the subsidy scheme to explore its desirable 
features. Also, the paper examines the possible role of monetary policy in 
boosting the banks’ incentive to write off.

* economist, Research Division I, Institute for Monetary and Economic Studies, Bank of Japan. 
  e-mail: naohiko.baba@boj.or.jp.

I am grateful to Takeshi Amemiya, Joseph Haubrich, Takeo Hoshi, Yukinobu Kitamura, Kotaro Tsuru, 
and staff members of the Bank of Japan for their helpful comments and suggestions. Any remaining 
errors are entirely mine. Views expressed in this paper are those of the author and do not reflect those of 
the Bank of Japan.
I. Introduction

The Japanese economy has been in a prolonged recession, with many banks loaded with large-scale non-performing loans. Its primary cause is a sharp fall in land prices that began in late 1991. The Ministry of Finance (MOF), once the primary regulatory agency\(^1\) in Japan, was slow in reacting to this problem\(^2\). Recently, harsh criticism is directed to the so-called “forbearance” or “buy-time” policy adopted by the MOF, which allowed the banks to keep non-performing loans on their balance sheets in the hope that the economy and the real estate market would recover in the not-too-distant future\(^3\).

As argued by many authors including Cargill (2000), the failure to promptly solve the non-performing loan problem generated a credit crunch. It has contributed to stagnant or declining real GDP growth for almost a decade and has interfered with the efforts by the Bank of Japan (BOJ) to stimulate the economy.

Note, however, that purely from the banks’ perspective, the forbearance policy itself can be a rational choice. It is because under the stochastic circumstances with a potentially large loss associated with write-off decisions, the option to wait (delay write-offs) should have a value. Hence, in deciding whether to write off their non-performing loans immediately the banks should weigh between the value of the option to wait and the (net) value of carrying out write-off immediately.

Hoshi (2000) pointed out that under the condition that the banks are not required by the authorities to disclose the true magnitude of their non-performing loans, they do not have an incentive to dispose of their non-performing loans. Rather, they tend to increase lending to riskier projects. The true problem caused by the non-performing loans, Hoshi (2000) argues, is that the banks lose an incentive to lend to (possibly manufacturing) corporations with prospective projects, which might damage the intermediation function of banks\(^4\). If the social

\(^1\) In June 1998, the Financial Supervisory Agency (FSA) was established, which is directly under the prime minister and independent of the MOF. The functions of monitoring financial markets and supervising financial institutions were transferred from the MOF to the FSA. In July 2000, the FSA was upgraded to the Financial Agency (FA), which is responsible for wide-ranging matters related to the financial system. The MOF became the Ministry of Treasury, which is mainly in charge of budgetary and taxation matters.

\(^2\) Ito (2000) pointed out that bank analysts at brokerage firms began to discuss a potential non-performing loan problem in 1992-93, but the MOF was reluctant to force banks to disclose the specific amounts of their non-performing loans.

\(^3\) As explained by Ueda (2000), with the exception of a brief period around 1975, postwar Japan had never experienced a decline in land prices. Thus, no time series analysis of Japanese land price through the late 1980s would have given a high probability of a sharp fall in land prices in 1990.

\(^4\) Hoshi (2000) also argues that even if the economy is in the state of a liquidity trap, the credit channel works to ensure the effectiveness of monetary policy.
costs caused by the damage of the intermediation function of banks outweigh the costs of a subsidy, then it might become justification for the use of public funds to urge self-help efforts of banks to clean up their non-performing loans. At last, in March 1998 and 1999, the Japanese government injected public funds into some banks as capital support. The use of public funds was justified on the basis that a prompt resolution of the non-performing loan problem is beneficial to the economy as a whole in the long run.

As pointed out by Corbett and Mitchell (2000), however, one of the most puzzling and interesting facts regarding the recent bank rescue package in Japan is that the government’s offer was not welcomed by the rescued banks. The key to understanding this seemingly puzzling fact lies in the existence of asymmetric information and the possible reputation problem.

Typically, asymmetric information exists between the banks and the public regarding the true magnitude of non-performing loans on their balance sheets. To be more specific, by the public I mean shareholders, depositors, and other participants in the financial asset markets. This asymmetric information creates an incentive for the banks to roll over their non-performing loans to disguise their true financial standing. The government, which is generally in a position to grasp the true standing of the banks’ balance sheets via bank examinations and monitoring on a regular basis, is able to rescue the banks by providing capital supports.

Corbett and Mitchell (2000) further argue that the banks may decline the rescue offer from reputation concerns. This is because the acceptance of such offer may force the banks to carry out write-offs of non-performing loans, which reveals the hidden information to the public.

More generally, however, the banks’ reputation depends on what the market participants infer from the banks’ write-off decision. In fact, in many occasions, stock prices rose for the Japanese banks that announced that they would be increasing their write-offs.

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5 It should be noted, however, that the bank regulator may pursue self-interest rather than social welfare. Boot and Thakor (1993) examines this possibility by introducing uncertainty about regulator’s ability to monitor banks’ asset choice.
6 In March 1998, a first wave of capital injection was made to 21 banks. Its amount was less than 2 trillion yen. The size of the injections, in the form of subordinated loans and preferred stocks, was almost the same across the banks and insufficiently. In April 1998, the scheme called “the prompt corrective action (PCA) rule” took effect, which requires the banks with capital ratios below certain levels to restructure or even cease operations. Also in October 1998, an agreement to appropriate 60 trillion yen of public funds to strengthen the financial system and recapitalize the banks were agreed. Recapitalization was done in March 1999. See Ito (2000) for further details.
7 In fact, attempts to identify the scale of the problem have been hampered by lack of disclosure and frequent changes in the definition of ‘non-performing’ loans.
8 For example, recall a surge in the stock prices of Sanwa, Tokai, and Asahi after they released the news of large-scale write-offs for FY 2000.
These episodes suggest that there could be reputational repercussions from not writing off non-performing loans. In this paper, I model the reputation concerns as a fear of a rise in the fund-raising (outside finance) cost. And based on these episodes, I introduce the fear only in specifying the value of the option to wait (delay write-offs), not in specifying the value of immediate write-offs.

Motivated by the discussion above, this paper attempts to evaluate (i) the optimal timing for the banks’ write-offs decisions and (ii) how much compensation or subsidy is needed for the banks to carry out write-offs as self-help efforts. In this paper, to measure the value of the option to wait, I use the so-called real options approach. More specifically, I treat the banks’ optimal write-off decisions as continuous-time problems with the infinite horizon.

I know well that the capital injections done by the Japanese government in 1998 and 1999 were not literally a subsidy for write-offs. Despite this, it is of some benefit to regulatory authorities to conduct this kind of experiments as far as they believe that cleaning up the non-performing loan problem is a prerequisite to stabilizing the financial system as a whole.

The sources of uncertainty assumed in this paper are as follows (see Figure 1):
(i) the re-investment return from freeing up funds (collected by liquidating collateral): the banks can lend (invest) the funds to prospective projects (in financial assets);
(ii) the loss caused by carrying out write-offs: this is closely linked to the land prices since real estate collateral has been extensively used when bank loans were contracted;
(iii) the future implementation of a subsidy scheme by the government;
(iv) the reputational repercussions from not writing off immediately, which takes the form of an upward jump in the fund-raising cost.

Another (secondary) aim of this paper is to provide a solid microeconomic foundation to the question of why loans to manufacturing corporations have stagnated in Japan. As explained by Hoshi (2001), during the bubble period, banks eagerly shifted into collateralized lending. For them, lending to real estate and construction corporations was particularly

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9 See, for example, Boot and Greenbaum (1993) in this regard.
10 In reality, the write-off procedure consists of the following two steps. The first step is termed an indirect write-off. In this step, the bank only reports an estimated loss, but does not actually liquidate collateral. The second step is a direct write-off, which makes the bank liquidate collateral and the actual amount of the associated loss is fixed. The approach in this paper skips the first step, the indirect write-off, for simplicity.
11 A simpler class of models with only two or three discrete decision points might suffice as far as the qualitative analysis is concerned. It should be noted, however, that that class of models is based on the unrealistic assumption that there is no uncertainty after the two or three units of time. In most markets, future returns are always uncertain, and the degree of uncertainty increases with the time horizon.
12 In other words, this is an opportunity cost arising from keeping non-performing loans on the balance sheet.
promising, because they owned real estate collateral. Thus, when the land prices collapsed in the early 1990s, a non-negligible portion of the loans to such industries became non-performing.

Figure 2 shows that bank loans to real estate and construction industries relative to the size of the real economic activity has almost unchanged even after the bursting of the bubble economy, while loans to manufacturing industry has declined significantly. In the meantime, Figure 3 shows that the profitability measured by the ratio of current profits to sales had been markedly lower in real estate industry than in other industries until recently. Put together, these facts imply that Japanese banks have rolled over non-performing loans to real estate industry and thus have not had an incentive to explore prospective projects in manufacturing industry.

The rest of the paper is organized as follows. Section II describes the theoretical foundation using the dynamic programming technique. Section III numerically analyzes the model. Section IV discusses some policy implications. Lastly, Section V concludes the study.

II. Theoretical Foundation

A. Assumptions

The problem that a typical bank faces is specified as whether or not the bank exercises the option to write off its non-performing loans. Figure 4 illustrates the simplified bank’s balance sheet on the premise that the bank lends all the collected money to good projects\(^\text{13}\). It shows that once the bank writes off its non-performing loans, it can lend collected money denoted \(L_{LB} + S - L\) to prospective projects\(^\text{14}\). Here, \(L\) denotes the amount of the loss associated with the write-off defined as the difference between the value of non-performing loans (\(L_{B}\)) and the current value of the collateral. And \(S\) is the government subsidy. At the same time, net worth changes from \(N\) to \(N + S - L\).

There exist informational asymmetries regarding the true magnitude of the non-performing loans between managers of the bank and the public including its shareholders\(^\text{15}\).

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\(^{13}\) Alternatively, the bank can re-invest it in the financial assets markets.

\(^{14}\) If one assumes that the bank lends the collected money to other projects, one should regard the loss \(L\) as including monitoring costs.

\(^{15}\) Some recent studies in corporate governance suggest that bank managers in Japan probably do not act to maximize bank share price. Instead, they engage in activities to entrench themselves. Particularly, Claessens, Djankov, and Lang (2000), and Morek and Nakamura (1999) documented that Japanese banks engage in substantial cross-shareholdings with other companies as an explicit defense against competitive changes in corporate control. Through crossholdings, companies can acquire control rights in banks without getting cash flow rights. Such controlling shareholders could have the incentive to encourage bank management to do things that maximize their own wealth, like continue to finance their company’s unprofitable operations, which does not maximize the bank’s stock price.
The incentive for the bank to carry out write-off lies in the fact that the bank can lend the money collected from liquidating collateral to other borrowers with prospective projects (or re-invest it in the financial markets). The return from the lending (re-investment) after netting out a possible rise in fund-raising cost from not writing off immediately is denoted \( R \).

In this paper, I assume that the re-investment return itself follows the standard geometric Brownian motion. But, due to the possible reputation problem when the bank delays write-off, there is a possibility that the net re-investment return will fall as a result of a rise in the fund-raising cost in the future. Hence, I assume that there exists a probability denoted \( \lambda \) that the re-investment return net of the fund-raising cost exhibits a downward jump.

Also, the bank suffers a loss denoted \( L \) in carrying out the write-off, particularly due to a fall in the value of collateral. It should be noted, however, that the loss \( L \) itself moves stochastically because it mainly reflects the land prices\(^{16}\). I assume that \( L \) also follows the standard geometric Brownian motion.

The stochastic processes of the re-investment return and the loss from the write-off can be summarized as

\[
dR = \alpha_R R dt + \sigma_R R dz_R - Rdq, \tag{1}
\]

and

\[
dL = \alpha_L L dt + \sigma_L L dz_L, \tag{2}
\]

where \( \alpha_R (\alpha_L) \) denotes the expected growth rate of \( R (L) \), \( \sigma_R (\sigma_L) \) the standard deviation parameter of \( R (L) \), \( dz_R (dz_L) \) the increment of a Wiener process of \( R (L) \), \( dq \) the increment of a Poisson process with the probability \( \lambda \). By assumption, \( E[ (dz_R)(dq)] = 0 \) holds, that is, \( dz_R \) and \( dq \) are assumed to be independent each other. Also, I assume \( E[ (dz_R)^2] = E[ (dz_L)^2] = dt \) and \( E[ (dz_R)(dz_L)] = \rho dt \), implying that the correlation \( \rho \) between \( R \) and \( L \) can be taken into consideration in the analysis below. Equation (1) states that if a jump occurs, then \( R \) falls by sum fixed ratio \( \phi \) (\( 0 \leq \phi \leq 1 \)).

Further, the bank faces another source of uncertainty from the future implementation of the government’s rescue scheme under which the government subsidies the bank’s write-off. The strategy in this paper is to express the possible policy implementation by a Poisson jump\(^{17}\).

\(^{16}\) The majority of non-performing loans are known to be those to real estate and construct industries.

\(^{17}\) This strategy basically follows Hasset and Metcalf (1999). They handle the case in which there are one underlying stochastic variable and a possible Poisson jump-type policy intervention to analyze the impact of uncertainty about tax policy on investment decision. The model in this paper augments their model in that there are two underlying stochastic variables, one of which entails a possible downward jump risk, as
Also, for simplicity, I assume that the amount of the subsidy is proportional to the loss from the write-off.

When the subsidy scheme is not in effect, the probability that the government will implement it in the next short period \( dt \) is denoted \( \lambda_i dt \). In that case, the amount of the subsidy is \( \theta L \). On the other hand, when the scheme is currently in effect, the probability that it will be removed in the next short period \( dt \) is \( \lambda_o dt \). In sum, the subsidy process is given by the following equation of motion:

\[
d\theta = \begin{cases} 
\Delta \theta & \text{with probability } \lambda_i dt \\
0 & \text{with probability } 1 - \lambda_i dt \\
-\Delta \theta & \text{with probability } \lambda_o dt \\
0 & \text{with probability } 1 - \lambda_o dt
\end{cases}
\]  

(3)

In what follows, first, I consider the case without the possible implementation of the subsidy scheme, and then, I take the government subsidy into consideration.

**B. The Case without the Subsidy Scheme**

**(i) Basic Setup**

First, the value of (immediate) write-off is given by considering only the part of the standard geometric Brownian motion of equation (1) such that\(^{18}\)

\[
V(R) = E \left[ \int_0^\infty R(t) e^{-\mu_R t} dt \right] = \int_0^\infty R e^{-(\mu_R - \alpha_R) t} dt = \frac{R}{\mu_R - \alpha_R} = \frac{R}{\delta_R},
\]  

(4)

where the relationship \( \mu_R = \alpha_R + \delta_R = r + \nu \rho(R, M) \sigma_R \) is assumed to hold\(^{19}\) as in Dixit and Pindyck (1994)\(^{20}\). Here, \( \mu_R \) denotes the risk-adjusted discount rate, \( \delta_R \) the rate of return well as a Poisson jump-type policy intervention.

\(^{18}\) Notice that once write-off is carried out, further uncertainty associated with the loss is irrelevant, hence the value of write-off depends only on the return from re-investing the collected money.

\(^{19}\) The relationship \( \mu_R = r + \nu \rho(R, M) \sigma_R \) is derived from the CAPM(Capital Asset Pricing Model). To get this relationship, one needs the stochastic fluctuations in \( R \) to be spanned by financial markets. Also, implicitly, I assume that the jump risk is non-systematic, that is, uncorrelated with the market portfolio.

\(^{20}\) See Chapter 4 for details.
shortfall in \( R \) (hereafter, shortfall rate), \( r \) the risk-free interest rate, \( \nu \) the market price of risk, and \( \rho(R, M) \) the coefficient of correlation between \( R \) and the market return \( M \). For the value of write-off \( V(R) \) to be bounded, the condition \( \delta_{R} > 0 \) must hold\(^{21}\). Otherwise, the bank never carries out write-off irrespective of uncertainty and sunk cost.

Second, let \( F(R, L) \) denote the value of keeping the option to write off alive in the future (hereafter, the value of waiting). The Bellman equation can be written as\(^{22}\)

\[
\mu F(R, L)dt = E[dF(R, L)].
\]

Expanding \( dF(R, L) \) in equation (5) by Ito’s Lemma for the combined geometric Brownian motion and Poisson jump\(^{23}\) yields

\[
\mu F(R, L)= \frac{1}{2}\left(\sigma_{R}^{2} R^{2} F_{RR} + 2\rho \sigma_{R} \sigma_{I} R F_{RL} + \sigma_{I}^{2} L^{2} F_{LL}\right)+\alpha_{R} R F + \alpha_{I} L F + \lambda \{F(0) - F(R, L)\}.
\]

where \( F_{RR} \equiv \frac{\partial^{2} F}{\partial R^{2}} \), \( F_{LL} \equiv \frac{\partial^{2} F}{\partial L^{2}} \), and \( F_{RL} \equiv \frac{\partial^{2} F}{\partial R \partial L} \).

Now, boundary conditions can be written as

\[
F(\hat{R}, \hat{L})=V(\hat{R})-\hat{L} = \frac{\hat{R}}{\delta_{R}} - \hat{L}, \tag{7}
\]

\[
F_{R}(\hat{R}, \hat{L})=V'(\hat{R})=\frac{1}{\delta_{R}}, \tag{8}
\]

\(^{21}\) In understanding the role of \( \delta_{R} \), it is helpful to draw upon the analogy with a financial call option in which \( R \) corresponds to the price of a share of common stock, and \( \delta_{R} \) the dividend rate. Thus, the total expected rate on the stock is written as \( \mu_{R} = \alpha_{R} + \delta_{R} \). In such a case, if the dividend rate \( \delta_{R} \) were zero, the call option would always be held to maturity, and never exercised since the opportunity cost to keep the option alive is zero.

\(^{22}\) In what follows, for simplicity, I drop the subscript \( R \) for \( \mu_{R} \).

\(^{23}\) In general, if the stochastic process is

\[
dx = a(x, t)dx + b(x, t)dz + g(x, t)dq,
\]

then the expected value of the change in any function \( H(x, t) \) can be given by

\[
E[dH] = \left[\frac{\partial H}{\partial t} + a(x, t)\frac{\partial H}{\partial x} + \frac{1}{2} b^{2}(x, t)\frac{\partial^{2} H}{\partial x^{2}}\right] dt + E_{\phi} \left[\lambda \{H + g(x, t)\phi, t\} - H(x, t)\right] dt,
\]

where \( \phi \) is the size of the jump when the event happens. For more details, see Dixit and Pindyck (1994),
and \( F_L(\hat{R}, \hat{L}) = -1, \) \( 9 \)

where condition (7) is the value-matching condition and conditions (8) and (9) are both the smooth-pasting conditions. Also, \( \hat{R} \) and \( \hat{L} \) indicate the threshold values of \( R \) and \( L \) at which the bank becomes indifferent between carrying out the write-off and waiting.

(ii) The Free boundary Problem

The problem in the last section is called a “free boundary” problem\(^{24}\). In such a case, it is very difficult to obtain clear-cut analytical solutions. But, the property of homogeneity of the net value function \( V(R) - L \) \(^{25}\) allows one to reduce the problem to one dimension. Thus, the optimal decision depend only on the ratio \( \frac{r_R}{R} = \frac{R}{L} \), which implies that the value of waiting \( F(R, L) \) should also be homogeneous of degree one with respect to \( R \) and \( L \). That is, the following set of relationships holds:

\[
\begin{align*}
F(R, L) &= Lf\left(\frac{R}{L}\right) = Lf(r_R), \quad (10) \\
F_R(R, L) &= f'(r_R), \quad (11) \\
F_L(R, L) &= f(r_R) - r_Rf'(r_R), \quad (12) \\
F_{RR}(R, L) &= f''(r_R)/L, \quad (13) \\
F_{RL}(R, L) &= -r_Rf''(r_R)/L, \quad (14) \\
\text{and} \quad F_{LL}(R, L) &= (r_R)^2 f''(r_R)/L. \quad (15)
\end{align*}
\]

Using equations (10)-(15), equation (6) can be rewritten in the special case of \( \phi = 1 \)\(^{26}\) as

\[
\frac{1}{2}(\sigma_R^2 - 2p\sigma_R\sigma_L + \sigma_L^2)(r_R)^2 f''(r_R) + (\alpha_R - \alpha_L)r_Rf'(r_R) + (\sigma_R - \mu - \lambda)f(r_R) = 0. \quad (16)
\]

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\(^{24}\) For more details, see Dixit and Pindyck (1994), Chapter 6, pp. 209.

\(^{25}\) Note that if the current values of both \( R \) and \( L \) are doubled, it will double the value \( V \) and the cost \( L \) simultaneously.

\(^{26}\) For computational facility, I assume \( \phi = 1 \) throughout the paper.
The solution to the second-order differential equation (16) takes the form:

\[ f(r_R) = A(r_R)^\beta, \quad (17) \]

where \( A \) and \( \beta \) are coefficients to be determined.

Direct substitution of the solution (17) into equation (16) yields

\[ \frac{1}{2} \left( \sigma_R^2 - 2\rho \sigma_R \sigma_L + \sigma_L^2 \right) \beta (\beta - 1) + (\alpha_R - \alpha_L) \beta + (\alpha_L - \mu - \lambda) = 0. \quad (18) \]

Thus, \( \beta \) can be solved analytically such that

\[ \beta = 1 - \frac{\alpha_R - \alpha_L}{\left( \sigma_R^2 - 2\rho \sigma_R \sigma_L + \sigma_L^2 \right)} + \sqrt{\left( \frac{\alpha_R - \alpha_L}{\left( \sigma_R^2 - 2\rho \sigma_R \sigma_L + \sigma_L^2 \right)} - \frac{1}{2} \right)^2 + \frac{2(\mu + \lambda - \alpha_L)}{\left( \sigma_R^2 - 2\rho \sigma_R \sigma_L + \sigma_L^2 \right)}. \quad (19) \]

Now boundary conditions (7)-(9) can be rewritten as

\[ f(\hat{r}_R) = A(\hat{r}_R)^\beta = \frac{\hat{r}_R}{\delta_R} - 1, \quad (20) \]
\[ f'(\hat{r}_R) = A\beta (\hat{r}_R)^{\beta-1} = \frac{1}{\delta_R}, \quad (21) \]
and

\[ f(\hat{r}_R) - \hat{r}_R f'(\hat{r}_R) = A(\hat{r}_R)^\beta (1 - \beta) = -1, \quad (22) \]

where \( \hat{r}_R \) denote the threshold ratio. Note that of these three boundary conditions, any one is not independent from the other two.

Equations (20) and (21) jointly imply

\[ \hat{r}_R = \frac{\beta}{\beta - 1} \delta_R. \quad (23) \]

Figure 5 illustrates a free boundary \( \hat{r}_R \) of the ratio of re-investment return to the loss from write-off. In regime I, current value of \( r_R \) is below the threshold value \( \hat{r}_R \) so that the
bank prefer waiting to writing off now. Also, Figure 6 depicts boundary conditions for \( f(r_R) \) and the determination of \( \hat{r}_R \). At the threshold ratio \( \hat{r}_R \), the value from write-off meets the value of waiting tangentially.

(iii) The Relationship between \( \hat{r}_R \) and the Required Re-Investment Rate of Return \( \bar{r}_R \)

Note that the threshold ratio \( \hat{r}_R \) is in terms of the loss from write-off, not in terms of the amount of the re-invested funds. Thus, in evaluating \( \hat{r}_R \) in line with realistic economic situations, it is helpful to translate \( \hat{r}_R \) into the usual rate of return form.

Figure 7 shows the relationship between \( \hat{r}_R \) and the required re-investment rate of return \( \bar{r}_R \). The following relationship is evident:

\[
\begin{align*}
\text{if } L \leq \frac{1}{2} L_B, & \quad \text{then } \bar{r}_R = \frac{L}{L_B - L} \hat{r}_R \leq \hat{r}_R \\
\text{otherwise,} & \quad \bar{r}_R > \hat{r}_R
\end{align*}
\]

(24)

For example, when the loss amounts to a quarter of the non-performing loan, \( \bar{r}_R \) is equal to \( 1/3 \hat{r}_R \). And when the loss is three quarters, \( \bar{r}_R \) is \( 3\hat{r}_R \).

C. The Case with the Possible Implementation of the Subsidy Scheme

Now let me consider the case in which the bank expects the implementation of a subsidy by the government with some probability. To begin with, let \( F_0(R, L) = Lf_0(r_R) \) denote the value of waiting in the absence of the subsidy scheme and \( F_1(R, L) = Lf_1(r_R) \) the value in the presence of the scheme, respectively. In this setting, one can divide the decision rule of the bank into the following three regimes, implying the existence of two threshold ratios. See Figures 8 and 9 for the illustration of the three regimes.

First, over the interval of low values of \( r_R \), denoted \( (0, \hat{r}_R) \), the bank will not write off irrespective of whether or not the subsidy scheme is in effect.

Second, over the interval denoted \( (\hat{r}_R, \bar{r}_R) \), the bank will write off if the subsidy scheme is in effect. Otherwise, the bank will prefer to waiting in the hope that the subsidy scheme will be implemented and/or land price will recover so that the liquidation loss will be
smaller in the future.

Third, over the interval denoted \( (\bar{r}_R, \infty) \), the bank is willing to write off irrespective of the subsidy scheme. Referring to Hassett and Metcalf (1999), I will find the two thresholds, \( \bar{r}_R \) and \( \underline{r}_R \) below.

(i) Regime 1 \( (0, \underline{r}_R) \): No Write-off Irrespective of the Subsidy Scheme

Over the interval \( (0, \underline{r}_R) \), the bank prefers to waiting irrespective of the subsidy scheme, and each regime can switch to the other. Thus, the following pair of equations holds:

\[
\frac{1}{2} \left( \sigma_R^2 - 2 \rho \sigma_R \sigma_L + \sigma_L^2 \right) \left( r_R \right)^2 f_0'(r_R) + (\alpha_R - \alpha_L) r_R f_0'(r_R) + (\alpha_L - \mu - \lambda) f_0(r_R) + \lambda_1 \left[ f_1(r_R) - f_0(r_R) \right] = 0
\]

(25)

\[
\frac{1}{2} \left( \sigma_R^2 - 2 \rho \sigma_R \sigma_L + \sigma_L^2 \right) \left( r_R \right)^2 f_1'(r_R) + (\alpha_R - \alpha_L) r_R f_1'(r_R) + (\alpha_L - \mu - \lambda) f_1(r_R) + \lambda_0 \left[ f_0(r_R) - f_1(r_R) \right] = 0
\]

(26)

Since both value functions \( f_0(r_R) \) and \( f_1(r_R) \) appear in each equation, one needs to consider the two linear combinations that can be solved easily. For example, consider new value functions \( f_a(r_R) \) and \( f_b(r_R) \) such that

\[
f_a(r_R) = \frac{f_0(r_R)}{\lambda_1} + \frac{f_1(r_R)}{\lambda_0},
\]

(27)

and

\[
f_b(r_R) = f_1(r_R) - f_0(r_R).
\]

(28)

Then, equations (25) and (26) can be rewritten in terms of \( f_a(r_R) \) and \( f_b(r_R) \) as

---

27 The derivation of equation (25) is made in the following way. When the subsidy is not in effect, over the next short interval of time \( dt \), the probability that the subsidy will be implemented is \( \lambda \, dt \). In this case, the value of the option to write off is \( dLL_dRRF^+ + 1 \). Otherwise, it is \( dLL_dRRF^+ + 0 \). Hence, \( F_0(R + dR, L + dL) = e^{-\lambda dt} \left\{ \lambda_1 dt \left[ F_1(R + dR, L + dL) \right] + (1 - \lambda_1 dt) \left[ F_0(R + dR, L + dL) \right] \right\} \) follows. Expanding the preceding equation by Ito's Lemma and using the assumption of homogeneity yields equation (25). Equation (26) can be derived in the same way.
\[
\frac{1}{2} \left( \sigma_R^2 - 2\rho \sigma_R \sigma_L + \sigma_L^2 \right) f_a'(r_R) + (\alpha_R - \alpha_L) r_R f_a'(r_R) + (\alpha_L - \mu - \lambda) f_a(r_R) = 0, \quad (29)
\]
\[
\frac{1}{2} \left( \sigma_R^2 - 2\rho \sigma_R \sigma_L + \sigma_L^2 \right) f_b'(r_R) + (\alpha_R - \alpha_L) r_R f_b'(r_R) + (\alpha_L - \mu - \lambda - \lambda_0 - \lambda_1) f_b(r_R) = 0. \quad (30)
\]

The solutions to the second-order differential equations take the forms\(^\text{28}\):

\[
f_a(r_R) = B(r_R)^{\beta_1}, \quad (31)
\]
and
\[
f_b(r_R) = C(r_R)^{\beta_2}, \quad (32)
\]

where \( B, C, \beta_1 \), and \( \beta_2 \) are coefficients to be determined. Here note that \( \beta_1 \) is the positive root of

\[
\frac{1}{2} \left( \sigma_R^2 - 2\rho \sigma_R \sigma_L + \sigma_L^2 \right) \beta (\beta - 1) + (\alpha_R - \alpha_L) \beta + (\alpha_L - \mu - \lambda) = 0, \quad (33)
\]
and \( \beta_2 \) is the positive root of

\[
\frac{1}{2} \left( \sigma_R^2 - 2\rho \sigma_R \sigma_L + \sigma_L^2 \right) \beta (\beta - 1) + (\alpha_R - \alpha_L) \beta + (\alpha_L - \mu - \lambda - \lambda_0 - \lambda_1) = 0. \quad (34)
\]

With this background information, the solutions for \( f_0(r_R) \) and \( f_1(r_R) \) over the interval \( (0, r_R) \) are given by

\[
f_0(r_R) = \frac{\lambda_0 \lambda_1 B(r_R)^{\beta_1} - \lambda_1 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1}, \quad (35)
\]
and
\[
f_1(r_R) = \frac{\lambda_0 \lambda_1 B(r_R)^{\beta_1} + \lambda_0 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1}. \quad (36)
\]

\(^{28}\) Note that since the interval of \( r_R \) extends to zero, only the positive root of a quadratic equation matters.
(ii) Regime 2 \( (r_R, \delta^*_R) \): Write-off Now if the Subsidy Scheme is in Effect

Over the interval \( (r_R, \delta^*_R) \), the bank will write off non-performing loans immediately if the subsidy scheme is in effect, otherwise not. Thus \( f_1 (r_R) \) is given by\(^{29}\)

\[
f_1 (r_R) = \frac{r_R}{\delta_R} - (1 - \theta),
\]

(37)

where \( \theta \) denotes the portion of the subsidy\(^{30}\) in the loss.

On the other hand, \( f_0 (r_R) \) is found in the same way as equations (25) and (26).

\[
\frac{1}{2} (\sigma_R^2 - 2 \rho \sigma_R \sigma_L + \sigma_L^2) f''_0 (r_R) + (\alpha_R - \alpha_L) f'_0 (r_R) + (\alpha_L - \mu - \lambda) f_0 (r_R) + \lambda_i [f_1 (r_R) - f_0 (r_R)] = 0
\]

(38)

Using equation (37), equation (38) can be rewritten as

\[
\frac{1}{2} (\sigma_R^2 - 2 \rho \sigma_R \sigma_L + \sigma_L^2) f''_0 (r_R) + (\alpha_R - \alpha_L) f'_0 (r_R) + (\alpha_L - \mu - \lambda - \lambda_i) f_0 (r_R) + \frac{\lambda_i}{\delta_R} r_R - \lambda_i (1 - \theta) = 0.
\]

(39)

The general solution takes the form:

\[
f_0 (r_R) = D (r_R)^{\beta_1} + E (r_R)^{\beta_2} + \frac{\lambda_i r_R}{\delta_R (\mu + \lambda + \lambda_i - \alpha_R)} - \frac{\lambda_i (1 - \theta)}{\mu + \lambda + \lambda_i - \alpha_L}.
\]

(40)

where \( D \) and \( E \) are constants to be determined and \( \beta_1 \) and \( \beta_2 \) are the positive and negative roots of the quadratic function of the form:

\[
\frac{1}{2} (\sigma_R^2 - 2 \rho \sigma_R \sigma_L + \sigma_L^2) \beta (\beta - 1) + (\alpha_R - \alpha_L) \beta + (\alpha_L - \mu - \lambda - \lambda_i) = 0.
\]

(41)

---

\(^{29}\) Notice that equation (37) is equivalent to \( F_1 (R, L) = R/\delta_R - (1 - \theta)L \).
(iii) Regime 3 \( \left( R_R, \infty \right) \): Write-off Now Irrespective of the Subsidy Scheme

Over the interval \( \left( R_R, \infty \right) \), the bank always writes off non-performing loans, irrespective of the subsidy scheme. Thus the following relationships hold:

\[
 f_0 (r_R) = \frac{r_R}{\delta_R} - 1 , \quad (42)
\]

and

\[
 f_1 (r_R) = \frac{r_R}{\delta_R} - (1 - \theta) , \quad (43)
\]

(iv) Boundary Conditions Linking each Regime\(^{31}\)

Now that each solution form is found, the next step is to find boundary conditions, which relate each value function derived above.

First, at the threshold \( r_R \), the bank will write off if the subsidy scheme is in effect. Thus for the expressions for \( f_i (r_R) \), equations (36) and (37) yield,

\[
 \frac{\lambda_0 \lambda_i B (r_R)^{\beta_i} + \lambda_0 C (r_R)^{\beta_2}}{\lambda_0 + \lambda_i} = \frac{r_R}{\delta_R} - (1 - \theta) , \quad (44)
\]

and

\[
 \frac{\lambda_0 \lambda_i B \beta_i (r_R)^{\beta_i-1} + \lambda_0 C \beta_2 (r_R)^{\beta_2-1}}{\lambda_0 + \lambda_i} = \frac{1}{\delta_R} , \quad (45)
\]

where equations (44) and (45) denote value-matching and smooth-pasting conditions.

Second, for \( f_0 (r_R) \), although this is not actually associated with a decision threshold, the function has to be continuously differentiable across it. Thus equations (35) and (40) yield

\[
 \frac{\lambda_0 \lambda_i B (r_R)^{\beta_i} - \lambda_1 C (r_R)^{\beta_2}}{\lambda_0 + \lambda_i} = D (r_R)^{\beta_i} + E (r_R)^{\beta_2} + \frac{\lambda_1 r_R}{\delta_R (\mu + \lambda + \lambda_i - \alpha_R)} - \frac{\lambda_1 (1 - \theta)}{\mu + \lambda + \lambda_i - \alpha_L} , \quad (46)
\]

\(^{30}\) See equation (3) for its definition.

\(^{31}\) Notice that the case without the government subsidy corresponds to a special case in which \( \lambda_0 = \lambda_1 = \theta = 0 \) of this model.
\[
\frac{\lambda_0 \lambda_1 B \beta_i (r_{R_0}^{\beta_i - 1} - \lambda_i C \beta_j (r_{R_1}^{\beta_j - 1})}{\lambda_0 + \lambda_1} = D \beta_3 (r_R)^{\beta_3 - 1} + E \beta_4 (r_R)^{\beta_4 - 1} + \frac{\lambda_1}{\delta_R (\mu + \lambda + \lambda_1 - \alpha_R)}. \tag{47}
\]

Third, at the threshold \( r_R \), the expressions for \( f_{0}(r_R) \) should satisfy the value-matching and smooth-pasting conditions. Hence, equations (40) and (42) yield

\[
D(r_R)^{\beta_1} + E(r_R)^{\beta_1} + \frac{\lambda_1 r_R}{\delta_R (\mu + \lambda + \lambda_1 - \alpha_R)} - \frac{\lambda_1 (1 - \theta)}{\mu + \lambda + \lambda_1 - \alpha_L} = \frac{r_R}{\delta_R} - 1, \tag{48}
\]

and

\[
D \beta_3 (r_R)^{\beta_3 - 1} + E \beta_4 (r_R)^{\beta_4 - 1} + \frac{\lambda_1}{\delta_R (\mu + \lambda + \lambda_1 - \alpha_R)} = \frac{1}{\delta_R}. \tag{49}
\]

where one can analytically find \( \beta \)'s from equations (33), (34), and (41) as follows:

\[
\begin{aligned}
\beta_1 &= \frac{1}{2} \frac{F}{G} + \sqrt{\left( \frac{F}{G} - \frac{1}{2} \right)^2 + \frac{2(\mu + \lambda - \alpha_L)}{G}} \\
\beta_2 &= \frac{1}{2} \frac{F}{G} + \sqrt{\left( \frac{F}{G} - \frac{1}{2} \right)^2 + \frac{2(\mu + \lambda + \lambda_0 + \lambda_1 - \alpha_L)}{G}} \\
\beta_3 &= \frac{1}{2} \frac{F}{G} + \sqrt{\left( \frac{F}{G} - \frac{1}{2} \right)^2 + \frac{2(\mu + \lambda + \lambda_1 - \alpha_L)}{G}} \\
\beta_4 &= \frac{1}{2} \frac{F}{G} + \sqrt{\left( \frac{F}{G} - \frac{1}{2} \right)^2 + \frac{2(\mu + \lambda + \lambda_1 - \alpha_L)}{G}} \\
\end{aligned}
\tag{50}
\]

\[
F = \alpha_R - \alpha_L, \quad G = \sigma_R^2 - 2 \rho \sigma_R \sigma_L + \sigma_L^2. \tag{51}
\]

It is evident that the relation \( \beta_4 < 0 < \beta_1 \leq \beta_3 \leq \beta_2 \) holds. In sum, there are six equations to determine two thresholds \( r_{R_0} \) and \( r_{R_1} \) and four constants \( B \), \( C \), \( D \), and \( E \).

Figures 10 and 11 illustrate these boundary conditions and the determination of the threshold ratios \( r_{R_0} \) and \( r_{R_1} \). They show that at the threshold ratio \( r_R \), two expressions of

\[\text{I used a Levenberg-Marquardt method included in Mathcad 2000 Professional as a solving algorithm. It is a variant of the usual quasi-Newton method. To make the Levenberg-Marquardt method more efficiently Mathcad modifies the following points:}
\]
\[\text{(i) The first time the solver stops at a point that is not a solution, Mathcad adds a small random amount to all the variables and tries again. This helps avoid getting stuck in local minima and other points from which there is no preferred direction.}
\]
\[\text{(ii) If inequality constraints are included as in the case of mine, Mathcad solves the subsystem consisting}
\]
\[\text{(52) to determine the constants.}
\]

\[\text{The Mathcad code looks like this:}
\]
\[\text{R}_{R_0} \text{ and } \text{R}_{R_1} \text{, and four constants } B, C, D, \text{ and } E.}\]
$f_0(r_R)$ representing regimes 1 and 2 (equations (35) and (40)) meet tangentially and at the same time, two expressions of $f_1(r_R)$ representing regimes 1 and 2 (equations (36) and (37)) meet in the same manner. And at the threshold ratio $r_R^*$, only the two expressions of $f_0(r_R)$ representing regimes 2 and 3 meet tangentially to ensure the continuity.

III. Numerical Analysis

This section reports the results of numerical analysis based on the theoretical model described in the last section. The baseline set of the parameters is set in annual terms as follows:

$$
\alpha_R = 0.02, \quad \alpha_L = -0.02, \quad \sigma_R = 0.2, \quad \sigma_L = 0.3, \quad \delta_R = 0.02, \quad \rho = 0.0, \text{ and } \lambda = 0.0.
$$

Here the negative value of the expected growth rate of the write-off loss $\alpha_L$ reflects the optimistic expectation of the banks about future conditions in the real estate market and the relative magnitude between the $\sigma_R$ and $\sigma_L$ reflects the larger volatility in real estate market. As for the correlation term $\rho$ and the probability of a jump in the fund-raising cost $\lambda$, it is difficult to find “plausible” values. Thus, I tentatively set both values at zero in the baseline case and change them over the wide range. Table 1 summarizes the qualitative results of numerical analysis and I will see details in what follows.

A. The Case without the Subsidy Scheme

Figures 12 (i)-(v) show the dependence of the threshold ratio $\hat{r}_R = \hat{R}/\hat{L}$ on various parameter values. Before examining the detailed numerical results, note that a very large rate of return is required for the banks to immediately write off their non-performing loans under various settings. For example, the value of $\hat{r}_R = 0.05$ means more than 5 percent annual re-investment rate of return if the loss is more than half of the non-performing loans. Judging from the low level of current investment rate of return in the Japanese financial markets, obtaining such a

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33 With the benefit of hindsight, we know that this expectation was not realized, but as Cargill, Hutchison, and Ito (1997) argues, the MOF also had such an optimistic view on the real estate market, not to mention the Japanese banks.

34 For example, in December 1999, the average contracted interest rate on new loans and discounts is 1.82 percent and the yield of 10-year government bonds, which occupy non-negligible part of the
high return seems almost impossible in reality.

Now, let me check how the basic model works. First, Figure 12 (i) shows that the more and more uncertain the bank’s economic environment becomes (larger values of $\sigma_R$ and $\sigma_L$), the larger the threshold ratio $\hat{r}_R$ is\(^{35}\). This result holds for uncertainty in both underlying stochastic variables, re-investment return $R$ and the loss from the write-off $L$.

Second, consider the effect of a rise in the coefficient of correlation $\rho$ between $R$ and $L$. As shown in Figure 12 (ii), a rise in $\rho$ results in lower $\hat{r}_R$. This result directly follows the fact that the variance of $r_R$ under the assumption of homogeneity of degree one can be expressed as $\sigma^2_{\hat{r}_R} = 2\rho\sigma_R\sigma_L + \sigma^2_L$ so that a larger value of $\rho$ implies a smaller volatility, which raises the incentive to write off immediately.

Third, look at the effects of a change in expected growth parameters, $\alpha_R$ and $\alpha_L$. It turns out that the effect of $\alpha_L$ is more straightforward than that of $\alpha_R$. As shown in Figures 12 (ii) and (iii), the larger (smaller) the expected loss from write-off in the future is, the stronger (weaker) incentive to write off immediately than later the bank has. This result corresponds to the familiar story about the forbearance policy taken by the government regarding write-offs of non-performing loans in the Japanese banking industry.

For the effect of $\alpha_R$, one should be careful about the results obtained under alternative assumptions regarding which parameter is adjustable, $\mu$ or $\delta_R$. Under the assumption that $\mu$ changes exactly as much as $\alpha_R$\(^{36}\)(Figure 12 (iii) (a)), the threshold ratio $\hat{r}_R$ rises as a result of a rise in $\alpha_R$. Intuitively, the underlying reason is that the present value of the loss from write-off carried out at a future time is discounted by the risk-adjusted discount rate $\mu$,\(^{37}\) while the present value of the re-investment of the collected money is discounted by $\delta_R$\(^{38}\), which is assumed to be constant. Hence, an increase in $\alpha_R$ (thus $\mu$) reduces the present value of the cost of write-off in the future, but does not reduce its payoff.

On the other hand, under the alternative assumption that $\mu$ is held constant while letting $\delta_R$ adjust in response to a change in $\alpha_R$, the direction of the effect of a rise in $\alpha_R$

---

\(^{35}\) As pointed out by Dixit and Pindyck (1994)(see Chapter 5, pp.153), an interesting point here is that write-off (investment) decision is highly sensitive to volatility in write-off (project) values, irrespective of investors’ or managers’ risk preferences. Thus, if one assumes that the bank is risk-neutral, the same result follows.

\(^{36}\) Recall the relationship $\mu = \alpha_R + \delta_R$.

\(^{37}\) See equation (5).
becomes opposite to the preceding case. The reason for this result can be found in the same logic as before. That is, while the present value of the loss of the future write-off is invariant, its payoff from the re-investment will rise.

Fourth, how does an increase in the downward jump risk of the re-investment return influence the optimal decision-making of rational banks? Generally, the effects of a positive value of the probability of the downward jump risk \( \lambda \) can be stated in the following. First, it reduces the expected rate of capital gain on \( R \), which decreases the value of waiting. On the other hand, it increases the variance of changes in \( R \) and thus raises the value of waiting. It turns out that in normal circumstances, the former effect is more dominant. Figure 12 (iv) reports the result, that is, the former effect is much larger than the latter one, thereby reducing the threshold ratio \( \hat{r}_R \). Further, notice that a small increase in \( \lambda \) lead to a substantial fall in \( \hat{r}_R \), prompting the bank to write off immediately than later.

Fifth, look at the effect of an increase in the shortfall rate \( \delta_R \) on the threshold ratio of \( \hat{r}_R \). Figure 12 (v) shows the result under the assumption that the risk-adjusted discount rate \( \mu \) moves in response to a change in \( \delta_R \). As I explained, the profit flow is discounted by \( \delta_R \), while the loss from the future write-off is discounted by \( \mu \). In the present model, although the effect via \( \mu \) have an effect through \( \beta \) (equation (19)), the effect via \( \delta_R \) influences more directly as shown by equation (23). Thus, the net effect is to increase the threshold ratio \( \hat{r}_R \).

B. The Case with the Possible Implementation of the Subsidy Scheme

Now let me examine the case with the possible implementation of the subsidy scheme by the government. As I mentioned earlier, this case is generalization of the last case in that \( \lambda_0, \lambda_1 \), and \( \theta \) take positive values. In fact, the threshold ratios \( r_R \) and \( r_{\overline{R}} \) should converge to \( \hat{r}_R \) as the values of \( \lambda_0, \lambda_1 \), and \( \theta \) approach zero. Thus, in this subsection, I focus on the numerical results when one changes the values of \( \lambda_0, \lambda_1 \), and \( \theta \).

First, consider the situation where the scheme is not currently in effect. Figure 13 (i)
shows that as the probability of the implementation $\lambda_i$ increases, the threshold ratio $\bar{r}_R$ increases. This result is very intuitive. The prospect of a reduced loss from write-off inevitably increases the value of waiting. One of the most impressive aspects to note here is the magnitude of the effect of an increase in $\lambda_i$. When the bank is 100% sure\(^{41}\) of the implementation of the subsidy scheme\(^{42}\) in the next period, the threshold ratio $\bar{r}_R$ becomes more than double than that when $\lambda_i = 0.1$. Also, notice that even when the implementation of the scheme is being discussed and is still uncertain, the effect is to strongly depress the incentive to write off immediately. Another important point is that even in the absence of the scheme, the threshold ratio $\bar{r}_R$ is influenced by the probability $\lambda_o$ of the removal of the scheme.

Next consider the situation in the presence of the scheme. Figure 13 (ii) shows that the threshold ratio $r_R$ decreases as $\lambda_0$ increases. This result is also intuitive because it is natural to think that the prospect of losing the scheme should induce the bank to write off immediately. Further, this figure shows that an increase in $\lambda_i$ also increases $r_R$.

Now let me look at the effect of an increase in the ratio of the subsidy to the loss $\theta$. Figure 13 (iii) shows the dependence of $\bar{r}_R$ and $r_R$ on the value of $\theta$. This figure shows that both threshold ratios $\bar{r}_R$ and $r_R$ are negatively related to $\theta$ over the low values of $\theta$. And the effect is much stronger on $r_R$ than on $\bar{r}_R$.

Rather, a more interesting point is that the threshold ratio $\bar{r}_R$ in the absence of the scheme is influenced by $\theta$, too. Numerical analysis suggests that there are two competing channels through which $\theta$ can influence $\bar{r}_R$. One channel raises the incentive to wait by lowering the last term of $f_0(\bar{r}_R)$ (equation (40)). The other channel works in the opposite direction via a fall in $D$ in the same equation. Which force is stronger depends on the range of parameter $\theta$. Generally, as shown in Figure 13 (iii), when $\theta$ is small, the latter effect is larger than the former effect, but at some value of $\theta$, net effect is reversed in its direction and thus a rise in $\theta$ raises the threshold ratio $\bar{r}_R$.

\(^{41}\) Note that the 100% probability of enactment of subsidy is just in the expectation of the bank and does not imply that the policy will be really enacted within the next year.

\(^{42}\) One can rephrase this condition as saying “if the regulatory authorities can make a fully credible commitment to implement the subsidy scheme in the near future”.

19
IV. Some Policy Discussions

A. Implications for the Implementation of the Subsidy Scheme

Uncertainty about the implementation of the subsidy scheme will give banks the incentive to delay write-offs. Put differently, if the government aims to accelerate the banks’ self-help efforts toward reducing their non-performing loans, the right policy is a combination of low $\lambda_i$, high $\lambda_0$, and large $\theta$. That is, the government should implement the subsidy scheme immediately, giving the banks a credible threat to abolish it right away and pledging never to restore it, although we cannot imagine that such a threat really exists.

B. Possible Implications for Monetary Policy

The BOJ directly controls the short-term interest rate such as the over-night call rate for the purpose of influencing the real economy. The over-night call rate is thought to be risk-less. Under the maintained assumption of $\mu = \alpha + \delta R = r + u\rho(R, M)\sigma_R$ in this paper, a rise in the call rate by the BOJ results in a rise in the risk-adjusted discount rate $\mu$ of the banks either via a rise in $\alpha_R$ or $\delta R$, other things being equal.

The analysis shows that a rise in the risk-adjusted discount rate $\mu$ via either $\alpha_R$ or $\delta R$ makes banks more hesitate to dispose of their non-performing loans immediately. Hence, if the central bank would like to accelerate the banks’ own efforts to clean up the balance sheets, it should lower the interest rate up to the point where banks regards it as long-lasting so that they revise their perceived risk-adjusted discount rate downward. In this regard, recent monetary policy conducted by the BOJ is worthy of attention. To stimulate the depressed economy, the BOJ lowered the short-term interest rate including the call rate to almost zero from 1995\textsuperscript{43}, which might have some effects to raise the incentive for the banks to write off\textsuperscript{44}.

\textsuperscript{43} Monetary easing of this nature can be characterized as a policy designed to “buy time”, that is, to buy time until the structural policy bears fruits. In fact, when it lowered the official discount rate to 0.5 percent in September 1995, the Policy Board of the BOJ issued a statement stressing that such monetary easing would only be effective if it were accompanied by structural policies.

\textsuperscript{44} But, it is also true that the enlargement of profit margin as result of the so-called zero-interest –rate policy gave banks room for retaining their non-performing loans. The analysis in this paper pays no attention to the effect of monetary policy on the interaction between the cost structure and the incentive to dispose of non-performing loans of banks. Thus, we should be careful in evaluating implications for monetary policy.
V. Concluding Remarks

This paper has investigated how rational banks’ optimal timing of write-offs is influenced by uncertainty stemming from various sources. A real options approach is employed to evaluate the value of the option to delay write-offs, that is the value of forbearance policy.

Numerical analysis shows that under normal circumstances, a very large rate of re-investment return is required for the banks to immediately write off their non-performing loans. Another important results is that uncertainty about the implementation of the subsidy scheme will give the banks an incentive to wait contrary to the government’s intention. If the government aims to encourage the banks’ self-help efforts toward reducing their non-performing loans, it should enact the subsidy scheme immediately, giving them a credible threat to abolish it right away and pledging never to restore it in the future. This kind of policy is theoretically possible, but seems almost impossible in reality.
References


Table 1: Summary of Numerical Analysis

<table>
<thead>
<tr>
<th>Exogenous Variables</th>
<th>Endogenous Variables</th>
<th>( \hat{r}_R )</th>
<th>( \begin{pmatrix} \hat{r}_R \ \bar{r}_R \end{pmatrix} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Underlying Parameters</strong></td>
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<tr>
<td>Volatility</td>
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<tr>
<td>( \sigma_R )</td>
<td>+</td>
<td>+ +</td>
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</tr>
<tr>
<td>( \sigma_L )</td>
<td>+</td>
<td>+ +</td>
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<tr>
<td>Correlation</td>
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<td>( \rho )</td>
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<td>Expected Growth</td>
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<td>( \alpha_R )</td>
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<td>Case (i)</td>
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<td>Case (ii)</td>
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<td>( \alpha_L )</td>
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<td>( \delta_R )</td>
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<td>( \lambda )</td>
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<tr>
<td>( \lambda_0 )</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \theta )</td>
<td>—</td>
<td>– ±</td>
<td></td>
</tr>
</tbody>
</table>

Notes: 1. + indicates that the endogenous variable goes up when the exogenous variable rises.
   – indicates vice versa. ± denotes that the direction depends on the other parameter values.
2. The case (i) denotes the case in which \( \delta_R \) is held constant, while letting \( \mu \) adjust freely.
   The case (ii) denotes the case in which \( \mu \) is held constant while letting \( \delta_R \) adjust freely.
Figure 1: Sources of Uncertainty Influencing the Decision by the Bank

- Loan or Financial Markets
  - Re-investment Return (Opportunity Cost)
- Real Estate Market
  - Loss from Write-off (Land Prices)
- Government
  - Possible Subsidy (Capital Injection)
- Public
  - Reputation Problem (Risk Premium)
Figure 2: Loans and Discounts Outstanding by Industry
(Percent of Nominal GDP)

Note: The definition of the domestic banks changed in 92/1Q. Thus, I used the data after 92/2Q. The data is taken from the Bank of Japan’s Financial and Economic data CD-ROM.

Figure 3: Ratio of Current Profits to sales

Note: The data is taken from Corporate Business Statistics quarterly issued by the MOF.
### Figure 4: A Simplified Balance Sheet

#### A. Before Write-Off

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans ((L_G + L_B))</td>
<td>Funds ((F_U))</td>
</tr>
<tr>
<td>Good Loans ((L_G))</td>
<td></td>
</tr>
<tr>
<td>Bad Loans ((L_B))</td>
<td></td>
</tr>
<tr>
<td>Other Assets ((OA))</td>
<td>Net Worth ((N))</td>
</tr>
</tbody>
</table>

#### B. After Write-Off

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans ((L_G + L_B + S - L))</td>
<td>Funds ((F_U))</td>
</tr>
<tr>
<td>Good Loans ((L_G + L_B + S - L))</td>
<td></td>
</tr>
<tr>
<td>Other Assets ((OA))</td>
<td>Net Worth ((N + S - L))</td>
</tr>
</tbody>
</table>

**Notes:**
1. \(S\): Government subsidy, \(L\): Liquidation loss
2. The figures are drawn under the assumption that collected money is lent to profitable projects.
Figure 5: Free Boundary of $\hat{r}_R$ without the Subsidy Scheme

Re-investment Return net of Fund-Raising Cost (R)

Free Boundary

Regime II
Write Off Now

Regime I
No Write-off

$\hat{r}_R$

Loss (L)
Figure 6: Boundary Conditions for $f(r_R)$ and the Determination of $\hat{r}_R$

(i) $f(r_R) = A(r_R)^\beta \ (equation \ (17))$

(ii) $f(r_R) = \frac{r_R}{\delta_R} - 1 \ (right \ hand \ side \ of \ equation \ (20))$
Figure 7: The Relationship between $\hat{r}_R$ and the Required Rate of Return $\bar{r}_R$

\[
\bar{r}_R = \frac{L}{L_B - L_{\hat{r}_R}}
\]
Figure 8: Three Regimes of the Bank's Optimal Decision

<table>
<thead>
<tr>
<th>Regime 1</th>
<th>Regime 2</th>
<th>Regime 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Write-off</td>
<td>Write off now</td>
<td>Write off now</td>
</tr>
<tr>
<td>irrespective of</td>
<td>if the Subsidy is</td>
<td>irrespective of</td>
</tr>
<tr>
<td>the Subsidy</td>
<td>in Effect</td>
<td>the Subsidy</td>
</tr>
</tbody>
</table>

\[ r_R \equiv \frac{R}{L} \]
Figure 9: Free Boundaries of the Ratio of the Re-investment Return to the Loss from the Write-off

Re-investment Return net of Fund-Raising Cost (R)

Free Boundaries

Regime 1

Regime 2

Regime 3

$\frac{R}{L}$
Figure 10: Boundary Conditions for $f_0(r_R)$ and the Determination of $r_R$ and $\bar{r}_R$

$$f_0(r_R)$$

Regime 1

Regime 2

Regime 3

Write Off Now

(iii): $f_0(r_R) = \frac{\lambda_0 \lambda_1 B(r_R)^{\beta_1} - \lambda_1 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1}$ \hspace{1cm} (equation (35))

(iv): $f_0(r_R) = D(r_R)^{\beta_1} + E(r_R)^{\beta_2} + \frac{\lambda_2 r_R}{\delta_a (\mu + \lambda + \lambda_1 - \alpha_R)} - \frac{\lambda_1 (1 - \theta)}{\mu + \lambda + \lambda_1 - \alpha_L}$ \hspace{1cm} (equation (40))

(v): $f_0(r_R) = \frac{r_R}{\delta_R} - 1$ \hspace{1cm} (equation (42))
Figure 11: Boundary Conditions for $f_1(r_R)$ and the Determination of $r_R$

\[(vi): f_1(r_R) = \frac{\lambda_0 \lambda_1 B(r_R)^{\beta_1} + \lambda_0 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1} \quad \text{(equation (36))} \]

\[(vii): f_1(r_R) = \frac{r_R}{\delta_R} - (1 - \theta) \quad \text{(equations (37) and (43))} \]
Figure 12: Threshold Value \( \hat{r}_R \) as a Function of Parameters

(i) Dependence of \( \hat{r}_R \) on \( \sigma_R \) and \( \sigma_L \)

Note: \( \alpha_R = 0.02, \ \alpha_L = -0.02, \ \delta_R = 0.02, \ \rho = 0.0, \) and \( \lambda = 0.0. \)

(ii) Dependence of \( \hat{r}_R \) on \( \rho \) and \( \alpha_L \)

Note: \( \alpha_R = 0.02, \ \delta_R = 0.02, \ \sigma_R = 0.2, \ \sigma_L = 0.3, \) and \( \lambda = 0.0. \)
(iii) Dependence of $\hat{R}_R$ on $\alpha_R$ and $\alpha_L$

(a) In the Case of Adjustable $\mu$ and Fixed $\delta_R$

Note: $\delta_R = 0.02, \sigma_R = 0.2, \sigma_L = 0.3, \rho = 0.0$, and $\lambda = 0.0$.

(b) In the Case of Adjustable $\delta_R$ and Fixed $\mu$

Notes: 1. $\mu = 0.04, \sigma_R = 0.2, \sigma_L = 0.3, \rho = 0.0$, and $\lambda = 0.0$.
2. The range of $\alpha_R$ must satisfy the constraint $\delta_R \geq 0$. 

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(iv) Dependence of $\hat{r}_R$ on $\lambda$

Note: Calculation is done under the assumption that $\alpha_R$ and $\mu$ are fixed irrespective of the level of $\lambda$. Parameters are set as follows: $\mu = 0.04$, $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\delta_R = 0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, and $\rho = 0.0$.

(v) Dependence of $\hat{r}_R$ on $\delta_R$ and $\sigma_R$

(The Case of Adjusting $\mu$ and Fixed $\alpha_R$)

Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\sigma_L = 0.3$, $\rho = 0.0$, and $\lambda = 0.0$. 

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Figure 13: Threshold Values $\bar{r}_R$ and $r_R$ as a Function of Parameters

(i) Dependence of $\bar{r}_R$ on $\lambda_1$ and $\lambda_0$

![Graph showing dependence of $\bar{r}_R$ on $\lambda_1$ and $\lambda_0$.]

Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, $\delta_R = 0.02$, $\lambda = 0.1$, and $\theta = 0.5$.

(ii) Dependence of $r_R$ on $\lambda_1$ and $\lambda_0$

![Graph showing dependence of $r_R$ on $\lambda_1$ and $\lambda_0$.]

Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, $\delta_R = 0.02$, $\lambda = 0.1$, and $\theta = 0.5$. 

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(iii) Dependence of $\overline{r_R}$ and $r_R$ on $\theta$.

Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, $\delta_R = 0.02$, $\lambda = 0.1$, $\lambda_0 = 0.3$, and $\lambda_1 = 0.3$. 