Optimal Sustainable Monetary Policy*

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Abstract

In recent monetary policy literature, optimal commitment policy or its variant from a timeless perspective has been studied with emphasis on welfare gains from policy commitment, regardless of its time consistency problem called a *stabilization bias*. With an optimizing model used in the literature, we examine sustainable equilibrium of Chari and Kehoe (1990) and study *optimal sustainable policy*, the policymaker's strategy in the best sustainable equilibrium. In the absence of commitment technologies, calibrated versions of the model show that the Ramsey equilibrium generated by optimal commitment policy is not achievable, suggesting that optimal sustainable policy is the desirable policy benchmark. From the viewpoint of policy operationality, the latter policy involves some issues due to its dependence on the entire history of Lagrange multipliers on a sustainability constraint. As a guidepost for actual policy conduct that contains no such issues and attains higher social welfare than discretionary policy does, we propose a sustainable policy conducted by following a policy rule that achieves the best Markov equilibrium.

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1 Introduction

Since the seminal paper of Kydland and Prescott (1977), the time consistency problem of optimal commitment policy has been well known. In the absence of commitment technologies, policymakers have temptations to deviate from their previous commitment on current policy and as a consequence, the optimal policy involves a problem with its credibility under rational expectations. In the real world no central bank possesses such technologies perfectly. Nevertheless, recent monetary policy literature, such as Clarida et al. (1999), McCallum and Nelson (2004), and Woodford (2003), has studied optimal commitment policy or its variant from a timeless perspective proposed by Woodford (1999), with emphasis on welfare gains from policy commitment relative to discretionary policy.[1] The recent literature has dealt with the time consistency problem called a *stabilization bias* (Svensson and Woodford, 2005), which arises from an inefficient trade-off in policymaking with private agents' *forward-looking* behavior and differs from the well-known inflation bias that has been studied in traditional literature starting from Kydland and Prescott (1977) and Barro and Gordon (1983a).[2]

In this paper we examine sustainable equilibrium of Chari and Kehoe (1990) in an optimizing model with the stabilization bias that has been used in the recent literature, and study the policymaker's strategy in the best sustainable equilibrium.[3] We call such a policy strategy *optimal sustainable policy*, which has not been studied in the recent literature. Chari and Kehoe proposed sustainable equilibrium for policy games played between competitive private agents

[1]For the timeless-perspective variant of optimal commitment policy, see also Woodford (2003), McCallum and Nelson (2004), and Jensen and McCallum (2002). Note that even this variant involves the time consistency problem.

[2]Two key assumptions inducing the inflation bias are that the policymaker has a desire to push output above its natural level and that some private agents make their decisions before policy is set. These assumptions lead the policymaker to have a temptation to choose a surprise policy, thereby inducing the time consistency problem. Note that such assumptions are never required for the stabilization bias. As McCallum (1997) and Clarida et al. (1999) point out, how important the inflation bias is in practice is a matter of controversy.

[3]Ireland (1997) examines sustainable equilibrium in an optimizing model with the inflation bias.

and their government in infinite-horizon economies. With a fiscal policy model, these authors addressed the question of how the concept of Kydland and Prescott's (1977) time consistent equilibrium in finite-horizon economies can be extended to infinite-horizon ones (see also Stokey, 1991). By adapting Abreu's (1988) optimal penal codes to their policy game, Chari and Kehoe obtained the following three key findings.[4] First, the entire set of outcomes of sustainable equilibria can be represented by private agents' optimality conditions, the government budget constraint, and a set of inequalities called a *sustainability constraint*. Second, any outcome in the entire set can be implemented by the government's strategy that specifies continuation with that outcome as long as it has been chosen in the past; otherwise, the strategy specifies to switch to the worst sustainable equilibrium outcome in the subsequent economy. Last, even in the absence of commitment technologies, the Ramsey equilibrium outcome generated by optimal commitment policy is attainable if private agents are sufficiently patient. Their first two findings suggest that once we can characterize the entire set of sustainable equilibrium outcomes, optimal sustainable policy can be found as the policymaker's strategy which specifies to continue a policy that yields the best sustainable equilibrium outcome as long as this policy has been adopted in the past; otherwise, the strategy specifies to switch to a policy that induces the worst sustainable equilibrium outcome in the subsequent economy. Their last finding provides a condition whereby we can examine whether optimal commitment policy or its timeless-perspective variant is a desirable policy benchmark even in the absence of commitment technologies.

We show first that the entire set of sustainable equilibrium outcomes of our model can

[4] Note that Abreu's (1988) method involves finding the worst equilibrium and so does that of Chari and Kehoe (1990). It may be more difficult to find the worst sustainable equilibrium in a more general model, although we conjecture that in a broad class of policy games, discretionary policy induces the worst sustainable equilibrium, as shown in our model. To overcome the difficulty, Chang (1998) and Phelan and Stacchetti (2001) adapt an approach of Abreu et al. (1990) to their policy games to feature the entire set of sustainable equilibrium outcomes. Judd et al. (2003) provide a computational method for this approach. Sleet (2001) uses it to analyze an optimizing model with the inflation bias in which the monetary authority has private information.

be fully represented by private agents' optimality condition and a sustainability constraint.[5] This constraint requires that for every history the present value of social welfare attained by an outcome in question be at least as high as the one by the worst sustainable equilibrium outcome, which is induced by discretionary policy in the model, as shown later.

In order to study optimal sustainable policy, we use a Lagrange method of Marcet and Marimon (1998), who develop the pioneering work of Kydland and Prescott (1980). This method leads to *optimal quasi-sustainable policy*, which yields the best sustainable equilibrium outcome in the *presence* of a commitment technology. Specifically, such a policy can be obtained by maximizing a social welfare function subject to private agents' optimality condition and the sustainability constraint, since these two conditions fully represent the entire set of sustainable equilibrium outcomes. Optimal sustainable policy is then given by the policymaker's strategy that specifies to continue optimal quasi-sustainable policy as long as it has been adopted in the past; otherwise, the strategy specifies to switch to discretionary policy forever. This implies that optimal sustainable policy is conducted by following optimal quasi-sustainable policy and implements the best sustainable equilibrium outcome in the *absence* of commitment technologies.[6] This is because private agents are policy takers and optimal sustainable policy leads the policymaker to have no temptation to deviate from optimal quasi-sustainable policy.[7] Thus,

[5]We follow the recently common practice in monetary policy analyses of leaving hidden fiscal policy and the government budget constraint. This would be the case if fiscal policy is "Ricardian", i.e. it appropriately accommodates consequences of monetary policy for the government budget constraint.

[6]The Lagrange method of Marcet and Marimon (1998) is for recursive contract theory, implying that optimal quasi-sustainable policy can be interpreted as an optimal self-enforcing contract in which the policymaker is expected to adopt a policy that attains higher social welfare than discretionary policy does. Although this interpretation of optimal quasi-sustainable policy is in stark contrast with optimal sustainable policy, which the policymaker willingly adopts so as to seek the highest social welfare in the absence of commitment technologies, it is of great interest that the optimal contract and optimal sustainable policy implement the same equilibrium outcome in the model.

[7]The policymaker has no such temptation for the following three reasons. First, optimal sustainable policy specifies to adopt discretionary policy forever once the policymaker deviates from optimal quasi-sustainable policy. Second, the sustainability constraint ensures that for every history the optimal quasi-sustainable policy

by analyzing optimal quasi-sustainable policy, we obtain the following three features of optimal sustainable policy. First of all, the best sustainable equilibrium outcome implemented by optimal sustainable policy is an intermediate one between the Ramsey equilibrium outcome and an equilibrium outcome under discretionary policy, since optimal quasi-sustainable policy is featured as an intermediate one between optimal commitment policy and discretionary policy. Second, optimal quasi-sustainable policy converges to optimal commitment policy in future periods, which implies that even in the absence of commitment technologies the policymaker can credibly adopt optimal commitment policy after he/she keeps optimal quasi-sustainable policy for a sufficiently long period.[8] Last but not least, if the sustainability constraint is never binding, optimal quasi-sustainable policy is consistent with optimal commitment policy and hence the time consistency problem does not matter in that optimal sustainable policy achieves the Ramsey equilibrium outcome. This holds if private agents are sufficiently patient, as shown later. In that case, sticking to optimal commitment policy yields such a large present value of future welfare that deviating from it never pays for the policymaker. However, this is not the case for a range of realistic calibrations of model parameters: a certain lower bound on the discount factor that can sustain the Ramsey equilibrium outcome is extremely close to one.[9] This result also applies to the timeless-perspective variant of optimal commitment policy. Therefore, the desirable benchmark for policy without commitment technologies in the model is not optimal commitment policy, nor its timeless-perspective variant, but optimal sustainable

attains at least as high a present value of social welfare as discretionary policy does. Last, adopting discretionary policy from any period on, together with an associated decision rule of private agents, constitutes a sustainable equilibrium in the subsequent economy.

[8]This finding seems to be related to basic ideas of Woodford's (1999) timeless-perspective variant of optimal commitment policy and of Jensen's (2003) delay in implementation of discretionary policy. Woodford regards the variant as one to which the policymaker would have wished to commit at a date far in the past. Jensen shows that the performance of discretionary policy improves by introducing a delay between the publication and implementation of it and approaches that of optimal commitment policy as the period of the delay is lengthened.

[9]This result is in contrast with Ireland (1997) and Albanesi et al. (2003), who show that the time consistency problem is unlikely to matter in optimizing models with the inflation bias.

policy.

From the viewpoint of operationality of monetary policy, optimal sustainable policy involves some issues. It is conducted by following optimal quasi-sustainable policy, which depends on the current sum of Lagrange multipliers on the sustainability constraint as shown later. This implies that an actual policymaker, i.e. a central bank, is required to trace the history of the Lagrange multipliers from the initial period of the policy design problem. We can then raise the following at least three issues regarding the operationality of optimal sustainable policy. The first issue is about the initial period. When does the central bank set the initial period? The second is how the central bank precisely knows values of the Lagrange multipliers, which are neither actual economic variables, such as inflation and output, nor variables that have explicit relationships with these actual variables. The last, but not least, issue is whether the best sustainable equilibrium outcome generated by optimal quasi-sustainable policy is unique. Taking account of these policy-operationality issues, we may consider optimal sustainable policy unreliable as the guidepost for actual policy conduct. As a reliable policy guidepost, the present paper suggests a sustainable policy that contains no such issues and attains higher social welfare than discretionary policy does. This sustainable policy is the policymaker's strategy that specifies to continue a policy rule, which achieves the best Markov equilibrium in the presence of a commitment technology,[10] as long as it has been adopted in the past; otherwise, the strategy specifies to switch to discretionary policy forever. Thus, as is the case with optimal sustainable policy, such a sustainable policy is conducted by following that policy rule and implements the best Markov equilibrium outcome in the absence of commitment technologies.

The remainder of the paper proceeds as follows. The next section describes a policy design problem used in recent monetary policy literature and reviews optimal commitment policy, its variant from a timeless perspective, and discretionary policy. Section 3 examines optimal

[10]This policy rule is also investigated by Clarida et al. (1999) and is consistent, in the model, with what Woodford (2003) calls the "optimal non-inertial plan". One crucial point of this paper is that the sustainable policy requires no commitment technology but achieves the same best Markov equilibrium outcome as the policy rule does in the presence of such a technology.

sustainable policy. Section 4 argues that neither optimal commitment policy nor its timeless-perspective variant is a desirable benchmark for policy without commitment technologies. Section 5 discusses operationality of optimal sustainable policy. Finally, Section 6 concludes.

2 A monetary policy design problem

In this section we present a policy design problem that has been used in recent monetary policy literature. The model is a simple optimizing model with the stabilization bias.

2.1 An optimizing model with the stabilization bias

The economy consists of a continuum of infinitely-lived private agents and a monetary authority. The private agents set prices of their products monopolistically competitively in the presence of price rigidities and as a consequence, the inflation rate π_t is determined by the following equation, which describes (a log-linear approximation of) an optimality condition for the private agents' problem.[11]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \tag{1}$$

where x_t is the output gap, u_t is an exogenous disturbance, E_t is the expectation operator conditional on the private agents' period-t information set, $\beta \in (0,1)$ is the discount factor, and $\kappa > 0$ is the output gap elasticity of inflation.

The monetary authority is assumed, for simplicity, to be able to control the output gap directly so as to maximize a social welfare function

$$-E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda (x_t - x^*)^2\right], \tag{2}$$

[11] As Roberts (1995) showed, (1) can be derived from a variety of price rigidity models, e.g., a staggered price setting model of Calvo (1983) or Taylor (1980) and a price adjustment cost model of Rotemberg (1982). The private agents' spending behavior is described by an Euler equation for optimal consumption decisions, which can be found in recent monetary policy literature. This Euler equation is not needed in the present analysis, since the output gap is assumed to be the policy instrument, as noted later.

where $\lambda > 0$ is a weight of output gap stabilization relative to inflation stabilization and x^* is an optimal output gap.[12] Note that period zero denotes the initial period of the policy design problem, but not the beginning of the economy. As shown by Woodford (2003), Erceg and Levin (2006), and Nisticó (2007), (2) can be obtained as a second-order approximation of a representative household's utility function in the presence of Calvo (1983) or Taylor (1980) style staggered price setting or Rotemberg (1982) style price adjustment cost of monopolistically competitive firms, whose behavior is described by (1). Also, the weight λ and the optimal output gap x^* can be determined by fundamental parameters of private agents' behavior.

The presence of the disturbance to (1), u_t, which is called the "cost-push shock" by Clarida et al. (1999) or the "inefficient supply shock" by Woodford (2003), is of all importance for this analysis. It induces a trade-off in policymaking between stabilization of inflation and the output gap, thereby causing the time consistency problem of optimal commitment policy in the presence of private agents' forward-looking behavior described by (1). Svensson and Woodford (2005) refer to the problem as a *stabilization bias*, which differs from the well-known inflation bias that has been studied in traditional literature starting from Kydland and Prescott (1977) and Barro and Gordon (1983a). In our model, although the inflation bias can be induced by a positive optimal output gap, we assume that $x^* = 0$, so that the inflation bias is absent. This is because how important the inflation bias is in practice is a matter of controversy, as McCallum (1997) and Clarida et al. (1999) point out. We also assume, as in recent monetary policy literature, that the monetary authority and private agents can observe the shocks u_t while making their decisions in period t and all of them know that the shocks follow a stationary

[12] In recent monetary policy literature, the short-term nominal interest rate is assumed to be the policy instrument, reflecting actual ones adopted by central banks in industrialized countries. Also, the transmission of policy from the interest rate to the output gap is given by the consumption Euler equation mentioned in Footnote 11. It then does not matter whether the instrument is either the interest rate or the output gap as long as there is no uncertainty about that transmission, since controlling either of them can reach identical results obtained below. Jensen (2003) shows how one can use the Euler equation to derive an interest-rate rule that attains the same equilibrium as a given output-gap rule does.

first-order autoregressive process

$$u_t = \rho u_{t-1} + \varepsilon_t, \tag{3}$$

where $\rho \in (-1, 1)$ is an autoregression parameter and ε_t is a white noise with a variance of $\sigma_\varepsilon^2 > 0$. This white noise ε_t is assumed to be bounded. It then follows that the shocks u_t are bounded, i.e. there is $B > 0$ such that $|u_t| < B \; \forall t$.

As Chari and Kehoe (1990) emphasized, it is important to stress that the monetary authority has strategic power while the competitive private agents do not. Therefore, the policy game played between them differs totally from games consisting of a finite number of players, each of whom has strategic power. Although the private agents' future expectations have an effect on current policymaking in equilibrium, this is not due to strategic behavior of the private agents but due to their rational expectations. Also, it is important to stress that the competitive private agents do not collude to "punish" the monetary authority while choosing their decisions, i.e. they are policy takers. This is in stark contrast with traditional literature with the inflation bias, such as Barro and Gordon (1983a, b) and Rogoff (1989), in which the authority plays a game against a coalition of noncompetitive private agents. Another point that is in stark contrast with the traditional literature is timing of decision-making. The traditional literature presumes that some private agents make their decisions before monetary policy is set. This leads the monetary authority to have a temptation to "cheat" such private agents, i.e. to choose a surprise policy after the private agents make the decisions. We assume, as in recent monetary policy literature with the stabilization bias, that the authority and private agents make their decisions simultaneously with their information sets that include past and present values of aggregate variables such as the shocks, inflation, and the output gap. Therefore, there is no such surprise policy in our policy game.

In the rest of this section, we use the model presented above to review three policies, each of which some recent monetary policy analyses have examined as desirable policy: optimal commitment policy, its variant from a timeless perspective proposed by Woodford (1999), and discretionary policy.

2.2 Optimal commitment policy and its timeless-perspective variant

In the presence of a commitment technology that compels the monetary authority to keep his/her previous commitment on current policy, the authority can credibly adopt optimal commitment policy, which makes policy commitment for the entire future in period zero, the initial period of the policy design problem. This policy maximizes the social welfare function (2) subject to private agents' optimality condition (1) from period zero on. Hence, in the presence of the commitment technology, the optimal policy is the most desirable in period zero. Letting ϕ_t be the Lagrange multiplier on (1) in period $t \geq 0$, the associated Lagrangian $\mathcal{L}^c$ can be written as

$$\mathcal{L}^c \equiv E_0 \sum_{t=0}^{\infty} \beta^t \big[- \left(\pi_t^2 + \lambda x_t^2 \right) + 2\phi_t (\pi_t - \beta \pi_{t+1} - \kappa x_t - u_t) \big] \, .$$

The first-order condition for the optimal policy is then given by

$$\pi_t - \phi_t + \phi_{t-1} = 0, \quad \lambda x_t + \kappa \phi_t = 0 \quad \forall t \geq 0$$

together with the initial condition that $\phi_{-1} = 0$, which indicates no previous commitment in period zero. Substituting out the Lagrange multipliers yields an output-gap rule that implements the optimal policy

$$x_0 = -(\kappa/\lambda)\, \pi_0, \quad x_t = -(\kappa/\lambda)\, \pi_t + x_{t-1} \quad \forall t > 0. \tag{4}$$

Ramsey equilibrium, a rational expectations equilibrium (REE) under optimal commitment policy, is a pair of stochastic processes of inflation and the output gap such that this pair is a solution to the system consisting of (1) and (4). This system yields a determinate (i.e. unique, nonexplosive) REE given by

$$\pi_t^c = a_\pi u_t + b_\pi x_{t-1}^c, \quad x_t^c = a_x u_t + b_x x_{t-1}^c \quad \forall t \geq 0 \tag{5}$$

together with $x_{-1}^c = 0$,[13] where $a_\pi \equiv 1/[\beta(b^+ - \rho)] > 0$, $b_\pi \equiv (\lambda/\kappa)(1 - b_x) > 0$, $a_x \equiv -(\kappa/\lambda)\, a_\pi < 0$, $b_x \equiv b^- \in (0,1)$, and $b^\pm \equiv [\,1+\beta+\kappa^2/\lambda \pm \sqrt{(1+\beta+\kappa^2/\lambda)^2 - 4\beta}\,]/(2\beta)$. Note

[13] This means no previous commitment in period zero, but not a zero output gap in period -1.

that $b^+ > 1$. As shown in Appendix A, optimal commitment policy attains the period-$t(\geq 0)$ value of social welfare from that period on, given by

$$\begin{aligned} W^c(u_t, x_{t-1}^c) \;&\equiv\; -E_t \sum_{s=t}^{\infty} \beta^{s-t} \Big[(\pi_s^c)^2 + \lambda \, (x_s^c)^2 \Big] \\ &= \; -\frac{1}{1-\beta\rho^2} \left[a_\pi^2 + \lambda a_x^2 + \frac{2\beta\rho a_x}{1-\beta\rho b_x} \left(a_\pi b_\pi + \lambda a_x b_x \right) + \frac{\beta a_x^2}{1-\beta b_x^2} \left(b_\pi^2 + \lambda b_x^2 \right) \right] \left(u_t^2 + \frac{\beta \sigma_\varepsilon^2}{1-\beta} \right) \\ &\quad - \left\{ \frac{b_\pi^2 + \lambda b_x^2}{1-\beta b_x^2} \, x_{t-1}^c + 2 \left[\frac{a_\pi b_\pi + \lambda a_x b_x}{1-\beta\rho b_x} + \frac{\beta a_x b_x \left(b_\pi^2 + \lambda b_x^2 \right)}{(1-\beta\rho b_x)(1-\beta b_x^2)} \right] u_t \right\} x_{t-1}^c . \end{aligned} \tag{6}$$

This consists of welfare losses due to the current and future shocks, $u_t^2 + \beta\sigma_\varepsilon^2/(1-\beta)$, and current welfare gains or losses due to previous commitment, x_{t-1}^c.

Woodford (1999) argues that optimal commitment policy lacks continuity in its form and thereby "fails to be time consistent *only* if the central bank considers 'optimality' at each point in time" (p. 293). He thus proposes its variant from a timeless perspective, which leads the equilibrium evolution from any period on to be optimal subject to an additional constraint that the evolution in period zero be one associated with the policy in question. In the model the timeless-perspective variant is implemented by an output-gap rule of the time invariant form

$$x_t = \; -(\kappa/\lambda)\, \pi_t + x_{t-1} \quad \forall t \geq 0. \tag{7}$$

Note that as is the case with optimal commitment policy, its timeless-perspective variant involves the time consistency problem in the original policy design problem and therefore a commitment technology is required for adopting it. The output-gap rule (7), together with (1), brings about a determinate REE in the same form as (5)

$$\pi_t^w = \; a_\pi u_t + b_\pi x_{t-1}^w, \quad x_t^w = \; a_x u_t + b_x x_{t-1}^w \quad \forall t \geq 0, \tag{8}$$

given some level of the output gap in period -1, $x_{-1}^w = x_{-1}$. Hence, it attains the period-$t(\geq 0)$ value of social welfare from that period on in the same form as (6)

$$W^w(u_t, x_{t-1}^w) = W^c(u_t, x_{t-1}^w). \tag{9}$$

2.3 Discretionary policy

In the absence of commitment technologies, optimal commitment policy involves the time consistency problem. The lack of such technologies thus leads the authority to choose policy sequentially as opposed to once and for all in period zero. Discretionary policy is then one of policy choices for the authority. This policy is determined in every period so as to maximize the present value of the social welfare function (2) subject to private agents' optimality condition (1) from the current period on, taking private agents' expectations as given. As Clarida et al. (1999) indicate, such a policy design problem can be reduced to a sequence of static problems in which the current output gap is chosen so as to maximize current welfare in (2), $-(\pi_t^2 + \lambda x_t^2)$, subject to (1) with private agents' expectations exogenously given. Then, an output-gap rule that implements the discretionary policy can be given by

$$x_t = -(\kappa/\lambda)\,\pi_t \quad \forall t \geq 0. \tag{10}$$

The discretionary policy implies (10), which differs from (4) in all periods except period zero, thereby causing the time consistency problem. The output-gap rule (10), together with (1), generates a determinate REE given by

$$\pi_t^d = c_\pi u_t, \quad x_t^d = c_x u_t \quad \forall t \geq 0, \tag{11}$$

where $c_\pi \equiv 1/(1 - \beta\rho + \kappa^2/\lambda) > 0$ and $c_x \equiv -(\kappa/\lambda)\,c_\pi < 0$, and therefore it attains, as shown in Appendix A, the period-$t(\geq 0)$ value of social welfare from that period on, given by

$$W^d(u_t) \equiv \, - E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[(\pi_s^d)^2 + \lambda (x_s^d)^2 \right] \; = \; - \frac{c_\pi^2 + \lambda c_x^2}{1 - \beta\rho^2} \left(u_t^2 + \frac{\beta\sigma_\varepsilon^2}{1-\beta} \right). \tag{12}$$

Because discretionary policy makes no policy commitment, (12) represents welfare losses only due to the current and future shocks, $u_t^2 + \beta\sigma_\varepsilon^2/(1-\beta)$.

We have so far reviewed the three policies, each of which some recent monetary policy analyses have studied as desirable policy. Optimal commitment policy is the most desirable in the presence of a commitment technology, but no central bank in the real world possesses such a technology perfectly and hence the policy involves the time consistency problem. Woodford's timeless-perspective variant of optimal commitment policy also contains the same problem, although the continuity or the time invariant form of the associated output-gap rule may bring some advantages in actual policy conduct. By contrast, discretionary policy is credible but attains relatively low social welfare as stressed in recent monetary policy literature such as Clarida et al. (1999), McCallum and Nelson (2004) and Woodford (2003).[14] It then seems quite natural to ask the question of whether there is any concept of optimal policy that is credible with no commitment technology and achieves higher social welfare than discretionary policy does. According to traditional literature with the inflation bias, such as Barro and Gordon (1983b) and Rogoff (1989), even in the absence of commitment technologies, infinite-horizon economies may possess "reputational" equilibrium, which is REE with higher welfare than that attained by discretionary policy. Theoretically accurately, Chari and Kehoe (1990) proposed *sustainable equilibrium* for policy games played between competitive private agents and their government in infinite-horizon economies. With a fiscal policy model, these authors addressed the question of how the concept of Kydland and Prescott's (1977) time consistent equilibrium in finite-horizon economies can be extended to infinite-horizon ones. To this end Chari and Kehoe adapted Abreu's (1988) optimal penal codes to their policy game. Specifically, they featured the entire set of outcomes of sustainable equilibria in their infinite-horizon economy by finding its worst sustainable equilibrium. They also showed that any outcome in the entire set can be implemented by the government's strategy that specifies continuation with that outcome as long as it has been chosen in the past; otherwise, the strategy specifies to switch to the worst sustainable equilibrium outcome in the subsequent economy. These results of Chari and Kehoe lead us to study the monetary authority's policy strategy in the best sustainable equilibrium as a promising alternative to the three policies reviewed above. We call such a policy strategy *optimal sustainable policy*. We investigate this policy in the following sections.

[14] In Section 4, we show welfare gains from optimal commitment policy relative to discretionary policy, using calibrated versions of the model.

3 Optimal sustainable policy

In this section we examine sustainable equilibrium in the model with the stabilization bias and study optimal sustainable policy.

3.1 Characterization of entire set of sustainable equilibrium outcomes

We begin by defining sustainable equilibrium of the model. Let h_t denote the history of the shocks and output gaps (i.e. monetary policy) up to the time of period t at which all the agents make their decisions, and it is defined recursively by $h_0 = u_0$, $h_t = (h_{t-1}, x_{t-1}, u_t)$ $\forall t > 0$.[15] Then, the assumption on timing of decision-making can be formulated as follows. The monetary authority, faced with a history h_t, sets the current output gap x_t as a function of the history, $x_t = \sigma_t(h_t)$, together with a contingent plan $(\sigma_s)_{s \geq t+1}$ for setting future output gaps for all possible future histories. Private agents, faced with a history (h_t, x_t), choose the current inflation rate π_t as a function of the history, $\pi_t = f_t(h_t, x_t)$, together with a contingent plan $(f_s)_{s \geq t+1}$ for choosing future inflation rates for all possible future histories. Note that given a history h_t, the monetary policy strategy for output gaps, $\sigma = (\sigma_t)_{t \geq 0}$, induces future histories in the recursive way such that $h_{t+1} = (h_t, \sigma_t(h_t), u_{t+1})\, \forall t \geq 0$.

The monetary policy strategy σ and private agents' decision rule for inflation rates, $f = (f_t)_{t \geq 0}$, are chosen as follows. Given a history (h_t, x_t) and given a monetary policy strategy σ, private agents determine $(f_s)_{s \geq t}$ so as to satisfy the optimality condition (1):

$$f_t(h_t, x_t) = \beta E_t[f_{t+1}(h_{t+1}, \sigma_{t+1}(h_{t+1}))] + \kappa x_t + u_t, \tag{13}$$

$$f_s(h_s, \sigma_s(h_s)) = \beta E_s[f_{s+1}(h_{s+1}, \sigma_{s+1}(h_{s+1}))] + \kappa \sigma_s(h_s) + u_s \tag{14}$$

for all possible future histories $h_s, s > t$ induced by σ. The monetary authority chooses $(\sigma_s)_{s \geq t}$

[15] The reader may wonder why histories h_t do not include past values of the inflation rate determined by private agents. For a discussion of this point, see Chari and Kehoe (1990).

so that given a history h_t and given a decision rule of private agents f, it solves the problem:

$$\begin{aligned} \max_{(\tilde{\sigma}_s)_{s\geq t}} \quad & -E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ [f_s(h_s, \tilde{\sigma}_s(h_s))]^2 + \lambda [\tilde{\sigma}_s(h_s)]^2 \right\} \\ \text{s.t.} \quad & f_s(h_s, \tilde{\sigma}_s(h_s)) = \beta E_s[f_{s+1}(h_{s+1}, \tilde{\sigma}_{s+1}(h_{s+1}))] + \kappa \tilde{\sigma}_s(h_s) + u_s \end{aligned} \tag{15}$$

for all possible histories $h_s, s \geq t$ induced by $(\tilde{\sigma}_s)_{s\geq t}$.

We can now define sustainable equilibrium of the model.

Definition 1 *A sustainable equilibrium of the model is a pair* (f, σ) *of private agents' decision rule for inflation rates and the monetary policy strategy for output gaps such that*

(D1) *given the monetary policy strategy* σ*, the continuation of private agents' decision rule* f *satisfies the optimality conditions* (13) *and* (14) *for every history* (h_t, x_t)*; and*

(D2) *given the decision rule of private agents* f*, the continuation of the monetary policy strategy* σ *solves the monetary authority's problem* (15) *for every history* h_t*.*

As is the case with time consistent equilibrium of Kydland and Prescott (1977), sustainable equilibrium requires both the private agents' decision rule and the monetary policy strategy to be sequentially rational (see Chari et al., 1989). With the definition of sustainable equilibrium, we have the next result about the REE under discretionary policy.

Proposition 1 *The REE under discretionary policy, given by* (11)*, is the worst sustainable equilibrium of the model.*

Proof. See Appendix B. ■

An intuition for this result is as follows. Discretionary policy is credible, so that the monetary authority can always adopt it and implement the associated REE (11). This implies that the authority never takes any policies that result in inferior social welfare relative to the REE (11). Then, because discretionary policy and the associated decision rule of private agents are sequentially rational, the REE (11) is the worst sustainable equilibrium of the model.

Like Chari and Kehoe (1990), we use Proposition 1 to characterize the entire set of outcomes generated by sustainable equilibria of the model. Recall that a sustainable equilibrium $(f, \sigma) =$

$\{(f_t), (\sigma_t)\}_{t\geq 0}$ is a sequence of functions that specify inflation rates and output gaps for all possible histories. Hence, each sustainable equilibrium yields a particular pair of contingent sequences of inflation rates and output gaps, $(\pi, x) = \{(\pi_t), (x_t)\}_{t\geq 0}$. This pair is called an outcome of that equilibrium. For an arbitrary pair of contingent sequences of inflation rates and output gaps, the next proposition provides the necessary and sufficient condition for the existence of a sustainable equilibrium whose outcome is such a pair.

Proposition 2 *An arbitrary pair (π, x) of contingent sequences of inflation rates and output gaps is an outcome of a sustainable equilibrium if and only if*

(S1) *the pair (π, x) satisfies* (1) *for every possible history h_t; and*

(S2) *the next inequality holds for every possible history h_t:*

$$-E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\pi_s^2 + \lambda x_s^2\right) \geq W^d(u_t). \tag{16}$$

Proof. See Appendix C. ■

The condition (S2) is called a *sustainability constraint.*[16] It is important to stress that it would be a gross mis-reading of Proposition 2 to think that this proposition only features the set of outcomes sustainable by particular trigger-like behavior. Traditional literature with the inflation bias, such as Barro and Gordon (1983b) and Rogoff (1989), uses some sorts of such behavior to seek reputational equilibrium. On the contrary, Proposition 2 characterizes the entire set of outcomes that can be supported by any conceivable sustainable equilibrium.

3.2 Features of optimal sustainable policy

With Proposition 1 and 2, we can find optimal sustainable policy, which is the monetary policy strategy in the best sustainable equilibrium. The argument in the proof of Proposition 2

[16] With an optimizing model with the inflation bias, Ireland (1997) obtains a similar result to Proposition 2. The inflation bias implies that in his sustainability constraint, the right-hand side of the counterpart to (16) consists of current welfare gained by the monetary authority's surprise policy plus a present value of future welfare induced by discretionary policy. In our model, in which there is no such surprise policy, the right-hand side of (16) is the present value of current and future welfare attained by discretionary policy.

suggests that optimal sustainable policy is the policy strategy that specifies continuation with the best sustainable equilibrium outcome as long as it has been chosen in the past; otherwise, the strategy specifies to adopt discretionary policy forever, which induces the worst sustainable equilibrium outcome in the subsequent economy as shown in Proposition 1. To examine optimal sustainable policy, we next derive a policy that generates the best sustainable equilibrium outcome in the presence of a commitment technology. We call such a policy *optimal quasi-sustainable policy.*

Optimal quasi-sustainable policy can be obtained by maximizing the social welfare function (2) subject to the set of (1) and (16) from period zero on, since this set completely represents the entire set of sustainable equilibrium outcomes as shown in Proposition 2. To solve this maximization problem, we use a Lagrange method of Marcet and Marimon (1998), who develop the pioneering work of Kydland and Prescott (1980). Note that this Lagrange method is for recursive contract theory and thus implies that optimal quasi-sustainable policy can be interpreted as an optimal self-enforcing contract in which the monetary authority is expected to adopt a policy that attains higher social welfare than discretionary policy does. Interestingly, this interpretation of optimal quasi-sustainable policy is in stark contrast with optimal sustainable policy, which the authority willingly adopts so as to seek the highest social welfare in the absence of commitment technologies. Although the optimal contract and optimal sustainable policy are based on exactly opposite perspectives on the design of monetary policy, these two implement the same equilibrium outcome in the model.

The Lagrangian associated with optimal quasi-sustainable policy, $\mathcal{L}^s$, can be written as

$$\begin{aligned}\mathcal{L}^s &\equiv E_0\sum_{t=0}^{\infty}\beta^t\left\{-(\pi_t^2+\lambda x_t^2)+2\phi_t(\pi_t-\beta\pi_{t+1}-\kappa x_t-u_t)-\psi_t\left[\sum_{s=t}^{\infty}\beta^{s-t}(\pi_s^2+\lambda x_s^2)+W^d(u_t)\right]\right\}\\ &= E_0\sum_{t=0}^{\infty}\beta^t\left[-\Psi_t(\pi_t^2+\lambda x_t^2)+2\phi_t\left(\pi_t-\beta\pi_{t+1}-\kappa x_t-u_t\right)-\psi_t W^d(u_t)\right],\end{aligned}$$

where ϕ_t and $\psi_t \geq 0$ are Lagrange multipliers on, respectively, (1) and (16) in period $t \geq 0$ and $\Psi_t \geq 1$ is defined recursively by $\Psi_{-1} = 1$, $\Psi_t = \Psi_{t-1} + \psi_t$ $\forall t \geq 0$ or sequentially by $\Psi_t = 1 + \sum_{s=0}^{t} \psi_s$ $\forall t \geq 0$. Thus, Ψ_t contains the current sum of Lagrange multipliers $\{\psi_s\}_{s=0}^{t}$

on the sustainability constraint (S2) and $\psi_t \geq 0$ implies that $\{\Psi_t\}$ is non-decreasing. The first-order condition is given by

$$\Psi_t \pi_t - \phi_t + \phi_{t-1} \;=\; 0, \quad \Psi_t \lambda x_t + \kappa \phi_t \;=\; 0 \quad \forall t \geq 0 \tag{17}$$

together with $\phi_{-1} = 0$, $\Psi_{-1} = 1$, $\Psi_t = \Psi_{t-1} + \psi_t$, and the complementary slackness condition for the sustainability constraint (S2)

$$\psi_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(\pi_s^2 + \lambda x_s^2 \right) + W^d(u_t) \right] = 0 \quad \forall t \geq 0,$$

so that $-\sum_{s=t}^{\infty} \beta^{s-t} (\pi_s^2 + \lambda x_s^2) = W^d(u_t)$ if $\psi_t > 0$. Substituting ϕ_t out from (17) yields an output-gap rule that would implement optimal quasi-sustainable policy

$$x_0 = -(\kappa/\lambda)\, \pi_0, \quad x_t = -(\kappa/\lambda)\, \pi_t + (\Psi_{t-1}/\Psi_t)\, x_{t-1} \quad \forall t > 0. \tag{18}$$

The next proposition summarizes three important properties of this output-gap rule.

Proposition 3 *In the output-gap rule* (18)*, the following three hold.*

(a) $0 < \Psi_{t-1}/\Psi_t \leq 1 \; \forall t \geq 0$*;*

(b) $\Psi_{t-1}/\Psi_t \to 1$ *as* $t \to \infty$*;*

(c) *If* $\Psi_t = 1 \; \forall t \geq 0$ *(i.e.* $\psi_t = 0 \; \forall t \geq 0$*), then* (18) *is consistent with* (4).

Proof. Property (a) immediately follows from the fact that $1 \leq \Psi_{t-1} \leq \Psi_t$ for all $t \geq 0$. This fact also implies that for some histories such that $\lim_{t\to\infty} \Psi_t < \infty$, property (b) immediately follows, and for other histories (i.e. $\lim_{t\to\infty} \Psi_t = \infty$) we have that $\Psi_{t-1}/\Psi_t = 1 - \psi_t/\Psi_t \to 1$ as $t \to \infty$. Property (c) holds because $\Psi_{t-1}/\Psi_t = 1 \; \forall t \geq 0$. ■

Optimal sustainable policy is now given by the monetary policy strategy that specifies to continue optimal quasi-sustainable policy as long as it has been adopted in the past; otherwise, the strategy specifies to switch to discretionary policy forever. Then, it follows that optimal sustainable policy is conducted by following optimal quasi-sustainable policy and implements the best sustainable equilibrium outcome in the absence of commitment technologies. This is because private agents are policy takers and optimal sustainable policy leads the monetary authority to have no temptation to deviate from optimal quasi-sustainable policy for the following three reasons. First, optimal sustainable policy specifies to switch to discretionary policy forever once the authority deviates from optimal quasi-sustainable policy. Second, the sustainability constraint ensures that for every history the optimal quasi-sustainable policy attains at least as high a present value of social welfare as discretionary policy does. Last, adopting discretionary policy from any period on, together with the associated decision rule of private agents, constitutes a sustainable equilibrium in the subsequent economy.

From Proposition 3, we can obtain the following three features of optimal sustainable policy. First of all, property (a) implies that the output-gap rule (18), which would implement optimal quasi-sustainable policy, is featured as an intermediate one between the output-gap rules (4) and (10), which implement, respectively, optimal commitment policy and discretionary policy. This in turn implies that the best sustainable equilibrium outcome implemented by optimal sustainable policy is an intermediate one between the Ramsey equilibrium outcome and the equilibrium outcome under discretionary policy, i.e. the worst sustainable equilibrium outcome.

Second, from property (b) we have that the output-gap rule (18) converges to the one (4) in future periods. Hence, it follows that even in the absence of commitment technologies the monetary authority can credibly adopt optimal commitment policy after he/she keeps optimal quasi-sustainable policy for a sufficiently long period. This finding seems to be related to Woodford's (1999) timeless-perspective variant of optimal commitment policy and Jensen's (2003) delay in implementation of discretionary policy. Woodford regards the timeless-perspective variant as one to which the authority would have wished to commit at a date far in the past. Then, if we set the initial period of the policy design problem at that date (i.e. $t = -\infty$), the basic idea of Woodford seems very close to the one behind property (b). Also, we can see a similar idea in Jensen, who shows that the performance of discretionary policy approaches that of optimal commitment policy as the period of the delay between the publication and implementation of discretionary policy is lengthened.

Last but not least, property (c) implies that if $\psi_t = 0 \ \forall t \geq 0$, which means that the sustainability constraint (S2) is never binding, then the time consistency problem does not matter in that optimal sustainable policy achieves the Ramsey equilibrium outcome. The question we address at this point is under what condition this holds. Because the shocks u_t induce the stabilization bias, optimal commitment policy attains higher social welfare in period zero than discretionary policy does, i.e. $W^c(u_0, x^c_{-1}) > W^d(u_0)$. Hence (16) obtains in period zero. Then, if for every history h_t, $t > 0$, optimal commitment policy also achieves a greater period-t value of social welfare from that period on than discretionary policy does, i.e. $W^c(u_t, x^c_{t-1}) > W^d(u_t)$, then the sustainability constraint (S2) is never binding. A condition for this is provided in the next proposition.

Proposition 4 *There exists* $\underline{\beta} \in (0,1)$ *such that for any discount factor* $\beta \in \left(\underline{\beta}, 1\right)$*, optimal sustainable policy achieves the Ramsey equilibrium outcome given by* (5)*.*

Proof. See Appendix D. ■

Proposition 4 suggests that if private agents are sufficiently patient, i.e., $\underline{\beta} < \beta < 1$, the Ramsey equilibrium outcome is attainable even in the absence of commitment technologies. Indeed, sticking to optimal commitment policy yields such a large present value of future welfare that deviating from it never pays for the monetary authority as long as the discount factor is sufficiently close to one.[17] A similar result to Proposition 4 holds for Woodford's (1999) timeless-perspective variant of optimal commitment policy, since this variant differs from optimal commitment policy only in period zero. The next corollary states the result.

Corollary 1 *If* $|x_{-1}| < B|a_x|/(1 - b_x)$*, where* a_x, b_x *are given in* (5) *and* B *is a bound for the shocks* u_t*,*[18] *then* $\underline{\beta}$ *given in Proposition 4 ensures that for any discount factor* $\beta \in \left(\underline{\beta}, 1\right)$*, the outcome of REE under the timeless-perspective variant of optimal commitment policy, given by* (8)*, is attainable. Otherwise, there exists* $\beta_w \in \left[\underline{\beta}, 1\right)$ *such that for any discount factor*

[17] This is an analogy to the reason why the folk theorem holds in infinitely repeated games.

[18] The boundedness of the shocks u_t follows from that of the white noise ε_t in the shock process (3).

$\beta \in (\beta_w, 1)$, *that outcome is attainable.*

Proof. This corollary immediately follows from a similar proof to that of Proposition 4. ■

From Proposition 4 and Corollary 1, we can consider a test that asks whether optimal commitment policy or its timeless-perspective variant is a desirable policy benchmark even in the absence of commitment technologies. If the actual value of the discount factor β is greater than $\underline{\beta}$ given in Proposition 4, the sustainability constraint is never binding, so that optimal quasi-sustainable policy is consistent with optimal commitment policy. This implies that optimal sustainable policy is conducted by following optimal commitment policy and hence we can regard the latter policy as the desirable policy benchmark. Otherwise, there would be no grounds for supporting optimal commitment policy. The same argument can apply to the timeless-perspective variant. In the next section, we investigate only optimal commitment policy, since the condition for x_{-1} in Corollary 1 is relevant with a sufficiently large B and hence the result obtained below also holds for the timeless-perspective variant.

4 Is optimal commitment policy the desirable benchmark?

In this section we address the question of whether optimal commitment policy is a desirable benchmark for policy without commitment technologies.

From the argument in the proof of Proposition 4, we have a lower bound $\underline{\beta}$ on the discount factor required for sustaining the Ramsey equilibrium outcome, given by

$$\underline{\beta} = \inf\{\beta \in (0,1) \, : \, F(\tilde{\beta}) \geq 0 \;\; \forall \tilde{\beta} \in (\beta, 1)\}, \tag{19}$$

where[19]

$$\begin{aligned} F(\beta) &= \frac{\beta\sigma_\varepsilon^2\Delta(\beta)}{(1-\beta)(1-\beta\rho^2)} - \frac{B^2 a_x^2(b_\pi^2+\lambda b_x^2)}{(1-\beta b_x^2)(1-b_x)^2} + \frac{2B^2 a_x}{1-b_x}\left[\frac{a_\pi b_\pi - \lambda a_x b_x}{1-\beta\rho b_x} - \frac{\beta a_x b_x(b_\pi^2+\lambda b_x^2)}{(1-\beta\rho b_x)(1-\beta b_x^2)}\right], \\ \Delta(\beta) &= (c_\pi^2+\lambda c_x^2) - (a_\pi^2+\lambda a_x^2) - \frac{2\beta\rho a_x}{1-\beta\rho b_x}(a_\pi b_\pi + \lambda a_x b_x) - \frac{\beta a_x^2}{1-\beta b_x^2}(b_\pi^2+\lambda b_x^2). \end{aligned}$$

[19]Note that a_π, b_π, a_x, b_x are functions of β and satisfy that for each $\beta \in (0,1]$, $0 < a_x < 1/(1-\beta\rho)$, $0 < b_x < \lambda/\kappa$, $-(\kappa/\lambda)/(1-\beta\rho) < a_x < 0$, and $0 < b_x < 1$.

As shown in the proof of Proposition 4, we have $\Delta(\beta) > 0$ for all $\beta \in (0, 1]$. This is because the function Δ takes a value proportional to the difference between welfare losses only due to the current and future shocks, induced by optimal commitment policy and by discretionary policy, i.e. $W^c(u_t, 0) - W^d(u_t)$, which is positive.[20] Also, the function F satisfies that for each $\beta \in (0, 1]$, $W^c(u_t, x^c_{t-1}) - W^d(u_t) > F(\beta)$ for every history h_t. Thus, for $\beta \in (0, 1)$ such that $F(\beta) \geq 0$, we have that $W^c(u_t, x^c_{t-1}) > W^d(u_t)$ for every history h_t, which implies that the sustainability constraint (S2) is never binding and hence optimal quasi-sustainable policy is consistent with optimal commitment policy.

As mentioned above, if the actual value of the discount factor β is greater than $\underline{\beta}$ given by (19), optimal sustainable policy is conducted by following optimal commitment policy. This provides a justification for regarding optimal commitment policy as the desirable policy benchmark even in the absence of commitment technologies. We now compute $\underline{\beta}$ to examine whether $\beta > \underline{\beta}$.

4.1 Calibration of model parameters

To carry out the test proposed above, we need realistic calibrations of parameters in (19), $\beta, \kappa, \lambda, \rho, \sigma_\varepsilon, B$. Table 1 summarizes calibrated values of the first four parameters. These values are taken from McCallum and Nelson (2004). Note that the calibrations are for the quarterly model with the annualized inflation rate and in terms of this inflation rate the value of $\lambda = (1/4)^2 = .0625$ means equal weights in the social welfare function (2). As for the bound for the shocks u_t, we set $B = 10\sigma_\varepsilon/(1 + |\rho|)$, which yields that $|\varepsilon_t| < B(1 + |\rho|) = 10\sigma_\varepsilon$. This choice of B seems reasonable, since the probability that $|\varepsilon_t| > 10\sigma_\varepsilon$ is almost zero when ε_t is normally distributed. Also, such a choice allows us to compute $\underline{\beta}$ given by (19) without specifying σ_ε, because we can write $F(\beta) = \sigma^2_\varepsilon \tilde{F}(\beta)$, where $\tilde{F}(\beta)$ is independent of σ_ε, and hence $F(\beta) \geq 0$ if and only if $\tilde{F}(\beta) \geq 0$.

[20] Note that $W^c(u_t, 0) - W^d(u_t) = [\Delta(\beta)/(1 - \beta\rho^2)][u_t^2 + \beta\sigma^2_\varepsilon/(1 - \beta)] > 0$.

4.2 Test results

Before presenting results of the test, we confirm the recent literature's emphasis on welfare gains from policy commitment, or equivalently, how severe the time consistency problem is in the model with the stabilization bias. Table 2 shows welfare gains from optimal commitment policy relative to discretionary policy, i.e. $W^{c/d} \equiv [W^c(u_0, x^c_{-1}) - W^d(u_0)]/|W^d(u_0)| \times 100$.[21] In this table, we can see that the welfare gains vary from 3.32% to 83.35%, depending on model parameters, κ, λ, ρ. One feature of Table 2 is that the welfare gains enlarge as ρ increases, that is, the shocks u_t (i.e. the source of the time consistency problem) become more persistent. Discretionary policy performs relatively well when the persistence of the shocks is sufficiently low. Otherwise, the time consistency problem is very severe in the model. This result is in contrast with recent analyses with the inflation bias, such as Ireland (1997) and Albanesi et al. (2003), who demonstrate that the time consistency problem is unlikely to matter in optimizing models with the inflation bias, i.e. a sticky price model of Ireland and some variants of a cash credit good model of Lucas and Stokey (1983) and of a limited participation model of Christiano et al. (1997).

We now discuss whether optimal commitment policy is a desirable benchmark for policy without commitment technologies. Table 3 shows values of $\underline{\beta}$ determined by (19) in all the calibration cases.[22] In this table the values with no shadows represent calibration cases in which $\underline{\beta}$ is less than .99, that is, optimal sustainable policy is conducted by following optimal commitment policy. In many of the realistic calibration cases, however, $\underline{\beta}$ is greater than .99. This suggests that there are no grounds for regarding optimal commitment policy as the desirable policy benchmark in the absence of commitment technologies, which no central bank in the real world possesses perfectly. Thus, the policy benchmark is not optimal commitment policy (nor its variant from a timeless perspective by Woodford) but optimal sustainable policy.

[21] Note that this measure of welfare gains depends on niether the period-zero shocks u_0 nor the variance σ^2_ε of the white noise ε_t in the shock process (3).

[22] In each calibration case, F is strictly increasing in β, so that there is no $\beta < \underline{\beta}$ such that $F(\beta) \geq 0$.

In Table 3, we can see the following three features. First, $\underline{\beta}$ is more likely to be greater than .99 when the output gap elasticity of inflation, κ, decreases. As McCallum and Nelson (2004) point out, the actual value of κ lies between .01 and .05 in recent years of low, stable inflation. Hence, it is likely $\underline{\beta}$ is greater than .99 and optimal commitment policy is not a desirable policy benchmark. Second, $\underline{\beta}$ is more likely to be greater than .99 as the social welfare function's weight of output gap stabilization relative to inflation stabilization, λ, increases. While Woodford (2003) indicates a small value of λ using Calvo's (1983) staggered price setting, Erceg and Levin (2006) present a large value of it with Taylor's (1980). Thus, optimal commitment policy is not a desirable policy benchmark when monopolistically competitive firms set prices of their products for fixed duration rather than for random duration. This is true because distortions due to the monopolistic competition are smaller with the fixed duration of unchanged prices than with the random duration. Last, $\underline{\beta}$ is more likely to be greater than .99 as the persistence of the shocks, ρ, decreases. This is because, as noted above, the welfare gains from optimal commitment policy relative to discretionary policy, $W^{c/d}$, decline with less persistent shocks, so that it is more likely to pay for the monetary authority to deviate from optimal commitment policy.[23]

5 On operationality of optimal sustainable policy

We have shown with calibrated versions of the model that the desirable benchmark for policy without commitment technologies is not optimal commitment policy, nor its timeless-perspective variant, but optimal sustainable policy. As noted above, optimal sustainable policy is conducted by following optimal quasi-sustainable policy. In such policy conduct, however, there are some issues regarding operationality. As shown above, optimal quasi-sustainable policy depends on the current sum $\Psi_t (= 1 + \sum_{s=0}^{t} \psi_s)$ of Lagrange multipliers $\{\psi_s\}_{s=0}^{t}$ on

[23]For the same reason, $\underline{\beta}$ is more likely to be greater than .99 when the variance σ_ε^2 of the white noise ε_t in the shock process (3) decreases (while keeping the value, but not the form, of the bound B for the shocks the same as specified above).

the sustainability constraint (S2). This implies that an actual monetary authority, i.e. a central bank, is required to trace the history of the Lagrange multipliers from the initial period of the policy design problem. We can then raise the following at least three issues with the operationality of optimal sustainable policy.

5.1 Issues regarding operationality of optimal sustainable policy

The first issue is about the initial period of the policy design problem. When does the central bank set the initial period? This issue is indeed the same as what recent monetary policy literature considers one of the most critical defects of optimal commitment policy, and hence Woodford (1999) proposes its variant from a timeless perspective. Optimal sustainable policy has additional difficulty with this issue. In the model, if the output gap happens to be zero in some period, we can see from (4) that the continuation of optimal commitment policy can be obtained by re-optimizing the associated problem in the next period. By contrast, we cannot necessarily obtain the continuation of optimal quasi-sustainable policy by such re-optimization, due to the dependence on the history of the Lagrange multipliers from some initial period. This contrast can be attributed to the fact that optimal commitment policy is partially history-dependent (and so is its timeless-perspective variant) while optimal sustainable policy is fully history-dependent.

The second issue is how the central bank precisely knows values of Lagrange multipliers on the sustainability constraint (S2). These multipliers are neither actual economic variables, such as inflation and output, nor variables that have explicit relationships with these actual variables. Moreover, the sustainability constraint (S2) is a sequence of inequalities, so that we cannot analytically compute an outcome of equilibrium under optimal quasi-sustainable policy, nor can we apply the usual Blanchard-Kahn method for numerically solving linear rational expectations models. In the existing literature there are two numerical methods for solving such a nonlinear model. One is the method of Christiano and Fisher (2000), who use first-order conditions for optimal policy with inequality constraints. Another is that of

Marcet and Marimon (1998), who use a Bellman equation called the "recursive saddle point functional equation". We then need some techniques to approximate expectation functions in the Christiano-Fisher method or a value function in the Marcet-Marimon method. This implies that we cannot obtain the exact values of the Lagrange multipliers and we know only their approximate values implicitly. Such lack of knowledge about the precise values of the Lagrange multipliers in turn would cause lack of transparency in monetary policy conduct.

The last, but not least, issue is whether the best sustainable equilibrium outcome generated by optimal quasi-sustainable policy is unique.[24] If so, private agents and the monetary authority could coordinate on that unique equilibrium outcome. Otherwise, we are faced with a critical problem of how a particular outcome can be chosen from multiple equilibrium outcomes with the same level of social welfare. The above-mentioned methods for solving the nonlinear model give no guarantee of a unique solution to the problem associated with optimal quasi-sustainable policy. To the best of our knowledge, there is no way to investigate whether the nonlinear model contains a unique solution.

5.2 A sustainable policy reliable as actual policy guidepost

Taking account of the policy-operationality issues raised above, we may consider optimal sustainable policy unreliable as the guidepost for actual policy conduct. The question we address at this point is whether there is a sustainable policy that contains no such issues and attains higher social welfare than discretionary policy does. To answer this question, it is reasonable to focus on monetary policy strategies played in Markov equilibrium of the model, in which equilibrium processes of inflation and the output gap depend only on the model's state variables,

[24]This possible multiplicity of the best sustainable equilibrium outcome differs totally from multiplicity of reputational equilibrium that has been discussed in traditional literature with the inflation bias, such as Rogoff (1989). The latter equilibrium multiplicity is induced by the fact that the traditional literature has studied games played between the monetary authority and a coalition of noncompetitive private agents, so that a coordination problem arises as the game-theory literature involves a cooperation problem. Such a problem never appears in our policy game in which competitive private agents are policy takers.

i.e. the shocks u_t. This is because the sustainability constraint (S2) given in Proposition 2 requires that for every history, social welfare attained by a policy under consideration be comparable with that by discretionary policy, which yields a Markov equilibrium.[25] Thus, if a proposed policy generates a Markov equilibrium, it is easy to examine whether an outcome of equilibrium under this policy is sustainable. Note that discretionary policy does not necessarily bring about the best Markov equilibrium, so that it may be possible to find a sustainable policy that achieves higher social welfare than discretionary policy does. Indeed, we can obtain the following sustainable policy that attains the highest social welfare among all Markov equilibria of the model including the one induced by discretionary policy. The sustainable policy is the monetary policy strategy that specifies to continue a policy rule of the form

$$x_t = -[(\kappa/\lambda)/(1-\beta\rho)]\,\pi_t \tag{20}$$

as long as this rule has been adopted in the past; otherwise, it specifies to switch to discretionary policy forever. As is the case with optimal sustainable policy, this sustainable policy is conducted by following the policy rule (20) and implements the best Markov equilibrium outcome of the form

$$\pi_t^s = d_\pi u_t, \quad x_t^s = d_x u_t \quad \forall t \geq 0, \tag{21}$$

where $d_\pi = 1/[1-\beta\rho + (\kappa^2/\lambda)/(1-\beta\rho)]$ and $d_x = -[(\kappa/\lambda)/(1-\beta\rho)]d_\pi$. Hence, it attains the period-$t(\geq 0)$ value of social welfare from that period on, given by

$$W^s(u_t) = -\frac{d_\pi^2 + \lambda d_x^2}{1-\beta\rho^2}\left(u_t^2 + \frac{\beta\sigma_\varepsilon^2}{1-\beta}\right). \tag{22}$$

[25] Another reason for the focus on monetary policy strategies played in Markov equilibrium is that the entire set of sustainable policies in this class is not sensitive to model parameters. We can alternatively consider a class of policy strategies that specify to continue a policy rule of an intermediate form between the output-gap rules (7) and (10), $x_t = -(\kappa/\lambda)\,\pi_t + c\,x_{t-1}$, where $0 \leq c \leq 1$, as long as this rule has been adopted in the past; otherwise, they specify to switch to discretionary policy forever. The entire set of sustainable policies in this class, however, would be very sensitive to the model parameters, particularly the discount factor β, as is the case with optimal commitment policy or its timeless-perspective variant.

Clarida et al. (1999) have established that in the presence of a commitment technology, the policy rule (20) achieves the best Markov equilibrium (21). One crucial point of this paper is that the sustainable policy requires no commitment technology but implements the same best Markov equilibrium outcome as the policy rule (20) does with such a technology.

The sustainable policy with (20) contains no such issues that optimal sustainable policy involves, since there are no problems with the initial period and with the transparancy and since the best Markov equilibrium outcome is unique in the model. Table 4 shows the performance of the sustainable policy with (20). Note that in the case of no persistence of the shocks, i.e. $\rho = 0$, the sustainable policy is consistent with discretionary policy, so that this case is omitted from Table 4. One point of this table is that as the shocks u_t become more persistent, welfare gains from the sustainable policy relative to discretionary policy, $W^{s/d}$, enlarge and the performance of the sustainable policy relative to optimal commitment policy, $W^{s/d}/W^{c/d}$, is ameliorated. As noted above, when the persistence of the shocks is sufficiently low, discretionary policy performs relatively well. Therefore, the sustainable policy exhibits a very good performance. This paper thus suggests the sustainable policy as a reliable guidepost for actual policy conduct.

6 Concluding remarks

In this paper we have examined optimal sustainable monetary policy, employing an optimizing model with the stabilization bias that has been used in recent monetary policy literature. We have shown with realistically calibrated versions of the model that the desirable benchmark for policy without commitment technologies, which no central bank in the real world possesses perfectly, is not optimal commitment policy, nor its variant from a timeless perspective proposed by Woodford (1999), but optimal sustainable policy. From the viewpoint of policy operationality, the latter optimal policy involves some issues and hence it may be unreliable as the guidepost for actual policy conduct. We then suggest as a reliable policy guidepost a sustainable policy conducted by a policy rule that implements the best Markov equilibrium outcome. This sustainable policy contains no such issues that optimal sustainable policy involves, and achieves higher social welfare than discretionary policy does.

Although optimal sustainable policy involves some issues regarding its operationality, there is still great interest in examining how this policy responds to shocks, particularly in the case in which the sustainability constraint is binding, i.e. optimal quasi-sustainable policy differs from optimal commitment policy. This is because we have no idea about the exact responses of optimal quasi-sustainable policy to shocks and because optimal sustainable policy is the desirable policy benchmark in the absence of commitment technologies. Thus, the numerical investigation of optimal (quasi-)sustainable policy is one future research topic.

Another topic of future research is to examine sustainability of outcomes of equilibria under particular monetary policy rules. Since the pioneering work of Taylor (1993), policy rules have received much attention in monetary policy literature. Many recent analyses assume that the monetary authority can credibly commit to a proposed policy rule. It is far from clear, however, exactly how or whether such credibility would arise. To address this question, we suggest examining whether the proposed policy rule leads to an outcome of a sustainable equilibrium. If the policy rule does so, the authority's commitment to it can be supported by a sustainable equilibrium in which the authority takes the policy strategy that specifies to continue the policy rule as long as it has been adopted in the past; otherwise, the strategy specifies to switch to a policy that induces the worst sustainable equilibrium outcome in the subsequent economy. Hence the commitment is credible. Otherwise, private agents know the authority's temptation to deviate from that policy rule, so that such commitment cannot be credible. Thus, the sustainability of equilibrium outcomes seems to be a requirement policy rules must meet. A companion paper by Kurozumi (2006) investigates sustainability of outcomes of equilibria under policy rules of the sort proposed by Taylor (1993).

Appendix

A Calculation of (6) and (12)

In each period $t \geq 0$, we have

$$W^c(u_t, x_{t-1}^c) = -E_t \sum_{s=t}^{\infty} \beta^{s-t} \Big\{ (E_t \pi_s^c)^2 + V_t \pi_s^c + \lambda \left[(E_t x_s^c)^2 + V_t x_s^c \right] \Big\}, \tag{23}$$

where $x_{-1}^c = 0$. Using (3) and (5), we have for $z = \pi, x$,

$$\begin{aligned} E_t z_s^c &= b_z b_x^{s-t} x_{t-1}^c + \rho^{s-t-1} \left[a_z \rho + a_x b_z \frac{1 - (b_x/\rho)^{s-t}}{1 - b_x/\rho} \right] u_t \quad \forall s \geq t, \\ V_t z_s^c &= \sigma_\varepsilon^2 \left\{ a_z^2 + (a_z \rho + a_x b_\pi)^2 + \cdots + \rho^{2(s-t-2)} \left[a_z \rho + a_x b_z \frac{1 - (b_x/\rho)^{s-t-1}}{1 - b_x/\rho} \right]^2 \right\} \ \forall s > t, \end{aligned}$$

and $V_t z_t^c = 0$. Substituting these into (23) yields (6).

Similarly, in each period $t \geq 0$, substituting (11) into $W^d(u_t)$ yields

$$W^d(u_t) = -(c_\pi^2 + \lambda c_x^2) \sum_{s=t}^{\infty} \beta^{s-t} \left[(E_t u_s)^2 + V_t u_s \right]. \tag{24}$$

From (3) we have $E_t u_s = \rho^{s-t} u_t$, $V_t u_s = \sigma_\varepsilon^2 [1 - \rho^{2(s-t)}]/(1-\rho^2)$ $\forall s \geq t$. Substituting these into (24) yields (12).

B Proof of Proposition 1

Let (f^d, σ^d) denote the REE under discretionary policy. From (11), it is given by

$$f_t^d(h_t, x_t) = c_\pi u_t \ \forall (h_t, x_t), t \geq 0; \quad \sigma_t^d(h_t) = c_x u_t \ \forall h_t, t \geq 0. \tag{25}$$

First let us show that (f^d, σ^d) is sustainable equilibrium. From the constraint condition of the monetary authority's problem (15) with the decision rule of private agents f^d, it immediately follows that $(\sigma_s^d)_{s \geq t}$ is its unique solution, that is, $(\sigma_s^d)_{s \geq t}$ is the unique best response to $(f_s^d)_{s \geq t}$, so that (f^d, σ^d) satisfies (D2). Let us consider the private agents' decision rule f^d. Given the discretionary policy strategy σ^d and given a history (h_t, x_t), we have that $(f_s^d)_{s \geq t}$ satisfies (13) and (14), since $x_t = \sigma_t^d(h_t) = c_x u_t$. Hence, (f^d, σ^d) satisfies (D1) and is therefore sustainable equilibrium. Note that $(f_s^d)_{s \geq t}$ is the unique best response to $(\sigma_s^d)_{s \geq t}$, since

$$f_s^d(h_s, \sigma_s^d(h_s)) = E_s \sum_{\tau=s}^{\infty} \beta^{\tau-s}(\kappa\, c_x + 1)u_\tau = c_\pi u_s.$$

This implies that for every history h_t, $\{(f_s), (\sigma_s)\}_{s \geq t}$ constitutes a sustainable equilibrium in the subsequent economy.

We next show that the sustainable equilibrium (f^d, σ^d) is the worst one. Suppose to the contrary that there is a worse sustainable equilibrium (f^*, σ^*) than (f^d, σ^d). Then, we have

$$-E_0 \sum_{t=0}^{\infty} \beta^t \{[f_t^d(h_t, \sigma_t^d(h_t))]^2 + \lambda[\sigma_t^d(h_t)]^2\} \;>\; -E_0 \sum_{t=0}^{\infty} \beta^t \{[f_t^*(h_t, \sigma_t^*(h_t))]^2 + \lambda[\sigma_t^*(h_t)]^2\}. \quad (26)$$

Suppose first that $\sigma_t^*(h_t) = \sigma_t^d(h_t)$ for all possible histories $h_t, t \geq 0$ induced by σ^*. Since f^d is the unique best response to σ^d, f^* must satisfy that $f_t^*(h_t, \sigma_t^d(h_t)) = f_t^d(h_t, \sigma_t^d(h_t))$ for all possible histories $h_t, t \geq 0$ induced by σ^*. Thus, we have

$$\begin{aligned} -E_0 \sum_{t=0}^{\infty} \beta^t \{[f_t^*(h_t, \sigma_t^*(h_t))]^2 + \lambda[\sigma_t^*(h_t)]^2\} &= -E_0 \sum_{t=0}^{\infty} \beta^t \{[f_t^*(h_t, \sigma_t^d(h_t))]^2 + \lambda[\sigma_t^d(h_t)]^2\} \\ &= -E_0 \sum_{t=0}^{\infty} \beta^t \{[f_t^d(h_t, \sigma_t^d(h_t))]^2 + \lambda[\sigma_t^d(h_t)]^2\}, \end{aligned}$$

which contradicts (26). Suppose next that there is a possible history h_{t^*} induced by σ^* such that $\sigma_t^*(h_t) = \sigma_t^d(h_t)$ $\forall t = 0, 1, \ldots, t^* - 1$ and $\sigma_{t^*}^*(h_{t^*}) \neq \sigma_{t^*}^d(h_{t^*})$. In the sustainable equilibrium (f^*, σ^*), consider the monetary authority's deviation from $(\sigma_t^*)_{t \geq t^*}$ to $(\sigma_t^d)_{t \geq t^*}$. Under the assumption on timing of decision-making, private agents know this deviation in period t^*. Then, because (f^*, σ^*) is sustainable equilibrium, it follows from an analogous argument to the one above that

$$\begin{aligned} -E_0 \sum_{t=0}^{\infty} \beta^t \{[f_t^*(h_t, \sigma_t^*(h_t))]^2 + \lambda[\sigma_t^*(h_t)]^2\} &\geq -E_0 \sum_{t=0}^{\infty} \beta^t \{[f_t^*(h_t, \sigma_t^d(h_t))]^2 + \lambda[\sigma_t^d(h_t)]^2\} \\ &= -E_0 \sum_{t=0}^{\infty} \beta^t \{[f_t^d(h_t, \sigma_t^d(h_t))]^2 + \lambda[\sigma_t^d(h_t)]^2\}, \end{aligned}$$

which contradicts (26). Consequently, (f^d, σ^d) is the worst sustainable equilibrium.

C Proof of Proposition 2

We begin by showing that if (π, x) is an outcome of a sustainable equilibrium (f, σ), then (S1) and (S2) are satisfied. Clearly, (S1) holds because any sustainable equilibrium satisfies (13) and (14) for every possible history. To show that (S2) is satisfied, suppose to the contrary that there is a possible history h_{t^*} such that

$$-E_{t^*}\sum_{s=t^*}^{\infty}\beta^{s-t^*}\left\{[f_s(h_s,\sigma_s(h_s))]^2+\lambda[\sigma_s(h_s)]^2\right\} \;<\; W^d(u_{t^*}). \qquad (27)$$

Consider the monetary authority's deviation at the history h_{t^*} from $(\sigma_s)_{s\geq t^*}$ to the discretionary policy strategy $(\sigma_s^d)_{s\geq t^*}$, which is given by (25) in Appendix B. From an analogous argument to that in the proof of Proposition 1, it follows that $f_s(h_s,\sigma_s^d(h_s)) = f_s^d(h_s,\sigma_s^d(h_s))$ for all possible histories h_s, $s \geq t^*$ induced by $(\sigma_s^d)_{s\geq t^*}$, and hence we have

$$\begin{aligned}
-E_{t^*}\sum_{s=t^*}^{\infty}\beta^{s-t^*}\{[f_s(h_s,\sigma_s(h_s))]^2+\lambda[\sigma_s(h_s)]^2\} \;&\geq\; -E_{t^*}\sum_{s=t^*}^{\infty}\beta^{s-t^*}\{[f_s(h_s,\sigma_s^d(h_s))]^2+\lambda[\sigma_s^d(h_s)]^2\}\\
&=\; -E_{t^*}\sum_{s=t^*}^{\infty}\beta^{s-t^*}\{[f_s^d(h_s,\sigma_s^d(h_s))]^2+\lambda[\sigma_s^d(h_s)]^2\}\\
&=\; W^d(u_{t^*}),
\end{aligned}$$

which contradicts (27).

We next show that if a pair of contingent sequences of inflation rates and output gaps, (π, x), satisfies (S1) and (S2), then this pair is an outcome of a sustainable equilibrium. To this end, we define a *switch-to-discretion* plan. For an arbitrary pair $(\tilde{\pi}, \tilde{x})$, the switch-to-discretion plan specifies continuation with that pair as long as it has been chosen in the past; otherwise, the plan specifies to switch to $\{(f_s^d), (\sigma_s^d)\}_{s\geq t}$ in the subsequent economy. We then show that for the given pair (π, x), the associated switch-to-discretion plan constitutes sustainable equilibrium. First consider histories h_t under which there has been no deviation from x up until period t. Then, (S1) implies that the continuation of π is optimal for private agents in period t when they are faced with the continuation of x. If the monetary authority deviates from x in period t, private agents switch to $(f_s^d)_{s\geq t}$. For the authority who expects this switch of private agents, it is optimal to choose $(\sigma_s^d)_{s\geq t}$. Hence, the most the authority's deviation in period t

can attain is the right-hand side of (16). Then, from (S2) it follows that it is optimal for the authority to stick to x. Next consider histories h_t in which there have been deviations from x before period t. The switch-to-discretion plan then specifies that the monetary authority and private agents choose, respectively, $(\sigma_s^d)_{s\geq t}$ and $(f_s^d)_{s\geq t}$, which is a sustainable equilibrium in the subsequent economy as shown in the proof of Proposition 1. Thus, the switch-to-discretion plan is a sustainable equilibrium and implements the outcome (π, x).

D Proof of Proposition 4

To clarify that the period-t values of social welfare attained by optimal commitment policy and by discretionary policy, (6) and (12), depend on the discount factor β, let these values be denoted by $W^c(u_t, x_{t-1}^c; \beta)$ and $W^d(u_t; \beta)$. Also, let the coefficients of REE (5) and (11) be represented as $e_j = e_j(\beta)$ for $e = a, b, c$ and for $j = \pi, x$. Note that for each $\beta \in (0, 1\,]$, $0 < a_\pi(\beta) < 1/(1-\beta\rho)$, $0 < b_\pi(\beta) < \lambda/\kappa$, $-(\kappa/\lambda)/(1-\beta\rho) < a_x(\beta) < 0$, and $0 < b_x(\beta) < 1$. To prove Proposition 4, it suffices to show that the difference between the period-t values of social welfare, $W^c(u_t, x_{t-1}^c; \beta) - W^d(u_t; \beta)$, goes to an infinity uniformly in h_t as β approaches one. From (6) and (12), we have that for each $\beta \in (0, 1\,]$ and for every history h_t,

$$
\begin{aligned}
& W^c(u_t, x_{t-1}^c; \beta) - W^d(u_t; \beta) \\
&= \frac{\Delta(\beta)}{1-\beta\rho^2}\left(u_t^2 + \frac{\beta\sigma_\varepsilon^2}{1-\beta}\right) - \frac{b_\pi^2(\beta) + \lambda\, b_x^2(\beta)}{1-\beta\, b_x^2(\beta)}(x_{t-1}^c)^2 \\
&\quad - 2\left\{\frac{a_\pi(\beta)\, b_\pi(\beta) + \lambda\, a_x(\beta)\, b_x(\beta)}{1-\beta\rho\, b_x(\beta)} + \frac{\beta\, a_x(\beta)\, b_x(\beta)[b_\pi^2(\beta) + \lambda\, b_x^2(\beta)]}{[1-\beta\rho\, b_x(\beta)][1-\beta\, b_x^2(\beta)]}\right\} u_t x_{t-1}^c,
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta(\beta) &= [c_\pi^2(\beta) + \lambda\, c_x^2(\beta)] - [a_\pi^2(\beta) + \lambda\, a_x^2(\beta)] \\
&\quad - \frac{2\beta\rho\, a_x(\beta)}{1-\beta\rho\, b_x(\beta)}[a_\pi(\beta)\, b_\pi(\beta) + \lambda\, a_x(\beta)\, b_x(\beta)] - \frac{\beta\, a_x^2(\beta)}{1-\beta\, b_x^2(\beta)}[b_\pi^2(\beta) + \lambda\, b_x^2(\beta)] \\
&= \frac{\beta(b^-)^2(1-b^-)[1-\beta b^- + \beta(b^-)^2(1-\beta\rho^2)]}{[(b^-)(1-\beta\rho b^-)(1-\beta\rho+\kappa^2/\lambda)]^2} + \frac{2\beta^2\rho(b^-)^3(1-b^-)}{(1-\beta\rho b^-)^3} \;>\; 0,
\end{aligned}
$$

where the last equality follows from the coefficients of REE (5) and (11). Because u_t is bounded, i.e. there is $B > 0$ such that $|\,u_t| < B$ for all t, we have from (5) that $|\,x_t^c| < B|\,a_x(\beta)|/[1 - b_x(\beta)]$

for all t and hence that for each $\beta \in (0, 1\,]$ and for every history h_t,

$$
\begin{aligned}
& W^c(u_t, x^c_{t-1}; \beta) - W^d(u_t; \beta) \\
> \;& \frac{\beta \sigma_\varepsilon^2\, \Delta(\beta)}{(1-\beta)(1-\beta\rho^2)} - \frac{b_\pi^2(\beta) + \lambda\, b_x^2(\beta)}{1 - \beta\, b_x^2(\beta)} \left[\frac{B\, a_x(\beta)}{1 - b_x(\beta)} \right]^2 \\
& - 2 \left\{ \frac{a_\pi(\beta)\, b_\pi(\beta) + \lambda\, |\, a_x(\beta)|\, b_x(\beta)}{1 - \beta\rho\, b_x(\beta)} + \frac{\beta\, |\, a_x(\beta)|\, b_x(\beta)\, [b_\pi^2(\beta) + \lambda\, b_x^2(\beta)]}{[1 - \beta\rho\, b_x(\beta)][1 - \beta\, b_x^2(\beta)]} \right\} \frac{B^2 |\, a_x(\beta)|}{1 - b_x(\beta)}. \qquad (28)
\end{aligned}
$$

Then, as β approaches one, the first term in the right-hand side of (28) goes to an infinity but the other two terms converge to finite numbers, so that the right-hand side of (28) goes to an infinity. Therefore, there is $\underline{\beta} \in (0, 1)$ such that for any $\beta \in (\underline{\beta}, 1\,]$, $W^c(u_t, x^c_{t-1}; \beta) - W^d(u_t; \beta) > 0$ for every history h_t. Because $W^c(u_t, x^c_{t-1}; \beta) < \infty$ and $W^d(u_t; \beta) < \infty$ for all $\beta \in (\underline{\beta}, 1)$, we have that for any $\beta \in (\underline{\beta}, 1)$, $W^c(u_t, x^c_{t-1}; \beta) > W^d(u_t; \beta)$ for every history h_t.

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Table 1:

Calibrations of model parameters, quarterly

β	discount factor	.99
κ	output gap elasticity of annualized inflation	.01, .05, or .10
λ	relative weight in social welfare function	.0010, .0100, or .0625
ρ	autoregression parameter in shock process	.00, .35, or .80

Table 2:

Welfare gains from optimal commitment policy

relative to discretionary policy ($W^{c/d}$), %

		λ		
	κ	.0010	.0100	.0625
$\rho = .00$	.01	19.38	8.18	3.32
	.05	17.79	23.51	14.44
	.10	7.62	23.40	21.73
		λ		
	κ	.0010	.0100	.0625
$\rho = .35$	.01	41.00	21.91	9.75
	.05	23.41	43.78	34.14
	.10	8.49	35.37	43.12
		λ		
	κ	.0010	.0100	.0625
$\rho = .80$	.01	83.35	68.40	40.14
	.05	31.76	78.43	81.57
	.10	9.69	54.15	81.78

Note: $W^{c/d} \equiv \frac{W^c(u_0, x^c_{-1}) - W^d(u_0)}{|W^d(u_0)|} \times 100$.

Table 3:

Values of $\underline{\beta}$ given by (19)

		λ		
	κ	.0010	.0100	.0625
$\rho = .00$	.01	.9996	.9999	.9999
	.05	.9983	.9993	.9998
	.10	.9981	.9986	.9995
		λ		
	κ	.0010	.0100	.0625
$\rho = .35$	.01	.9981	.9997	.9999
	.05	.9954	.9970	.9990
	.10	.9957	.9956	.9976
		λ		
	κ	.0010	.0100	.0625
$\rho = .80$	.01	.9807	.9949	.9987
	.05	.9760	.9747	.9877
	.10	.9803	.9732	.9773

Table 4:

Welfare gains from sustainable policy with (20)

relative to discretionary policy ($W^{s/d}$), %

		λ					
		.0010		.0100		.0625	
$\rho = .35$	κ	$W^{s/d}$	$\frac{W^{s/d}}{W^{c/d}}$	$W^{s/d}$	$\frac{W^{s/d}}{W^{c/d}}$	$W^{s/d}$	$\frac{W^{s/d}}{W^{c/d}}$
	.01	2.07	5.05	.27	1.24	.05	.46
	.05	2.93	12.51	3.55	8.10	.99	2.90
	.10	1.05	12.33	4.21	11.89	2.82	6.54
		λ					
		.0010		.0100		.0625	
$\rho = .80$	κ	$W^{s/d}$	$\frac{W^{s/d}}{W^{c/d}}$	$W^{s/d}$	$\frac{W^{s/d}}{W^{c/d}}$	$W^{s/d}$	$\frac{W^{s/d}}{W^{c/d}}$
	.01	39.80	47.76	11.66	17.05	2.23	5.56
	.05	17.62	55.46	42.78	54.54	28.97	35.52
	.10	5.68	58.60	30.06	55.52	42.56	52.05

Note: $W^{s/d} \equiv \frac{W^s(u_0) - W^d(u_0)}{|W^d(u_0)|} \times 100,$

$W^{c/d} \equiv \frac{W^c(u_0, x^c_{-1}) - W^d(u_0)}{|W^d(u_0)|} \times 100.$