Credit Conditions of Financial Intermediaries and Entrepreneurs and Financial Accelerators
(Preliminary)

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Abstract

Based on financial accelerator model of Bernanke et al. (1999), we develop a dynamic general equilibrium model in which financial intermediaries (hereafter FIs) as well as entrepreneurs are subject to credit constraints. We study the effect of interactions between these two borrowing sectors on the financial accelerator mechanism. Calibrated to the U.S. data, our model shows the following two features about credit market: (i) the sectoral shock propagation mechanism is enhanced when the shock hits FIs compared to the case when the shock hits entrepreneurs. (ii) the aggregate shock amplification mechanism can be reduced if net worth distribution between the sectors are less biased. Key features for the results are asymmetry of the two borrowing sectors in terms of agency problems. Net worth of FIs are fewer and bankruptcy costs associated with FIs are higher than those of entrepreneurs, making aggregate economy more vulnerable to adverse shocks.

Keywords: Net worth of Financial Intermediary; Cross-sectional Net Worth Distribution; Financial Accelerator Effect;

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1 Introduction

Accumulating empirical evidence suggests that a deterioration of the financial intermediaries’ (hereafter FIs) credit conditions generates a macroeconomic downturn (Ashcraft, 2005; Calomiris and Mason, 2003; Peek and Rosengren, 1997; 2000; and Anari, Kolari and Mason, 2005). However, theoretical research on the role of FIs’ credit conditions is limited, in particular, in a general equilibrium macroeconomics framework.

In this paper, we develop a financial accelerator model à la Bernanke, Gertler and Gilchrist (1999, hereafter BGG) in which FIs as well as entrepreneurs are credit constrained. Here, FIs’ loans to the entrepreneurs are financed by FIs’ own net worth and the borrowing from investors. Similarly, capital investments by entrepreneurs are financed by the entrepreneurial own net worth and borrowing from FIs. Both two borrowing contracts are subject to the information asymmetry problem and similarly to BGG, developments of the net worth in the two sectors help credit market propagate sectoral shock to the aggregate economy (sectoral shock propagation mechanism) and amplify the aggregate adverse shocks to the economy (aggregate shock amplification mechanism). Since the two borrowing contracts are vertically chained, these financial accelerator mechanisms of credit market are affected not only by the credit condition of each borrowing sector but also by their interaction.

Based on the calibration to the U.S. data including those about FIs and entrepreneurs, we investigate the quantitative implication of our model to the financial accelerator mechanism. There are two main findings. First, regarding sectoral shock propagation mechanism, a propagation is enhanced when the shock hits FIs compared to the case when the shock hits the entrepreneurs. Namely, downturns of investment and output are larger following shocks to FIs sector. Second, regarding aggregate shock amplification mechanism, the amplification can be reduced if net worth distribution between FIs and entrepreneurs are less biased.

Central mechanism for our result is given by the interaction between the two credit market imperfections. In our model, monopolistic FIs determine the contents of borrowing contracts and lending contracts jointly, based on the credit conditions of their own and those of entrepreneurs. Since the two contracts work complementarily, a contract with more severe agency problem, such as the one in which borrowers are more severely credit constrained or the one in which lenders need to pay higher bankruptcy cost is more likely to affect FIs’ decision. According to the calibration to the U.S. data, FI is a such sector. That is, net worth is less distributed to FIs and FIs’ bankruptcy cost is higher than entrepreneurs’. Consequentially, the adverse shock to entrepreneurial sector is greater than that to entrepreneurial sector and amplification mechanism is enhanced compared to the case when more net worth is distributed to FIs.

Our paper is related to the researches including Chen (2001), Aikman and Paustian (2006) and Meh and Moran (2004, 2008), as well as BGG and following work by Christiano, Motto and Rostagno (2004) (Hereafter CMR). These studies extend the model of
Holmstrom and Tirole (1997), and quantitatively examine the link between the FIs’ net worth and the macroeconomy. Their model is build upon the moral hazard problem in the credit contract between the entrepreneurs and FIs. Here, only FIs pay the monitoring cost since there are no FIs-specific default risks. Our model in contrast is build upon the two separate costly state verification problems, where both FIs and entrepreneurs have their own idiosyncratic default risks. Both investors and FIs need to monitor their borrowers in the corresponding credit contract, so that two credit contracts reflect more of the economic conditions of the corresponding borrowers. Thus our model gives a theoretical relationship among the market spreads, bank capital, entrepreneurial capital and macroeconomy in a unified way. Other related research is Van den Heuvel (2008), in which welfare of regulatory requirements for FI’s capital is analyzed. Gerali et al. (2008) and Dib (2009) discuss monopolistically competitive banks in deposit and loan markets.

The rest of the paper is organized as follows. Section 2 presents our model in which there are two types of financial frictions in the credit market. The important feature of our model is the role of interaction between net worth held by FIs sector and that held by entrepreneurial sector in generating credit market imperfection. Section 3 provides the model’s response to sectoral shocks and aggregate shocks. We find that propagation of sectoral shock is enhanced when the shock hits a sector with fewer net worth. We also find that the cross-sectional distribution of net worth in the U.S. contributes amplification of aggregate shock. Section 4 concludes.

2 The Model Economy

This section describes the structure of our model and the optimization problems that the economy’s agents solve. The economy consists of credit market and goods market, and seven types of agents; a household, investors, FIs, entrepreneurs, capital goods producers, final goods producer and government. The participants of credit market are investors, FIs and entrepreneurs. Entrepreneurs are final borrower of fund in the economy, and they own the net worth by themselves, but they do not own enough amount of net worth to finance their projects. They thus engage in credit contracts with FIs in which they borrow the rest of the fund from FIs. FIs also own the net worth by themselves but they do not have enough amount of net worth to finance their loans to entrepreneurs. They engage in one another credit contracts with investors in order to borrow the rest of the funds. Investors collect deposits from household in a competitive market, and invest what they collect on the loan to FIs. There are information asymmetry problems in credit contracts between FIs and entrepreneurs (hereafter FE contract) and the credit contracts between investors and FIs (hereafter IF contract), and this makes borrowing costs determined in the two credit markets depend on borrowers’ credit conditions. Among the three participants of credit market, investors are competitive and they earn zero profit, FIs and entrepreneurs earn positive profits and they accumulate their net worth. FIs are
monopolistic lender in FE contract. FIs then maximize their profits by solving costly state verification problems associated with the IF contracts and FE contracts, ensuring the participation constraints of entrepreneurs and investors so that the all of the credit contracts are incentive-compatible.

Our goods market consists of input market and output market for final goods, and capital goods market. These markets are competitive, and prices of goods are all flexible. Final goods producers own Cobb-Douglas production technology that convert capital and labor into final goods. Capitals are supplied by entrepreneurs. Entrepreneurs purchase capital goods from capital goods producer thanks to the fund they borrowed from credit market, and sell capital goods to the final goods producers. Labor inputs are supplied by household, FIs and entrepreneurs. Once produced, final goods are allocated to consumption and investment at the competitive final goods market.

2.1 Credit Contracts and Net Worth

The Environment
There are continuum number of investors, FIs and entrepreneurs, and two types of credit contracts are signed by them. In addition, there are three kinds of the interest rates, $R(s^t), R^F(s^t)$ and $R^E(s^t)$, that are relevant for the credit contracts, where $s^t$ is state at $t$. $R(s^t)$ is risk free rate of return in the economy, $R^F(s^t)$ is the ex post return on the loans to entrepreneurs, and $R^E(s^t)$ is the ex post aggregate return to capital. At period $t$, investors collect deposits from a household at the competitive market and lend them to continuum number of FIs (IF contract). Investors face an opportunity cost of deposits equal to the economy’s risk free rate of return, $R(s^t)$ so that their returns on the loans to FIs are equalized to this opportunity cost. FIs monopolistically supply loans to a continuum of entrepreneurs. Each FI, say type $i$ FI, makes loan contracts with specific group of entrepreneurs, say group $j_i$ entrepreneurs, that are attached to the FI. By lending to a continuum of group $j_i$ entrepreneurs, type $i$ FI diversify the loan risk associated with a specific entrepreneur and obtain the return equal to $R^F(s^t)$.

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1This contrasts with the setting employed in Holmstrom and Tirole (1997). Modelling differences are that, in Holmstrom and Tirole (1997), it is entrepreneurs who optimize a contract among FIs and investors. Entrepreneurs maximize their expected profits subject to zero profit conditions of FIs and investors who face perfect competition. But because it is assumed that all projects are perfectly correlated, in the ex-post, FIs can earn positive profits if a project succeeds. In our model, FIs are assumed to be monopolistic and it is the FIs who optimize a contract among FIs and investors. Projects are not correlated, so lenders can exempt from idiosyncratic uncertainty, and without aggregate uncertainty, lenders ex-post profits are the same as ex-ante expected profits.

2See Klein (1971) and Monti (1972) and related discussions provided in Freixas and Rochet (2008) for theoretical discussion about monopolistic FIs.

3In our model, there is no credit contract between households and entrepreneurs. We assume that bankruptcy cost $\mu$ associated with household-entrepreneur credit contract is high enough, so that it is desirable for household not to make direct credit contracts with entrepreneurs.
Entrepreneurs are final borrower in the economy. They invest their loans on the purchase of capital goods and receive the return to capital \( R^E (s^t) \).

At the beginning of each period, each type \( i \) FI offers a loan contract to group \( j_i \) entrepreneurs. Each entrepreneur in group \( j_i \) owns net worth \( N_{ji}^E (s^t) \) and purchase capital an amount \( Q (s^t) K_{tji} (s^t) \), where \( Q (s^t) \) is the price paid per unit of capital and \( K_{tji} (s^t) \) is the quantity of capital purchased by group \( j_i \) entrepreneur. Following BGG, we assume that entrepreneurs are subject to an idiosyncratic productivity shock \( \omega_{ji}^E (s^{t+1}) \) so that net return to capital is \( \omega_{ji}^E (s^{t+1}) R^E (s^{t+1}) \). Here, the IE contract specifies (1) an amount of debt that group \( j_i \) entrepreneur borrows from type \( i \) FI, \( Q (s^t) K_{ji} (s^t) - N_{ji}^E (s^t) \), and (2) a cut-off value of idiosyncratic shock \( \omega_{ji}^E (s^{t+1}) \), which we denote by \( \overline{\omega}_{ji}^E (s_{t+1}|s^t) \), such that entrepreneurs repay their debt for \( \omega_{ji}^E (s^{t+1}) \geq \overline{\omega}_{ji}^E (s_{t+1}|s^t) \) and they declare the default for \( \omega_{ji}^E (s^{t+1}) < \overline{\omega}_{ji}^E (s_{t+1}|s^t) \), (3) a loan rate when group \( j_i \) entrepreneurs do not default, \( Z_{ji}^E (s_{t+1}|s^t) \). Here, ex-post, non-default entrepreneur \( j_i \) receives \( (\omega_{ji}^E (s^{t+1}) - \overline{\omega}_{ji}^E (s_{t+1}|s^t)) R^E (s^{t+1}) Q (s^t) K_{ji} (s^t) \) and default entrepreneur receives nothing from the contract. The relationship between cut-off value \( \overline{\omega}_{ji}^E (s_{t+1}|s^t) \) and non-default rate \( Z_{ji}^E (s_{t+1}|s^t) \) is given by

\[
\overline{\omega}_{ji}^E (s_{t+1}|s^t) R^E (s_{t+1}|s^t) Q (s^t) K_{ji} (s^t) = Z_{ji}^E (s_{t+1}|s^t) (Q (s^t) K_{ji} (s^t) - N_{ji}^E (s^t)).
\] (1)

Alternatively, group \( j_i \) entrepreneurs can purchase capital goods by their own net worth \( N_{ji}^E (s^t) \), without participating loan contracts with FIs. In this alternative case, the ex-post return to their investments equals to \( \omega_{ji}^E (s^{t+1}) R^E (s^{t+1}) N_{ji}^E (s^t) \). Thus FE contract between FI and entrepreneurs is agreed only when the following inequality is expected to hold;

\[
\{1 - \Gamma_t^E (\overline{\omega}_{ji}^E (s_{t+1}|s^t)) \} R^E (s_{t+1}|s^t) Q (s^t) K_{ji} (s^t) \geq R (s_{t+1}|s^t) N_{ji}^E (s^t)
\] for \( \forall j_i, s^{t+1}|s^t \),

(2)

where

\[
\Gamma_t^E (\overline{\omega}_{ji}^E (s_{t+1}|s^t)) \equiv G_t^E (\overline{\omega}_{ji}^E (s_{t+1}|s^t)) + \overline{\omega}_{ji}^E (s_{t+1}|s^t) \int_{\overline{\omega}_{ji}^E (s_{t+1}|s^t)}^\infty dF_t^E (\omega^E),
\]

\[
G_t^E (\overline{\omega}_{ji}^E (s_{t+1}|s^t)) \equiv \int_0^{\overline{\omega}_{ji}^E (s_{t+1}|s^t)} \omega^E dF_t^E (\omega^E).
\]

Note that \( 1 - \Gamma_t^E \) is expected share of profits from purchasing capital goods that goes to the lenders of FE contract. Left hand side of the inequality (2) shows the expected return from FE contract for group \( j_i \) entrepreneurs, and right hand of inequality (2) shows the expected return from investing the entrepreneurial net worth \( N_{ji}^E (s^t) \). Credit contracts are signed only when the inequality holds.
Next we examine FIs’ profits. According to left hand side of the inequality (2), expected profit that each type \( i \) FI earns from each of IF contract is given by

\[
\Phi_{j_i,t}^E \left( \omega_{j_i}^E \left( s_{t+1} | s^t \right) \right) R^E \left( s_{t+1} | s^t \right) Q \left( s^t \right) K_{ji} \left( s^t \right)
\]

where

\[
\Phi_{j_i,t}^E \left( \omega_{j_i}^E \left( s_{t+1} | s^t \right) \right) = \Gamma_t^E \left( \omega_{j_i}^E \left( s_{t+1} | s^t \right) \right) - \mu^E G_t^E \left( \omega_{j_i}^E_{t+1} \right) \\
= \int_0^{\omega_{j_i}^E \left( s_{t+1} | s^t \right)} \mu^E \omega^E dF^E \left( \omega^E \right) .
\]

Note that \( \mu^E \omega^E_{j_i} \left( s_{t+1} \right) R^E \left( s_{t+1} \right) Q \left( s^t \right) K_{ji} \left( s^t \right) \), with \( 0 < \mu^E < 1 \), is the ex-post monitoring cost that FIs pay whenever group \( j_i \) entrepreneurs declare the default. Since each type \( i \) FI lends a continuum number of entrepreneurs in group \( j_i \), the loan risk of the FI is perfectly diversified. For convenience, we define the expected return on the loans to entrepreneurs, \( R^E \left( s_{t+1} | s^t \right) \) by

\[
\int_{j_i} \{ \Gamma_t^E \left( \omega_{j_i}^E \left( s_{t+1} | s^t \right) \right) - \mu^E G_t^E \left( \omega_{j_i}^E \left( s_{t+1} | s^t \right) \right) \} R^E \left( s_{t+1} | s^t \right) Q \left( s^t \right) K_{ji} \left( s^t \right) d_{ji} \\
= R_i^E \left( s_{t+1} | s^t \right) (Q \left( s^t \right) K_i \left( s^t \right) - N_i^E \left( s^t \right)) \text{ for } \forall s_{t+1} | s^t . \tag{3}
\]

where

\[
K_i \left( s^t \right) \equiv \int_{j_i} K_{ji} \left( s^t \right) d_{ji},
\]

\[
N_i^E \left( s^t \right) \equiv \int_{j_i} N^E_{ji} \left( s^t \right) d_{ji}.
\]

The left hand side of equation (3) is the gross profit that a specific type \( i \) FI receives from a continuum number of FE contracts with group \( j_i \) entrepreneurs. Since type \( i \) FI’s loans to entrepreneurs are financed by the FI’s net worth and their borrowing from investors, type \( i \) FI splits this gross profit with investors according to another credit contract, in order to repay the loans to investors. The IF contract has the same costly state verification structure as does FE contract, except that FIs are now borrowers of the contract. In IF contract, investors lend the loans to a continuum number of FIs. Each type \( i \) FI owns the net worth \( N_i^E \left( s^t \right) \) and it invests on the loans to group \( j_i \) entrepreneurs an amount \( Q \left( s^t \right) K_i \left( s^t \right) \). It then borrows the rest \( Q \left( s^t \right) K_i \left( s^t \right) - N_i^E \left( s^t \right) \) from investors, and repay the loan from its profit of its FE contracts. We assume that type \( i \) FI is subject
to idiosyncratic productivity shock $\omega_i^F (s^{t+1})$ and its ex post gross return on the loans to entrepreneurs is $\omega_i^F (s^{t+1}) R^F (s^{t+1})$. Here, the IF contract specifies (1) an amount of debt that type $i$ FI borrows from investors, $Q (s^t) K_i (s^t) - N_i^E (s^t) - N_i^F (s^t)$, and (2) a cut-off value of idiosyncratic shock $\omega_i^F (s^{t+1})$, which we denote by $\underline{\omega}_i^F (s_{t+1}|s^t)$, such that FIs repay their debt for $\omega_i^F (s^{t+1}) \geq \underline{\omega}_i^F (s_{t+1}|s^t)$ and they declare the default for $\omega_i^F (s^{t+1}) < \underline{\omega}_i^F (s_{t+1}|s^t)$. (3) return rate of the loan when type $i$ FI does not default, $Z_i^F (s_{t+1}|s^t)$. Here, ex-post, non-default FI $i$ receives $(\omega_i^F (s^{t+1}) - \underline{\omega}_i^F (s_{t+1}|s^t)) R^F (s^{t+1}) Q (s^t) K_i (s^t)$ and default FI receives nothing from the contract. The relationship between cut-off value $\underline{\omega}_i^F (s_{t+1}|s^t)$ and non-default rate $Z_i^F (s_{t+1}|s^t)$ is given by

$$\underline{\omega}_i^F (s_{t+1}|s^t) R^F (s_{t+1}|s^t) (Q (s^t) K_i (s^t) - N_i^E (s^t)) = Z_i^F (s_{t+1}|s^t) (Q (s^t) K_i (s^t) - N_i^F (s^t) - N_i^E (s^t))$$

(4)

Given the risk free rate of return in the economy $R (s^{t+1})$, investors profit from the invest on the loans to FIs must equal to opportunity cost of lending. That is

$$\{\Gamma^F (\underline{\omega}_i^F (s_{t+1}|s^t)) - \mu^F G^F_i (\underline{\omega}_i^F (s_{t+1}|s^t)) \} R^F (s_{t+1}|s^t) (Q (s^t) K_i (s^t) - N_i^E (s^t)) \geq R (s^{t+1}) (Q (s^t) K_i (s^t) - N_i^F (s^t) - N_i^E (s^t)),$$

(5)

where

$$\Gamma^F_i (\underline{\omega}_i^F (s_{t+1}|s^t)) \equiv G^F_i (\underline{\omega}_i^F (s_{t+1}|s^t)) + \underline{\omega}_i^F (s_{t+1}|s^t) \int_0^\infty dF^F_t (\omega^F),$$

$$G^F_i (\underline{\omega}_i^F (s_{t+1}|s^t)) \equiv \int_{\underline{\omega}_i^F (s_{t+1}|s^t)}^\infty \omega^F dF^F_t (\omega^F).$$

Expected net profit for type $i$ FI is expressed by

$$\sum_{s_{t+1}} \Pi (s_{t+1}|s^t) \{1 - \Gamma^F_i (\underline{\omega}_i^F (s_{t+1}|s^t)) \} R^F (s_{t+1}|s^t) (Q_i (s^t) K_i (s^t) - N_i^E (s^t)),$$

(6)

$\Pi (s_{t+1}|s^t)$ is a probability weight for state $s_{t+1}$, depending on the information set available at period $t$.

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4We assume that two variables $\omega_j^F$ and $\omega_i^F$ are unit mean, lognormal random variables distributed independently over time and across entrepreneurs and FIs. We express density function of these variables by $f_j^F (\omega_j^F)$ and $f_i^F (\omega_i^F)$, and cdf of them by $F_j^F (\omega_j^F)$ and $F_i^F (\omega_i^F)$. Following CMR, we further assume that the standard-deviation of log ($\omega_j^F$) and log ($\omega_i^F$), denoted by $\sigma_j^F$ and $\sigma_i^F$ respectively, follow stochastic process.
Optimally choosing the variables, conditions yield the following equation.

\[ 0 = \sum_{s^{t+1}|s^t} \Pi(s_{t+1}|s^t) \left( 1 - \Gamma^F_t \left( \omega^F_t(s_{t+1}|s^t) \right) \right) \Phi^E_{t,t}(s_{t+1}|s^t) \mathcal{X}_E \left( s_{t+1}|s^t \right) \]

Combining first order conditions yield the following equation.

\[
\begin{align*}
0 &= \sum_{s^{t+1}|s^t} \Pi(s_{t+1}|s^t) \left( 1 - \Gamma^F_t \left( \omega^F_t(s_{t+1}|s^t) \right) \right) \Phi^E_{t,t}(s_{t+1}|s^t) \mathcal{X}_E \left( s_{t+1}|s^t \right) \\
&+ \sum_{s^{t+1}|s^t} \frac{\Pi(s_{t+1}|s^t) \Gamma^F_t \left( \omega^F_t(s_{t+1}|s^t) \right)}{\Phi^E_{t,t}(s_{t+1}|s^t)} \Phi^E_{t,t}(s_{t+1}|s^t) \Phi^E_{t,t}(s_{t+1}|s^t) \mathcal{X}_E \left( s_{t+1}|s^t \right) \\
&- \sum_{s^{t+1}|s^t} \frac{\Pi(s_{t+1}|s^t) \Gamma^F_t \left( \omega^F_t(s_{t+1}|s^t) \right)}{\Phi^E_{t,t}(s_{t+1}|s^t)} \mathcal{X}_E \left( s_{t+1}|s^t \right) \\
&+ \sum_{s^{t+1}|s^t} \frac{\Pi(s_{t+1}|s^t) \Gamma^F_t \left( \omega^F_t(s_{t+1}|s^t) \right) \Phi^E_{t,t}(s_{t+1}|s^t) \Phi^E_{t,t}(s_{t+1}|s^t)}{\Phi^E_{t,t}(s_{t+1}|s^t) \mathcal{X}_E \left( s_{t+1}|s^t \right)} \mathcal{X}_E \left( s_{t+1}|s^t \right) \\
&+ \frac{\Pi(s_{t+1}|s^t) \Gamma^F_t \left( \omega^F_t(s_{t+1}|s^t) \right) \Phi^E_{t,t}(s_{t+1}|s^t) \Phi^E_{t,t}(s_{t+1}|s^t)}{\Phi^E_{t,t}(s_{t+1}|s^t) \mathcal{X}_E \left( s_{t+1}|s^t \right)} \mathcal{X}_E \left( s_{t+1}|s^t \right)
\end{align*}
\]

for \( \forall j_i \), \( j \), \( i \).

Net Worth of Each Sector and Choice of Capital

Type \( i \) FI maximizes the equation profit equation (6) from the two credit contracts by optimally choosing the variables \( \omega^F_i, K_i, \omega^E_{j,i}, K_{j,i} \), subject to the investors’ participation constraint (5) and entrepreneurial participation constrain (2). Combining first order conditions yield the following equation.

\[
\begin{align*}
E_i \left\{ R^E \left( s_{t+1} \right) \right\} &= \left( \Phi^F_t \left( \frac{N^F \left( s^t \right)}{Q \left( s^t \right) K \left( s^t \right)} - \frac{N^E \left( s^t \right)}{Q \left( s^t \right) K \left( s^t \right)} \right) \right)^{-1} \left( \Phi^E_t \left( \frac{N^E \left( s^t \right)}{Q \left( s^t \right) K \left( s^t \right)} \right) \right)^{-1} \\
&\times \left( 1 - \frac{N^F \left( s^t \right)}{Q \left( s^t \right) K \left( s^t \right)} - \frac{N^E \left( s^t \right)}{Q \left( s^t \right) K \left( s^t \right)} \right) \\
&= S \left( n^F \left( s^t \right), n^E \left( s^t \right) \right)
\end{align*}
\]

where \( n^F \left( s^t \right) \) and \( n^E \left( s^t \right) \) are ratio of each of FIs’ net worth and entrepreneurial net worth to the total amount of capital.

This equation describes the important relationship between the two net worth \( N^F \left( s^t \right) \), \( N^E \left( s^t \right) \) and external finance premium, the latter of which affects aggregate capital accumulation. Here, we show the property of this relationship using the two numerical exercises about the structure of function \( S \left( \cdot \right) \). We first study the relationship between
each of the two net worth $N_t^F(s^t), N_t^E(s^t)$ and external finance premium. We then investigate how a relative size of the two net worth, or equivalently a distribution of net worth across the two sectors, is related to external finance premium.

Figure 1 displays the cost-of-fund curve in our economy. This curve presents the relationship between the external risk premium, or equivalently, the expected discounted return to capital, and net-worth capital ratio in each of the sectors, based on the function $S(\cdot)$. Net-worth capital ratio in each sector is depicted on the horizontal axis and external finance premium is depicted on the vertical axis. In the left panel of Figure 1, we depict the values of external finance premium for various sizes of FIs’ net worth/capital ratio, maintaining the entrepreneurial net-worth capital ratio equal to constant. According to the panel, the external finance premium is decreasing in the FIs’ net-worth capital ratio. As capital investment becomes larger relative to FIs’ net worth, expected bankruptcy costs associated with IF contract rise. This is demonstrated in the top left panel of Figure 2 as the quantitative relationships between the expected default costs and the net worth in the two sectors. A fall in FIs’ net worth is followed by more borrowing, which results in higher borrowing rates and higher expected default rates of FIs. In contrast, the fall in FIs’ net worth does not affect expected default in the FE contract, as it does not change entrepreneurs’ participation constraint in the FE contract. Given a higher external finance premium, additional capital investment is beneficial for FIs only if expected discounted return to capital is sufficiently high.

Another important feature of this curve is the role of net worth held by entrepreneurs. According to the left panel of Figure 1, the external finance premium is decreasing in the entrepreneurial net-worth capital ratio. As the bottom right panel of Figure 2 shows, a rise in entrepreneurial net worth reduces the expected default cost associated with FE contract, causing the FIs’ cost of capital investment to increase. Furthermore, the bottom left panel of Figure 2 suggests that it also reduces the expected default cost associated with IF contract as credit conditions of entrepreneurs are improved.

We next discuss the role of the net worth distribution. Unlike the set up of BGG, our net worth is distributed across distinct agents FIs and entrepreneurs, and the relative size of the net worth in each of the sectors is also important determinant of the capital investment. Figure 3 displays the share of the net worth held by FIs sector on the horizontal axis and the external finance premium on the vertical axis. Here, we set the ratio of total net worth to the total amount of capital equal to .6, and investigate how an increase in FIs’ share changes the level of external finance premium that FIs require for choosing this size of capital. Solid line in the two panels of Figure 3 presents the cost of fund curve that gives relationship between the FIs’ net share and the external finance

\footnote{For the exercises displayed in Figure 1 and Figure 2, we set model parameters pertaining to the two credit contracts following BGG. Namely, we set the values for parameters $\mu^E, \sigma^E$ and $1 - \gamma^E$ equal to the values of bankruptcy cost, variance of entrepreneurial idiosyncratic productivity and death rate reported in BGG, respectively. We further assume that $\mu^F = \mu^E, \sigma^F = \sigma^E$ and $\gamma^F = \gamma^E$ so that the two credit contracts are symmetric in terms of these parameters.}
premium, when the bankruptcy costs and the variance of idiosyncratic productivity are symmetric across the two contracts. Here, we set $\mu^F = \mu^E = \mu$ and $\sigma^F = \sigma^E = \sigma$, where $\mu$ and $\sigma$ are values of bankruptcy cost and the variance used in BGG. This cost of curve has U-shape with respect to the net worth distribution. That is, the required expected discounted return to capital is decreasing in the FI's share while the FIs' share takes value smaller than 40%, and it is increasing in the FI's share for the FIs' share value is above around 40%. Consequentially, FIs may choose moderate size of capital even in the case that entrepreneurial net worth is large, if expected default cost of IF contract is sufficiently large.

The net worth in a sector that owns less net worth affects the capital size more, because an increase in default probability caused by a decrease in net worth/capital ratio dominates a decrease in default probability caused by an increase in net worth/capital ratio of the same amount. Figure 4 confirms this. It demonstrates that in the region of lower share of FIs' net worth, as FIs' net worth share decreases, the expected default cost of IF contract increases hugely while moderately decreasing expected default cost of FE contract. On the other hand, in the region of high share of FIs' net worth, as FIs' net worth share increases, the expected default cost of IF contract drops moderately while increasingly raising expected default cost of FE contract.

Of course, this argument depends crucially on how expected default cost of the two contracts are affected by the net worth/capital ratio of borrowers of the corresponding contracts. Since the expected default costs are product of the monitoring cost and the default probability that is subject to the distribution of borrowers' idiosyncratic productivity and these technology parameters and distribution parameters are not necessarily symmetric across two contracts, the effects of net worth/capital ratios on the expected default costs may be different across contracts.

We thus discuss the cases in which either technologies or distribution is different between the two contracts. We study the case in which FIs' bankruptcy cost is more expensive, $\mu^F = \mu$ and $\mu^E = 0.5\mu$, and the case in which entrepreneurial bankruptcy cost is more expensive, $\mu^E = \mu$ and $\mu^F = 0.5\mu$, respectively. Line with black circle in Figure 3 and Figure 4 shows the case when bankruptcy cost in IF contract is lower than that in FE contract. Under this environment, the external finance premium becomes increasing function of the FI's net worth share. The higher FI's net worth share, the lower the default probability of FIs and the higher the default probability of entrepreneurs. As Figure 4 shows, bankruptcy cost in FE contract is more expensive than that in IF contract, so a rise in default probability of entrepreneurs in FE contract dominates a decline in default probability of FIs in IF contract. Consequentially, FIs requires higher expected discounted return to capital as net worth is more distributed to the entrepreneurial sector from FIs sector. Line with black circle in the upper panel of Figure 3 and Figure 4 shows the opposite case in which bankruptcy cost in IF contract is lower than that in FE contract. In this case, the external finance premium becomes decreasing function of the FI's net worth share by the similar mechanism.
Finally, we discuss the case in which the variance of borrowers' idiosyncratic productivity are different between IF contract and FE contract. We study the case in which the variance of FIs' idiosyncratic productivity is higher, \( \sigma^F = \sigma \) and \( \sigma^E = 0.5\sigma \), and the case in which the variance of entrepreneurial idiosyncratic productivity is higher, \( \sigma^E = \sigma \) and \( \sigma^F = 0.5\sigma \), respectively. Figure 5 and Figure 6 present the outcomes of these exercises. Similarly to the quantitative results for changing bankruptcy costs, asymmetric variances across the borrowers of two credit contracts shift the cost of fund curve downwards. But in contrast to the case of changing bankruptcy costs, U-shape of the curve is only slightly modified under changes of variances across credit contract.

**Dynamic Behavior of Net Worth**

The net worth of FIs and entrepreneurs, \( N^F(s^t) \) and \( N^E(s^t) \), depend on their earnings from the credit contracts and their labor income. In addition to the profits coming from entrepreneurial projects, both FIs and entrepreneurs inelastically supply a unit of labor to final goods producer and receive labor income \( W^F(s^t) \) and \( W^E(s^t) \). We assume that each FI and entrepreneur survives to the next period with a constant probability \( \gamma^F \) and \( \gamma^E \), then the aggregate net worth of FIs and entrepreneurs are given by

\[
N^F(s^{t+1}) = \gamma^F V^F(s^t) + W^F(s^t), \tag{9}
\]

\[
N^E(s^{t+1}) = \gamma^E V^E(s^t) + W^E(s^t), \tag{10}
\]

with

\[
V^F(s^t) \equiv (1 - \Gamma_t^F(\mathcal{W}(s^{t+1})))(\Gamma_t^E(\mathcal{W}(s^{t+1}))) - \mu^E G_t^E(\mathcal{W}(s^{t+1})) R^E(s^{t+1}) Q(s^t) K(s^t),
\]

\[
V^E(s^t) \equiv (1 - \Gamma_t^E(\mathcal{W}(s^{t+1}))) R^E(s^{t+1}) Q(s^t) K(s^t).
\]

FIs and entrepreneurs that fail to survive at period \( t \) consume \((1 - \gamma^F)V^F(s^t)\) and \((1 - \gamma^E)V^E(s^t)\), respectively.

**2.2 Rest of the Economy**

**Household**

A representative household is infinitely lived, and maximizes the following utility function subject to the budget constraint

\[
\max_{C(s^t), H(s^t), D(s^t)} \beta^{t+1} \mathbb{E}_t \left\{ \log C(s^{t+1}) - \frac{H(s^{t+1})^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}} \right\} \tag{11}
\]

\(^6\)See BGG and CMR for the technical reason of this specification.
subject to

\[ C(s^{t+1}) + D(s^{t+1}) \leq W(s^{t+1}) H(s^{t+1}) + R(s^{t+1}) \cdot D(s^{t+1}) - T(s^{t+1}), \]

where \( C(s^t) \) is final goods consumption, \( H(s^t) \) is hours worked, \( D(s^t) \) is real amount of deposits held by investors, \( W(s^t) \) is real wage measured by the final goods, \( R(s^{t+1}) \) is real risk-free return from the deposit \( D(s^t) \) between time \( t \) and \( t + 1 \), and \( T(s^t) \) is lump-sum transfer. \( \beta \in (0, 1) \), \( \eta \) and \( \chi \) are subjective discount factor, the elasticity of leisure, and utility weight on leisure.

First order conditions associated with the household’s problem are

\[
\frac{1}{C(s^t)} = \beta E_t \left\{ \frac{1}{C(s^{t+1})} R(s^{t+1}) \right\}, \tag{12}
\]

\[ W(s^t) = \chi H(s^t)^{\frac{1}{\eta}} C(s^t). \tag{13} \]

**Final goods producer**

Final goods producer are price takers in both input market and output market. They hire three types of labor inputs \( H(s^t), H^F(s^t) \) and \( H^E(s^t) \), from household, FIs and entrepreneurs, and pay real wage \( W(s^t), W^F(s^t) \) and \( W^E(s^t) \) to each type of labor input respectively. Capital \( K(s^{t-1}) \) is supplied from entrepreneurs with rental price \( R^E(s^t) \). At the end of each period, the capital is sold back to entrepreneurs with price \( Q(s^{t-1}) \).

A maximization problem for final goods producer is given by

\[
\max_{Y(s^t),K(s^{t-1}),H(s^t),H^F(s^t),H^E(s^t)} Y(s^t) + Q(s^{t-1}) K(s^{t-1}) (1 - \delta)
\]

\[-R^E(s^t) Q(s^{t-1}) K(s^{t-1}) - W(s^t) H(s^t) - W^F(s^t) H^F(s^t) - W^E(s^t) H^E(s^t) \]

subject to

\[ Y(s^t) = A \exp(e^A(s^t)) K(s^{t-1})^\alpha L(s^t)^{1 - \alpha}, \]

\[ L(s^t) \equiv (H(s^t))^{1 - \Omega_E - \Omega_F} (H^F(s^t))^{\Omega_F} (H^E(s^t))^{\Omega_E}, \]

where \( Y(s^t) \) is the final goods produced and \( A \exp(e^A(s^t)) \) denotes the level of technology of final goods production. \( \delta \in (0, 1] \), \( \alpha \), \( \Omega_E \) and \( \Omega_F \) are a depreciation rate of capital goods, a capital share, a share of FIs’ labor inputs and a share of entrepreneurial labor inputs. First order conditions for final goods producers are

\[
\alpha \frac{Y(s^t)}{K(s^{t-1})} = R^E(s^t) Q(s^{t-1}) Q(s^{t-1}) (1 - \delta) = 0, \tag{14}
\]
\[ (1 - \alpha) (1 - \Omega_F - \Omega_E) \frac{Y(s^t)}{H(s^t)} = W(s^t), \quad (15) \]
\[ (1 - \alpha) \Omega_F \frac{Y(s^t)}{HF(s^t)} = WF(s^t), \quad (16) \]
\[ (1 - \alpha) \Omega_E \frac{Y(s^t)}{HE(s^t)} = WE(s^t). \quad (17) \]

**Capital producer**

Capital producer owns technology that converts final goods to capital goods. They sell capital goods at competitive market with price \( Q(s^{t-1}) \). At each period, it purchases \( I(s^t) \) amount of final goods from final goods producer. It also receives \( K(s^{t-1}) (1 - \delta) \) amount of used capital goods from final goods producer with price \( Q(s^{t-1}) \). It then produce capital goods \( K(s^t) \), using technology \( F_I \). Capital Producer’s problem is to maximize the profit function below.

\[
\max_{I_t} \sum_{t=0}^{\infty} \Pi(s^{t+1} \mid s^t) \Lambda_{t+1}(s^{t+1}) \left[ Q_{t+1}(s^{t+1}) (1 - F_I(I_{t+1}(s^{t+1}), I_{t+1-1}(s^{t+1-1}))) I_{t+1}(s^{t+1}) - I_{t+1}(s^{t+1}) \right] 
\]

where \( F_I \) is defined as follows:

\[
F_I(I_{t+1}(s^{t+1}), I_{t+1-1}(s^{t+1-1})) \equiv \frac{\kappa}{2} \left( \frac{I_{t+1}(s^{t+1})}{I_{t+1-1}(s^{t+1-1})} - 1 \right)^2.
\]

Note \( \kappa \) is a parameter that is associated with investment technology with adjustment cost\(^7\).

Since capital depreciates at each period, evolvement of total capital available at period \( t \) is given by

\[
K(s^t) = (1 - F_I(I(s^t), I(s^{t-1}))) I(s^t) + (1 - \delta) K(s^{t-1}). \quad (19)
\]

**Government**

The government collects lump-sum tax from a household \( T(s^t) \), and spends \( G(s^t) \). Budget balance is maintained for each period \( t \).

\(^7\)The equation (18) does not have a term for used capital \( K_{t-1} \) that is sold by entrepreneurs at the end of the last period. This is because following BGG, we assume that the price of capital that entrepreneurs sell to the capital producer at the end of period, say \( Q_t \), is close to the price of newly produced capital \( Q_t \) around the steady state.
\[ G(s^t) = T(s^t). \] (20)

**Resource constraint**

Resource constraint for final goods is written as

\[
Y(s^t) = C(s^t) + I(s^t) + G(s^t) + \mu^E G_i^E(\omega^E(s^t)) R^E(s^t) Q(s^{t-1}) K(s^{t-1})
+ \mu^F G_i^F(\omega^F(s^t)) R^F(s^t) Q(s^{t-1}) K(s^{t-1}) - N^E(s^{t-1}).
\]
\[ + (1 - \gamma^F) (1 - \Gamma_i^F(\omega^F(s^t))) \Gamma_i^E(\omega^E(s^t)) R^E(s^t) Q(s^{t-1}) K(s^{t-1})
- (1 - \gamma^F) (1 - \Gamma_i^F(\omega^F(s^t))) \mu^E G_i^E(\omega^E(s^t)) R^E(s^t) Q(s^{t-1}) K(s^{t-1})
+ (1 - \gamma^E) (1 - \Gamma_i^E(\omega^E(s^t))) R^E(s^t) Q(s^{t-1}) K(s^{t-1}). \] (21)

Note that the fourth and the fifth terms in the right hand side of the equation correspond to the monitoring costs spent by FIs and a household, respectively.

**Exogenous variables**

The exogenous shocks to the model, the technology shock, the riskiness shocks in FIs, the riskiness shock in entrepreneurs follow the process

\[
\varepsilon^A(s^t) = \rho_A \varepsilon^A(s^{t-1}) + \varepsilon^A(s^t),
\]
(22)
\[
\log \left( \frac{\sigma^F(s^t)}{\sigma^F} \right) = \rho_{\sigma^F} \log \left( \frac{\sigma^F(s^{t-1})}{\sigma^F} \right) + \varepsilon^\sigma_F(s^t),
\]
(23)
\[
\log \left( \frac{\sigma^E(s^t)}{\sigma^E} \right) = \rho_{\sigma^E} \log \left( \frac{\sigma^E(s^{t-1})}{\sigma^E} \right) + \varepsilon^\sigma_E(s^t),
\]
(24)

where \( \rho_A, \rho_{\sigma^F} \) and \( \rho_{\sigma^E} \in (0,1) \) are autoregressive roots of the exogenous variables, and \( \varepsilon^A(s^t), \varepsilon^\sigma_F(s^t) \) and \( \varepsilon^\sigma_E(s^t) \) are innovations that are mutually independent, serially uncorrelated and normally distributed with mean zero and variances \( \sigma_A^2, \sigma_F^2 \) and \( \sigma_E^2 \), respectively.

### 2.3 Equilibrium Condition

An equilibrium consists of a set of prices, \{\( R(s^t), R^F(s^t), R^E(s^t), W(s^t), W^F(s^t), W^E(s^t), Q(s^t), R^E(st_{t+1}|s^t), R^E(st_{t+1}|s^t), Z^E(st_{t+1}|s^t) \}\), and the allocations \{\{\omega_i^F(st_{t+1}|s^t)\}_{i=1}^{\infty} {i=0}, \{\{N_i^F(s^t)\}_{i=1}^{\infty} {i=0}, \{\{N_i^E(s^t)\}_{j=1}^{\infty} {j=0}, \{\{Y_i(s^t), C(s^t), D(s^t), I(s^t), K(s^t), H(s^t)\}_{i=1}^{\infty} {i=0}, \}

for a given government policy \{G(s^t), T(s^t)\}_{i=0}^{\infty}, realization of exogenous variables \{\varepsilon^A(s^t), \varepsilon^\sigma_F(s^t), \varepsilon^\sigma_E(s^t)\}_{i=0}^{\infty} \} and initial conditions \{N_i^F(s^t)\}_{i=1}^{\infty}, \{N_i^E(s^t)\}_{j=1}^{\infty}, \{K_{-1}\} \} such that for all \( t, i, j_i \) and \( h : (i) \) a household maximizes
utility given the prices; (ii) financial intermediary maximizes its profit given the prices; (iii) entrepreneur maximizes its profit given the prices; (iv) final goods producers maximizes its profit given the prices; (vi) investment goods producers maximizes its profit given the prices; (vii) the government budget constraint holds; (viii) markets clear.

3 Simulation

We now report the simulation outcomes of our model. For simulation, we calculate the steady state of the model, and linearize the system (7), (9), (10), (12), (13), (19), (14), (15), (16), (17) around the steady state. We then compute the equilibrium response of the economy to several adverse shocks that are analyzed in the literature. We study five types of adverse shocks: (1) a net worth shock in FIs sector, (2) a net worth shock in entrepreneurial sector, (3) a shock to the standard error of idiosyncratic productivity in FIs sector, (4) a shock to the standard error of idiosyncratic productivity in entrepreneurial sector, and (5) a shock to the technology in final goods sector. (1), (2), (3) and (4) are sectoral shocks that hit each of the participants of the credit market, and (5) is an aggregate shock.

We have two goals for simulation exercises. Our first goal is to show the quantitative implication of our model to the sectoral shock propagation mechanism. We study how this propagation mechanism differs across two borrowing sectors and what causes the difference. Our second goal is to investigate the aggregate shock amplification mechanism. Our credit market amplifies the aggregate shock through the endogenous reactions of the net worth in the two sectors. In comparing to BGG, our model has two sectors that have net worth. The two net worth work separately and jointly to propagate and amplify the adverse shocks to the economy.

As we show below, these mechanisms are much affected by the net worth distribution across sectors. To see this clearly, we study three alternative models with different net worth distribution. In the first model, the steady state values of net worth distribution as well as other parameters are calibrated to the U.S. data. We call this baseline model. As we see in the next subsection, according to the U.S. data, the net worth of the economy is unequally distributed and entrepreneurs have richer net worth than do FIs. In the second model, we hypothetically alter net worth distribution from the setting of baseline model so that each sector owns an equal amount of net worth. Here, the equality $N^F = N^E$ holds at the steady state of the model. In order to isolate the effect of net worth distribution, we hold all of the technology and distribution parameters pertaining to the credit contracts, two bankruptcy costs $\mu^F$, $\mu^E$ and two variances of borrowers’ idiosyncratic productivities $\sigma^F$, $\sigma^E$ fixed at the values of baseline model\(^8\). We adopt the

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\(^8\)We re-calculate the exit parameters of FIs and entrepreneurs $\gamma^F$ and $\gamma^E$ for each of the three models so that all of the equilibrium conditions (1), (2), (4), (5), (7), (9), (10) and the model specific net worth distribution hold at the steady state.
similar setting in the third model. We alter net worth distribution from that of baseline model so that FIs own more net worth than entrepreneurs. Again we maintain the parameters related to the credit contracts to the same value of the baseline model.

3.1 Calibration

We choose several parameter values of our benchmark model to those that are used in BGG. These include, quarterly discount factor $\beta$, labor supply elasticity $\eta$, capital share $\alpha$, quarterly depreciation rate $\delta$, and steady state share of government expenditure in total output $G/Y$. We set values for parameter that are linked to the IF contract and FE contract so that these are consistent with the following six conditions: (1) A risk spread, $R^E - R$, equal to 200 basis points annually, (2) a spread between FIs’ lending rate and FI’s borrowing rate $Z^E - Z^F$ equal to 230 basis points annually, the historical average spread between the prime lending rate and the six-month Certificates of Deposit rate from 1980 to 2006; (3) a spread between FIs’ borrowing rate and risk free, $Z^F - R$, equal to 60 basis points annually, approximately the historical average spread between the six-month Certificates of Deposit rate and the six-month Treasury bill rate from 1980 to 2006; (4) an annualized failure rate of FIs and entrepreneurs, $F^F(\omega^F)$ and $F^E(\omega^E)$ equal to 2%; (5) a ratio of net worth held by FIs to capital, $N^F/QK$ is 0.1, the approximate value in the data; (6) a ratio of net worth held by entrepreneurs to capital, $N^E/QK$ is 0.50, the approximate value in the data. The estimated parameters from these steady state conditions include lenders’ bankruptcy cost in IF contract $\mu^F$, lenders’ bankruptcy cost in FE contract $\mu^E$, variance of idiosyncratic productivity shock in FIs sector $\sigma^F$, variance of idiosyncratic productivity shock in entrepreneurial sector $\sigma^E$, survival rate of FIs $\gamma^F$ and survival rate of entrepreneurs $\gamma^E$. See Appendix B and Appendix C for details.

Figure 7 displays the quantitative relationships between the external finance premium, net worth distribution and level of capital investment, similar to those shown Figures 3, 4, 5 and 6, but now they are evaluated by these calibrated parameters. Calibrated cost of fund curve looks a mixture of curves shown in the sections above. Two observation are made. First, external finance premium can be reduced by distributing net worth from entrepreneurs to FIs, according to the U.S. data, share of FIs’ net worth is $0.1/(0.1+0.5)=0.17$. The external finance premium keeps decreasing with FIs’ net-worth capital ratio until the value of share reaches around 60%. The premium then starts to increase as FIs’ approaches 100% but its increase is considerably limited. Second, the Figure has a U-shape looking similar to those depicted by the solid line in Figure 3 and

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9 We calculate the steady state value of $N^E/QK$ based on the Flow of Fund data, released from Federal Reserve Board. We calculate the historical series of the sum of Corporate Equities and Equity in noncorp. business held by Financial Sectors divided by Total Liability and Equity of Nonfinancial Business Sector, and set the steady state value of $N^E/QK$ equal to .09, which is the historical average from 1990 to 2005.
5, but this curve is tilted to the right. One of the reasons for this observation is that we have $\mu^F > \mu^E$ based on the U.S. data. As we saw from Figure 3, with bankruptcy costs associated with IF contract that is higher than that with FE contract, a change in expected default cost for IF contract dominates an opposite change in expected default contract for FE contract in a wide range of net-worth capital ratios.

### 3.2 Shocks to Credit Market

We first discuss the propagation mechanism of our model to sectoral shocks, net worth shocks and riskiness shock. As for net worth shock, we introduce a shock either in equation (9) or (10), following Gilchrist and Leahy (2002). A shock to the standard error of idiosyncratic productivity in entrepreneurial sector is to our knowledge, first introduced in financial accelerator model of CMR. Following their specification for these riskiness shocks, we consider these shocks to the standard error occurs both in FIs sector and entrepreneurial sector.

First, we consider an experiment where the baseline economy is subjected to an unexpected, 1% increase in the standard error of FIs’ idiosyncratic productivity. The standard error gradually returns to its steady state exogenously at the rate $\rho_{\sigma_{FE}}$ following (23). Solid line with black circle in Figure 8 presents the baseline economy’s response to the shock. The shock generates a drop in capital goods price $Q(s_t)$. As (14) implies, a fall in the capital goods price causes the return to capital to decrease that results in the decrease of net worth of the two borrowers sectors. Here, although the adverse shock primarily hits the participants of IF contract, it also affects participants of FE contracts through an endogenous evolvement of entrepreneurial net worth. Declines of the two net worth worsen the agency problems of the two credit contracts. Consequentially, the external finance premium rises, which causes aggregate investment to fall. For comparison, we also consider the similar experiment in which economy is subject to a shock to the standard error of the entrepreneurial idiosyncratic productivity by 1%. Dashed line in the panels of Figure 8 presents the baseline economy’s responses to this shock. The shock generates a qualitatively similar dynamics in the endogenous variables. Its quantitative impact is, however, clearly more moderate compared to the impact of the shock to FIs.

Next we consider an experiment where the baseline economy is subject to an unexpected, once-and-for-all decline of the FIs’ net worth by one unit of net worth. Solid line in Figure 9 presents the economy’s response to the shock under the baseline model. In response to the shock to FIs’ net worth, entrepreneurial net worth also decreases, although the depth of decline is small compared to that of FIs’ net worth. Similarly to the experiments using the two shocks to the borrowers’ variances, net worth shock to FIs sector is propagated to the other sector through the movements of capital goods price $Q(s_t)$. As two net worth declines, the external finance premium rises that causes investment to fall. This decreases FIs’ net worth further, which amplifies the propagation
mechanism. Again for comparison, we show the economy's response to the once-and-for-all decline in the entrepreneurial net worth by one unit of net worth, with dashed line. This shock also generates a qualitatively similar dynamics in the endogenous variables in the economy. Its quantitative impacts on aggregate variables are, however, moderate compared to the that of the shock to FIs' net worth.

The reason for this asymmetric propagation effects across sectors is related to the net worth distribution under the baseline model. Recall Figure 7, that the effect of a one unit change in FIs’ net worth on aggregate capital investment (or external finance premium) is larger than the effect of a one unit change in entrepreneurial net worth when the net worth distribution is biased to FIs. An expected default cost of IF contract rises drastically with a decline in FIs’ net worth while an expected default cost of FE contract increases moderately with a decline in entrepreneurial net worth. Because of this asymmetry across the two contracts, the aggregate investment is more affected by the adverse shocks to FIs than the adverse shocks to entrepreneurs.

This results depend on the setting about the net worth distribution. Figure 10 shows the responses of investment to the four adverse shocks we discussed so far under three alternative settings of net worth distribution. Solid line with black circle depicts the model’s response under baseline net worth distribution. Solid line depicts the case in which net worth are equally distributed between FIs and entrepreneurs, so that \( n^F(s^t) = n^E(s^t) = 0 \) at the steady state. The dotted line depicts the case in which net worth is distributed more to the FIs sector, so that \( n^F(s^t) = 0.5 \) and \( n^E(s^t) = 0.1 \). The panels show that as more net worth is allocated to the entrepreneurial sector, the investment decline in the baseline model.

### 3.3 Technology Shock

We also consider an experiment where the baseline economy is subject to an unexpected, temporary decrease in productivity in final goods sector. The productivity of final goods then gradually returns to its steady state at the rate \( \rho_A \). Solid line with black circle in Figure 11 presents the economy’s response to the shock. As equation implies (14), this productivity shock decreases ex-post discounted return to capital. Consequently, an expected demand towards capital goods drops, causing capital goods price \( Q(s^t) \) to fall. Through the same mechanism we discuss in the subsection above, net worth of the two sectors decline. These endogenous evolvements of net worth affect the credit market imperfections of the two credit contract, causing the rise in the external finance premium that drives down the aggregate investment. This amplification mechanism of the credit market is affected by the net worth distribution of the economy. To see this, we compare the baseline model’s responses with those under alternative two models we studied above. Solid line in Figure 11 depicts the case in which net worth are equally distributed between FIs and entrepreneurs, so that \( n^F(s^t) = n^E(s^t) = 0.3 \) at the steady state. The dotted line in Figure 11 depicts the case in which net worth is distributed more
to the FIs sector, so that \( n^F(s^t) = 0.5 \) and \( n^E(s^t) = 0.1 \). The downturns of investment, output and capital price, and the rise in the external finance premium become moderate as FIs’ net worth share increases. Clearly, as FIs have relatively higher net worth than entrepreneurs, amplification of the technology shock is less enhanced.

4 Conclusion

Based on financial accelerator model of Bernanke et al. (1999), we have developed a dynamic general equilibrium model in which financial intermediaries as well as entrepreneurs are subject to credit constraints. We study the effect of interactions between the two borrowing sectors on the financial accelerator mechanism of the credit market. Especially, we focus on the role played by net worth distribution across sectors. Calibrated to the U.S. data, our model implies that (i) among the sectoral shocks to the credit market, shock to financial intermediaries is more propagated to the aggregate economy than shock to the entrepreneurs, and (ii) the amplification mechanism of the aggregate shock is reduced if net worth distribution between FIs and entrepreneurs are less biased. We find that the key feature of our model generating these results is the net worth distribution across sectors. Scarcity of net worth in FIs sector makes its own sector and aggregate economy more vulnerable to the adverse shocks.
A Analytical Expressions for the variables appearing in the credit contracts

In this section, we provide the analytical expressions for $G_t^F (\omega_t^F)$, $G_t^E (\omega_t^E)$, $\Gamma_t^F (\omega_t^F)$, $\Gamma_t^E (\omega_t^E)$, and their differentials with respect to their cut-off values. Following BGG and CMR, we assume that both $\omega_t^F$ and $\omega_t^E$ obey different log-normal distributions, with $E(\omega_t^F) = 1$ and $E(\omega_t^E) = 1$, respectively, and we denote the Cdf of the two distributions by $F_t (\omega_t^F)$ and $F_t (\omega_t^E)$, and denote the variance of $\log \omega_t^F$ and $\log \omega_t^E$ by $\sigma_{t,F}^2$ and $\sigma_{t,E}^2$.

Variables $G_t^F (\omega_t^F)$ ($G_t^E (\omega_t^E)$) are expected return from the default FIs (the default entrepreneurs) in IF contract (FE contract). Using the assumption about the distribution of $\omega_t^F$ and $\omega_t^E$, they are expressed as

$$G_t^F (\omega_t^F) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\log \omega_t^F - 0.5\sigma_{t,F}^2} \exp \left( -\frac{u^2}{2} \right) du_F,$$

$$G_t^E (\omega_t^E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\log \omega_t^E - 0.5\sigma_{t,E}^2} \exp \left( -\frac{v^2}{2} \right) dv_E.$$

Note that $G_t^F (\omega_t^F)$ and $G_t^E (\omega_t^E)$ are functions of current value of time-varying riskiness $\sigma_{t,F}^2$ and $\sigma_{t,E}^2$. Differentials of $G_t^F (\omega_t^F)$ and $G_t^E (\omega_t^E)$ with respect to $\omega_t^F$ and $\omega_t^E$ are given by

$$G_t^{\frac{d}{dt}} G_t^F (\omega_t^F) = \left( \frac{1}{\sqrt{2\pi}} \right) \left( \frac{1}{\omega_t^F \sigma_{t,F}} \right) \exp \left( -0.5 \left( \frac{\log \omega_t^F - 0.5\sigma_{t,F}^2}{\sigma_{t,F}} \right)^2 \right),$$

$$G_t^{\frac{d}{dt}} G_t^E (\omega_t^E) = \left( \frac{1}{\sqrt{2\pi}} \right) \left( \frac{1}{\omega_t^E \sigma_{t,E}} \right) \exp \left( -0.5 \left( \frac{\log \omega_t^E - 0.5\sigma_{t,E}^2}{\sigma_{t,E}} \right)^2 \right).$$

$\Gamma_t^F (\omega_t^F)$ ($\Gamma_t^E (\omega_t^E)$) are the net share of profit going to investors (FIs) in the IF contract (FE contract). These are expressed as

$$\Gamma_t^F (\omega_t^F) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\log \omega_t^F - 0.5\sigma_{t,F}^2} \exp \left( -\frac{u^2}{2} \right) du_F + \frac{\omega_t^F}{\sqrt{2\pi}} \int_{\log \omega_t^F - 0.5\sigma_{t,F}^2}^{\infty} \exp \left( -\frac{u^2}{2} \right) du_F,$$

$$\Gamma_t^E (\omega_t^E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\log \omega_t^E - 0.5\sigma_{t,E}^2} \exp \left( -\frac{v^2}{2} \right) dv_E + \frac{\omega_t^E}{\sqrt{2\pi}} \int_{\log \omega_t^E - 0.5\sigma_{t,E}^2}^{\infty} \exp \left( -\frac{v^2}{2} \right) dv_E.$$
Similarly, differentials of $\Gamma^F_t (\overline{w}^F_t)$ and $\Gamma^E_t (\overline{w}^E_t)$ with respect to $\overline{w}^F_t$ and $\overline{w}^E_t$ are given by

$$\Gamma^F_t (\overline{w}^F_t) = \frac{1}{\sqrt{2\pi}\overline{w}^F_t \sigma_{t,F}} \exp \left(-.5 \left( \frac{\log \overline{w}^F_t - 0.5\sigma^2_{t,F}}{\sigma_{t,F}} \right)^2 \right) dx + \frac{1}{\sqrt{2\pi}} \int_{\log \overline{w}^F_t + 0.5\sigma^2_{t,F}}^{\infty} \exp \left(- \frac{v^2_F}{2} \right) dv_F$$

$$- \frac{1}{\sqrt{2\pi}\sigma_{t,F}} \exp \left(- \frac{\log \overline{w}^F_t + 0.5\sigma^2_{t,F}}{2} \right) \frac{1}{\sqrt{2\pi}\overline{w}^F_t \sigma_{t,F}} \exp \left(-.5 \left( \frac{\log \overline{w}^F_t - 0.5\sigma^2_{t,F}}{\sigma_{t,F}} \right)^2 \right) dx$$

$$\Gamma^E_t (\overline{w}^E_t) = \frac{1}{\sqrt{2\pi}\overline{w}^E_t \sigma_{t,E}} \exp \left(-.5 \left( \frac{\log \overline{w}^E_t - 0.5\sigma^2_{t,E}}{\sigma_{t,E}} \right)^2 \right) dx + \frac{1}{\sqrt{2\pi}} \int_{\log \overline{w}^E_t + 0.5\sigma^2_{t,E}}^{\infty} \exp \left(- \frac{v^2_E}{2} \right) dv_E$$

$$- \frac{1}{\sqrt{2\pi}\sigma_{t,E}} \exp \left(- \frac{\log \overline{w}^E_t + 0.5\sigma^2_{t,E}}{2} \right) \frac{1}{\sqrt{2\pi}\overline{w}^E_t \sigma_{t,E}} \exp \left(-.5 \left( \frac{\log \overline{w}^E_t - 0.5\sigma^2_{t,E}}{\sigma_{t,E}} \right)^2 \right) dx.$$
B Parameterization I

This appendix provides parameterization of the variables associated with household, wholesalers, capital goods producers, retailers, final goods producers, government and monetary authority. Following precedent studies including BGG and CMR, we choose conventional values for these parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>.99</td>
<td>Discount Factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>.025</td>
<td>Depreciation Rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>.35</td>
<td>Capital Share</td>
</tr>
<tr>
<td>$R$</td>
<td>$.99^{-1}$</td>
<td>Risk-free Rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>3</td>
<td>Elasticity of Labor</td>
</tr>
<tr>
<td>$\chi$</td>
<td>.3</td>
<td>Utility Weight on Leisure</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>2.5</td>
<td>Adjustment Cost of Investment</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>.85</td>
<td>Autoregressive Parameter for TFP</td>
</tr>
</tbody>
</table>

Figures are quarterly unless otherwise noted.

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10Figures are quarterly unless otherwise noted.
C Parameterization II

This appendix provides parameterization of the variables that are related to the credit contracts among investors, FIs and entrepreneurs. We choose six parameters so that they are consistent with the equilibrium conditions (1), (2), (4), (5), (7), (9) and (10) evaluated by the steady state values for risk free rate \( R \), FIs’ lending rate \( Z^E \), FIs’ borrowing rate \( Z^F \), entrepreneurial default probability \( F(\bar{\pi}^E) \), FIs default probability \( F(\bar{\pi}^F) \), entrepreneurial net worth/capital ratio \( n^E \) and FIs’ net worth/capital ratio \( n^F \) shown in the lower table.

Calibrated Parameters\(^{11}\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_F )</td>
<td>0.107366</td>
<td>Variance of FIs Idiosyncratic Productivity at Steady State</td>
</tr>
<tr>
<td>( \sigma_E )</td>
<td>0.312687</td>
<td>Variance of Entrepreneurial Idiosyncratic Productivity at Steady State</td>
</tr>
<tr>
<td>( \mu_F )</td>
<td>0.033046</td>
<td>Bankruptcy Cost associated with FIs</td>
</tr>
<tr>
<td>( \mu_E )</td>
<td>0.013123</td>
<td>Bankruptcy Cost associated with entrepreneurs</td>
</tr>
<tr>
<td>( \gamma_F )</td>
<td>0.963286</td>
<td>Survival Rate of FIs</td>
</tr>
<tr>
<td>( \gamma_E )</td>
<td>0.983840</td>
<td>Survival Rate of Entrepreneurs</td>
</tr>
</tbody>
</table>

Steady State Conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R = .99^{-1} )</td>
<td>Risk-free rate is inverse of subjective discount factor.</td>
</tr>
<tr>
<td>( Z^E = Z^F + .023^{.25} )</td>
<td>Premium for FIs’ lending rate is .023^{.25}.</td>
</tr>
<tr>
<td>( Z^F = R + .006^{.25} )</td>
<td>Premium for FIs’ borrowing rate is .006^{.25}.</td>
</tr>
<tr>
<td>( F(\bar{\pi}^F) = .02 )</td>
<td>Default probability in IF contract is .02.</td>
</tr>
<tr>
<td>( F(\bar{\pi}^E) = .02 )</td>
<td>Default probability in FE contract is .02.</td>
</tr>
<tr>
<td>( n^F = .1 )</td>
<td>FIs’ net-worth capital ratio is set to .1</td>
</tr>
<tr>
<td>( n^E = .5 )</td>
<td>Entrepreneurial net-worth capital ratio is set to .5.</td>
</tr>
</tbody>
</table>

\(^{11}\)Figures are quarterly unless otherwise noted.
References


Figure 1. Effect of net worth in FIs sector (left panel) and in entrepreneurial sector (right panel). The ratio of net worth to capital in each sector is depicted on the horizontal axis and external finance premium is depicted on the vertical axis.
Figure 2. Effect of net worth in FIs sector (left panel) and in entrepreneurial sector (right panel). The ratio of net worth to capital in each sector is depicted on the horizontal axis and the expected default rate is depicted on the vertical axis.
Figure 3. Effect of net worth distribution on the external finance premium under information asymmetry problem. The ratio of FIs’ net worth over total net worth is depicted on the horizontal axis and external finance premium is depicted on the vertical axis. Solid line depicts the case in which monitoring costs and variances of idiosyncratic productivities are symmetric in IF contract and FE contract. Solid line with black circle (Dashed line) depicts the case in which FIs’ (entrepreneurial) monitoring cost is lower than entrepreneurial (FIs) monitoring cost and variances of idiosyncratic productivities are symmetric in IF contract and FE contract.
Figure 4. Effect of net worth distribution on the expected default costs. The share of FIs’ net worth over total net worth is depicted on the horizontal axis and the expected default cost of IF contract (FE contract) is depicted on the vertical axis in the left (right) panel. Solid line depicts the case in which monitoring costs and variances of idiosyncratic productivities are symmetric in IF contract and FE contract. Solid line with black circle (Dashed line) depicts the case in which FIs’ (entrepreneurial) monitoring cost is lower than entrepreneurial (FIs) monitoring cost and variances of idiosyncratic productivities are symmetric in IF contract and FE contract.
Figure 5. Effect of net worth distribution on the external finance premium. The share of FIs’ net worth over total net worth is depicted on the horizontal axis and the external finance premium is depicted on the vertical axis. Solid line depicts the case in which monitoring costs and variances of idiosyncratic productivities are symmetric in IF contract and FE contract. Solid line with black circle (Dashed line) depicts the case in which variance of FIs’ (entrepreneurial) idiosyncratic productivity is lower than that of entrepreneurs (FIs) and monitoring costs are symmetric in IF contract and FE contract.
Figure 6. Effect of net worth distribution on the expected default costs. The share of FIs’ net worth over total net worth is depicted on the horizontal axis and the expected default cost of IF contract (FE contract) is depicted on the vertical axis in the left (right) panel. Solid line with black circle (Dashed line) depicts the case in which variance of FIs’ (entrepreneurial) idiosyncratic productivity is lower than that of entrepreneurs (FIs) and monitoring costs are symmetric in IF contract and FE contract.
Figure 7. Effect of net worth distribution on the external finance premium and expected default costs under baseline calibration.
Figure 8. Effect of shock to the variance of idiosyncratic productivity of borrower in credit contract. Impulse response of variables to positive riskiness shock in FIs and entrepreneurs ($\sigma^F$ shock, $\sigma^E$ shock, respectively) are depicted on the y-axis.
Figure 9. Effect of net worth shocks. Impulse response of variables to once-and-for-all decline ($N^F$ shock, $N^E$ shock, respectively) are depicted on the vertical axis.
Figure 10. Effect of net worth distribution on how the economy response to adverse shocks in the credit market. Impulse responses of investment after an unexpected rise in variance of FLs' idiosyncratic productivity (upper left panel), an unexpected rise in variance of entrepreneurial idiosyncratic productivity (upper right panel), an unexpected decline in FLs' net worth (lower right panel) and an unexpected decline in entrepreneurial net worth (lower right panel) are depicted.
Figure 11. Effect of negative technology shocks. Impulse response of variables to a temporary decline in the productivity of final goods sector are depicted on the vertical axis.