Liquidity Constraints in a Monetary Economy

Leo Ferraris Makoto Watanabe

Departamento de Economía, Universidad Carlos III de Madrid

December, 2008

Objective

- Money: the medium to transfer resources on the spot
- Liquidity: the availability of a medium to transfer resources over time

Explore a (monetary) model to study the issue of liquidity.

Key ingredients

(i) Use of money in spot exchange (Kiyotaki and Wright (1989)):

- Anonymity;
- Absence of double coincidence of wants.

$\Longrightarrow$ Pledgeability of returns: the fundamental impediments arising in spot trade may seep into the credit market.

(ii) Liquidity (Holmstrom and Tirole (1998)):

- liquid project;

$$\text{pledgeable returns} = \text{expected returns}$$

- illiquid project;

$$\text{pledgeable returns} < \text{expected returns}.$$

Preview of main results:

- The same frictions generating an essential role for money may also make firms liquidity constrained;
- Money can perform two roles - as a provider of liquidity service and exchange service;
- The binding liquidity constraint constitutes a channel through which under-investment occurs.

Literature

- Kiyotaki and Wright (1989)
- Kiyotaki and Moore (2001)

Model

A divisible/competitive version of monetary model, Lagos and Wright (2005), with a consumption and an investment market

- Time: discrete, infinite w./ three sub-periods (morning, afternoon and evening)

- Agents: entrepreneurs, investors; homogeneous, unit mass, infinitely lived

- Goods: consumption goods, investment goods; all production costs are normalized to one.

- Morning market (investment market):

- Investors produce an investment good;
- Entrepreneurs and investors meet randomly and bilaterally;
- An investment good q_1 generates returns, *early returns* and *late returns*, to entrepreneurs with technology $g(q_1)$;
- $g(\cdot)$ is continuously differentiable, strictly increasing and concave with $g(0) = 0$, $g'(0) = \infty$, $g'(\infty) = 0$;
- The investment is a one-period event.

- Afternoon market (consumption market):

- Anonymous trading;

- Uncertainty in production and consumption opportunities; a buyer with prob δ; a seller with prob $1-\sigma$;

- A consumption good q_2 yields utility $u(q_2)$ to buyers. $u(\cdot)$ is differentiable and strictly increasing and concave with $u'(0)=\infty$, $u'(\infty)=0$;

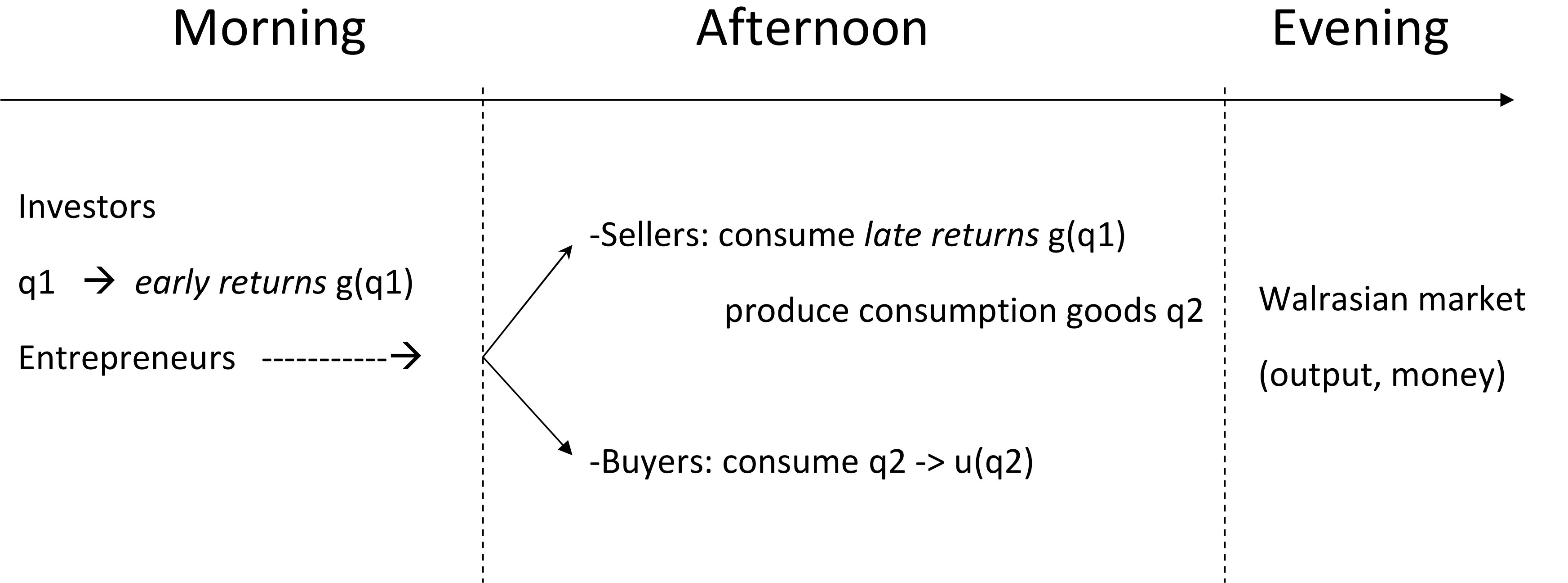
- Sellers have production technologies.

- Evening market (Walrasian market):

Agents can produce and trade an output whose market price is normalized to one.

Fiat money can also be traded at a market price, denoted by ϕ.

Timing



Morning
Afternoon
Evening
Investors
q1 → *early returns* g(q1)
Entrepreneurs ----------→
-Sellers: consume *late returns* g(q1)
produce consumption goods q2
-Buyers: consume q2 -> u(q2)
Walrasian market
(output, money)

Efficiency

The planner solves

$$\max_{q_1,q_2\geq 0}\ [g(q_1)-q_1]+[(1-\sigma)g(q_1)+\sigma(u(q_2)-q_2)].$$

The optimal solution $q_1^*, q_2^* > 0$ satisfies

$$\begin{aligned}(2-\sigma)g'(q_1^*) &= 1,\\ u'(q_2^*) &= 1.\end{aligned}$$

Contract with investors

- Long term contracts are not available;
- Only early returns of investment are pledgeable;

A contract between an entrepreneur and an investor specifies the amount q_1 of investment goods, generating output with technology $g(q_1)$, and its payment z, θ that satisfies

$$\begin{aligned} z + \theta\phi m &= q_1 && (1) \\ z &\leq g(q_1) && (2) \\ \theta &\in [0, 1]. && (3) \end{aligned}$$

Berman equations

[Evening market]:

$$W(\widehat{m}) = \max_{x,e,m_{+1} \geq 0} \left[x - e + \beta V(m_{+1}) \right]$$

$$\text{s.t.} \quad x - e = \phi(\widehat{m} - m_{+1}) + \tau$$

where $\widehat{m} = (1-\theta)m - pq_2$ or $\widehat{m} = (1-\theta)m + pq_2^s$.

[Afternoon market]:

$$Z(q_1,(1-\theta)m) = \sigma \left\{ \begin{array}{c} \text{max}_{\ q_2 \geq 0} \ \left[u(q_2) + W((1-\theta)m - pq_2)\right] \\ \text{s.t. } pq_2 \leq (1-\theta)m \end{array} \right\}$$

$$+(1-\sigma)\left\{ \text{max}_{\ q_2^s \geq 0} \ \left[g(q_1) - q_2^s + W((1-\theta)m + pq_2^s)\right] \right\}$$

[Morning market]:

$$
\begin{aligned}
V(m) &= \max_{q_1, z, \theta \geq 0} \left[g(q_1) - z + Z(q_1, (1-\theta)m) \right] \\
&\quad \text{s.t.} \quad \text{(1)-(3)}
\end{aligned}
$$

or by $z = q_1 - \theta \phi m$,

$$
V(m) = \max_{q_1, \theta \geq 0} \left[g(q_1) - (q_1 - \theta \phi m) + Z(q_1, (1-\theta)m) \right]
$$

$$
\begin{aligned}
\text{s.t.} \quad q_1 - \theta \phi m &\leq g(q_1) \\
\theta &\in [0, 1]
\end{aligned}
$$

First order conditions

$$(2-\delta)g'(q_1) = 1 + \mu(1 - g'(q_1))$$

$$\mu + \frac{\gamma}{\phi m} = \delta(u'(q_2) - 1)$$

Complementary slackness condition

$$\mu \left[g(q_1) - q_1 + \theta\phi m\right] = 0$$

$$\gamma\theta = 0$$

Two situations are possible:

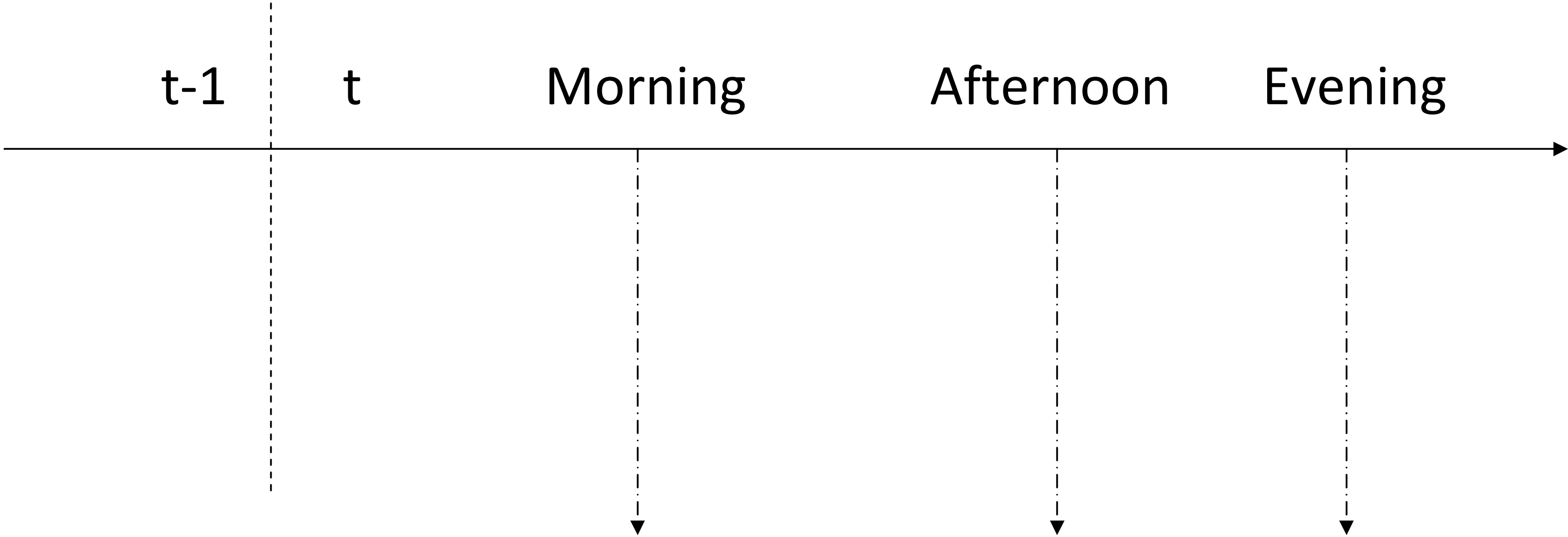
1. Binding liquidity constraint.

2. Non-binding liquidity constraint.

Euler equation

$$\phi = \beta\phi_{+}\left[(1-\theta)(\delta u'(q_2) + 1 - \delta) + \theta(\mu + 1)\right]$$

Euler Equation



$$1 \rightarrow \beta \, \frac{\varphi_{+1}}{\varphi} \, [\theta(1+\mu) + (1-\theta)(\delta u'(q_2) + 1 - \delta)]$$

Stationary monetary equilibrium

$$\mu + \frac{\gamma}{q_2} = \delta(u'(q_2) - 1) = \frac{\pi}{\beta} - 1$$

Three possible cases for $\pi > \beta$:

[1] liquidity constraint is not binding $\mu = 0$ and no money is pledged $\theta = 0$;

[2] liquidity constraint is binding $\mu > 0$ and no money is pledged $\theta = 0$;

[3] liquidity constraint is binding $\mu > 0$ and a positive amount of money is pledged $\theta > 0$.

Proposition 1 *Suppose* $g(q_1^*)/q_1^* \geq 1$*. Then, a unique equilibrium exists for all* $\pi > \beta$ *in which the liquidity constraint is not binding,* $\mu = 0$*, and no money is pledged,* $\theta = 0$*. Further, it satisfies:* $q_1 = q_1^*$ *for all* $\pi > \beta$*;* $q_2 \in (0, q_2^*)$ *is strictly decreasing in* $\pi \in (\beta, \infty)$*;* $q_1 \to q_1^*$*,* $q_2 \to q_2^*$ *as* $\pi \to \beta$*.*

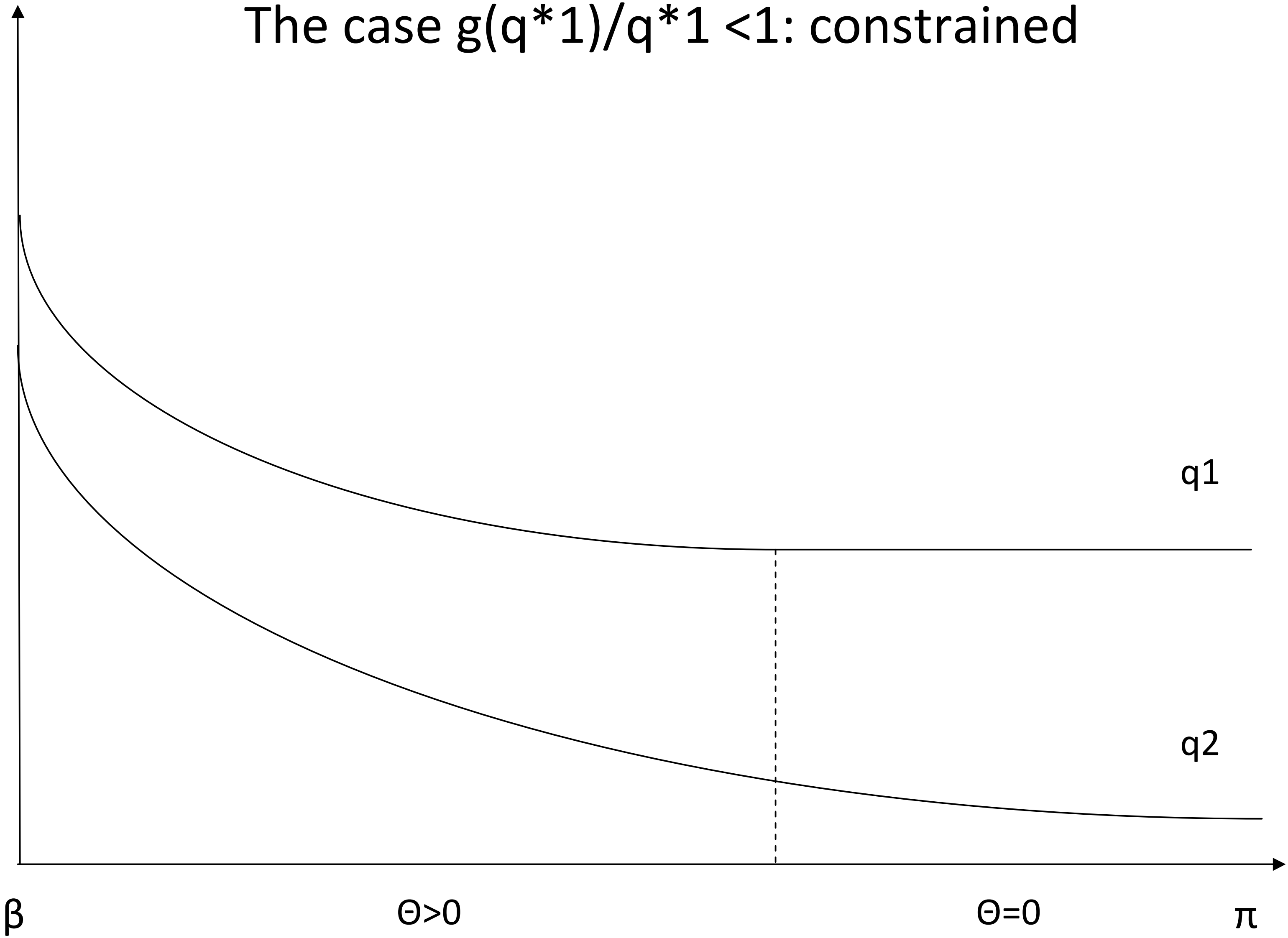
The case $g(q^*1)/q^*1 \geq 1$: unconstrained

Proposition 2 *Suppose* $g(q_1^*)/q_1^* < 1$. *Then, a unique equilibrium exists for all* $\pi > \beta$ *in which the liquidity constraint is binding,* $\mu > 0$. *It satisfies:* $q_2 \in (0, q_2^*)$ *is strictly decreasing in* $\pi \in (\beta, \infty)$; $q_1 \to q_1^*$, $q_2 \to q_2^*$ *as* $\pi \to \beta$. *Further, there exists a unique* $\hat{\pi} \in (\beta, \infty)$ *such that* $q_1 = \hat{q}_1 \in (0, q_1^*)$ *at* $\pi = \hat{\pi}$ *and:*

1. $\theta > 0$ *for* $\pi \in (\beta, \hat{\pi})$ *and* $\theta = 0$ *for* $\pi \in [\hat{\pi}, \infty)$;

2. $q_1 \in (\hat{q}_1, q_1^*)$ *is strictly decreasing in* $\pi \in (\beta, \hat{\pi})$ *and* $q_1 = \hat{q}_1$ *for all* $\pi \in [\hat{\pi}, \infty)$.

The case g(q*1)/q*1 <1: constrained



Discussion 1: money and credit

“Evil is the root of all money” (Kiyotaki and Moore (2001))

versus

“Money is the root of all evil” (The Bible, 1 Timothy 6:10)

Discussion 2: policy and empirical implications

- Impact of inflation on investment according to the stage of country development (Gertler and Rogoff (1990))

- Negative but decreasing effect of inflation on investment (Boyd, Levine and Smith (2001))

Discussion 3: definition of liquidity

- Completemness of markets (Holmstrom and Titole (1998))
- Means of payment (Shubik (1999), Kiyotaki and Moore (2000))
- Thinnes of market (Diamond (1986), Jones and Ostroy (1984), Morris and Shin (2003))
- Agents' ability to sell contingent promises of future deliveries (Diamond and Rajan (2001), Caballero and Krishnamurthy (2001))
- Flexibility to move goods (Fostel and Geanakoplos (2008))

Conclusion

- Liquidity constraints
- Money can play two roles - as a provide of liquidity services and exchange services
- Interaction of an investment and a consumption market