Income Risk, Consumption Inequality, and Macroeconomy in Japan*

[Very Preliminary and Very Incomplete]

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Abstract

In this paper, using an overlapping generations model with heterogeneous households, we investigate economic inequality in recent decades in Japan. We decompose the causes of the economic inequality into three factors: (a) macroeconomic variables factor, (b) a demographic factor, and (c) the idiosyncratic income risks factor. We find that the income and wage inequality generated by the model closely replicates the actual evolution of the inequality in Japan. On the contrary, the consumption inequality in the data is much more volatile than that predicted by the model. Based on a counterfactual simulation, we demonstrate that in addition to the demographic factor, time-varying macroeconomic factors play an important role in the evolution of economic inequality. In particular, we show that the low growth rate of total factor productivity in the 1990s in Japan limited the dispersion of economic inequality.

Keywords Income risk, Consumption inequality, Population aging, Labor supply

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1 Introduction

The life cycle permanent income hypothesis (LC-PIH) is a key concern for considering the consumption and saving problem of households. According to the LC-PIH, income inequality measured at one period does not necessarily reflect economic inequality over the life cycle. Since the income inequality includes temporary variation of income, even if the income inequality rises, consumption inequality may not rise when households are prepared for such a temporary income shock. Therefore, in order to investigate the economic inequality and consider welfare evaluation and policy implications, we need to simultaneously consider the characteristics of income inequality and consumption inequality.

Households face various idiosyncratic risks on labor income such as career success/failure, a bonus cut, and unemployment. Although these income risks are seen as income inequality by exogenous observers, a household that receives these shocks reacts differently depending on the nature of the income shocks. If the income shock is temporary, a household easily shares the risk through precautionary savings, help from relatives, or public insurance. However, if the income shock is very persistent or permanent, a household must lower its standard of living. This is a well known consequence according to the permanent income hypothesis.[1] Moreover, macroeconomic environment like factor prices and technical change may affects the economic inequality through households' decision making.

In this paper, we consider the following question: what kinds of factors affect the evolution of economic inequality? There are many factors that cause macroeconomic inequality in addition to the idiosyncratic income risks that households face. In order to investigate the causes of economic inequality, we decompose the causes into three factors: (a) macroeconomic factors, (b) the demographic factor and (c) the idiosyncratic income risks factor. The first factor that affects the economic inequality is the time-varying

[1] The characteristics of idiosyncratic risks have direct implications on policy making. If the income shock is temporary, opening insurance markets or risk sharing within family may be included in the economic policy. On the other hand, if the shock is permanent, a different policy may be required. For example, Genda et al. (2007) show that cohorts who enter the economy in recession receive a negative shock on life time income. This is a typical permanent income shock, which cannot be shared through market transaction. Therefore, in order to improve the household welfare, the government needs to introduce a redistribution policy.

macroeconomic variables such as the TFP growth rate, labor share, or interest rate.[2] For considering the relationship between the macroeconomic factors and the economic inequality, Japanese economy is a good sample because Japan have experienced big boom and deep recession in recent decades. Moreover, there are several substantial changes in several variables (See Table 1). Hayashi and Prescott (2002) demonstrate that a standard neoclassical growth model with calibrated parameters could explain Japan's deep recession including the so-called lost decade of the 1990s.[3] Many subsequent papers have been published that investigate the lost decade using a dynamic general equilibrium framework. For example, Chen et al. (2006,2007) and Braun et al. (2007) construct a general equilibrium model and explain the history of Japanese saving rates based on macroeconomic factors and demographic change. If the change in the saving rate is explained by the macroeconomic factors, the same may be true about the economic inequality. Thus, we examine economic inequality using the same approach.

Empirical researches such as Ohtake and Saito (1998) have pointed out that dispersion of economic inequality in the Japanese economy in recent times is largely due to population aging. Thus, even when considering economic inequality with a dynamic general equilibrium model, we cannot ignore the effect of a demographic factor. Economic inequality that are not explained by the above factors may be due to the change of the idiosyncratic income risk factors. Thus, we can infer whether the idiosyncratic income risks can indirectly explain consumption inequality in Japan. As mentioned above, the income risks affect consumption differently depending on their respective characteristics. Since consumption inequality has direct implications on economic welfare, we need to decompose its causes. Therefore, by controlling the macroeconomic factors and a demographic factor, we consider the relationship between income risks and consumption inequality using a dynamic general equilibrium model.

Usually, modeling a stochastic income risk process requires panel data over a long

[2] Ríos-Rull and Santaeulàlia-Llopis (2008) and Choi and Ríos-Rull (2008) show that the labor share is volatile and countercyclical with the TFP. They find that the bivariate specification of productivity shock (TFP) and redistributive shock (labor share) generate the observed dynamics of busuness cycle in the US. We include the time-varying labor share to consider whether reallocation of labor earners and capital earners affect the economic inequality.

[3] In Kehoe and Prescott (2007), in addition to Hayashi and Prescott (2002), many researchers investigate deep recession of several countries based on dynamic general equilibrium framefork. See also Cole and Ohanian (1999).

period or detailed microdata.[4] If panel data is available, it will be possible to investigate the nature of the idiosyncratic income risks, and determine whether the idiosyncratic risks are affected by business cycles. Thus, we could analyze the evolution of the risks and economic inequality over the period including business cycles. Heathcote et al. (2004) decompose the property of income shock by using the Panel Study of Income Dynamics (PSID). However, there are no comparable panel data in Japan.[5] Our approach in this paper is as follows. Based on a general equilibrium model in which the LC-PIH holds, we generate simulated economic inequality paths after controlling some factors. In other words, we construct a DGE model and decompose the income/consumption inequality into *explained* and *unexplained* components through counterfactual simulations.

Our analysis is at the cross-roads of two studies. The first is a quantitative research on the Japanese economy after Hayashi and Prescott (2002). As stated above, Hayashi and Prescott (2002) show that the time variation of TFP growth and reduction in work hours have a great impact on the lost decade by using a standard neoclassical growth model. Moreover, Chen et al. (2006,2007) and Braun et al. (2007) extend their results in order to explain the history of Japan's saving rate. Our research is considered as an extension of this literature by investigating the second moment in the model. In the existing literature, many authors explain macroeconomic variables such as GDP or saving rate (*first moments*) of Japanese economy using the general equilibrium model. We consider the economic inequality (variance or *second moments*) using quantitative analysis. The second study is a research on household consumption/saving decision. In particular, our approach is similar to that of Heathcote et al. (2004,2008) who investigate the link between income risk and consumption inequality in the U.S. Since there are many researches on this topic, in particular in the U.S., we review the literature in the next section.

It is impossible to study the present economic inequality in Japan without considering demographic structure. Thus, we construct an overlapping generations (OLG)

[4] See Abowd and Card (1989), Attanasio and Davis (1996), Blundell and Preston (1998), Blundell, Pistaferri and Preston (2006), Guvenen (2007,2008), Heathcote et al. (2004,2005,2008), Meghir and Pistaferri (2004), Primiceri and van Rens (2008), and Storesletten et al. (2004b).

[5] Using long term income tax statistics, Moriguchi and Saez (2007) investigate Japanese income inequality over 100 years. Since they depend on the income tax statistics, they could not consider consumption inequality. As stated above, both income and consumption inequalities are needed for considering economic inequality based on the permanent income hypothesis.

model with long-living households. Details of the model are as follows. There are many households with different ages, who make decisions on consumption and saving. In each cohort, a continuum of households exists and they face idiosyncratic income risk. Although they are identical when young, the idiosyncratic income risks are realized. As a result, the households differ in their wealth, labor supply, and consumption in their middle and old age. Aggregating the heterogeneous households, we consider a general equilibrium of the model. Including a demographic structure and exogenous macroeconomic variables, we calibrate our model to the Japanese economy and use it as an experimental tool.

Our results are as follows. First, using the OLG model with calibrated parameters, the macroeconomic variables (first moments) and earning/consumption inequality over life cycle is replicated between 1980 and 2000. Second, rise of earning inequality in recent decades is at least partially explained by our model. Third, it is difficult to explain the consumption inequality using the benchmark model, particularly in 1980s. Based on the counterfactual experiments, we find that the earning, income, and consumption inequalities are reduced by the low TFP growth rate in the lost decade. Fourth, although the demographic factor gives rise to the positive trend of earning inequality, the macroeconomic factors also contribute to a positive trend of inequality. Lastly, we find that the preference change and credibility of social security system may be causes of recent rises of consumption inequality.

The remainder of this paper is organized as follows. Before constructing our dynamic general equilibrium model, we begin with the precise definitions of income risks, which are applied in our model, in Section 2. In Section 3, we construct an OLG model with heterogeneous households and idiosyncratic income risks. We calibrate parameters for Japanese economy in Section 4. In Section 5, based on the numerical results, we discuss income and consumption inequality in Japan. Section 7 concludes the paper.

2 Empirical Facts on Earning, Income and Consumption Inequality in Japan and the U.S.

2.1 Idiosyncratic Labor-Market Risk

Before constructing a dynamic general equilibrium model, we review the empirical researches on the relationship between income risk and consumption inequality. The specification of the idiosyncratic labor income risk stated below will be included in the model specified in Section 3. First, we focus on the research on income risk estimation.

Estimation of the income risks that households face is extensively examined in the literature. [6] Since panel data over a long period such as the PSID is available in the U.S., many researchers estimate the income risks by specifying stochastic process. For example, Storesletten et al. (2004b) estimate the income risk by developing a generalized method of moments (GMM) estimator and a variation in the cross-sectional variance between households. Using the PSID, Meghir and Pistaferri (2004) also estimate the income shocks as an autoregressive conditional heteroskedastic (ARCH) process from the conditional variance of income shocks.

Labor income for individual i at age j is written as y_j^i, and we denote the individual's observable variables such as labor market experience or sex as $X_{i,t}$. Then, we define the stochastic component of labor income e_j^i as follows:

$$\ln y_{j,t}^i = \beta_{0,t} + f(X_{i,t}, \beta_{1,t}) + e_j^i.$$

In keeping with the existing literature, we decompose the residual e_j^i as the permanent income component z_j^i and the transitory shock ε_j^i.

$$e_j^i = z_j^i + \varepsilon_j^i,\ \varepsilon_j \sim \mathcal{N}\left(0, \sigma_{\varepsilon,t}^2\right), \tag{1}$$

where the transitory shock follows the normal distribution with zero mean and variance $\sigma_{\varepsilon,t}^2$. The permanent income is assumed to follow a random walk:

$$z_j^i = z_{j-1}^i + \eta_j^i,\ \eta_j \sim \mathcal{N}(0, \sigma_{\eta,j,t}^2), \tag{2}$$

[6]The research stated below focuses on *natures* of the income process, not on the *causes* of the income inequality. With regard to the causes of the income dispersion, for example, Acemoglu (2002) emphasizes the role of technology change. Further, Kambayashi et al. (2007) and Ohtake (2005) explain the wage inequality in Japan based on skill biases among workers.

where η_j^i is the permanent shock which also follows the normal distribution. We assume that these shocks are independent across households. The variances of each shock generally depend on time t. Since the transitory shock, by definition, is not persistent, such an income risk is easily shared through time by precautionary saving. Moreover, the consumption may not affected by the transitory shock ε_j^i. On the contrary, since the permanent shock η_j^i has a persistent effect on the income level of a household, as is well known from theoretical result by Constantinides and Duffie (1996), it has a permanent effect on the consumption level. Therefore, the variance of the transitory shock and the permanent shock has different implications on consumption inequality. However, it is difficult to separate these shocks based on income data alone.[7]

Storesletten et al. (2004a,b) extend equation (1) and decompose the idiosyncratic risk into three factors: (a) a fixed effect α^i, (b) a persistent shock η_j^i, and (c) a transitory shock z_j^i. They estimate variances of these shocks $\{\sigma_\alpha^2, \sigma_{\eta,j}^2, \sigma_\varepsilon^2\}$ and a persistence parameter of the shock ρ from the following equations:

$$e_j^i = \xi_t^f \alpha^i + z_j^i + \xi_t^t \varepsilon_j^i,\ \alpha \sim \mathcal{N}\left(0, \sigma_\alpha^2\right),\ \varepsilon_j \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \tag{3}$$

$$z_j^i = \rho z_{j-1}^i + \xi_t^p \eta_j^i,\ \eta_j \sim \mathcal{N}(0, \sigma_{\eta,j}^2), \tag{4}$$

where $\{\xi_t^f, \xi_t^p, \xi_t^t\}$ are time-varying loading factors for measuring the size of each shock over the business cycle. By normalizing these factors to one at year $t = 1$, size of the other years's shocks are represented by these parameters. The fixed effect α^i represents a shock that a household receives on the timing of entry in an economy. It is impossible to share such a shock through insurance contracts because the household still does not enter the market. Note that equation (4) is a generalization of equation (2), and these equations coincide when the persistence parameter ρ is equal to one. Using the PSID, Storesletten et al. (2004b) show that the autocorrelation coefficient ρ is at a highly persistent level of 0.95 in the U.S., which is consistent with the findings by Deaton and Paxson (1994). Ohtake and Saito (1998) show that the variance of logarithm of income increases with age in Japan as in the U.S. Moreover, the slope of the variance also increases over age 50. This observation implies that the autocorrelation coefficient ρ may be over one in Japan. In fact, Abe and Yamada (2006) estimate the stochastic process (4) using the variance structure in Japan, and find that it is over one.

[7] It would be possible to include time effects and cohort effects in equation (1). Heathcote et al. (2005) show that the time effects largely account for the observed trends in inequality in the US, whereas the cohort effects are less important.

In order to estimate the variances of each shock over business cycle, we need to have panel data over a long period. Storesletten et al. (2004b) show that in the U.S., the idiosyncratic income risk is strongly countercyclical with business cycle. Due to limited data, we were unable to estimate such an income process using Japanese data set.

2.2 LC-PIH and Consumption Inequality

Following the income risk specification of equations (3) and (4), it is well known from the LC-PIH that the each shock has different implications on consumption inequality. If fixed effects such as education background are large, there may be high consumption inequality because such a shock is uninsurable through saving or insurance markets. For example, if both income and consumption inequalities of some cohort are high, we can attribute this to the high fixed effect shock. On the contrary, the transitory shock has little effect on consumption inequality because of consumption smoothing over a period, if insurance markets function effectively or the amount of precautionary saving is sufficient.[8][9]

Following these ideas, Heathcote et al. (2004) investigate the relationship between income risk and consumption inequality in the U.S. using a dynamic general equilibrium framework. In the U.S., there was a sharp rise in income inequality in the 1980s. However, according to the Consumer Expenditure Survey, consumption inequality did not increase significantly in the same period.[10] Based on the income risk model stated above, these facts can be explained by the increase in the size of transitory shock ξ_t^t. If the transitory shock is large, the income inequality appears to rise; however the corresponding consumption inequality does not change from the LC-PIH. On the other hand, the period of rising consumption inequality can be explained by the increase in the size

[8] If insurance markets are complete, marginal utilities of all households coincide and growth rates of the individual's consumption are proportional to aggregate consumption. Based on this theoretical result, Hayashi, Altonji and Kotlikoff (1996) test the complete market hypothesis in the U.S. Attanasio and Davis (1996) reveal that the insurance market works if the shock is temporary, but infrequent shocks are not shared. Moreover, Kohara et al. (2002) test the complete market hypothesis in Japan.

[9] Blundell and Preston (1998) summarize the relationship between income shock and consumption inequality, from a simple model, as the follows. The increment of consumption inequality is explained by the permanent shock, and the difference of the variance of log income and consumption become the transitory shock.

[10] See also Attanasio et al. (2004).

of the fixed effect or the persistent shock, i.e., $\{\xi_t^f, \xi_t^p\}$. Heathcote et al. (2004) conclude that both the variance of fixed effect and transitory shocks increased, but variance of the persistent shock component declined up to the mid 1970s. The persistent shock component increased sharply in the 1980s, and the transitory shock was high in the 1990s. From the same observation, Blundell et al. (2008) show that the income dispersion and the lack of dispersion in the consumption inequality stem from partial insurance of the permanent shocks.

These studies offer ideas for investigating the link between income and consumption inequality when panel data (and cross-section data) for a long period is available. Unfortunately, in Japan, panel data is not available for a long period; thus, it is difficult to estimate consumption inequality for every year. However it may be possible to investigate the link between income and consumption inequality using a dynamic general equilibrium model with counterfactual simulation. The simulation study identifies the factors that have had a crucial impact on economic inequality in recent decades in Japan.

3 Overlapping Generations Model

The model we used in this paper is an OLG model with a continuum of heterogeneous households, which is developed by Aiyagari (1994) and extended to a life cycle model by Huggett (1996).

3.1 Demographics

We consider the OLG model with long-living households.[11] Households enter the economy at age 20 and live up to a maximum age of 100. However, they face mortality risk and may die before the age of 100. $\mu_{j,t}$ denotes the population at age $j \in \mathcal{J} = \{0, \dots, 100\}$ at time t. In the next period, a fraction of the households $(1 - \phi_{j,t})$ dies and exits the economy. Thus, the size of each cohort evolves as follows:

$$\mu_{j+1,t+1} = s_{j,t}\mu_{j,t}$$

[11] Our model includes population dynamics and time variation of total factor productivity. Thus, in order to numerically solve the model, we need to detrend all macroeconomic and microeconomic variables by the growth rate. For details, see Appendix.

We denote the population growth rate of age 0 from time t to $t+1$ as ψ_t. Thus, the new population at period $t+1$ is determined from $\mu_{0,t+1} = (1+\psi_t)\mu_{0,t}$. Because households are in their childhood at $j = 0, 1, \ldots, 19$, they do not engage in consumption or employment but are included in the population dynamics for computing the future fertility rate. The total population including the children at time t is $N_t = \sum_{j=0}^{100} \mu_{j,t}$.

3.2 Household Behavior

A household that entered the economy at time t elastically supplies labor until age 65.[12] The utility function of the household that entered the economy at age 20 in period t is as follows:[13]

$$U_t = E_{20,t}\left\{\sum_{j=20}^{100}\beta^{j-20}\left(\prod_{i=20}^{j-1}\phi_{i,t}\right)\frac{\left[c_{j,t}^{\sigma}(\bar{h}_t - h_{j,t})^{1-\sigma}\right]^{1-\gamma}}{1-\gamma}\right\},\ s_{19,t} = 1,$$

where $\beta > 0$ is a discount factor, γ represents a parameter for intertemporal elasticity of substitution, σ is a parameter for the share of consumption and leisure, $\bar{h}_t$ is the time endowment and $h_{j,t} \in [0, \bar{h}_t]$ is a labor supply at age j. We assume that the time endowment depends on period t because we want to consider the effect of reduction in work hours (referred as *jitan*) in the late 1980s and early 1990s.

Households comprise workers of age $j \in \{20, \ldots, 65\}$ and supply labor elastically. Thereafter, $j \in \{66, \ldots, 100\}$, they must retire and cannot supply labor anymore. The households must pay the social security tax when they are employed, and receive the public pension after retirement.

[12] Braun et al. (2007) consider time-variation in the family scale in their model. Moreover, Heathcote et al. (2008) include education and intra-family time allocation in their model, and investigate the wage inequality. We assume that household consists of the male household head and omit the female labor supply and family structure from the model. Therefore, parameters in our model will be calibrated for the husband's labor. The survival probability also taken from that of male.

[13] Some empirical researches report that, based on microeconomic data, labor-leisure choice of households is consistent with a separable utility function between consumption and leisure (See Browining et al., 1999). However, because we consider a growth economy, in order to maintain consistency of the macroeconomic data, we need to adopt the Cobb-Douglas utility function. As is well known, the aggregate labor supply has no trend even if the economy grows. If we adopt a separable utility function with consumption and leisure, the aggregate labor supply declines as the economy grows. The only exception is the separable type with log utitlity function on consumption. We explore the separability on our results in Section 6.

Each household faces idiosyncratic risks with regard to the labor skill. The labor skill realized at age j is denoted by $e_j \in E$. The labor skill e_j is assumed to follow the stochastic process of equation (3) and (4). The labor skill, which consists of the fixed effect the persistent shock and the transitory shock, represents the state of a household, $s_j = \{\alpha, z, \varepsilon\}$. On the other hand, because the average earning is observed to be hump-shaped across age groups, the average productivity of labor for each age is denoted as $\{\kappa_j\}_{j=20}^{65}$. Therefore, when a household supplies hours $h_{j,t}$, the earning before tax is $y_{j,t} = w_t \kappa_j e_j h_{j,t}$, where w_t is the wage rate of macroeconomy.

The government grants social security benefits through a constant payroll tax on labor earnings, and retired households receive the social security benefit. The payroll tax rate is τ_t^{ss} and the retired households receive a constant amount $\varphi_t w_t H_t$. As will be explained later, the population-adjusted average earning of workers is denoted by $w_t H_t$, and φ_t is a replacement rate. Thus, we assume that the all retired households receive a constant fraction of the working household's labor income. Since the households face the mortality risk, they may die with accidental bequests. For simplicity, we assume that the government levies 100% tax on the accidental bequest and distributes it among all households equally. The redistributed accidental bequest is denoted as b_t. We impose the capital income tax τ_t^{cap} on the interest rate.

The budget constraints of workers and retirees are as follows:

$$c_{j,t} + a_{j+1,t+1} = (1 + (1 - \tau_t^{\text{cap}}) r_t)(a_{j,t} + b_t) + (1 - \tau_t^{\text{ss}}) w_t \kappa_j e_j h_{j,t}, \quad : \text{Worker}$$
$$c_{j,t} + a_{j+1,t+1} = (1 + (1 - \tau_t^{\text{cap}}) r_t)(a_{j,t} + b_t) + \varphi_t w_t H_t, \quad : \text{Retiree}$$

where r_t is the interest rate at time t. We impose the liquidity constraint $a_{j,t} \geq 0$ on saving $a_{j,t}$.

3.3 Firm's Behavior and Factor Prices

The aggregate production function is of a Cobb-Douglas type function:

$$Y_t = A_t K_t^{\theta_t} H_t^{1-\theta_t}$$

where A_t is the time-varying total factor productivity (TFP) at time t, K_t is an aggregate capital, H_t is the aggregate labor which coincides with the average labor income of workers. θ_t is a capital share, which is also dependent on time t. We assume that the sequence of TFP is deterministic and predictable. Thus, there is no macroeconomic uncertainty in the model.

Because of the idiosyncratic income risk, households of the same cohort may have different wealth holdings and different labor supply depending on the realization of the income shocks. Denote the density function of age j in time t with asset holding a and the idiosyncratic income state s as $\Phi_t(a, s, j)$. The aggregate capital K_t and aggregate labor H_t are integral to all households as follows:

$$K_t = \sum_{j=20}^{100} \mu_{j,t} \int a_{j,t} d\Phi_t(a, s, j), \; H_t = \sum_{j=20}^{65} \mu_{j,t} \int \kappa_j e_j h_{j,t} d\Phi_t(a, s, j). \quad (5)$$

The aggregate capital depreciates at the rate δ_t. Then, the interest rate r_t and wage w_t of time t are determined as follows:

$$r_t = \theta_t A_t (K_t/H_t)^{\theta_t - 1} - \delta_t, \; w_t = (1 - \theta_t) A_t (K_t/H_t)^{\theta_t}. \quad (6)$$

3.4 Government's Budget Constraints

In our model, the government has three roles: (1) collecting payroll tax of social security and managing the social security system: (2) collecting capital income tax and using it for government expenditure: and (3) distributing accidental bequests. The payroll tax for social security is redistributed among retirees depending on the replacement rate φ_t. Thus, the government budget must satisfy following equation:

$$\sum_{j=20}^{65} \mu_{j,t} \int \tau_t^{\text{ss}} w_t \kappa_j e_j h_{j,t} d\Phi_t(a, s, j) = \sum_{j=66}^{100} \mu_{j,t} \varphi_t w_t H_t. \quad (7)$$

The government expenditure, which does not yield utility, is based on the capital income tax.[14] Thus, the government expenditure must satisfy the following condition:

$$G_t = \sum_{j=20}^{100} \mu_{j,t} \int \tau_t^{\text{cap}} r_t a_{j,t} d\Phi_t(a, s, j).$$

The accidental bequests are taxed away and redistributed among all households equally:

$$b_t = \sum_{j=20}^{100} \int (1 - s_{j,t}) a_{j,t} d\Phi_t(a, s, j). \quad (8)$$

[14] In the specification, increase in the capital income tax rate directly implies reduction of household utility. However, welfare comparison is out of range for our purpose. Thus, our specification does not cause any problem in our analysis. The capital income tax rate plays a crucial role on matching the saving rate of the model to the data.

3.5 Definition of a Competitive Equilibrium

Our concern in this paper is to identify the transition path between stationary states. The definition of the competitive equilibrium is as follows.

Definition: Recursive Competitive Equilibrium Given the paths of TFP $\{A_t\}$, the capital share $\{\theta_t\}$, the depreciation rate $\{\delta_t\}$, the capital income tax rate $\{\tau_t^{\text{cap}}\}$, the time endowment $\{\bar{h}_t\}$ and the replacement rate $\{\varphi_t\}$, the recursive competitive equilibrium is a set of the policy functions $\{g_t^c, g_t^h, g_t^a\}$, aggregate capital $\{K_t\}$, aggregate labor $\{H_t\}$, factor prices $\{r_t, w_r\}$, payroll tax rates $\{\tau_t^{\text{ss}}\}$ and the accidental bequest $\{b_t\}$ that satisfy following conditions:

- A Household's Optimality: Given the factor prices $\{r_t, w_r\}$ and the payroll tax rates $\{\tau_t^{\text{ss}}\}$, the household maximizes expected utility, and the functions $\{g_t^c, g_t^h, g_t^a\}$ are the associated policy functions.
- A Firm's Optimality: The factor prices are competitively determined by equation (6).
- Market Clearing: The market clearing conditions of equation (5) are satisfied.[15]
- The Government's Budget: The government budget constraint (7) is satisfied.
- Accidental Bequest: The accidental bequests are redistributed as in (8).
- Transition Law of Motion: The distribution function $\Phi_t(a, s, j)$ transits consistently with the policy function g_t^a.

Although our model is more realistic as compared to a simple two-period OLG model, its complexity makes it impossible to compute the model analytically. Therefore, we need to resort to numerical analysis. In this paper, following Conesa and Krueger (1999), we compute initial and final stationary states as a first step, and after that, we compute the transition path between them.

[15] According to Walras' law, a good market clears if the remaining capital and labor markets clear.

4 Calibration

4.1 Fundamental Parameters

The purpose of this paper is to investigate implications of macroeconomic factors, a demographic factor and income risk factors on economic inequality in Japan by replicating Japanese economy from 1980 to 2000. The reason we choose the target period as 1980–2000 is that there are comparable data on earning, income and consumption inequality only for this period. In order to replicate the Japanese economy between 1980 and 2000, we need to calibrate the model parameters for the Japanese economy. As a target, we choose the Japanese economy in 1980 because the targeted consumption inequality data, which is estimated from the National Survey of Family Income and Expenditure, starts from the 1979 survey.

Although we assume that households must retire at age 65, the actual mandatory retirement age in Japan in 1980 was around 60. However, because of the steep decline in average labor productivity over 60, many households voluntarily retire around age 60. The available time endowment for labor supply $\bar{h}_t$ is assumed that all households have 16 hours $\times$ 5.5 days $\times$ 4 weeks $\times$ 12 months in the early 1980s. Since the reduction in work hours was legally introduced in the late 1980s, Hayashi and Prescott (2002) show that the average work hours per week reduced from 44 hours to 40 hours. To replicate this fact, we assume that the time endowment reduces from 1988 to 1993, and after 1993, all households worked five days per week, i.e., 16 hours $\times$ 5 days $\times$ 4 weeks $\times$ 12 months. We set the share parameter for consumption and leisure at $\sigma = 0.55$, which implies that average work hours per year becomes $2000 - 2200$ hours in our model.

We set the intertemporal elasticity parameter γ at 2. In this calibration, the intertemporal elasticity of substitution becomes 0.5, and the relative risk aversion is $1-\sigma(1-\gamma) = 1.55$. These values are within the plausible range in the existing literature on business cycles. The discount factor β is chosen to be $\beta = 0.9871$ for matching the capital-output ratio of the model between 1980 and 2000 to the actual data.

Following Oshio and Yashiro (1997), the replacement rate of the social security in Japan is about 40% in 1990s. Thus, we set the replacement parameter φ_t to match this fact. Since we consider the government's budget clearing condition, endogenously determined payroll tax rate τ_t^{ss} is smaller than the actual data.

4.2 Income Risk and Age-Efficiency Profile

As stated in section 2, the idiosyncratic skill shock e_j follows equation (3) and (4). The income risk parameters $\{\sigma_\alpha^2, \sigma_{\eta,j}^2, \sigma_\varepsilon^2\}$, which are most important parameters in this paper, are calibrated to match the cross-section variance profiles of logarithms of consumption and income estimated by Abe and Yamada (2006).[16] Since the age profile of variance of log consumption is convex over age, Abe and Yamada (2006) report that they do not reject the possibility of $\rho \geq 1$. However, since it would be difficult to compute the model with $\rho \geq 1$, we assume that the autocorrelation coefficient is very close to one, and approximate it to the Markov chain. We choose $\rho = 0.98$ and $\sigma_{z_{20}} = 0.25$ and assume that the standard deviation of the persistent shock increases by 0.0025. After the specification of the AR(1) process, we approximate the process by the seven-state Markov chain from Tauchen's (1986) method. Standard deviation of the fixed effect σ_α and the transitory shock σ_ε are set at 0.25 and 0.07 respectively, and both are approximated by two-states as $\{e^{-\sigma}, e^{\sigma}\}$. For tractability of the model, we assume that the idiosyncratic income risks parameters are constant for all time. Although it seems strong assumption, Ohtake and Saito (1998) and Abe and Yamada (2006) report that the variance profiles of log income/consumption over life cycle do not change so much from 1984 to 1999. We will discuss whether this calibration really explains Japanese inequality in Section 5.2.

The average productivity profile over age $\{\kappa_j\}$ corresponds to the average hourly wage for each age in our model. Following Hansen (1993) and Braun et al. (2005), using the *Basic Survey in Wage Structure* by the Ministry of Health, Labor, and Welfare, we compute the average hourly wage (efficiency-unit wage) for all age classes. The average hourly wages for every five-year period are presented in Table 2. We compute $\{\kappa_j\}$ by smoothing the average hourly wage. In our model, the male head of the household supplies labor; thus, the average hourly wage is based on male's wage. Although the wage profile has flattened in recent times, it is very difficult to estimate the future wage profile. For simplicity, we use the average wage profile in the 1990s.

[16] For details, see Appendix Table 2 in Abe and Yamada (2006).

4.3 Demographic Structure

We choose the demographic parameters to replicate actual and projected population dynamics. The National Institute of Population and Social Security Research (NIPSSR) provides recent population projections from 2006 to 2055. We set the survival probability $\{s_{j,t}\}_{t=2006}^{2055}$ from the life table estimated using the NIPSSR's estimates in 2005, and $\{s_{j,t}\}_{t=2000}^{2005}$ are taken from the NIPSSR's estimates in 2002. The survival probabilities from 1980 to 1999 are extrapolated from the survival probability of the life table.[17] Since the population growth $\{\psi_t\}$ in our model is represented by the growth rate of 0-year old children, we use the ratio of the projected population of new born people between period t and $t+1$. The population projection displays *three* variants of estimation; low, medium and high variants for fertility and mortality rates.[18] Therefore, there are nine variations in all. We use medium variants of both the fertility and mortality rates. The calibrated population growth rates between 1980 and 2005 are that of the realized values.

The population distribution in the model moves from 1980 to 2055 following the projected and the realized values, and the population growth rate is assumed to converge to zero after the transition period. However, the convergence rate is slow and it takes approximately 100 years to reach a new stationary population distribution. Thus, we choose the final stationary state in 2200. Following Braun et al. (2007), we assume that the population growth rate converges to zero, $\psi_t = 0$, between 2056 and 2065.

One problem that arises here is how to choose an initial population distribution in the initial stationary state. The actual population distribution in 1980 does not appear to be stationary because of the existence of the baby-boomer generations. However, a population distribution is required for computing the initial stationary state. Therefore, we assume that the households in our model believe that the actual population in 1980 is stationary.

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[17] Another way of obtaining the survival probability of realized years is to calculate them from population distribution of recent year. However, since the new population distribution includes immigrants, the population size of age $j+1$ at period $t+1$ may be larger than that of age j at period t. This contradicts the model's assumption. Thus, we use the life table.

[18] For details, see the following website: http://www.ipss.go.jp/syoushika/tohkei/Popular/Popular2008.asp?chap=0.

4.4 Macroeconomic Factors

We need to determine exogenously given macroeconomic variable paths. For the purpose of comparison, the paths of all macroeconomic factors are taken from Hayashi and Prescott (2002). The TFP factor growth rates $A_{t+1}^{1/(1-\theta_{t+1})}/A_t^{1/(1-\theta_t)} = 1 + g_t$ between 1980 are 2000 are estimated by Hayashi and Prescott (2002).[19] Although the TFP factor growth after 2000 could be estimated by macro data, we unfortunately have no data on economic inequality after 2000 in Japan. Therefore, we simply assume that the TFP factor growth rate converges after 2000. The converged TFP factor growth rate is the average TFP growth rate of 1960-2000, 2%, following Braun et al. (2007). The TFP level A_{1980} is normalized to make the equilibrium wage w_{1980} equal to one. The capital share $\{\theta_t\}$, the depreciation rate $\{\delta_t\}$ and the capital income tax $\{\tau_t^{\text{cap}}\}$ are also taken from estimated data of Hayashi and Prescott (2002).[20] These values are assumed to converge to their average in the 1990s for ten years after 2000.[21] All calibrated parameters are summarized in Table 3.

5 Quantitative Results

5.1 Macroeconomic Variables: First Moments

Before discussing the economic inequality after 1980 including the lost decade, we confirm whether the average paths such as interest rate and saving rate in the model replicates Japanese economy. Panel (a) of Figure 1 plots the after tax interest rate in the model and the data. Since the capital income tax rate taken from Hayashi and Prescott (2002)

[19]There is a skeptism about low TFP growth in the 1990s. Kawamoto (2004) points out that the estimation by Hayashi and Prescott (2002) does not necessarily imply the pure effect of *productivity.* Moreover, Chari, Kehoe, and McGrattan (2007) demonstrate that a liquidity constraint on investment, for example, may lead to the decline of the TFP, which apparently does not reflect productivity. Therefore, it may not be correct to consider time-varying TFP as the macroeconomic productivity shock. However, for our purposes, it does not matter because the TFP movement in our model may cause inequality whether or not is actually a productivity shock. We will consider the interpretation of the TFP shock on economic inequality in the next research. Our calibration targets such as interest rate are taken from the estimation data of Hayashi and Prescott (2002). Thus, for consistency, we use all the macroeconomic variables from Hayashi and Prescott (2002).

[20]Hayashi and Prescott (2002) define the capital income tax as direct tax on corporate income, 50% of indirect business tax, and 8% of operating surplus.

[21]We confirm that small change in the converged points does not change our results.

is over 40%, the before-tax interest rate is over 10% in the late 1980s. Compared with the data, the endogenously determined interest rate closely replicates the data, although the model slightly exceeds the data in late 1980s and is lower in the mid 1990s. Panel (b) of Figure 1 plots the capital-output ratio K/Y. The model replicates the flat capital-output ratio in 1980s, and capital deepening process in the lost decade.

In addition to the time variation of the TFP growth rate, Hayashi and Prescott (2002) emphasis on the effect of reduction in work hours in the late 1980s for explaining the macroeconomic dynamics in Japan. Panel (c) of Figure 1 represents the yearly work hours. Because the level of work hours in the model is slightly higher than the data, we normalize the wage of year 1980 to be one. Since the available time endowment $\bar{h}_t$ are assumed to reduce from 1988 to 1993, the endogenously determined work hours decreases in this period. From Hayashi and Prescott (2002), since the work hours per week were reduced from 44 hours/week to 40 hours/week, the reduction amounts to about 9.1%. In our model, since there is a corresponding reduction of about 10.4%, our model closely replicates the reduction of work hours. In addition, the work hours of the model after 1990 also trace the data. However, because we assume perfect foresight, the work hours in the model increase between 1986 and 1988. From intertemporal substitution purpose, households increase labor supply in the period since they predict the future reduction in time endowment. Thus, for the period of before-time-reduction, the time path of the model deviates from the data.

Lastly, we focus on the saving rate of the data and the model. Chen et al. (2006,2007) and Braun et al. (2007) reveal that time variation of TFP growth and population dynamics are crucial factors for explaining the Japanese saving rate after WW II. Compared to Braun et al. (2007), we extend the model by including idiosyncratic income risk.[22] We plot time series of macro saving rate in Panel (d) of Figure 1. The model appears to explain the saving rate before the mid 1990s very clearly. One discrepancy between the data and model is the period around 1995. In conclusion, using the overlapping generations model with idiosyncratic income risk and exogenously given macroeconomic factors, the dynamic general equilibrium model can explain the data very clearly.

[22] On the contrary, Braun et al. (2007) considers family structure in their analysis.

5.2 Income and Consumption Inequality over Life Cycle

Figures 2 plot the variance of logarithm of income and consumption over age given a calibrated parameter set of idiosyncratic income risks $\{\rho, \sigma_\alpha^2, \sigma_{\eta,j}^2, \sigma_\varepsilon^2\}$. As a target of the calibration, we also plot the variance of log income and consumption in 1984 and 1999 provided by Abe and Yamada (2006). From Panel (a) of Figure 2, we confirm that our numerical model closely replicates the income inequality.

As Ohtake and Saito (1998) and Abe and Yamada (2006) discuss, the variance of log profiles do not differ considerably from the 1980s.[23] From Panel (a) of Figure 2, it is apparent that the variance of log income increases by age, and the slope also increases after 50. In our model, except for the sharp rise after age 55, the income inequality is clearly explained. On the contrary, from Panel (a) of Figure 2, although the model's consumption inequality around age 25–40 closely matches the data, the consumption inequality of old age is too small in the model. Thus, the idiosyncratic income risks in the model are effectively shared by saving, which implies that the consumption inequality becomes slightly lower. Although it is difficult to explain the consumption inequality of life cycle aspect perfectly, the overall level of the consumption variance and slopes of the profiles in the model are similar to that of the estimates.

5.3 Evolution through Time of Cross Section Inequality of Wage and Income

We plot the earning and income inequality between 1980 and 2004 in Panel (a) and (b) of Figure 3. For computing the economic inequality, as in the previous section, we use the variance of logarithm of variables. Because the variance of log is used in the previous literature, we adopt the same criteria for the purpose of comparison. Moreover, Blundell and Preston (1998) reveal that the variance of log consumption can be used as a criterion on evaluating welfare using a simple model. In Figure 3(a)–3(c), we also plot the evolution of the variance of log income and consumption estimated from Ohtake and Saito (1998) and Kohara and Ohtake (2006). It is difficult to exactly match the *level* of these variances because of the definitions of consumption and income.[24] Therefore, we use the time paths of the deviation from mean.

[23] Recently, it is pointed out that the economic inequality of youth generation disperses.

[24] For example, family structure, female labor supply, and difference between durables and nondurables are omitted from the model. Moreover, we do not consider progressive income tax. Thus, it is impossible

Ohtake and Saito (1998) estimate the variance of log income and consumption based on the National Survey of Family Income and Expenditure (NSFIE) of 1979, 1984, and 1989. They use samples of households with two or more members, engaged in non-agricultural sectors, and heads aged 22–75. Each sample contains over 40,000 households. As is well known, the income inequality in Japan has risen in recent decades.[25] Using the same data from the NSFIE of 1984, 1989, 1994, and 1999, Kohara and Ohtake (2006) also estimate the (before and after tax) income and consumption inequality. Since our model does not include the progressive income tax code due to its complexity, we use the before-tax income inequality estimated by Kohara and Ohtake (2006). The targeted data is plotted in Panel (a) and (c) of Figures 3. The definition of income used by Kohara and Ohtake (2006) includes, in addition to the earnings, interest and dividend income.[26] In order to compare our model with Kohara and Ohtake's (2006) result, the definition of *income* in our model adds interest income $(1-\tau_t^{\mathrm{cap}})r_t a_{j,t}$ to the labor earning $y_{j,t}$ (Figure 3(b)). The *earning* inequality in Figure 3(a) is the variance of logarithm of the labor earning $y_{j,t}$.

Based on two previous researches, we know that the variance of log income has positive trend since year 1980. Unfortunately, we have only four points of observation. Based on the observation, the slope of the data and model are close to each other. From Panel (b) of Figure 3, we find that the simulation data between 1980 and 1998 shows a weak positive trend; thus, the evolution of income inequality matches the data in this period. Although both the model and the data show positive trend from the 1980s to 1990s, the inequality in data rises sharply compared to the model. The actual data estimated by Kohara and Ohtake (2006) indicates that the income inequality from 1989 to 1999 increases rapidly as compared to the model as in Figure 3(b). The income inequality in the model declines in the late 1990s. In this period, the corresponding saving rate in the model is higher than the data. In Japan in the late 1990s, many

to exactly compare the data and model, because of the differences in definition. In the model, the average of variance of log income is about 0.38 and the corresponding data is 0.32. The average of the variance of log consumption in the data is about 0.25, whereas the corresponding value in the model is merely 0.20.

[25]Ohtake and Saito (1998) empirically show that a large fraction of rise in income inequality is due to population ageing.

[26]Since the interest and dividend income in the NSFIE in the sample is too small, Kohara and Ohtake (2006) compute the capital income from asset holdings (bank deposit) and the average interest rate. Thus, the definition of income in their estimation is close to that in our model.

households suffered due to the serious recession and decumulated their wealth. As a result, the income inequality in the data may show a dispersion in this period.

Next, although the definition used by Kohara and Ohtake (2006) is not consistent with our model, we will focus on the transition of the earning inequality. As is clear from Panel (b) of Figure 3, the time variation of the earning inequality in the model is larger than that of income inequality. The model fails to explain the rise of inequality in the early 1980s. One possible reason for the failure of matching wage inequality is that the households can predict the reduction of the time endowment in the model. Thus, households prepared for the reduction by increasing labor supply in the early 1980s. This implies a concentration of high labor supply and small wage inequality. On the other hand, from 1984 to 1999, the evolution of earning inequality closely matches to data compared with that of the income inequality. Because the interest rate in this period is near zero due to the zero interest policy adopted by Bank of Japan, the income inequality estimated by Kohara and Ohtake (2006) may be close to the earning inequality. If so, our model explains the transition of earning inequality in Japan in the 1990s.

There still remain unexplained components in our model. Based on the construction of our model, we guess that the remaining components of inequality may be explained by three factors; the fixed effect, persistent shock, and transitory shock.[27] In other words, we guess that the size of the idiosyncratic income shocks $\{\xi_t^f, \xi_t^p, \xi_t^t\}$ changed in this period. As stated in Section 2, without investigating consumption inequality, it is difficult to analyze what component affects inequality .

5.4 Transition of Consumption Inequality

The transition path of consumption inequality is represented in Panel (c) of Figure 3, which depicts the data obtained by Ohtake and Saito (1998), Kohara and Ohtake (2006)

[27]The income inequality in the model is also affected by wealth inequality and the interest rate. Although we omit these figures, we compute the wealth inequality in the model and the Gini coefficient is about 0.65. Thus, the wealth inequality in the model is close to that in Japanese economy. As stated above, since the Bank of Japan adopted a zero interest policy, the actual capital income of households in the late 1990s may be much lower than that of the model. This difference may be the reason why the model cannot explain the income inequality in the late 1990s. However, it is difficult to separately define the macroeconomic interest rate and the interest rate of bank deposit.

and the simulated model.[28,29] This figure also plots the deviation from mean. According Ohtake and Saito (1998) and Kohara and Ohtake (2006), although the consumption inequality also has a positive trend, the path is flatter than that of income inequality. In our simulation, excluding the rise between the late 1980s and the early 1990s, the consumption inequality does not appear to have a positive trend. This seems to be surprising because the population aging and the macroeconomic variables changed in this period, and as a result, the earning inequality fluctuated. This implies that households in the model perform consumption smoothing very effectively and the idiosyncratic shocks are shared through saving, which is also observed in Figures 2(a) and 2(b). From Figure 2(b), the age-profile of log consumption variance in the model is flatter than the data. As a result, the effect of population aging may be underestimated. We will discuss this point later. Although the income inequality from the 1980s to mid 1990s rises in the model, it does not lead to consumption inequality. Thus, the inequality in the 1980s is not clearly explained by the model. On the other hand, the model performs well for the period between 1994 and 1999.

From the above, we can safely say that the income and eaning inequality in Japan is clearly explained by the dynamic general equilibrium model. Compared with these two inequalities, it is difficult to match the consumption inequality of the data and model. Since the consumption inequality has a direct implication on economic welfare, we need to consider the omitted factors on consumption inequality. One possible explanation is that the size parameter of each shock $\{\xi_t^f, \xi_t^p, \xi_t^t\}$ has changed through time and the uninsurable component of the idiosyncratic income risk has become larger.

5.5 On the Effect of Macroeconomic Factors: A Counterfactual Simulation

In Section 5.1, we showed that the first moments of the model, i.e. aggregate variables, move close to the data, but the second moments of the model, i.e., the economic inequal-

[28] As noted above, it is very difficult to define consumption precisely because we need to consider the family structure, durable goods, housing, etc. For details on the definition of consumption, see Ohtake and Saito (1998) and Kohara and Ohtake (2006).

[29] We define the consumption inequality in the model for households aged between 20 and 80. Since the NSFIE surveys are based on household heads, there are very few samples where the household head is aged over 80. We confirm that including households aged over 80 in the model does not change our results.

ity, is not so close to the data. Since our model is complex, it is difficult to analyze the model theoretically. Instead, using counterfactual simulations, we consider the link between the income and consumption inequalities if the macroeconomic factors such as the TFP factor growth rate, the capital share, the capital income tax and time endowment are constant over time.

In Figures 4 and 5, we depict the evolution of the earning, income, and consumption inequalities if the TFP factor growth rates are constant at 2%, or each exogenously give path $\{\theta_t, \tau_t^{\text{cap}}, \bar{h}_t\}$ is constant at the average of the 1990s.[30] In keeping with Hayashi and Prescott (2002), the TFP growth rate and the reduction in work hours have large impacts on the path (Figures 4(a)–(c)). From Panel (a) of Figure 4, we find that the rise in the earning inequality between 1988 and 1993 is small and the path over the entire period is flat, if the time endowment is constant over time. Thus, a reason why the model does not match the data in the 1980s is the model specification of the reduction in work hours. If the TFP growth rate is constant, except for the early 1980s, the earning inequality becomes high compared to the benchmark case. The income inequality is not significantly affected by the macroeconomic variables from Panel (b) of Figure 4, although the constant time endowment case flattens the income inequality path. A surprising result is that the capital share has a large effect on the economic inequality (Figures 5(a) and 5(b)). The capital share raises the earning inequality in the early 1980s and sharply rises in the late 1980s. On the contrary, the capital income tax has little effect on the economic inequality. This is because the capital income rate has a small effect on total income.

We compare the consumption inequality in Panel (c) of Figure 4 and 5. The reduction in work hours has a large impact on consumption inequality. The sharp rise of consumption inequality between the late 1980s and the early 1990s is due to the reduction in work hours. Reduction of the time endowment implies that there is less room for smoothing marginal utility by changing the leisure. It leads to the consumption inequality. If the TFP growth rate is constant at 2%, the consumption inequality in the period of so-called lost decade is higher than that of the benchmark case. Thus, the low TFP growth rate in the lost decade implies low earning and consumption inequality. Time variation of the capital share lowered the consumption inequality in the 1980s and

[30]The total factor productivity is re-estimated if the capital share is time-varying. We also examine the constant depreciation rate path, but its effect is very small. Thus, we omit it in Figures 4 and 5.

increased consumption inequality around 1990 (Figure 5(c)).

5.6 Demographics vs Macroeconomic Variables

Next, we discuss the effects of the demographic factor and the macroeconomic factors on the economic inequality. The label denoted as “Population” in Figures 6 is the counterfactual simulation with constant population distribution, but the macroeconomic variables are exogenously given calibrated parameters, and the label denoted as “Macro” is the simulated results in which the macroeconomic variables are constant over time but the population aging.

If there is no population aging, the income inequality flattens over the period; the income inequality slightly declines in the 1980s. The earning inequality becomes more U-shaped, i.e., it rises in the early 1980s and declines in the 1990s. This implies that the sharp rise in the income and earning inequality in the 1990s is partially due to the population aging effect. However, even without population aging, there remains positive trend of the eaning inequality. On the contrary, if all the macroeconomic factors are constant over time, the earning inequality peaks at mid 1990s and declined in the late 1990s. In conclusion, both the aging and macroeconomic factors makes positive trend of earning inequality. Consumption inequality in Panel (c) of Figure 6 also reveals the importance of the macroeconomic factors. Without macroeconomic fluctuations, the time path of consumption inequality becomes hump-shaped at the peak of mid 1990s. The time path becomes very smooth and declines after the peak. On the other hand, the consumption inequality sharply rises after 1990 in the simulation labeled “Population”. Thus, the positive trend of consumption inequality is also explained by the aging and the macroeconomic factors.

6 Sensitivity Analysis

6.1 Separability, the Frisch Elasticity, and the Intertemporal Elasticity of Substitution

Our specification of utility function is quite standard in the macroeconomics literature as we used of Cobb=Douglas form. However, because we employ time varying time endowment, non-separability of the marginal utility of consumption and leisure may cause problems for considering consumption inequality. Note that because we consider growth economy, the utility function needs to be consistent with no aggregate labor trend. For investigating the robustness of our findings, we consider the separable utility function. If the parameter γ is equal to 1, the utility function becomes as follows:

$$u(c_{j,t}, \bar{h}_t - h_{j,t}) = \sigma \log c_{j,t} + (1-\sigma) \log(\bar{h}_t - h_{j,t}).$$

In this form, due to its separability, the leisure has no effect on the marginal utility of consumption.[31] The Frisch elasticity changes through the endogenous labor supply and the time endowment. Although the Frisch elasticity is a significant parameter for reseach on business cycle resarch or economic inequality, estimates on the elasticity is in the wide range between 0 and 1.0.[32] In addition to the log-log case, we consider the case that $\gamma = 4$.[33]

In Figure 7, we plot the simulated earning, income and consumption inequality with the log-log utility and $\gamma = 4$. If the temporary utility function is of log type, the earning, income and consumption inequality profiles become more steep. Thus, compared to the benchmark case in Figure 3, explanatory power of our model may improves, especially in 1990s. In the benchmark case, rise of the consumption inequality in data cannot be well explained by the model. However, in the log utility case, the sharp rise of the consumption inequality in 1990s is close to actual data. On the contrary, high γ implies less positive trend of inequality. Because the changes in gamma is consistently moves in all cases, the separability of the utility function does not have significant effect in our analysis.

6.2 On the Effect of Social Security

Lastly, we confirm whether our specification of the social security form have significant effect for considering the time path of economic inequality. In Figure 8, we consider half reduction of the replacement rate compared to the benchmark case, i.e., $\varphi = 0.2$. Our specification of the social security form have redistribution and insurance effect,

[31] For more general separable utility forms, see King et al. (1998) and Basu and Kimball (2002). Employing structural estimation, French (2005) estimates the preference parameters.

[32] See Browning et al. (1999). For estimates of the elasticity in a model with borrowing constraints, see Domeij and Flodén (2006).

[33] Based on the separable utility function, İmrohoroğlu and Kitao (2008) show that teh IES has small impacts on social security reform.

thus it may lead to smaller economic inequality. Panel (a) of Figure 8 show that the eaning inequality does not change even if the replacement rate is small. Therefore, the social security reform does not change the distribution of earnings. On the contrary, time path of the consumption decreases in 1980s and increases in the mid 1990s and after 2000. This is interesting because recently it is widely recognized that rapid aging worsen sustainability and credibility of the social security systemi in Japan. If households predict that the social security payment they can receive become small, the consumption inequality disperse although the corresponding earning inequality is almost constant over time.

7 Concluding Remarks

In this paper, we investigated the evolution of the economic inequality between 1980 and 2000 in Japan using a dynamic general equilibrium model. For analyzing the inequality, we consider quantitative impacts of the three factors: (a) macroeconomic factors, (b) a demographic factor, and (c) idiosyncratic income risk factors. Our model explains time variation of the macroeconomic variables such as the interest rate and the saving rate; thus, an OLG model with heterogeneous households would be suitable as a benchmark for understanding the transition in the Japanese economy. In this respect, our result is considered to be an extension of the result obtained by Hayashi and Prescott (2002), which explains causes of the lost decade through a standard neoclassical growth model. Moreover, our model has implications on the earning and income inequality.

Using the counterfactual simulation, we analyzed the reason why the economic inequality in Japan has risen. We found that the time variation of the TFP growth and the reduction in work hours have implications not only on the macroeconomic variables but also on economic inequality. In particular, lower TFP growth rate in the 1990s leads to low economic inequality. Moreover, we revealed that even without the demographic factor and the constant idiosyncratic income risk factors, the earning and income inequality dispersed because of the macroeconomic factors. Based on our analysis, the preference change and the distrust of the social security system in Japan also contribute deterioration of consumption inequality.

There still remain unexplained components on the positive trend of the consumption inequality. In fact, there are some unresolved questions on income risk and consumption inequality in Japan. Ohtake and Saito (1998) and Abe and Yamada (2006) show that the life cycle profiles of the income inequalities have not changed through time. This implies that properties of the idiosyncratic income risk have not changed considerably since 1979. On the other hand, corresponding income and consumption inequalities of macroeconomy show a positive trend. Ohtake and Saito (1998) empirically showed that population aging best explain such a trend, because old households are more unequal as compared to young households, since population aging implies that the fraction of highly unequal households has increased. Thus, the macroeconomic inequality rises. This point is consistent with our quantitative results. However, there still remains an unexplained aspect. This unexplained component should be regarded as the changes brought about by the fixed, persistent, and transitory effects. In addition, we do not consider the reason why the earnings risk and inequality rises. For welfare analysis, we need to consider the causes of rising wage inequality and consumption inequality, for example, skill-biased technological change or human capital accumulation may explain the causes.

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	$70s$	$80s$	$90s$
TFP Factor Growth Rate (%)	1.40	2.68	0.57
Adjusted TFP Factor Growth Rate (%)	1.77	2.21	0.67
Depreciation Rate (%)	10.19	8.97	8.40
Capital Share	0.3512	0.3536	0.3627
Capital Income Tax Rate (%)	41.55	47.68	44.92

Table 1: Macroeconomic Variables in Japan

Age	Hourly Wage	Age	Hourly Wage
20-24	$1,349$	45-49	$3,075$
25-29	$1,777$	50-54	$3,145$
30-34	$2,187$	55-59	$2,797$
35-39	$2,548$	60-64	$1,923$
40-44	$2,842$	65-	$1,617$

Table 2: Average Hourly Wage for Each Age Group; Yen

Stationary State in Year 1980		
	Value	Target and Previous Research
β	0.9871	$K/Y \approx 1.74$
γ	2	Frisch Elasticity ≈ 0.67
σ	0.55	Average Hours Worked $\approx$ 2,200h
φ	0.4	Ohio and Yashiro (1997)
$\bar{h}_{1980}$	$4,224$	16h $\times$ 5.5days $\times$ 4weeks $\times$ 12month
θ_{1980}	0.3452	Hayashi and Prescott (2002)
δ_{1980}	0.0949	Hayashi and Prescott (2002)
τ^{cap}_{1980}	0.4636	Hayashi and Prescott (2002)
Stationary State in Year 2200		
β	0.9871	Constant over time
γ	2	Frisch Elasticity ≈ 0.63
σ	0.55	Average Hours Worked $\approx$ 2,000h
φ	0.4	Ohio and Yashiro (1997)
$\bar{h}_{2200}$	$3,840$	16h $\times$ 5days $\times$ 4weeks $\times$ 12month
θ_{2200}	0.362	Average of 1990-2000, Hayashi and Prescott (2002)
δ_{2200}	0.083	Average of 1990-2000, Hayashi and Prescott (2002)
τ^{cap}_{2200}	0.450	Average of 1990-2000, Hayashi and Prescott (2002)

Table 3: Calibrated Parameters in 1980 and 2200

Year	TFP Growth (g_t)	Adjusted TFP Growth (g_t)	Capital Share (θ_t)	Depreciation Rate (δ_t)	Capital Income Tax (τ_t^{cap})
1980	1.008	1.013	0.345	0.095	46.4%
1981	1.021	1.023	0.342	0.090	47.4%
1982	1.002	1.003	0.340	0.090	47.9%
1983	1.044	1.033	0.340	0.089	47.4%
1984	1.046	1.035	0.346	0.092	48.5%
1985	1.021	1.012	0.354	0.091	48.9%
1986	1.036	1.030	0.360	0.088	45.4%
1987	1.050	1.039	0.364	0.088	47.7%
1988	1.044	1.040	0.371	0.087	47.6%
1989	1.035	1.037	0.374	0.088	49.7%
1990	1.023	1.026	0.372	0.087	47.2%
1991	0.998	1.008	0.370	0.087	45.7%
1992	1.005	1.006	0.363	0.087	46.7%
1993	0.993	1.016	0.363	0.085	44.0%
1994	1.002	0.997	0.351	0.084	45.2%
1995	1.054	1.032	0.354	0.088	48.2%
1996	1.010	1.009	0.365	0.082	42.5%
1997	0.955	0.967	0.366	0.082	42.0%
1998	1.018	1.009	0.360	0.079	43.6%
1999	0.992	1.005	0.364	0.079	44.1%
2000	–	–	0.358	0.077	45.9%

Table 4: Exogenously Given Macroeonomic Variables

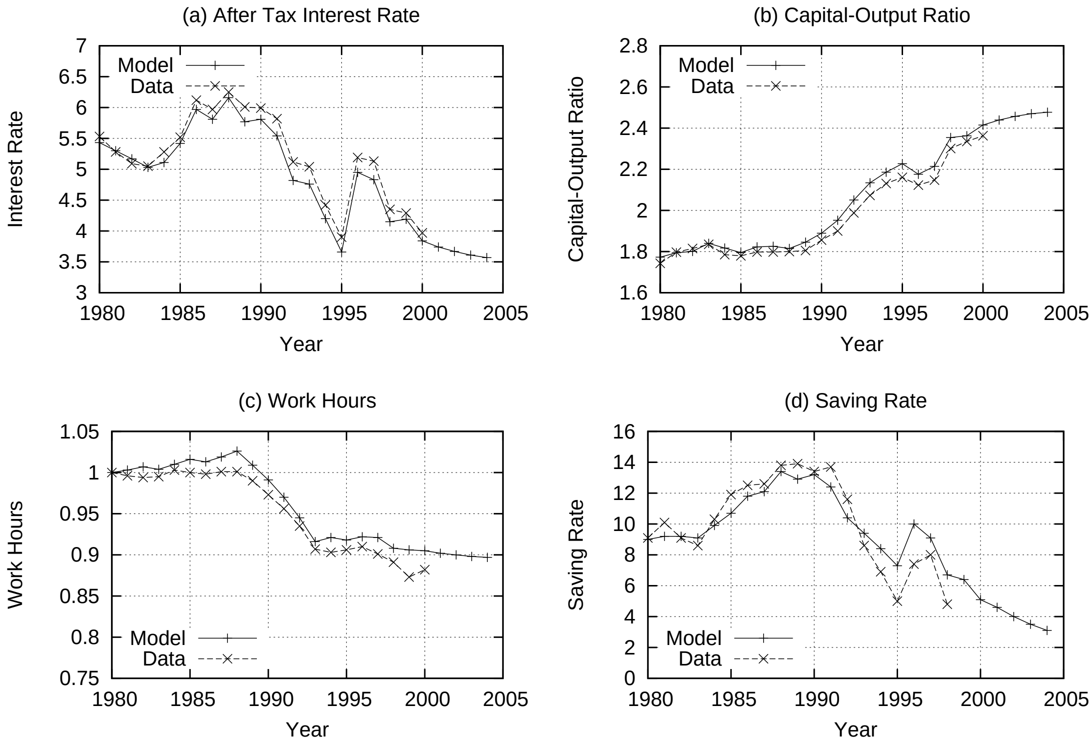

Figure 1: Paths of Macroeconomic Variables

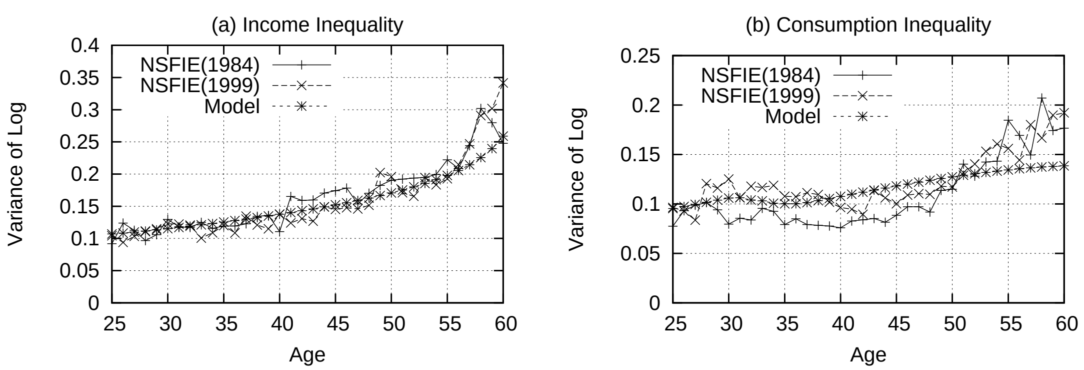

Figure 2: Cross Sectional Variance Profiles

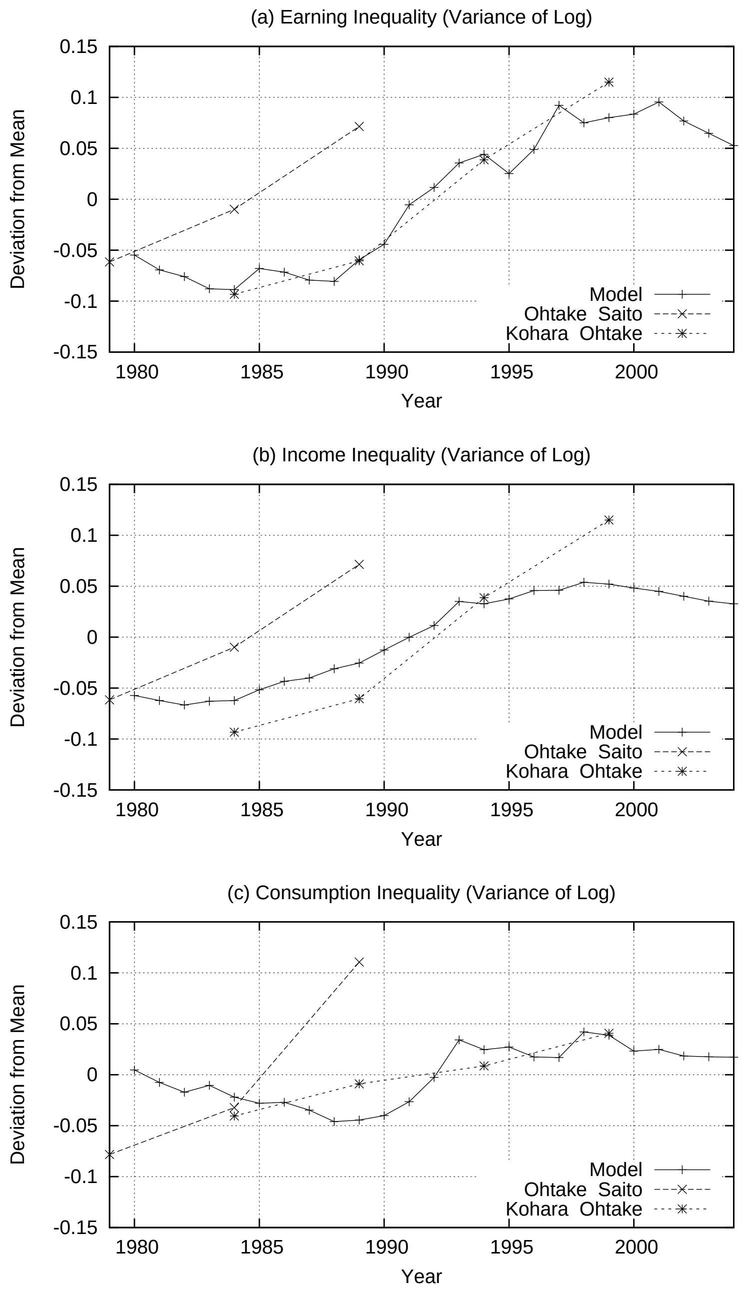

Figure 3: Inequality in Earning, Income and Consumption: Model versus Data

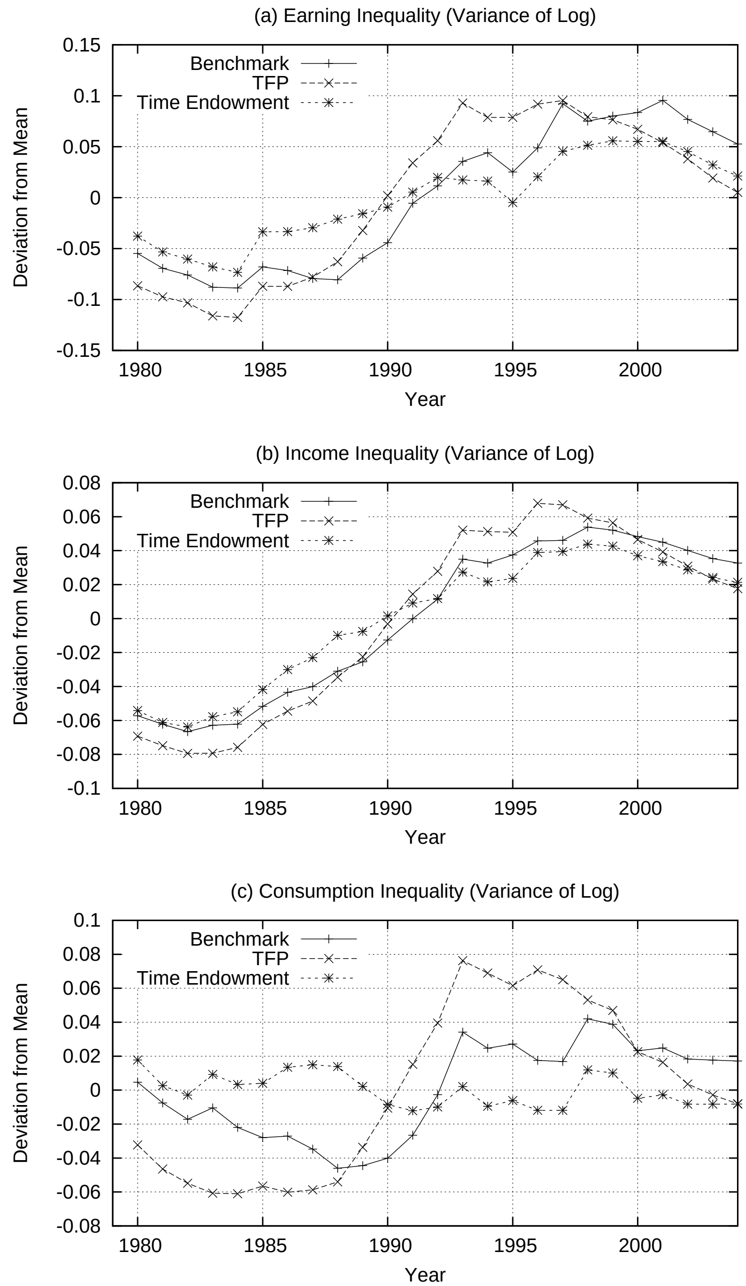

Figure 4: Counterfactual Simulation: Macroeconomic Effect (1)

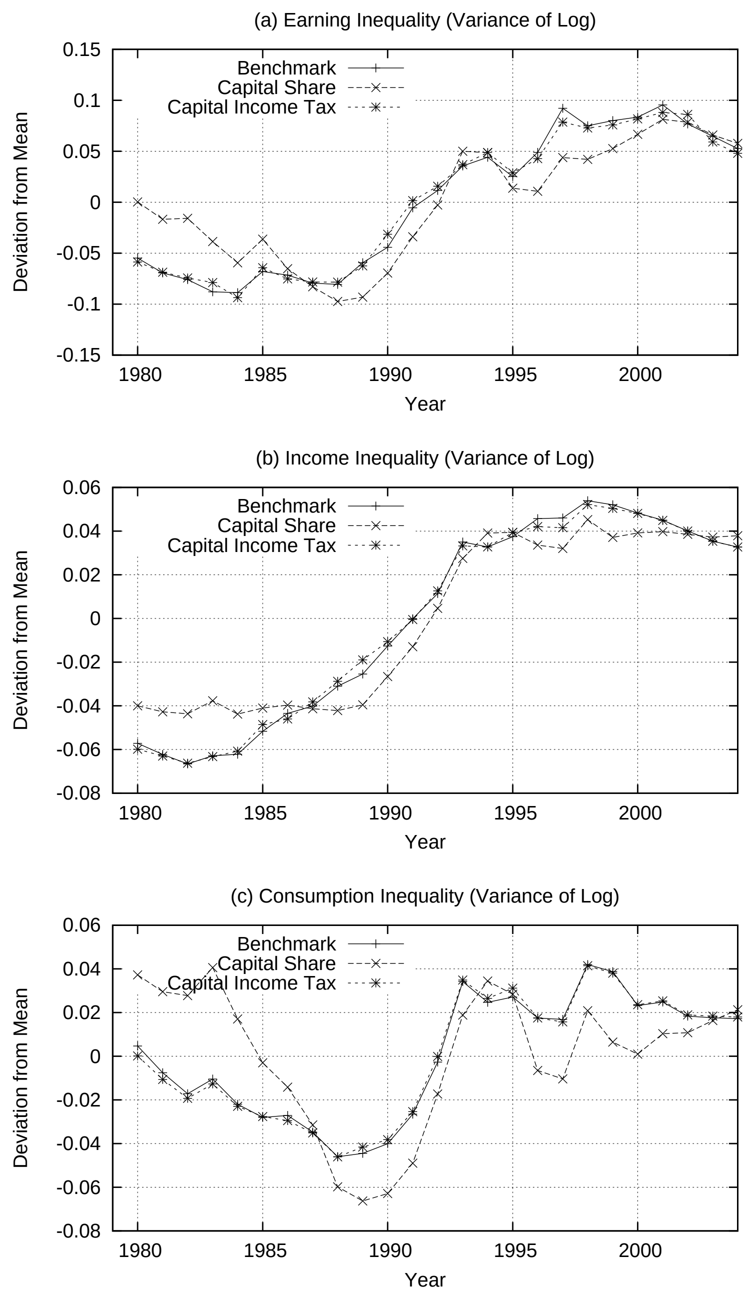

Figure 5: Counterfactual Simulation: Macroeconomic Effect (2)

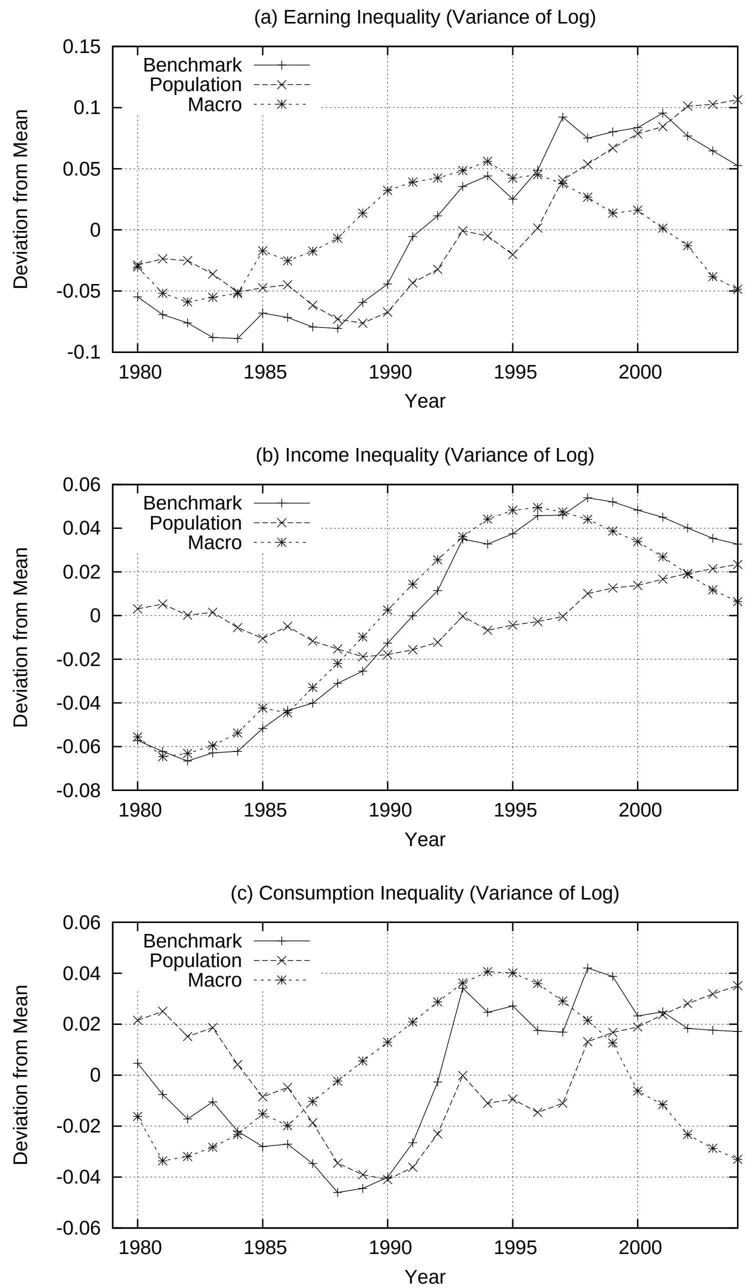

Figure 6: Counterfactual Simulation: Demographics vs Macroeconomic Variables

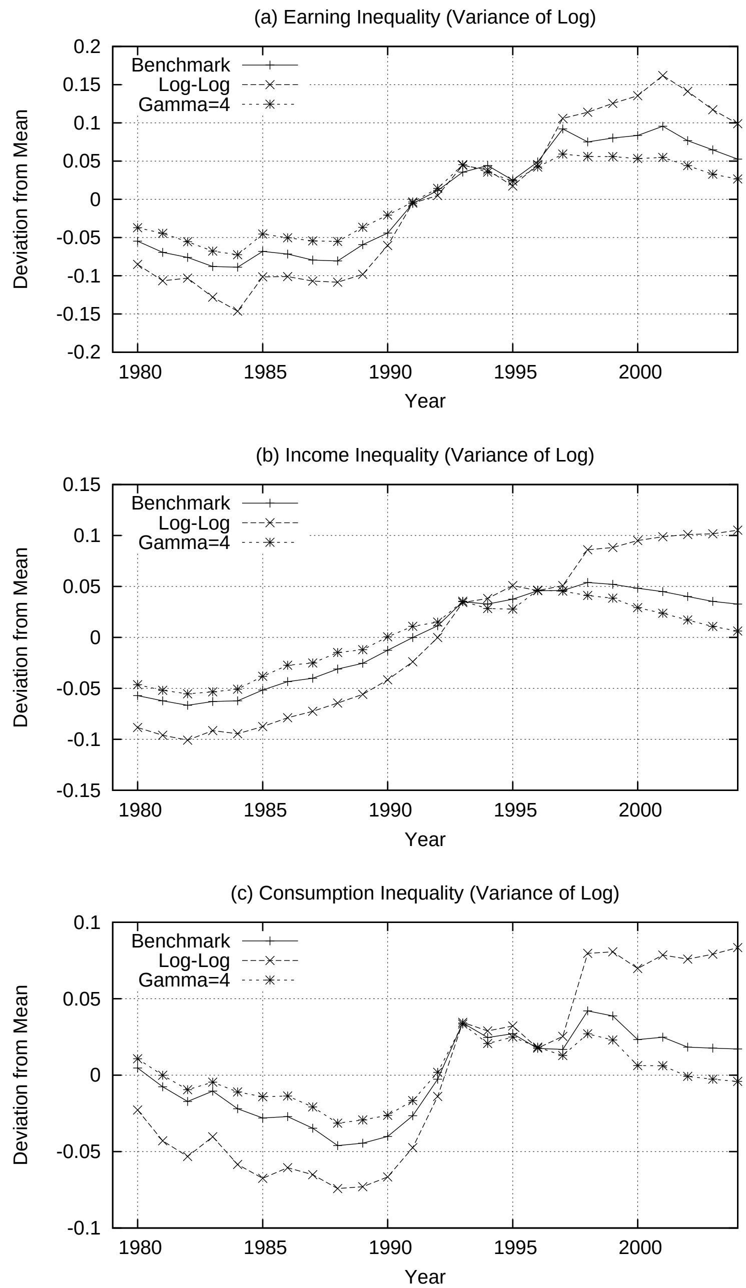

Figure 7: Robustness Analysis: Utility Function

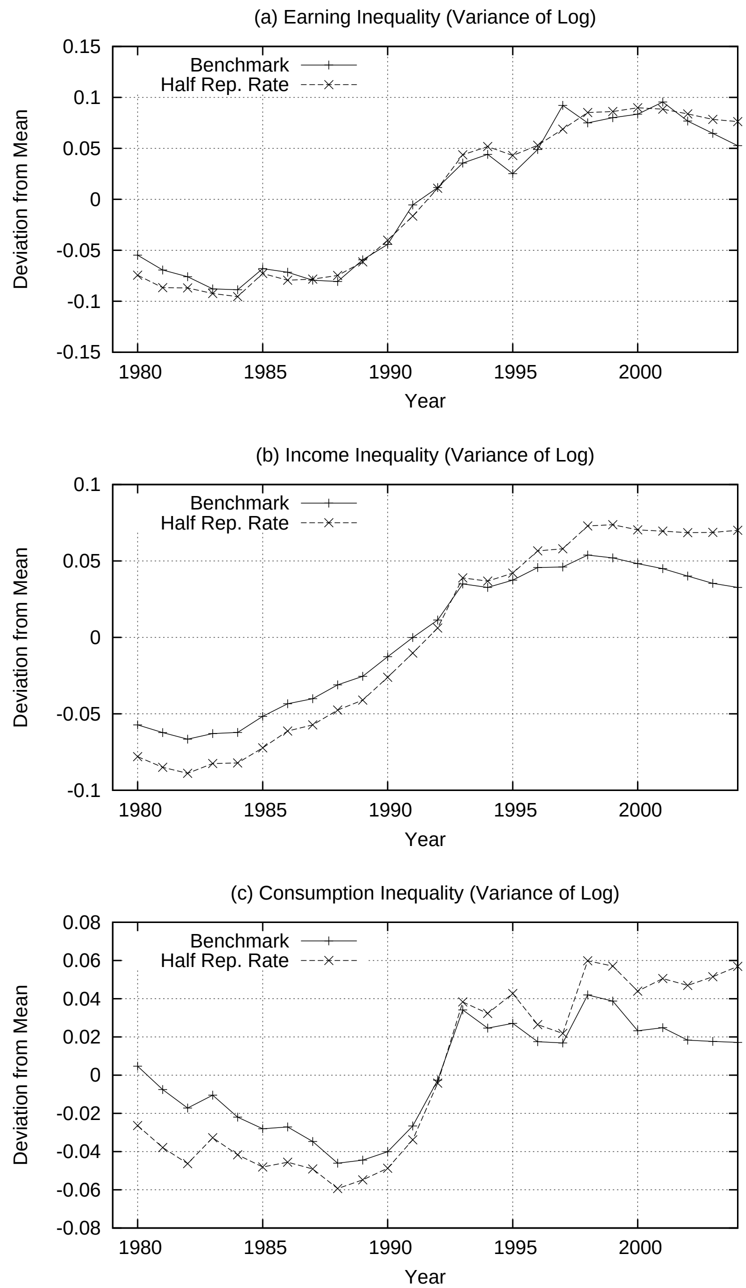

Figure 8: Robustness Analysis: Replacement Rate

A Details of the Model [Not For Publication]

A.1 Macroeconomic Variables

Because we consider an economy with time-varying TFP and population dynamics, to numerically solve the equilibrium, we need to remove the trend. Define the TFP factor growth rate and population growth rate by

$$\frac{A_{t+1}^{1/(1-\theta_{t+1})}}{A_t^{1/(1-\theta_t)}} \equiv 1+g_t,\ \frac{N_{t+1}}{N_t} \equiv 1+n_t.$$

Note that the adjustment parameter is not the TFP level A_t but the TFP factor $A_t^{1/(1-\theta_t)}$. We divide all macroeconomic variables by $A_t^{1/(1-\theta_t)}N_t$ for detrending excluding the aggregate labor supply. After the normalization, the macroeconomic variables are re-defined as follows:

$$\tilde{Y}_t = Y_t/(A_t^{1/(1-\theta_t)}N_t),\ \tilde{K}_t = K_t/(A_t^{1/(1-\theta_t)}N_t),\ \tilde{H}_t = H_t/N_t.$$

The factor prices are

$$r_t = \theta_t\frac{\tilde{Y}_t}{\tilde{K}_t} - \delta_t,\ \tilde{w}_t = \frac{w_t}{A_t^{1/(1-\theta_t)}}.$$

Thus, the interest rate has no trend on the productivity and population growth, but the wage grows with the productivity. The saving rate is

$$\frac{Y_t - C_t - G_t - \delta_t K_t}{Y_t - \delta_t K_t},$$

where G_t is a government expenditure financed by the capital income tax.

A.2 Normalization of Household Problem

A household's optimization problem defined in this paper is as follows:

$$V_{j,t}(a,s) = \max\left\{u(c_{j,t}, \bar{h}_t - h_{j,t}) + \phi_{j,t}\beta E V_{j+1,t+1}(a', s')\right\}, \tag{1}$$

subjec to

$$c_{j,t} + a_{j+1,t+1} = (1+(1-\tau_t^{\text{cap}})r_t)(a_{j,t}+b_t) + (1-\tau_t^{\text{ss}})w_t\kappa_j e_j h_{j,t},$$

$$c_{j,t} + a_{j+1,t+1} = (1+(1-\tau_t^{\text{cap}})r_t)(a_{j,t}+b_t) + \varphi_t w_t H_t.$$

Because microeconomic variables are not affected by the population trend, we need to detrend them by the TFP factor growth rate alone. Thus, we define $c_{j,t}/A_t^{1/(1-\theta_t)} = \tilde{c}_{j,t}$, $a_{j,t}/A_t^{1/(1-\theta_t)} = \tilde{a}_{j,t}$, and $h_{j,t} = \tilde{h}_{j,t}$.

Then, the normalized Bellman equation becomes as follows:

$$v_{j,t}(\tilde{a},s)=\max\left\{u(\tilde{c}_{j,t},\bar{h}_t-\tilde{h}_{j,t})+\phi_{j,t}\beta(1+g_t)^{\sigma(1-\gamma)}Ev_{j+1,t+1}(\tilde{a}',s')\right\}, \tag{2}$$

subject to

$$\tilde{c}_{j,t}+(1+g_t)\tilde{a}_{j+1,t+1}=(1+(1-\tau_t^{\text{cap}})r_t)(\tilde{a}_{j,t}+\tilde{b}_t)+(1-\tau_t^{\text{ss}})\tilde{w}_t\kappa_j e_j\tilde{h}_{j,t},$$

$$\tilde{c}_{j,t}+(1+g_t)\tilde{a}_{j+1,t+1}=(1+(1-\tau_t^{\text{cap}})r_t)(\tilde{a}_{j,t}+\tilde{b}_t)+\tilde{\varphi}\tilde{w}_t\tilde{H}_t$$

where $\tilde{b}_t=b_t/A_t^{1/(1-\theta_t)}$. Moreover, by defining $\tilde{\varphi}\equiv\varphi_t/N_t$, we could formulate that the normalized social security payment is a fraction of the average earning. Thus, it does not depend on period t.

A.3 First Order Conditions

From the Envelope theorem and Cobb-Douglas utility function, the Euler equation is as follows:

$$\frac{[\tilde{c}_{j,t}^{\sigma}(\bar{h}_t-\tilde{h}_{j,t})^{1-\sigma}]^{1-\gamma}}{\tilde{c}_{j,t}}=\phi_{j,t}\beta(1+g_t)^{\sigma(1-\gamma)-1}(1+(1-\tau_{t+1}^{\text{cap}})r_{t+1})E_j\left\{\frac{[\tilde{c}_{j+1,t+1}^{\sigma}(\bar{h}_{t+1}-\tilde{h}_{j+1,t+1})^{1-\sigma}]^{1-\gamma}}{\tilde{c}_{j+1,t+1}}\right\}, \tag{3}$$

$$(1-\sigma)\frac{[\tilde{c}_{j,t}^{\sigma}(\bar{h}_t-\tilde{h}_{j,t})^{1-\sigma}]^{1-\gamma}}{\bar{h}_t-\tilde{h}_{j,t}}\le\sigma\frac{[\tilde{c}_{j,t}^{\sigma}(\bar{h}_t-\tilde{h}_{j,t})^{1-\sigma}]^{1-\gamma}}{\tilde{c}_{j,t}}(1-\tau_t^{\text{ss}})\tilde{w}_t\kappa_j e_j,$$

Moreover, from the intratemporal first order condition, the labor supply function is

$$\tilde{h}_{j,t}=\max\left[\bar{h}_t-\left(\frac{1-\sigma}{\sigma}\right)\frac{\tilde{c}_{j,t}}{(1-\tau_t^{\text{ss}})\tilde{w}_t\kappa_j e_j},0\right].$$

Note that even when $\gamma=1$, the labor supply function does not change.

A.4 Law of Motion

From the policy function and the transition probability of labor endowment $\pi(s'|s)\equiv\text{Prob}(\varepsilon')\times\text{Prob}(z'|z)$, transition function $Q_t(\cdot,\cdot)$ of the states (a,s,j) and the distribution function $\Phi_{j,t}(a,s,j)$ over the states is computable.[1] Define the probability space as $((\mathcal{A}\times\mathcal{S}\times\mathcal{J}),\mathcal{B}((\mathcal{A}\times\mathcal{S}\times\mathcal{J})),\Phi_j)$ where $\mathcal{B}((\mathcal{A}\times\mathcal{S}\times\mathcal{J}))$ is a Borel σ-field and $\Phi_t(X)$ is a probability measure over $X\in\mathcal{B}((\mathcal{A}\times\mathcal{S}\times\mathcal{J}))$. The probability measure is defined over individual state and

[1] For details, see Stokey et al. (1989). For compuation of the distribution function, see Young (2004).

also represents fraction of households with state $X \in \mathcal{B}((\mathcal{A} \times \mathcal{S} \times \mathcal{J}))$. Because we assume that household of age $j = 20$ have zero asset, Φ_{20} is equal to one on $a_{20,t} = 0$. The transition function $Q_j : (\mathcal{A} \times \mathcal{S} \times \mathcal{J}) \times \mathcal{B}((\mathcal{A} \times \mathcal{S} \times \mathcal{J}))) \to [0, 1]$ is defined as

$$Q_j\left((\mathcal{A} \times \mathcal{S} \times \mathcal{J}), S\right) = \sum_{e' \in S} \begin{cases} \pi\left(s'|s\right) & \text{if } g^a_{j,t}\left(\tilde{a}, s\right) \in S \\ 0 & \text{else} \end{cases}, \text{ for all } j = 20, \ldots, 100.$$

From the initial distribution $\Phi_{20,t}$, distribution function $\{\Phi_{j,t}\}_{j=20}^{100}$ for each j transit by the following equation.

$$\Phi_{j+1,t+1}\left(X\right) = \int Q_j\left((\mathcal{A} \times \mathcal{S} \times \mathcal{J}), S\right) d\Phi_{j,t}, \;\; \left(\forall B \in \mathcal{B}(\mathcal{A} \times \mathcal{S} \times \mathcal{J})\right), \; j = 20, \ldots, 100,$$

Population dynamics is adjusted by μ_t, and the growth of TFP is already included. Thus, this distribution is purely a wealth distribution for each generation.

B Numerical Procedure

B.1 Endogenous Gridpoint Method

Among many available procedures for computing the policy function, we apply the Endogenous Gridpoint Method (EGM) by Carroll (2006) because it is a safe and relatively faster method.[2]

Define the right hand side of the Bellman equation (2) and the Euler equation (3) as

$$\Gamma_{j,t}(\tilde{a}', s) = \phi_{j,t}\beta(1 + g_t)^{\sigma(1-\gamma)} E_{j+1} v_{j+1,t+1}(\tilde{a}', s'),$$
$$\Gamma'_{j,t}(\tilde{a}', s) = \phi_{j,t}\beta(1 + g_t)^{\sigma(1-\gamma)-1}(1 + (1 - \tau^{\text{cap}}_{t+1})r_{t+1}) E_j u'_c(\tilde{c}_{j+1,t+1}, \bar{h}_{t+1} - \tilde{h}_{j+1,t+1}),$$

and take discretized grids on $\tilde{a}' \in [\underline{a}, \overline{a}]$. We set the number of grids to be 80. From the equation (3), the intertemporal first-order condition is rewritten as follows:

$$u'_c(\tilde{c}_{j,t}, \bar{h}_t - \tilde{h}_{j,t}) = \Gamma'_{j,t}(\tilde{a}', s) \tag{4}$$

Suppose that next period's consumption and labor supply function is already known as

$$\tilde{c}_{j+1,t+1} = \tilde{g}^c_{j+1,t+1}(\tilde{x}_{j+1,t+1}, s'),$$
$$\tilde{h}_{j+1,t+1} = \tilde{g}^h_{j+1,t+1}(\tilde{x}_{j+1,t+1}, s'), \text{ if } j \leq 65,$$

[2]For details on the endogeneous gridpoint method with endogenous labor supply, see appendix in Krueger and Ludwig (2006) and Barillas and Ferńandez-Villaverde (2006).

where $\tilde{x}_{j+1,t+1} \equiv (1+(1-\tau^{\text{cap}}_{t+1})r_{t+1})(\tilde{a}_{j+1,t+1,i}+\tilde{b}_{t+1}) + (1-\tau^{\text{ss}}_{t+1})\tilde{w}_{t+1}\kappa_{j+1}e_{j+1}\bar{h}_{t+1}$ is a cash on hand, then, we can compute the $\Gamma'_{j,t}(\tilde{a}', s)$ for each grids $\{\tilde{a}'_{j+1,t+1,i}\}_{i=1}^{80}$ for each age by backward induction. When we compute Γ'_t for each discretized state $(\tilde{a}_j, s)$, if the marginal utility function is invertible, we obtain the equilibrium consumption $\tilde{c}_j$ for each state.

Note that the marginal utility function is defined as follows:

$$u'_c(\tilde{c}_{j,t}, \bar{h}_t - \tilde{h}_{j,t}) = \sigma \frac{\left[\tilde{c}_j^{\sigma}(\bar{h}_t - \tilde{h}_{j,t})^{1-\sigma}\right]^{1-\gamma}}{\tilde{c}_j}.$$

From the first order condition (4), by taking inverse of the utility function $u'_c(\tilde{c}_{j,t}, \bar{h}_t - \tilde{h}_{j,t})$ with respect to $\tilde{c}_j$, we obtain $\tilde{c}_j$ for each choice variable $\tilde{a}_j$. Using the Euler equation for leisure and removing $\tilde{h}_{j,t}$, we have

$$\begin{aligned} u'_c(\tilde{c}_{j,t}, \bar{h}_t - \tilde{h}_{j,t}) &= \tilde{c}_{j,t}^{-\gamma}\sigma\left(\sigma\frac{1-\sigma}{\sigma}\frac{1}{(1-\tau^{\text{ss}}_t)\tilde{w}_t\kappa_j e_j}\right)^{(1-\sigma)(1-\gamma)} &&: \tilde{h}_{j,t} > 0 \\ u'_c(\tilde{c}_{j,t}, \bar{h}_t - \tilde{h}_{j,t}) &= \tilde{c}_{j,t}^{\sigma(1-\gamma)-1}\sigma\bar{h}_t^{(1-\sigma)(1-\gamma)} &&: \tilde{h}_{j,t} = 0 \end{aligned}$$

This equation is apparently invertible. Thus, we have

$$\tilde{c}^i_{j,t} = u^{-1}\cdot\Gamma'_{j,t}(\tilde{a}', e),.$$

From the consumption, we can directly induce $\tilde{h}^i_j$. From the set of $\{\tilde{c}^i_{j,t}, \tilde{h}^i_{j,t}, \tilde{a}^i_{j,t}\}$, we have new cash on hand $\tilde{x}^i_j = (1+g_t)\tilde{a}^i_{j+1,t+1} + \hat{c}^i_{j,t}$, where $\check{c}^i_j \equiv \tilde{c}^i_j + (1-\tau^{\text{ss}}_t)\tilde{w}_t\kappa_j e_j(\bar{h}_t - \tilde{h}_{j,t})$.

B.2 Computation of Stationary State

Computation of the stationary state is the same as in Aiyagari (1994) and Huggett (1996). There are three markets in the model: goods, labor and capital. However the factor prices (r, w) are determined from capital-labor ration $\tilde{K}/\tilde{H}$. By the Walras law, we concentrate on $\tilde{K}/\tilde{H}$.[3]

1. Given an initial guess of $(\tilde{K}^0, \tilde{H}^0)$, compute a pair of $(r^0, \tilde{w}^0)$.
2. Given $(r^0, \tilde{w}^0, \tilde{K}^0, \tilde{H}^0)$, compute the payroll tax rate τ^{ss} from the government budget condition.

[3] We take 80 grids on asset a for computing policy function, and to compute the distriution we take 5000 grids.

3. Given $(r^0, \tilde{w}^0, \tau^{\mathrm{ss}})$, compute the policy function using the EGM and get distribution function Φ^0 for each age.

4. Integrating the distribution function Φ^0, we can get the aggregate capital and labor $(\tilde{K}^1, \tilde{H}^1)$.

5. If new $(\tilde{K}^1, \tilde{H}^1)$ and old $(\tilde{K}^0, \tilde{H}^0)$ does not close to each other, refine the factor prices and repeat step $2-4$.

6. If new $(\tilde{K}^1, \tilde{H}^1)$ and old $(\tilde{K}^0, \tilde{H}^0)$ are sufficiently close to each other, stop computation. We have an equilibrium.

Note that all computation above are already detrended by $A_t^{1/(1-\theta_t)} N_t$.

B.3 Transitional Dynamics

Aftre computation of the steady state in 1980 and 2200, we compute transtion path of each equilibrium. Basic idea here is the same as Conesa and Krueger (1999) and Nishiyama and Smetters (2005).

1. Given an exogenous path of $\{\tilde{\varphi}, g_t, \theta_t, \delta_t, \bar{h}_t, \tau_t^{\mathrm{cap}}\}_{t=1980}^{2200}$, guess an equilibrium sequence of $\{r_t, \tilde{w}_t, \tau_t^{\mathrm{ss}}, \tilde{H}_t, \tilde{b}_t\}_{t=1980}^{2200}$, which are needed to a solve household's problem.[4] We assume that the replacement rate, TFP growth rate, capital share, depreciation rate, time endowment, and capital income tax are all perfect foresight and exogenously given.

2. Because we have stationary state in 2200, we compute a sequence of policy functions using the EGM by backward induction.

3. Given the policy functions, compute the distribution function from 1980 forwardly and compute aggregate variables, $\{\tilde{K}_t, \tilde{H}_t, r_t, \tilde{w}_t\}_{t=1980}^{2200}$.

4. Check whether each market clearing conditions and government budget balances are satisfied. If those are not in equilibrium, up-date the price sequences and repeas steps $2-3$.[5]

[4] For simplicity, we start a linear case.

[5] There are many efficient methods for update the price sequence. For example, Krueger and Ludwig (2006) and Ludwig (2006) uses a modified version of Gauss-Zeidel method for computing the transition path.

5. If all markets clear in all periods, stop computation.

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