Customer Search, Monopolistic Competition, and Relative Price Dispersion

Andrew T. Levin Tack Yun
Board of Governors of the Federal Reserve System

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Abstract

In much of recent macroeconomic models, product differentiation that arises from household preferences has been the sole source of the monopoly power of firms. It also has been recognized that imperfect information would result in firms having some degree of monopoly power. In this paper, we introduce sequential customer search into a dynamic general equilibrium model with differentiated goods and firm-specific productivity shocks, and we show that this framework enables us to reconcile very low micro-econometric estimates of the own-price elasticity of demand (obtained from household expenditure surveys) with the relatively small price markups that are typically implied by firm-level and industry data. Furthermore, each firm's demand curve has a quasi-kinked shape that arises directly from the customer search mechanism rather than from an arbitrary specification of household preferences. We also demonstrate that when a fixed cost of price adjustment is added to the model, inflation increases price dispersion and thereby aggregate total search cost as well as relative price distortion.

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Andrew T. Levin: (202) 452-3541; Email: andrew.levin@frb.gov.

Tack Yun: (202) 452-3606; Email: tack.yun@frb.gov.

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1 Introduction

Monopolistic competition has been an important building block of many macroeconomic models that helps to generate nominal price rigidity as a result of optimal behaviors of firms under various types of pricing friction. In particular, product differentiation that arises from household preferences has been the sole source of the monopoly power of firms in much of recent macroeconomic models. Meanwhile, it has long been recognized that imperfect information would result in firms having some degree of monopoly power.[1]

In this paper, we introduce sequential customer search into a dynamic general equilibrium model with differentiated goods and firm-specific productivity shocks, and we show that this mechanism has crucial implications for interpreting microeconomic evidence regarding the elasticity and curvature of demand. In this environment, the monopolistic power of each firm depends on the characteristics of customer search as well as the degree of product differentiation; that is, if customers have relatively low search costs, then the individual firm's demand elasticity may be much greater than the price elasticity of household demand for that item. Thus, this framework enables us to reconcile very low micro-econometric estimates of the own-price elasticity of demand (obtained from household expenditure surveys) with the relatively small price markups that are typically implied by firm-level and industry data.

In particular, we allow for firm-specific productivity shocks in order to produce a non-degenerate equilibrium price dispersion even in the case of full price flexibility, which is necessary to invoke consumer search for the lowest price. In addition, our analysis proceeds with the Dixit-Stiglitz (1977) preference of households that is applied to the aggregation of differentiated goods in the production of composite goods. As a result, our modelling strategy is to construct a dynamic general equilibrium model that allows for both relatively general inelastic household expenditures and elastic demand curves facing individual firms.

In other dimension, strategic complementarities have been used to generate the insensitivity of prices with respect to changes in marginal cost, thereby leading to persistent macroeconomic effects of monetary disturbances, especially in economies with a relatively small amount of price rigidity. An important source of strategic complementarities is the presence of quasi-kinked demand curves.[2] In relation to this, our modelling framework implies that each firm's demand curve has a quasi-kinked shape that arises directly from the customer search mechanism rather than from an arbitrary specification of household

[1] See, among a lot of existing papers on consumer search, Diamond (1971), MacMinn (1980), Stigler (1961), Stiglitz (1979, 1987, 1989) and Woglom (1982) for related theoretic issues and the generation of equilibrium price distribution that is consistent with consumer search.

[2] Kimball (1995) showed that demand curves facing firms can be more elastic as prices rise by generalizing the Dixit-Stiglitz specification of household preferences - so called quasi-kinked demand curves, which has been followed by numerous subsequent studies including Bergin and Feenstra (2000), Woodford (2003), Eichenbaum and Fisher (2004), Dotsey and King (2005), and Klenow and Kryvstov (2006).

preferences.

The use of consumer search in nominal rigidity models is not new. In particular, Ball and Romer (1990) embed consumer search into a nominal rigidity model. The primary concern of Ball and Romer falls on a highly stylized model in which all firms set prices at the same time and the number of consumer search is restricted to be one. Instead, we permit the optimal sequential search which does not impose any restriction on the number of search, while the price-setting of firms is staggered over time because of fixed cost of price adjustment.

Benabou (1988) and Diamond (1993) combine consumer search with (S, s) pricing in order to analyze the relationship between inflation and price dispersion and its welfare consequences. A difference from theirs is that we characterize optimal search behaviors of households by allowing for explicit demand functions of individual households on differentiated goods derived from the Dixit-Stiglitz preference.

Furthermore, a set of recent papers have studied dynamic general equilibrium models of menu-cost such as Dotsey, King, and Wolman (1999), Danziger (1999), Gertler and Leahy (2005), and Golosov and Lucas (2007). In particular, we present an exact characterization of a simple two-sided (s, S) strategy for the ratio of current-period's price to its optimal price in the inelastic household demands, following the method adopted in Danziger (1999). We then demonstrate that inflation increases price dispersion and thereby aggregate search cost as well as relative price distortion.

The rest of out paper is organized as follows. In the next section, we briefly highlight existing empirical works that motivate out modelling strategy. In section 3, we spell out how an equilibrium price distribution is determined using a complete dynamic general equilibrium model of full price flexibility in which all consumers adopt optimal reservation price strategies for their searches. In section 4, we add a fixed cost of price adjustment to the model, in order to make equilibrium price distribution dependent on inflation. In section 5, we discuss directions for future research.

2 Motivation

It has been well-known in the literature on empirical consumer demand that price elasticities of individual consumers' expenditures tend to be less than one in many cases. For example, Deaton and Muellbauer (1980) estimated a system of demand equations derived explicitly from consumer theory using British data from 1954 to 1974 including eight non-durable groups of consumers' expenditure. As shown in Table 1, their results on own-price elasticities indicate the general price inelasticity of demand, though transport and communication tend to be price elastic. They also conclude in their text book that there is fairly consistent evidence that own-elasticities are less than unity, at least at the sort of

Table 1: Own-Price Elasticities and Markup

Product	Own-Price Elasticities	Markup U.K.	Markup U.S.A.
Clothing	-0.92	1.098	1.065
Housing	-0.31	N/A	N/A
Fuel	-0.28	1.119	1.111
Food and Beverages	-0.60	1.087	1.070
Transport and Communication	-1.21	1.016	1.111

Sources: *Own-Price Elasticities:* Deaton and Muellbauer (1980); *Markups in U.K.:* Boulhol (2005); Clothing Markup (U.S.A.) is that of *Textiles, Textile Products, Leather and Footwear* in Boulhol (2005); Fuel Markup (U.S.A.) is that of *Retail Gasoline* in Slade (1987); Food and Beverages Markup (U.S.A.) is that of *Food Products, Beverages and Tobacco* in Boulhol (2005); Transport and Communication Markup (U.S.A.) is that of *Automobile* in Bresnahan (1981).

levels of commodity disaggregation.

Meanwhile, various empirical studies on industrial organization indicate that markup - the ratio of price to markup - is greater than one in most of industries. As shown in table 1, estimates of markup in nondurable goods are grater than one. In relation to this, it is well-known from microeconomics textbooks that price elasticities of demand curves facing individual firms should be greater than one in order to have their markups greater than one. We thus find that if expenditure functions of individual goods derived from consumer theory are exactly the same as the demand curves facing individual firms, one cannot reconcile it with the empirical facts included in Table 1.

The reason why we are concerned about this potential inconsistency is associated with calibrated price elasticities of expenditure functions of individual households in the recent macro literature. The left-column in Table 2 shows that calibrated price elasticities of expenditure functions of individual households are greater than one in many recent macro models, which is necessary to guarantee that average markup should greater than one. In particular, comparing Tables 1 and 2, we can see that calibrated price elasticities of expenditure functions in the literature are greater than those estimated from consumers' budget data.

In other dimension, recent papers on macro monetary models have emphasized the importance of real rigidities in order to explain persistent real effects of monetary policy

Table 2: Calibrated Price Elasticities and Curvatures of Demand Curve in Literature

	Price Elasticities	Curvatures
Kimball (1995)	-11	471
Bergin and Feestra (2000)	-3	1.33
Eichenbaum and Fisher (2004)	-11	10, 33
Coenen and Levin (2004)	-5, - 20	10, 33
Woodford (2005)	-7.67	6.67
Klenow and Willis (2006)	-5	10

Sources: Dossche, Heylen and Dirk Van den Poel (2007). Curvature is defined as the elasticity of the price elasticity of demand with respect to the relative price at the steady state.

changes. In relation to this, much of recent papers allow for the possibility that price elasticities of demand curves facing individual firms rise with their relative prices, thus leading to smaller responses of prices to changes in the marginal cost of production. But, since these kinked demand curves are based on the preference structures of households, they still require price elasticities of expenditure functions to be greater than one, as shown in Table 2.

A possible way to reconcile inelastic expenditure functions with elastic demand curves of firms is to introduce extensive margin into the demand curve of an individual firm. More specifically, one can allow for the possibility that price changes affect the number of customers as well as the amount of goods that a customer purchases. We do this by incorporating customer search into an otherwise standard model with differentiated goods.

3 Consumer Search and Quasi-Kinked Demand Curve

In this section, we derive a quasi-kinked demand curve from the optimizing behavior of consumers when they have imperfect information about location of different prices. It is shown that the demand curve facing an individual firm depends on not only the purchasing behavior of individual households but also their search behavior.

3.1 Economic environment

The economy is populated by a lot of infinitely-lived households, while different types of goods are produced and sold in different islands. Thus, members of each household visit different islands to purchase different goods. Labor services are traded in a perfectly competitive labor market and wages are fully flexible. Moreover, production of each firm in an island is subject to idiosyncratic shocks whose distribution is identically and independently distributed across islands and over time. The presence of such shocks creates a non-degenerate price distribution in each type of different goods.

Consumers do not know realized values of productivity shocks that hit individual firms. A non-degenerate price distribution then gives households incentive to find a seller with the lowest price for each type of goods, while a continuum of firms, indexed by [0, 1], produce the same type of goods. Although the number of search is not limited, we assume that any visit to a particular seller requires a fixed cost. Specifically, each visit to a seller incurs a fixed nominal amount of $z_{jt} = z_j L_t$ for type j goods where L_t is the total cost of purchasing goods and $Z_j(z_j)$ be the distribution function for search cost. For simplicity, we assume that search costs are uniformly distributed on $[\underline{z}_j, \bar{z}_j]$, so that $Z_j(z_j) = (z_j - \underline{z}_j)/(\bar{z}_j - \underline{z}_j)$.

Furthermore, we assume that members of individual household do not communicate each other while they visit different islands. Although this assumption may be rather restrictive, it leads consumers to follow a simple reservation-price strategy for each type of goods even when consumers are supposed to search a lot of different goods in each period. Specifically, search continues until each consumer finds a seller who quote price at or below his or her reservation price for each type of goods.

Finally, a fraction of firms can have their prices grater than the maximum reservation price of consumers if their productivity shocks are very low. In this case, firms that undergo these situations are assumed to shut down their production activities temporarily until their prices go back within the range of reservation prices of consumers.

3.2 Household Optimization

Each period is divided into two sub-periods. The first-half is search stage and the second half is spending stage. In the spending stage, households make actual purchases of goods after they have decided on which sellers they trade with. The level of actual spending is determined as a result of utility optimization. Specifically, households choose the use of time and the level of consumption to maximize their utilities and then their searches for sellers help to minimize the cost of maintaining the optimized level of consumption without deterring the efficient use of time.

3.2.1 Decisions on Consumption Spending and Use of Time

The preference of each household at period 0 is given by

$$\sum_{t=0}^{\infty} E_0[U(C_t, \bar{H} - H_t)], \tag{1}$$

where C_t is the consumption at period t, $\bar{H}$ is the amount of time endowment available for each household, H_t is the amount of hours worked at period t. The instantaneous utility function $U(C_t, \bar{H} - H_t)$ is continuously twice differentiable and concave in consumption and leisure.

Households aggregate differentiated goods to produce composite goods using the Dixit-Stiglitz aggregator. Specifically, composite goods are produced by the following Dixit-Stiglitz type aggregator:

$$C_t = (\int_{j=0}^{1} C_{jt}(i)^{\frac{\theta-1}{\theta}} dj)^{\frac{\theta}{\theta-1}}, \tag{2}$$

where C_t is the real amount of the composite goods and $C_{jt}(i)$ is the amount of type j goods that each household purchases from a seller i.

We also assume that there is a complete financial market in which all agents trade contingent claims. In addition, wages are fully flexible in a perfectly competitive market. Given these assumptions, the period budget constraint of each household can be written as

$$\int_0^1 (P_{jt}(i)C_{jt}(i) + z_{jt}X_{jt})dj + E_t[Q_{t,t+1}B_{t+1}] \leq W_t^N H_t + B_t + \Phi_t, \tag{3}$$

where $P_{jt}(i)$ is the dollar price at period t of good j at seller i, X_{jt} is the number of search that an individual household has made in order to determine a seller i, z_{jt} is the nominal cost of each visit to a seller, $Q_{t,t+1}$ is the stochastic discount factor used for computing the dollar value at period t of one dollar at period $t+1$, W_t^N is the nominal wage rate, and Φ_t is the dividend distributed to households.

The demand of each different type of goods is then determined by solving a cost-minimization of the form:

$$\min\{\int_{j=0}^{1} P_{jt}(i)C_{jt}(i)dj + \Lambda_t\{C_t - (\int_{j=0}^{1} C_{jt}(i)^{\frac{\theta-1}{\theta}} dj)^{\frac{\theta}{\theta-1}}\}\}, \tag{4}$$

where $P_t(i)$ is the dollar price at period t of good j at seller i and Λ_t is the Lagrange multiplier of this cost minimization. As a result of this cost-minimization, the demand curve facing a seller i that sells type j goods can be written as

$$C_{jt}(i) = (P_{jt}(i)/\Lambda_t)^{-\theta} C_t. \tag{5}$$

The cost-minimization also implies that the Lagrange multiplier can be written as

$$\Lambda_t = (\int_{j=0}^{1} P_{jt}(i)^{1-\theta} dj)^{\frac{1}{1-\theta}}. \tag{6}$$

In addition, letting $L_t(\{P_{jt}(i)\}) = \int_{j=0}^{1} P_{jt}(i)C_{jt}(i)dj$ denote the nominal consumption expenditures of each household, we can see that the following equation holds:

$$L_t(\{P_{jt}(i)\}) = \Lambda_t C_t. \tag{7}$$

It then follows from this equation that the specification of the period budget constraint described above is consistent with the Dixit-Stiglitz type aggregator. The demand function for individual goods as well as optimization condition of households have been widely used in much of the recent macro-economic literature that allows for monopolistic competition in goods markets.

Furthermore we can rewrite the nominal flow budget constraint of the household as follows:

$$\Lambda_t C_t + \int_0^1 z_{jt} X_{jt} dj + E_t[Q_{t,t+1} B_{t+1}] \leq W_t^N H_t + B_t + \Phi_t. \tag{8}$$

As a result, the utility maximization of each household leads to the following optimization conditions:

$$U_2(C_t, \bar{H} - H_t) = (W_t^N / \Lambda_t) U_1(C_t, \bar{H} - H_t), \tag{9}$$

$$Q_{t,t+1} = \beta \frac{U_1(C_{t+1}, \bar{H} - H_{t+1})\Lambda_t}{U_1(C_t, \bar{H} - H_t)\Lambda_{t+1}}, \tag{10}$$

where $U_1(C_t, \bar{H} - H_t)$ is the marginal utility of consumption and $U_2(C_t, \bar{H} - H_t)$ is the marginal utility of leisure.

Finally, it should be noted that the demand function specified above is valid only under the condition that households do not change sellers. More precisely, as noted earlier, each household does not know exact locations of individual prices, though their true distribution is publicly known. Therefore, this demand function is valid after households finish their searches for the lowest price for each type of goods. In the next, we analyze the search behavior of each household under the assumption that each type of goods has the intensive margin demand curve specified above once household determines a seller for each type of differentiated goods.

3.2.2 Search Decision

In this section, we consider search behaviors of households. It is important to note that their search behaviors should be fully consistent with their spending decisions described above, though households complete their searches for the lowest price in the first sub-period. The reason for this is that households should not deviate from their decisions made at the the first sub-period when actual spending is carried out at the next stage.

In order to see this, it is necessary to demonstrate that households have incentive to find a seller that gives the lowest price for each type of goods, given that they purchase each type of goods according to the demand function specified in (5). In particular, notice that the cost-minimization of households in the spending period leads to the following consumption expenditure function: $L_t(\{P_{jt}(i)\}) = \Lambda_t C_t$. In addition to this, we point out that the partial derivative of the consumption expenditure function with respect to the relative price of an individual price is positive: $\partial L_t(\{P_{jt}(i)\})/\partial P_{jt}(i) = (P_{jt}(i)/\Lambda_t)^{-\theta}\ C_t$. Hence, to the extent that θ is not very big and search does not require an arbitrary large amount of costs, households have incentive to search for sellers with the lowest price for each type of goods. As a result, the standard Dixit-Stiglitz model without search can be viewed as implicitly assuming that search costs are arbitrarily large so that no consumer wants to search.

Household's Information on Prices: Having shown that individual households have incentive to search for the lowest price of each type of goods, we briefly discuss the information of each household on prices. Individual households are supposed to know the true distribution of nominal prices denoted by $F_{jt}(P_{jt})$, where $F_{jt}(P_{jt})$ represents the measure of firms that set their nominal prices equal to or below P_{jt}.

We assume that each individual household has a lot of shopping members. Since each shopping member is supposed to search the lowest price for each type of goods, each individual household send a continuum of shopping members to each island. For example, a shopping member visits an island j in order to search for the lowest price for type j goods. In this case, observations on prices are independent random variables drawn from the true price distribution. After each observation, each shopping member decides to continue search or determine a particular seller, while a positive search cost limits the number of observations. But these shopping members do not communicate each other after they depart from their households until all of them determine a seller for each type of goods. Because of this assumption, households' search decisions turn out to resemble the search process of one single product, though they purchase a continuum of goods at the same time.

Derivation of the Objective Function of Search: Notice that the nominal consumption expenditure function for each household at the spending stage is given $L_t(\{P_{jt}(i)\}) = \Lambda_t C_t$. Since we assume that shopping members do not communicate each other, a shopping member's behavior affects only a slice of the consumption expenditure function while the consumption expenditure function is defined in terms of integral. In order to formulate this feature, notice that the consumption expenditure function can be viewed as a function of P_{jt} if only nominal price of type j goods changes but all other prices are fixed at $P_{st}(i)$ $= P_{st}$ for all i. We thus define a new function $L(P_{jt})$ reflecting this situation. Then, $L(P_{jt})$ is affected by a shopping member's decision on the determination of a particular seller. In addition, $z_{jt} = z_j \Lambda_t C_t$ is a realized level of nominal search cost for type j goods that a particular household should pay each time the household visits a seller at period t.

Determination of Reservation Prices: We now explain how households determine reservation prices for their sequential searches, denoted by $R_{jt}(z_j)$. We assume that individual households adopt a reservation price strategy for their searches. Specifically, shopping members stop searching for any price observation $P_{jt} \leq R_{jt}(z_j)$, while for any $P_{jt} \geq R_{jt}(z_j)$, they continue to search.

We describe the determination of reservation strategy in the context of dynamic programming. In order to do this, we let $V_j(P_{jt})$ represent the value function of search cost for each type of goods when the relative price of his or her first visit is P_{jt}. Then, the optimization of a shopping member whose objective is to find a seller with the lowest price can be written as

$$V_j(P_{jt}) = \min\{L_t(P_{jt}), z_j \Lambda_t C_t + \int_0^{R_{jt}(z_j)} V_j(K_{jt}) dF(K_{jt})\}. \tag{11}$$

The reservation price then satisfies the following condition:

$$z_j \Lambda_t^{1-\theta} = \int_0^{R_{jt}(z_j)} \{P_{jt}^{-\theta} F_{jt}(P_{jt})\} dP_{jt}. \tag{12}$$

The left-hand side of (12) corresponds to the cost of an additional search, while the right-hand side is its expected benefit.[3]

It would be worthwhile to discuss a couple of issues associated with the determination of reservation strategy discussed above. First, it is the case in the search literature that prices do not affect the real amount of goods that each customer purchases, which corresponds to setting $\theta = 0$. In this case, the reservation price for each level of z_j turns out to be

$$z_j \Lambda_t = \int_0^{R_{jt}(z_j)} F_{jt}(P_{jt}) dP_{jt}. \tag{13}$$

Second, when the maximum reservation price is higher than the maximum of actual transaction prices, the reservation price equation specified above can be rewritten as follows:

$$\bar{z}_j \Lambda_t^{1-\theta} = \frac{\bar{R}_{jt}^{1-\theta}}{1-\theta} - \frac{P_{\max,jt}^{1-\theta}}{1-\theta} + \int_{P_{\min,jt}}^{P_{\max,jt}} P_{jt}^{-\theta} F_{jt}(P_{jt}) dP_{jt}. \tag{14}$$

[3] The expected benefit of an additional search is $\int_0^{R_{jt}} \{L(R_{jt}) - L(P_{jt})\} dF_{jt}(P_{jt})$. The reservation price for consumers whose search cost is z_j should satisfy $z_j \Lambda_t^{1-\theta} C_t = \int_0^{R_{jt}} \{L(R_{jt}) - L(P_{jt})\} dF_{jt}(P_{jt})$. The integration by parts then leads to the formula specified in (12). If the maximum reservation price is hihger than the maximum transaction price, it should satisfy $\bar{z}_j \Lambda_t^{1-\theta} C_t = L(\bar{R}_{jt}) - L(P_{\max,jt}) + \int_0^{P_{\max,jt}} \{L(P_{\max,jt}) - L(P_{jt})\} dF_{jt}(P_{jt})$.

Average Cost of Search: Having specified the determination of reservation strategy, we now discuss the level of search cost that each individual household spends. In doing so, we begin with the assumption that a particular level of search cost is randomly assigned to each household for each type of differentiated goods at the beginning of each period. Meanwhile, a continuum of differentiated goods exists in the economy. We thus rely on the law of large numbers in order to make the expected level of total search cost identical across households.

We now discuss the level of total search cost that each individual household is expected to pay. In particular, we allow for the possibility that households can make infinite number of sequential search. It means that the expected number of search is $1/F(R_{jt}(z_j))$ when the fraction of search cost for type j goods is z_j. The expected nominal cost of search is therefore given by $\{z_j/F(R_{jt}(z_j))\}\Lambda_t C_t$ when the real search cost for type j goods is z_{jt} $= z_j \Lambda_t C_t$.

We also define the ex-ante expected search cost as the level of search cost, denoted by X^e_{jt}, which each household is expected to pay at the time point before realization of search cost. The aggregate expected search cost is then given by $S_t = \int_{j=0}^{1} X^e_{jt} dj$. In order to compute X^e_{jt}, notice that there exists a level of z_j denoted by z^*_{jt} such that if $\bar{R}_{jt} > P_{\max,jt}$, z^*_{jt} satisfies

$$z^*_{jt}\Lambda_t^{1-\theta} = \int_{P_{\min,jt}}^{P_{\max,jt}} \{P_{jt}^{-\theta} F_{jt}(P_{jt})\} dP_{jt}, \tag{15}$$

while $z^*_{jt} = \bar{z}_j$, when $\bar{R}_{jt} = P_{\max,jt}$. Given the definition of z^*_{jt}, the ex-ante expected search cost can be written as

$$X^e_{jt} = (\overline{z}_j - \underline{z}_j)^{-1}\{\int_{z^*_{jt}}^{\overline{z}_j} z_j dz_j + \int_{\underline{z}_j}^{z^*_{jt}} \frac{z_j}{F(R_t(z_j))} dz_j\}. \tag{16}$$

3.3 Demand Curves of Individual Sellers

Having described the reservation strategy of households, we now move onto the discussion on the demand curve facing a seller whose nominal price is P_{jt}. Since consumers use reservation price strategies, any consumers whose reservation prices are greater than P_{jt} are potential customers for the seller.

In order to derive a demand curve facing an individual seller whose price is P_{jt}, it is necessary ro compute the expected number of the seller's potential customers. In order to do so, we choose a set of consumers whose relative price, denoted by $R_{jt}(z_j)$, is equal to or greater than P_{jt}. It is then important to note that consumers whose reservation price is $R_{jt}(z_j)$ are randomly distributed to a set of sellers whose relative prices are less than $R_{jt}(z_j)$. Moreover, given the uniform distribution of search cost, the measure of consumers whose reservation price is $R_{jt}(z_j)$ is $(\bar{z}_j - \underline{z}_j)^{-1}$. As a result, $(\bar{z}_j - \underline{z}_j)^{-1}\{f_{jt}(P_{jt})/F(R_{jt}(z_j))\}$ is the measure of consumers with their reservation price

$R_{jt}(z_j)$, who are assigned to a group of sellers whose relative price is P_{jt} $(\leq R_{jt}(z_j))$, where $f_{jt}(P_{jt})$ is the measure at period t of sellers whose relative price is P_{jt}. Since this matching process holds for any consumers whose reservation price is greater than P_{jt}, the total expected number of consumers who purchase their products at a shop with price P_{jt} is

$$\frac{1}{\bar{z}_j - \underline{z}_j} \int_{P_{jt}}^{\bar{R}_{jt}} \frac{f_{jt}(P_{jt})}{F(R_{jt}(z_j))} dz_j, \tag{17}$$

where $\bar{R}_{jt}$ is the maximum reservation price at period t.

Having specified the total expected number of consumers in terms of search-cost distribution, we express it in terms of price distribution. Specifically, the total differentiation of the optimal reservation price (12) and then the aggregation of the resulting equations over individual households yields the following relation between search cost and reservation price: $dz_j = \Lambda_t^{\theta-1}\{F_{jt}(R_{jt}(z_j))R_{jt}(z_j)^{-\theta}\}dR_{jt}(z_j)$. Substituting this equation into (17) and solving the resulting integral, one can show that the expected total number of consumers for P_{jt} can be written as $f_{jt}(P_{jt})\Lambda_t^{\theta-1}(\bar{z}_j - \underline{z}_j)^{-1}$ $(\bar{R}_{jt}^{1-\theta} - P_{jt}^{1-\theta})/(1-\theta)$. Since $f_{jt}(P_{jt})$ is the measure of sellers who charge P_{jt}, it implies that the expected number of consumers for each individual seller with its relative price P_{jt}, denoted by $N_{jt}(P_{jt})$, is given by

$$N_{jt}(P_{jt}) = (\bar{z}_j - \underline{z}_j)^{-1}(\frac{\bar{R}_{jt}^{1-\theta}}{1-\theta} - \frac{P_{jt}^{1-\theta}}{1-\theta})\Lambda_t^{\theta-1}. \tag{18}$$

Here, it should be noted that if the maximum reservation price is greater than the maximum of actual prices, an individual seller who sets the maximum of actual prices has a significantly positive measure of customers. But it is possible that a fraction of firms can have their prices grater than the maximum reservation price of consumers if their productivity shocks are very low. Then, firms that undergo these situations are assumed to shut down their production activities temporarily until their prices go back within the range of reservation prices of consumers.

Furthermore, it is necessary to show that the total expected number of consumers should be equal to one because the measure of households is set to one. It means that $\int_{\underline{R}_{jt}}^{\bar{R}_{jt}} N_{jt}(P_{jt})f(P_{jt})dP_{jt} = 1$, where $\underline{R}_{jt}$ is the minimum reservation price for type j goods. In addition, the minimum reservation price satisfies the following equation:

$$\underline{z}_j\Lambda_t^{1-\theta} = \int_0^{\underline{R}_{jt}} \{P_{jt}^{-\theta}F_{jt}(P_{jt})\}dP_{jt}. \tag{19}$$

Subtracting this minimum reservation price equation from the maximum reservation price equation, we have

$$(\bar{z}_j - \underline{z}_j)\Lambda_t^{1-\theta} = \int_{\underline{R}_{jt}}^{\bar{R}_{jt}} \{P_{jt}^{-\theta}F_{jt}(P_{jt})\}dP_{jt}. \tag{20}$$

Consequently, we can see that $\int_{\underline{R}_{jt}}^{\bar{R}_{jt}} N_{jt}(P_{jt})f(P_{jt})dP_{jt} = 1$.

Having derived the expected number of consumers for each seller, we now move onto the demand curve facing an individual seller. Before proceeding, we assume throughout the paper that after determining sellers in the search process, equation (5) determines the amount of goods that each consumer purchases, namely the intensive margin of total demand.[4] As shown before, the intensive margin depends on relative prices. It would be more convenient to express total demand in terms of relative price. In doing so, we deflate individual nominal prices by households' marginal valuation on composite consumption goods, Λ_t, so that we denote the real price of P_{jt} by $\tilde{P}_{jt}$. We now combine equations (5) and (18) are combined to yield

$$D_{jt}(\tilde{P}_{jt}) = \tilde{P}_{jt}^{-\theta}(\frac{\tilde{R}_{jt}^{1-\theta}}{1-\theta} - \frac{\tilde{P}_{jt}^{1-\theta}}{1-\theta})C_t/(\overline{z}_j - \underline{z}_j), \tag{21}$$

where $D_{jt}(\tilde{P}_{jt})$ is the demand function at period t when a seller sets its relative price at $\tilde{P}_{jt}$ and $\tilde{R}_{jt}$ is the relative price of the maximum reservation price.

An immediate implication of the demand curve (21) is that the elasticity of demand for each good, denoted by $\epsilon(P_{jt})$, depends on its relative price. The main reason for this is associated with the presence of the maximum relative price. In particular, the expected number of consumers turns out to be nil when relative price of each firm exceeds the maximum reservation price. Thus, the logarithm of the expected number of consumers is not linear in the logarithm of the relative price. Specifically, the elasticity of demand can be written as follows:

$$\epsilon(P_{jt}) = \theta + \frac{P_{jt}^{1-\theta}}{\bar{R}_{jt}^{1-\theta}/(1-\theta) - P_{jt}^{1-\theta}/(1-\theta)}. \tag{22}$$

3.4 Discussion on Sources of Monopoly Powers of Firms

In order to see where the monopoly power of firms originates, we compute the value at which demand elasticities converge as idiosyncratic shocks have an arbitrarily small support.[5]

[4] When there are both of intensive and extensive margins, the reservation price that is determined as a result of minimizing unit-cost plus search-cost may be the same as the reservation price that minimizes actual transaction cost plus search cost. In our paper, consumers choose a seller for each type of goods that maximizes indirect utility function after they solve their utility maximization problem. Each individual consumer then uses equation (5) to determine the amount that each consumer purchases. But one may wonder if the same consumption demand can be derived when consumers are allowed to solve the utility maximization problem after they choose their sellers. It does not change the functional form of demand function for each seller as specified in (5) because consumers take as given the list of prices posted by sellers.

[5] Specifically, when prices are fully flexible, the equilibrium distribution of prices degenerates in a symmetric equilibrium especially when there are no idiosyncratic elements among firms. It is therefore subject to the Diamond's paradox. In order to avoid the Diamond's paradox, one can include idiosyncratic cost shocks.

Before going further, the demand function specified above can be used to show that the following relation holds:

$$\frac{(\bar{R}_{jt}/\Lambda_t)^{1-\theta}}{1-\theta} - \frac{(P_{jt}/\Lambda_t)^{1-\theta}}{1-\theta} = ((P_{jt}D_{jt})/(\Lambda_t C_t))(P_{jt}/\Lambda_t)^{-(1-\theta)}(\bar{z}_j - \underline{z}_j). \tag{23}$$

Substituting this equation into the elasticity of demand specified above, we have the following equation:

$$\epsilon(P_{jt}) = \theta + \frac{(P_{jt}/\Lambda_t)^{2(1-\theta)}}{\bar{z}_j - \underline{z}_j}((P_{jt}D_{jt})/(\Lambda_t C_t))^{-1}. \tag{24}$$

As a result, we can see that when the support of idiosyncratic shocks is arbitrarily small, the demand elasticity turns out to be

$$\epsilon_j = \theta + 1/(\overline{z}_j - \underline{z}_j). \tag{25}$$

It is now worthwhile to mention that the demand of an individual firm comes from not only the demand of an individual consumer but also the number of consumers who decides to purchase. We call the former *the demand at the intensive margin* and the latter at *the demand at the extensive margin*. The demand elasticity therefore reflects both of the elasticity of demand at the intensive margin and the elasticity of demand at the extensive margin. For example, the first-term in the right-hand side of (25) is the elasticity of demand at the intensive margin, while the second-term corresponds to the elasticity of demand at the extensive margin.

It is also clear from (25) that the elasticity of demand approaches infinity as $\bar{z}_j$ gets close to zero. It means that all firms are subject to perfect competition in the absence of consumer search frictions. As a result, we can find that the important source of the monopoly power of firms is the presence of search costs together with imperfect information of consumers about the location of prices.

3.5 Equilibrium Distribution of Prices

In order to generate an equilibrium price dispersion for each type of differentiated goods, we introduce idiosyncratic productivity shocks into the model. Specifically, firm i in island j produce its output using a production function of the form:

$$Y_{jt}(i) = H_{jt}(i)/A_t(i), \tag{26}$$

where $A_t(i)$ is the firm-specific shock at period t, $H_{jt}(i)$ is the amount of labor hired by firm i, and $Y_{jt}(i)$ is the output level at period t of firm i. In addition, we assume that $A_t(i)$ is an i.i.d. random variable over time and across individual firms and its distribution is a uniform distribution whose support is $[\underline{A}, \overline{A}]$.

Table 3: Example on the Determination of the Equilibrium Distribution of Prices (Uniform Distribution of Idiosyncratic Productivity Shocks)

Perceived Cumulative Distribution of Real Prices

$$F(P_{jt}) = (M^e(P_{j,t}) - M^e(P_{\min,jt}))/(M^e(P_{\max,jt}) - M^e(P_{\min,jt}))$$

$$M^e(P_{jt}) = P_{j,t}(2 - (\bar{R}_{jt}/P_{jt})^{1-\theta})/\{\theta((\bar{R}_{jt}/P_{jt})^{1-\theta} - 1)/(1-\theta) + 1\}$$

Maximum Reservation (Real) Price

$$\bar{z}_j \Lambda_t^{1-\theta} = (\bar{R}_{jt}^{1-\theta} - P_{\max,jt}^{1-\theta})/(1-\theta) + \int_{P_{\min,jt}}^{P_{\max,jt}} P_{jt}^{-\theta} F_{jt}(P_{jt}) dP_{jt}$$

Expected Number of Customers

$$N_{jt}(P_{jt}) = \{(\bar{R}_{jt}^{1-\theta} - P_{jt}^{1-\theta})/(1-\theta)\}\{\Lambda_t^{\theta-1} C_t/(\overline{z}_j - \underline{z}_j)\}.$$

Demand Function

$$D_{jt}(\tilde{P}_{jt}) = \{\tilde{P}_{jt}^{-\theta}(\tilde{R}_{jt}^{1-\theta} - \tilde{P}_{jt}^{1-\theta})/(1-\theta)\}\{C_t/(\overline{z}_j - \underline{z}_j)\}.$$

Profit Maximization Conditions with respect to Prices

$$AW_t(\theta \tilde{R}_{jt}^{1-\theta} \tilde{P}_{j,t}^{-(1+\theta)} - (2\theta - 1)\tilde{P}_{j,t}^{-2\theta}) = (1-\theta)(2\tilde{P}_{j,t}^{1-2\theta} - \tilde{P}_{j,t}^{-\theta} \tilde{R}_{jt}^{1-\theta})$$

Realized Cumulative Distribution of Prices

Maximum Real Price: Solution to the Following Equation

$$A_{\max} W_t(\theta \tilde{R}_{jt}^{1-\theta} \tilde{P}_{\max,jt}^{-(1+\theta)} - (2\theta - 1)\tilde{P}_{\max,jt}^{-2\theta}) = (1-\theta)(2\tilde{P}_{\max,jt}^{1-2\theta} - \tilde{P}_{\max,jt}^{-\theta} \tilde{R}_{jt}^{1-\theta})$$

Minimum Real Price: Solution to the Following Equation

$$A_{\min} W_t(\theta \tilde{R}_{jt}^{1-\theta} \tilde{P}_{\min,jt}^{-(1+\theta)} - (2\theta - 1)\tilde{P}_{\min,jt}^{-2\theta}) = (1-\theta)(2\tilde{P}_{\min,jt}^{1-2\theta} - \tilde{P}_{\min,jt}^{-\theta} \tilde{R}_{jt}^{1-\theta})$$

Notation: $\tilde{P}_{jt}$ is a profit-maximizing real price at period t of type j goods; $\tilde{P}_{\max,jt}$ is the maximum level of profit-maximizing real prices of type j goods; $\tilde{P}_{\min,jt}$ is the minimum level of profit-maximizing real prices of type j goods; W_t is the real wage ($= W_t^N/\Lambda_t$); C_t is the aggregate consumption level; $\tilde{R}_{jt}$ is the relative price of the maximum nominal reservation price; $\bar{R}_{jt}$ is the maximum of nominal reservation prices.

Given the demand curve and the production function specified above, the instantaneous profit at period t of firm i can be written as

$$\Phi_{jt}(\tilde{P}_{jt}) = \tilde{P}_{jt}^{-\theta}(\bar{R}_{jt}^{1-\theta} - \tilde{P}_{jt}^{1-\theta})(\tilde{P}_{jt} - AW_t)C_t/(\overline{z}_j - \underline{z}_j), \tag{27}$$

when the realized value at period t of the idiosyncratic shock is $A_t(i) = A$. The maximization of this one-period profit with respect to price can be written as

$$AW_t^N(\theta \bar{R}_{jt}^{1-\theta} - (2\theta - 1)P_{j,t}^{1-\theta}) = (1-\theta)P_{jt}(2P_{j,t}^{1-\theta} - \bar{R}_{jt}^{1-\theta}). \tag{28}$$

Furthermore, the optimization condition for prices specified above can be rewritten as

$AW_t^N = M(P_{jt}, \bar{R}_{jt})$, where $M(P_{jt}, \bar{R}_{jt})$ is defined as

$$M(P_{jt}, \bar{R}_{jt}) = P_{j,t}\frac{2 - (\bar{R}_{jt}/P_{jt})^{1-\theta}}{1 + \theta((\bar{R}_{jt}/P_{jt})^{1-\theta} - 1)/(1-\theta)}. \quad (29)$$

Thus, we can use this representation of profit maximization condition to characterize the distribution of nominal prices denoted by $F(P_{jt})$. For example, suppose that firm-specific shocks are uniformly distributed over a compact interval, as we did above. Then, the resulting distribution of prices can be written as follows:

$$F(P_{jt}) = \frac{M(P_{j,t}, \bar{R}_{jt}) - M(P_{\min,jt}, \bar{R}_{jt})}{M(P_{\max,jt}, \bar{R}_{jt}) - M(P_{\min,jt}, \bar{R}_{jt})}, \quad (30)$$

where $P_{\max,jt}$ is the price that satisfies the profit maximization condition when $A = A_{\max}$ and $P_{\min,jt}$ is the price that satisfies the profit maximization condition when $A = A_{\min}$.

3.6 Relative Price Distortion

Having described the distribution of prices, we discuss the distortion that arises because of the price dispersion induced by firm-specific shocks, namely relative price distortion. Specifically, the relative price distortion is defined as the part of output that is foregone because of price dispersion.

Before proceeding further, we define the real aggregate output of type j goods in terms of the shadow value of composite consumption goods denoted by Λ_t. In order to do this, we deflate the nominal output of individual firms by Λ_t and then aggregate these deflated outputs across firms to yield

$$Y_{jt} = \frac{\Lambda_t^{2\theta-2} C_t}{\bar{z}_j - \underline{z}_j} \int_{P_{\min,jt}}^{P_{\max,jt}} P_{jt}^{1-\theta}\left(\frac{\bar{R}_{jt}^{1-\theta}}{1-\theta} - \frac{P_{jt}^{1-\theta}}{1-\theta}\right) dF(P_{jt}), \quad (31)$$

where Y_{jt} is the real aggregate output for type j goods. The real aggregate output is thus defined as $Y_t = \int_0^1 Y_{jt} dj$. In addition, when the aggregate market clearing condition, $C_t(1 + S_t) = Y_t$, holds at an equilibrium, the definition of the aggregate output specified above implies that the following condition holds:

$$1 = \frac{\Lambda_t^{2\theta-2}}{1 + S_t} \int_0^1 \int_{P_{\min,jt}}^{P_{\max,jt}} \frac{P_{jt}^{1-\theta}}{\bar{z}_j - \underline{z}_j}\left(\frac{\bar{R}_{jt}^{1-\theta}}{1-\theta} - \frac{P_{jt}^{1-\theta}}{1-\theta}\right) dF(P_{jt}) dj. \quad (32)$$

Furthermore, the relationship between the aggregate output in island j and its total labor input can be written as $Y_{jt}\Delta_{jt} = H_{jt}$, where Δ_{jt} denotes the measure of relative price distortion and H_{jt} denotes the aggregate labor input for type j goods. Given that individual market clearing conditions hold, the following equation should hold

$$\Delta_{jt} Y_{jt} = \frac{\Lambda_t^{2\theta-1} C_t}{\bar{z}_j - \underline{z}_j} \int_{P_{\min,jt}}^{P_{\max,jt}} A P_{jt}^{-\theta}\left(\frac{\bar{R}_{jt}^{1-\theta}}{1-\theta} - \frac{P_{jt}^{1-\theta}}{1-\theta}\right) dF(P_{jt}). \quad (33)$$

Combining these two equations, we have the following equation for the relative price distortion:

$$\Delta_{jt} = \Lambda_t \frac{\int_{P_{\min,jt}}^{P_{\max,jt}} AP_{jt}^{-\theta}(\bar{R}_{jt}^{1-\theta} - P_{jt}^{1-\theta})dF(P_{jt})}{\int_{P_{\min,jt}}^{P_{\max,jt}} P_{jt}^{1-\theta}(\bar{R}_{jt}^{1-\theta} - P_{jt}^{1-\theta})dF(P_{jt})}. \quad (34)$$

In addition, the aggregate production function can be written as

$$Y_t = H_t/\Delta_t, \quad (35)$$

where H_t $(= \int_0^1 H_{jt}dj)$ is the aggregate amount of hours worked and Δ_t is the relative price distortion:

$$\Delta_t = \int_0^1 (Y_{jt}/Y_t)\Delta_{jt}dj. \quad (36)$$

3.7 Numerical Example on Quasi-Kinked Demand Curve

In this section, we present a numerical example of the demand curve that is implied by the model. In doing so, we assume that the preference of each household is represented by an additively separable utility between consumption and leisure of the form:

$$U(C_t, \bar{H} - H_t) = \log C_t + b(\bar{H} - H_t). \quad (37)$$

The utility maximization of each household then leads to the following equation:

$$C_t = bW_t. \quad (38)$$

It is also possible to have an exact closed-form solution to the model in the case of $\theta = 0$. The resulting equilibrium conditions are described in Table 4.

As shown in Figure 1, we compare search-based and utility-based demand curves of individual firms. In order to do this, we compute equilibrium price distributions for cases in which $\theta = 0$ and $\theta = 1/2$, respectively. The left column corresponds to $\theta = 0$ and the right column corresponds to $\theta = 1/2$. In addition, as a benchmark calibration, we set $\bar{A} = 1.80$, $\underline{A} = 0.20$ for the support of idiosyncratic cost shocks and $z_{\max} = 0.035$ and $z_{\min} = 0.025$ for the support of search cost parameter. Under these parameter values, the share of the aggregate search cost in real output turns out to be around 10% for the case of $\theta = 1/2$.

Figure 1 indicates that the price-elasticity of the demand curve facing individual firms is larger than that of household's expenditures on each type of differentiated goods. For example, $\theta = 0$ generates $\epsilon = 3.91$, while $\theta = 1/2$ generates $\epsilon = 3.52$ where ϵ is the price-elasticity measured at the mean of equilibrium relative prices. We thus find that adding search behavior of customers to the model enables us to reconcile the general price inelasticity of household expenditures with measures of markup greater than one in

Table 4: Symmetric Equilibrium Conditions in the Case of $\theta = 0$

Relative Price Distortion

$$\Delta_t = \frac{1}{W_t}\frac{9\tilde{R}_t(\tilde{P}_{\max,t}+\tilde{P}_{\min,t})-6\tilde{R}_t^2-4(\tilde{P}_{\max,t}^2+\tilde{P}_{\min,t}^2+\tilde{P}_{\max,t}\tilde{P}_{\min,t})}{3\tilde{R}_t(\tilde{P}_{\max,t}+\tilde{P}_{\min,t})-2(\tilde{P}_{\max,t}^2+\tilde{P}_{\min,t}^2+\tilde{P}_{\max,t}\tilde{P}_{\min,t})}$$

Marginal Value of Composite Consumption Goods

$$1 = (6(\bar{z}-\underline{z})(1+X_t))^{-1}(3\tilde{R}_t(\tilde{P}_{\max,t}+\tilde{P}_{\min,t}) - 2(\tilde{P}_{\max,t}^2+\tilde{P}_{\min,t}^2+\tilde{P}_{\max,t}\tilde{P}_{\min,t}))$$

Share of Search Cost in Aggregate Real Consumption

$$S_t = \{(1/2)(\bar{z}^2 - z_t^{*2}) + (2^{1/2}/3)(z_t^{*3/2} - \underline{z}^{3/2})\sqrt{\tilde{P}_{\max,t}-\tilde{P}_{\min,t}}\}/(\bar{z}-\underline{z})$$

$$z_t^* = (1/2)(\tilde{P}_{\max,t}-\tilde{P}_{\min,t})$$

Aggregate Production Function

$$Y_t = H_t/\Delta_t$$

Aggregate Market Clearing

$$Y_t = C_t(1+S_t)$$

Aggregate Labor Supply

$$C_t = bW_t$$

Maximum Real Price

$$A_{\max}W_t = 2\tilde{P}_{\max,t} - \tilde{R}_t$$

Minimum Real Price

$$A_{\min}W_t = 2\tilde{P}_{\min,t} - \tilde{R}_t$$

Maximum Reservation Price

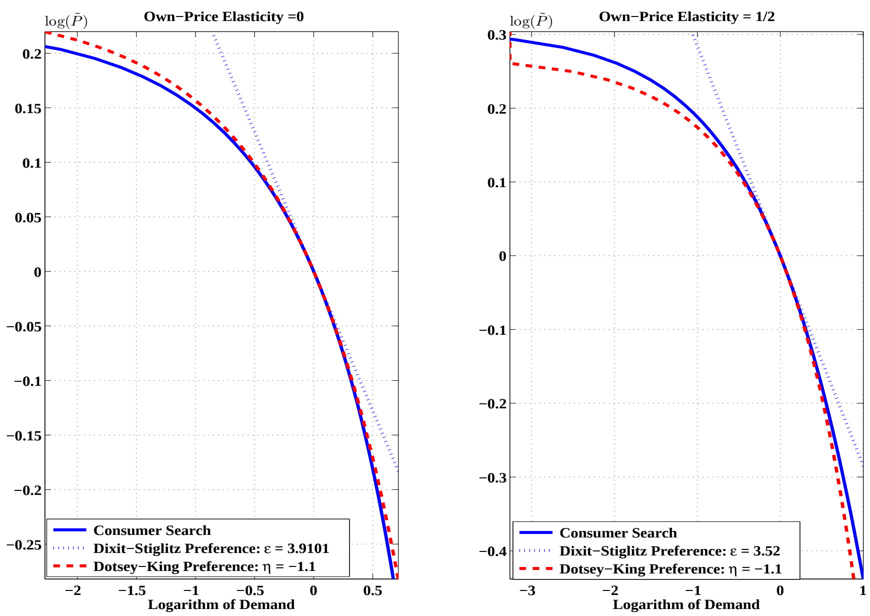
$$\bar{z} = \tilde{R}_t - (\tilde{P}_{\max,t}+\tilde{P}_{\min,t})/2$$

Note: this table includes 9 equations for 9 variables such as $\tilde{R}_t$ $\tilde{P}_{\max,t}$, $\tilde{P}_{\min,t}$, W_t, Y_t, C_t, Δ_t, S_t, and H_t, when $\theta = 0$.

industry data. Figure 1 also shows that the elasticity of each seller's demand is higher in the case of $\theta = 0$ than in the case of $\theta = 1/2$. The reason for this is that the number of customers can be more responsive to changes in prices in models with lower own-price elasticities of household expenditures, especially when $\theta \leq 1$.

Furthermore, we can see that as price rises, the elasticity of seller's demand curve becomes more elastic. In particular, the model's implied demand curves tend to be similar with those derived from the Dotsey-King's aggregator by setting its curvature parameter

Figure 1: Search-Based and Utility-Based Quasi-Kinked Demands



Note: this figure compares quasi-kinked demand curves that are derived from the Dotsey-King aggregator and the search model analyzed in this paper. In order to compare three different demand curves, we adopt a normalization to make the logarithm of demand in the search model become zero when $\log \tilde{P} = 0$, where $\log \tilde{P}$ denotes the logarithm of real price. The own-price elasticity of household expenditure is denoted by θ in the text.

η = - 1.1. In relation to this, a negative value of η amounts to the presence of a satiation level for each type of differentiated goods under the Dotsey-King's aggregator and this satiation level helps to reduce increases of consumption expenditures on goods in response to the reduction in their relative prices. This summarizes the mechanism behind quasi-kinked demand curves derived from the Dotsey-King's aggregator. Meanwhile, potential customers do not know exact locations of price decreases of sellers who they do not trade in models with customer search. As a result, the number of customers who gather because of price decreases is smaller than the number of existing customers who flee from sellers when they raise their prices, thereby leading to quasi-kinked demand curves.

4 Equilibrium Price Dispersion under Search and Menu Costs

In this section, we add a fixed cost of price adjustment to the benchmark model described above, so that equilibrium price dispersion arises from not only firm-specific productivity shocks but also fixed costs of price changes. In addition, we focus on the case where demands of households are completely inelastic ($\theta = 0$), in order to obtain exact solutions of the model. It is then shown that there exists a staggered Markov perfect equilibrium in which prices are determined by a simple two-sided (s,S) strategy for the ratio of current-period's price to the optimal price following the approach used in Danziger (1999).

4.1 Characterization of Optimal (s, S) Strategy

In order to see this, we define $\mu_{jt}(A)$ as the ratio of a firm's nominal price to the sum of the maximum reservation price of its potential customers and its marginal production cost when the firm's idiosyncratic cost is realized as $A_t = A$:

$$\mu_{jt}(A) = 2P_{jt}/(\bar{R}_{jt} + AW_t^N). \tag{39}$$

In addition, recall that under full price flexibility, the profit-maximizing nominal price is $P_{jt}^* = (\bar{R}_{jt} + AW_t^N)/2$. It thus follows from this equation that $\mu_{jt}(A)$ can be interpreted as the ratio of the current nominal price to the optimal price under full price flexibility. The real fixed cost of price adjustment is given by ξW_t for a positive number ξ, which means that menu-cost is proportional to real wage. Furthermore, we assume that the nominal wage is the same as the nominal money stock, following Golosov and Lucas (2007). We also continue to assume that the period utility function of households is $U(C_t, H_t) = \log C_t + b(\bar{H} - H_t)$.

Given this set of assumptions, we deflate one-period real profit by real wage where the one-period real profit is defined as the ratio of one-period nominal profit to the money stock.[6] We also abstract from subscript j for each variable hereafter in order to maintain the simplicity of notation. Then, the resulting real profit, net of menu cost, can be written as

$$\phi(\mu, A, R) = -(\frac{R + A}{2})^2\mu^2 + 2(\frac{R + A}{2})^2\mu, \tag{40}$$

where $R = \bar{R}/W^N$. In order to describe pricing behaviors of firms, let v_{nc} be the value function when an individual firm does not change their prices and v_c be the value function when an individual firm changes its price. In addition, when firms do not change prices in

[6] More precisely, one-period nominal profit is $\Phi_{jt} = \{-P_{jt}^2 + (\bar{R}_{jt} + AW_t^N)P_{jt} + A\bar{R}_{jt}W_t^N\}\frac{C_t}{\Lambda_t}$. We then define ϕ_{jt} as the one satisfying the following equation: $U_1(C_t, H_t)(\Phi_{jt}/W_t^N) = (\phi_{jt} + AR_{jt})W_t$.

the next period, the following relation holds:

$$\mu' = \frac{\mu}{1+g}\frac{R+A}{R+A'}, \tag{41}$$

where μ' is the next-period value of μ for firms that do not change and g is the growth rate of money supply. Hence, the two value functions discussed above can be written as follows.

$$v_{nc}(\mu, A, R) = \phi(\mu, A, R) + \beta \int \max\{v_{nc}(\frac{\mu}{1+g}\frac{R+A}{R+A'}, A', R), v_c(A', R)\}dG(A'), \tag{42}$$

where $G(A)$ is the distribution of A and $v_c(A, R)$ is defined as

$$v_c(A, R) = \max_{\mu}\{v_{nc}(\mu, A, R) - \xi\}. \tag{43}$$

We now guess that if prices are determined by a simple two-sided (s,S) strategy for μ, the value function of those firms that do not change prices can be written as

$$v_{nc}(\mu, A, R) = (\frac{R+A}{2})^2\{Q - \tau(\mu - s)(\mu - S)\}, \tag{44}$$

where Q and τ will be determined. The undetermined coefficient method is then used to determine S and s as follows:

$$\int (R+A)S\ dG(A) = \kappa(R+A^e) + \frac{\sqrt{\xi/\tau}}{2}; \quad \int (R+A)s\ dG(A) = \kappa(R+A^e) - \frac{\sqrt{\xi/\tau}}{2}, \tag{45}$$

where $A^e = \int A\ dG(A)$ and parameters κ and τ are given by

$$\kappa = \frac{(1+g)^2 - \beta}{(1+g)(1+g-\beta)}; \quad \tau = \frac{(1+g)^2}{(1+g)^2 - \beta}. \tag{46}$$

For example, if A is a permanent characteristic of an individual firm so that $A = A^e$, then upper and lower bounds of μ turn out to be

$$S = \kappa + \frac{\sqrt{\xi/\tau}}{2(R+A)}; \quad s = \kappa - \frac{\sqrt{\xi/\tau}}{2(R+A)}. \tag{47}$$

4.2 Numerical Example

We now present simulation results of simple numerical examples. Table 5 summarizes a set of steady-state equilibrium conditions used in simulations, which hold in the absence of idiosyncratic cost shocks.

In order to obtain simulation results, we assign numerical values to a set of parameters. As the benchmark calibration, we set $\xi = 0.007$ which implies that the fixed cost of price adjustment is 0.7 % of real wage. The weight for leisure in the utility function set to be b

Table 5: Steady-State Equilibrium Conditions in the Case of $\theta = 0$

Relative Price Distortion

$$\Delta = \frac{6\tilde{R} - 3(\tilde{P}_{\max} + \tilde{P}_{\min})}{3\tilde{R}(\tilde{P}_{\max} + \tilde{P}_{\min}) - 2(\tilde{P}^2_{\max} + \tilde{P}^2_{\min} + \tilde{P}_{\max}\tilde{P}_{\min})}$$

Marginal Value of Composite Consumption Goods

$$1 = (6(\bar{z} - \underline{z})(1 + X + \alpha\xi/b))^{-1}(3\tilde{R}(\tilde{P}_{\max} + \tilde{P}_{\min}) - 2(\tilde{P}^2_{\max} + \tilde{P}^2_{\min} + \tilde{P}_{\max}\tilde{P}_{\min}))$$

Share of Search Cost in Aggregate Real Consumption

$$V = \{(1/2)(\bar{z}^2 - z^{*2}) + (2^{1/2}/3)(z^{*3/2} - \underline{z}^{3/2})\sqrt{\tilde{P}_{\max} - \tilde{P}_{\min}}\}/(\bar{z} - \underline{z})$$

$$z^* = (1/2)(\tilde{P}_{\max} - \tilde{P}_{\min})$$

Fraction of Firms that Change Prices

$$\alpha = (g/(1+g))\{\kappa - (1/2)(W/(\tilde{R} + W))\sqrt{\xi/\tau}\}$$

$$\tau = (1+g)^2/((1+g)^2 - \beta)$$

Aggregate Production Function

$$Y = H/\Delta$$

Aggregate Market Clearing

$$Y = C(1 + X + \alpha\xi/b)$$

Aggregate Labor Supply

$$C = bW$$

Maximum Real Price

$$\tilde{P}_{\max} = \kappa(\tilde{R} + W)/2 + (W/4)\sqrt{\xi/\tau}$$

Minimum Real Price

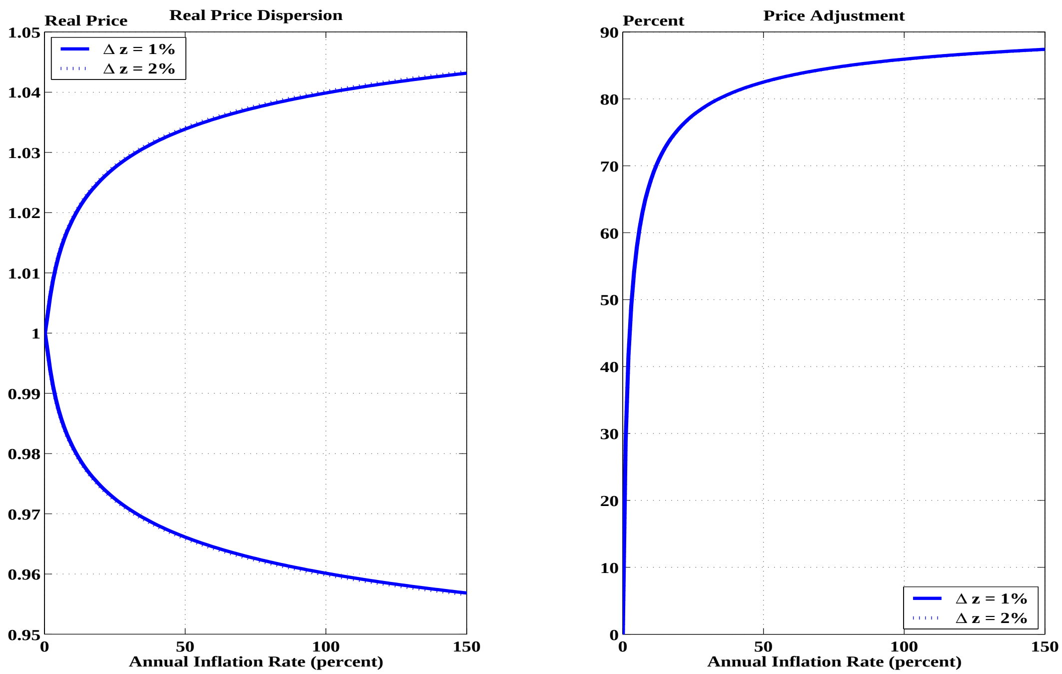
$$\tilde{P}_{\max} = \kappa(\tilde{R} + W)/2 - (W/4)\sqrt{\xi/\tau}$$

Maximum Reservation Price

$$\bar{z} = \tilde{R} - (\tilde{P}_{\max} + \tilde{P}_{\min})/2$$

Note: this table includes 10 equations for 10 variables such as $\tilde{R}$, $\tilde{P}_{\max}$, $\tilde{P}_{\min}$, W, Y, C, Δ, V, α, and H. The parameter κ is defined as $\kappa = \frac{(1+g)^2 - \beta}{(1+g)(1+g-\beta)}$, These steady-state equilibrium conditions hold true in the absence of firm-specific cost shocks. In addition, the consumer price index, denoted by Λ, is used to describe these equilibrium conditions, while the mean of firms' prices can be used as an alternative price index. The equilibrium conditions at the steady-state with zero inflation can be written as $\Delta = 1$, $W = 1 - \bar{z}$, $\tilde{R} = 1 + \bar{z}$, $Y = H = C = b(1 - \bar{z})$, and $\tilde{P}_{\max} = \tilde{P}_{\min} = 1$.

Figure 2: Effect of Inflation on Price Dispersion and Adjustment



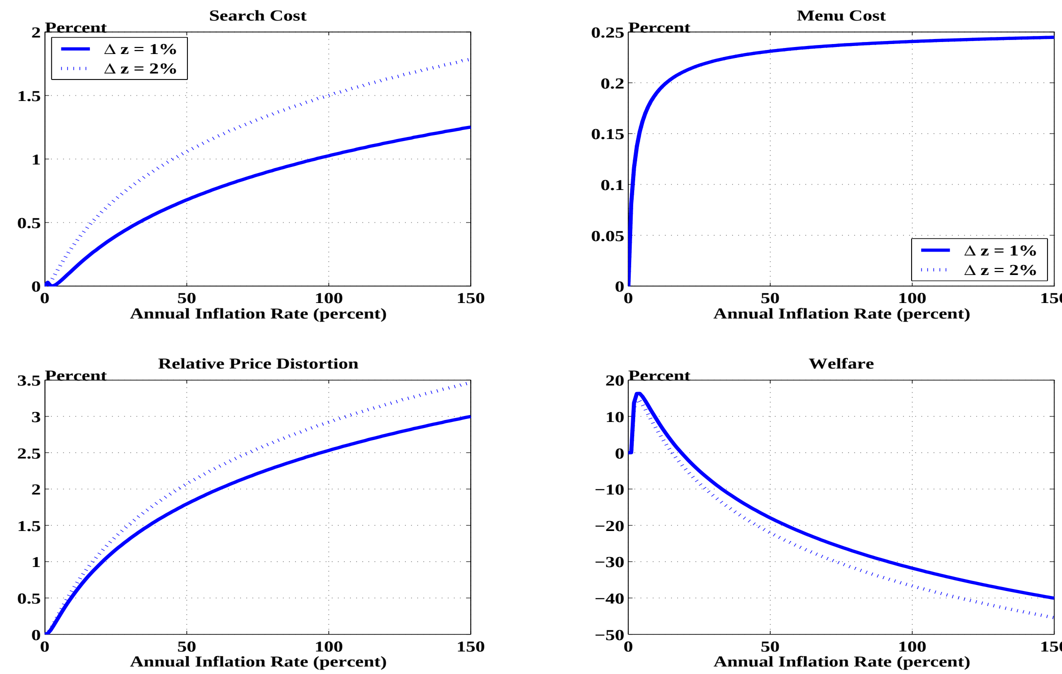
Note: this figure depicts effects of inflation on real price dispersion and price adjustment. In this figure, we allow for the mean-preserving spread of search costs where the mean of search costs is fixed to be 1 percent of household expenditures. Δz denotes the spread between the maximum and minimum of search costs. The solid line represents a difference of 1 percent and the dotted line corresponds to 2 percent. It is assumed in this figure that firm-specific cost shocks do not exist.

= 2.5 and the time-discount factor is given by $\beta = 0.99$ that corresponds to 4 % annual rate of real interest. The mean of search costs is set to be 1 % of household expenditures. All of these numbers may not be realistic but we point out that the purpose of simulation in this section is not to match actual date but falls onto having some intuition about how the model economy described in this section works.

Figure 2 depicts effects of mean preserving spread of search costs on real price dispersion and price adjustment. Δz denotes the spread between the maximum and minimum of search costs. The solid line represents a difference of 1 percent and the dotted line corresponds to 2 percent. As shown in Figure 2, as the rate of steady-state inflation rises, real price dispersion increases and prices are changed more frequently. In addition, the effect of mean preserving spread of search cost on the real price dispersion seems to be small[7]

[7] Konieczny and Skrzypacz (2006) show that the ratio of the initial to terminal real price - the ratio of S to s - can be increasing in search costs represented by their maximum value under a set of conditions. But they do not take into account possible interactions between search cost and marginal cost of production

Figure 3: Effect of Search Costs on Inflation Costs and Welfare



Note: the upper panel of this figure depicts fractions of search and menu costs in the aggregate consumption respectively. The right column in the lower panel describes the percentage deviation of relative price distortion from its level at zero inflation and the left column displays the percentage deviation of welfare from its level at zero inflation. Δz denotes the spread between the maximum and minimum of search costs. The solid line represents a difference of 1 percent and the dotted line corresponds to 2 percent. The mean of search costs is set to be 1 percent of household expenditures. It is assumed in this figure that firm-specific cost shocks do not exist.

Since inflation increases price dispersion, a higher inflation results in increased search and hence more resources are devoted to search as discussed in Benabou (1992). As shown in Figure 3, we also confirm that the fraction of the aggregate search cost in the aggregate consumption rises as the rate of inflation increases. Moreover, in this example, the real price dispersion increases as mean-preserving spread rises, so that the ratio of the aggregate search cost to the aggregate consumption increases as the maximum value of search costs rises. Another consequence of increased real price dispersion is that relative price distortion rises as inflation increase, where the relative price distortion is defined as the fraction of output that is foregone because of relative price dispersion. In relation to this, Figure 3 shows that relative price distortion rises as inflation rises.

We now turn to the relationship between inflation and welfare that is implied by the

that can take place in a general equilibrium model. The effect of mean-preserving spread of search costs on price dispersion and other variables can change depending on parameter values.

model. In Figure 3, the left column displays the percentage deviation of welfare from its level at zero inflation. Figure 3 indicates that the utility function of households achieves its peak at a mild positive inflation and then declines as inflation increases further. In particular, this result is consistent with Diamond (1993) who demonstrates that consumer welfare is inverse U-shaped in inflation with a strictly positive optimal inflation rate in an equilibrium model where consumer search and firms set prices. But it would be worthwhile to mention that in our example, preference parameters critically affect whether the steady state welfare is maximized at a positive inflation. For example, the steady-state welfare tends to be maximized at a positive inflation when the coefficient of leisure in the utility function, denoted by b, is higher than around 2 given the set of parameter values used in the simulation.[8]

5 Directions for Future Research

We have incorporated product differentiation and consumer search in a general equilibrium model. In the model of this paper, own-price elasticities of household expenditures are completely determined by an elasticity of substitution over differentiated goods. It is thus interesting to allow for the possibility of non-zero cross-price elasticities in household expenditures. An advantage of doing this would be that one can use disaggregated data on household expenditures and transaction prices in the estimation of parameters of the model in order to develop an equilibrium model that better describes responses of households with respect to changes in prices as well as the behavior of price changes.

In this paper, we have presented a very simple example of a state-dependent pricing model in order to focus on the exact solution of the model. But since characteristics of consumer search can affect the timing of price changes for individual firms, it would be interesting to extend the analysis to a more complicated model that include stochastic exogenous shocks.

Furthermore, this type of modeling strategy can be used in time-dependent pricing models such as Calvo-type, Fisher-type, and Taylor-type staggered price-setting models. In this case, as noted earlier, the incorporation of consumer search into such models can affect endogenous responses of prices with respect to marginal cost.

[8] Benabou (1992) demonstrates that whether inflation is beneficial or harmful to social welfare depends on preferences and market structure, and in particular, on whether search costs are low or high relative to consumer surplus, when welfare is defined as the sum of aggregate consumer and producer surplus.

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