# The Kinked Demand Curve and Price Rigidity: Evidence from Scanner Data\*

Maarten Dossche<sup>†</sup> Freddy Heylen<sup>‡</sup> Dirk Van den Poel<sup>§</sup>

January 26, 2007

#### Abstract

This paper uses scanner data from a large euro area retailer. We extend Deaton and Muellbauer's Almost Ideal Demand System to estimate the price elasticity and curvature of demand for a wide range of products. Our results support the introduction of a kinked (concave) demand curve in general equilibrium macro models. We find that the price elasticity of demand is on average higher for price increases than for price decreases. However, the degree of curvature in demand is much lower than is currently imposed. Moreover, for a significant fraction of products we observe a convex demand curve.

**JEL**: C33, D12, E3

Key words: price setting, real rigidity, kinked demand curve, behavioral AIDS

<sup>\*</sup>This paper has been written with the financial support of the National Bank of Belgium in preparation of its biennial conference "Price and Wage Rigidities in an Open Economy", October 2006. We are grateful to Filip Abraham, Luc Aucremanne, Gerdie Everaert, Joep Konings, Daniel Levy, Morten Ravn, Dirk Van de gaer and Raf Wouters for helpful comments and suggestions. We have also benefited from discussions during seminars at Universitat Pompeu Fabra, University of Antwerp and the National Bank of Hungary. All errors are our own. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the National Bank of Belgium.

<sup>&</sup>lt;sup>†</sup>Corresponding author. National Bank of Belgium and SHERPPA, Ghent University, Maarten.Dossche@nbb.be, http://www.sherppa.be

<sup>&</sup>lt;sup>‡</sup>SHERPPA, Ghent University, Freddy.Heylen@UGent.be, http://www.sherppa.be

<sup>§</sup>Marketing Department, Ghent University, Dirk.VandenPoel@UGent.be, http://www.crm.ugent.be

#### 1 Introduction

A large literature documents the persistent effects of monetary policy on real output and inflation (Christiano et al., 1999, 2005; Peersman, 2004). Given the key role of price rigidity to explain this persistence, micro-based models of price setting have been developed for macro models. A first approach has been to introduce frictions to nominal price adjustment (e.g. Taylor, 1980; Calvo, 1983; Mankiw, 1985). However, as shown by several authors, the real effects of nominal frictions do not last much longer than the average duration of a price (Chari et al., 2000; Bergin and Feenstra, 2000). Taking into account recent microeconomic evidence that the mean price duration in the United States is only about 1.8 quarters, while in the euro area it is only 4 to 5 quarters (Bils and Klenow, 2004; Dhyne et al., 2006), nominal frictions alone clearly fail to generate the real persistence observed in the data.

The failure of nominal frictions has led to the development of models which combine nominal and real price rigidities (Ball and Romer, 1990). Real rigidities refer to a firm's reluctance to adjust its price in response to changes in economic activity if other firms do not change their prices. Either supply side or demand side factors can explain this reluctance to carry out significant price changes. Blanchard and Galí (2006), among others, obtain real rigidities from the supply side by modelling rigid real wages. Bergin and Feenstra (2000) adopt the production structure proposed by Basu (1995). Real price rigidity follows from the assumption that firms use the output of all other firms as materials in their own production. Many other authors point to firm-specific factors of production (e.g. Galí and Gertler, 1999; Sbordone, 2002; Woodford, 2003; Altig et al., 2005). Although these supply side assumptions generally raise the capacity of calibrated models to match the data, they never are completely convincing. The stylized fact that real wages are procyclical may be a problem for models emphasizing wage rigidity. Prices seem to change even less than wages in response to changes in economic activity (Rotemberg and Woodford, 1999). Models putting firm-specific factors of production at the center only seem

to match the micro evidence on price adjustment by assuming either an unrealistically steep marginal cost curve or an unrealistically high price elasticity of demand.<sup>1</sup> Bergin and Feenstra (2000) do not need unrealistic price elasticities. However, their model performs best when they also introduce Kimball (1995) preferences and a concave demand curve.

The specification of Kimball preferences has become the most successful way to obtain real price rigidity from the demand side in recent research.<sup>2</sup> In contrast to the traditional Dixit and Stiglitz (1977) approach, Kimball (1995) no longer assumes a constant elasticity of substitution in demand. The price elasticity of demand becomes a function of relative prices. A key concept is the so-called curvature, which measures the relative price elasticity of the price elasticity. When the curvature is positive, Kimball preferences generate a concave or smoothed "kinked" demand curve in a log price/log quantity framework. This may create real price rigidity. Intuitively, assume an increase in aggregate demand which raises a firm's marginal cost due to higher wages. If the firm were free to change its price, it would raise it. However, if a price above the level of its competitors strongly increases the elasticity of demand for the firm's product, the firm can lose profits from strong price changes. Inversely, in the case of a fall in marginal cost, if a reduction in the firm's price strongly reduces the elasticity of demand, the firm can again lose profits from drastic price changes. Price rigidity is a rational choice.

Despite its attractiveness, the literature suffers from a remarkable lack of empirical evidence on the existence of the kinked (concave) demand curve and on the size of its curvature. In Table 1 we report the parameter values for the price elasticity of demand and for the curvature, both at steady state, as imposed in recent model calibrations. Values for the (positive) price elasticity range from 3 to 20. Values for the curvature range from less than 2 to more than 400.

<sup>&</sup>lt;sup>1</sup>For example Altig et al. (2005) require a (positive) price elasticity of demand above 20 for their model to match the micro evidence on price adjustment. Most of the empirical studies, however, reveal much lower elasticities. Bijmolt et al. (2005) present a meta-analysis of the price elasticity of demand. Across a set of 1851 estimated price elasticities based on 81 studies, the median (positive) price elasticity is 2.2. The empirical evidence that we will report in this paper confirms that the price elasticity of demand is much lower than the elasticity required by Altig et al. (2005).

<sup>&</sup>lt;sup>2</sup>See e.g. Bergin and Feenstra (2000), Coenen and Levin (2004), Eichenbaum and Fisher (2004), de Walque, Smets and Wouters (2006), Dotsey and King (2005), Dotsey, King and Wolman (2006), Klenow and Willis (2006).

Table 1: Price Elasticity and Curvature of Demand in the Literature

	price elasticity	curvature
Kimball (1995)	11	$471^{(a)}$
Chari, Kehoe and McGrattan (2000)	10	$385^{(a)}$
Bergin and Feenstra (2000)	3	$1.33^{(a)}$
Eichenbaum and Fisher (2004)	11	10, 33
Coenen and Levin (2004)	5 - 20	10, 33
Woodford (2005)	7.67	$6.67^{(a)}$
de Walque, Smets and Wouters (2006)	3	20, 60
Klenow and Willis (2006)	5	10

Note: Curvature is defined as the elasticity of the price elasticity of demand with respect to the relative price at steady state. Several authors characterize curvature differently. In Appendix 1 we derive the relationships between alternative definitions of curvature. The numbers indicated with (a) have been calculated using these relationships. It is often argued in the literature that Kimball (1995) would have imposed a curvature equal to 33 (see Eichenbaum and Fisher, 2004; Coenen and Levin, 2004). Our calculations show however that Kimball's curvature, as we have consistently defined it, must be much larger.

Our contribution in this paper is twofold. First, we test the theory of the kinked (concave) demand curve. We investigate whether the price elasticity of demand does indeed rise in the relative price. Our second contribution is to estimate this price elasticity and especially the curvature of the demand curve. Our results should be able to reduce the uncertainty in the literature surrounding these parameters. To do this, we need data on both prices and quantities. We use a scanner dataset from a large euro area supermarket chain. The strength of this dataset is that it contains information about prices and quantities sold of about 15,000 items in 2002-2005.<sup>3</sup> Moreover, since a supermarket supplies many substitutes for each item at the same place, it may constitute the ideal environment to estimate price elasticities and curvatures. Correspondence to the Dixit-Stiglitz and Kimball setting where consumers hold preferences over a continuum of differentiated goods can hardly be closer. Section 2 of the paper describes the dataset in greater detail. We also analyze key properties of the data like the size and frequency of price changes, the correlation between price and quantity changes as one indicator for the importance of demand versus supply shocks and the (a)symmetry in the observed price elasticity of demand for price increases versus decreases. Section 3 of the paper presents a much more

<sup>&</sup>lt;sup>3</sup>Note that the *items* that are sold by our retailer can be differently packaged goods of the same *brand*. All items and/or brands in turn belong to a particular *product category* (e.g. potatoes, detergent).

rigorous econometric analysis of price elasticities and curvature parameters for individual items. To that end we extend the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) by introducing assumptions drawn from behavioral decision theory. Our behavioral AIDS model allows for a more general curvature, which is necessary to answer our research questions. We follow Hausman (1997), using a panel data model, to estimate our demand system. Section 4 concludes the paper.

Our main results are as follows. First, we find wide variation in the estimated price elasticity and the curvature of demand among items/product categories. Although demand for the median item is concave, the fraction of items showing convex demand is substantial. Second, our results support the introduction of a kinked (concave) demand curve in general equilibrium macro models. However, the degree of curvature is much lower than is currently imposed. Our suggestion would be to impose a curvature parameter around 4. Third, with curvature being much lower than generally imposed, the kinked demand curve alone fails to generate sufficient real price rigidity. There must be complementary ingredients. Fourth, we find no correlation between the estimated price elasticity/curvature and the observed size or frequency of price adjustment in our data. Our specific context of a multi-product retailer may however explain this lack of correlation.

#### 2 Basic Facts about the Data

#### 2.1 Description of Dataset

We use scanner data for a sample of six outlets of an anonymous large euro area supermarket chain. This retailer carries a very broad assortment of about 15,000 different items (stockkeeping units). The products in the total dataset correspond to approximately 40% of the euro area CPI. The data that we use in this paper are prices and total quantities sold per outlet of 2274 individual items belonging to 58 randomly selected product categories. Appendix 2 describes these categories and the number of items in each product category. The time span of our data

runs from January 2002 to April 2005. Observations are bi-weekly. Prices are constant during each period of two weeks. They are the same in each of the six outlets. The quantities are the number of packages of an item that are sold during a time period.

#### 2.2 Nominal Price Adjustment

The nominal price friction in our dataset is that prices are predetermined for periods of at least two weeks. If they are changed at the beginning of a period of two weeks, they are not changed again before the beginning of the next period of two weeks, irrespective of demand. A second characteristic of our data is the high frequency of temporary price markdowns. We define the latter as any sequence of three, two or one price(s) that is below both the most left adjacent price and the most right adjacent price.<sup>4</sup> The median item is marked down for 8% of the time, whereas 27% of the median item's output is sold at times of price markdowns. In line with the previous, price markdowns are valid for an entire period, and not just for a few days.

Using the prices in the dataset, we can estimate the size of price adjustment, the frequency of price adjustment and median price duration as has been done in Bils and Klenow (2004) and Dhyne et al. (2006). Table 2 contains these statistics. The total number of items involved is 2274. Note that due to entry or exit we do not observe data for all items in all periods. We calculate price adjustment statistics including and excluding temporary price markdowns. When an observed price is a markdown price, we replace it by the last observed regular price (see also Klenow and Kryvtsov, 2005). We illustrate our procedure in Appendix 3.

Conditional on price changes taking place and including markdowns, we see in Table 2 that 25% of the items have an average absolute price change of less than 5%. At the other end, 25% have an average absolute price change of more than 17%. The median item has an average absolute price change of 9%. Filtering out markdowns, the latter falls to 5%. The size of price changes in our dataset is slightly smaller than is typically observed in the US.<sup>5</sup> As to price

<sup>&</sup>lt;sup>4</sup>This definition puts us in between Klenow and Kryvtsov (2005) and Midrigan (2006).

<sup>&</sup>lt;sup>5</sup>Excluding markdowns, Klenow and Kryvtsov (2005) report a mean absolute price change of 8%. In our data the mean price change excluding markdowns is 7%.

duration, the median item's price lasts 0.9 quarters when we include markdown periods. It lasts 6.6 quarters excluding markdown periods. Price duration in our data is longer than is typically observed in the US.<sup>6</sup>

Table 2: Nominal Price Adjustment Statistics

	Incl. markdowns		Excl.	markd	owns	
Percentile	25%	50%	75%	25%	50%	75%
Average Absolute Size	5%	9%	17%	3%	5%	8%
Implied Median Price Duration (quarters)	0.4	0.9	2.8	2.4	6.6	$\infty$

Note: The statistics reported in this table are based on bi-weekly price data for 2274 items belonging to 58 product categories from January 2002 to April 2005. The data show the average absolute percentage price change (conditional on a price change taking place) and the median price duration of the items at the 25th, 50th and 75th percentile, ordered from low to high.

#### 2.3 Real Price and Quantity Adjustment

#### Relative Importance of Demand and Supply Shocks

Table 3 presents summary statistics on real (relative) price and quantity changes over the six outlets in our dataset. All changes are again in comparison with the previous period of two weeks. The nominal price  $p_i$  of individual item i is common across the outlets. All the other data are different per outlet. Real (relative) item prices  $p_i/P^*$  have been calculated by deflating the nominal price of item i by the outlet-specific Stone price index  $P^*$  for the product category to which the item belongs.<sup>7</sup> Algebraically, the Stone price index is calculated as

$$\ln P^* = \sum_{i=1}^{N} s_i \ln p_i \tag{1}$$

with N the number of items in the product category to which i belongs,  $s_i = \frac{p_i q_i}{X}$  the outlet-specific share of item i in total nominal expenditures X on the product category,  $q_i$  the total quantity of item i sold at the outlet and  $X = \sum_{i=1}^{N} p_i q_i$ . Total outlet-specific real expenditures Q on the product category have been obtained as  $Q = X/P^*$ . Relative quantities  $q_i/Q$  show much

<sup>&</sup>lt;sup>6</sup>Bils and Klenow (2004) report a median price duration of about 1.1 quarter in US data. The rise in their median duration to about 1.4 quarters when temporary markdowns are netted out is much smaller than in our data, confirming stylized facts on price rigidity in the euro area versus the US. Furthermore, the median price duration including markdowns in our data is shorter than the 2.6 quarters for the euro area reported by Dhyne et al. (2006). Clearly, this may be related to supermarket prices being more flexible than prices in other outlets, e.g. corner shops.

<sup>&</sup>lt;sup>7</sup>As an alternative to the Stone index we have also worked with the Fisher index. The results based on this price index are reported in Appendix 4. They confirm our main findings here.

higher and much more variable percentage changes than relative prices. Including markdowns, the average absolute percentage change in relative quantity equals 59% for the median item, with a standard deviation of 77%. The average absolute percentage relative price change for the median item equals only 9%, with a standard deviation of 12%.

The underlying individual goods data also allow for a first explorative analysis of the importance of supply and demand shocks. To that aim we first calculate simple correlations per item and per outlet between the change in real (relative) item prices and the change in relative quantities sold. In case demand shocks dominate supply shocks, we should mainly find positive correlations between items' price and quantity changes. In case supply shocks are dominant, we should observe negative correlations. Next we split up the calculated variance in individual items' real price and quantity changes into a fraction due to supply shocks and a fraction due to demand shocks. The bottom rows of Table 3 show the fractions due to supply shocks. Concentrating on price changes, this fraction has been computed as

% Supply shocks to 
$$\Delta \ln(p_i/P^*) = \frac{\sum_{SS} (\Delta \ln(p_i/P^*) - \pi_i)^2}{\sum_{SS} (\Delta \ln(p_i/P^*) - \pi_i)^2} * 100$$

where  $\pi_i$  is the mean of  $\Delta \ln(p_i/P^*)$  over all periods. The numerator of this ratio includes only observations where price and accompanying quantity changes in a period have the opposite sign, revealing a supply shock (SS). The denominator includes all observations. The fraction of the variance in real price changes due to demand shocks, can be calculated as 1 minus the fraction due to supply shocks. Our results reveal that price and quantity changes are mainly driven by supply shocks. Including all data, the median item shows a clearly negative correlation between price and quantity changes equal to -0.23. Moreover, about 65% of the variance in price and quantity changes of the median item seems to follow from supply shocks.

The right part of Table 3 presents results obtained from data excluding markdown periods.

Temporary price markdowns are interesting supply shocks to identify a possibly kinked demand curve, but we do not consider them as representing idiosyncratic supply shocks such as shifts

in costs or technology.<sup>8</sup> As can be seen, the results at the right hand side of the table are fully in line with those at the left hand side.

Table 3: Importance of Demand and Supply Shocks

	Including markdowns			Excl. markdowns		
Percentile	25%	50%	75%	25%	50%	75%
Average absolute $\Delta \ln(p_i/P^*)$	6%	9%	15%	5%	8%	15%
Average absolute $\Delta \ln(q_i/Q)$	39%	59%	80%	38%	59%	79%
Standard Deviation $\Delta \ln(p_i/P^*)$	7%	12%	21%	7%	12%	21%
Standard Deviation $\Delta \ln(q_i/Q)$	52%	77%	102%	51%	77%	101%
Correlation $(\Delta \ln(p_i/P^*); \Delta \ln(q_i/Q))$	-0.49	-0.23	0.02	-0.50	-0.24	0.01
% Supply Shocks to $\Delta \ln(p_i/P^*)^{(a)}$	48%	68%	86%	48%	69%	87%
% Supply Shocks to $\Delta \ln(q_i/Q)^{(a)}$	45%	64%	81%	45%	64%	82%

Note: The statistics reported in this table are based on changes in bi-weekly data for 2274 items belonging to 58 product categories in six outlets. Individual nominal item prices  $(p_i)$  are common across the outlets, all the other data  $(P^*, q_i, Q)$  can be different per outlet. For the statistical analysis we have excluded items that are mentioned in the supermarket's circular. Items in the circular are often sold at lower price. Including them may bias the results in favor of supply shock dominance (high quantity sold, low price). For a proper interpretation, note that the median item can be different in each row of this table. (a) The contribution of demand shocks to price and quantity variability equals 1 minus the contribution of supply shocks. Computation methods are described in the main text.

An analysis of the relative importance of supply versus demand shocks is important for more than one reason. First, this is important to know in order to do a proper econometric demand analysis. One needs enough variation in supply to be able to identify a demand curve. Our results in Table 3 are obviously encouraging in this respect. The minor contribution of demand shocks should not be surprising given that prices are being set in advance or in the very beginning of the period. As long as the supplier does not know demand in advance, demand shocks cannot have an effect on prices. Second, the results of a decomposition of the variance of price changes into fractions due to demand and supply shocks may be important for a proper calibration of theoretical macro models. In order to explain large price changes, a number of authors have introduced idiosyncratic shocks in their models, affecting prices and

<sup>&</sup>lt;sup>8</sup>Note that we only exclude the item whose price is marked down, while keeping the other items. The effects of the (excluded) marked down item on the other items are thus not filtered out. If we excluded all items in periods where at least one item in the product category is marked down, we would be left with almost no observations.

<sup>&</sup>lt;sup>9</sup>When we use the concept 'supplier' we mean the retailer and the producer together. Usually prices in the retail sector are set in an agreement between the retailer and the producer, so that there is not one easily identifiable party that sets prices.

<sup>&</sup>lt;sup>10</sup>Of course, one could argue that the supplier does know in advance that demand will be high or low, so that he can already at the moment of price setting fix an appropriate price. Considering the large majority of negative correlations in Table 3, however, there is little evidence that this hypothesis would be important.

quantities (Golosov and Lucas, 2003; Dotsey, King and Wolman, 2006; Klenow and Willis, 2006). As Klenow and Willis (2006) point out, there is not much empirical evidence available that tells us whether these idiosyncratic shocks are mainly supply-driven or demand-driven. Evidence like ours on the importance of demand and supply shocks *excluding* markdowns, as well as the extent of supply and demand shocks, may be very indicative.

#### Preliminary Evidence on Asymmetric Price Sensitivity

An explorative analysis of our data may also provide a first test of the kinked demand curve hypothesis that the price elasticity of demand rises in a product's relative price. Figure 1 may be helpful to clarify our identification. Per item we relate real (relative) prices to quantities in natural logs. All relative price and quantity data have been demeaned to account for item specific fixed effects. The average is thus at the origin.

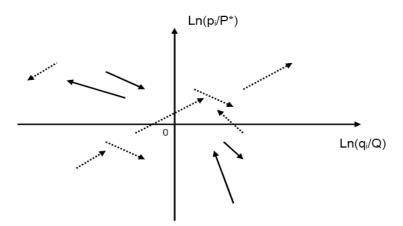


Figure 1: Identification of Asymmetry in the Demand Curve

An important element is then to use supply shocks to identify the demand curve and potential asymmetries in demand. Supply shocks should imply shifts in prices and quantities that go into opposite directions. Our approach to identify the asymmetry in the demand curve is to use only the price-quantity information that is consistent with movements along the bold arrows. In particular, we use all couples of consecutive (log relative) price-quantity observations that lie in the second or fourth quadrant and that reflect a negative slope. Each couple allows us to

calculate a corresponding price elasticity as the inverse of this slope. Price-quantity observations that do not respect this double condition (see the dotted arrows) are not taken into account. Observations along negatively sloped arrows in the first or third quadrant are not considered since it is unclear whether they took place along the (potentially) low or high elasticity part of the demand curve. The last step is to compute the median of all price elasticities that meet our conditions in the second quadrant, where the relative price is high, and to repeat this in the fourth quadrant where the relative price is low.

The data in Table 4 contain the results for the difference between these two median elasticities in absolute value ( $\varepsilon^H$  and  $\varepsilon^L$  respectively). The interpretation of the Table is analogous to earlier tables. The price elasticity of demand at high relative price is higher than at low relative price for most of the items analyzed, which would be consistent with the existence of a kinked demand curve. For the median item  $\varepsilon^H$  is about 1.3 higher than  $\varepsilon^L$ . Excluding markdowns hardly affects this result. Note however that a large fraction of items show a convex demand curve. Including markdowns this fraction is 41%, excluding markdowns it is 42%.

Table 4: Asymmetric Price Sensitivity: Difference between  $\varepsilon^H$  and  $\varepsilon^L$ 

	Including markdowns			Exclu	ding markd	owns
Percentile	25%	50%	75%	25%	50%	75%
Median $\varepsilon^H - \varepsilon^L$	-3.58	1.26	7.47	-3.75	1.17	7.27

Note:  $\varepsilon^H$  and  $\varepsilon^L$  are the absolute values of the price elasticity of demand at high and low relative prices respectively.  $\varepsilon^H > \varepsilon^L$  suggests that the demand curve is concave (smoothed "kinked"). Reported data refer to the items at the 25th, 50th and 75th percentile ordered from low to high. Items mentioned in the supermarket's circular have again been excluded from the analysis (see our note to Table 3).

Our approach here is rudimentary. A more rigorous econometric analysis, which allows us to control for other potential determinants of demand, is necessary. Yet, our results in Table 4 may shed first light on an important issue, while imposing only limited conditions on the data and without requiring any specific functional form assumptions. The evidence may already be useful for models like the one of Burstein et al. (2006), where the difference between  $\varepsilon^L$  and  $\varepsilon^H$  plays a key role in their calibration.<sup>11</sup> For the other models (e.g. Bergin and Feenstra, 2000; de

<sup>11</sup> In their basic calibration Burstein et al. impose  $\varepsilon^H = 9$  and  $\varepsilon^L = 3$ , yielding an equilibrium elasticity of

Walque et al., 2006) with a curvature parameter, we need to do a structural analysis.

## 3 How Large is the Curvature? An Econometric Analysis

In this section we estimate the price elasticity and the curvature of demand for a broad range of goods in our scanner dataset described above. We extend the Almost Ideal Demand System (AIDS) developed by Deaton and Muellbauer (1980) by introducing assumptions drawn from behavioral decision theory. Our "behavioral" AIDS model allows for a more general curvature, which is necessary to answer our research question. The model still has the original AIDS nested as a special case. For several reasons we believe the AIDS is the most appropriate for our purposes: (i) it is flexible with respect to estimating own- and cross-price elasticities; (ii) it is simple, transparent and easy to estimate, allowing us to deal with a large number of product categories; (iii) it is most appropriate in a setup like ours where consumers may buy different items of given product categories; (iv) it is not necessary to specify the characteristics of all goods, and use these in the regressions. The latter three characteristics particularly distinguish the AIDS from alternative approaches like the mixed logit model used by Berry et al. (1995). Their demand model is based on a discrete-choice assumption under which consumers purchase at most one unit of one item of the differentiated product. This assumption is appropriate for large purchases such as cars. In a context where consumers might have a taste for diversity and purchase several items, it may be less suitable. Moreover, to estimate Berry et al. (1995)'s mixed logit model, the characteristics of all goods/items must be specified. In the case of cars this is a much easier task to do than for instance for cement or spaghetti. Computational requirements of their methodology are also very demanding.

We follow the approach of Broda and Weinstein (2006) to cover as many goods as possible in order to get a reliable estimate for the aggregate curvature, useful in calibrated macro models. In Section 3.1 we first describe our extension of the AIDS model. Section 3.2 discusses our

<sup>6,</sup> and a steady state mark-up of 1.2. Considering our preliminary evidence in this Section and the evidence on the price elasticity that we referred to in Footnote 1, both the level of the imposed elasticities in Burstein et al. (2006) and the difference between  $\varepsilon^H$  and  $\varepsilon^L$  are high.

econometric setup and identification and estimation. Section 3.3 presents the results. In Section 3.4 we discuss their robustness.

#### 3.1 Model

Our extension of Deaton and Muellbauer's AIDS model is specified in expenditure share form as

$$s_i = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln p_j + \beta_i \ln \left(\frac{X}{P}\right) + \sum_{j=1}^{N} \delta_{ij} \left(\ln(\frac{p_j}{P})\right)^2$$
 (2)

for i = 1, ..., N. In this equation X is total nominal expenditure on the product category of N items being analyzed (e.g. detergents), P is the price index for this product category,  $p_j$  is the price of the jth item within the product category and  $s_i$  is the share of total expenditures allocated to item i (i.e.  $s_i = p_i q_i / X$ ). Deaton and Muellbauer define the price index P as

$$\ln P = \alpha_0 + \sum_{j=1}^{N} \alpha_j \ln p_j + \frac{1}{2} \sum_{j=1}^{N} \sum_{i=1}^{N} \gamma_{ij} \ln p_i \ln p_j$$
 (3)

Our extension of the model concerns the last term at the right hand side of Equation (2). The original AIDS model has  $\delta_{ij} = 0$ . Although this model is generally recognized to be flexible, it is not flexible enough for our purposes. As we demonstrate below, the curvature parameter, which carries our main interest, is not free in the original AIDS model. It is a very restrictive function of the price elasticity, implying that in the original AIDS model it would not be possible to obtain a convex demand curve empirically.

In extending the AIDS model we are inspired by relatively recent contributions to the theory of consumer choice, which draw on behavioral decision theory and also have asymmetric consumer reactions to price changes. Seminal work has been done by Kahneman, Tversky and Thaler. An important idea in these contributions is that consumers evaluate choice alternatives not only in absolute terms, but as deviations from a reference point (e.g. Tversky and Kahneman, 1991; Thaler, 1985). A popular representation of this idea is that consumers form a reference price, with deviations between the actual price and the reference price conveying utility, and thus influencing consumer purchasing behavior for a given budget constraint (see

Putler, 1992). We translate this idea to the context of standard macro models where consumers base their decisions on the price of individual goods relative to the aggregate price, as in Dixit and Stiglitz (1977) or Kimball (1995). The aggregate price would thus be the reference price. Within this broader approach, consumers will not only buy less of a good when its price rises above the aggregate price due to standard substitution and income effects, but also because a price rise may shift preferences away from the good that increased in price. The consumer may for example feel being treated unfairly, like in Okun (1981) or Rotemberg (2002). Inversely, preferences may shift towards a good when its price decreases below the aggregate price.

Figure 2 illustrates this argument. A key element is that the slope of an indifference curve through a single point in a good 1 and good 2 space will depend on whether the actual price is relatively high or low compared to the relevant aggregate (reference) price. Initial prices of goods 1 and 2 are  $p_1^a$  and  $p_2^a$ . Both are equal to the aggregate price. The consumer maximizes utility when she buys  $q_1^a$  (point a). Then assume a price increase for good 1 to  $p_1^b$ , rotating the budget line downwards. Traditional income and substitution effects will make the consumer move to point b, reducing the quantity of good 1 to  $q_1^b$ . Additional relative (or reference) price effects, however, will now shift the indifference surface. With  $p_1$  now relatively high, the indifference curve through point b will become flatter. Intuitively, since buying good 1 conveys utility losses, the consumer is willing to give up less of good 2 for more of good 1. The consumer reaches a new optimum at point d. Relative price effects on utility therefore induce an additional drop in  $q_1$  to  $q_1^d$ . Note that a similar graphical experiment can be done for a fall in  $p_1$ . Tversky and Kahneman's (1991) loss aversion hypothesis would then predict opposite, but smaller relative price effects, implying a kink in the demand curve (see also Putler, 1992).

The implication of this argument is that relative price effects on the indifference surface should be accounted for in demand analysis. The added term  $\sum_{j=1}^{N} \delta_{ij} \left( \ln(\frac{p_j}{P}) \right)^2$  in Equation (2) allows us to capture these additional effects. Provided that standard adding up  $\left( \sum_{i=1}^{N} \alpha_i = 1, \frac{1}{N} \right)^2$ 

$$\sum_{i=1}^{N} \gamma_{ij} = 0, \sum_{i=1}^{N} \beta_i = 0, \sum_{i=1}^{N} \delta_{ij} = 0), \text{ homogeneity } (\sum_{j=1}^{N} \gamma_{ij} = 0) \text{ and symmetry } (\gamma_{ij} = \gamma_{ji})$$
 restrictions hold, our extended equation is a valid representation of preferences.

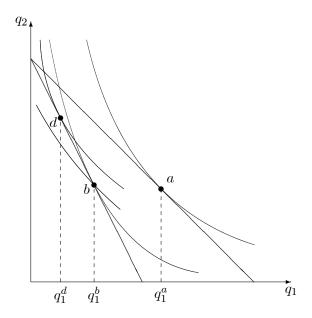


Figure 2. The Effects of Increasing the Price of Good 1

A general definition of the (positive) uncompensated own price elasticity of demand for good i is:

$$\varepsilon_{i} = -\frac{\partial \ln q_{i}}{\partial \ln p_{i}}$$

$$= 1 - \frac{\partial \ln s_{i}}{\partial \ln p_{i}}$$
(4)

where  $q_i = s_i X/p_i$ . Applied to our behavioral AIDS model,  $\varepsilon_i$  can then be derived from Equation (2) as

$$\varepsilon_{i(B-AIDS)} = 1 - \frac{1}{s_i} \left( \gamma_{ii} - \beta_i \frac{\partial \ln P}{\partial \ln p_i} + 2\delta_{ii} \ln(\frac{p_i}{P}) - 2\sum_{j=1}^{N} \delta_{ij} \ln(\frac{p_j}{P}) \frac{\partial \ln P}{\partial \ln p_i} \right)$$
 (5)

where we hold total nominal expenditure on the product category X as well as all other prices  $p_j$   $(j \neq i)$  constant. In the AIDS model the correct expression for the elasticity of the group price P with respect to  $p_i$  is

$$\frac{\partial \ln P}{\partial \ln p_i} = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln p_j \tag{6}$$

However, since using the price index from Equation (3) often raises empirical difficulties (see e.g. Buse, 1994), researchers commonly use Stone's geometric price index  $P^*$ , given by (1). The model is then called the "linear approximate AIDS" (LA/AIDS). To obtain the own price elasticity for the LA/AIDS model, one has to start from Stone's  $P^*$  and derive

$$\frac{\partial \ln P^*}{\partial \ln p_i} = s_i + \sum_{j=1}^N s_j \ln p_j \frac{\partial \ln s_j}{\partial \ln p_i}$$
 (7)

Green and Alston (1990) and Buse (1994) discuss several approaches to computing the LA/AIDS price elasticities depending on the assumptions made with regard to  $\frac{\partial \ln s_j}{\partial \ln p_i}$  and therefore  $\frac{\partial \ln P^*}{\partial \ln p_i}$ . A common approach is to assume  $\frac{\partial \ln s_j}{\partial \ln p_i} = 0$ , such that  $\frac{\partial \ln P^*}{\partial \ln p_i} = s_i$ . Monte Carlo simulations by Alston et al. (1994) and Buse (1994) reveal that this approximation is superior to many others (e.g. smaller estimation bias). In our empirical work we will also use Stone's price index and this approximation. The (positive) uncompensated own price elasticity implied by this approach then is

$$\varepsilon_{i(LA/B-AIDS)} = 1 - \frac{\gamma_{ii}}{s_i} + \beta_i - \frac{2\delta_{ii}\ln(\frac{p_i}{P^*})}{s_i} + 2\sum_{j=1}^N \delta_{ij}\ln(\frac{p_j}{P^*})$$
 (8)

Equation (8) incorporates several channels for the relative price of an item to affect the price elasticity of demand. The contribution of our behavioral extension of the AIDS model is obvious given the prominence of  $\delta_{ii}$  in this equation. Since  $s_i$  is typically far below 1, observing  $\delta_{ii} < 0$  will most likely imply a concave demand curve, with  $\varepsilon_i$  rising in the relative price  $\frac{p_i}{P^*}$ . When  $\delta_{ii} > 0$ , it is more likely to find convexity in the demand curve.

At steady state, for all relative prices equal to 1, the price elasticity becomes

$$\varepsilon_{i(LA/B-AIDS)(1)} = 1 - \frac{\gamma_{ii}}{s_i} + \beta_i \tag{9}$$

Finally, starting from Equation (8) we show in Appendix 5 that the implied curvature of the demand function at steady state is

$$\epsilon_{i(LA/B-AIDS)} = \frac{\partial \ln \varepsilon_i}{\partial \ln p_i}$$
(10)

$$= \frac{1}{\varepsilon_i} \left( (\varepsilon_i - 1) \left( \varepsilon_i - 1 - \beta_i \right) - \frac{2\delta_{ii} (1 - s_i)}{s_i} + 2(\delta_{ii} - s_i \sum_{j=1}^N \delta_{ij}) \right)$$
(11)

Also in this equation the key role of  $\delta_{ii}$  stands out. For given price elasticity, the lower  $\delta_{ii}$ , the higher the estimated curvature.

A simple comparison of the above results with the price elasticity and the curvature in the basic LA/AIDS model underscores the importance of our extension. Putting  $\delta_{ii} = \delta_{ij} = 0$ , one can derive for the basic LA/AIDS model that

$$\varepsilon_{i(LA/AIDS)} = 1 - \frac{\gamma_{ii}}{s_i} + \beta_i \tag{12}$$

$$\epsilon_{i(LA/AIDS)} = \frac{(\varepsilon_i - 1)(\varepsilon_i - 1 - \beta_i)}{\varepsilon_i}$$
(13)

With  $\beta_i$  mostly close to zero (and zero on average) the curvature then becomes a restrictive and rising function of the price elasticity, at least for  $\varepsilon_i > 1$ . Moreover, positive price elasticities  $\varepsilon_i$  almost unavoidably imply positive curvatures, which excludes convex demand curves. In light of our findings in Table 4 this seems too restrictive.

#### 3.2 Identification/Estimation

The sample that we use for estimation contains data for 28 product categories sold in each of the six outlets (supermarkets). The time frequency is a period of two weeks, with the time series running from the first bi-week of 2002 until the 8th bi-week of 2005. The selection of the 28 categories, coming from 58 in Section 2, is driven by data requirements and motivated in Appendix 2.

To keep estimation manageable we include five items per product category. Four of these items have been selected on the basis of clear criteria to improve data quality and estimation capacity. The fifth item is called "other". It is constructed as a weighted average of all other items. We include "other" to fully capture substitution possibilities for the four main items. Specifying "other" also enables us to deal with entry and exit of individual items during the sample period. We discuss the selection of the four items and the construction of "other" in Appendix 2 as well. For each item i within a product category the basic empirical demand

<sup>&</sup>lt;sup>12</sup>The specification of "other" may however also imply a cost. Including "other" imposes a large number of restrictions on the regression. In Section 3.4. we briefly reconsider this issue.

specification is:

$$s_{imt} = \alpha_{im} + \sum_{j=1}^{5} \gamma_{ij} \ln p_{jt} + \beta_{i} \ln \left(\frac{X_{mt}}{P_{mt}^{*}}\right) + \sum_{j=1}^{5} \delta_{ij} \left(\ln \left(\frac{p_{jt}}{P_{mt}^{*}}\right)\right)^{2} + \sum_{j=1}^{5} \varphi_{ij} C_{jt} + \lambda_{it} + \varepsilon_{imt}$$

$$i = 1, \dots, 5 \qquad m = 1, \dots, 6 \qquad t = 1, \dots, 86$$

$$(14)$$

where  $s_{imt}$  is the share of item i in total product category expenditure at outlet m and time t,  $X_{mt}$  is overall product category expenditure at outlet m and time t,  $P_{mt}^*$  is Stone's price index for the category at outlet m and  $p_{jt}$  is the price of the jth item in the category. As we have mentioned before, individual item prices are equal across outlets and predetermined. They are not changed during the period. This is an important characteristic of our data, which strongly facilitates identification of the demand curve (cf. infra). Furthermore,  $\alpha_{im}$  captures item specific and outlet specific fixed effects.<sup>13</sup> Finally, we include dummies to capture demand shocks with respect to item i at time t which are common across outlets. Circular dummies  $C_{jt}$  are equal to 1 when an item j in the product category to which i belongs, is mentioned in the supermarket's circular. The circular is common to all outlets. Also, for each item we include three time dummies  $\lambda_{it}$  for New Year, Easter and Christmas. These dummies should capture shifts in market share from one item to another during the respective periods.

Our estimation method is SUR. A key assumption underlying this choice is that prices  $p_{it}$  are uncorrelated with the error term  $\varepsilon_{imt}$ . For at least two reasons we believe this assumption is justified. Problems to identify the demand curve, as discussed by e.g. Hausman et al. (1994), Hausman (1997) and Menezes-Filho (2005), should therefore not exist. First, since our retailer sets prices in advance and does not change them to equilibrate supply and demand in a given period, prices can be considered predetermined with respect to Equation (14). Second, prices are set equal for all six outlets. We assume that outlet specific demand shocks for an item do not affect the price of that item at the chain level. Of course, against these explanations one

To control for item specific fixed effects, note that we have also de-meaned  $\ln(\frac{p_{jt}}{P_{mt}^*})$  when introducing the additional term  $\sum \delta_{ij} (\ln(\frac{p_{jt}}{P_{mt}^*}))^2$  in the regression.

<sup>&</sup>lt;sup>14</sup>Hausman et al. (1994) and Hausman (1997) make a similar assumption. See our brief discussion in Section 3.4.

could argue that the supplier may know in advance that demand will be high or low, so that he can already at the moment of price setting fix an appropriate price. However, our results in Section 2.3. do not provide strong evidence for this hypothesis. Demand shocks are of relatively minor importance in driving price and quantity changes. Moreover, many demand shocks may be captured by the circular dummies  $(C_{jt})$  and the item specific time dummies  $(\lambda_{it})$  included in our equations. They will not show up in the error term. In the same vein, the included fixed effect  $\alpha_{im}$  captures the influence on expenditure shares of time-invariant product specific characteristics which may also affect the price charged by the retailer. Therefore, item specific characteristics will not show up in the error term of the regressions either. A robustness test that we discuss in Section 3.4. provides additional support for our assumption that prices  $p_{it}$  are uncorrelated with the error term  $\varepsilon_{imt}$ . Using IV methods we obtain very similar results as the ones reported below.

Following Hausman et al. (1994) we estimate Equation (14) imposing homogeneity and symmetry from the outset (i.e.  $\sum_{j=1}^{5} \gamma_{ij} = 0$  and  $\gamma_{ij} = \gamma_{ji}$ ). We also impose symmetry on the effects of the circular dummies (i.e.  $\varphi_{ij} = \varphi_{ji}$ ). Finally, note that the adding up conditions  $(\sum_{i=1}^{5} \alpha_{im} = 1, \sum_{i=1}^{5} \gamma_{ij} = 0, \sum_{i=1}^{5} \beta_{i} = 0, \sum_{i=1}^{5} \delta_{ij} = 0, \sum_{i=1}^{5} \varphi_{ij} = 0)$  allow us to drop one equation from the system. We drop the equation for "other".

#### 3.3 Results

Estimation of Equation (14) for 28 product categories over six outlets, with each product category containing four items, generates 672 estimated elasticities and curvatures. Since 6 of these elasticities were implausible, we decided to drop them, leaving 666 plausible estimates.<sup>15</sup>

First, as we cannot discuss explicitly the 666 estimated elasticities and curvatures, we present our results in the form of a histogram in Figure 3. We find that the unweighted median price elasticity is 1.4. The unweighted median curvature is 0.8. If we weight our results with the

<sup>&</sup>lt;sup>15</sup>These 6 price elasticities were lower than -10 (where our definition is such that the elasticity for a negatively sloped demand curve should be a positive number). Note that we do not include the estimated elasticities and curvatures for the composite "other" item in our further discussion. Due to the continuously changing composition of this "other" item over time, any interpretation of the estimates would be delicate.

turnover each item generates, we do not find very different results. We find a median weighted elasticity of 1.2 and a median weighted curvature of 0.8. Considering the values that general equilibrium modelers impose when calibrating their models, these are low numbers (see Table 1). The elasticities that we find are also low in comparison with the existing empirical literature (see Bijmolt et al., 2005). The main reason for our relatively low price elasticity seems to be the overrepresentation of necessities (e.g. cornflakes, baking flour, mineral water) in the product categories that we could draw from our dataset. The estimated price elasticities for luxury goods, durables and large ticket items (e.g. smoked salmon, wine, airing cupboards) are generally much higher.

Figure 4 and Table 5 bring more structure in our estimation results. Excluding some extreme values for the curvature, Figure 4 reveals that the estimated price elasticity and curvature are strongly positively correlated. The correlation coefficient is  $0.53.^{16}$  In Table 5 we report the unweighted median elasticity and curvature, and their correlation, conditional on the elasticity taking certain values. The condition that the elasticity is strictly higher than 1 corresponds to the approach in standard macroeconomic models. When we impose this condition, the median estimated price elasticity is 2.4, the median estimated curvature 1.7. Imposing that the elasticity is strictly higher than 3 further raises the median curvature to 5.7. Estimated price elasticities between 3 and 6 go together with a median curvature of 3.5.

We can now reduce the uncertainty surrounding the curvature parameter to be used in calibrated macro models. The empirical literature on the price elasticity of demand surveyed by Bijmolt et al. (2005) reveals a median elasticity of about 2.2. Only 9% of estimated elasticities exceed 5. More or less in line with these results, the recent industrial organization literature reports price-cost mark-ups that are consistent with price elasticities between 3 and 6 (see e.g. Domowitz et al., 1988; Konings et al., 2001; Dobbelaere, 2004). Combining these results with

<sup>&</sup>lt;sup>16</sup>Figure 4 excludes 38 observations with an estimated curvature higher than 40 or lower than -40. If we exclude only observations with a curvature above +60 or below -60, the correlation is +0.51. Note that most of the extreme estimates for the curvature occur when the estimated price elasticity is very close to zero. Relatively small changes in the absolute value of the elasticity then result into huge percentage changes in the elasticity and, according to our definition, extreme curvature.

our findings in Table 5, a sensible value to choose for the curvature would be around 4. Note that this value is fairly robust to changes in our selection of product categories. Our approach in Figure 4 and Table 5 allows us to overcome the bias on our median estimates that may result from any overrepresentation of certain product categories. Clearly, a value for the curvature of 4 is far below current practice (see again Table 1). Only Bergin and Feenstra (2000) impose a lower value. Moreover, considering our results, the values for the curvature imposed by most macro modelers hardly fit their values for the elasticity. Only Woodford's (2005) choice to impose a curvature of about 7 and a price elasticity of about 8 is consistent with our results, if we condition on a price elasticity between 6 and 10 (see Table 5).

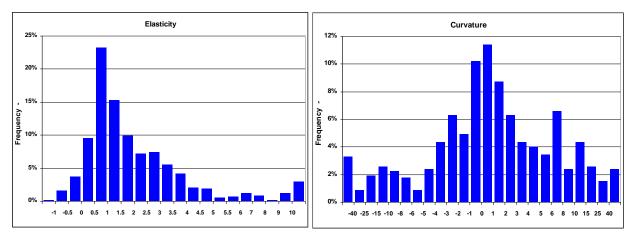


Figure 3: Histograms of Estimated Elasticity and Curvature

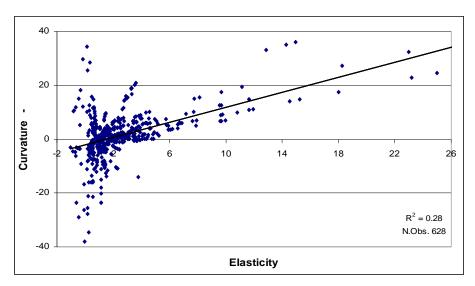


Figure 4: Correlation between Estimated Elasticity and Curvature

Table 5: Estimated Price Elasticity and Curvature

	Unconditional	Conditional on				
		$\varepsilon > 1$	$\varepsilon > 3$	$1 < \varepsilon \le 3$	$3 < \varepsilon \le 6$	$6 < \varepsilon \le 10$
Median Elasticity	1.4	2.4	4.2	1.8	3.7	7.8
Median Curvature	0.8	1.7	5.7	0.8	3.5	6.8
Correlation $(\varepsilon, \epsilon)$	0.12	0.45	0.40	0.33	0.02	0.53
Fraction $\epsilon < 0$	42%	26%	6%	38%	8%	0%
N.obs.	666	410	144	266	101	23

Second, our estimated curvatures show that the constant elasticity Dixit-Stiglitz (1977) benchmark is too simplistic. Over the broad range of product categories that we have studied, convex and concave demand curves coexist. Fully in line with our results in Table 4, we observe a negative curvature for 42% of the items. About 27% of our estimated curvatures are below -2, about 38% are above +2. The high frequency of non-zero estimated curvatures, including many negative curvatures, supports our argument that the original AIDS model is too restrictive to answer our research question. A key parameter in our behavioral extension is  $\delta_{ii}$  (see our discussion of Equation (8)). Additional tests show that this extension makes sense. We find the estimated  $\delta_{ii}$  to be statistically different from zero at the 10% significance level for 43% of the items. Furthermore, a Wald test rejects the null hypothesis that  $\delta_{11} = \delta_{22} = \delta_{33} = \delta_{44} = 0$  at the 5% significance level for two thirds of the included product categories. Appendix 6 provides details. A macroeconomic model that fits the microeconomic evidence well should thus ideally allow for sectors with differing elasticities and curvatures. However, conditioning on values for the price elasticity between 3 and 6, which may be more in line with the consensus in the literature, we also have to recognize that the large majority of demand curves is concave.

Third, in order to find out whether a concave demand curve gives rise to stickier prices, we check whether there is a link between our results on the curvature/elasticity and the size/frequency of price adjustment. In other words, does the supplier act differently for products with a high curvature compared to products with a low curvature. We calculated the corre-

<sup>&</sup>lt;sup>17</sup>See also the evidence on heterogeneous sectoral price rigidity presented in Angeloni et al. (2006) and Nakamura and Steinsson (2006) to support this conclusion.

lation between the statistics on nominal price adjustment presented in Table 2 with the 666 estimated elasticities and curvatures. Table 6 reports the results. Our estimated curvatures are not correlated with either the frequency or the size of price adjustment. This finding applies irrespective of including or excluding markdowns. It also applies irrespective of any condition on the level of the curvature (e.g.  $\epsilon > 0$ ) or the elasticity (e.g.  $\epsilon > 1$ ). This may cast doubt on whether the curvature of the demand curve is really an additional source of price rigidity. However, an issue that might drive the absent correlation between the curvature and the frequency and size of price adjustment is the fact that our data refer to a multi-product firm. Midrigan (2006) documents that multi-product stores tend to adjust prices of goods in narrow product categories simultaneously. This kind of coordination is likely breaking the potential relation between individual items' curvatures and frequency and size of price adjustment. It cannot be excluded that for single product firms, or firms in other sectors than the retail sector, the curvature of the demand curve has an effect on price rigidity. Our results for the relationship between the price elasticity of demand and the size and frequency of price adjustment are not very different. Excluding markdowns, correlation is negative. This result may provide some evidence in favor of the role of firm-specific production factors to create additional price rigidity, but the evidence is weak. The correlation is far from statistically significant.

Table 6: Correlation with Nominal Price Adjustment Statistics

	Including M	$\operatorname{arkdowns}$	Excluding Markdowns		
	Frequency Size		Frequency	Size	
Elasticity	0.04	-0.09	-0.10	-0.15	
Curvature	0.02	0.00	0.00	0.02	

Note: The correlations in this Table are calculated using the 666 item elasticity/curvature estimates and their corresponding size and frequency of price adjustment. The column "Excluding Markdowns" indicates that the size and frequency of price adjustment were calculated discarding periods of temporary price markdowns.

#### 3.4 Robustness

We have tested the robustness of our estimation results in various ways. First, we have changed the estimation methodology. A key assumption underlying the use of SUR is that prices  $p_{it}$ 

in Equation (14) are uncorrelated to the error term  $\varepsilon_{imt}$ . Although we believe we have good reasons to make this assumption, we have dropped it as a robustness check, and re-estimated our model using an IV method. Ideally, one can use information on costs, e.g. material prices, as instruments. However, data on a sufficient number of input prices with a high enough frequency is generally not available. Hausman et al. (1994) and Hausman (1997), who also use prices and quantities in different outlets, solve this problem by exploiting the panel structure of their data. They make the identifying assumption that prices in all outlets are driven by common cost changes which are themselves independent of outlet specific variables. Demand shocks that may affect the price of an item in one outlet are assumed not to affect the price of that item in other outlets. Prices in other outlets then provide reliable instruments for the price in a specific outlet. This procedure cannot work in our setup however since prices are identical across outlets. As an alternative we have used once to three times lagged prices  $p_i$  and once lagged relative prices  $\frac{p_i}{p_i}$  as instruments. Re-estimating our model for a large subset of the included product categories with the 3SLS methodology, we obtained very similar results for the elasticities and curvatures.

As a second robustness check we have introduced seasonal dummies to capture possible demand shifts related to the time of the year. As we have mentioned before, when suppliers are aware of such demand shifts they may fix their price differently. Not accounting for these demand shifts may then introduce correlation between the price and the error term, and undermine the quality of our estimates. Re-estimating our model with additional seasonal dummies did not affect our results in any serious way either.

Third, we allowed for gradual demand adjustment to price changes by adding a lagged dependent variable to the regression. Although often statistically significant, we generally found the estimated parameter on this lagged dependent variable to be between +0.1 and -0.1. Gradual adjustment seems to be no important issue in our dataset.

Fourth, our results are based on the assumption that the aggregate price  $(P_t^*)$  is the relevant

reference price when consumers make their choice. This assumption is in line with the approach in standard macro models. In marketing literature however it is often assumed that reference prices are given at the time of choice (see e.g. Putler, 1992; Bell and Latin, 2000). As a fourth robustness test we have therefore assumed the reference price to be equal to the one-period lagged aggregate price  $P_{t-1}^*$ . Re-estimating our model for a subset of product categories we found that this alternative had no influence on the estimated price elasticities. It implied slightly higher estimated curvatures for most items, however without affecting any of our conclusions drawn above<sup>18</sup>.

A final check on the reliability of our results considers potential implications of the way we have specified and introduced "other". Although necessary to make estimation manageable, introducing "other" imposes a large number of restrictions on the regression. In Appendix 7 we report additional statistics showing that there is no correlation at all between the market share of "other" in a product category and the average estimated elasticity and curvature for the four items in that product category. The estimated elasticity and curvature are not correlated either with the total number of items in the category. Limiting the fraction of items included in the estimation would not seem to bias our estimation results in any specific way.

#### 4 Conclusions

The failure of nominal frictions to generate persistent effects of monetary policy shocks has led to the development of models which combine nominal and real price rigidities. Many researchers have recently introduced a kinked (concave) demand curve as an attractive way to obtain real rigidities. However, the literature suffers from a lack of empirical evidence on the existence of the kinked demand curve and on the size of its curvature. This paper uses scanner data from a large euro area supermarket chain. Since a supermarket supplies many substitutes for each item at the same place, it may constitute the ideal environment to estimate price elasticities

<sup>18</sup> Assuming that the reference price equals  $P_{t-1}^*$  affects the equation for the curvature. Instead of Equation (11) it then holds that  $\epsilon_i = \frac{\partial \ln \varepsilon_i}{\partial \ln p_i} = \frac{(\varepsilon_i - 1)(\varepsilon_i - 1 - \beta_i) - 2\delta_{ii}/s_i}{\varepsilon_i}$ .

and curvatures. However, having the capacity to coordinate price changes, a supermarket may not be the best place to test the link between curvature and real price rigidity.

Our main conclusions are as follows. First, we find wide variation in the estimated price elasticity and the curvature of demand among different products. Although demand for the median product is concave, the fraction of products showing convex demand is significant. Our finding of wide heterogeneity, with negative curvature for a large fraction of products, forms a challenge for the relevant literature. It would suggest the need to model at least two - or even more - sectors, some with real price flexibility, and others with real price rigidity.

Second, our results support the introduction of a kinked (concave) demand curve in general equilibrium macro models. We find that the price elasticity of demand is on average higher for price increases than for price decreases. However, the degree of curvature is much lower than is currently imposed. Our suggestion is to impose a curvature parameter around 4. In this respect, our results are consistent with Klenow and Willis (2006) when they find that the joint assumption of realistic idiosyncratic shocks and a curvature of 10 is incompatible with observed nominal and relative price changes in US data. Realistic curvature must be lower.

Third, finding lower curvature than generally imposed, it seems clear from our results that the kinked demand curve alone may fail to generate sufficient real price rigidity. With a representative price elasticity of demand in the literature around 3 to 6, a curvature of 4 implies that demand remains elastic even when prices are reduced by 15%. Total revenue would still rise. The observation in our data that the median item is marked down for 8% of the time, whereas 27% of the median item's output is sold at times of price markdowns, illustrates this fact.

If concavity in the demand curve is empirically not strong enough, there must be other ingredients of real price rigidity at work. A promising approach may be to combine the kinked demand curve with the input-output structure proposed by Basu (1995), as in Bergin and Feenstra (2000). After all, Bergin and Feenstra (2000) do not need such a high curvature.

#### References

- [1] Alston J., Foster A. and R.Green, 1994, Estimating Elasticities with the Linear Approximate Almost Ideal Demand System: Some Monte Carlo Results, Review of Economics and Statistics, 76, 351-356
- [2] Altig D., Christiano L., Eichenbaum M. and J. Lindé, 2005, Firm-Specific Capital, Nominal Rigidities and the Business Cycle, CEPR Discussion Paper, No 4858
- [3] Angeloni I., Aucremanne L., Ehrmann M., Galí J., Levin A. and F. Smets, 2006, New Evidence on Inflation Persistence and Price Stickiness in the Euro Area: Implications for Macro Modelling, Journal of the European Economic Association, 4, 562-574
- [4] Ball L. and D. Romer, 1990, Real Rigidities and the Non-Neutrality of Money, Review of Economic Studies, 57, 183-203
- [5] Basu S., 1995, Intermediate Goods and Business Cycles: Implications for Productivity and Welfare, American Economic Review, 85, 512-531
- [6] Bell D.R. and J.R. Lattin, 2000, Looking for Loss Aversion in Scanner Panel Data: The Confounding Effect of Price Response Heterogeneity, Marketing Science, 19, 185-200
- [7] Bergin P. and R. Feenstra, 2000, Staggered Price Setting, Translog Preferences and Endogenous Persistence, Journal of Monetary Economics, 45, 657-680
- [8] Berry S., Levinsohn J. and A. Pakes, 1995, Automobile Prices in Market Equilibrium, Econometrica, 63, 841-890
- [9] Bijmolt T., Van Heerde H. and R. Pieters, 2005, New Empirical Generalisations on the Determinants of Price Elasticity, *Journal of Marketing Research*, 42, 141-156
- [10] Bils M. and P.J. Klenow, 2004, Some Evidence on the Importance of Sticky Prices, Journal of Political Economy, 112, 947-985

- [11] Blanchard O.J. and J. Galí, 2006, Real Wage Rigidities and the New Keynesian Model, Journal of Money Credit and Banking, forthcoming
- [12] Broda C. and D. Weinstein, 2006, Globalization and the Gains from Variety, Quarterly Journal of Economics, 121, 541-585
- [13] Burstein A., Eichenbaum M. and S. Rebelo, 2006, Modeling Exchange Rate Passthrough after Large Devaluations, *Journal of Monetary Economics*, forthcoming
- [14] Buse A., 1994, Evaluating the Linearized Almost Ideal Demand System, American Journal of Agricultural Economics, 76, 781-793
- [15] Calvo G., 1983, Staggered Prices in a Utility-Maximizing Framework, Journal of Monetary Economics, 12, 383-398
- [16] Chari V. Kehoe P. and E. McGrattan, 2000, Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem, *Econometrica*, 68, 1151-1179
- [17] Christiano L, Eichenbaum M. and C. Evans, 1999, Monetary Policy Shocks: What Have We Learned and to What End?, *Handbook of Macroeconomics*, 1A, eds, Michael Woodford and John Taylor, Elsevier Science
- [18] Christiano L., Eichenbaum M. and Evans C., 2005, Nominal Rigidities and the Dynamic Effects of Shock to Monetary Policy, Journal of Political Economy, 113, 1-45
- [19] Coenen G. and A. Levin, 2004, Identifying the influences of nominal and real rigidities in aggregate price setting behavior, ECB Working Paper, No 418
- [20] Deaton A. and J. Muellbauer, 1980, An Almost Ideal Demand System, American Economic Review, 70, 312-326
- [21] de Walque G., Smets F. and R. Wouters, 2006, Firm-Specific Production Factors in a DSGE Model with Taylor Price Setting, ECB Working Paper, No 648

- [22] Dhyne E., Álvarez L., Le Bihan H., Veronese G., Dias D., Hoffman J., Jonker N., Lünnemann P., Rumler F., and J. Vilmunen, 2006, Price setting in the euro area: Some stylized facts from individual consumer price data, *Journal of Economic Perspectives*, 20, 171-192
- [23] Dobbelaere S., 2004, Estimation of Price-Cost Margins and Union Bargaining Power for Belgian Manufacturing, International Journal of Industrial Organization, 22, 1381-1398
- [24] Domowitz I., Hubbard G. and B. Petersen, 1988, Market Structure and Cyclical Fluctuations in US Manufacturing, Review of Economics and Statistics, 70, 55-66
- [25] Dotsey M. and R. King, 2005, Implications of State-Dependent Pricing for Dynamic Macroeconomic Models, Journal of Monetary Economics, 52, 213-242
- [26] Dotsey M., King R. and A. Wolman, 2006, Inflation and Real Activity with Firm-Level Productivity Shocks: A Quantitative Framework, mimeo
- [27] Dixit A. and J. Stiglitz, 1977, Monopolistic Competition and Optimum Product Diversity, American Economic Review, 67, 297-308
- [28] Eichenbaum M. and J. Fisher, 2004, Evaluating the Calvo Model of Sticky Prices, NBER Working Paper, No 10617
- [29] Galí J. and M. Gertler, 1999, Inflation Dynamics: A Structural Econometric Analysis, Journal of Monetary Economics, 44, 195-222
- [30] Golosov M. and R.E. Lucas Jr., 2003, Menu Costs and Phillips Curves, NBER Working Paper, No 10187
- [31] Green R and J. Alston, 1990, Elasticities in AIDS Models, American Journal of Agricultural Economics, 72, 442-445
- [32] Hausman J., 1997, Valuation of New Goods under Perfect and Imperfect Competition, in The Economics of New Goods, Bresnahan and Gordon, eds., Chicago Press

- [33] Hausman J., Leonard G. and J.D. Zona, 1994, Competitive Analysis with Differentiated Products, Annales d'Économie et de Statistique, 34, 159-180
- [34] Kimball M., 1995, The Quantitative Analytics of the Basic Neomonetarist Model, Journal of Money, Credit and Banking, 27, 1241-1277
- [35] Klenow P.J. and O. Kryvtsov, 2005, State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?, NBER Working Paper, No 11043
- [36] Klenow P.J. and J.L. Willis, 2006, Real Rigidities and Nominal Price Changes, Research Working Paper, Federal Reserve Bank of Kansas City, No 06-03
- [37] Konings J., Van Cayseele P. and F. Warzynski, 2001, The Dynamics of Industrial Mark-Ups in Two Small Open Economies: Does National Competition Policy Matter? *International Journal of Industrial Organization*, 19, 841-859
- [38] Mankiw N.G., 1985, Small Menu Costs and Large Business Cycles, Quarterly Journal of Economics, 100, 529-537
- [39] Midrigan V., 2006, Menu Costs, Multi-Product Firms, and Aggregate Fluctuations, Ohio State University, mimeo
- [40] Menezes-Filho N., 2005, Is the Consumer Sector Competitive in the U.K.? A Test Using Household-Level Demand Elasticities and Firm-Level Price Equations, Journal of Business and Economic Statistics, 23, 295-304
- [41] Nakamura E. and J. Steinsson, 2006, Monetary Non-Neutrality in a Multi-Sector Menu Cost Model, Harvard University, mimeo
- [42] Okun A., 1981, Prices and Quantities, The Brookings Institution, Washington DC
- [43] Peersman G., 2004, The Transmission of Monetary Policy in the Euro Area: Are the Effects

  Different Across Countries?, Oxford Bulletin of Economics and Statistics, 66, 285-308

- [44] Putler D.S., 1992, Incorporating Reference Price Effects into a Theory of Consumer Choice, Marketing Science, 11, No 3, 287-309
- [45] Rotemberg J., 2002, Customer Anger at Price Increases, Time Variation in the Frequency of Price Changes and Monetary Policy, NBER Working Paper, No 9320
- [46] Rotemberg J. and M. Woodford, 1999, The Cyclical Behavior of Prices and Costs, Handbook of Macroeconomics, 1B, eds. J. Taylor and M. Woodford
- [47] Sbordone A.M., 2002, Prices and Unit Labor Costs: A New Test of Price Stickiness, Journal of Monetary Economics, 49, 265-292
- [48] Taylor J., 1980, Aggregate Dynamics and Staggered Contracts, Journal of Political Economy, 88, 1-23
- [49] Thaler M., 1985, Mental Accounting and Consumer Choice, Marketing Science, 4, 199-214
- [50] Tversky A. and D. Kahneman, 1991, Loss Aversion in Riskless Choice: A Reference-Dependent Model, Quarterly Journal of Economics, 106, 1039-1061
- [51] Woodford M, 2003, Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton, Princeton University Press
- [52] Woodford M., 2005, Firm-specific Capital and the New Keynesian Phillips Curve, *International Journal of Central Banking*, 1, September, 1-46

#### Appendix 1: Different Curvatures

Curvature is not defined homogeneously across the different papers in the literature on price rigidity. In this appendix we derive the relationships between the alternative definitions. These relationships underly some of the parameter values that we report in Table 1 in the main text. We use the following notation:  $x_i = q_i/Q$  is firm i's relative output,  $p_i$  is its price,  $\varepsilon(x_i)$  is the (positive) price elasticity of demand,  $\mu(x_i) = \frac{\varepsilon(x_i)}{\varepsilon(x_i)-1}$  is the firm's desired markup. Assuming an aggregate price level equal to 1,  $p_i$  also indicates the firm's relative price.

Eichenbaum and Fisher (2004) and de Walque et al. (2006) define curvature as we have done as the elasticity of the price elasticity of demand with respect to the relative price at steady state:

$$\epsilon = \left[ \frac{\partial \varepsilon(x_i)}{\partial p_i} \frac{p_i}{\varepsilon(x_i)} \right]_{x_i = 1} \tag{15}$$

Coenen and Levin (2004) define the curvature of the demand curve as the relative slope of the price elasticity of demand around steady state:

$$\epsilon = \left[ -\frac{\partial \varepsilon(x_i)}{\partial x_i} \right]_{x_i = 1} \tag{16}$$

It can be shown that in steady state both approaches are identical:

$$\frac{\partial \varepsilon(x_i)}{\partial p_i} \frac{p_i}{\varepsilon(x_i)} = \frac{\partial \varepsilon(x_i)}{\partial p_i} \frac{p_i}{\varepsilon(x_i)} \frac{\partial x_i}{\partial x_i} \frac{x_i}{x_i} = \frac{\partial \varepsilon(x_i)}{\partial x_i} \frac{p_i}{x_i} \frac{\partial x_i}{\partial p_i} \frac{x_i}{\varepsilon(x_i)} = -\frac{\partial \varepsilon(x_i)}{\partial x_i} \varepsilon(x_i) \frac{x_i}{\varepsilon(x_i)}$$

Evaluated at steady state  $(x_i = 1)$ , this is equal to  $-\frac{\partial \varepsilon(x_i)}{\partial x_i}$ .

Kimball (1995) and Woodford (2005) characterize the curvature in the demand curve by the elasticity of the firm's desired markup with respect to relative output at steady state, i.e.

$$\xi = \left[ \frac{\partial \mu(x_i)}{\partial x_i} \frac{x_i}{\mu(x_i)} \right]_{x_i = 1} \tag{17}$$

The relationship between  $\epsilon$  and  $\xi$  is as follows:

$$\xi = \left[\frac{\partial \mu(x_i)}{\partial x_i} \frac{x_i}{\mu(x_i)}\right]_{x_i=1} = \left[\epsilon \frac{\partial \mu(x_i)}{\partial \varepsilon(x_i)} \frac{\partial p_i}{\partial x_i} \frac{x_i}{p_i} \frac{\varepsilon(x_i)}{\mu(x_i)}\right]_{x_i=1}$$

$$= \left[\epsilon \frac{1}{(\varepsilon(x_i) - 1)^2} \frac{1}{\varepsilon(x_i)} (\varepsilon(x_i) - 1)\right]_{x_i=1} = \frac{\epsilon}{(\varepsilon(1) - 1) \varepsilon(1)}$$

Kimball (1995) assumes  $\xi = 4.28$  and  $\varepsilon(1) = 11$ . Woodford imposes  $\xi = 0.13$  and  $\varepsilon(1) = 7.67$ .

The approach in Chari et al. (2000) is very close to Eichenbaum and Fisher (2004), Coenen and Levin (2004) and de Walque et al. (2006). Cost minimization by households buying differentiated products i to achieve optimal composite consumption Q yields the following first order condition for demand:

$$p_i = \frac{\lambda}{Q}G'(x_i)$$

with  $\lambda$  the Lagrangian lambda on the constraint relating household composite consumption Q to individual quantities  $q_i$ , G the Kimball (1995) aggregator function for composite consumption and (as defined before)  $x_i = q_i/Q$ . Rewriting this first order condition, we obtain the demand curve  $x_i = D(p_iQ/\lambda)$  with  $D = (G')^{-1}$ . The price elasticity of demand equals

$$\varepsilon(x_i) = -\frac{D'(G'(x_i))G'(x_i)}{x_i}$$

Evaluated at steady state this is  $\varepsilon(1) = -D'(G'(1))G'(1)$ . The curvature of the demand curve at steady state can then be obtained as:

$$\epsilon = \left[ -\frac{\partial \varepsilon(x_i)}{\partial x_i} \right]_{x_i = 1} = D''(G'(1))G''(1)G'(1) + G''(1)D'(G'(1)) - D'(G'(1))G'(1)$$

Since D'(G'(1)) = 1/G''(1) it follows that

$$\epsilon = \frac{D''(G'(1))G'(1)}{D'(G'(1))} + 1 + \varepsilon(1)$$

Chari et al. (2000) define their curvature parameter  $\chi$  as

$$\chi = -\frac{D''(G'(1))G'(1)}{D'(G'(1))},\tag{18}$$

from which the relationship with  $\epsilon$  is:

$$\epsilon = -\chi + 1 + \varepsilon(1) \tag{19}$$

Chari et al. (2000) state a value of -289 for  $\chi$  and 10 for  $\varepsilon(1)$ . According to Equation (19) this would imply  $\epsilon = 300$ . The discrepancy with the value of 385 that we report in Table 1 is

due to the fact that Chari et al. (2000) use a first order Taylor series expansion of the demand elasticity around the steady state to calculate their curvature parameter  $\chi$  associated with the Kimball (1995) parameterisation. The exact value of  $\chi$  would be -374.

Finally, Bergin and Feenstra (2000) derive a concave demand curve from assuming preferences with a translog functional form. The (positive) own price elasticity of demand is  $\varepsilon_i = 1 - \frac{\gamma_{ii}}{s_i}$  with  $s_i$  the expenditure share of good i and  $\gamma_{ii} = \partial s_i/\partial \ln p_i < 0$ . Along the lines set out in Section 3.1. of this paper it can be derived that  $\epsilon = \frac{(\varepsilon_i - 1)^2}{\varepsilon_i}$ . Starting from the imposed  $\varepsilon(1) = 3$ ,  $\epsilon$  should be 1.33.

#### Appendix 2: Description of Dataset

Table 7 gives an overview of the 58 product categories that are in the dataset that we use in this paper. Between brackets we indicate the number of items within each category. The available data for all these categories have been used to compute the basic statistics in Section 2. Product categories in italic are also included in the econometric analysis in Section 3.

Table 7: Product Categories and Number of Items

Drinks: tea (67), coke (39), chocolate milk (9), lemonade (33), mineral water (66), wine (17)

port wine (54), gin (21), fruit juice (54), beer (6), whiskey (82)

Food: cornflakes (49), tuna (46), smoked salmon (18), biscuit (9), mayonnaise (45), tomato soup (5), emmental cheese (56), gruyere cheese (19), spinach (29), margarine (62), potatoes (26), liver torta (98), baking flour (18), spaghetti (30), coffee biscuits (5), minarine (2)

Equipment: airing cupboard (61), knife (19), hedge shears (32), dishwasher (43), washing machine (36), tape measure (15), tap (24), dvd recorder (20), casserole (74), toaster (40)

Clothes and related: jeans (79), jacket (88) Cleaning products: dishwasher detergent (43), detergent (43), soap powder (98), floorcloth (11) toilet soap (34)

**Leisure and education**: hometrainer (52), football (32), cartoon (86), dictionary (32), school book (34)

Personal care: plaster (33), nail polish (15), handkerchief (63), nappy (64), toilet paper (13)

Other: potting soil (33), cement (43), bath mat (48), aluminium foil (5)

Note: The number of items in a particular product category is stated in brackets. Only the product categories in italic are included in the econometric analysis in Section 3.

Our econometric analysis in Section 3 includes four items per product category and a composite of all other items in the category, called "other". Including more than four items could make sense from the perspective of covering a larger share of the market. However, it would also imply an inflation of coefficients to be estimated. Moreover, since the price of each item occurs as an explanatory variable in the expenditure share equation of all included items within the product category, raising the number of items could limit estimation capacity when additional items have shorter or non-overlapping data availability. Our criteria to select the four items per product category reflect these concerns. These criteria are (long) data availability and (relatively high) market share within the category. More precisely, we ranked all items within the category on the basis of the total number of observations available (the maximum being 86), and chose those items with the highest number of observations. Among items with an

<sup>&</sup>lt;sup>19</sup>Note that both these criteria are strongly (positively) correlated.

equal number of observations we selected those with the highest market share. If this procedure implied different selections among the six available outlets, we chose those products with the best ranking in most outlets.

The market share of "other" has been constructed as

$$s_{other} = \frac{X_{other}}{X} = \frac{\displaystyle\sum_{j \notin S4}^{N} p_{j}q_{j}}{X}$$

with S4 the selected four items, and all other variables as defined in the main text. The price index of "other" is the Stone index for all items included in "other".

$$p_{other} = \sum_{j \notin S4}^{N} s_j p_j$$

with  $s_j = p_j q_j / X_{other}$ . Due to different weights  $p_{other}$  will differ across the six outlets.

The reduction to 28 product categories in the econometric analysis in Section 3, coming from 58, has been driven by the following criteria. For a category to be included in the econometric analysis we required (i) data availability in all six outlets, (ii) the four selected items to have a total market share of at least 20% in their product category and (iii) the four selected items to show sufficient price variation. Over the whole time span the four items together should show at least 20 price changes of at least 5%, where we counted the typical V-pattern of a price markdown as 1 price change. At least 3 of these price changes should be regular price changes. The minimum market share requirement should make certain that the chosen four items are important within their category. This should raise the relevance of our estimates. Sufficient price variation is an obvious requirement if one wants to estimate a demand curve accurately.

## Appendix 3: Identification of Markdowns

Figure 5 illustrates the identification of markdowns for an individual item of potatoes. A markdown is a sequence of three, two or one price(s) that are/is below both the most left adjacent price and the most right adjacent price. To calculate our "excluding markdowns" statistics in Section 2, we have filtered out markdown prices. We have replaced them by the last observed regular price.

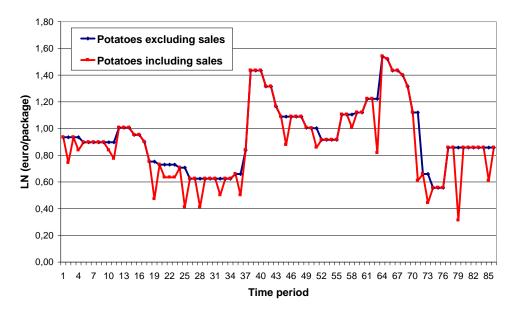


Figure 5: Price for Potato Item Including and Excluding Temporary Markdowns

## Appendix 4: Robustness (Fisher price index)

Table 8: Importance of Demand and Supply Shocks

	Including markdowns			Excl. markdowns		
Percentile	25%	50%	75%	25%	50%	75%
Average absolute $\Delta \ln(p_i/P^*)$	2%	3%	4%	1%	2%	3%
Average absolute $\Delta \ln(q_i/Q)$	38%	57%	76%	38%	57%	76%
Standard Deviation $\Delta \ln(p_i/P^*)$	3%	4%	7%	2%	3%	5%
Standard Deviation $\Delta \ln(q_i/Q)$	50%	75%	98%	50%	74%	97%
Correlation $(\Delta \ln(p_i/P^*); \Delta \ln(q_i/Q))$	-0.45	-0.22	-0.02	-0.48	-0.24	-0.04
% Supply Shocks to $\Delta \ln(p_i/P^*)^{(a)}$	48%	71%	88%	50%	72%	89%
% Supply Shocks to $\Delta \ln(q_i/Q)^{-(a)}$	50%	70%	86%	50%	71%	87%

Note: The statistics reported in this table are based on bi-weekly data for 2274 items belonging to 58 product categories in six outlets. Individual nominal items prices  $(p_i)$  are common across the outlets, all the other data  $(P^*, q_i, Q)$  can be different per outlet.(a) The contribution of demand shocks to price and quantity variability equals 1 minus the contribution of supply shocks. Computation methods are described in the main text.

Table 9: Asymmetric Price Sensitivity: Difference between  $\varepsilon^H$  and  $\varepsilon^L$ 

	Including markdowns			Exclud	ing mar	kdowns
Percentile	25%	50%	75%	25%	50%	75%
Median $\varepsilon^H - \varepsilon^L$	-20.14	-1.27	13.60	-23.27	-2.14	12.74

Note:  $\varepsilon^H$  and  $\varepsilon^L$  are the absolute values of the price elasticity of demand at high and low relative prices respectively.  $\varepsilon^H > \varepsilon^L$  suggests that the demand curve is concave (smoothed "kinked"). The reported data refer to the items at the 25th, 50th and 75th percentile, ordered from low to high.

# Appendix 5: Derivation of Curvature in the Behavorial AIDS model

Starting from Equation (8)

$$\varepsilon_{i(LA/B-AIDS)} = 1 - \frac{\gamma_{ii}}{s_i} + \beta_i - \frac{2\delta_{ii} \ln(\frac{p_i}{P^*})}{s_i} + 2\sum_{i=1}^{N} \delta_{ij} \ln(\frac{p_j}{P^*})$$

the derivation of the curvature goes as follows:

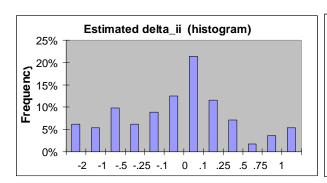
$$\begin{split} \epsilon_{i(LA/B-AIDS)} &= \frac{\partial \ln \varepsilon_{i}}{\partial \ln p_{i}} \\ &= -\frac{\partial}{\varepsilon_{i}} \frac{\left(\frac{\gamma_{ii}+2\delta_{ii} \ln(\frac{p_{i}}{P^{*}})}{s_{i}} - 2\sum_{j=1}^{N} \delta_{ij} \ln(\frac{p_{j}}{P^{*}})\right)}{\partial \ln p_{i}} \\ &= -\frac{1}{\varepsilon_{i}} \left(\frac{2\delta_{ii}(1-s_{i})s_{i} - (\partial s_{i}/\partial \ln p_{i})(\gamma_{ii} + 2\delta_{ii} \ln(\frac{p_{i}}{P^{*}}))}{s_{i}^{2}} - 2(\delta_{ii} - s_{i}\sum_{j=1}^{N} \delta_{ij})\right) \\ &= -\frac{1}{\varepsilon_{i}} \left(\frac{2\delta_{ii}(1-s_{i})}{s_{i}} + (\varepsilon_{i}-1)\left(1-\varepsilon_{i}+\beta_{i}+2\sum_{j=1}^{N} \delta_{ij} \ln(\frac{p_{j}}{P^{*}})\right) - 2(\delta_{ii} - s_{i}\sum_{j=1}^{N} \delta_{ij})\right) \end{split}$$

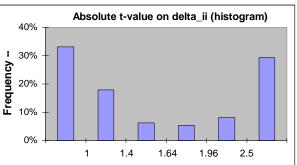
In the third line we again use the (empirically supported) assumption that  $\frac{\partial \ln P^*}{\partial \ln p_i} = s_i$ . The fourth line relies on the definition that  $-\frac{\partial s_i/s_i}{\partial \ln p_i} = (\varepsilon_i - 1)$  and the result derived from Equation (8) that  $\frac{\gamma_{ii}}{s_i} + \frac{2\delta_{ii} \ln(\frac{p_i}{P^*})}{s_i} = 1 - \varepsilon_i + \beta_i + 2\sum_{j=1}^{N} \delta_{ij} \ln(\frac{p_j}{P^*})$ . Rearranging and imposing the steady state assumption that all relative prices are 1, we find for the curvature that

$$\epsilon_{i(LA/B-AIDS)} = \frac{1}{\varepsilon_i} \left( (\varepsilon_i - 1) \left( \varepsilon_i - 1 - \beta_i \right) - \frac{2\delta_{ii}(1 - s_i)}{s_i} + 2(\delta_{ii} - s_i \sum_{j=1}^N \delta_{ij}) \right)$$

# Appendix 6: Estimation results for $\delta_{ii}$

The two figures below show the distribution of the 112 (=28x4) estimated values for  $\delta_{ii}$  and the distribution of the related absolute t-values. The table contains the results of a Wald test for each of the 28 product categories of the joint hypothesis that  $\delta_{11} = \delta_{22} = \delta_{33} = \delta_{44} = 0$ . The results are briefly discussed in the main text.





	Wald Test for $\delta_{11} = \delta_{22} = \delta_{33} = \delta_{44} = 0$				
<i>p</i> -value	$p \le 0.05$	$0.05$	$0.1$	0.2 < p	
Number of product categories	19	2	4	3	

## Appendix 7: Size of "Other" and Estimation Results

This appendix reveals that there is no specific relationship between our estimation results for the elasticity and the curvature in a product category and the number of items *not* included in the regressions. The table below contains all relevant correlation coefficients, the figures illustrate two of the results involving curvature.

	Pairwise correlation coefficients over 28 observations (product categories)							
	market share "other"   number of items   median elasticity   median c							
market share "other"	1							
number of items	0.61	1						
median elasticity	-0.06	-0.19	1					
median curvature	-0.03	0.06	0.56	1				
median $\delta_{ii}$	+0.12	-0.19	-0.29	-0.78				

