Choice, Opportunities, and Procedures: Collected Papers of Kotaro Suzumura

Part VI Consequentialism versus Non-consequentialism

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Chapter 21
Consequences, Opportunities, and Procedures*

1 Introduction

In his characteristically lucid exposition of the celebrated general possibility theorem, Kenneth Arrow observed that "[e]conomic or any other social policy has consequences for the many and diverse individuals who make up the society or economy. It has been taken for granted in virtually all economic policy discussions since the time of Adam Smith, if not before, that alternative policies should be judged on the basis of their consequences for individuals (Arrow [3, p.124])." As a matter of fact, the informational parsimony of traditional normative economics is even more restrictive than consequentialism as such, as it allows for the description of consequences only in terms of welfares which "the many and diverse individuals who make up the society or economy" entertain under each and every consequence.[1] It is true that Amartya Sen's justly famous criticism against the welfaristic foundations of normative economics, which capitalizes on his *impossibility of a Paretian liberal*, brought back the importance of non-welfaristic features of consequences into proper perspective, yet it still keeps us within the broad boundary of consequentialism.[2]

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[1] On the informational parsimony of traditional welfare economics and social choice theory, see Sen [54; 55; 63] and Suzumura [77].

[2] The amount of work that followed Sen's [51; 53] original criticism against welfarism in terms of the impossibility of a Paretinan liberal is really enormous. See Sen [55; 56; 61; 63] and Suzumura [72, Chapter 7; 75] for succinct overview and evaluation of this large literature.

Starkly opposing to the *consequentialist approach* to normative economics is the *procedural approach* vigorously advocated most notably by James Buchanan [10; 11; 12; 13], Robert Nozick [40] and Robert Sugden [67; 68; 69; 70]. The contrast is really sharp, as "[Buchanan's] emphasis on procedural judgements may be taken to suggest ... that we should abandon altogether consequence-based evaluation of social happenings, opting instead for a procedural approach. In its pure form, such an approach would look for 'right' institutions rather than 'good' outcomes and would demand the priority of appropriate procedures (including the acceptance of what follows from these procedures) (Sen [63, p.2])."[3]

The purpose of the present chapter is to fill in the gap between these two seemingly irreconcilable stances by making the following two elementary points. In the first place, we contend that procedural considerations do matter in many important contexts of real substance, where welfare economics and social choice theory should have much to say, so that the traditional informational basis of normative economics cannot but be expanded to accommodate procedural information along with consequential information. To make this point, we need not look for far-fetched and/or concocted examples. Quite to the contrary, an embarrassment of riches is a real possibility here, and Sections 3-5 will be devoted to the three concrete problems that are meant to shed light on the separate aspects of the same coin. In the second place, there seems to exist a widely held presumption that procedural considerations can be combined with consequential considerations by re-characterizing the concept of social states more extensively. This is demonstrably the case indeed, but the necessary conceptual extension is neither trivial nor mechanical, and it is worthwhile to work out at least one example to see what is really involved in this conceptual extension. Section 6 is devoted to this task, focussing on the classical issue of welfare and rights posed by Sen's impossibility theorem as the field of our exercise. Section 7 concludes this chapter with a few final observations.

Before setting about pursuing the proper tasks of this chapter, a preliminary observation on the concept of *opportunities* and *procedures* might be in order.

2 Opportunities and Procedures

Proper recognition of the importance of procedural considerations along with consequential considerations, particularly when one is concerned with the issue of individual rights and freedom, may not infrequently be found in the literature. To cite a salient example, in his Joseph Schumpeter Lecture on *individual freedom and welfare state policy*, Assar Lindbeck aptly observed that "the issue of 'individual freedom,' which is closely related not only to actual achievements but also to the process by which these are realized, is seldom squarely confronted in economic analysis (Lindbeck [38, p.295])." However, Lindbeck's

[3]See, also, Agnar Sandom's review article on Buchanan, according to whom "[Buchanan's] view has strong implications for the way in which economists conceive of their own role in society. Rather than attempting to find 'optimal' solutions to economic problems they should concentrate on finding good decision rules, which all individuals and interest groups will find it in their own long-run interest to adopt for the solution of still unidentified conflicts over resource allocation (Sandmo [49, p.52])."

emphasis on the process by which consequential outcomes are realized quickly recedes to the background. Instead of formulating the *mechanism* or *procedure* that determines the process of choice explicitly, his attention becomes focussed on the *opportunity set* from which the consequential outcomes are to be chosen. Presumably, this almost imperceptible shift of emphasis is motivated by his observation, which is certainly legitimate in itself, to the effect that "individual freedom has to do with the tightness of various types of constraints on the decisions and actions of individuals (ibid., p.295)," so that "[s]uch freedom of choice will be analyzed ... in terms of the *opportunity set* and/or the *potential options* of the individual (ibid., p.297)." Thus, the procedural considerations enter into Lindbeck's analytical scenario only through the following two indirect channels: (i) the opportunity sets differ depending on how the various choice procedures constrain the decisions and actions of individuals; (ii) "the attitude of the individual to constraints on his freedom of choice differ depending on *who* has imposed them (ibid., p.298)."

The advocacy of using the opportunity set to capture the idea of freedom of choice in general, and the role of procedural considerations in analysing freedom of choice in particular, finds additional support in Sen [65, p.202]: "[T]he value of [an opportunity] set need not invariably be identified with the value of the best – or the chosen – element of it. Importance can also be attached to having opportunities that are *not* taken up. This is the natural direction to go if the process through which outcomes are generated is of a significance of its own. Indeed, 'choosing' itself can be seen as a valuable functioning, and having an $\mathbf{x}$ when there is no alternative may be sensibly distinguished from choosing $\mathbf{x}$ when substantial alternatives exist."[4,5]

Instead of subsuming procedural considerations indirectly through opportunity sets as in the Lindbeck-Sen approach, we adopt in this chapter a direct approach whose origin can be traced back to Arrow [2, pp.90-91]: "[A]mong the variables which taken together define the social state, one is the very process by which the society makes its choice. This is especially important if the mechanism of choice itself has a value to the individuals in the society. For example, an individual may have a positive preference for achieving a given distribution through the free market mechanism over achieving the same distribution through rationing by the government."[6]

We may give analytical substance to Arrow's insightful observation in the following way. Instead of sticking to the traditional informational basis of normative economics

[4]See, also, Sen [57; 58]

[5]Sen's [57; 58; 59; 60; 62] due emphasis on the importance of opportunity set led to a huge literature on the evaluation of freedom in terms of the worth of opportunity sets. See, among many others, Arneson [1], Basu [6], Bossert et al. [9], Fleurbaey [18], Gravel [22], Klemisch-Ahlert [34], Pattanaik and Xu [46], Puppe [47] and Sugden [71]. See, also, related earlier studies by Dworkin [16] and Jones and Sugden [32].

[6]See, also, Sandmo who wrote as follows: "Judging allocation mechanism either by results or by the allocation processes themselves may not be a very fruitful contrast. ... One problem with this approach is that it is not always clear how one should draw the distinction between outcomes and processes, because the individuals in the economy may have preferences that are partly defined over the processes themselves. Thus, one may prefer competitive markets to central planning because markets result in more consumption goods for everybody, but one may also have a preference for having one's consumption goods delivered by the market rather than by the central planning bureau (Sandmo [49, p.58])."

which focusses on individual preference orderings over the set X of conventionally defined social alternatives, we use an extended informational basis that consists of individual preference orderings over the Cartesian product of X and Θ, where Θ denotes the set of social decision-making mechanisms through which the actual choices are going to be made. Let Q_i denote the extended preference ordering of individual i, and (x, θ^1) and (y, θ^2) be two representative elements of $X \times \Theta$. Then $(x, \theta^1) Q_i (y, \theta^2)$ means that, according to i's view, having an outcome x through the mechanism θ^1 is at least as good as having another outcome y through another mechanism θ^2.[7]

People seem prepared to make welfare judgements of this extended type, and it is in terms of this extended conceptual framework that we are going to incorporate procedural considerations along with consequential considerations.

3 Discrimination by Procedural Considerations: Problem of Fair Cake Division

Back, then, to our main scenario. The first problem we examine is meant to shed light on the role procedural considerations may play in complementing purely consequential considerations when the latter fail to distinguish two situations which are relevantly different from the viewpoint of social welfare.

Example 1: (Suzumura [75, p.31])

A father is to divide a cake among three children fairly. There are two methods to be examined. Method I is that the father divides this cake into three equal pieces, and tell the children to take a piece each, or leave it. Method II is that the children are given the opportunity to discuss how this cake should be divided fairly among them, and cut it into three pieces in accordance with the conclusion they agree on. If they happen to conclude that the equal division is the fair outcome, and if we are informed only of the outcomes, we cannot but conclude that these two methods of division are the same. It is clear, however, that this is certainly inappropriate. In the case of Method I, children are not provided with any right to participate in the process through which their distributive shares are determined, whereas in the case of Method II, they are indeed endowed with such an important right of participation. ‖

The gist of this example is that the right to participate is an important procedural feature which we are unable to capture if our informational basis is narrowly confined to consequential outcomes and nothing else. To rectify this defect, we must enlarge the description of social states in such a way that, not only the social *outcomes*, but also the *procedures* or *mechanisms* through which such outcomes are chosen, are included.

[7] In what follows, what we call a *mechanism* is to be interpreted as synonymous with a set of *institutional arrangements* which, in its turn, is identified with a family of *game forms*. We will explain the meaning of these cryptic expressions in the specific context of individual libertarian rights in Section 5, leaving the general analytical details to Hurwicz [31]. See also Arrow and Hurwicz [4, Part I and Part IV] and Schotter [50] for related analyses.

The same point can be made in terms of a more serious-looking example to the following effect.

Example 2: (Pattanaik and Suzumura [44, p.437])

Consider a society endowed with a fixed bundle of commodities. Suppose that individuals are entitled to engage in voluntary exchange among themselves, and the outcome in the core will be socially chosen. We assume that the core is always non-empty and consists of a single allocation. Suppose that the social decision-making procedure is changed into the one where the central planning board assigns to each individual a consumption bundle which he/she would have chosen if the previous procedure were used. As far as the outcome of social decision-making procedures are concerned, the two situations are the same, yet the structure of individual freedom and rights are substantially different between the two situations.[8] $\|$

Thus, procedural considerations can help us discriminate two situations which cannot but be identified if our conceptual framework is narrowly confined to consequential considerations only.

4 Rectification by Procedural Considerations: Welfare and Competition

The second problem we analyse in order to shed further light on the services rendered by procedural considerations is the classic issue of competition and economic welfare. It goes without saying that competition is assigned an indispensable role in economics in general, and in welfare economics in particular. In the absence of market failures, it is the *market-oriented competition* which is emphasized as the efficient and decentralized allocator of scarce resources, whereas in the presence of market failures, it is the *contest-based competition* within the public decision-making mechanism to cope with market failures which receives emphasis as a built-in safeguard against government failures.[9] In any case, there is a strong conventional belief in the welfare-enhancing effects of competition. This widespread belief may be traced back to Adam Smith who wrote in *The Wealth of Nations* that "[i]n general, if any branch of trade, or any division of labour, be advantageous to the public, the freer and more general the competition, it will always be the more so." Our first order of business is to see if and how this conventional wisdom can be vindicated.

With this crucial task in mind, we consider an oligopolistic industry producing a homogeneous product, where n firms compete in terms of quantities. Let $\pi_i(q_i; Q_{-i})$ be the profit of firm i, which is defined by

[8] See, also, Sen [64, p.14, footnote 20].

[9] See Suzumura [76] for further details on this dual role of competition in the special context of developing market economies. See Stiglitz [66] and Vickers [81] for the theory of contest-based competition.

$$\pi_i(q_i; Q_{-i}) := q_i f(Q) - c(q_i) \quad (i = 1, 2, \ldots, n), \tag{4.1}$$

where f is the inverse demand function for the product of this industry, q_i is the output of firm $i, Q_{-i} := \Sigma_{j \neq i} q_j$ is the aggregate output of firms other than $i, Q := Q_{-i} + q_i$ is the industry output, and c is the cost function that is identical for all firms.

We assume that:

(A.1): $f'(Q) < 0$ *holds for all* $Q > 0$.

(A.2): $c'(q) > 0$ *and* $c''(q) > 0$ *hold for all* $q > 0$.

(A.3): $(\partial^2/\partial q_i \partial q_j)\pi_i(q_i; Q_{-i}) < 0$ $(i \neq j; i, j = 1, 2, \ldots, n)$ *holds for all* $(q_i; Q_{-i}) > 0$.

Note that these assumptions are not pathological at all. To understand what is involved in (A.3), which is called the assumption of *strategic substitutability*, let the *reaction function* of firm i, to be denoted by $r_i(Q_{-i})$, be defined by

$$(\partial/\partial q_i)\pi_i(r_i(Q_{-i}), Q_{-i}) = 0. \tag{4.2}$$

It is easy to check that *the reaction function is downward sloping if and only if the assumption of strategic substitutability is satisfied*.[10]

In what follows, we assume that *the symmetric Cournot-Nash equilibrium* exists and is unique for each number of firms n. Let $q^{\mathrm{N}}(n)$ and $Q^{\mathrm{N}}(n)$ be, respectively, the individual firm's output and the industry output at the symmetric Cournot-Nash equilibrium with n firms. To see how $q^{\mathrm{N}}(n)$ and $Q^{\mathrm{N}}(n)$ will be affected by a change in n, we define the *cumulative reaction function*, to be denoted by $R_i(Q)$, by

$$q_i = R_i(Q) \ \textit{ if and only if } \ q_i = r_i(Q - q_i). \tag{4.3}$$

It is easy to confirm that the *cumulative reaction function is downward sloping if and only if the reaction function is downward sloping*. Note also that, by definition, we have $q^{\mathrm{N}}(n) = R_i(Q^{\mathrm{N}}(n))$ for all $i = 1, 2, \ldots, n$. Adding up these n equations, we may obtain $Q^{\mathrm{N}}(n) = \Sigma_{i=1}^n R_i(Q^{\mathrm{N}}(n))$, which means that $Q^{\mathrm{N}}(n)$ is nothing but the fixed point of the aggregate cumulative reaction function $\Sigma_{i=1}^n R_i$.

Figure 1 describes the displacement of the Cournot-Nash equilibrium when the number of firms increases from n to $n + \Delta n$. As is clearly seen from this figure, we have $q^{\mathrm{N}}(n) > q^{\mathrm{N}}(n + \Delta n)$ and $Q^{\mathrm{N}}(n) < Q^{\mathrm{N}}(n + \Delta n)$. Analytical proof of these properties is available in Suzumura [74] if necessary.

[10]This crucial assumption was first introduced by Bulow et al. [14] and soon became a standard tool in the analysis of oligopolistic interactions. See Suzumura [74] for the use of this assumption in various contexts of oligopoly and economic welfare.

Turning to the long-run performance of the Cournot market, let $\pi^{\mathrm{N}}(n)$ be the profit earned by each incumbent firm at the Cournot-Nash equilibrium with n firms, which is defined by

$$\pi^{\mathrm{N}}(n) := q^{\mathrm{N}}(n)f(Q^{\mathrm{N}}(n))-c(q^{\mathrm{N}}(n)). \tag{4.4}$$

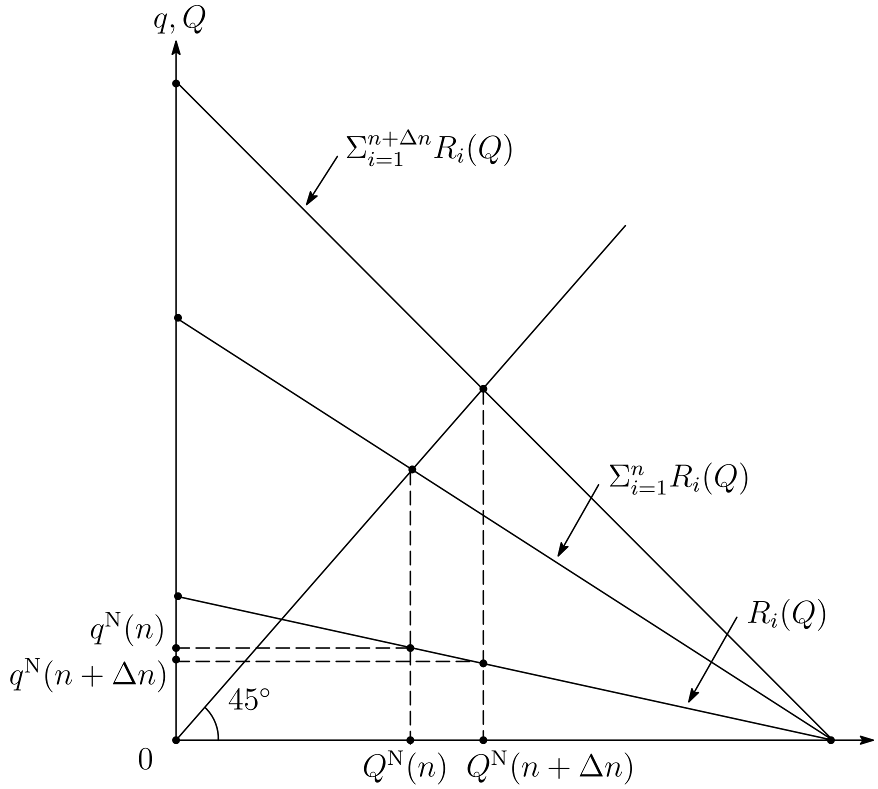

Figure 1. Firm entry and displacement of Cournot-Nash equilibrium

If $\pi^{\mathrm{N}}(n) >$ (resp. $<$) 0 holds, potential firms (resp. incumbent firms) will be motivated to enter into (resp. exit from) this industry, so that the *long-run Cournot-Nash equilibrium* will be attained when there are n_e firms in the industry, each producing the output $q^{\mathrm{N}}(n_e)$, where n_e is defined by $\pi^{\mathrm{N}}(n_e) = 0$. If this industry is left unregulated, this is the outcome we should expect to observe in the long-run Cournot market.

To understand the welfare implication of the long-run competition in the Cournot market, let

$$V(x, y) = u(x) + y \tag{4.5}$$

denote the utility function of the representative consumer, where x and y stand, respectively, for the consumption of the good produced by the industry in question and that of the *numerative* good. The budget constraint of the representative consumer is given by $M = px + y$, where p is the price of the good produced by this industry, which not only gives us $u'(x) = p$ as the first order condition for utility maximization, but also allows us to rewrite (4.5) as $v(x) := V(x, M - px) = u(x) + M - px$, where M consists of the external income I and profits generated in this industry $\Sigma_{i=1}^{n} \pi_i(q_i; Q_{-i})$. Evaluating $v(x)$ at the Cournot-Nash equilibrium, and neglecting a term which is exogenous in our present analysis, we obtain the following welfare criterion:

$$W^{\mathrm{N}}(n) := u(Q^{\mathrm{N}}(n)) - nc(q^{\mathrm{N}}(n)). \tag{4.6}$$

Suppose now that we are in the long-run Cournot-Nash equilibrium, and let there be a small decrease in the number of firms for whatever reason. This marginal change in n will affect the value of our welfare function by

$$(\mathrm{d}/\mathrm{d}n)W^{\mathrm{N}}(n_e) = \pi^{\mathrm{N}}(n_e) + n_e\{f(Q^{\mathrm{N}}(n_e)) - c'(q^{\mathrm{N}}(n_e))\}(\mathrm{d}/\mathrm{d}n)q^{\mathrm{N}}(n_e) \tag{4.7}$$

at the margin. It is clear that the first term in the RHS of (4.7) is zero by the definition of n_e, whereas the second term thereof is negative by virtue of (A.1), (A.2) and $(\mathrm{d}/\mathrm{d}n)q^{\mathrm{N}}(n) < 0$. Thus, the social welfare measured by (4.6) *increases* when there is a marginal *decrease* in the number of firms from the long-run equilibrium level n_e, which vindicates that there is a *socially excessive* firm entry at the long-run Coutnot-Nash equilibrium.[11]

If we are committed to the welfaristic, hence consequentialist, normative framework pure and simple, this simple result cannot but suggest that the traditional belief in the role of competition in the market economy must be rejected as a general truth. Should we acquiesce in a resigned view that "regulation by enlightened, but not omniscient, regulators could in principle achieve greater efficiency than deregulation (Panzar [42, p.313])?" It is in this context that we should remind ourselves of the role we expect of competition much more in detail.

There are three *instrumental* roles of competition which seem to underlie the traditional belief. First, competition abhors wastes, just as nature abhors a vacuum. If wastes exist anywhere in the current allocation of scarce resources, an unambiguous signal is sent that there are unexploited profit opportunities, and unfettered interfirm competition will soon drive them out. Thus, competition plays an instrumental role in promoting an intrinsic value of economic efficiency. Second, competition is a spontaneous mechanism

[11] This result, which came to be known as the *excess entry theorem*, was discovered independently by Mankiw and Whinston [39] and Suzumura and Kiyono [79]. Since then, it has been generalized along several lines. To cite just a few salient examples, Konishi et al. [35] extended this proposition by allowing for the general equilibrium interactions through factor markets, whereas Okuno-Fujiwara and Suzumura [41] extended the theorem to the three stage game subsuming strategic commitment with R&D in the second stage of oligopolistic interactions. These and other extensions are systematically expounded in Suzumura [74].

through which new innovations are introduced into the market economy. Indeed, firms will be motivated to introduce new innovations in order to sharpen their competitive edge against their rivals, and also to protect themselves from rival's challenges. In this sense, competition plays an instrumental role in promoting an intrinsic value of long-run economic progress. Third, competition is a *discovery procedure* in the sense of Friedrich von Hayek [28; 29; 30] through which a decentralized economic system may find a way to make an efficient use of dispersed and privately owned information. Since no agent in the decentralized economic system owns relevant information for calculating efficient use of resources at one stroke, it is left to market competition to find out who is most capable of meeting market demand in accordance with profit incentives and price signals. It is in this sense that competition plays an instrumental role in promoting an intrinsic value of privacy-respecting decentralized decision-making.

In addition to these instrumental values, competition allegedly has an *intrinsic* value of its own. It is the unique mechanism through which each economic agent can try out his/her own life chance on his/her own initiative and responsibility. It is true that the mechanism of competition can be very cruel and wasteful in weeding out the winners from the losers. It is also true that freedom rendered by this mechanism is crucially conditioned by initial distribution of assets and capabilities. However, these valid reservations do not change an iota of the fact that competition is a process which provides each agent with an equal and free opportunity to pursue his/her own aspirations in life. It is this intrinsic value of competition which the welfaristic concentration on consequential outcomes forces us to neglect.[12]

John Harsanyi once made an acute observation on value judgements to the following effect: "[I]f I approve of using A as a means to achieve some end B, I will do this on the assumption that A is causally effective in achieving B. Hence, my approval will be mistaken if this assumption is incorrect. Likewise, if I approve of A as an intrinsically desirable goal, I will do this on the assumption that A has some qualities I find intrinsically attractive. My approval will be mistaken if in fact A does not possess these qualities (Harsanyi [27, pp.792-793])." If we assume that A stands for the mechanism of competition and B for the enhancement of well-being of the individuals, his argument helps us summarize the gist of our foregoing analysis briefly as follows: If we are committed to consequential and instrumental approach to the value of competition, we cannot but disapprove this mechanism generally for its failure to achieve the proposed goal, whereas if we approve the same mechanism for its intrinsic value in terms of individual freedom, we are obliged to show that the competitive market mechanism is in fact capable of embodying such a value. It goes without saying that the normative analysis to this effect requires the expansion of our conceptual framework beyond consequentialism pure and simple.

[12] Sen was absolutely right when he wrote that "the economic theory of market allocation has tended to be firmly linked with a 'welfarist' normative framework. The success and failures of competitive markets are judged entirely by achievements of individual welfare ..., rather than by accomplishments in promoting individual freedom (Sen [62, p.519])." See, also, Lindbeck [38, p.314].

5 Endorsement by Procedural Considerations: Restricted Trade and Welfare

The third context where procedural considerations play an essential role in normative economics is the bilateral trade restrictions and economic welfare. Among several possible issues we may select in this broad area, we focus in what follows on the welfare implications of *voluntary export restraints* (VERs). VER is a trade-restricting device which has been introduced at the request of the government of the importing country and is accepted by the exporting country to forestall other trade restrictions. It is a highly costly protective measure to the importing country. Indeed, a VER is a variant of an import quota where license is assigned to foreign exporters rather than to domestic importers.[13] This being the case, a natural question suggests itself: Why should this bilateral agreement be requested by the government of importing country, and why should it be at all "voluntarily" accepted by exporting country? The answer is intuitively clear, yet depressing enough.

In the first place, by agreeing voluntarily to a VER, the foreign country may secure that rents will accrue to the exporters rather than remaining in the importing country. In the second place, domestic producers, who would otherwise have to face challenge by foreign exporters, are protected by the VER-induced cartel, and can reap extra profits from the domestic price increase, which foreign exporters can likewise entertain. Thus, domestic producers as well as foreign exporters may by able to entertain simultaneous increase in their profits *at the sacrifice of overwhelming decrease in domestic consumes' benefit*. This is the reason why a VER arrangement, being strongly promoted by domestic (import-competing) producers, is agreed to in complete neglect of domestic consumers' concomitant loss.

To show that this intuitive conclusion is indeed inevitable, let us consider a simple duopoly model where a domestic firm h and a foreign firm f supply their products to the domestic market and compete in terms of prices.[14] Let $d_i(p_h, p_f)$ be demand function for the good produced by firm i $(i = h, f)$. It is assumed that:

(B.1): *d_i is twice continuously differentiable. The goods produced by firms h and f are substitutes with each other, viz.* $(\partial/\partial p_i)d_i(p_h, p_f) < 0$ *and* $(\partial/\partial p_j)d_i(p_h, p_f) > 0\,(i \neq j; i, j = h, f)$ *holds for all* $(p_h, p_f) > 0$.

Let $c_i(q_i)$ be the cost function of firm i, which is assumed to satisfy the following:

[13] Instead of requesting a VER agreement, the importing country could use a tariff or import quota that would limit imports by the same amount. If these alternative import-restricting measures were invoked, rents earned by foreign exporters under VER would remain in the importing country either as tariff revenue or import license fee.

[14] The following analysis on the welfare effects of a VER agreement is a simplified version of Suzumura and Ishikawa [78]. Although we concentrate in the main text on the case of price competition, Suzumura and Ishikawa [78] have shown that essentially the same result holds true in the case of quantity competition as well. See, also, Bhagwati [8], Ethier [17], Harris [26] and Krishna [36; 37] for related analyses of the effects of a VER.

(B.2): $c_i'(q_i) > 0$ *and* $c_i''(q_i) \geq 0$ *hold for all* $q_i > 0$ $(i = h, f)$.

The profit function of firm i $(i = h, f)$ is defined to be $\pi_i(p_h, p_f) := p_i d_i(p_h, p_f) - c_i(d_i(p_h, p_f))$, which each firm i maximizes with respect to its own strategic variable, viz. price p_i. The first order condition for profit maximization is given by:

$$(\partial/\partial p_i)\pi_i(p_h, p_f) = d_i(p_h, p_f) + \{p_i - c_i'(d_h(p_h, p_f))\}(\partial/\partial p_i)d_i(p_h, p_f)$$
$$= 0\ (i \neq j; i, j = h, f).^{15} \tag{5.1}$$

Let $\{p_h^F, p_f^F\}$ be the Nash equilibrium in prices, which is defined by the simultaneous solution for the first order conditions (5.1) for $i = h$ and f. This is the *free trade price equilibrium* in our two country model. Let us now suppose that a VER is imposed on the foreign firm at the free trade equilibrium level of exports, viz. $q_f^F := d_f(p_h^F, p_f^F)$, so that the feasible price configuration $\{p_h, p_f\}$ must satisfy the following constraint: $d_f(p_h, p_f) = q_f^F$. Under this VER constraint, we may define a function $\zeta(p_h)$ by

$$p_f = \zeta(p_h) \text{ \textit{if and only if} } d_f(p_h, \zeta(p_h)) = q_f^F \tag{5.2}$$

for all $p_h > 0$. The meaning of the function $p_f = \zeta(p_h)$ should be clear. Since the VER cannot be violated once it is agreed to, the foreign firm cannot but choose $p_f = \zeta(p_h)$ when the domestic firm chooses p_h. In effect, the function $p_f = \zeta(p_h)$ describes the way how the foreign firm follows the domestic firm in choosing prices so as not to violate the VER. It is assumed that:

(B.3): $(\partial/\partial p_h)d_h(p_h, \zeta(p_h)) + \zeta'(p_h)(\partial/\partial p_f)d_h(p_h, \zeta(p_h)) < 0$ *holds for all* $p_h > 0$.

This assumption requires that an increase in the price of domestic good results in the decrease in the demand for domestic good even when the price of foreign product undergoes a concomitant change so as to keep the VER unviolated. Differentiating $\zeta(p_h)$ and taking (B.1) into consideration, we can verify that

$$\zeta'(p_h) = -\,(\partial/\partial p_h)d_f(p_h, \zeta(p_h))/(\partial/\partial p_f)d_f(p_h, \zeta(p_h)) > 0 \tag{5.3}$$

holds.

[15] Our analysis could be generalized by the introduction of conjectural variations. See Suzumura and Ishikawa [78].

If firm h behaves in full awareness of the VER agreement, the profit function of firm h becomes $\pi_h^V(p_h) := \pi_h(p_h, \zeta(p_h))$, which is assumed to be a strictly concave function in p_h. If we define $p_h^V :=$ arg max $\pi_h^V(p_h)$, then $\{p_h^V, \zeta(p_h^V)\}$ represents the *VER equilibrium in prices*. Since p_h^V is characterized by $(d/dp_h)\pi_h(p_h^V, \zeta(p_h^V)) = 0$, we may obtain

$$d_h(p_h^V, \zeta(p_h^V)) + \{p_h^V - c_h'(d_h(p_h^V, \zeta(p_h^V)))\}\{(\partial/\partial p_h)d_h(p_h^V, \zeta(p_h^V)) + \zeta'(p_h^V)(\partial/\partial p_f)d_h(p_h^V, \zeta(p_h^V))\} = 0. \tag{5.4}$$

On the other hand, invoking (5.1) for $i = h$ at the free trade price equilibrium $\{p_h^F, p_f^F\}$, we may obtain

$$(d/dp_h)\pi_h(p_h^F, \zeta(p_h^F)) = \zeta'(p_h^F)\{p_h^F - c_h'(d_h(p_h^F, \zeta(p_h^F)))\}(\partial/\partial p_f)d_h(p_h^F, \zeta(p_h^F)). \tag{5.5}$$

Since $p_f^F > c_h'(d_h(p_h^F, \zeta(p_h^F)))$ follows from (5.1) for $i = h$, we can conclude on the basis of (5.5) that $(d/dp_h)\pi_h(p_h^F, \zeta(p_h^F)) > 0$, where use is made of (B.1) and (5.3), which implies in turn that $p_h^F < p_h^V$ by virtue of the strict concavity of $\pi_h^V(p_h)$. Since we have $d_f(p_h^V, \zeta(p_h^V)) = d_f(p_h^F, p_f^F) = q_f^F$ by virtue of (5.2), we may obtain

$$\pi_f(p_h^F, p_f^F) < \pi_f(p_h^V, \zeta(p_h^V)), \tag{5.6}$$

where use is also made of (5.3). Therefore, *profits of the foreign firm increase when a VER is imposed at the free trade equilibrium level of exports.* This being the case, we may assert that, *in an international duopoly with product differentiation, where domestic firm and foreign firm compete in terms of prices, a VER imposed at the free trade equilibrium level of exports is voluntarily compiled with.*[16]

Turning to the consumer side of the home country, let $V_h(q_h, q_f) = u_h(q_h, q_f) + M_h - \Sigma_{i=h,f}\ p_i q_i$ be the utility function of the domestic representative consumer, where $M_h = I_h + \pi_h(p_h, p_f)$ denotes the income of the representative consumer, and I_h is his external income. By virtue of the first order condition for utility maximization, u_h satisfies $(\partial/\partial q_i)u_h(q_h, q_f) = p_i$ $(i = h, f)$. Thus, our welfare measure can be specified by

$$W_h(d_h(p_h, p_f), d_f(p_h, p_f)) := u_h(d_h(p_h, p_f), d_f(p_h, p_f)) - p_f d_f(p_h, p_f) - c_h(d_h(p_h, p_f)). \tag{5.7}$$

[16]This part of our result vindicates the conclusion of Harris [26] in a generalized framework.

It then follows that

$$
\begin{aligned}
&(d/dp_h)W_h(d_h(p_h,\zeta(p_h)),d_f(p_h,\zeta(p_h)))\\
&=\{p_h-c_h'(d_h(p_h,\zeta(p_h)))\}\{(\partial/\partial p_h)d_h(p_h,\zeta(p_h))\\
&+\zeta'(p_h)(\partial/\partial p_f)d_h(p_h,\zeta(p_h))\}\\
&-\zeta'(p_h)d_f(p_h,\zeta(p_h)), \qquad (5.8)
\end{aligned}
$$

where $(\partial/\partial q_h)u_h(q_h,q_f)=p_h$, $(\partial/\partial q_f)u_h(q_h(p_h,q_f))=p_h=\zeta(p_h)$, and $d_f(p_h,\zeta(p_h))=q_f^F$ are all made use of.

As an auxiliary step in evaluating the effect of a VER on domestic welfare, we need a simple lemma:

Lemma: *Let $\rho(p_h)$ be defined by $\rho(p_h):=p_h-c_h'(d_h(p_h,\zeta(p_h)))$. If $\rho(p_h)>0$ $(t=1,2)$ holds, then $\rho(\theta p_h^1+(1-\theta)p_h^2)>0$ holds for any θ satisfying $0<\theta<1$.*

Proof. Differentiating $\rho(p_h)$ we may verify that

$$
\begin{aligned}
p'(p_h)=1-c_h''(d_h(p_h,\zeta(p_h)))\{(\partial/\partial p_h)d_h(p_h,\zeta(p_h))\\
+\zeta'(p_h)(\partial/\partial p_f)d_h(p_h,\zeta(p_h))\}>0, \qquad (5.9)
\end{aligned}
$$

where use is made of (B.2) and (B.3). Since $\rho(p_h^t)>0$ $(t=1,2)$ by assumption and $\rho(p_h)$ is an increasing function, the assertion of the lemma immediately follows. $\|$

In order to compare the level of domestic welfare at the free trade equilibrium and that at the VER equilibrium, we now invoke the mean value theorem to assert that there exists a θ satisfying $0<\theta<1$ such that

$$
\begin{aligned}
&W_h(d_h(p_h^F,p_f^F),d_f(p_h^F,p_f^F))-W_h(d_h(p_h^V,\zeta(p_h^V)),d_f(p_h^V,\zeta(p_h^V)))\\
&=(p_h^F-p_h^V)(d/dp_h)W_h(d_h(p_h(\theta),\zeta(p_h(\theta))),d_f(p_f(\theta),\zeta(p_h(\theta)))) \qquad (5.10)
\end{aligned}
$$

holds, where $p_h(\theta):=\theta p_h^F+(1-\theta)p_h^V$. It is clear that (5.1) for $i=h$ evaluated at $\{p_h^F,p_f^F\}$ entails $\rho(p_h^F)>0$, whereas (5.4) entails $\rho(p_h^V)>0$. Combining (5.8) for $p_h=p_h(\theta)$ with (5.10) and invoking Lemma as well as the inequality $p_h^V>p_h^F$ which we have already established, we may finally obtain an unambiguous result:

$$
W_h(d_h(p_h^F,p_f^F),d_f(p_h^F,p_f^F))>W_h(d_h(p_h^V,\zeta(p_h^V)),d_f(p_h^V,\zeta(p_h^V))). \qquad (5.11)
$$

We have thus established the following remarkable result: *In an international duopoly with product differentiation, where domestic firm and foreign firm compete in terms of prices, a VER imposed at the free trade equilibrium level of exports cannot but decrease domestic welfare*.[17] This result is strongly reminiscent of Vilfredo Pareto's insightful observation to the following effect: "A protectionist measure provides large benefits to a small number of people, and causes a very great number of consumers a slight loss. This circumstance makes it easier to put a protection measure into practice (Pareto [43, p.379])."

In addition to the *welfaristic* criticism against the bilateral VER arrangements presented above, there is a *non-welfaristic* or *procedural* criticism which also casts a serious doubt on the legitimacy of bilateral trade restriction as a fair measure for international conflict resolution. More often than not, bilateral agreements are prepared and concluded in complete neglect of the third parties which have no way of representing themselves in the procedure through which such agreements are designed and implemented between the two countries involved, even though the third parties would most likely be affected by spillover effects of the bilateral agreements thereby concluded. This problem would not be alleviated even when the nature of bilateral agreements in question were such that the third parities could equally entertain the beneficial *outcomes* of bilateral agreements in full accordance with the most-favoured-nation treatment stipulated in the GATT/WTO agreements.[18] As was aptly observed by Sir Isaiah Berlin, "[t]he desire to be governed by myself, or at any rate to participate in the process by which my life is to be controlled, may be as deep a wish as that of a free area for action, and perhaps historically older (Berlin [7, pp.15-16])." Recent proliferation of bilateralism is deeply lamentable, as it tends to deny this innate desire to the third parties which are left outside the bilateral negotiation procedure. The problematic nature of bilateralism becomes even more lamentable when it is enforced by means of unilateral aggression and/or excessive extraterritorial application of domestic laws. Needless to say, this concern is also rooted in the procedural fairness considerations.

We have thus exemplified that there are cases where procedural considerations can add force to consequential considerations to strengthen the case for or against an economic or social policy under serious debate.

[17] A recent study of Kemp et al. [33] examined the conditions under which this result holds in the presence of general equilibrium interactions.

[18] Our argument, which emphasizes the procedural fairness of trade dispute settlement mechanism, is in sharp contrast with Rudiger Dornbusch who put forward a strongly consequentialist argument in favour of bilateralism to the following effect: "Bilateralism received a bad name when it was an instrument for restricting trade, but open bilateralism ... can be an effective instrument for securing more open trade. Indeed, if trade is open in the sense of allowing conditional most-favoured-nation access, a bilateral initiative can become a vehicle for freer trade on a multilateral basis. Third countries excluded from an initial agreement should be welcome to enjoy its benefits on condition they adhere to its terms (Dornbusch [15, p.107])." See also Garten [21] who likewise tries to justify bilateralism along the line we are arguing against in the text. Jeffrey Garten served as US Under Secretary of Commerce for International Trade until October 1995.

6 A Taste of Pudding: Welfare, Rights, and Social Choice Procedure

Enough have been said about the importance of procedural considerations in complementing, rectifying, and endorsing the social welfare judgements based on consequential considerations. What remains to be done is to exemplify the type of analysis which can be pursued on the basis of extended conceptual framework introduced in Section 2.

Among many possible directions to explore, we have chosen to focus in what follows on the issue of logical compatibility between individual libertarian rights and Pareto efficiency, which was originally posed by Sen [51; 52] in terms of his *impossibility of a Paretian liberal*. However, unlike Sen who articulated the concept of individual rights in terms of an individual's power (*local decisiveness*) to constrain the social choice rule, we articulate it as (i) the complete freedom of each individual to choose any admissible strategy, and (ii) the obligation of each individual not to choose an inadmissible strategy for him/herself, and not to prevent anyone from choosing his/her admissible strategy. To be more precise, the concept of individual libertarian rights is articulated in our approach through a *game form* which is a specification of a set $N := \{1, 2, \ldots, n\}$ of *players*, a set S_i of *admissible strategies* for each player $i \in N$, a set A of *feasible outcomes*, and an *outcome function* g_A, which maps each strategy profile $\mathbf{s}^N = (s_1, s_2, \ldots, s_n) \in \times_{i=1}^n S_i$ into a social outcome $g_A(\mathbf{s}^N) \in A$, viz. $G_A = (N, \{S_i\}, A, g_A)$.[19]

Let us begin with a general remark. In any discussion on individual rights, three distinct issues should be carefully distinguished and separately addressed. The first issue is the *formal structure* of rights. As we have already observed, the game form approach captures the formal structure of rights in terms of a game form G_A. Game form being nothing other than a game prior to the specification of players' preferences over outcomes, the formal structure of rights in the game form articulation is completely independent of players' preferences over outcomes. The second issue is the *realization* of conferred rights. The game form approach treats this issue in terms of a *game* which is defined by a pair $(G_A, \boldsymbol{R})$ of a game form G_A, and a profile $\boldsymbol{R} = (R_1, R_2, \ldots, R_n)$ of players' preference orderings over social outcomes. The game form approach regards the conferred individual rights to be fully realized if each and every player can freely exercise his/her admissible strategy in the actual play of the game $(G_A, \boldsymbol{R})$.[20] The third issue is that of the *initial conferment* of rights. It is in the context of treating this third issue within the game form approach that our extended conceptual framework plays an essential role.

To show how this issue can be treated in our framework, let us suppose that the decision-making mechanism is specified by the *rights-system* $\mathbf{G}$ which specifies a game form G_A for each set of feasible outcomes $A \subset X$, viz. $\mathbf{G} = \{G_A | A \in \Sigma\}$, where Σ is

[19]This *game form approach to individual rights*, so-called, originates in Nozick [40] and has been explored by Sugden [68], Gaertner et al. [20], Pattanaik and Suzumura [44; 45] and Suzumura [73; 75].

[20]It may well be the case that the game $(G_A, \boldsymbol{R})$ may not have an equilibrium when an equilibrium concept $\mathscr{E}$ is externally specified. This poses a problem for a theorist who wishes to analyse the interactions among individuals through this equilibrium concept, but the non-existence of equilibrium does not mean in itself that the realization of individual rights is somehow hindered.

the family of all feasible sets of outcomes. Let $\mathbf{Q} = (Q_1, Q_2, \ldots, Q_n)$ be the profile of extended individual preference orderings over the pairs of outcomes and rights-systems, viz. $(x, \mathbf{G}^1), (y, \mathbf{G}^2)$, etc. Recollect that the intended meaning of $(x, \mathbf{G}^1)Q_i(y, \mathbf{G}^2)$ is that, according to the judgements of i, it is at least as good having an outcome x through a rights-system $\mathbf{G}^1$ as having an alternative outcome y through an alternative rights-system $\mathbf{G}^2$.

What do we precisely mean when we say "having an outcome x through a rights-system $\mathbf{G}$"? In particular, how do we formalize the concept of feasibility in our extended framework? To clarify our meaning, note that a rights-system $\mathbf{G}$ and an extended profile $\mathbf{Q}$ induce a restricted profile $\mathbf{Q_G} = (Q_{1\mathbf{G}}, Q_{2\mathbf{G}}, \ldots, Q_{n\mathbf{G}})$ over the set of conventionally defined social states by $xQ_{i\mathbf{G}}y$ holds if and only if $(x, \mathbf{G})Q_i(y, \mathbf{G})$ holds for all $x, y \in X$ and all $i \in N$. Given a game form $G_A = (N, \{S_i\}, A, g_A)$ that articulates the conferred rights when the feasible set $A \in \Sigma$ prevails, we have a *game* $(G_A, \mathbf{Q_G})$. The play of this game will determine a set $T(G_A, \mathbf{Q_G})$ of *realizable extended social states*.[21] For the sake of simplicity in exposition, we assume in what follows that $T(G_A, \mathbf{Q_G})$ consists of a single outcome, say $\tau(G_A, \mathbf{Q_G})$. In this case, a feasible extended social state is represented by a pair of outcome and a rights-system $(\tau(G_A, \mathbf{Q_G}), \mathbf{G})$. We are now in the position to answer our previous question. Given a physically feasible set of states $A \in \Sigma$ and an extended profile $\mathbf{Q}$, an extended social state $(x, \mathbf{G})$ is feasible in the sense that an outcome x is actually attainable through a rights-system $\mathbf{G}$, given $(A, \mathbf{Q})$, if and only if $x = \tau(G_A, \mathbf{Q_G})$ holds true.[22]

We are now ready to explain how the game form approach treats the issue of initial conferment of rights. Let Ψ be the *extended social welfare function* which maps each profile $\mathbf{Q} = (Q_1, Q_2, \ldots, Q_n)$ of extended individual preference orderings into an extended social welfare ordering: $Q = \Psi(\mathbf{Q})$. Given a set $A \in \Sigma$ of feasible social outcomes, the *socially optimal conferment of rights* is nothing other than the rights-system $\mathbf{G}^*$ such that

$$(\tau(G^*_A, \mathbf{Q}_{\mathbf{G}^*}), \mathbf{G}^*)\boldsymbol{\Psi}(\mathbf{Q})(\tau(G_A, \mathbf{Q_G}), \mathbf{G}) \tag{6.1}$$

holds for any feasible rights-system $\mathbf{G}$.

An example may be of help to make this abstract framework more intuitively understandable.[23] Consider a specific situation where $N = \{C, D\}$ ($C =$"consequentialist"; $D =$ "deontologist"). There are two issues to be decided on. The first issue is the choice of religion, and there are two feasible options: $b =$ "Buddhism" and $c =$ "Christianity".

[21]Let $C_{\mathscr{E}}(G_A, \mathbf{Q_G})$ be the set of pure strategy equilibria of the game $(G_A, \mathbf{Q_G})$, where $\mathscr{E}$ is the relevant equilibrium concept. If the set $g_A[C_{\mathscr{E}}(G_A, \mathbf{Q_G})]$ of pure strategy equilibrium outcomes is non-empty, it is quite natural to assume that $T(G_A, \mathbf{Q_G}) = g_A[C_{\mathscr{E}}(G_A, \mathbf{Q_G})]$. See Pattanaik and Suzumura [45, pp.199-200] for the possible definition of $T(G_A, \mathbf{Q_G})$ when there exists no pure strategy equilibrium of the game $(G_A, \mathbf{Q_G})$.

[22]If $x \neq \tau(G_A, \mathbf{Q_G})$ holds, the extended alternative $(x, \mathbf{G})$ is not feasible, given $(A, \mathbf{Q})$. However, it still makes sense to talk about the preference for or against $(x, \mathbf{G}^1)$ *vis-à-vis* $(y, \mathbf{G}^2)$ just as we may meaningfully talk about preferences over the set of consumption plans irrespective of whether they can be attained under budget constraints.

[23]The following analysis capitalizes on Suzumura [75, Section 5].

The second issue is whether or not a given book is to be read, and there are two options: r = "to read the book" and n = "not to read the book".

The set A of feasible social outcomes consists of 16 states. A typical element of A is denoted by $(c, n; b, r)$, which is the state where Mr. C believes in Christianity and does not read the book, and Mr. D believes in Buddhism and reads the book.

There are two feasible rights-systems: $\mathbf{G}^1 = \{G^1\}$ and $\mathbf{G^2} = \{G^2\}$. The game form $G^1 = (N, \{S_i^1\}, A, g^1)$, where $S_i^1 = \{b, c\} \times \{r, n\}$ for $i = C, D$ and $g^1(\mathbf{s}^N) = \mathbf{s}^N$ for all $\mathbf{s}^N = (s_1, s_2)$ such that $s_i \in S_i^1$ for $i = C, D$, is the one where the two persons are empowered to choose their religion as well as reading or not reading the book freely. In contrast, the game form $G^2 = (N, \{S_i^2\}, A, g^2)$, where $S_i^2 = \{r, n\}$ for $i = C, D$ and $g^2(\mathbf{s}^N) = \mathbf{s}^N$ for all $\mathbf{s}^N = (s_1, s_2)$ such that $s_i \in S_i^2$ for $i = C, D$, is the one where the two persons are only allowed to choose reading or not reading the book freely, the matter of choosing common religion being decided by the society. If the social choice of common religion is $t \in \{b, c\}$ and the strategy pairs $\mathbf{s}^N = (s_1, s_2)$ is chosen, the social state will be given by $(t, s_1; t, s_2)$.

Let $\mathbf{Q} = (Q_C, Q_D)$ be the profile of extended individual preference orderings. We assume that Mr. C is a die hard *consequentialist*, who cares only about the outcomes of social interactions and nothing else. Thus, for all social state x in A, $(x, \mathbf{G}^1)I(Q_C)(x, \mathbf{G}^2)$ holds true, where $I(Q_C)$ is the indifference relation generated by Q_C. For each pair (u, v), where u (resp. v) refers to Mr. C's (resp. Mr. D's) religion, and for each $\mathbf{G} = \mathbf{G}^1$ and $\mathbf{G}^2$, let $Q_{C\mathbf{G}}(u, v)$ be defined by:[24]

$$Q_{C\mathbf{G}}(u, v) : (u, r; v, r), (u, n; v, r), (u, r; v, n), (u, n; v, n), \tag{6.2}$$

which, in turn, is used to define $Q_{C\mathbf{G}}$ that orders 16 alternatives altogether by:

$$Q_{C\mathbf{G}} : Q_{C\mathbf{G}}(b, c), Q_{C\mathbf{G}}(b, b), Q_{C\mathbf{G}}(c, b), Q_{C\mathbf{G}}(c, c). \tag{6.3}$$

We assume that Mr. D is a *deontologist* whose belief in the procedural justice in allowing people to choose their religion has such predominance that, whatever alternatives $x, y \in A$ are in fact at stake, he holds that $(x, \mathbf{G}^1)P(Q_D)(y, \mathbf{G}^2)$. For each pair (u, v) of religions of Mr. C and Mr. D and for each $\mathbf{G} = \mathbf{G}^1$ and $\mathbf{G}^2$, let us define $Q_{D\mathbf{G}}(u, v)$ by:

$$Q_{D\mathbf{G}}(u, v) : (u, n; v, n), (u, n; v, r), (u, r; v, n), (u, r; v, r), \tag{6.4}$$

which, in turn, is used to define $Q_{D\mathbf{G}}$ by:

$$Q_{D\mathbf{G}} : Q_{D\mathbf{G}}(c, c), Q_{D\mathbf{G}}(b, c), Q_{D\mathbf{G}}(c, b), Q_{D\mathbf{G}}(b, b). \tag{6.5}$$

Let us examine the game $(G^1, \mathbf{Q}_{\mathbf{G}^1})$. It is easy, if tedious, to check that (b, r) is the dominant strategy for Mr. C, and (c, n) is the dominant strategy for Mr. D. Thus, $(b, r; c, n)$ in A is the dominant strategy equilibrium of the game $(G^1, \mathbf{Q}_{\mathbf{G}^1})$. In the situation where there exists a dominant strategy equilibrium, it is very natural to assume

[24]Preference orderings are represented horizontally, with the less preferred alternative to the right of the more preferred alternative.

that $\tau(G^1, \mathbf{Q}_{\mathbf{G}^1}) = (b, r; c, n)$. Turning to the game $(G^2, \mathbf{Q}_{\mathbf{G}^2})$, it is again easy to confirm that r (resp. n) is the dominant strategy for Mr. C (resp. Mr. D) irrespective of whether the social choice of religion turns out to be b or c. Thus, $\tau(G^2, \mathbf{Q}_{\mathbf{G}^2}) = (b, r; b, n)$ or $(c, r; c, n)$ depending on the social choice of b or c. Recollect that Mr. D holds a lexicographic preference for $(x, \mathbf{G}^1)$ against $(y, \mathbf{G}^2)$ whatever may be x and y. Thus, he must surely prefer $(\tau(G^1, \mathbf{Q}_{\mathbf{G}^1}), \mathbf{G}^1)$ to $(\tau(G^2, \mathbf{Q}_{\mathbf{G}^2}), \mathbf{G}^2)$. Mr. C being a consequentialist, he is indifferent between $((b, r; c, n), \mathbf{G}^1)$ and $((b, r; c, n), \mathbf{G}^2)$ and he prefers $((b, r; c, n), \mathbf{G}^2)$ to $((b, r; b, n), \mathbf{G}^2)$ as well as to $((c, r; c, n), \mathbf{G}^2)$. By virtue of transitivity of Q_C, Mr. C must then prefer $(\tau(\mathbf{G}^1, Q_{G^1}), \mathbf{G}^1)$ to $(\tau(\mathbf{G}^2, Q_{G^2}), \mathbf{G}^2)$. Thus, as long as the extended social welfare function Ψ satisfies the Pareto principle, $\mathbf{G}^1$ must be the rights-system to be conferred. However, if $\mathbf{G}^1$ is conferred and the game $(G^1, \mathbf{Q}_{\mathbf{G}^1})$ is played, $(b, r; c, n)$ will be the social outcome, which is Pareto dominated by another feasible social state $(b, n; c, r)$.

We have thus exemplified in terms of the issue of initial conferment of game form rights what kind of analysis can be conducted through our extended conceptual framework which endogenizes procedural considerations along with consequential considerations. In so doing, we have shown that Sen's Pareto libertarian paradox recurs in the context of initial conferment of game form rights in the sense that the Pareto principle imposed on the extended social welfare function, which is invoked in the initial conferment of game form rights, and the Pareto principle imposed on the conventionally defined social outcomes cannot be satisfied simultaneously in general. In this sense, Sen's original criticism against welfaristic foundations of normative economics survives even if his articulation of libertarian rights in terms of individuals' local decisiveness is replaced by the allegedly more proper articulation through game forms. We would like to conclude this section by repeating our conviction that Sen's impossibility problem "persists under virtually every plausible concept of individual rights (Gaertner et al. [20, p.161])."

7 Concluding Remarks

No attempt will be made to summarize our main arguments presented in this chapter. Instead, we would like to conclude this chapter with several final observations.

In the first place, since the choice of a decision-making mechanism or procedure itself has to be made by a decision-making mechanism or procedure, it may be asserted that there must be some circularity in our conceptual framework. For example, in the concrete example through which we exemplified the working of this framework, the rights-system is chosen through the extended social welfare function which, in itself, is a social decision-making mechanism. However, this does not cause any serious problem of logical circularity. To verify this fact, let x_1 denote the component of a social state which is not a decision-making mechanism. Let x_2 be the component of a social state which is a social decision-making mechanism to decide on x_1's. In general, let x_t be the component of a social state which is a social decision-making mechanism to decide on $(x_1, x_2, \ldots x_{t-1})$. Suppose that there exists a t^* and a social decision-making mechanism $x^*_{t^*}$ such that there exists unanimous support for $x^*_{t^*}$. In this case, the seeming logical circularity can be safely broken, and our exercise is on the firm logical ground.[25] One cannot but recollect in this context a memorable passage from Jean-Jacques Rousseau to the following effect: "The law of plurality of votes is itself established by agreement, and supposes unanimity at least in the beginning."[26]

In the second place, the initial conferment of individual rights is determined in the game form approach by extended individual preference orderings through the extended social welfare function. A question may be asked if we are indeed prepared to change the initial conferment of rights, each time people change their mind and express different preferences. Our answer is in the affirmative in a qualified sense. It would be absurd if we were to change the social choice of rights-system, each time people changed their preferences over *conventionally defined social outcomes*, but it would be likewise absurd if our social decision-making mechanism were totally insensitive to changes in people's preferences over *the pairs of outcome and decision-making mechanism*. People's perception of what should be the conferred rights do change over time, and it would be an unsatisfactory social decision-making mechanism indeed if it were unable to keep these changes properly on record.

In the third place, our method of incorporating procedural considerations, which capitalizes on an insightful observation due to Arrow, is by no means the only feasible method to serve for that purpose. To cite just one salient example, Sven Hansson's [24; 25] recent attempt to explore social choice theory with procedural preferences is another method to capture the related, but distinct idea. According to Hansson, "[p]rocedural preferences are no mere trifles in social decisions. To the contrary, they provide mechanisms for decisional stability in cases when preferences restricted to outcomes do not do this. Preferences for consensus, or preferences against ties, induce participants to take part in compromises that would not be motivated by their strictly consequential preferences (Hansson [24, p.273])." The gist of our attempt in this chapter is not to advocate any particular method of analysis, but to call the readers' attention to the broad class of procedural issues which awaits our vigorous exploration.[27]

In the fourth and last place, we would like to keep on record our full agreement with Sen's assertion to the following effect: "The contrast between the procedural and consequential approaches is ... somewhat overdrawn, and it may be possible to combine them, to a considerable extent, in an adequately rich characterization of states of affairs. The dichotomy is far from pure, and it is mainly a question of relative concentration (Sen [63, p.12])." The purpose of this chapter would be served if we could bring this simple point home by exemplifying how such "adequately rich characterization of states of affairs" could be implemented.

[25] This reasoning is a simple adaptation of Arrow's argument [2, p.90].

[26] J.-J. Rousseau, *The Social Contract*, English translation, New York and London: G. P. Putnam's Sons, second edition, revised, 1906, pp.165-166.

[27] See, also, related earlier work by Friedland and Cimbala [19] as well as Grofman and Uhlaner [23].

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Chapter 22
Characterizations of Consequentialism and Non-Consequentialism*

1 Introduction

It is undeniable that most, if not all, welfare economists are *welfaristic* in their conviction in the sense that they regard an economic policy or economic system to be satisfactory if and only if it is warranted to generate outcomes which score high in the measuring rod of social welfare[1]. It is equally undeniable, however, that there do exist people who care not only about welfaristic features of the consequences, but also about non-welfaristic features of the consequences, or even non-consequential features of the procedure through which these consequences are brought about. Even those welfare economists with strong welfaristic conviction should be ready to take the judgements of people with non-welfaristic convictions into account in order not to be paternalistic in their welfare analysis. The purpose of this chapter is to develop several analytical frameworks which enable us to examine the choice behaviour of non-welfaristic people. More specifically, we develop several frameworks which can accommodate situations where an individual expresses his preferences of the following type: it is better for me that an outcome x is realized from the opportunity set A than another outcome y being realized from the opportunity set B.[2] Note, in particular, that he is expressing his intrinsic valuation of the opportunity

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[1]For a general observation on the concept and content of wefarism, see, among others, Sen [10; 11].

[2]Much attention has been focussed on the opportunity set evaluation, beginning with Sen [12; 13]. See, among many others, Bossert, Pattanaik and Xu [2], Pattanaik and Xu [8; 9], Sen [14; 15], and Suzumura [16]. To the best of our knowledge, Gravel [4; 5] remains the unique precursor in analysing the extended preference ordering on $X \times K$, where X is the set of social states and K is the set of opportunity sets. However, his approach is quite different from ours in that he assumes that an individual has two preference orderings, one for ordering outcomes in X and another for ordering the choice situations in $X \times K$. His analysis is focused on the possibility of conflict between these two orderings, and it has nothing to do with the consequentialism and non-consequentialism. Capitalizing on Arrow's [1; pp.89-91] insightful observation, Pattanaik and Suzumura [6; 7] and Suzumura [16; 17] developed a conceptual framework for the analysis of non-consequential features of the decision-making procedures through which consequential outcomes are brought about.

for choice if he prefers choosing x from the opportunity A, where $x \in A$, rather than choosing x from the sigleton opportunity set $\{x\}$. Using this analytical framework, we can put forward a concise definition of consequentialism and non-consequentialism, and we can also characterize these concepts in terms of a few simple axioms.

The structure of this chapter is as follows. In Section 2, we present the basic notation and definitions. Section 3 discusses the basic axioms which are assumed throughout this chapter. Some simple implications of these axioms are also identified in this section. In Section 4, we define the concept of an *extreme consequentialist* and a *strong consequentialist*, and characterize them axiomatically. We then turn in Section 5 to the concepts of an *extreme non-consequentialist* and a *strong non-consequentialist* and their axiomatic characterizations. Section 6 concludes this chapter with some remarks.

2 Basic Notations and Definitions

Let X, where $3 \leq |X| < \infty$, be the set of all mutually exclusive and jointly exhaustive *social states*. The elements of X will be denoted by $x, y, z, \cdots$, and they are called *outcomes*. K denotes the set of all finite non-empty subsets of X. The elements in K will be denoted by $A, B, C, \cdots$, and they are called *opportunity sets*. Let $X \times K$ be the Cartesian product of X and K. Elements of $X \times K$ will be denoted by $(x, A), (y, B), (z, C), \cdots$, and they are called *extended alternatives*. Let $\Omega := \{(x, A) | A \in K \text{ and } x \in A\}$. That is, Ω contains all (x, A) such that A is finite and x is an element of A. It should be clear that $\Omega \subseteq X \times K$, and for all $(x, A) \in \Omega$, $x \in A$ holds. For all $(x, A) \in \Omega$, the intended interpretation is the following: the alternative x is chosen from the opportunity set A.

Let $\succeq$ be a reflexive, complete and transitive binary relation over Ω. The asymmetric and symmetric parts of $\succeq$ will be denoted by $\succ$ and $\sim$, respectively. For any $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B)$ is interpreted as "choosing x from the opportunity set A is at least as good as choosing y from the opportunity set B." Thus, in the extended framework, it is possible to give an expression to the intrinsic value of the opportunity set in addition to the instrumental value thereof. Indeed, the decision maker recognizes the intrinsic value of the opportunity of choice if there exists an extended alternative $(x, A) \in \Omega$ such that $(x, A) \succ (x, \{x\})$.

3 Basic Axioms and Their Implication

In this section, we introduce two basic axioms for the ordering $\succeq$, and examine the implication of combining them together.

Independence (IND): For all $(x, A), (y, B) \in \Omega$, and all $z \in X \setminus (A \cup B)$, $(x, A) \succeq (y, B) \Leftrightarrow (x, A \cup \{z\}) \succeq (y, B \cup \{z\})$.

Simple Indifference (SI): For all $x \in X$, and all $y, z \in X \setminus \{x\}$, $(x, \{x, y\}) \sim (x, \{x, z\})$.

(IND) corresponds to the standard independence axiom used in the literature (see, for example, Pattanaik and Xu [8]. Its requirement is simple: for all opportunity sets A and B, if an alternative z is not in both A and B, then the extended preference ranking over $(x, A \cup \{z\})$ and $(x, B \cup \{z\})$ corresponds to that over (x, A) and (y, B), independent of the nature of the added alternative z. (SI) requires that choosing x from "simple" cases, each involving two alternatives, is regarded as indifferent to each other.

The following result summarizes the implication of the above two axioms.

Theorem 3.1. If $\succeq$ satisfies (IND) and (SI), then for all $(x, A), (x, B) \in \Omega$, $\#A = \#B \Rightarrow (x, A) \sim (x, B)$.

Proof. Let $\succeq$ satisfy (IND) and (SI). Let $(x, A), (x, B) \in \Omega$ be such that $\#A = \#B$.

First, consider the case where $A \cap B = \{x\}$. Let $A = \{x, a_1, \ldots, a_m\}$ and $B = \{x, b_1, \ldots, b_m\}$. Since opportunity sets are finite, $m < +\infty$ holds. From (SI), clearly, $(x, \{x, a_i\}) \sim (x, \{x, b_j\})$ for all $i, j = 1, \ldots, m$. Two simple applications of (IND) lead us to have $(x, \{x, a_1, a_2\}) \sim (x, \{x, a_1, b_1\})$ and $(x, \{x, a_1, b_1\}) \sim (x, \{x, b_1, b_2\})$. By the transitivity of $\succeq, (x, \{x, a_1, a_2\}) \sim (x, \{x, b_1, b_2\})$ follows easily. By using an argument similar to the above, from (IND) and the transitivity of $\succeq$, we obtain $(x, A) \sim (x, B)$.

Next, consider the case that $A \cap B = \{x\} \cup C$, where C is nonempty. From the above, noting that $\#(A \setminus B) = \#(B \setminus C)$, we must have $(x, (A \setminus C) \cup \{x\}) \sim (x, (B \setminus C) \cup \{x\})$. Since opportunity sets are finite, $(x, A) \sim (x, B)$ can then be obtained from (IND). ■

4 Consequentialism

In this section, we define and characterize two versions of consequentialism: *extreme consequentialism* and *strong consequentialism*. First, we define the extreme consequentialism and strong consequentialism, respectively, as follows.

Definition 4.1. $\succeq$ is said to be *extremely consequential* if, for all $(x, A), (x, B) \in \Omega$, $(x, A) \sim (x, B)$.

Definition 4.2. $\succeq$ is said to be *strongly consequential* if, for all $(x, A), (y, B) \in \Omega$, $(x, \{x\}) \sim (y, \{y\})$ implies $[(x, A) \succeq (y, B) \Leftrightarrow \#A \geq \#B]$, and $(x, \{x\}) \succ (y, \{y\})$ implies $(x, A) \succ (y, B)$.

Thus, according to the extreme consequentialism, two choice situations (x, A) and (y, B) are judged exclusively on their consequences x and y, and the opportunity sets A and B from which these consequences are chosen are irrelevant. On the other hand, the strong consequentialism stipulates that opportunity sets do not matter when the indiviudual has a strict preference over $(x, \{x\})$ and $(y, \{y\})$. Only when the individual is indifferent between $(x, \{x\})$ and $(y, \{y\})$ do opportunities matter.

To characterize the extreme consequentialism and strong consequentialism, the following axioms will prove useful.

Local Indifference (LI): For all $x \in X$, there exists $(x, A) \in \Omega \setminus \{(x, \{x\})\}$ such that $(x, X) \sim (x, A)$.

Local Strict Monotonicity (LSM): For all $x \in X$, there exists $(x, A) \in \Omega \setminus \{(x, \{x\})\}$ such that $(x, A) \succ (x, \{x\})$.

(LI) is a minimal and local requirement of extreme consequentialism: there exists an opportunity set A in K, which is distinct from $\{x\}$, such that choosing an alternative x from A is regarded as being indifferent to choosing x from the sigleton set $\{x\}$. (LSM) requires that there exists an opportunity set A such that choosing x from the opportunity set A is strictly better than choosing x from the singleton opportunity set $\{x\}$. In other words, the individual values opportunities per se at least in this very limited sense.

Theorem 4.1. $\succeq$ satisfies (IND), (SI) and (LI) if and only if it is extremely consequential.

Proof. If $\succeq$ is extremely consequential, then it clearly satisfies (IND), (SI), and (LI). Therefore, we have only to prove that, if $\succeq$ satisfies (IND), (SI), and (LI), then, for all $(x, A), (x, B) \in \Omega, (x, A) \sim (x, B)$ holds.

Let $\succeq$ satisfy (IND), (SI), and (LI). First, note that from Theorem 3.1 we have the following:

$$\text{For all } (x, A), (x, B) \in \Omega, \#A = \#B \Rightarrow (x, A) \sim (x, B). \tag{4.1}$$

Hence, we have only to show that

$$\text{For all } (x, A), (x, B) \in \Omega, \#A > \#B \Rightarrow (x, A) \sim (x, B). \tag{4.2}$$

To begin with, we show that:

$$\text{For all } x \in X \text{ and all } y \in X \setminus \{x\}, \quad (x, \{x, y\}) \sim (x, \{x\}). \tag{4.3}$$

Let $x \in X$. Suppose for some $a \in X \setminus \{x\}, (x, \{x, a\}) \succ (x, \{x\})$. Given (SI), by the transitivity of $\succeq$ we have:

$$\text{For all } y \in X \setminus \{x\} : (x, \{x, y\}) \succ (x, \{x\}). \tag{4.4}$$

Then, by (IND), we obtain:

$$\text{For all } z \in X \setminus \{x, y\} : \ (x, \{x, y, z\}) \succ (x, \{x, z\}). \tag{4.5}$$

Using an argument similar to that for (4.4) and (4.5), we can show that:

$$\text{For all } (x, A) \in \Omega \text{ and all } m = 4, \dots \ : \#A = \ m \quad \Rightarrow (x, A) \succ (x, \{x\}). \tag{4.6}$$

Equation (4.6), together with (4.4) and (4.5), is in contradiction with (LI). Therefore, we cannot have $(x, \{x, a\}) \succ (x, \{x\})$ for some $a \in X \setminus \{x\}$. Similarly, if $(x, \{x\}) \succ (x, \{x, b\})$ for some $b \in X$, we can show that $(x, \{x\}) \succ (x, A)$ holds for all $(x, A) \in \Omega$

with $A \neq \{x\}$, another contradiction with (LI). Hence, by the completeness of $\succeq$, Eq. (4.3) holds.

From (4.3), noting the finiteness of opportunity sets, and by the repeated use of (IND), (4.1) and the transitivity of $\succeq$ (4.2) obtains. ■

Before turning to the full characterization of the strong consequentialism, we note the following result which will prove useful in establishing the remainder of our results in this paper.

Lemma 4.1. If $\succeq$ satisfies (IND), (SI) and (LSM), then, for all $(x, A), (x, B) \in \Omega$, $\#A \geq \#B \Leftrightarrow (x, A) \succeq (x, B)$.

Proof. Let $\succeq$ satisfy (IND), (SI) and (LSM). Note that, from Theorem 3.1, we have the following:

$$\text{For all } (x, A), (x, B) \in \Omega : \#A = \#B \Rightarrow (x, A) \sim (x, B). \tag{4.7}$$

Therefore, we have only to show that:

$$\text{For all } (x, A), (x, B) \in \Omega : \#A > \#B \Rightarrow (x, A) \succ (x, B). \tag{4.8}$$

We first note that, by following an argument simlar to that in the proof of Theorem 4.1, the following can be established.

$$\text{For all } x \in X, \text{ all } y \in X : x \neq y \Rightarrow (x, \{x, y\}) \succ (x, \{x\}). \tag{4.9}$$

Now, from (4.9), by the repeated use of (IND), we can derive the following:

$$\text{For all } (x, A) \in \Omega \setminus \{(x, X)\} \text{ and } y \in X \setminus A : (x, A \cup \{y\}) \succ (x, A). \tag{4.10}$$

Then, given the finiteness of the opportunity sets, note (4.7) and (4.10), (4.8) follows from the transitivity of $\succeq$. ■

To characterize the strong consequentialism, we need an additional condition which requires that, for all $(x, A), (y, B) \in \Omega$ and all $z \in X$, if the indiviudual ranks (x, A) higher than (y, B), then adding z to B while maintaining y being chosen from $B \cup \{z\}$ will not affect the individual's ranking: (x, A) is still ranked higher than $(y, B \cup \{z\})$. Formally:

Robustness (ROB): For all $(x, A), (y, B) \in \Omega$ and all $z \in X$, if $(x, \{x\}) \succ (y, \{y\})$ and $(x, A) \succ (y, B)$, then $(x, A) \succ (y, B \cup \{z\})$.

We are now ready to put forward the following full characterization of strong consequentialism.

Theorem 4.2. $\succeq$ satisfies (IND), (SI), (LSM) and (ROB) if and only if it is strongly consequential.

Proof. If $\succeq$ is strongly consequential, then it clearly satisfies (IND), (SI), (LSM) and (ROB). Therefore, we have only to prove that if $\succeq$ satisfies (IND), (SI), (LSM) and (ROB), then, for all $(x, A), (y, B) \in \Omega$, $(x, \{x\}) \sim (y, \{y\})$ implies $[(x, A) \succeq (y, B) \Leftrightarrow \#A \geq \#B]$, and $(x, \{x\}) \succ (y, \{y\})$ implies $(x, A) \succ (y, B)$.

Let $\succeq$ satisfy (IND), (SI), (LSM) and (ROB). Note that, from Lemma 4.1, we have the following:

$$\text{For all } (x, A), (x, B) \in \Omega : \#A \geq \#B \Leftrightarrow (x, A) \succeq (x, B). \tag{4.11}$$

Now, for all $x, y \in X$, consider $(x, \{x\})$ and $(y, \{y\})$. If $(x, \{x\}) \sim (y, \{y\})$, then, since X contains at least three alternatives, by (IND), for all $z \in X \setminus \{x, y\}$, we must have $(x, \{x, z\}) \sim (y, \{y, z\})$. From (4.11) and by the transitivity of $\succeq$, we then have $(x, \{x, y\}) \sim (y, \{x, y\})$. Then, by (IND), we have $(x, \{x, y, z\}) \sim (y, \{x, y, z\})$. Since the opportunity sets are finite, by repeated application of (4.11), the transitivity of $\succeq$ and (IND), we obtain

$$\text{For all } (x, A), (y, B) \in \Omega : \text{ if } (x, \{x\}) \sim (y, \{y\}), \text{ then } \#A \geq \#B \Leftrightarrow (x, A) \succeq (y, B). \tag{4.12}$$

If, on the other hand, $(x, \{x\}) \succ (y, \{y\})$, then, for all $z \in X$, by (ROB), $(x, \{x\}) \succ (y, \{y, z\})$. Since the opportunity sets are finite, by repeated use of (ROB), we then have $(x, \{x\}) \succ (y, A)$ for all $(y, A) \in \Omega$. Therefore, from (4.11) and the transitivity of $\succeq$, we obtain that:

$$\text{For all } (x, A), (y, B) \in \Omega, \text{ if } (x, \{x\}) \succ (y, \{y\}), \text{ then } (x, A) \succ (y, B). \tag{4.13}$$

Equation (4.13), together with (4.11) and (4.12), completes the proof. ■

In Suzumura and Xu [18], we have exemplified that the axioms (IND), (SI), and (LI) are independent and also that the axioms (IND), (SI), (LSM), and (ROB) are independent. Thus, our characterization theorems, viz., Theorem 4.1 for extreme consequentialism and Theorem 4.2 for strong consequentialism, do not contain any redundancy.

5 Non-Consequentialism

In this section, we define and characterize two versions of non-consequentialism, to be called the *extreme non-consequentialism* and *strong non-consequentialism*, respectively. Their definitions are given below.

Definition 5.1. $\succeq$ is said to be *extremely non-consequential* if, for all $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B) \Leftrightarrow \#A \geq \#B$.

Definition 5.2. $\succeq$ is said to be *strongly non-consequential* if, for all $(x, A), (y, B) \in \Omega$, $\#A > \#B \Rightarrow (x, A) \succ (y, B)$, and $\#A = \#B \Rightarrow [(x, \{x\}) \succeq (y, \{y\}) \Leftrightarrow (x, A) \succeq (y, B)]$.

According to extreme non-consequentialism, consequences do not matter at all, and what is valued is the richness of opportunity involved in the choice situation. Thus, two extended alternatives, (x, A) and (y, B), are ranked exclusively according to the cardinality of A and B and the consequences do not have any influence at all. In its complete neglect of consequences, extreme non-consequentialism is indeed extreme, but it captures the sense in which people may claim: "Give me liberty, or give me death." On the other hand, strong non-consequentialism pays attention to consequences if and only if two opportunity sets contain the same number of alternatives.

To give characterizations of extreme non-consequentialism and strong non-consequentialism, the following axioms will be used.

Indifference of No-Choice Situations (INS): For all $x, y \in X$, $(x, \{x\}) \sim (y, \{y\})$.

Simple Preference for Opportunities (SPO): For all distinct $x, y \in X$, $(x, \{x, y\}) \succ \{y, \{y\})$.

(INS) is simple and easy to interpret. It says that in facing two choice situations in which each outcome is restricted to choices from singleton sets, the individual is indifferent between them. (INS) thus conveys the message that in these simple cases, the individual feels that there is no real freedom of choice in each choice situation and is ready to express his indifference among these cases *regardless of the nature of the outcomes.* In a sense, it is the lack of freedom of choice that "forces" the individual to be indifferent among these situations. This idea is similar to an axiom proposed by Pattanaik and Xu [8] for ranking opportunity sets in terms of freedom of choice, which stipulates that all singleton sets offer the same amount of freedom of choice. On the other hand, (SPO) stipulates that it is always better for the individual to choose an outcome from the set containing two elements (one of which is the chosen outcome) than to choose an outcome from the singleton set. (SPO) thus displays the individual's desire to have some genuine opportunities of choice.

Theorem 5.1. $\succeq$ satisfies (IND), (SI), (LSM) and (INS) if and only if it is extremely non-consequential.

Proof. If $\succeq$ is extremely non-consequential, then it clearly satisfies (IND), (SI), (LSM) and (INS). Therefore, we have only to prove that, if $\succeq$ satisfies (IND), (SI), (LSM) and (INS), then, for all $(x, A), (y, B) \in \Omega$, $\#A \geq \#B \Leftrightarrow (x, A) \succeq (y, B)$.

To begin with, from Lemma 4.1, we have the following:

$$\text{For all } (x, A), (x, B) \in \Omega, (x, A) \succeq (x, B) \Leftrightarrow \#A \geq \#B. \tag{5.1}$$

Now, for all $x, y \in X$, by (INS), $(x, \{x\}) \sim \{y, \{y\})$. For all $z \in X \setminus \{x, y\}$, by (IND), $(x, \{x, z\}) \sim (y, \{y, z\})$. It follows from (5.1) that $(x, \{x, y\}) \sim (y, \{x, y\})$, where use is made of the transitivity of $\succeq$. By the repeated use of (5.1), (IND) and the transitivity of $\succeq$, and noting that all opportunity sets are finite, we can show that:

$$\text{For all } (x, A), (y, B) \in \Omega, (x, A) \succeq (y, B) \Leftrightarrow \#A \geq \#B. \tag{5.2}$$

Equation (5.2) completes the proof of Theorem 5.1. ■

Before presenting the characterization theorem for strong non-consequentialism, we note the following result which will prove useful in establishing the characterization theorem.

Lemma 5.1. If $\succeq$ satisfies (IND), (SI) and (SPO), then it also satisfies (LSM).

Proof. Let $\succeq$ satisfies (IND), (SI) and (SPO). Let $x \in X$. For all $y \in X \setminus \{x\}$, by (SPO), $(x, \{x, y\}) \succ (y, \{y\})$. Then, (IND) implies $(x, \{x, y, z\}) \succ (y, \{y, z\})$ for all $z \in X \setminus \{x, y\}$. By (SI), $(y, \{y, z\}) \sim (y, \{x, y\})$. The transitivity of $\succeq$ now implies $(x, \{x, y, z\}) \succ (y, \{x, y\})$. By (SPO), $(y, \{x, y\}) \succ (x, \{x\})$. Then, $(x, \{x, y, z\}) \succ (x, \{x\})$ follows from the transitivity of $\succeq$. That is, (LSM) holds. ■

Theorem 5.2. $\succeq$ satisfies (IND), (SI) and (SPO) if and only if it is strongly non-consequential.

Proof. If $\succeq$ is strongly non-consequential, then it clearly satisfies (IND), (SI) and (SPO). Therefore, we have only to prove that, if $\succeq$ clearly satisfies (IND), (SI) and (SPO), then, for all $(x, A), (y, B) \in \Omega, \#A > \#B \Rightarrow (x, A) \succ (y, B)$ and $\#A = \#B \Rightarrow [(x, \{x\}) \succeq (y, \{y\}) \Leftrightarrow (x, A) \succeq (y, B)]$.

Conversely, from Lemma 5.1 and Lemma 4.1, we have (5.1). For all distinct $x, y \in X$, by (SPO), $(x, \{x, y\}) \succ (y, \{y\})$. Then, from (5.1) and the transitivity of $\succeq$, for all $z \in X \setminus \{x\}$, $(x, \{x, z\}) \succ (y, \{y\})$. By (IND), from $(x, \{x, y\}) \succ (y, \{y\})$, for all $z \in X \setminus \{x, y\}$, $(x, \{x, y, z\}) \succ (y, \{y, z\})$. Noting (5.1) and the transitivity of $\succeq$, we then have:

$$\text{For all } (x, A), (y, B) \in \Omega, \text{ if } \#A = \#B + 1 \text{ and } \#B \leq 2, \text{then } (x, A) \succ (y, B). \quad (5.3)$$

From (5.3), by the repeated use of (IND), (5.1) and the transitivity of $\succeq$, coupled with the finiteness of all opportunity sets, we can obtain the following:

$$\text{For all } (x, A), (y, B) \in \Omega, \text{ if } \#A = \#B + 1, \text{ then } (x, A) \succ (y, B). \quad (5.4)$$

From (5.4), the transitivity of $\succeq$ and (5.1), we have

$$\text{For all } (x, A), (y, B) \in \Omega, \text{ if } \#A > \#B, \text{ then } (x, A) \succ (y, B). \quad (5.5)$$

Consider now $(x, \{x\})$ and $(y, \{y\})$. If $(x, \{x\}) \sim \{y, \{y\}\}$, following a similar argument as in the proof of Theorem 5.1, we can obtain

$$\text{For all } (x, A), (y, B) \in \Omega, \text{ if } (x, \{x\}) \sim (y, \{y\}) \text{ and } \#A = \#B, \text{ then } (x, A) \sim (y, B). \quad (5.6)$$

If, on the other hand, $(x, \{x\}) \succ \{y, \{y\}\}$, we can follow a similar argument as in the proof of Theorem 4.2 to obtain

$$\text{For all } (x, A), (y, B) \in \Omega, \text{ if } (x, \{x\}) \succ (y, \{y\}) \text{ and } \#A = \#B, \text{ then } (x, A) \succ (y, B). \quad (5.7)$$

Equation (5.7), together with (5.5) and (5.6), completes the proof. ■

In Suzumura and Xu [18], we have exemplified that the axioms (IND), (SI), (LSM) and (INS) are independent and also that the axioms (IND), (SI), and (SPO) are independent. Thus our characterzation theorems, viz., Theorem 5.1 for extreme non-consequentialism and Theorem 5.2 for strong non-consequentialism, do not contain any redundancy.

6 Concluding Remarks

Using the analytical framework of extended preference orderings, where individuals express preferences over the pairs of outcomes and opportunity sets from which outcomes are chosen, we developed in this chapter a simple analysis of consequentialism and non-consequentialism. We have identified two types of consequentialism, extreme consequentialism and strong consequentialism, and two types of non-consequentialism, extreme non-consequentialism and strong non-consequentialism. Although these identified types are rather extreme, they are meant to illustrate the kind of analysis in which we may talk about the similarity and dissimilarity of individual attitude toward outcomes and opportunities. Such an analysis is presented in our companion paper, Suzumura and Xu [19], where we examined how and to what extent Arrow's general impossibility theorem hinges on his basic assumption of welfarist-consequentialism, on the one hand, and whether or not Arrow's suggestion that "the possibility of social welfare judgements rests upon a similarity of attitudes toward social alternative (Arrow [1, p.69])" can be sustained within the wider conceptual framework than Arrow's own.

$$(\text{IND}) \oplus (\text{SI}) \begin{cases} \oplus\ (\text{LI}) = \text{extreme consequentialism} \\[2em] \oplus\ (\text{LSM}) \begin{cases} \oplus\ (\text{ROB}) = \text{strong consequentialism} \\ \oplus\ (\text{INS}) = \text{extreme non-consequentialism} \\ \oplus\ (\text{SPO}) = \text{strong non-consequentialism} \end{cases} \end{cases}$$

IND: Independence
SI: Simple Indifference
LI: Local Indifference
LSM: Local Strict Monotonicity
ROB: Robustness
INS: Indifference of No-choice Situations
SPO: Simple Preference for Opportunities

Note: $(A) \oplus (B)$ indicates the logical combination of the two axioms A and B.

Figure 1: Characterization Theorems

It should be noted that, although individual attitudes toward outcomes and opportunities reflected in extreme and strong consequentialism, and extreme and strong non-consequentialism are quite diverse, they all satisfy (IND) and (SI). More remarkbly, the strong consequentialism, extreme and strong non-consequentialism have more in common: they all satisfy not only (IND) and (SI), but also (LSM). With our axiomatic characterizations of the extreme and strong consequentialism, and the extreme and strong non-consequentialism, we hope that the contrast and similarity of these concepts have received some clarifications. These characterization theorems are summarized in Figure 1, where $(A) \oplus (B)$ indicates the logical combination of the two axioms A and B.

It is interesting to note that, despite their diverse attitudes toward opportunities and outcomes, the extreme and strong consequentialism, and the extreme and strong non-consequentialism all satisfy the following property:

Monotonicity (MON): For all $(x, A), (x, B) \in \Omega$, $B \subseteq A \Rightarrow (x, A) \succeq (x, B)$.

According to (MON), the individual is not averse to richer opportunities; that is, choosing an outcome x from the opportunity set A is at least as good as choosing the same x from the opportunity set B which is a subset of A.[3] Clearly, (MON) and (LI), and (MON) and (LSM) are independent.

It is hoped that our attempt in this chapter will be suggestive enough to motivate further exploration of the analytical framework of extended preference orderings. To identify some possible directions to be explored, two final remarks may be in order.

First, the two basic axioms, viz., (IND) and (SI), which are commonly invoked in our axiomatic characterzations of consequentialism and non-consequentialism, are not in fact beyond any dispute. Indeed, it is fairly common that an added alternative may have "epistemic value" in that it tells us something important about the nature of the choice situation. Sen [15, p.753] provides us with a telling example: "If invited to tea (t) by an acquaintance you might accept the invitation rather than going home (O), that is, pick t from the choice over $\{t, O\}$, and yet turn the invitation down if the acquaintance, whom you do not know very well, offers you a *wider* menu of having either tea with him or some heroin and cocain (h); that is, you may pick O, rejecting t, from the larger set $\{t, h, O\}$. The expansion of the menu offered by this acquaintance may tell you something about the kind of person he is, and this could affect your decision even to have tea with him." This means a clear violation of (IND) when $A = B$. (SI) is also not immune to possible exceptions. Take, for example, the case where X denotes the set of alternative measures of transportation for moving from city A to city B. If x and y stand, respectively, for exactly the same car except for the serial number, you may feel indifferent between choosing x and y, so that you may feel fine even if you must choose x rather than y from the opportunity set $\{x, y\}$. However, when the choice is between x and z where z is a comfortable train connecting A and B, you may feel very unhappy if you are forced to choose x in the presence of z. Thus, you may express a preference for $(x, \{x, z\})$ against $(x, \{x, y\})$, which is a clear violation of (SI).

[3] Note that we are neglecting decision-making cost and other factors which may make a larger opportunity set a liability rather than a credit. In this context, see Dworkin [3].

Second, our axiomatizations of consequentialism and non-consequentialism were concerned with rather extreme cases where unequivocal priority is given to consequences (resp. opportunities) not only in the case of extreme consequentialism (resp. extreme non-consequentialim) but also in the case of strong consequentialism (resp. strong non-consequentialism). It goes withoug saying that further research should be pursued so that active interactions between consequential considerations and procedural considerations are allowed to play an essential role. One possible approach retains the axioms (IND) and (SI) and invokes the axiom (MON), which we have just observed to be the common property of extreme consequentialism, strong consequentialism, etreme non-consequentialism, and strong non-consequentialism. It can be shown that, for finite X and K being the set of all nonempty subsets of X, $\succeq$ satisfies (IND), (SI), and (MON) if and only if there exists a function $u : X \to \mathbb{R}$ and a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that (a) for all $x, y \in X, u(x) \succeq u(y) \Leftrightarrow (x, \{x\}) \succeq (y, \{y\})$; (b) for all $(x, A), (y, B) \in \Omega, (x, A) \succeq (y, B) \Leftrightarrow f(u(x), \#A) \geq f(u(y), \#B)$; and (c) f is non-decreasing in each of its arguments. It should be clear that the characterization theorems on consequentialism and non-consequentialism which are developed in this chapter can be located as special cases of this general theorem which allows trade-off between the value of consequences and the richness of opportunities. However, the full exploration of this general approach must be left for a future occation.

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Chapter 23
Consequences, Opportunities, and Generalized Consequentialism and Non-Consequentialism*

1 Introduction

In recent years, people have gradually come to realize the importance of non-welfaristic features of the consequences in forming economic policy recommendations as well as in performing economic systems analysis. Some salient examples of non-welfaristic features of the consequences are individual and group rights, procedures through which consequences are realized, and opportunities from which outcomes are chosen.[1]

In a recent paper, Suzumura and Xu [20] developed several analytical frameworks which can accommodate situations where an individual expresses his extended preferences of the following type: it is better for him that an alternative x is brought about from the opportunity set A rather than another alternative y being brought about from the opportunity set B. An important feature of these frameworks is the novel concept of *extended alternatives* in the form of (x, A) with the intended interpretation of x being realized from the opportunity set A, which is used to evaluate alternative economic policies. Within these proposed frameworks, they defined various concepts of consequentialism and non-consequentialism and gave axiomatic characterizations of these concepts. The various concepts of consequentialism and non-consequentialism defined and characterized in Suzumura and Xu [20] are the *extreme consequentialism, strong consequentialism, extreme non-consequentialism* and *strong non-consequentialism*.

However, their axiomatizations of consequentialism and non-consequentialism were concerned only with rather extreme cases where unequivocal priority was given to consequences (resp. opportunities) not only in the case of extreme consequentialism (resp.

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[1]There is an extensive literature addressing non-welfaristic features of consequences. See, among others, Baharad and Nitzan [1], Bossert et al., [3], Dworkin [4], Gravel [5; 6] Pattanaik and Suzumura [7; 8], Pattanaik and Xu [9; 10; 11], Sen [12; 13; 14; 15; 16; 17], Suzumura [18; 19], and Suzumura and Xu [20; 21].

extreme non-consequentialism) but also in the case of strong consequentialism (resp. strong non-consequentialism). The purpose of this chapter is to pursue a more general framework so that active interactions between consequential considerations and procedural considerations are allowed to play essential role. In other words, we would like to develop a framework which allows trade-off between the value of consequences and the richness of opportunities.

The structure of the chapter is as follows. In Section 2, we present the basic notations and definitions. Section 3 discusses the generalized consequentialism and non-consequentialism in a simple framework in which the universal set is finite. In Section 4, we consider the generalized consequentialism and non-consequentialism in a context in which the universal set is infinite, but opportunity sets are finite. Section 5 concludes the chapter with some final remarks.

2 Basic Notations and Definitions

Let $\mathscr{Z}$ and $\mathscr{R}$ denote the set of all positive integers and the set of all real numbers, respectively. Let X, where $3 \leq \#X$, be the set of all mutually exclusive and jointly exhaustive alternatives.[2] The elements of X will be denoted by $x, y, z, \ldots$. K denotes a collection of non-empty subsets of X. The elements in K will be denoted by $A, B, C, \ldots$, and they are called *opportunity sets*. Let $X \times K$ be the Cartesian product of X and K. Elements of $X \times K$ will be denoted by $(x, A), (y, B), (z, C), \ldots$, and they are called *extended alternatives*. Let $\Omega = \{(x, A) \mid A \in K \text{ and } x \in A\}$. That is, Ω contains all (x, A) such that A is an element of K and x is an element of A. It should be clear that $\Omega \subset X \times K$, and for all $(x, A) \in \Omega, x \in A$ holds. For all $(x, A) \in \Omega$, the intended interpretation is that the alternative x is chosen from the opportunity set A.[3]

Let $\succeq$ be a reflexive, complete and transitive binary relation over Ω, viz., $\succeq$ is an ordering over Ω. The asymmetric and symmetric parts of $\succeq$ will be denoted by $\succ$ and $\sim$, respectively. For any $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B)$ is interpreted as "choosing x from A is at least as good as choosing y from B." Thus, in the extended framework, it is possible to give an expression to the intrinsic value of opportunity set in addition to the instrumental value thereof. Indeed, the decision-maker recognizes the intrinsic value of the opportunity of choice if there exists an extended alternative $(x, A) \in \Omega$ such that $(x, A) \succ (x, \{x\})$.

[2]Throughout this chapter, and for any finite subset A of the universal set X, $\#A$ denotes the cardinality of A.

[3]Depending on the context, one could also have alternative interpretations of the extended alternatives. For example, in the single agent case, (x, A) could be interpreted as having the alternative x from the set of alternatives that are produced by the procedure A. We owe this observation to the referee of *Journal of Economic Theory*.

3 A Simple Framework

In this section, we confine our attention to the case where $\#X < \infty$, and K is the set of all non-empty subsets of X. We consider the following three axioms on the ordering $\succeq$, which are proposed by Suzumura and Xu [20].

Independence (IND): For all $(x, A), (y, B) \in \Omega$, and all $z \in X \setminus (A \cup B)$, $(x, A) \succeq (y, B) \Leftrightarrow (x, A \cup \{z\}) \succeq (y, B \cup \{z\})$.

Simple indifference (SI): For all $x \in X$, and all $y, z \in X \setminus \{x\}$, $(x, \{x, y\}) \sim (x, \{x, z\})$.

Simple monotonicity (SM): For all $(x, A), (x, B) \in \Omega$, if $B \subset A$, then $(x, A) \succeq (x, B)$.

(IND) has a parallel in the literature on ranking opportunity sets in terms of freedom of choice; see, for example, Pattanaik and Xu [9]. It requires that, for all opportunity sets A and B, if an alternative z is not in both A and B, then the extended preference ranking over $(x, A \cup \{z\})$ and $(y, B \cup \{z\})$ should correspond to the extended preference ranking over (x, A) and (y, B), regardless of the nature of the added alternative z. This axiom has been criticized in the literature. For example, one may argue that freedom of choice offered by an opportunity set is concerned about how diverse the alternatives are in the opportunity set. It may be the case that the added alternative z is very similar to some alternative in A, but quite different from all the alternatives in B. In consequence, the addition of z to A may not change the degree of real freedom of choice already offered by A, while adding z to B may vastly increase the degree of freedom of choice offered by B (see Pattanaik and Xu [10] for some formal analysis). If the individual cares about this aspect of opportunities, the individual may rank $(y, B \cup \{z\})$ strictly above $(x, A \cup \{z\})$ even though the same individual ranks (x, A) and (y, B) equally. (SI) requires that the individual ranks $(x, \{x, y\})$ and $(x, \{x, z\})$ equally as long as x and y are distinct, and x and z are distinct. It is clear that this property is vulnerable to similar criticisms as (IND). Finally, (SM) is a monotonicity property requiring that choosing an alternative x from the set A cannot be worse than choosing the same alternative x from the subset B of A. It essentially reflects the conviction that the individual is not averse to richer opportunities. It may be argued that, in some cases, richer opportunities can be a liability rather than a virtue; see, for example, Dworkin [4]. In such cases, the individual may prefer choosing x from a smaller set to choosing the same x from a larger set. Thus, (IND), (SI) and (SM) are *not* axioms of uncompromising appeal, but they are *not* pathological axioms either. At the very least, they leave us with sufficient room for analyzing generalized consequentialism and non-consequentialism within a simple operational framework.

The following simple implication of (IND), (SI) and (SM) proves useful.

Lemma 3.1. *Let $\succeq$ be an ordering over Ω and satisfy* (IND), (SI) *and* (SM). *Then, for all $(a, A), (b, B) \in \Omega$, and all $x \in X \setminus A, y \in X \setminus B$, $(a, A) \succeq (b, B) \Leftrightarrow (a, A \cup \{x\}) \succeq (b, B \cup \{y\})$.*

Proof. We first denote that, by Theorem 3.1 of Suzumura and Xu (2001), the following is true:

Claim 3.2. For all $(a, A), (a, A') \in \Omega$, if $\#A = \#A'$, then $(a, A) \sim (a, A')$.

Let $(a, A), (b, B) \in \Omega$, $x \in X \setminus A$, $y \in X \setminus B$ and $(a, A) \succeq (b, B)$. Because $\succeq$ is an ordering, we have only to show that $(a, A) \sim (b, B) \Rightarrow (a, A \cup \{x\}) \sim (b, B \cup \{y\})$ and $(a, A) \succ (b, B) \Rightarrow (a, A \cup \{x\}) \succ (b, B \cup \{y\})$.

First, we show that

$(*)$ $(a, A) \sim (b, B) \Rightarrow (a, A \cup \{x\}) \sim (b, B \cup \{y\})$.

Since $x \in X \setminus A$ and $y \in X \setminus B$, it is clear that $A \neq X$ and $B \neq X$. We consider three sub-cases: (i) $A = \{a\}$; (ii) $B = \{b\}$; and (iii) $\#A > 1$ and $\#B > 1$.

(i) $A = \{a\}$: In this case, we distinguish two sub-cases: (i.1) $x \notin B$; and (i.2) $x \in B$. Consider (i.1). Since $x \notin B$, it follows from $(a, \{a\}) \sim (b, B)$ and (IND) that $(a, \{a, x\}) \sim (b, B \cup \{x\})$. By Claim 3.2, $(b, B \cup \{x\}) \sim (b, B \cup \{y\})$. Transitivity of $\sim$ then implies that $(a, \{a, x\}) \sim (b, B \cup \{y\})$. Consider now (i.2) where $x \in B$. To begin with, consider the sub-case where $B \cup \{y\} = \{a, b\}$. Given that $x \in X \setminus A$ and $y \in X \setminus B$, we have $x = b$ and $y = a$, hence $B = \{b\}$. Since $\#X \geq 3$, there exists $c \in X$ such that $c \notin \{a, b\}$. It follows from $(a, \{a\}) \sim (b, \{b\}) = (b, B)$ and (IND) that $(a, \{a, c\}) \sim (b, \{b, c\})$. From Claim 3.2, $(a, \{a, b\}) \sim (a, \{a, c\})$, and $(b, \{b, c\}) \sim (b, \{a, b\})$. Then, transitivity of $\sim$ implies $(a, \{a, b\}) \sim (b, \{a, b\})$; that is, $(a, \{a, x\}) \sim (b, B \cup \{y\})$. Turn now to the sub-case where $B \cup \{y\} \neq \{a, b\}$. If $y \neq a$, starting with $(a, \{a\}) \sim (b, B)$, by (IND), $(a, \{a, y\}) \sim (b, B \cup \{y\})$. By Claim 3.2, $(a, \{a, x\}) \sim (a, \{a, y\})$. Transitivity of $\sim$ implies that $(a, \{a, x\}) \sim (b, B \cup \{y\})$. If $y = a$, given that $\#X \geq 3$ and $B \cup \{y\} \neq \{a, b\}$, there exists $z \in B$ such that $z \notin \{a, b\}$. By Claim 3.2, $(b, B) \sim (b, (B \cup \{y\}) \setminus \{z\})$. From $(a, \{a\}) \sim (b, B)$, transitivity of $\sim$ implies $(a, \{a\}) \sim (b, (B \cup \{y\}) \setminus \{z\})$. Now, noting that $z \neq a$, by (IND), $(a, \{a, z\}) \sim (b, B \cup \{y\})$ holds. From Claim 3.2, $(a, \{a, x\}) \sim (a, \{a, z\})$. Transitivity of $\sim$ now implies $(a, \{a, x\}) \sim (b, B \cup \{y\})$.

(ii) $B = \{b\}$: This case can be dealt with similarly as case (i).

(iii) $\#A > 1$ and $\#B > 1$: Consider $A', A'' \in K$ such that $\{a, b\} \subset A'' \subset A'$, $\#A'' = \min\{\#A, \#B\} > 1$, $\#A' = \max\{\#A, \#B\} > 1$. Since $A \neq X$ and $B \neq X$, the existence of such A' and A'' is guaranteed. It should be clear that there exists $z \in X$ such that $z \in A'$. If $\#A \geq \#B$, consider (a, A') and (b, A''). From Claim 3.2, $(a, A') \sim (b, A'')$ follows from the construction of A' and A'', the assumption that $(a, A) \sim (b, B)$, and transitivity of $\sim$. Note that there exists $z \in X \setminus A'$. By (IND), $(a, A' \cup \{z\}) \sim (b, A'' \cup \{z\})$. By virtue of Claim 3.2, noting that $\#(A \cup \{x\}) = \#(A' \cup \{z\})$ and $\#(B \cup \{y\}) = \#(A'' \cup \{z\})$, $(a, A \cup \{x\}) \sim (b, B \cup \{y\})$ follows easily from transitivity of $\sim$. If $\#A < \#B$, consider (a, A'') and (b, A'). Following a similar argument as above, we can show that $(a, A \cup \{x\}) \sim (b, B \cup \{y\})$. Thus, $(*)$ is proved. The next order of our business is to show that

$(**)$ $(a, A) \succ (b, B) \Rightarrow (a, A \cup \{x\}) \succ (b, B \cup \{y\})$.

As in the proof of $(*)$, we distinguish three cases: (a) $A = \{a\}$; (b) $B = \{b\}$; and (c) $\#A > 1$ and $\#B > 1$.

(a) $A = \{a\}$: (a.1) $x \notin B$. In this sub-case, from $(a, \{a\}) \succ (b, B)$, by (IND), we obtain $(a, \{a, x\}) \succ (b, B \cup \{x\})$. By Claim 3.2, $(b, B \cup \{x\}) \sim (b, B \cup \{y\})$. Transitivity of $\succeq$ implies that $(a, \{a, x\}) = (a, A \cup \{x\}) \succ (b, B \cup \{y\})$. (a.2) $x \in B$. If $B \cup \{y\} = \{a, b\}$, then, given that $x \notin A$ and $y \notin B$, we have $x = b$ and $y = a$. Since $\#X \geq 3$, there exists $c \in X$ such that $c \notin \{a, b\}$. It follows from $(a, \{a\}) \succ (b, \{b\}) = (b, B)$ and (IND) that $(a, \{a, c\}) \succ (b, \{b, c\})$. From Claim 3.2, $(a, \{a, b\}) \sim (a, \{a, c\})$ and $(b, \{b, c\}) \sim (b, \{a, b\})$. Transitivity of $\succeq$ implies $(a, \{a, b\}) \succ (b, \{a, b\})$; i.e., $(a, \{a, x\}) = (a, A \cup \{x\}) \succ (b, B \cup \{y\})$. If $B \cup \{y\} \neq \{a, b\}$, we consider (a.2.i) $y \neq a$ and (a.2.ii) $y = a$. Suppose that (a.2.i) $y \neq a$. From $(a, \{a\}) \succ (b, B)$, by (IND), $(a, \{a, y\}) \succ (b, B \cup \{y\})$. By Claim 3.2, $(a, \{a, x\}) \sim (a, \{a, y\})$. Transitivity of $\succeq$ now implies $(a, \{a, x\}) = (a, A \cup \{x\}) \succ (b, B \cup \{y\})$. Suppose next that (a.2.ii) $y = a$. Since $\#X \geq 3$ and $B \cup \{y\} \neq \{a, b\}$, there exists $c \in B$ such that $c \notin \{a, b\}$. By Claim 3.2, $(b, B) \sim (b, (B \cup \{y\}) \setminus \{c\})$. From $(a, \{a\}) \succ (b, B)$, by transitivity of $\succeq$, $(a, \{a\}) \succ (b, (B \cup \{y\}) \setminus \{c\})$. Now, noting that $c \neq a$, by (IND), $(a, \{a, c\}) \succ (b, B \cup \{y\})$. From Claim 3.2, $(a, \{a, c\}) \sim (a, \{a, x\})$. Transitivity of $\succeq$ implies $(a, \{a, x\}) = (a, A \cup \{x\}) \succ (b, B \cup \{y\})$.

(b) $B = \{b\}$. If $x \notin B$, it follows from $(a, A) \succ (b, B)$ and (IND) that $(a, A \cup \{x\}) \succ (b, \{b, x\})$. By Claim 3.2, $(b, \{b, x\}) \sim (b, \{b, y\}) = (b, B \cup \{y\})$. Transitivity of $\succeq$ now implies $(a, A \cup \{x\}) \succ (b, \{b, y\}) = (b, B \cup \{y\})$. If $x \in B$, then $x = b$. Consider first the case where $y = a$. If $A = \{a\}$, it follows from (a) that $(a, \{a, x\}) = (a, A \cup \{x\}) \succ (b, \{b, y\}) = (b, B \cup \{y\})$. Suppose $A \neq \{a\}$. Given that $x = b$, $y = a$, $x \notin A$, and $y \notin B$, and noting that $\#X \geq 3$, there exists $c \in A \setminus \{a, b\}$. From Claim 3.2, $(a, (A \cup \{x\}) \setminus \{c\}) \sim (a, A)$. From transitivity of $\succeq$ and noting that $(a, A) \succ (b, \{b\})$, $(a, (A \cup \{x\}) \setminus \{c\}) \succ (b, \{b\})$ holds. By (IND), $(a, A \cup \{x\}) \succ (b, \{b, c\})$. From Claim 3.2, $(b, \{b, y\}) \sim (b, \{b, c\})$. Therefore, $(a, A \cup \{x\}) \succ (b, \{b, y\}) = (b, B \cup \{y\})$ follows easily from transitivity of $\succeq$. Consider next that $y \neq a$. If $y \notin A$, then, by (IND) and $(a, A) \succ (b, \{b\})$, we obtain $(a, A \cup \{y\}) \succ (b, \{b, y\})$ immediately. By Claim 3.2, $(a, A \cup \{y\}) \sim (a, A \cup \{x\})$. Transitivity of $\succeq$ implies $(a, A \cup \{x\}) \succ (b, \{b, y\}) = (b, B \cup \{y\})$. If $y \in A$, noting that $y \neq a$, $y \notin B$, and $x = b$, we have $\#((A \cup \{x\}) \setminus \{y\}) = \#A$. By Claim 3.2, $(a, A) \sim (a, (A \cup \{x\}) \setminus \{y\})$. Transitivity of $\succeq$ implies $(a, (A \cup \{x\}) \setminus \{y\}) \succ (b, \{b\})$. By (IND), it then follows that $(a, A \cup \{x\}) \succ (b, \{b, y\}) = (b, B \cup \{y\})$.

(c) $\#A > 1$ and $\#B > 1$. This case is similar to the case (iii) above, and we may safely omit it.

Thus, $(**)$ is proved. $(*)$ together with $(**)$ completes the proof of Lemma 3.1. ■

We are now ready to characterize completely the class of all orderings which satisfy (IND), (SI) and (SM).

Theorem 3.3. $\succeq$ *satisfies* (IND), (SI) *and* (SM) *if and only if there exist a function* $u : X \to \mathscr{R}$ *and a function* $f : \mathscr{R} \times \mathscr{Z} \to \mathscr{R}$ such that

(3.1) *For all* $x, y \in X, u(x) \geq u(y) \Leftrightarrow (x, \{x\}) \succeq (y, \{y\})$;
(3.2) *For all* $(x, A), (y, B) \in \Omega, (x, A) \succeq (y, B) \Leftrightarrow f(u(x), \#A) \geq f(u(y), \#B)$;

(3.3) *f is non-decreasing in each of its arguments and has the following property: For all integers $i, j, k \geq 1$ and all $x, y \in X$, if $i + k, j + k \leq \#X$, then*
(3.3.1) $f(u(x), i) \geq f(u(y), j) \Leftrightarrow f(u(x), i + k) \geq f(u(y), j + k)$.

Proof. We first show that if $\succeq$ satisfies (IND), (SI) and (SM), then there exist a function $f : \mathscr{R} \times \mathscr{Z} \to \mathscr{R}$ and a function $u : X \to \mathscr{R}$ such that (3.1)-(3.3) of Theorem 3.3 hold.[4] Let $\succeq$ satisfy (IND), (SI) and (SM). Since $\succeq$ is an ordering, X is finite and so is Ω as well, there exist $u : X \to \mathscr{R}$ and $F : \Omega \to \mathscr{R}$ such that

(3.4) For all $x, y \in X$, $(x, \{x\}) \succeq (y, \{y\}) \Leftrightarrow u(x) \geq u(y)$;

(3.5) For all $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B) \Leftrightarrow F(x, A) \geq F(y, B)$.

Clearly, (3.1) of Theorem 3.3 is satisfied. To show that (3.2) holds, let $(x, A), (y, B) \in \Omega$ be such that $u(x) = u(y)$ and $\#A = \#B$. From $u(x) = u(y)$, we must have $(x, \{x\}) \sim (y, \{y\})$. Then, if necessary, by the repeated use of Lemma 3.1 and noting that $\#A = \#B$, $(x, A) \sim (y, B)$ can be easily obtained. Now, define $\Sigma \subset \mathscr{R} \times \mathscr{Z}$ as follows: $\Sigma := \{(t, i) \in \mathscr{R} \times \mathscr{Z} \mid \exists (x, A) \in \Omega : t = u(x) \text{ and } i = \#A\}$. Next, define a binary relation $\geq^*$ on Σ as follows: For all $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B) \Leftrightarrow (u(x), \#A) \geq^* (u(y), \#B)$. From the above discussion and noting that $\succeq$ satisfies (SM) and (IND), the binary relation $\geq^*$ defined on Σ is an ordering, and it has the following properties:

(SM$'$): For all $(t, i), (t, j) \in \Sigma$, if $j \geq i$ then $(t, j) \geq^* (t, i)$;

(IND$'$): For all $(s, i), (t, j) \in \Sigma$, and all integer k, if $i + k \leq \#X$ and $j + k \leq \#X$, then $(s, i) \geq^* (t, j) \Leftrightarrow (s, i + k) \geq^* (t, j + k)$.

Since Σ is finite and $\geq^*$ is an ordering on Σ, there exists a function $f : \mathscr{R} \times \mathscr{Z} \to \mathscr{R}$ such that, for all $(s, i), (t, j) \in \Sigma$, $(s, i) \geq^* (t, j)$ if and only if $f(s, i) \geq f(t, j)$. From the definition of $\geq^*$ and Σ, we must have the following: For all $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B) \Leftrightarrow (u(x), \#A) \geq^* (u(y), \#B) \Leftrightarrow f(u(x), \#A) \geq f(u(y), \#B)$. To prove that f is non-decreasing in each of its arguments, we first consider the case in which $u(x) \geq u(y)$ and $\#A = \#B$. Given $u(x) \geq u(y)$, it follows from the definition of u that $(x, \{x\}) \succeq (y, \{y\})$. Noting that $\#A = \#B$, by the repeated use of Lemma 3.1, if necessary, we must have $(x, A) \succeq (y, B)$. Thus, f is non-decreasing in its first argument. To show that f is non-decreasing in its second argument as well, we consider the case in which $u(x) = u(y)$ and $\#A \geq \#B$. From $u(x) = u(y)$, we must have $(x, \{x\}) \sim (y, \{y\})$. Then, from the earlier argument, $(x, A') \sim (y, B)$ for some $A' \subset A$ such that $\#A' = \#B$. Now, by (SM), $(x, A) \succeq (x, A')$. Then, $(x, A) \succeq (y, B)$ follows from the transitivity of $\succeq$. Therefore, f is non-decreasing in each of its arguments. Finally, (3.3.1) follows clearly from (IND$'$).

[4]According to (3.1), the function u can be construed as a utility function defined on the set of social states, whereas the function f weighs the value of consequential states, which is measured in terms of the utility of consequential states, vis-à-vis the value of richness of opportunities, which is measured in terms of the cardinality of opportunity sets.

To check the necessity part of the theorem, suppose $u : X \to \mathscr{R}$ and $f : \mathscr{R} \times \mathscr{Z} \to \mathscr{R}$ are such that (3.1)-(3.3) of Theorem 3.3 are satisfied.

(SI): For all $x \in X$ and all $y, z \in X \setminus \{x\}$, we note that $\#\{x, y\} = \#\{x, z\}$, therefore, $f(u(x), \#\{x, y\}) = f(u(x), \#\{x, z\})$, which in turn implies that $(x, \{x, y\}) \sim (x, \{x, z\})$ holds.

(SM): For all $(x, A), (x, B) \in \Omega$ such that $B \subset A$, $f(u(x), \#A) \geq f(u(x), \#B)$ holds, since f is non-decreasing in each of its arguments and $\#A \geq \#B$. Therefore, $(x, A) \succeq (x, B)$.

(IND): For all $(x, A), (y, B) \in \Omega$, and all $z \in X \setminus (A \cup B)$, it follows from (3.3.1) that

$$f(u(x), \#A) \geq f(u(y), \#B) \Leftrightarrow F(u(x), \#A + 1) \geq f(u(y), \#B + 1)$$

$$\Leftrightarrow f(u(x), \#(A \cup \{z\})) \geq f(u(y), \#(B \cup \{z\})).$$

Therefore, $(x, A) \succeq (y, B) \Leftrightarrow (x, A \cup \{z\}) \succeq (y, B \cup \{z\})$. ■

Remark 3.4. Theorem 3.3 allows us to treat all the cases where the utility of consequential states and the value of richness of opportunities actively interact; it also covers the polar extreme cases of consequentialism and non-consequentialism defined in Suzumura and Xu [20]. Indeed, they can be obtained as special cases of Theorem 3.3 as follows:

Extreme consequentialism: For all $(x, A) \in \Omega, f(u(x), \#A) = u(x)$.

Strong consequentialism: For all $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B) \Leftrightarrow [u(x) > u(y)$ or $(u(x) = u(y)$ and $\#A \geq \#B)]$.

Extreme non-consequentialism: For all $(x, A) \in \Omega$, $f(u(x), \#A) = \#A$.

Strong non-consequentialism: For all $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B) \Leftrightarrow [\#A > \#B$ or $(\#A = \#B$ and $u(x) \geq u(y))]$.

4 Finite Opportunity Sets in the Infinite Universe

Although Theorem 3.3 provides us with a full characterization result for all orderings satisfying (IND), (SI) and (SM), it hinges squarely on the restrictive assumption to the effect that the universal set X is finite. In many settings, for example, in microeconomics, the universal set X is often the consumption set which is typically infinite. In this section, therefore, we discuss the case in which X contains an infinite number of alternatives, but K consists of the set of all finite non-empty subsets of X. Consequently, Ω contains all (x, A) such that A is finite and x is an element of A.

In this arena, the necessary and sufficient condition for an ordering over Ω to satisfy (IND), (SI) and (SM) can be identified as follows.

Theorem 4.1. $\succeq$ *satisfies* (IND), (SI) *and* (SM) *if and only if there exists an ordering* $\succeq^{\#}$ *on* $X \times \mathscr{Z}$ *such that*

(4.1) *For all* $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B) \Leftrightarrow (x, \#A) \succeq^{\#} (y, \#B)$;
(4.2) *For all integers* $i, j, k \geq 1$ *and all* $x, y \in X$, $(x, i) \succeq^{\#} (y, j) \Leftrightarrow (x, i+k) \succeq^{\#} (y, j+k)$, *and* $(x, i+k) \succeq^{\#} (x, i)$.

Proof. First, we note that Claim 3.2 still holds; that is, for all $(a, A), (a, B) \in \Omega$, if $\#A = \#B$, then $(a, A) \sim (a, B)$. Next, we show the following:

Claim 4.2. For all $x, y \in X$ and all $(x, A), (y, A) \in \Omega$, $(x, \{x\}) \succeq (y, \{y\}) \Leftrightarrow (x, A) \succeq (y, A)$.

Let $x, y \in X$ and $(x, A), (y, A) \in \Omega$. If $x = y$, then Claim 4.2 follows immediately from reflexivity of $\succeq$. Let $x \neq y$. Suppose that $(x, \{x\}) \sim (y, \{y\})$. Let $z \in X \setminus \{x, y\}$. By (IND), we have $(x, \{x, z\}) \sim (y, \{y, z\})$. From Claim 3.2, we must have $(x, \{x, z\}) \sim (x, \{x, y\})$ and $(y, \{y, z\}) \sim (y, \{x, y\})$. Then, by transitivity of $\sim$, we obatin $(x, \{x, y\}) \sim (y, \{x, y\})$. Noting that A is finite, by the repeated use of (IND), we have $(x, A) \sim (y, A)$. Similarly, we can show that if $(x, \{x\}) \succ (y, \{y\})$ then $(x, A) \succ (y, A)$. Since $\succeq$ is an ordering, Claim 4.2 is proved.

We now show that, for all $(x, A), (y, B) \in \Omega$, if $(x, \{x\}) \sim (y, \{y\})$ and $\#A = \#B$, then $(x, A) \sim (y, B)$. Let $C \in K$ be such that $\#C = \#A = \#B$ and $\{x, y\} \subset C$. From Claim 4.2, we have $(x, C) \sim (y, C)$. Note that $(x, C) \sim (x, A)$ and $(y, C) \sim (y, B)$ follow from Claim 3.2. By transitivity of $\sim$, we have $(x, A) \sim (y, B)$. Define a binary relation $\succeq^{\#}$ on $X \times \mathscr{Z}$ as follows: For all $x, y \in X$ and all positive integers i, j, $(x, i) \succeq^{\#} (y, j) \Leftrightarrow [(x, A) \succeq (y, B)$ for some $A, B \in K$ such that $x \in A$, $y \in B$, $i = \#A$, $j = \#B]$. From the above discussion, $\succeq^{\#}$ is well-defined and is an ordering. A similar method of proving (3.3) can be invoked to prove that (4.2) holds. ■

Remark 4.3. Based on Theorem 4.1, the concepts of extreme consequentialism, strong consequentialism, extreme non-consequentialism and strong non-consequentialism defined in Suzumura and Xu [20] can be expressed similarly as in the last section. Details may be left to the interested readers to spell out.

It should be clear that without imposing any other condition on the extended preference ordering $\succeq$, the value of consequential states may not be representable by any numerical function. In order for the value of consequential states to be numerically representable, further restrictions on $\succeq$ must be imposed. Assume therefore that $X = \mathscr{R}^n_+$ in the rest of this section,[5] and define the following property:

Continuity (CON): For all $x^i \in X$ $(i = 1, 2, \dots)$ and all $x, y \in X$, if $\lim_{i \to \infty} x^i = x$, then $[(x^i, \{x^i\}) \succeq (y, \{y\})$ for $i = 1, 2, \ldots] \Rightarrow (x, \{x\}) \succeq (y, \{y\})$, and $[(y, \{y\}) \succeq (x^i, \{x^i\})$ for $i = 1, 2, \ldots] \Rightarrow (y, \{y\}) \succeq (x, \{x\})$.

Then we may assert the following proposition:

[5] $\mathscr{R}^n_+$ stands for the non-negative orthant of the Euclidean n-space, where n is a specified natural number.

Theorem 4.4. *Suppose that $X = \mathscr{R}^n_+$. Suppose also that $\succeq$ satisfies* (IND), (SI), (SM) *and* (CON). *Then there exist a continuous function $u : X \to \mathscr{R}$ and an ordering $\succeq^*$ on $\mathscr{R} \times \mathscr{Z}$ such that*

(a) *For all $(x, A), (y, B) \in \Omega, (x, A) \succeq (y, B) \Leftrightarrow (u(x), \#A) \succeq^* (u(y), \#B)$;*
(b) *For all $x, y \in X, (x, \{x\}) \succeq (y, \{y\}) \Leftrightarrow u(x) \geq u(y)$;*
(c) *For all integers $i, j, k \geq 1$, and all $x, y \in X, (u(x), i) \succeq^* (u(y), j) \Leftrightarrow (u(x), i + k) \succeq^* (u(y), j + k)$;* and *$(u(x), i + k) \succeq^* (u(x), i)$.*

Proof. From Theorem 4.1, we know that there exists an ordering $\succeq^{\#}$ on $X \times \mathscr{Z}$ such that

(F.1) For all $(x, A), (y, B) \in \Omega, (x, A) \succeq (y, B) \Leftrightarrow (x, \#A) \succeq^{\#} (y, \#B)$;
(F.2) For all integers $i, j, k \geq 1$ and all $x, y \in X$, $(x, i) \succeq^{\#} (y, j) \Leftrightarrow (x, i+k) \succeq^{\#} (y, j+k)$; and $(x, i + k) \succeq^{\#} (x, i)$.

Now, for all $x, y \in X$, define the binary relation R on X as follows: For all $x, y \in X, xRy \Leftrightarrow (x, \{x\}) \succeq (y, \{y\})$. Since $\succeq$ is an ordering and satisfies (CON), R on $X = \mathscr{R}^n_+$ is an ordering and satisfies the following continuity property: For all $x \in X, \{y \in X | yRx\}$, and $\{y \in X | xRy\}$ are closed. Therefore, there exists a continuous function $u : X \to \mathscr{R}$ such that for all $x, y \in X, xRy \Leftrightarrow u(x) \geq u(y)$. From the definition of R, it is clear that for all $x, y \in X, (x, \{x\}) \succeq (y, \{y\}) \Leftrightarrow u(x) \geq u(y)$. Therefore, (b) of Theorem 4.4 holds. Given that (b) holds, it is straightforward to check that (a) and (c) hold as well. ■

If we go one step further and strengthen the condition of Continuity in Theorem 4.4 to the following condition, to be called *Strong Continuity*, the binary relation $\succeq$ will be numerically representable altogether.

Strong continuity (SCON): For all $(x, A) \in \Omega$, all $y, y^i \in X$ $(i = 1, 2, \ldots)$ and all $B \in K \cup \{\emptyset\}$, if $B \cap \{y^i\} = B \cap \{y\} = \emptyset$ for all $i = 1, 2, \ldots$, and $\lim_{i\to\infty} y^i = y$, then $[(y^i, B \cup \{y^i\}) \succeq (x, A)$ for $i = 1, 2, \ldots] \Rightarrow (y, B \cup \{y\}) \succeq (x, A)$, and $[(x, A) \succeq (y^i, B \cup \{y^i\})$ for $i = 1, 2, \ldots] \Rightarrow (x, A) \succeq (y, B \cup \{y\})$.[6]

Theorem 4.5. *Suppose that $X = \mathscr{R}^n_+$ and that $\succeq$ satisfies* (IND), (SI), (SM) *and* (SCON). *Then, there exists a function $v : X \times \mathscr{Z} \to \mathscr{R}$, which is continuous in its first argument, such that*

(a) *For all $(x, A), (y, B) \in \Omega, (x, A) \succeq (y, B) \Leftrightarrow v(x, \#A) \geq v(y, \#B)$,*
(b) *For all $i, j, k \in \mathscr{Z}$ and all $x, y \in X$, $v(x, i) \geq v(y, j) \Leftrightarrow v(x, i + k) \geq v(y, j + k)$ and $v(x, i + k) \geq v(x, i)$.*

Proof. From Theorem 4.1, we know that there exists an ordering $\succeq^{\#}$ on $X \times \mathscr{Z}$ such that

[6]It is clear that (CON) is a special case of (SCON) where $B = \emptyset$.

(G.1) For all $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (y, B) \Leftrightarrow (x, \#A) \succeq^{\#} (y, \#B)$;

(G.2) For all integers $i, j, k \geq 1$ and all $x, y \in X$, $(x, i) \succeq^{\#} (y, j) \Leftrightarrow (x, i+k) \succeq^{\#} (y, j+k)$; and $(x, i+k) \succeq^{\#} (x, i)$.

For all $(x, i) \in X \times \mathscr{Z}$ and all $j \in \mathscr{Z}$, consider the sets:

$$U(x; i, j) = \{y \in X \mid (y, j) \succeq^{\#} (x, i)\}, L(x; i, j) = \{y \in X \mid (x, i) \succeq^{\#} (y, j)\}.$$

From (G.1), by (SCON), it is clear that both $U(x; i, j)$ and $L(x; i, j)$ are closed. Note that $X = \mathscr{R}^n_+$. By Theorem 3 of Blackorby et al. [2], there is a function $v : X \times \mathscr{Z} \to \mathscr{R}$, which is continuous in its first argument, that represents $\succeq^{\#}$. Therefore, part (a) of Theorem 4.5 follows. Part (b) of Theorem 4.5 follows from (a), (G.1) and (G.2). ■

5 Concluding Remarks

This chapter has generalized our previous analysis reported in Suzumura and Xu [20] on extended preferences over consequences of choice and opportunities of choice which lie behind these consequences. Our previous analysis was the first attempt in the literature to axiomatize the concepts of consequentialism and non-consequentialism, and it was concerned only with the special situations where no trade-off relationship exists between consequential considerations, which reflect the decision-maker's intrinsic preferences over consequential outcomes, and non-consequential considerations, which reflect his concern over richness of opportunities from which alternatives are chosen. In this restricted analytical framework, axiomatic characterizations were given to the concepts of extreme and strong consequentialism, as well as of extreme and strong non-consequentialism. Going beyond this concern with polar extreme cases of consequentialism and non-consequentialism, we have characterized in this chapter the extended preferences over consequences and opportunities of choice which embody active interactions between consequential considerations and non-consequential considerations.

The universe of our discourse can be a finite set or an infinite set, but the family of opportunity sets is restricted to consist solely of finite sets. This analytical framework can be useful in generalizing some classical analyses of individual and social choices, where there is a pervasive, yet implicit, assumption to the effect that all individuals as well as society are concerned only with consequences of their choices. As a matter of fact, the traditional analysis is even more restrictive than consequentialism as such, since all individuals as well as society are implicitly assumed to evaluate consequences only through their effects on individual or social welfare. The effects of the existence of non-consequentialists in the society on the Arrovian impossibility theorem are examined in Suzumura and Xu [21] on the basis of Suzumura and Xu [20], and it is an interesting exercise to see what difference, if any, will the existence of active interactions between consequential considerations and non-consequential considerations exert on the results obtained in Suzumura and Xu [21]. It is also interesting to examine how the traditional theory of consumer's behavior and general equilibrium analysis will have to be modified in the presence of non-consequential considerations. To conduct this latter analysis, however, we must go beyond our current analytical framework which restricts the family of opportunity sets to consist of finite sets. This task must be left for our future research.

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Chapter 24
Welfarist-Consequentialism, Similarity of Attitudes, and Arrow's General Impossibility Theorem*

1 Introduction

The purpose of this chapter is to re-examine Arrow's *general impossibility theorem* with special reference to the following two basic features of his celebrated *Social Choice and Individual Values*. The first feature is the *welfarist-consequentialism*, which claims that the social judgements on right or wrong actions should be based on the assessment of their consequential states of affairs, where the assessment of consequences is conducted exclusively in terms of people's welfare, their preference satisfaction, or people getting what they want.[1, 2] Not only is Arrow's own analysis based squarely on the welfarist-consequentialism in this sense, but also this basic feature permeates through the entire

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[1]See Sen [16; 17; 19], and Sen and Williams [22] for more detailed account of welfarist-consequentialism and criticisms thereof. See also Arrow [2], Hammond [8], and Suzumura [25; 26; 27].

[2]Welfarist-consequentialism has been under intensive scrutiny and harsh criticism in recent years by, e.g., Bossert, Pattanaik and Xu [4], Pattanaik and Xu [11; 12], Sen [18; 19; 20], and Suzumura [25; 26; 27]. It is often argued that individuals are ready to express preferences not only over outcomes, but also over opportunity sets from which outcomes are chosen. It is not infrequently suggested that such an extended preference should be duly taken into consideration in the analysis of how socially right or wrong actions should be determined. However, most of the preceding attempts to respond to these arguments and suggestions are concerned with the *ranking of opportunity sets* in terms of the freedom of choice and/or overall well-being of individuals. To the best of our knowledge, the question as to how individuals' extended preference orderings over outcomes and opportunities should be aggregated into the social preference ordering has been left unexplored in this literature.

edifice of contemporary social choice theory.[3] The second feature is the perception that "the possibility of social welfare judgements rests upon a similarity of attitudes toward social alternatives (Arrow [1, p.69])." To substantiate this claim analytically, Arrow [1, p.81] showed that "it is possible to construct suitable social welfare functions if we feel entitled to say in advance that the tastes of individuals fall within certain prescribed realms of similarity." It goes without saying that a large portion of the subsequent developments in social choice theory is devoted to the exploration of Arrow's important insight to this effect.[4]

To gauge the extent to which Arrow's impossibility theorem and the resolution thereof hinge on these two basic features of his framework, this chapter develops two extended frameworks in which individuals are supposed to express their preferences not only about consequential outcomes, but also about opportunity sets from which outcomes are chosen. Two such frameworks are identified below: a *consequentialist* framework and a *non-consequentialist* framework. It is shown that the counterpart of Arrow's impossibility theorem still holds in the consequentialist framework if the society is composed exclusively of individuals who show similar attitudes toward social alternatives, whereas a resolution of Arrow's impossibility theorem can be found if there is a diversity of attitudes among individuals. Thus, in the consequentialist conceptual framework, it is in fact a *dissimilarity* rather than a *similarity* among individuals that serves as a *deux ex machina* vis-à-vis Arrow's general impossibility theorem. In contrast with this verdict on the consequentialist framework, an interesting resolution of Arrow's general impossibility theorem exists in the non-consequentialist framework, which may work even in the *homogeneous* society where all individuals exhibit a similarity of attitudes toward outcomes and opportunities.

The structure of this chapter is as follows. In Section 2, we lay the foundation of our analysis by introducing some basic notation and definitions. Section 3 is devoted to examining how Arrow's general impossibility theorem fares in the consequentialist framework, whereas Section 4 conducts the corresponding analysis in the non-consequentialist framework. Section 5 describes the way how these results can be generalized, and Section 6 concludes this chapter with several observations.

2 Basic Notation and Definitions

Let X be the set of all conventionally defined social states, which are mutually exclusive and jointly exhaustive. It is assumed that X satisfies $3 \leq \#X < \infty$. The elements of X are denoted by $x, y, z, \cdots$, and they are called *outcomes*. K denotes the set of all non-empty subsets of X. The elements of K are denoted by $A, B, C, \cdots$, and they are called *opportunity sets*. Let $X \times K$ be the Cartesian product of X and K. The

[3] It is true that Sen's [14, Chapter 6*; 15; 16; 17; 19] well-known criticism against the welfaristic foundations of normative economics and social choice theory, which capitalizes on his *impossibility of a Paretian liberal*, sharply brought the importance of non-welfaristic features of consequences to the fore. However, it still keeps us within the broad territory of consequentialism. See, also, Suzumura [23; 25].

[4] See, among others, Black [3], Kuga and Nagatani [10], and Sen [14, Chapter 10*].

elements of $X \times K$ are denoted by $(x, A), (y, B), (z, C), \cdots$, and they are called *extended alternatives*. The intended meaning of $(x, A) \in X \times K$ is that the outcome x is chosen from the opportunity set A, but this interpretation will be vacuous if $x \notin A$. Thus, let $\Omega \subseteq X \times K$ be such that $x \in A$ whenever $(x, A) \in \Omega$.

Let $N = \{1, 2, \cdots, n\}$ be the set of all individuals in the society, where $2 \leq n = \#N < \infty$. Each individual $i \in N$ is assumed to have an extended preference ordering R_i over Ω, which is *reflexive, complete* and *transitive*. For any $(x, A), (y, B) \in \Omega, (x, A)R_i(y, B)$ is meant to imply that i feels at least as good when choosing x from A as when choosing y from B. The asymmetric part and the symmetric part of R_i are denoted by $P(R_i)$ and $I(R_i)$, respectively, which denote the strict preference relation and the indifference relation of $i \in N$.

Let $\wp$ be the set of all logically possible orderings over Ω. Then a *profile* $\mathbf{R} = (R_1, R_2, \cdots, R_n)$ of extended individual preference orderings, one extended ordering for each individual, is an element of $\wp^n$. An *extended social welfare function* (ESWF) is a function f which maps each and every profile in some subset D_f of $\wp^n$ into $\wp$. When $R = f(\mathbf{R})$ holds for some $\mathbf{R} \in D_f$, $I(R)$ and $P(R)$ stand, respectively, for the social indifference relation and the social strict preference relation corresponding to R. Given an ESWF f, the problem of social choice we envisage in this chapter can be phrased as follows. Suppose that a profile $\mathbf{R} \in D_f$ and a set $S \subseteq X$ of feasible social alternatives are given. Then the best social choice from S can be identified to be an $x^* \in S$ such that $(x^*, S)R(x, S)$ holds for all $x \in S$, where $R = f(\mathbf{R})$. To make this interpretation natural as well as sensible, we assume that each and every $x \in X$ denotes a *public* alternative such as a list of public goods to be provided in the society, or the description of a candidate in the public election.

2.1 Domain Restriction

In order to make our problem both analytically tractable and interesting, we assume that each individual's extended preference ordering R_i $(i \in N)$, which defines an admissible profile $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in D_f$, satisfies the following two conditions:

Independence (IND): For all $(x, A), (y, B) \in \Omega$ and all $z \in X \setminus (A \cup B)$,

$$(x, A)R_i(y, B) \Leftrightarrow (x, A \cup \{z\})R_i(y, B \cup \{z\}).$$

Simple Indifference (SI): For all $x \in X$ and all $y, z \in X \setminus \{x\}$,

$$(x, \{x, y\})I(R_i)(x, \{x, z\}).$$

(**IND**) corresponds to the standard independence axiom used in the literature (see, for example, Pattanaik and Xu [11]). It requires that, for all opportunity sets $A, B \in K$, if an alternative $z \in X$ is not in both A and B, then the preference ranking over $(x, A \cup \{z\})$ and $(y, B \cup \{z\})$ corresponds to the preference ranking over (x, A) and (y, B). (**SI**) requires that choosing x from the two simple cases consisting of two alternatives each is regarded as indifferent no matter what alternative is added to x.

We may as well assume the following condition:

Monotonicity (MON): For all $(x, A), (x, B) \in \Omega$,

$$B \subseteq A \Rightarrow (x, A)R_i(x, B).$$

(**MON**) makes an explicit use of information about the opportunity aspect of choice situations. It requires that choosing an outcome x from the opportunity set A is at least as good as choosing the same x from the opportunity set B which is a subset of A. In the present context, this axiom seems very reasonable.

The following result summarizes the implication of the above three conditions.

Lemma 1. If R_i satisfies (**IND**), (**SI**) and (**MON**), then for all $(x, A), (x, B) \in \Omega$, $\#A \geq \#B \Rightarrow (x, A)R_i(x, B)$.

Proof. Let R_i satisfy (**IND**), (**SI**), and (**MON**). Let $(x, A), (x, B) \in \Omega$ be such that $\#A \geq \#B$.

If $\#A = \#B = 1$, then $A = B = \{x\}$. By reflexivity of R_i, it follows immediately that $(x, A)I(R_i)(x, B)$. If $\#A = \#B = 2$, then by (**SI**), $(x, A)I(R_i)(x, B)$ follows from (**SI**) directly. Thus, we have proved the following:

$$\text{For all } (x, A), (x, B) \in \Omega, \text{ if } \#A = \#B \leq 2, \text{ then } (x, A)I(R_i)(x, B). \tag{2.1}$$

To prove that $\#A = \#B = m > 2 \Rightarrow (x, A)I(R_i)(x, B)$, we use the induction method. Suppose

$$\text{For all } (x, S), (x, T) \in \Omega \text{ such that } \#S = \#T < m, (x, S)I(R_i)(x, T). \tag{2.2}$$

If there exists $y \in A \cap B$ such that $y \neq x$, then, from (2.2), $(x, A \setminus \{y\})I(R_i)(x, B \setminus \{y\})$. By (**IND**), $(x, A)I(R_i)(x, B)$ follows immediately. If $A \cap B = \{x\}$, then consider $C = (A \setminus \{a\}) \cup \{b\}$ where $a \in A \setminus \{x\}$ and $b \in B \setminus \{x\}$. From the previous argument, clearly, $(x, A)I(R_i)(x, C)$ and $(x, B)I(R_i)(x, C)$. Thus, by the transitivity of R_i, $(x, A)I(R_i)(x, B)$.

From (2.1) and (2.2), noting the finiteness of X, we have

$$\text{For all } (x, A), (x, B) \in \Omega, \text{ if } \#A = \#B, \text{ then} (x, A)I(R_i)(x, B). \tag{2.3}$$

Consider now that $\#A > \#B$. If $\#B = 1$, that is, $B = \{x\}$, by (**MON**), $(x, A)R_i(x, B)$ follows immediately. Similarly, if $A = X$, then, by (**MON**), $(x, A) = (x, X)R_i(x, B)$. Let $\#X > \#A > \#B > 1$. Clearly, in this case, there exists $C \in K$ such that $\#(B \cup C) = \#A$. From (2.3), $(x, C \cup B)I(R_i)(x, A)$. By (**MON**), $(x, C \cup B)R_i(x, B)$. Hence, $(x, A)R_i(x, B)$ follows from the transitivity of R_i immediately. ■

Thus, these simple conditions impose a mild restriction on each individual's extended preference ordering to the effect that *each individual is not averse to richer opportunities,* *viz., a larger opportunity set does not do any harm to him.*[5, 6]

Throughout Sections 3 and 4, we assume that each and every profile $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in D_f$ is such that R_i satisfies the conditions (**IND**), (**SI**) and (**MON**) for all $i \in N$.

2.2 Arrovian Conditions in the Extended Framework

In addition to the domain restriction on D_f introduced above, we introduce several conditions on f, which are slight modifications of Arrow's own conditions in Arrow [1]. The first two conditions are well known, and require no further explanation.

Strong Pareto Principle (SP): For all $(x, A), (y, B) \in \Omega$, and for all $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in D_f$, if $(x, A)P(R_i)(y, B)$ holds for all $i \in N$, then we have $(x, A)P(R)(y, B)$, and if $(x, A)I(R_i)(y, B)$ holds for all $i \in N$, then we have $(x, A)I(R)(y, B)$, where $R = f(\mathbf{R})$.

Non-Dictatorship (ND): There exists no $i \in N$ that satisfies $[(x, A)P(R_i)(y, B) \Rightarrow (x, A)P(R)(y, B)$ for all $(x, A), (y, B) \in \Omega]$ holds for all $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in D_f$, where $R = f(\mathbf{R})$.

There are various ways of formulating Arrow's Independence of Irrelevant Alternatives in our present context. Consider the following:

Independence of Irrelevant Alternatives (i) (IIA(i)): For all $\mathbf{R^1} = (R_1^1, R_2^1, \cdots, R_n^1), \mathbf{R^2} = (R_1^2, R_2^2, \cdots, R_n^2) \in D_f$, and for all $(x, A), (y, B) \in \Omega$, if $[(x, A)R_i^1(y, B) \Leftrightarrow (x, A)R_i^2(y, B)$ and $(x, \{x\})R_i^1(y, \{y\}) \Leftrightarrow (x, \{x\})R_i^2(y, \{y\})]$ holds for all $i \in N$, then $[(x, A)R^1(y, B) \Leftrightarrow (x, A)R^2(y, B)]$ holds, where $R^1 = f(\mathbf{R^1})$ and $R^2 = f(\mathbf{R^2})$.

Independence of Irrelevant Alternatives (ii) (IIA(ii)): For all $\mathbf{R^1} = (R_1^1, R_2^1, \cdots, R_n^1), \mathbf{R^2} = (R_1^2, R_2^2, \cdots, R_n^2) \in D_f$, and for all $(x, A), (y, B) \in \Omega$ with $\#A = \#B$, if $[(x, A)R_i^1(y, B) \Leftrightarrow (x, A)R_i^2(y, B)]$ holds for all $i \in N$, then $[(x, A)R^1(y, B) \Leftrightarrow (x, A)R^2(y, B)]$ holds, where $R^1 = f(\mathbf{R^1})$ and $R^2 = f(\mathbf{R^2})$.

Full Independence of Irrelevant Alternatives (FIIA): For all $\mathbf{R^1} = (R_1^1, R_2^1, \cdots, R_n^1), \mathbf{R^2} = (R_1^2, R_2^2, \cdots, R_n^2) \in D_f$, and for all $(x, A), (y, B) \in \Omega$, if $[(x, A)R_i^1(y, B) \Leftrightarrow (x, A)R_i^2(y, B)]$ holds for all $i \in N$, then $[(x, A)R^1(y, B) \Leftrightarrow (x, A)R^2(y, B)]$ holds, where $R^1 = f(\mathbf{R^1})$ and $R^2 = f(\mathbf{R^2})$.

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[5] It may be argued that the measurement of opportunity in terms of the cardinality of the opportunity set is naive, and one should take such information as similarities among outcomes into consideration. This can be done as in Pattanaik and Xu [13] using the minimum of the cardinalities of informationally equivalent classes rather than the cardinality of the opportunity set *per se*. It is for the purpose of keeping our framework as simple as possible that we are using in this chapter the cardinality approach in measuring opportunity. See, however, Section 5 below.

[6] Recollect that we are neglecting decision-making cost and other factors that make a larger opportunity set a liability rather than a credit. For some arguments which may cast reasonable doubts on the universal worth of having a larger opportunity set rather than a smaller one, the interested readers are referred to Dworkin [5].

(**IIA(i)**) says that the extended social preference between any two extended alternatives (x, A) and (y, B) depends on each individual's extended preference between them as well as each individual's extended preference between $(x, \{x\})$ and $(y, \{y\})$. (**IIA(ii)**), on the other hand, says that the extended social preference between any two extended alternatives (x, A) and (y, B) with $\#A = \#B$ depends on each individual's extended preference between them. Finally, (**FIIA**) says that the extended social preference between *any* two extended alternatives (x, A) and (y, B) depends on each individual's extended preference between them. It is clear that (**IIA(i)**) is logically independent of (**IIA(ii)**), and both (**IIA(i)**) and (**IIA(ii)**) are logically weaker than (**FIIA**).

3 Arrovian Impossibility Theorems in the Consequentialist Framework

In this section, we discuss Arrovian impossibility theorems in the framework which is broader than welfarist-consequentialism, yet lies within consequentialism. Following Suzumura and Xu [28], let us identify two types of an individual whom we have a reason to call a *consequentialist*:

Extreme Consequentialist: An individual $i \in N$ is said to be an *extreme consequentialist* if, for all $(x, A), (x, B) \in \Omega$, it is true that $(x, A)I(R_i)(x, B)$.

Strong Consequentialist: An individual i is said to be a *strong consequentialist* if, for all $(x, A), (y, B) \in \Omega$,
(a) if $(x, \{x\})I(R_i)(y, \{y\})$, then $\#A \geq \#B \Leftrightarrow (x, A)R_i(y, B)$; and
(b) if $(x, \{x\})P(R_i)(y, \{y\})$, then $(x, A)P(R_i)(y, B)$.

Thus, an extreme consequentialist ranks two extended alternatives (x, A) and (y, B) simply in terms of their outcomes x and y, giving no relevance to the opportunity sets A and B from which x and y are chosen. In contrast, a strong consequentialist ranks two alternatives (x, A) and (y, B) in complete accordance with their outcomes x and y only if he has a strict preference between choosing x from the singleton set $\{x\}$, and choosing y from the singleton set $\{y\}$. If he is indifferent between choosing x from the singleton set $\{x\}$, and choosing y from the singleton set $\{y\}$, his preference ranking between (x, A) and (y, B) is in accordance with the cardinality comparison between A and B. It is to this limited extent that a strong consequentialist reveals his preference for opportunity, thereby exhibiting his impure consequentialist side of attitude.

As the following lemma shows, for an extreme as well as a strong consequentialist, imposing the conditions (**IND**), (**SI**) and (**MON**) does not in fact restrict his/her preferences at all.

Lemma 2. An extreme as well as a strong consequentialist's extended preference orderings must always satisfy the three conditions (**IND**), (**SI**) and (**MON**).

Proof. It is easy to check that both an extreme and a strong consequentialist's extended preference orderings satisfy (**SI**) and (**MON**). We now prove that (**IND**) is satisfied by an extreme and a strong consequentialist's extended preference orderings.

An Extreme Consequentialist: Let i be an extreme consequentialist and R_i be his extended preference ordering. Let $(x, A), (y, B) \in \Omega$ and $z \in X \setminus (A \cup B)$ be such that $(x, A)R_i(y, B)$ holds. By definition of an extreme consequentialist, we must have $(x, A)I(R_i)(x, A \cup \{z\})$ and $(y, B)I(R_i)(y, B \cup \{z\})$. Then transitivity of R_i implies $(x, A \cup \{z\})R_i(y, B \cup \{z\})$. The converse implication may be similarly verified. Therefore, (**IND**) holds for an extreme consequentialist's extended preference orderings.

A Strong Consequentialist: Let j be a strong consequentialist and R_j be his extended preference ordering. Let $(x, A), (y, B) \in \Omega$ and $z \in X \setminus (A \cup B)$ be such that $(x, A)R_j(y, B)$ holds. We distinguish three cases that exhaust all possibilities, viz. (a) $(x, A)I(R_j)(y, B)$; (b) $x = y$ and $(x, A)P(R_j)(y, B)$; (c) $x \neq y$ and $(x, A)P(R_j)(y, B)$. In case (a), according to the definition of a strong consequentilist, it must be that $(x, \{x\})I(R_j)(y, \{y\})$ and $\#A = \#B$. Then, by the definition of strong consequentialism, it follows that $(x, A \cup \{z\})I(R_j)(y, B \cup \{z\})$. In case (b), since $x = y$, by the definition of a strong consequentialist, we must have $\#A > \#B$. Clearly, $\#(A \cup \{z\}) > \#(B \cup \{z\})$. Hence, $(x, A \cup \{z\})P(R_j)(y, B \cup \{z\})$ follows from the definition of strong consequentialism. Finally, in case (c), we must have that $(x, \{x\})P(R_j)(y, \{y\})$. Then, $(x, A \cup \{z\})P(R_j)(y, B \cup \{z\})$ follows immediately from the definition of a strong consequentialist. The converse may be similarly verified. Hence, a strong consequentialist's extended preference orderings satisfy (**IND**). ■

Let us now introduce three domain restrictions on f by specifying some appropriate subsets of D_f. In the first place, let $D_f(E)$ be the set of all profiles in D_f such that all individuals are extreme consequentialists. In the second place, let $D_f(E \cup S)$ be the set of all profiles in D_f such that at least one individual is an extreme consequentialist *uniformly* for all profiles $\mathbf{R} = (R_1, \cdots, R_n) \in D_f$ and at least one individual is a strong consequentialist *uniformly* for all profiles $\mathbf{R} = (R_1, \cdots, R_n) \in D_f$. Finally, let $D_f(S)$ be the set of all profiles in D_f such that all individuals are strong consequentialists.

We are now ready to present our results, beginning with the consequentialist framework. The first result is a simple restatement of Arrow's general impossibility theorem save for the restriction on the domain of the extended social welfare function and a slight strengthening of the Pareto principle.

Theorem 1. Suppose that all individuals are extreme consequentialists. Then, there exists no extended social welfare function f with the domain $D_f(E)$ which satisfies (**SP**), (**ND**) and either (**IIA(i)**) or (**IIA(ii)**).

Proof. Suppose that there exists an ESWF f on $D_f(E)$ which satisfies (**SP**) as well as (**IIA(i)**). Since all individuals are extreme consequentialists,

$$\forall i \in N : (x, A)R_i(y, B) \Leftrightarrow (x, X)R_i(y, X) \tag{3.1}$$

holds for all $(x, A), (y, B) \in \Omega$ and for all $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in D_f(E)$. Note that the conditions (**IND**), (**SI**) and (**SM**) impose no restriction whatsoever on the profile $\mathbf{R} = (R_1, R_2, \cdots, R_n)$ even when, for each and every $i \in N, R_i$ is restricted on $\Omega_X := \{(x, X) \in X \times K | x \in X\}$. Note also that (**SP**) and (**IIA(i)**) imposed on f imply that the same conditions must be satisfied on the restricted space Ω_X. By virtue of the Arrow impossibility theorem, therefore, there exists a dictator, say $d \in N$, for f on the restricted space Ω_X. That is, for all $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in D_f(E)$ and all $(x, X), (y, X) \in \Omega_X$, $(x, X)P(R_d)(y, X) \Rightarrow (x, X)P(R)(y, X)$, where $R = f(\mathbf{R})$. We now show that for all $(x, A), (y, B) \in \Omega$, $(x, A)P(R_d)(y, B) \Rightarrow (x, A)P(R)(y, B)$, viz., d is a dictator for f on the full space Ω. Note that, since d is an extreme consequentialist, we must have that $(x, A)P(R_d)(y, B)$ iff $(x, X)P(R_d)(y, X)$. Since all individuals are extreme consequentialists, it must be true that $(x, A)I(R_i)(x, X)$ and $(y, B)I(R_i)(y, X)$ for all $i \in N$. Therefore, by (**SP**), $(x, A)I(R)(x, X)$ and $(y, B)I(R)(y, X)$. By virtue of the transitivity of R, it follows that $(x, X)P(R)(y, X) \Rightarrow (x, A)P(R)(y, B)$. That is, we have shown that $(x, A)P(R_d)(y, B) \Rightarrow (x, A)P(R)(y, B)$. In other words, d is a dictator for f on the full space Ω. Therefore, there exists no ESWF that satisfies (**SP**), (**IIA(i)**) and (**ND**).

A similar argument as above can be used to show that there exists no ESWF that satisfies (**SP**), (**IIA(ii)**) and (**ND**). ■

Thus, the similarity of attitudes among individuals in the sense that all individuals are extreme consequentialists brings back an essentially Arrovian impossibility result. The message of this theorem can be strengthened by proving the next theorem which asserts that the impossibility result disappears if an extreme consequentialist and a strong consequentialist coexist in the society.

Theorem 2. Suppose that there exist at least one uniform extreme-consequentialist over $D_f(E \cup S)$ and at least one uniform strong-consequentialist over $D_f(E \cup S)$ in the society.[7] Then, there exists an extended social welfare function f with the domain $D_f(E \cup S)$ which satisfies (**SP**), (**IIA(i)**),(**IIA(ii)**) and **ND**.

Proof. Let $e \in N$ be a uniform extreme-consequentialist and $s \in N$ be a uniform strong-consequentialist. By definition,

$$\forall (x, A), (x, B) \in \Omega : (x, A)I(R_e)(x, B), \tag{3.2}$$

$$\forall (x, A), (y, B) \in \Omega : (x, \{x\})I(R_s)(y, \{y\}) \Rightarrow [(x, A)R_s(y, B) \Leftrightarrow \#A \geq \#B] \tag{3.3}$$

and

$$\forall (x, A), (y, B) \in \Omega : (x, \{x\})P(R_s)(y, \{y\}) \Rightarrow (x, A)P(R_s)(y, B) \tag{3.4}$$

hold. Now consider the following ESWF:

[7] A *uniform* extreme-consequentialist over $D_f(E \cup S)$ is a person who is an extreme-consequentialist uniformly for all profiles in $D_f(E \cup S)$. The definition of a uniform strong-consequentialist is similar.

$\forall (x, A), (y, B) \in \Omega$:

$$(x, \{x\})P(R_s)(y, \{y\}) \Rightarrow [(x, A)R(y, B) \Leftrightarrow (x, A)R_s(y, B)];$$
$$(x, \{x\})I(R_s)(y, \{y\}) \Rightarrow [(x, A)R(y, B) \Leftrightarrow (x, A)R_e(y, B)],$$

where $R = f(\mathbf{R})$.

It may easily be verified that the above ESWF satisfies (**SP**) and (**ND**). To verify that it satisfies both (**IIA(i)**) and (**IIA(ii)**), we consider $(x, A), (y, B) \in \Omega$, and $\mathbf{R} = (R_1, R_2, \cdots, R_n), \mathbf{R}' = (R'_1, R'_2, \cdots, R'_n) \in D_f(E \cup S)$. Let $R = f(\mathbf{R})$ and $R' = f(\mathbf{R}')$.

To begin with, suppose that we have $(x, A)R_i(y, B) \Leftrightarrow (x, A)R'_i(y, B)$ as well as $(x, \{x\})R_i(y, \{y\}) \Leftrightarrow (x, \{x\})R'_i(y, \{y\})$ for all $i \in N$. If $(x, \{x\})P(R_s)(y, \{y\})$, then $(x, \{x\})P(R'_s)(y, \{y\})$, $(x, A)P(R_s)(y, B)$, as well as $(x, A)P(R'_s)(y, B)$. Then, the ESWF gives us $(x, A)P(R)(y, B)$ and $(x, A)P(R')(y, B)$. Secondly, if $(y, \{y\})P(R_s)(x, \{x\})$, then $(y, \{y\})P(R'_s)(x, \{x\})$, $(y, B)P(R_s)(x, A)$, and $(y, B)P(R'_s)(x, A)$. Then, the ESWF gives us $(y, B)P(R)(x, A)$ and $(y, B)P(R')(x, A)$. Thirdly, if $(x, \{x\})I(R_s)(y, \{y\})$, then $(x, \{x\})I(R'_s)(y, \{y\})$. The ESWF implies that $(x, A)R(y, B) \Leftrightarrow (x, A)R_e(y, B)$ and $(x, A)R'(y, B) \Leftrightarrow (x, A)R'_e(y, B)$. Note that individual e is an extreme consequentialist. It is therefore clear that, in this case, if $(x, A)R_e(y, B) \Leftrightarrow (x, A)R'_e(y, B)$, then $(x, A)R(y, B) \Leftrightarrow (x, A)R'(y, B)$. Therefore, (**IIA(i)**) is satisfied.

Next, suppose that $\#A = \#B$ and that $[(x, A)R_i(y, B) \Leftrightarrow (x, A)R'_i(y, B)]$ for all $i \in N$. To show that $(x, A)R(y, B) \Leftrightarrow (x, A)R'(y, B)$ in this case, we observe that, when $\#A = \#B$, $(x, A)R_s(y, B) \Leftrightarrow (x, \{x\})R_s(y, \{y\})$ and $(x, A)R'_s(y, B) \Leftrightarrow (x, \{x\})R'_s(y, \{y\})$. Then the proof that the above ESWF satisfies (**IIA(ii)**) is similar to the proof showing that the ESWF satisfies (**IIA(i)**). We have only to note that the individual e is an extreme consequentialist.

The binary relation R generated by this ESWF is clearly reflexive and complete. We now show that R is transitive. Let $(x, A), (y, B)$ and $(z, C) \in \Omega$ be such that $(x, A)R(y, B)$ and $(y, B)R(z, C)$. Note that, since $(x, A)R(y, B)$, by the ESWF constructed above, we cannot have $(y, \{y\})P(R_s)(x, \{x\})$. Then, by the completeness of R_s, there are only two cases to be distinguished and separately considered: (a) $(x, \{x\})I(R_s)(y, \{y\})$; and (b) $(x, \{x\})P(R_s)(y, \{y\})$.

Case (a): In this case, we must have $(x, A)R_e(y, B)$. If $(y, \{y\})I(R_s)(z, \{z\})$, then it follows from $(y, B)R(z, C)$ that $(y, B)R_e(z, C)$. Then, the transitivity of R_e implies $(x, A)R_e(z, C)$. By the transitivity of R_s, $(x, \{x\})I(R_s)(z, \{z\})$. Therefore, $(x, A)R(z, C)$ if and only if $(x, A)R_e(z, C)$. Hence, $(x, A)R(z, C)$ follows from $(x, A)R_e(z, C)$. If $(y, \{y\})P(R_s)(z, \{z\})$, then, by the transitivity of R_s, it follows that $(x, \{x\})P(R_s)(z, \{z\})$. Therefore, $(x, A)R(z, C)$ if and only if $(x, A)R_s(z, C)$. Since s is a strong consequentialist, given that $(x, \{x\})P(R_s)(z, \{z\})$, we must have $(x, A)P(R_s)(z, C)$. Therefore, $(x, A)P(R)(z, C)$. Hence, $(x, A)R(z, C)$ holds. Note that, given $(y, B)R(z, C)$, we cannot have $(z, \{z\})P(R_s)(y, \{y\})$. Therefore, the transitivity of R holds in the case (a).

Case (b): In this case, we must have $(x, A)P(R_s)(y, B)$, hence $(x, A)P(R)(y, B)$. Since $(y, B)R(z, C)$, we must then have $(y, \{y\})R_s(z, \{z\})$. By the transitivity of R_s, it follows that $(x, \{x\})P(R_s)(z, \{z\})$. Thus, $(x, A)P(R_s)(z, C)$ follows from s being a strong consequentialist. By construction, in this case, $(x, A)R(z, C)$ if and only if $(x, A)R_s(z, C)$. Hence, $(x, A)P(R)(z, C)$. Therefore, the transitivity of R holds in the case (b).

Combining the cases (a) and (b), the transitivity of R is proved. ■

How about the society consisting only of strong consequentialists? Consistent with Theorem 1 as well as Theorem 2, we may assert the following:

Theorem 3. Suppose that all individuals are strong consequentialists. Then, there exists no extended social welfare function f with the domain $D_f(S)$ which satisfies (**SP**), (**IIA**) and (**ND**).

Proof. Since all individuals are strong consequentialists, we have the following for all $i \in N$: For all $(x, A), (y, B) \in \Omega$, if $(x, \{x\})I(R_i)(y, \{y\})$, then $\#A \geq \#B \Leftrightarrow (x, A)R_i(y, B)$, whereas if $(x, \{x\})P(R_i)(y, \{y\})$, then $(x, A)R_i(y, B) \Leftrightarrow (x, X)R_i(y, X)$. Suppose that an ESWF f satisfies (**SP**) and (**FIIA**), and consider all triples $(x, A), (y, B)$ and $(z, C) \in \Omega$ such that x, y and z are all distinct. Since all individuals are strong consequentialists and f has the domain $D_f(S)$, there exists no restriction on each individual's strict extended preference orderings over $\{(x, A), (y, B), (z, C)\}$. Thus, there is a dictator over the triple $\{(x, A), (y, B), (z, C)\}$. Note that the triple $\{(x, A), (y, B), (z, C)\}$, where $A \neq X$, coincides with the triple $\{(x, X), (y, B), (z, C)\}$ over the pair $\{(y, B), (z, C)\}$. Hence the dictator over the triple $\{(x, A), (y, B), (z, C)\}$ must in fact be independent of the set $A \in K$. The same argument can be applied to $B \in K$ as well as $C \in K$. Hence, for *all* triples $\{(x, A), (y, B), (z, C)\}$, we must have a single dictator. Call him $d \in N$ and consider a triple $(x, A), (y, B)$ and $(z, C) \in \Omega$ such that x, y and z are all distinct. Consider any $(x, A^*) \in \Omega$, where $A \neq A^*$. If $A^* \subset A$, all individuals being strong consequentialists, (**SP**) implies that $(x, A)P(R)(x, A^*)$, where $R = f(\mathbf{R})$. Similarly, if $A \subset A^*$, all individuals being strong consequentialists, (**SP**) implies that $(x, A^*)P(R)(x, A)$, where $R = f(\mathbf{R})$. If neither A is a subset of A^*, nor A^* is a subset of A, all individuals being strong consequentialists, we must have $(x, A)R_i(x, A^*)$ iff $\#A \geq \#A^*$. Then, (**SP**) implies that $(x, A)R(x, A^*)$ iff $\#A \geq \#A^*$, where $R = f(\mathbf{R})$. Hence, d is a dictator over Ω. Therefore, there exists no ESWF satisfying (**SP**), (**FIIA**) and (**ND**). ■

The message of these simple results seems very clear. Within the consequentialist framework, if all individuals are either extreme consequentialists or strong consequentialists, we have essentially Arrovian impossibility results. As the society becomes diverse by having at least one uniform extreme consequentialist and at least one uniform strong consequentialist simultaneously, however, it is possible to design an extended social welfare function that satisfies a variant of the Arrow conditions. Thus, *it is the diversity of the society, or the heterogeneity of population in the society, that plays a crucial role in resolving the Arrow impossibility theorem within the consequentialist framework.*

4 Arrovian Impossibility Theorem in the Non-Consequentialist Framework

Let us now turn to the examination of Arrow's impossibility theorem in a non-consequentialist framework. Our first task is to clarify what precisely we mean by non-consequentialism. Following Suzumura and Xu [28], let us define an individual to be a non-consequentialist as follows:[8]

Non-Consequentialist: An individual $i \in N$ is said to be a *non-consequentialist* if, for all $(x, A), (y, B) \in \Omega$, (a) $\#A > \#B \Rightarrow (x, A)P(R_i)(y, B)$; and (b) $\#A = \#B \Rightarrow [(x, A)R_i(y, B) \Leftrightarrow (x, \{x\})R_i(y, \{y\})]$.

Thus, a non-consequentialist is a person whose preference ranking over two extended alternatives $(x, A), (y, B) \in \Omega$ are such that, whenever the opportunity set A contains more alternatives than the opportunity set B, (x, A) is ranked higher than (y, B). It is only when A and B contain the same number of alternatives that (x, A) and (y, B) are ranked exactly the same as $(x, \{x\})$ and $(y, \{y\})$. In this sense, a non-consequentialist is in sharp contrast with both an extreme consequentialist and a strong consequentialist.

The following lemma shows that, for a non-consequentialist, imposing the conditions (**IND**), (**SI**) and (**MON**) introduced in Section 2 does not in fact restrict his/her preferences at all.

Lemma 3. A non-consequentialist's extended preference orderings must satisfy the three conditions (**IND**), (**SI**) and (**MON**).

Proof. It can be checked easily that a non-consequentialist's extended preference orderings satisfy (**SI**) and (**MON**). We now show that (**IND**) is also satisfied by a non-consequentialist's extended preference orderings.

Let i be a non-consequentialist. Let $(x, A), (y, B) \in \Omega$ and $z \in X \setminus A \cup B$. Suppose $(x, A)R_i(y, B)$. There are two cases to consider: (a) $\#A > \#B$ and (b) $\#A = \#B$. In case (a), clearly, $(x, A)P(R_i)(y, B)$ and $\#(A \cup \{z\}) > \#(B \cup \{z\})$. Hence, $(x, A \cup \{z\})P(R_i)(y, B \cup \{z\})$ follows from i being a non-consequentialist. In case (b), we have $(x, A)R_i(y, B)$ if and only if $(x, \{x\})R_i(y, \{y\})$ and $\#(A \cup \{z\}) = \#(B \cup \{z\})$. Then, $(x, A \cup \{z\})R_i(y, B \cup \{z\})$ if and only if $(x, \{x\})R_i(y, \{y\})$ follows from i being a non-consequentialist. Noting that R_i is complete, (**IND**) is therefore satisfied by R_i. ■

Let $D_f(N)$ be the domain of an extended social welfare function f such that there exists at least one person, say $n^* \in N$, who is a non-consequentialist *uniformly* for all profiles $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in D_f(N)$. Such a person will be called a *uniform non-consequentialist* over $D_f(N)$.

[8] In the terminology coined by Suzumura and Xu [28], a non-consequentialist defined here is called a *strong non-consequentialist*. Since this is the only category of non-consequentialism which is relevant in the present chapter, we have simplified our circumlocution by avoinding the adjective "strong".

Theorem 4. Suppose that there exists at least one person who is a uniform non-consequentialist over $D_f(N)$. Then, there exists an extended social welfare function f with the domain $D_f(N)$ that satisfies (**SP**), (**FIIA**) and (**ND**).

Proof. Let $n^* \in N$ be a uniform non-consequentialist over $D_f(N)$. Then, for all $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in D_f(N)$ and all $(x, A), (y, B) \in \Omega$, $\#A > \#B$ implies $(x, A)P(R_{n^*})(y, B)$. Consider now the following ESWF f: For all $(x, A), (y, B) \in \Omega$,

if $\#A > \#B$, then $(x, A)P(R)(y, B)$;
if $\#A = \#B = 1$, then $(x, \{x\})R(y, \{y\})$ iff $(x, \{x\})R_1(y, \{y\})$;
if $\#A = \#B = 2$, then $(x, A)R(y, B)$ iff $(x, A)R_2(y, B)$;
$\vdots$

if $A = B = X$, then $(x, A)R(y, A)$ iff $(x, A)R_k(y, B)$, where $k = \min\{\#N, \#X\}$,

where $R = f(\mathbf{R})$. It is easy to verify that this f satisfies **SP, IIA** and **ND**.[9] It is also clear that R generated by this ESWF is reflexive and complete. We now show that R is transitive as well. Let $(x, A), (y, B), (z, C) \in \Omega$ be such that $(x, A)R(y, B)$ and $(y, B)R(z, C)$. Then, clearly, $\#A \geq \#B$ and $\#B \geq \#C$. If $\#A > \#B$ or $\#B > \#C$, then $\#A > \#C$. By the constructed ESWF, $(x, A)P(R)(z, C)$ follows easily. Thus, the transitivity of R holds for this case. Now, suppose $\#A = \#B = \#C$. Note that in this case, for all $(a, G), (b, H) \in \Omega$ such that $\#G = \#H = \#A$, $(a, G)R(b, H)$ iff $(a, G)R_k(b, H)$ where $k \in N$ and $k = \min\{\#N, \#A\}$. Therefore, the transitivity of R follows from the transitivity of R_k. The above two cases exhaust all the possibilities. Therefore, R is transitive. ■

It is worthwhile to emphasize that, unlike the extreme consequentialist or the strong consequentialist, a non-consequentialist is able to guarantee the existence of an Arrovian extended social welfare function *by oneself*, and his ability is not nullified even in the homogeneous society where all individuals are non-consequentialists.

[9] It may be worthwhile to note that the extended social welfare function constructed in this proof has some nice features. When $\#X + 1 \geq n$, every individual can dictate one "layer" of the extended social alternatives. Indeed, we assign the non-consequentialist to dictate over (x, A) and (y, B) in Ω such that $\#A \neq \#B$. For each of all other individuals, we assign him to dictate over (x, A) and (y, B) in Ω such that $\#A = \#B$ coincides with his "index". When $\#X + 1 < n$, the number of "layers" is less than sufficient to assign each individual a "layer" for him to dictate. In this case, however, we can divide the population into two groups, Group 1 and Group 2. Group 2, which consists of $(\#X + 1)$ individuals including the non-consequentialist, will dictate over specific extended social alternatives assigned to him. The remainder of individuals form Group 1, which consists of $(n - \#X - 1)$ individuals, decide which individual in Group 2 should dictate which "layer" of alternatives, allowing the non-consequentialist to dictate over (x, A) and (y, B) with $\#A \neq \#B$. In this fashion, all individuals are assigned to actively participate in the process of social decision-making.

5 Generalizations

Although our analysis so far invoked the simple cardinality measure of the richness of opportunities, which is often criticized for its naivety, some of our results can go far beyond this special measure. To see this, let Θ be a complete orderings over K such that, for all $A, B \in K, A\Theta B$ holds if and only if A contains no less opportunity than B. $P(\Theta)$ and $I(\Theta)$ stand, respectively, for the asymmetric part and the symmetric part of Θ. The set X of all conventionally defined social states can be partitioned by $I(\Theta)$. Let $\mathcal{K}_\Theta$ denote the family of equivalence classes in accordance with Θ. For each $A \in K$, let $E_\Theta(A) \in \mathcal{K}_\Theta$ be the equivalence class determined by A. We can then define a linear ordering Θ^* on $\mathcal{K}_\Theta$ by

$$\text{For all } E_\Theta(A), E_\Theta(B) \in \mathcal{K}_\Theta, E_\Theta(A)\Theta^* E_\Theta(B) \Leftrightarrow A\Theta B. \tag{5.1}$$

In what follows, we assume that $(\Theta, \mathcal{K}_\Theta)$ satisfies the following two basic requirements:

Assumption U: The richness measure of opportunities, Θ, is unanimously held by all individuals in the society.

Assumption R: There exist at least two equivalence classes in $\mathcal{K}_\Theta$.

The definitions of consequentialists and non-consequentialists now read as follows:

Extreme Consequentialist: An individual $i \in N$ is said to be an *extreme consequentialist* if, for all $(x, A), (x, B) \in \Omega$, it is true that $(x, A)I(R_i)(x, B)$.

Strong Consequentialist: An individual $i \in N$ is said to be a *strong consequentialist* if, for all $(x, A), (y, B) \in \Omega$,

(a) if $(x, \{x\})I(R_i)(y, \{y\})$, then $A\Theta B \Leftrightarrow (x, A)R_i(y, B)$; and

(b) if $(x, \{x\})P(R_i)(y, \{y\})$, then $(x, A)P(R_i)(y, B)$.

Non-Consequentialist: An individual $i \in N$ is said to be a *non-consequentialist* if, for all $(x, A), (y, B) \in \Omega$,

(a) $AP(\Theta)B \Rightarrow (x, A)P(R_i)(y, B)$; and

(b) $AI(\Theta)B \Rightarrow [(x, A)R_i(y, B) \Leftrightarrow (x, \{x\})R_i(y, \{y\})]$.

We can now generalize our results in Sections 3 and 4 for the framework discussed in this section as follows. The proofs of these results are similar to those of Theorems 1, 2, 3 and 4, and we may safely omit them.

Theorem 5. Suppose that all individuals are extreme consequentialists. Then, there exists no extended social welfare function f with the domain $D_f(E)$ which satisfies (**SP**), (**ND**) and either (**IIA(i)**) or (**IIA(ii)**).

Theorem 6. Suppose that there exists at least one person who is a uniform extreme-consequentialist over $D_f(E \cup S)$ and at least one person who is a uniform strong-consequentialist over $D_f(E \cup S)$ in the society. Then, there exists an extended social welfare function f with the domain $D_f(E \cup S)$ which satisfies (**SP**), (**IIA(i)**), (**IIA(ii)**) and (**ND**).

Theorem 7. Suppose that all individuals are strong consequentialists. Then, there exists no extended social welfare function f with the domain $D_f(S)$ which satisfies (**SP**), (**FIIA**) and (**ND**).

Theorem 8. Suppose that there exists at least one person who is a uniform non-consequentialist over $D_f(N)$. Then, there exists an extended social welfare function f with the domain $D_f(N)$ which satisfies (**SP**), **IIA** and (**ND**).

Thus, our basic results in this chapter do not in fact hinge on somewhat controversial cardinality measure of the richness of opportunities.

6 Concluding Remarks

This chapter developed two extended analytical frameworks of social choice theory in order to check how and to what extent Arrow's general impossibility theorem hinges on his basic assumption of welfarist-consequentialism. Another motivation of our analysis was to see whether or not Arrow's observation that "the possibility of social welfare judgments rests upon a similarity of attitudes toward social alternatives" could be substantiated in the arena which is wider than welfarist-consequentialism.

The starting point of our analysis was an extended individual preference ordering defined over the pairs of social states and opportunity sets to which these social states belong.[10] It seems to us that people are prepared to say that choosing an alternative x from an opportunity set A is at least as good as choosing an alternative y from an opportunity set B. Negating the possibility of expressing such an extended preference ordering altogether is tantamount to saying that *there is no intrinsic value in the act of choice as such,* since we are then not in the position to say that choosing x from A, which includes x among others, is better than choosing x from $\{x\}$, which in fact means no effective choice at all. The concept of extended preference orderings enabled us to formulate a wider conceptual framework for analyzing social choice, and we could identify two such frameworks: the *consequentialist framework* and the *non-consequentialist framework*. The former is concerned with a society in which at least one consequentialist, either extreme or strong, exists, whereas the latter is concerned with a society in which at least one non-consequentialist exists.

[10] As far as we are aware, Gravel [6; 7] was the first who analyzed the concept of extended preference orderings. However, he assumed the existence of two individual preference orderings, viz. the preference ordering r on the set of options X, on the one hand, and the extended preference ordering R on the Cartesian product of X and K, where K is the set of opportunity sets from which the individual chooses. His analysis was focussed on the possible conflict between these preference orderings, and no role whatsoever is played in his analysis by the concept of consequentialism and non-consequentialism. See also Sen [21] for an implicit approach of using extended preference orderings in social choice theory.

Within the consequentialist framework, it was shown that the Arrovian impossibility theorem strenuously comes back if all individuals are either extreme consequentialists or strong consequentialists, whereas a more diverse society resided simultaneously by at least one extreme consequentialist and at least one strong consequentialist admits the existence of an Arrovian extended social welfare function. In this sense, it is the *diversity* rather than *similarity* of individual attitudes towards social alternatives in the society that contributes to resolve the Arrow impossibility theorem within the consequentialist framework. The logical fate of the non-consequentialist society is rather different. Indeed, within the non-consequentialist framework, it was possible to guarantee the existence of an Arrovian extended social welfare function as long as there exists at least one non-consequentialist in the society, and this ability is not nullified even if the society is *homogeneous* so that all individuals are non-consequentialists. Although these results are first established by using a naive cardinality measure of the richness of opportunities, their validity does not hinge on this arguably controversial measure.

It is hoped that our results, though simple, would be suggestive enough to motivate further exploration of the wider conceptual frameworks of social choice theory.

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