TAX FINANCED GOVERNMENT HEALTH EXPENDITURE AND GROWTH WITH CAPITAL DEEPENING EXTERNAILITY*

By Kei Hosoya†

Abstract

This paper develops a two-sector endogenous growth model with health capital and examines the impact tax financed health expenditure has on long-run growth. In this model, health capital is accumulated through government spending as a flow channel and a capital deepening externality as a stock channel. When arguing about the problem of growth maximizing flat tax, the latter channel plays a significant role for determining tax rate.

Keywords: Health capital; Capital deepening externality.

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†The Japan Society for the Promotion of Science, and Hitotsubashi University. Naka 2-1, Kunitachi, Tokyo 186-8601, JAPAN (e-mail: pg01109@srv.cc.hit-u.ac.jp).
1 Introduction

The aim of this paper is to examine the relationship between growth of the nations and government health expenditure under existing a capital deepening externality on health capital production. An important branch in the endogenous growth literature presents models in which government public investment positively influences the macroeconomic activities and improves its performances. Barro (1990) first investigates the endogenous growth model in which government spending contributes to the economic activities and plays an important role for determining long-run growth rate of the economy. Notable extensions of the Barro’s model are presented by Futagami et al. (1993), Greiner and Hamusch (1998) and so on. They employ a dynamic equation of public capital stock and derive some interesting results differ from the Barro’s model.

In the viewpoint of (macroeconomic) human capital theory, many researchers have examined the effects government educational spending have on a country’s growth and development; examples include Capolupo (2000) and Glomm and Ravikumar (2001). However, human capital consists of two components; one is “education”, and the other is “health” (see Mushkin, 1962). In this context, it is surprising that health, which is another component of human capital, has been largely ignored in the growth studies. Therefore, we should not forget that health is also a significant factor for long-run growth. To treat the health aspect in macroeconomic analysis, we introduce health capital to the model and construct a two-sector representative agent model of endogenous growth with the government which undertakes productive public health expenditure.\(^1\)

Outstanding features in this paper relate to the technology of health capital production. That is, we introduce a physical capital deepening externality to the health production function. The accumulation process of health capital also depends on this externality effect next to government health expenditure. We can suppose the government spending channel has a “flow” effect while the externality channel has a “stock” effect for health capital production.\(^2\) The former channel represents a direct activity of the government and contributes to the improvements of public health environment. Consequently, an improved health environment positively affects the individuals’ health status. Medical practice in a public hospital and the maintenance of an water purification plant, which are provided by government health expenditure, are mentioned as suitable examples. In contrast, the latter channel represents an indirectly but desirable influence to the health status. Such the effects are derived from an improvement in living standards. As a good example and an important channel, we can propose the following avenue: a development in the level of economic activity

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\(^1\)Previous papers based on a two-sector framework containing physical and human capital by Bond et al. (1996), Ortigueira and Santos (2002), among others investigated the fundamental properties of a two-sector model and especially concentrate on the analysis of dynamic behavior of the model (i.e. “transitional dynamics”).

\(^2\)The government spending channel, which is characterized by the flow effects, is often employed in the literature of economic growth. Capolupo (2000) uses a two-sector endogenous growth model with human capital and specifies a human capital production technology whose productive input only depends on government educational expenditure.
contributes to keep a better public health environment, so this will prevent an epidemic disease. By employing the external effects of capital deepening, we characterize the stock effects in health capital production.

An additional feature of the present model is that the flow effects are external factor for individuals’ economic behaviors in particular for their investment decisions as well as the case of the stock effects. Therefore, the agents perform their economic activities with ignoring the accumulation processes of own health capital.\(^3\) This reflects the fact that the public health policies of the government in many countries have played a central role for healthy management of individuals. Such a tendency will be observed especially in the developing countries. Perhaps it is natural to assume that the large part of public health is provided by the government. Within this framework, we analyze theoretically the impacts tax financed health expenditure have on long-run growth of the economy and discuss about the problem of growth maximizing flat tax rate.

To confirm an explicit relationship between growth rate and tax level which finances government health spending, we need for calibrating the present model. However, in this paper, we only propose the theoretical result of the model as a preliminary result. In the immediate paper succeeding to the present analysis, we will proceed the calibration studies of the model.

The rest of the paper is organized as follows. In Section 2 we present a simple model and investigate its equilibrium properties. Section 3 summarizes the preliminary results of the model.

\section{Simple model}

\subsection{Production technologies}

We present in this section a two-sector endogenous growth model with health capital accumulation \((H)\) and investigate the decentralized economy. Our model is an extended version of Capolupo (2000). The two sectors are composed of the goods production sector and the health capital creation sector, respectively. The evolution of health capital accumulation is an autonomous process which is accelerated by both government health expenditure and the external effects from capital deepening. The goods production function is given by

\[ Y(t) = K(t)^\alpha [H(t)L(t)]^{1-\alpha}, \quad \text{where } \alpha \in (0,1). \quad (1) \]

In Eq.(1), \(Y(t)\) is the total output, \(K(t)\) represents the aggregate stock of physical capital, \(H(t)\) is the level of health capital and \(L(t)\) denotes the total labor force which is identical with the total population in this model.\(^4\) This production function is also assumed to exhibit constant returns to scale.

Produced goods may be any consumed, invested to the accumulation of physical capital or expended to the maintenance of health capital as a governmental activity. We then assume that the evolution of physical capital is

\(^3\)In contrast with our model, van Zon and Muysken (2001) investigated the model incorporating the agents' investment choices for own health capital accumulation.

\(^4\)When there is no danger of misinterpretation, we omit the time argument \(t\).
governed by the familiar process of $\dot{K} = Y - C - G$ and that government health expenditure is financed by a proportional tax (an exogenous flat tax rate) on output $\tau Y$; i.e. $G = \tau Y$ ($\tau \in (0,1)$). The government balances its budget at each point in time. From Eq.(1)

$$\dot{K} = K^\alpha (HL)^{1-\alpha} - C - G = (1 - \tau)K^\alpha (HL)^{1-\alpha} - C. \quad (2)$$

As noted above, health capital accumulation is covered by both government expenditure $G$ and the capital deepening externality $f(\hat{k})$. Therefore, the evolution of health capital is as follows:

$$\dot{H} = AGf(\hat{k}), \quad (3)$$

where $f(0) = 0$, $f'(\hat{k}) > 0$ and $f''(\hat{k}) < 0$. $A > 0$ is a constant parameter related to the efficiency of health capital production. Moreover, for $f(\hat{k})$, we assume it well-behaved function and impose the Inada conditions: $\lim_{\hat{k} \to 0} f'(\hat{k}) = +\infty$ and $\lim_{\hat{k} \to +\infty} f'(\hat{k}) = 0$. We now define $\hat{k} = \hat{K}/\hat{HL}$. $\hat{k}$ is the social average level of the physical capital/effective labor ratio which brings about the external effects of capital deepening for health capital accumulation. Such the effects represent a social benefit derived from an improvement in living standards. As an earlier contribution, we should mention the following paper. Marvin Frankel (1962) employed an economy-wide development index to the goods production function at firm level, which is similar to the specification of the external effects we defined in this paper. However, in the present model, such the index only affects the evolution of health capital. The “learning-by-doing” models of Arrow (1962), Sheshinski (1967) and Romer (1986) also developed the similar ideas. Due to the properties of $f(\hat{k})$, we specify the following functional form:

$$f(\hat{k}) = f\left(\frac{\hat{K}}{\hat{HL}}\right) = \left(\frac{\hat{K}}{\hat{HL}}\right)^\epsilon, \quad \text{where} \quad \epsilon \in (0,1). \quad (4)$$

Using the relation of Eq.(4), we can rewrite Eq.(3) to have

$$\dot{H} = AG\left(\frac{K}{HL}\right)^\epsilon. \quad (5)$$

### 2.2 Preference

The representative agent chooses the level of consumption and the investment level of physical capital to maximize own intertemporal utility. For analytical convenience, we employ the logarithmic preference. Formally, the agent’s dynamic optimization problem is given as

$$\max_{C(t)} V(K(0)) = \int_0^{+\infty} \ln C(t)e^{-\rho t} dt, \quad (6)$$

subject to Eq.(2), $K(0) = K_0 > 0$, where $\rho > 0$ is a constant subjective rate of time preference.

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5The first paper to give much attention to the importance of Frankel’s paper in the relevant field was Cannon (2000).

6In contrast with our model, just like Frankel (1962), they also employ the goods production function containing the “social level” of capital as well as the private capital.
2.3 Solving the model

In our model environment, both government expenditure and the effects of capital deepening are external for the agent’s economic behavior. The agent then disregards the creation process of own health capital. As a result, the present model is similar to the conventional one-sector neoclassical growth model. For analytical simplicity, we now set the total labor force normalized to unity \((L = 1)\). To solve the corresponding dynamic optimization problem, we define the current-value Hamiltonian \(\mathcal{H}\):

\[
\mathcal{H} \equiv \ln C + \lambda [(1 - \tau) K^\alpha H^{1-\alpha} - C],
\]

where \(\lambda\) corresponds to the co-state variable related to physical capital \(K\). For obtaining an interior solution, the first-order conditions are listed below:

\[
\frac{1}{C} = \lambda, \quad (8)
\]
\[
\dot{\lambda} = -\lambda \alpha (1 - \tau) K^{\alpha - 1} H^{1-\alpha} + \lambda \rho, \quad (9)
\]

plus the usual transversality condition,

\[
\lim_{t \to +\infty} \lambda(t) K(t) e^{-\rho t} = 0. \quad (10)
\]

Using Eqs.(8) and (9), we can derive the formula related to the rate of growth of consumption:

\[
g_C \equiv \frac{\dot{C}}{C} = \alpha (1 - \tau) \left(\frac{K}{H}\right)^{\alpha - 1} - \rho, \quad (11)
\]

where \(g_x\) denotes the equilibrium growth rate of placeholder \(x\).

2.4 Equilibrium path

Let us characterize here the equilibrium of the model. Since \(L = 1\), substituting the relation of \(G = \tau Y = \tau K^\alpha H^{1-\alpha}\) into Eq.(5) leads to the following dynamical process of health capital accumulation:

\[
\dot{H} = A \tau K^\alpha H^{1-\alpha} \left(\frac{\bar{K}}{\bar{H}}\right)^\epsilon. \quad (12)
\]

Rearranging Eq.(12)

\[
\frac{\dot{H}}{H} = A \tau \left(\frac{K}{H}\right)^\alpha \left(\frac{\bar{K}}{\bar{H}}\right)^\epsilon. \quad (13)
\]

At the equilibrium, of course, \(\bar{K}, \bar{H}\) must be set equal to \(K, H\), respectively. Applying these expressions to Eq.(13) to obtain

\[
g_H \equiv \frac{\dot{H}}{H} = A \tau \left(\frac{K}{H}\right)^{\alpha + \epsilon}. \quad (14)
\]
On the balanced growth path (BGP), \( g_Y, g_C, g_K \) and \( g_H \) are all equal to \( g \); i.e. \( g = g_Y = g_C = g_K = g_H \). Therefore, from Eq.(14), we can derive
\[
\frac{K}{H} = \left( \frac{g}{A\tau} \right)^{\frac{1}{\alpha+\epsilon}}.
\] (15)

Putting Eq.(15) into Eq.(11), we obtain the equilibrium growth rate along the BGP. That is,
\[
g = \alpha(1 - \tau) \left( \frac{g}{A\tau} \right)^{\frac{\alpha-1}{\alpha+\epsilon}} - \rho.
\] (16)

From Eq.(16), we find that the equilibrium growth rate of the economy \( g \) depends on the parameters \( \alpha, \tau, A, \epsilon, \rho \).

**Proposition 1** The equilibrium growth rate at the BGP is determined by the structural parameters of \( \alpha, \tau, A, \epsilon, \rho \).

**Proof:** See Eq.(16).

As for the equilibrium of the model, we can refer to the following result.

**Proposition 2 (Existence and Uniqueness of the Equilibrium)** There exists a unique equilibrium with a positive solution in this model.

**Proof:** First, rewriting Eq.(16) to obtain \( g + \rho = \alpha(1 - \tau) \left( \frac{g}{A\tau} \right)^{(\alpha-1)/(\alpha+\epsilon)} \).

Here, we denote the LHS and the RHS of this equation by \( \chi(g) \) and \( \Gamma(g) \), respectively. At the first quadrant in \( (g, \chi) \)-plane, \( \chi \) is a very simple linear function of \( g \) with a positive slope. On the other hand, \( \Gamma \) is a strictly decreasing and a strictly convex function of \( g \) in the same quadrant. That is, simple calculation leads to \( \Gamma'(g) = \alpha(1 - \tau) \left( \frac{\alpha-1}{\alpha+\epsilon} \right) \left( A\tau \right)^{(1-\alpha)/(\alpha+\epsilon)} g^{-(1+\epsilon)/(\alpha+\epsilon)} \). Therefore we have \( \lim_{g \to 0} \Gamma(g) = +\infty, \lim_{g \to +\infty} \Gamma(g) = 0, \lim_{g \to 0} \Gamma'(g) = -\infty \) and \( \lim_{g \to +\infty} \Gamma'(g) = 0 \). From these functional properties, two functions \( \chi \) and \( \Gamma \) inevitably intersect in the first quadrant only once.

Typical case is shown in Figure 1. From these papers, for example, the relevant range for \( \rho \) is approximately 0.01-0.04. Benchmark parameters we used here are listed below; \( (\rho, \alpha, \tau, A, \epsilon) = (0.02, 0.30, 0.10, 0.10, 0.20) \). These parameterization problems are taken

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7From Eq.(8), we obtain \( \lambda(t) = \lambda(0)e^{-\rho t} \). On the other hand, the relation \( \hat{K}/K = g \) yields \( K(t) = K(0)e^{\rho t} \). These relations imply \( \lambda(t)K(t)e^{-\rho t} = \lambda(0)K(0)e^{(\rho t + gt - \rho t)} = \lambda(0)K(0)e^{-\rho t} \). Since it assumed earlier \( \rho > 0 \), a necessary condition for an optimum in Eq.(10) is surely satisfied.

8Note that the function \( \Gamma \) is plotted the relation between the horizontal axis and the left-vertical axis, while the function \( \chi \) is plotted the relation between the horizontal axis and the right-vertical axis.

9From these papers, for example, the relevant range for \( \rho \) is approximately 0.01-0.04.
Figure 1: Existence and uniqueness of positive solution

up again later. According to author’s calculation under these parameter values, the equilibrium growth rate is approximately equal to 3.2%.\(^{10}\)

Further examining Eq.(16), we ask what effect an increase in \(\tau\) has on the growth rate at the BGP. By total differentiating Eq.(16), we obtain

\[
\frac{dg}{d\tau} = \frac{\alpha \left( \frac{g}{A\tau} \right)^{\alpha - 1}}{1 + \Gamma \left( \frac{1 - \alpha}{\alpha + \epsilon} \right) \left( \frac{1}{g} \right)},
\]

(17)

where \(\Gamma \equiv \alpha (1 - \tau) (\frac{g}{A\tau})^{(\alpha - 1)/(\alpha + \epsilon)} > 0\). In the RHS of Eq.(17), the denominator is definitely positive. Then the effects of changes in tax rate on growth rate are determined by the signs of the numerator. The results are summarized as follows.

\[
\begin{align*}
\frac{dg}{d\tau} > 0 & \iff \text{if } \tau < \frac{1 - \alpha}{1 + \epsilon}, \\
\frac{dg}{d\tau} = 0 & \iff \text{if } \tau = \frac{1 - \alpha}{1 + \epsilon}, \\
\frac{dg}{d\tau} < 0 & \iff \text{if } \tau > \frac{1 - \alpha}{1 + \epsilon}.
\end{align*}
\]

From these results, we find that the relation between growth rate and proportional tax rate has a hump-shape in \((\tau, g)\)-plane. Note that the growth maximizing rate of tax is \(\tau = (1 - \alpha)/(1 + \epsilon).\)^{11}\)

\(^{10}\)As an additional example, we present the case in which the degree of externality is relatively high. For an increased value \(\epsilon = 0.40\) (other parameter values are unchanged), the growth rate is approximately equal to 4.3%.

\(^{11}\)Contrary to the Barro’s (1990) result, in general, the tax rate which maximizes economic growth does not agree with the tax rate which maximizes welfare (see for example Futagami et al., 1993; Greiner and Hanusch, 1998). However, in this paper, we will not take up the welfare aspect.
Finally, we should refer to the stability of the equilibrium. As will be presented in Appendix, the equilibrium of the present model is saddle-path stable. This result is directly derived from the Jacobian properties in the reduced two-dimensional dynamical system. As a result, the following holds:

**Proposition 3 (Stability of the Equilibrium)** Since Det$J^*$ < 0 and the Jacobian is $2 \times 2$, the equilibrium is locally unique, and then the system is saddle-path stable.

**Proof**: See Appendix.

### 3 Results

Based on the previous analysis, we can calculate numerically the growth maximizing tax rate. The tax rate we obtained theoretically is $\tau = (1 - \alpha)/(1 + \epsilon)$. Therefore, the relation between physical capital share in goods production and the external effects from capital deepening directly determines the growth maximizing flat tax rate in this economy. Following Lucas (1988), Capolupo (2000) and others, we assume that the range for physical capital share is 0.25-0.35. In such a range, we select three-benchmark cases: $\alpha = 0.25, 0.30, 0.35$. On the other hand, the relevant value for externality is unclear. To our knowledge, the empirical evidence on this value has never been reported in the literature so far. Consequently, as a tentative assumption, we set the range for externality is $\epsilon < 0.5$. This range follows the value for human capital externality (in the goods production sector) reported by Lucas (1988). Roughly speaking, it seems reasonable to suppose that the capital deepening externality is not so large. Three-benchmark capital shares as given, Figure 2 represents the relation between the growth maximizing tax rate and the degree of external effects.

Figure 2 shows the basic properties of the model. When the degree of external effects is fixed, the lower (higher) physical capital share case needs a higher (lower) rate of tax for a country’s growth rate maximizes. These contrasting results are explained as follows. When the case of lower capital share (e.g. $\alpha = 0.25$), it is necessary for maximizing output growth to input the higher level of health capital into goods production. Then the degree of capital deepening externality as given, the flat tax rate must be a higher value. In the same way, we can find that the case of higher capital share (e.g. $\alpha = 0.35$) does not need so a higher tax rate compared with the case of lower capital share.

Finally, when we focus on the degree of capital deepening externality, another important result is obtained (see also Figure 2). If the level of externality is relatively high, the government does not need to set a higher tax rate for maximizing growth rate. Government size may be then “small”. Hence, the higher the capital deepening externality in health production the lower the flat tax rate for growth maximization. Realistically speaking, it is likely that the

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12 Specifically, his estimate on the degree of human capital externality is 0.417.
13 Consider the case $\alpha = 0.30$ as an example. It is shown that a rise in the degree of capital deepening externality from 0.20 to 0.40 reduces the growth maximizing tax rate by 8.3%
external effects, which represent an improvement in living standards, have a considerable impact on health capital accumulation. Therefore, under the situation that productive health expenditure is implemented, if the policymaker wants to maximize a country’s growth rate as a principal policy target, his adequate evaluation on this externality will lead to the determination of an appropriate tax rate.

Appendix

Note on local stability

We define the following new stationary variables which will be constant along the BGP:

\[ X \equiv \frac{C}{K}, \]
\[ Z \equiv \frac{K}{H}. \]

From these definitions, we obtain the following expressions:

\[ \frac{\dot{X}}{X} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K}, \quad (A1) \]
\[ \frac{\dot{Z}}{Z} = \frac{\dot{K}}{K} - \frac{\dot{H}}{H}. \quad (A2) \]

Using Eqs.(2), (11) and (14), Eqs.(A1) and (A2) are transformed by

\[ \frac{\dot{X}}{X} = X + (\alpha - 1)(1 - \tau)Z^{\alpha-1} - \rho, \quad (A3) \]
\[ \frac{\dot{Z}}{Z} = -X + (1 - \tau)Z^{\alpha-1} - A\tau Z^{\alpha+\epsilon}. \quad (A4) \]
Original three-dimensional dynamical system \((C, K, H)\) is completely transformed by Eqs.\((A3)\) and \((A4)\). On the BGP, the relations of \(g_x = g_Z = 0\) are satisfied. Here, let the values for \(X\) and \(Z\) at the BGP denote \(X^*\) and \(Z^*\), respectively. From Eqs.\((A3)\) and \((A4)\), these values must be satisfied the following simultaneous equations:

\[
X^* + (\alpha - 1)(1 - \tau)(Z^*)^{\alpha-1} = \rho, \\
X^* = (1 - \tau)(Z^*)^{\alpha-1} - A\tau(Z^*)^{\alpha+\epsilon}.
\]

Consequently, the Jacobian evaluated at the BGP \((J^*)\) is given by

\[
J^* = \begin{bmatrix}
\frac{\partial \dot{X}}{\partial X} & \frac{\partial \dot{X}}{\partial Z} \\
\frac{\partial \dot{Z}}{\partial X} & \frac{\partial \dot{Z}}{\partial Z}
\end{bmatrix} = \begin{bmatrix}
X^* & (\alpha - 1)^2(1 - \tau)X^*(Z^*)^{\alpha-2} \\
-Z^* & [(\alpha - 1)(1 - \tau)(Z^*)^{\alpha-1} - A\tau(\alpha + \epsilon)(Z^*)^{\alpha+\epsilon}]
\end{bmatrix}.
\]

The determinant of \(J^*\) is calculated as follows:

\[
\text{Det}\, J^* = -[\alpha(1 - \alpha)(1 - \tau)X^*(Z^*)^{\alpha-1} + A\tau(\alpha + \epsilon)X^*(Z^*)^{\alpha+\epsilon}] < 0,
\]

where \([\alpha(1 - \alpha)(1 - \tau)X^*(Z^*)^{\alpha-1} + A\tau(\alpha + \epsilon)X^*(Z^*)^{\alpha+\epsilon}]\) is positive. Hence, at least one eigenvalue is negative (or has negative real part). Since \(\text{Det}\, J^*\) is always negative, we find that the equilibrium is locally unique, so that the dynamical system is saddle-path stable at the neighborhood of the BGP. As noted in Harrison and Weder (2002), if \(\text{Det}\, J^* < 0\) and the Jacobian is \(2 \times 2\) matrix, the saddle-path stability does not depend on the signs of trace \((\text{Tr}\, J^*)\).

**References**


