EK-110xxiii

Optimal Monetary and Fiscal Policy Mix in a Currency Union with Nontradable Goods*

OKANO, Eiji†

This Draft: Oct. 2006

Abstract

By constructing a form of dynamic stochastic general equilibrium model describing a currency union that consists of two countries with nontradable goods, this paper shows the importance of an optimal monetary and fiscal policy mix with regard to social welfare.

Keywords: Currency Union, DSGE, Balassa–Samuelson theorem, Optimal Monetary Policy, Monetary and Fiscal Policy Mix
JEL Classification: E52; E62; F41

---

*I would like to thank Eiji Ogawa and Ippei Fujiwara for helpful suggestions. All errors remain my responsibility.

†Correspondence to: Faculty of Economics, Chiba Keizai University, 59-5, Todorokicho 3-chome, Inage-ku, Chiba-shi, Chiba, 263-0021, JAPAN. Tel: +81-43-253-9738 (Direct); Fax: +81-43-254-6600; E-mail: e.okano@cku.ac.jp

1 Introduction

A currency union that had previously been pure academic speculation became a reality when the European Monetary Union (EMU) was established. Needless to say, the creation of the EMU has led to new challenges for policy makers. The present paper provides a tractable framework suitable for the analysis of fiscal and monetary policy in a currency union and studies its implications for the optimal design of such policies, not only from the viewpoint of the union-wide economy but also from the viewpoint of the individual countries that comprise a currency union.

Discussions of optimal monetary policy in a currency union become brisk in such a situation. Assuming that all goods are tradable, Benigno[4] derived an interest policy implication that optimal monetary policy, a synonym for inflation targeting in a simple situation, can maximize social welfare in a currency union that consists of two countries, assuming that in addition to perfect risk sharing at both domestic and international levels, the economies in the two countries are identical on the demand side. Thus, the solitary central bank in a currency union can achieve welfare maximization. On the other side, Gali and Monacelli[7] insist on a monetary and fiscal policy mix using a currency union model that consists of not two countries but infinite infinitesimal countries. Under this framework, the solitary central bank can maximize welfare at a union-wide level whereas it needs some support brought about by fiscal authority to maximize welfare. There is great difference in the policy implications between the two superb studies.

We should pay attention to the presuppositions of these policy implications. While canonical studies consider the existence of nontradable goods, these studies do not consider the existence of nontradable goods.[1] While the definition of nontradable goods is not simple, as was mentioned by McKinnon[9], nontradables in general correspond to services toward goods in an actual economy. Following the definition that regards goods produced in the manufacturing industry, agriculture, forestry, fishery and mining as tradables and regards goods produced in other industries as nontradables, as used by Canzoneri, Cumby and Diba[5], nontradables in terms of current and purchaser's price accounted for 50.3% of the sum of nontradables and tradables in major Euro area countries such as Belgium, Germany, France, Greece, Italy, the Netherlands, Portugal and Spain in 1999. It is obvious that the share of nontradables should not be ignored in analyzing monetary policy.

In consideration of the existence of nontradable goods, the present paper constructs a form of DSGE model that describes a currency union that consists of two countries with nontradable goods to analyze an optimal monetary policy, and an optimal monetary and fiscal policy mix. The model developed in this paper has two distinctive features to analyze an optimal policy design based on the existence of nontradable goods in the currency union, the Euro area. First, because of the existence of nontradables, we focus on the Balassa–Samuelson theorem, which explains a nominal exchange rate deviating from purchasing power parity. Needless to say, nontradable goods have a disregarded effect on

[1] Neither papers on monetary policy in a currency union nor papers on monetary policy in an open economy, such as Benigno[4], Benigno and Benigno[3], Gali and Monacelli[8] and Okano[13], consider the existence of nontradable goods, although these papers derive some important implications.

an open economy. Analyzing not monetary policy but exchange rate volatility, Stockman and Tesar[16], Benigno and Thoenissen[2] and Selaive and Tuesta[?] focus on nontradable goods in the consumption-real exchange rate anomaly. These papers on the Balassa–Samuelson theorem point out the relationship between the anomaly and the theorem. Whereas a nominal exchange rate does not appear in our model because the model is a closed system, the Balassa–Samuelson theorem explains a disparity in the Consumer Price Indices (CPIs) between two countries composing a currency union. We show the difficulties of conducting a monetary policy and the necessity of a monetary and fiscal policy mix in a currency union with nontradables. Second, we allow implementation of fiscal policy by a centralized government. We investigate the appropriateness of a centralized government, which is advocated by Mundel[11] and McKinnon[9] in welfare maximization. This is one of the important agenda items in recent DSGE literature in the currency area.

This paper refers to some secondary issues, too. While we point out an inconsistency between Benigno[4] and Gali and Monacelli[7] on policy implications above, we resolve these disparities between two studies. As the Maastricht Treaty is strictly applied in the Euro area, this paper refers to the suitability of the treaty in a currency union with nontradables with a view to welfare maximization.

The paper is organized as follows. Section 2 constructs the model. Section 3 defines and analyzes monetary policy qualitatively without a fiscal policy regime and a mixed optimal monetary and fiscal policy regime. Section 4 is a numerical analysis including a welfare analysis. Section 5 concludes this paper.

2 The Model

We construct a closed-system currency union model belonging to the class of DSGE models with nominal rigidities and imperfect competition and refer to Obstfeld and Rogoff[12], Gali and Monacelli[8]. Following Stockman and Tesar[16], we allow imperfect substitution between tradables and nontradables, while Obstfeld and Rogoff[12] implicitly assume that these goods are a perfect substitution. The union-wide economy consists of two equally sized countries, countries H and F. Country H produces an array of differentiated tradable goods indexed by the interval $[0, 1]$, while country F produces an array of differentiated goods indexed by $[1, 2]$. In addition, each country produces an array of differentiated nontradables indexed by $[0, 1]$.

2.1 Households

Preference of the representative household in country H is given by:

$$
\begin{aligned}
U_t &\equiv \mathrm{E}_t \sum_{t=0}^{\infty} \delta^t \left\{ \ln C_t D_t - \frac{Z_t}{1+\varphi} N_t^{1+\varphi} \right\} \\
U_t^* &\equiv \mathrm{E}_t \sum_{t=0}^{\infty} \delta^t \left\{ \ln C_t^* D_t - \frac{Z_t}{1+\varphi} \left(N_t^*\right)^{1+\varphi} \right\} \qquad (1)
\end{aligned}
$$

where E_t denotes the expectation conditional on the information set at period t, $\delta \in (0, 1)$ denotes the subjective discount factor, C_t denotes consumption in country H, N_t denotes hours of work in country H, D_t denotes a union-wide consumption preference shifter, Z_t denotes a union-wide disutility of work shifter and φ denotes the inverse of a labor supply elasticity. We note that quantities and prices peculiar to country F are denoted by asterisks while quantities and prices without asterisks are those in country H or common to both countries.

More precisely, private consumption is a composite index defined by:

$$\begin{aligned} C_t &\equiv \left\{ \gamma^{\frac{1}{\eta}} C_{T,t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{N,t}^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\ C_t^* &\equiv \left\{ \gamma^{\frac{1}{\eta}} \left(C_{T,t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} \left(C_{N,t}^*\right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \end{aligned} \tag{2}$$

where $C_{T,t} \equiv 2C_{H,t}^{\frac{1}{2}} C_{F,t}^{\frac{1}{2}}$ denotes the consumption index for tradables in country H, $C_{T,t}^* = C_{T,t}$ denotes the consumption index for tradables in country F, $C_{N,t}$ and $C_{N,t}^*$ denote Dixit–Stiglitz-type indices of consumption across the nontradable goods produced in countries H and F, respectively, $C_{H,t}$ and $C_{F,t}$ denote Dixit–Stiglitz-type indices of consumption across the tradable goods produced in countries H and F, respectively, γ denotes the share of tradables in the consumer price index (CPI), $\theta > 1$ denotes the elasticity of substitution across goods produced within a country and $\eta > 0$ denotes the elasticity of substitution between tradable and nontradable goods.

Total consumption expenditures by households in country H are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{N,t}C_{N,t} = P_tC_t$, while their counterparts in country F are given by $P_{F,t}C_{F,t} + P_{H,t}C_{H,t} + P_{N,t}^*C_{N,t}^* = P_t^*C_t^*$, with $P_{H,t}$ and $P_{F,t}$ being Dixit–Stiglitz-type indices of price of tradable goods produced in countries H and F, respectively, and $P_{N,t}$ and $P_{N,t}^*$ being Dixit–Stiglitz-type indices of price of nontradable goods produced in countries H and F. A sequence of budget constraints on the form is given by:

$$\begin{aligned} B_t + W_tN_t + S_t &\geq P_tC_t + \mathrm{E}_tQ_{t,t+1}B_{t+1} \\ B_t + W_t^*N_t^* + S_t^* &\geq P_t^*C_t^* + \mathrm{E}_tQ_{t,t+1}B_{t+1} \end{aligned} \tag{3}$$

where $Q_{t,t+1}$ denotes the stochastic discount factor, B_t denotes the nominal payoff of the portfolio, W_t denotes the nominal wage and S_t denotes the lump-sum transfers.[2]

The optimal allocation of any given expenditure within each category of goods implies the demand functions as follows:

$$\begin{aligned} C_{H,t} = \frac{1}{2}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C_{T,t} \quad &; \quad C_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C_{T,t} \\ C_{T,t} = \left(\frac{P_{T,t}}{P_t}\right)^{-\eta} C_t \quad &; \quad C_{N,t} = \left(\frac{P_{N,t}}{P_t}\right)^{-\eta} C_t \end{aligned} \tag{4}$$

where $P_{T,t} \equiv P_{H,t}^{\frac{1}{2}} P_{F,t}^{\frac{1}{2}}$ denotes the tradables price index (TPI), and

$$\begin{aligned} P_t &\equiv \left[\gamma P_{T,t}^{1-\eta} + (1-\gamma) P_{N,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \\ P_t^* &\equiv \left[\gamma P_{T,t}^{1-\eta} + (1-\gamma) \left(P_{N,t}^*\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \end{aligned} \tag{5}$$

[2]Individuals in country F face a parallel constraint.

denote the CPIs in countries H and F, respectively. We also note that the producer price index (PPI) is defined by:

$$\begin{aligned} P_{P,t} &\equiv \left[\gamma P_{H,t}^{1-\eta} + (1-\gamma) P_{N,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \\ P_{P,t}^{*} &\equiv \left[\gamma P_{F,t}^{1-\eta} + (1-\gamma) \left(P_{N,t}^{*}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \end{aligned} \tag{6}$$

with $P_{P,t}$ being the PPI in country H.

The representative household maximizes Eq.(1) subject to Eq.(??). Optimality conditions are given by:

$$\delta \mathrm{E}_t \frac{C_{t+1}^{-1} P_t D_{t+1}}{C_t^{-1} P_{t+1} D_t} = \frac{1}{R_t} \quad ; \quad \delta \mathrm{E}_t \frac{\left(C_{t+1}^{*}\right)^{-1} P_t^{*} D_{t+1}}{\left(C_t^{*}\right)^{-1} P_{t+1}^{*} D_t} = \frac{1}{R_t} \tag{7}$$

$$\frac{C_t N_t^{\varphi} Z_t}{D_t} = \frac{W_t}{P_t} \quad ; \quad \frac{C_t^{*} \left(N_t^{*}\right)^{\varphi} Z_t}{D_t} = \frac{W_t^{*}}{P_t^{*}} \tag{8}$$

where $R_t \equiv 1 + r_t$ satisfying $R_t^{-1} = Q_{t,t+1}$ denotes the gross nominal return on a riskless one-period discount bond paying off on one unit of the common currency (for short, the gross nominal interest rate), and r_t denotes the net nominal interest rate. Eq.(7) is an intertemporal optimality condition, namely the Euler equation, while Eq.(8) is an intratemporal optimality condition.[3] Combining and iterating both of Eq.(7), we have an optimal risk-sharing condition as follows:

$$C_t = \vartheta C_t^{*} \mathsf{Q}_t \tag{9}$$

with $\mathsf{Q}_t \equiv \frac{P_t^{*}}{P_t}$ denoting the CPI differential between the two countries and ϑ denoting a constant depending on initial value. Following Chari, Kehoe and McGrattan[6], we assume that $\vartheta = 1$.[4]

2.2 Firms

Each firm is classified into one of two groups: tradables producers and nontradables producers. Each producer can use a linear technology to produce a differentiated good as follows:

$$\begin{aligned} Y_{H,t}(h) = N_t(i) \quad &; \quad Y_{N,t}(i) = N_t(i) \\ Y_{F,t}(f) = N_t^{*}(i) \quad &; \quad Y_{N,t}^{*}(i) = N_t^{*}(i) \end{aligned} \tag{10}$$

where $Y_{H,t}(h)$ denotes the output of tradable goods i produced in country H, $Y_{N,t}$ denotes the output of nontradables i produced in country H.[5]

Each firm of a single differentiated good prices its goods in response to the elasticity of substitution across goods produced within the SOE given the CPI. This is because each firm plays an active part in the monopolistically competitive market. We assume that Calvo–Yun-style price-setting behavior

[3]Optimality conditions analogous to Eqs.(7) and (8) must hold in country F.

[4]When $C_{-1} = C_{-1}^{*} = P_{-1} = P_{-1}^{*} = 1$, we have $\vartheta = 1$.

[5]The nature of the production technology implies that nominal unit cost is equal to nominal marginal cost.

applies, and therefore that each firm resets its price with a probability of $1-\alpha$ in each period, independently of the time elapsed since the last adjustment.

When setting a new price in period t, firms seek to maximize the expected discounted value of profits. The FONCs are as follows:

$$
\begin{aligned}
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\Lambda_{t+k}\tilde{C}_{H,t+k}\left(\tilde{P}_{H,t}-\zeta(1-\tau)P_{P,t+k}MC_{H,t+k}\right)\right] &= 0, \\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\Lambda_{t+k}\tilde{C}_{N,t+k}\left(\tilde{P}_{N,t}-\zeta(1-\tau)P_{P,t+k}MC_{N,t+k}\right)\right] &= 0 \\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\Lambda^*_{t+k}\tilde{C}_{F,t+k}\left(\tilde{P}_{F,t}-\zeta(1-\tau)P^*_{P,t+k}MC_{F,t+k}\right)\right] &= 0 \\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\Lambda^*_{t+k}\tilde{C}^*_{N,t+k}\left(\tilde{P}^*_{N,t}-\zeta(1-\tau)P^*_{P,t+k}MC^*_{N,t+k}\right)\right] &= 0 \quad (11)
\end{aligned}
$$

where Λ_t and Λ^*_t denote the marginal utility of nominal income in countries H and F, respectively, $MC_{H,t} \equiv \frac{W_t}{P_{P,t}A_{H,t}}$, $MC_{N,t} \equiv \frac{W_t}{P_{P,t}A_{N,t}}$, $MC_{F,t} \equiv \frac{W^*_t}{P^*_{P,t}A_{F,t}}$ and $MC^*_{N,t} \equiv \frac{W^*_t}{P^*_{P,t}A^*_{N,t}}$ denote the marginal costs associated with tradables produced in country H, nontradables produced in country H, tradables produced in country F and nontradables produced in country F, respectively, $\tilde{C}_{H,t+k}$, $\tilde{C}_{N,t+k}$, $\tilde{C}_{F,t+k}$ and $\tilde{C}^*_{N,t+k}$ denote the total demands when the prices are changed of tradables produced in country H, nontradables produced in country H, tradables produced in country F, and nontradables produced in country F, respectively, $A_{H,t}$, $A_{N,t}$, $A_{F,t}$ and $A^*_{N,t}$ denote stochastic productivity shifters associated with tradables produced in country H, nontradables produced in country H, tradables produced in country F and nontradables produced in country F, respectively, $\tilde{P}_{H,t}$, $\tilde{P}_{N,t}$, $\tilde{P}^H_{F,t}$ and $\tilde{P}^*_{H,t}$ denote the adjusted prices of tradables produced in country H and nontradables produced in country H, respectively, and $\zeta \equiv \frac{\theta}{\theta-1}$ is a constant markup. We take it as given that the law of one price (LOOP) always holds.

We also note that using Eq.(8), marginal cost can be rewritten as follows.

$$
\begin{aligned}
MC_{H,t} &= \frac{C_t N_t^{\varphi} Z_t}{D_t}\frac{P_t}{P_{P,t}A_{H,t}} \quad ; \quad MC_{N,t} = \frac{C_t N_t^{\varphi} Z_t}{D_t}\frac{P_t}{P_{P,t}A_{N,t}} \\
MC_{F,t} &= \frac{C^*_t (N^*_t)^{\varphi} Z_t}{D_t}\frac{P^*_t}{P^*_{P,t}A_{F,t}} \quad ; \quad MC^*_{N,t} = \frac{C^*_t (N^*_t)^{\varphi} Z_t}{D_t}\frac{P^*_t}{P_{P,t}A^*_{N,t}} \quad (12)
\end{aligned}
$$

2.3 Centralized Government

As mentioned above, we verify alternative policy regimes, i.e., an optimal monetary policy without a fiscal policy regime and an optimal monetary and fiscal policy mix regime. In the former case, in this union, no government absolutely defrays its fiscal deficit in the first place. This reflects an actual phenomenon that an excess of government expenditure beyond 3% of GDP is not allowed under the Maastricht Treaty, and this phenomenon is indicated by $G_t = G^*_t = 0$ for all t in our model, where G_t and G^*_t denote government expenditures on goods produced in countries H and F, respectively.[6] In the latter case, a centralized rather than decentralized government conducts fiscal policy as a policy authority as well as the central bank. We refer to this centralized government as merely "government" and only distinguish the two as the need arises.

The government expenditure index is given by the Dixit–Stiglitz type. For simplicity, we assume that government purchases are fully allocated to a domestically produced good. For any given level of public consumption, the government allocates expenditures across goods in order to minimize total cost. Thus, a set of government demand schedules is analogous to and associated with private consumption. Because our attention is focused on the determination of its aggregate level and its effects, we assume that government spending is entirely financed by means of lump sum taxes.

2.4 Market Clearing

The market in country H for tradables clears when domestic demand equals domestic supply as follows.

$$\begin{aligned} Y_{H,t}(i) &= C_{T,t}(h) + C^*_{T,t}(h) + G_t(i) \\ Y_{F,t}(i) &= C_{T,t}(f) + C^*_{T,t}(f) + G^*_t(i) \end{aligned} \tag{13}$$

where $Y_{H,t}(i)$ and $Y_{H,t}(i)$ denote the outputs of tradables produced by generic firms in countries H and F, respectively.

As for nontradables, equilibrium requires that:

$$\begin{aligned} Y_{N,t}(i) &= C_{N,t}(i) + G_t(i) \\ Y^*_{N,t}(i) &= C^*_{N,t}(i) + G^*_t(i) \end{aligned} \tag{14}$$

where $Y_{N,t}(i)$ and $Y^*_{N,t}$ denote the outputs by generic firms for domestic demand in countries H and F, respectively.

Let $Y_{H,t}$ and $Y_{F,t}$ denote Dixit–Stiglitz-type indices of the aggregate output of tradables produced in countries H and F, respectively. Combining this definition and Eqs.(4), (9) and (62), Eq.(13) can be rewritten as:

$$\begin{aligned} Y_{H,t} &= \frac{1}{2}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C_t \left[\left(\frac{P_{T,t}}{P_t}\right)^{-\eta} + \left(\frac{P_{T,t}}{P^*_t}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_t \\ Y_{F,t} &= \frac{1}{2}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C_t \left[\left(\frac{P_{T,t}}{P_t}\right)^{-\eta} + \left(\frac{P_{T,t}}{P^*_t}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G^*_t \end{aligned} \tag{15}$$

where we use the fact that $C^*_t = \frac{C_t}{\mathsf{Q}_t}$, which is derived from Eq.(9).

Eq.(15) implies that:

$$\frac{Y_{H,t} - G_t}{Y_{F,t} - G^*_t} = \mathsf{T}_t$$

where $\mathsf{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the terms of trade (TOT). Thus, the differential of output of tradables between country H and country F is equal to the TOT.

[6] To be exact, this phenomenon is indicated by $\frac{dG_t}{Y} = \frac{dG^*_t}{Y} = 0$ in our log-linearized model, where Y denotes a steady-state value of output.

Let $Y_{N,t} \equiv$ and $Y_{N,t}^*$ denote Dixit–Stiglitz-type indices of the aggregate output of tradables produced in countries H and F, respectively. Combining this definition and Eqs.(4), (9) and (62), Eq.(14) can be rewritten as follows.

$$\begin{aligned} Y_{N,t} &= \left(\frac{P_{N,t}}{P_t}\right)^{-\eta} C_t + G_t \\ Y_{N,t}^* &= \left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\eta} C_t \mathsf{Q}_t^{-1} + G_t^* \end{aligned} \tag{16}$$

Eq.(16) implies that:

$$\frac{Y_{N,t} - G_t}{Y_{N,t}^* - G_t^*} = \mathsf{N}_t^{\eta} \mathsf{Q}_t^{-(\eta-1)}$$

where $\mathsf{N}_t \equiv \frac{P_{N,t}^*}{P_{N,t}}$ denotes a nontradables price differential between countries H and F (NPD). Analogous to the differential of output of tradables, the differential of output of nontradables between the two countries is equal to the price differential of nontradables between them.

We define the aggregate domestic indices as:

$$\begin{aligned} Y_t &\equiv \left\{ \gamma^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} Y_{N,t}^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\ Y_t^* &\equiv \left\{ \gamma^{\frac{1}{\eta}} Y_{F,t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} \left(Y_{N,t}^*\right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \end{aligned} \tag{17}$$

analogous to that introduced for consumption index Eq.(2). Combining Eqs.(17) and (60), we have aggregate production functions as follows:

$$Y_t = \Gamma N_t \ ; \ Y_t^* = \Gamma N_t^* \tag{18}$$

where $\Gamma \equiv \left[\gamma^{\frac{1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ can be interpreted as a productivity amplifier associated with nontradables. When all goods are tradable, i.e. $\gamma = 1$, Eq.(18) reduces to $Y_t = N_t$ and $Y_t^* = N_t^*$, which are familiar expressions.

2.5 Current Account

Following Gali and Monacelli[8], the current accounts in the two countries deflated by the PPI are defined as follows:

$$\begin{aligned} CA_t &\equiv Y_t - \frac{P_t}{P_{P,t}} C_t - G_t \\ CA_t^* &\equiv Y_t^* - \frac{P_t^*}{P_{P,t}^*} C_t^* - G_t^* \end{aligned} \tag{19}$$

where CA_t and CA_t^* denote the current accounts in countries H and F, respectively.

3 Log-linearization of the Model

This section describes the stochastic equilibrium that arises from perturbations around the deterministic equilibrium.[7] Lowercase letters denote percentage de-

[7] See Appendix B for details about the nonstochastic steady state.

viations of steady-state values for respective uppercase letters when there is no note to the contrary, i.e., $v_t \equiv \frac{dV_t}{V}$, where V_t denotes the voluntary variable and V denotes the steady-state value of V_t. Lowercase letters accompanied with R as superscript mean logarithmic differential between the two countries for respective uppercase letters, i.e., $v_t^R \equiv v_t - v_t^*$. Lastly, Small letters accompanied with Δ mean changes in the large-letter variable, i.e., $\Delta v_t \equiv v_t - v_{t-1}$.

3.1 Aggregate Demand and Output

Log-linearizing Eqs.(7) and (9), we obtain the following:

$$\begin{aligned} c_t &= \mathrm{E}_t c_{t+1} - \hat{r}_t + \mathrm{E}_t \pi_{t+1} + d_t \\ c_t^R &= \mathsf{q}_t \end{aligned} \tag{20}$$

where c_t and c_t^* denote percentage deviations of consumption from its steady-state value in countries H and F, respectively, $\hat{r}_t \equiv \ln R_t$ denotes the logarithmic union-wide gross nominal interest rate, π_t denotes the CPI inflation rate in country H, q_t denotes the logarithmic CPI differential between the two countries, and d_t denotes a logarithmic consumption preference shifter. Notice that the second equality in Eq.(20) implies that the logarithmic consumption differential between the two countries depends on the logarithmic CPI differential.

Log-linearizing and manipulating Eqs.(5) and (6), we obtain:

$$\pi_t = \gamma \pi_{T,t} + (1-\gamma)\,\pi_{N,t+1} \quad ; \quad \pi_t^* = \gamma \pi_{T,t} + (1-\gamma)\,\pi_{N,t+1}^* \tag{21}$$

$$\pi_{P,t} = \gamma \pi_{H,t} + (1-\gamma)\,\pi_{N,t} \quad ; \quad \pi_{P,t}^* = \gamma \pi_{F,t} + (1-\gamma)\,\pi_{N,t}^* \tag{22}$$

with $\pi_{T,t} = \frac{1}{2}\pi_{H,t} + \frac{1}{2}\pi_{F,t}$, where $\pi_{T,t}$ denotes the TPI inflation rate, $\pi_{H,t}$ and $\pi_{F,t}$ denotes the inflation rate of tradables produced in countries H and F, respectively, $\pi_{N,t}$ and $\pi_{N,t}^*$ denote the inflation rates of nontradables produced in countries H and F, respectively, and $\pi_{P,t}$ and $\pi_{P,t}^*$ denote the PPI inflation rates in countries H and F, respectively.

Log-linearizing Eq.(17), we have:

$$\begin{aligned} y_t &= \gamma y_{H,t} + (1-\gamma)\,y_{N,t} \\ y_t^* &= \gamma y_{F,t} + (1-\gamma)\,y_{N,t}^* \end{aligned} \tag{23}$$

where $y_{H,t}$, $y_{F,t}$, $y_{N,t}$ and $y_{N,t}^*$ denote percentage deviations from the steady-state values of $Y_{H,t}$, $Y_{F,t}$, $Y_{N,t}$ and $Y_{N,t}^*$. Log-linearizing Eqs.(15) and (16) and plugging these equalities into Eq.(23), we have:

$$\begin{aligned} y_t &= c_t + \frac{\gamma}{2}\mathsf{t}_t + \frac{\psi}{2}\mathsf{n}_t + \hat{g}_t \\ y_t^* &= c_t^* - \frac{\gamma}{2}\mathsf{t}_t - \frac{\psi}{2}\mathsf{n}_t + \hat{g}_t^* \end{aligned} \tag{24}$$

with $\psi \equiv (1-\gamma)\,\gamma\,(\eta-1)$, where t_t denotes logarithmic TOT, n_t denotes logarithmic NPD, and $\hat{g}_t \equiv \frac{dG_t}{Y}$ and $\hat{g}_t^* \equiv \frac{dG_t^*}{Y^*}$ denote percentage deviations of government spending from steady-state output levels in countries H and F, respectively,

Eq.(24) implies that:

$$y_t^R = \gamma \mathsf{t}_t + (1-\gamma)\,\varpi \mathsf{n}_t + \hat{g}_t^R \tag{25}$$

with y_t^R being the output differential between the two countries, $\hat{g}_t^R$ being the government expenditure differential between them and $\varpi \equiv 1 + (\eta - 1)\gamma$. Because of existing nontradables, the output differential between the two countries depends not only on the TOT but also on the NPD. When $\gamma = 1$, implying that there are no nontradables, this equality is reduced to $y_t^R = \mathsf{t}_t + g_t^R$, which is familiar from many NOEM studies. This equality shows that an increase of domestic nontradables price diminishes domestic output when we ignore the effect of η.

Using the definition of the TOT and Eqs.(21) and (22), we have:

$$\Delta \mathsf{t}_t = -\frac{1}{\gamma}\pi_{P,t}^R - \frac{1-\gamma}{\gamma}\Delta \mathsf{n}_t \tag{26}$$

with $\pi_{P,t}^R$ being the PPI inflation differential between the two countries. When $\gamma = 1$, Eq.(29) is reduced to $\Delta \mathsf{t}_t = -\pi_{P,t}^R$, which implies that the TOT depreciation has an exact relationship with the PPI inflation differential.

Plugging Eqs.(29) and (24) into Eq.(20), we have aggregate demand curves so-called new Keynesian IS curve (NKIS) as follows:

$$\begin{aligned} y_t &= \mathrm{E}_t y_{t+1} - \hat{r}_t + \mathrm{E}_t \pi_{P,t+1} - \frac{\psi}{2}\Delta \mathrm{E}_t \mathsf{n}_{t+1} - \Delta \mathrm{E}_t \hat{g}_{t+1} + d_t \\ y_t^* &= \mathrm{E}_t y_{t+1}^* - \hat{r}_t + \mathrm{E}_t \pi_{P,t+1}^* + \frac{\psi}{2}\Delta \mathrm{E}_t \mathsf{n}_{t+1} - \Delta \mathrm{E}_t \hat{g}_{t+1}^* + d_t \end{aligned} \tag{27}$$

with $\Delta \mathsf{n}_t$, $\Delta \hat{g}_t$ and $\Delta \hat{g}_t^*$ denoting percentage change of n_t, $\hat{g}_t$ and $\hat{g}_t^*$, respectively.[8] When all goods are tradable, i.e. $\gamma = 1$, Eq.(27) is reduced to:

$$\begin{aligned} y_t &= \mathrm{E}_t y_{t+1} - \hat{r}_t + \mathrm{E}_t \pi_{P,t+1} - \Delta \mathrm{E}_t \hat{g}_{t+1} + d_t \\ y_t^* &= \mathrm{E}_t y_{t+1}^* - \hat{r}_t + \mathrm{E}_t \pi_{P,t+1}^* - \Delta \mathrm{E}_t \hat{g}_{t+1}^* + d_t. \end{aligned}$$

These equalities show no expected percentage change of the NPD in the NKISs and are familiar expressions when all goods are tradable.

Now, we refer to government expenditure constraints according to the Maastricht treaty. At least, our currency union has the government expenditure constraint as follows:

$$\hat{g}_t^W = 0 \tag{28}$$

for all t with $g_t^W \equiv \frac{1}{2}g_t + \frac{1}{2}g_t^*$ denoting union-wide public expenditure. Eq.(28) implies that there are no union-wide government surpluses or deficits. In fact, the Maastricht treaty does not allow fiscal deficits beyond 3% of GDP in each country. This phenomenon allows us to say that the government expenditure constraint is not only Eq.(28) but the following equality:

$$\hat{g}_t^R = 0 \tag{29}$$

which implies that the government expenditure differential is zero. Thus, when both Eqs.(28) and (29) are imposed, government expenditure is zero in each country. In a later section, we analyze monetary and fiscal policy under alternative government expenditure constraints.

[8] Eq.(29) can be rewritten as $\pi_t - \frac{\gamma}{2}\Delta \mathsf{t}_t = \pi_{P,t}$ or $\pi_t^* + \frac{\gamma}{2}\Delta \mathsf{t}_t = \pi_{P,t}^*$, which implies that there is no difference between CPI and PPI when there are no tradables, i.e. $\gamma = 0$.

3.2 Aggregate Supply and Inflation

Log-linearizing Eq.(11) and rearranging, we can describe the dynamics of inflation in terms of marginal cost as follows:

$$\begin{aligned}
\pi_{H,t} &= \delta \mathrm{E}_t \pi_{H,t+1} + \lambda (1-\gamma) p_{N,t} - \lambda (1-\gamma) p_{H,t} + \lambda mc_{H,t} \\
\pi_{N,t} &= \delta \mathrm{E}_t \pi_{N,t+1} - \lambda \gamma p_{N,t} + \lambda \gamma p_{H,t} + \lambda mc_{N,t} \\
\pi_{F,t} &= \delta \mathrm{E}_t \pi_{F,t+1} + \lambda (1-\gamma) p^*_{N,t} - \lambda (1-\gamma) p_{F,t} + \lambda mc_{F,t} \\
\pi^*_{N,t} &= \delta \mathrm{E}_t \pi^*_{N,t+1} - \lambda \gamma p^*_{N,t} + \lambda \gamma p_{F,t} + \lambda mc^*_{N,t}
\end{aligned} \tag{30}$$

with $\lambda \equiv \frac{(1-\alpha)(1-\alpha\delta)}{\alpha}$, where $mc_{H,t}$, $mc_{N,t}$, $mc_{F,t}$ and $mc^*_{N,t}$ denote percentage deviations of real marginal costs from their steady-state values associated with tradables produced in country H, nontradables produced in country H, tradables produced in country F and nontradables produced in country F respectively.

Plugging Eq.(30) into Eq.(22), we have PPI-based inflation dynamics equations in two countries as follows:

$$\begin{aligned}
\pi_{P,t} &= \delta \mathrm{E}_t \pi_{P,t+1} + \lambda mc_t \\
\pi^*_{P,t} &= \delta \mathrm{E}_t \pi^*_{P,t+1} + \lambda mc^*_t
\end{aligned} \tag{31}$$

where:

$$\begin{aligned}
mc_t &\equiv \gamma mc_{H,t} + (1-\gamma) mc_{N,t} \\
mc^*_t &\equiv \gamma mc_{F,t} + (1-\gamma) mc^*_{N,t}
\end{aligned} \tag{32}$$

where mc_t denotes logarithmic domestic marginal cost in country H.

Combining the second and fourth equalities of Eq.(30), the nontradables inflation differential is given by:

$$\pi^R_{N,t} = \delta \mathrm{E}_t \pi^R_{N,t+1} + \lambda \gamma \mathsf{n}_t - \lambda \gamma \mathsf{t}_t + \lambda mc^R_{N,t} \tag{33}$$

where $\pi^R_{N,t} \equiv \pi_{N,t} - \pi^*_{N,t}$ denotes a nontradables price inflation differential and $mc^R_{N,t} \equiv mc_{N,t} - mc^*_{N,t}$ denotes a logarithmic marginal cost differential associated with nontradables. Eq.(33), which is a sort of NKPC that at first glance evolves into this version of a Balassa–Samuelson theorem equality and can be named the New Keynesian Balassa–Samuelson theorem equation (NKBS). Our model is a closed system while a two-country economy is assumed; however, alike the Balassa–Samuelson theorem, Eq.(33) explains the CPI disparity between the two countries although the Balassa–Samuelson theorem address the problem of why the nominal exchange rate deviates from purchasing power parity in the canonical international money and finance literature. Details on Eq.(33) are mentioned in a later section.

Log-linearizing Eq.(18), we have:

$$y_t = n_t \ ; \ y^*_t = n_t \tag{34}$$

with n_t and n^*_t being percentage deviations of hours of work from their steady-state values in countries H and F, respectively, and z_t being the union-wide logarithmic labor effort disutility shifter, in which we use the fact that $\frac{\Gamma N}{Y} = 1$

to simplify without loss of generality, with $\frac{N}{Y}$ being steady-state hours of labor to produce.

Combining log-linearized Eq.(8), Eqs.(24) and (34), we have:

$$\begin{aligned} mc_{H,t} &= (1+\varphi)\, y_t - \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t - a_{H,t} + z_t - d_t \\ mc_{N,t} &= (1+\varphi)\, y_t - \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t - a_{N,t} + z_t - d_t \\ mc_{F,t} &= (1+\varphi)\, y_t^* + \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t^* - a_{F,t} + z_t - d_t \\ mc_{N,t}^* &= (1+\varphi)\, y_t^* + \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t^* - a_{N,t}^* + z_t - d_t \end{aligned} \tag{35}$$

where $a_{H,t}$, $a_{N,t}$, $a_{F,t}$, and $a_{N,t}^*$ denote logarithmic productivity shifters associated with tradables produced in country H, nontradables produced in country H, tradables produced in country F and nontradables produced in country F, respectively. Eq.(35) implies that marginal cost depends not only on domestic output but also on the NPD.

Using Eq.(32), Eq.(35) can be rewritten as follows.

$$\begin{aligned} mc_t &= (1+\varphi)\, y_t - \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t - \gamma a_{H,t} - (1-\gamma)\, a_{N,t} + z_t - d_t \\ mc_t^* &= (1+\varphi)\, y_t^* + \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t^* - \gamma a_{F,t} - (1-\gamma)\, a_{N,t}^* + z_t - d_t \end{aligned} \tag{36}$$

Eq.(36) implies that domestic marginal cost depends on the DPN. Needless to say, when $\gamma = 1$, the first equality of Eq.(36) reduces to:

$$\begin{aligned} mc_t &= (1+\varphi)\, y_t - \hat{g}_t - a_{H,t} + z_t - d_t \\ mc_t^* &= (1+\varphi)\, y_t^* - \hat{g}_t^* - a_{F,t} + z_t - d_t \end{aligned}$$

because of $\psi = 0$ when $\gamma = 1$. These equalities are familiar expressions in DSGE applied to NOEM literature.

Combining the second and last equalities in Eq.(35), the logarithmic marginal cost differential associated with nontradables is given by:

$$mc_{N,t}^R = (1+\varphi)\, y_t^R - \psi\mathsf{n}_t - \hat{g}_t^R - a_{N,t} + a_{N,t}^*. \tag{37}$$

3.3 Dynamics of Relative Price and Current Account

Log-linearizing Eq.(5) and rearranging yields:

$$\mathsf{q}_t = (1-\gamma)\, \mathsf{n}_t \tag{38}$$

with $\mathsf{n}_t \equiv \ln \mathsf{N}_t$ being logarithmic NPD. It is clear by paying attention to Eqs.(20) and (38) that the logarithmic consumption differential depends not only on logarithmic CPI differential but also on logarithmic NPD. When $\gamma = 1$, Eq.(38) is altered as $\mathsf{q}_t = 0$ implying that the CPI between the two countries has an identity. In ordinary international finance literature, this means that purchasing power parity holds.

Combining Eqs.(20), (21) and (38), and rearranging, we have:

$$\Delta \mathrm{E}_t \mathsf{n}_{t+1} = \frac{1}{1-\gamma} \Delta \mathrm{E}_t c_{t+1}^R$$

where $\Delta c_t^R \equiv c_t^R - c_{t-1}^R$ denotes percentage changes in the consumption differential. This equality implies that expected changes in the NPD are exactly related to expected changes in logarithmic consumption differential between the two countries. Combining this equality and Eq.(23), we obtain:

$$\Delta \mathrm{E}_t \mathsf{n}_{t+1} = \frac{1}{\psi} \mathrm{E}_t \pi_{P,t+1}^R + \frac{1}{\psi} \Delta \mathrm{E}_t y_{t+1}^R - \frac{1}{\psi} \Delta \mathrm{E}_t \hat{g}_{t+1}^R \tag{39}$$

where $\Delta y_t^R \equiv y_t^R - y_{t-1}^R$ and $\Delta \hat{g}_t^R \equiv \hat{g}_t^R - \hat{g}_{t-1}^R$ denote percentage changes in logarithmic output differential and the ratio of government expenditure ratio to steady-state output.

Using the definition of the NPD and the inflation rate of nontradables, expected changes in the NPD can be written as:

$$\Delta \mathrm{E}_t \mathsf{n}_{t+1} = -\mathrm{E}_t \pi_{N,t+1}^R \tag{40}$$

with $\pi_{N,t}^R \equiv \pi_{N,t}^* - \pi_{N,t}$ being the nontradables inflation differential between the two countries.

There is some relationship between the NPD and current account. Loglinearizing Eq.(19), we have:

$$\begin{aligned} \widehat{ca}_t &= y_t - c_t - \hat{g}_t - \frac{\gamma}{2} \mathsf{t}_t \\ \widehat{ca}_t^* &= y_t^* - c_t^* - \hat{g}_t^* + \frac{\gamma}{2} \mathsf{t}_t \end{aligned}$$

which implies that $\widehat{ca}_t = -\widehat{ca}_t^*$ because of the union-wide market clearing condition and Eq.(28). Using Eq.(24), these equalities can be reduced to:

$$\widehat{ca}_t = \frac{\psi}{2} \mathsf{n}_t. \tag{41}$$

Eq.(41) implies that a relative increase of nontradables price in country F brings about a current account surplus in country H and vice versa. When the nontradables price increases relative to tradables, demand for tradables increases while that for nontradables decreases. Hence, when the nontradables price in country F increases relative to that in country H, demand for tradable goods in country F, including tradables produced in country H, increases, and the current account in country H goes into the black. When $\gamma = 1$, implying that all goods are tradable, Eq.(41) is reduced to $\widehat{ca}_t = 0$ implying balanced trade. In this model, the degree of relative risk aversion and the elasticity of substitution between tradables produced in countries H and F are implicitly assumed to be unity. These assumptions are adopted in Gali and Monacelli[8]: assuming that all goods are tradable they showed that balanced trade is achieved under such parameter constraints. Thus, our argument is consistent with Gali and Monacelli[8].

There is another case that $\widehat{ca}_t = 0$ holds: when $\eta = 1$ implying the elasticity of substitution between tradables and nontradables is unity, namely, both tradables and nontradables are a perfect substitution.

3.4 Marginal Cost and Output gap

In this section, we show that the linearized equilibrium dynamics have a representation in terms of an output gap. That representation has provided a basis for the analysis and evaluation of alternative policy regimes in much of the DSGE and NOEM literature. Following Gali and Monacelli[8], we define the relationship between output, its natural level and its gap as follows:

$$\begin{aligned} y_t &\equiv \bar{y}_t + \tilde{y}_t \\ y_t^* &\equiv \bar{y}_t^* + \tilde{y}_t^* \end{aligned}$$

where $\tilde{y}_t$ denotes logarithmic output gap at its natural level, and $\bar{y}_t$ denotes logarithmic natural level output. Under flexible price, $\tilde{y}_t = \tilde{y}_t^* = 0$ must hold.

When fiscal authorities design their policies to dissolve distortion generated by monopolistically competitive markets, real marginal costs under flexible price equilibrium are unity, and their logarithm is given by:

$$mc_t = mc_t^* = 0.$$

Also, under flexible price equilibrium, all relative prices are unity. Thus, logarithmic NPD under flexible price equilibrium is given by:

$$\mathsf{n}_t = 0.$$

Combining these facts, Eq.(36) implies that:

$$\begin{aligned} \bar{y}_t &= \varphi_\omega \hat{g}_t + \varphi_\tau a_{H,t} + \varphi_\nu a_{N,t} - \varphi_\omega z_t + \varphi_\omega d_t \\ \bar{y}_t^* &= \varphi_\omega \hat{g}_t^* + \varphi_\tau a_{F,t} + \varphi_\nu a_{N,t}^* - \varphi_\omega z_t + \varphi_\omega d_t \end{aligned} \tag{42}$$

with $\varphi_\tau \equiv \frac{\gamma}{1+\varphi}$, $\varphi_\nu \equiv \frac{1-\gamma}{1+\varphi}$ and $\varphi_\omega \equiv \frac{1}{1+\varphi}$. Eq.(42) implies that the natural level of output consists of productivity, consumption disparity and government spending.

Using Eq.(42), the log-linear approximated model can be rewritten in terms of output gap. Eq.(27) can be rewritten as:

$$\begin{aligned} \tilde{y}_t &= \mathrm{E}_t \tilde{y}_{t+1} - \hat{r}_t + \mathrm{E}_t \pi_{P,t+1} - \frac{\psi}{2} \Delta \mathrm{E}_t \mathsf{n}_{t+1} - \varphi_\varphi \Delta \mathrm{E}_t \hat{g}_{t+1} - \varphi_\tau a_{H,t} - \varphi_\nu a_{N,t} \\ &\quad + \varphi_\omega z_t + \varphi_\varphi d_t \\ \tilde{y}_t^* &= \mathrm{E}_t \tilde{y}_{t+1}^* - \hat{r}_t + \mathrm{E}_t \pi_{P,t+1}^* + \frac{\psi}{2} \Delta \mathrm{E}_t \mathsf{n}_{t+1} - \varphi_\varphi \Delta \mathrm{E}_t \hat{g}_{t+1}^* - \varphi_\tau a_{F,t} - \varphi_\nu a_{N,t}^* \\ &\quad + \varphi_\omega z_t + \varphi_\varphi d_t \end{aligned} \tag{43}$$

with $\varphi_\varphi \equiv \frac{\varphi}{1+\varphi}$.

NKPCs in terms of output gap are given by:

$$\begin{aligned} \pi_{P,t} &= \delta \mathrm{E}_t \pi_{P,t+1} + \lambda_\varphi \tilde{y}_t - \frac{\psi\lambda}{2} \mathsf{n}_t \\ \pi_{P,t}^* &= \delta \mathrm{E}_t \pi_{P,t+1}^* + \lambda_\varphi \tilde{y}_t^* + \frac{\psi\lambda}{2} \mathsf{n}_t \end{aligned} \tag{44}$$

with $\lambda_\varphi \equiv (1+\varphi)\lambda$. These expressions become familiar when $\gamma = 1$. In this case, Eq.(44) can be rewritten as:

$$\begin{aligned} \pi_{P,t} &= \delta \mathrm{E}_t \pi_{P,t+1} + \lambda_\varphi \tilde{y}_t \\ \pi^*_{P,t} &= \delta \mathrm{E}_t \pi^*_{P,t+1} + \lambda_\varphi \tilde{y}^*_t \end{aligned}$$

which are derived by Gali and Monacelli[7], who insist that inflation–output trade-offs can be dissolved simultaneously in a small open economy under severe deep parameter restrictions by inflation targeting. Indeed, when inflation targeting such as $\pi_{P,t} = \pi^*_{P,t} = 0$ for all t is introduced in our currency union with special restrictions i.e. $\gamma = 1$ and $g_t = g^*_t = 0$, these equalities imply that $\tilde{y}_t = \tilde{y}^*_t = 0$ for all t implying that output gap is dissolved.

Similar to NKPCs, we derive NKISs in terms of output gap. Because Eq.(39) can be rewritten as:

$$\begin{aligned} \Delta \mathrm{E}_t \mathsf{n}_{t+1} &= \frac{1}{\psi}\mathrm{E}_t \pi^R_{P,t+1} + \frac{1}{\psi}\Delta \mathrm{E}_t \tilde{y}^R_{t+1} - \frac{\varphi_\varphi}{\psi}\Delta \mathrm{E}_t \hat{g}^R_{t+1} - \frac{\varphi_\tau}{\psi} a_{H,t} + \frac{\varphi_\tau}{\psi} a_{F,t} \\ &\quad - \frac{\varphi_\nu}{\psi} a_{N,t} + \frac{\varphi_\nu}{\psi} a^*_{N,t}. \end{aligned} \tag{45}$$

NKISs are altered as:

$$\begin{aligned} \tilde{y}_t &= \mathrm{E}_t \tilde{y}_{t+1} - 2\hat{r}_t + \mathrm{E}_t \pi_{P,t+1} + \mathrm{E}_t \pi^*_{P,t+1} + \Delta \mathrm{E}_t \tilde{y}^*_{t+1} + \bar{r}_t \\ \tilde{y}^*_t &= \mathrm{E}_t \tilde{y}^*_{t+1} - 2\hat{r}_t + \mathrm{E}_t \pi^*_{P,t+1} + \mathrm{E}_t \pi_{P,t+1} + \Delta \mathrm{E}_t \tilde{y}_{t+1} + \bar{r}_t \end{aligned} \tag{46}$$

by plugging Eq.(45) into Eq.(43) with $\bar{r}_t \equiv -\varphi_\tau a_{H,t} - \varphi_\tau a_{F,t} - \varphi_\nu a_{N,t} - \varphi_\nu a^*_{N,t} + 2\varphi_\omega z_t + 2\varphi_\varphi d_t$ denoting a version of real natural interest rate. Eq.(46) implies that under optimal risk sharing, Eq.(9) or the second equality of (20), NKISs in two countries are homogeneous because the first and second equalities of Eq.(46) are identical.

3.5 Canonical Balassa–Samuelson Theorem and NKBS

As mentioned in the former subsection, we now turn to the relationship between the canonical Balassa–Samuelson theorem and the NKBS. Using Eq.(42), NKBS Eq.(33) can be rewritten as:

$$\begin{aligned} \pi^R_{N,t} &= \delta \mathrm{E}_t \pi^R_{N,t+1} + \lambda\varphi \tilde{y}^R_t + \lambda \mathsf{n}_t + \varphi_\varphi \lambda \hat{g}^R_t + \varphi_\varphi \lambda\gamma a_{H,t} - \varphi_\varphi \lambda\gamma a_{F,t} - \varphi_\sigma \lambda a_{N,t} \\ &\quad + \varphi_\sigma \lambda a^*_{N,t} \end{aligned} \tag{47}$$

with $\varphi_\sigma \equiv \frac{1+\varphi\gamma}{1+\varphi}$. Now we refer to Eq.(47) as NKBS. Using Eq.(38), Eq.(47) can be rewritten as follows.

$$\begin{aligned} \mathsf{q}_t &= \frac{1-\gamma}{\lambda}\pi^R_{N,t} - \frac{(1-\gamma)\,\delta}{\lambda}\mathrm{E}_t \pi^R_{N,t+1} - (1-\gamma)\,\varphi \tilde{y}^R_t - (1-\gamma)\,\varphi_\varphi \hat{g}^R_t \\ &\quad - (1-\gamma)\,\varphi_\varphi \gamma a_{H,t} + (1-\gamma)\,\varphi_\varphi \gamma a_{F,t} + (1-\gamma)\,\varphi_\sigma a_{N,t} - (1-\gamma)\,\varphi_\sigma a^*_{N,t} \end{aligned} \tag{48}$$

When the currency union has no nontradables, i.e., as $\gamma = 1$, Eq.(48) implies that the CPI disparity is dissolved between the two countries, namely, $\mathsf{q}_t = 0$ holds. A problem with the CPI disparity is resolved since each country has the same CPI. This implies that purchasing power parity holds under an ordinary open economy model. Even the fact that $\mathsf{q}_t = 0$ holds when $\gamma = 1$ can be depicted; however, a character of Eq.(48) as canonical Balassa–Samuelson theorem in international money and finance literature is obscure. This stems from the fact that Eq.(48) is a dynamic equation, as in the New Keynesian literature, that has rightfully assumed nominal rigidities. To easily understand this character, we inspect Eq.(48) in a flexible price equilibrium. Under a flexible price equilibrium, Eq.(48) can be rewritten as:

$$\begin{aligned}\mathsf{q}_t &= -(1-\gamma)\,\varphi_\varphi \hat{g}_t^R - (1-\gamma)\,\varphi_\varphi \gamma a_{H,t} + (1-\gamma)\,\varphi_\varphi \gamma a_{F,t} + (1-\gamma)\,\varphi_\sigma a_{N,t} \\ &\quad - (1-\gamma)\,\varphi_\sigma a_{N,t}^*\end{aligned}$$

because $\alpha = 0$ and $\tilde{y}_t = \tilde{y}_t^* = 0$ holds. By neglecting the first and last terms in the RHS in this equality, the CPI disparity, which can be called the "real exchange rate" when a nominal exchange rate exists, is determined by productivity shifters in a currency union. In this equality, increasing productivity of tradables produced in country H causes a decrease in the CPI disparity q_t. As the canonical Balassa–Samuelson theorem explains, a rise in productivity of the tradables sector in the home country causes a decreasing real exchange rate through an increase in nontradables prices in the home country, which stems from an increase in wages not only in the tradables but also in the nontradables sector because of perfect labor mobility between each sector.[9] This equality, the flexible price version of NKBS, can explain a decrease in the CPI differential stemming from an increase in productivity of tradables produced in country H. Thus, Eq.(48) and similar equalities can be called an NKBS. Existing nontradables that cause disparities in the CPI and consumption is the principal friction taking rank with nominal rigidities in our currency union model.

4 Monetary and Fiscal Policy

In the present section, we analyze the macroeconomic implications of an alternative policy regime for the currency union: an optimal monetary policy without fiscal policy regime and an optimal monetary and fiscal policy mix regime. Under an optimal monetary policy without fiscal policy regime, because Eqs.(28) and (29) are imposed as government expenditure, government expenditure in each country is given by:

$$\hat{g}_t = \hat{g}_t^* = 0 \tag{49}$$

which implies that government expenditure in each country is zero. This reflects the actual Maastricht treaty.

Under an optimal monetary and fiscal policy mix regime, only Eq.(28) is available as a government expenditure constraint. Government expenditure constraint under this regime can be written as:

$$\hat{g}_t = -\hat{g}_t^* \tag{50}$$

which represents an imaginary Maastricht treaty which is relaxed. Under this regime, government expenditure is allowed while each government keeps zero union-wide government expenditure.

[9] Labor mobility is not allowed between countries H and F while perfect labor mobility between sectors tradables and nontradables in each country.

4.1 Optimal Monetary Policy without Fiscal Policy

Under an optimal monetary policy without fiscal policy regime, only the central bank takes part as the authority because of Eq.(49). The central bank seeks to minimize the social loss function subject to our structural model.[10] The period loss function is derived by second-order Taylor approximated Eq.(1), which is given by:

$$U_t^W = -\frac{1}{4}\sum_{t=0}^{\infty}\delta^t\left\{\frac{\theta}{\lambda}\pi_{P,t}^2 + (1+\varphi)\,\tilde{y}_t^2 + \frac{\theta}{\lambda}\left(\pi_{P,t}^*\right)^2 + (1+\varphi)\left(\tilde{y}_t^*\right)^2\right\} + \text{t.i.p.} + o\left(\|\xi\|^3\right) \quad (51)$$

with $U_t^W \equiv U_t + U_t^*$ being union-wide utility, t.i.p. denoting the terms independent policy, and $o\left(\|\xi\|^3\right)$ denoting terms that are higher than third order. Hence, the period loss function is given by:

$$L_t = \frac{1}{4}\left\{\frac{\theta}{\lambda}\pi_{P,t}^2 + (1+\varphi)\,\tilde{y}_t^2 + \frac{\theta}{\lambda}\left(\pi_{P,t}^*\right)^2 + (1+\varphi)\left(\tilde{y}_t^*\right)^2\right\} \quad (52)$$

where L_t denotes the period loss function.[11] Using the FONC of the Lagrangian, which consists of Eq.(52) and our structural model, we obtain the optimal monetary policy rule as follows:

$$\hat{r}_t = \frac{1}{2}\bar{r}_t + \frac{\phi}{2}\pi_{P,t} + \frac{\phi}{2}\pi_{P,t}^* \quad (53)$$

or:

$$\hat{r}_t = \frac{1}{2}\bar{r}_t + \phi\pi_t^W$$

with $\phi \equiv \theta$ being the Taylor principle.[12] Gali and Monacelli[8] analyzed the subject using a policy rule similar to Eq.(53), whereas they did not derive an interest rate policy rule from the optimization problem.[13] As mentioned above (Gali and Monacelli[7] and Gali and Monacelli[8]), this policy rule implies a union-wide inflation targeting policy.

We can investigate the features of optimal monetary policy without fiscal policy by observing a structural model. Paying attention to real natural interest $\bar{r}_t$, which is common to the two countries, it is clear that all shifters affect the output gap in the same direction. Plugging Eq.(53) into Eq.(46), NKISs are altered as:

$$\begin{aligned}
\tilde{y}_t &= \mathrm{E}_t\tilde{y}_{t+1} - \phi\pi_{P,t} - \phi\pi_{P,t}^* + \mathrm{E}_t\pi_{P,t+1} + \mathrm{E}_t\pi_{P,t+1}^* + \Delta\mathrm{E}_t\tilde{y}_{t+1}^* \\
\tilde{y}_t^* &= \mathrm{E}_t\tilde{y}_{t+1}^* - \phi\pi_{P,t} - \phi\pi_{P,t}^* + \mathrm{E}_t\pi_{P,t+1}^* + \mathrm{E}_t\pi_{P,t+1} + \Delta\mathrm{E}_t\tilde{y}_{t+1}
\end{aligned}$$

implying that optimal monetary policy insulates output gap from any shifters without preference differential shifter d_t^R on the demand side because when the central bank's interest rate rule is Eq.(53), the central bank seeks to make the nominal interest rate identical with the real natural rate.

[10] Our structural model consists of Eqs.(40), (44), (47), and (46).

[11] See Appendix E for details on deriving Eq.(52).

[12] Derivation of Eq.(53) is shown in Appendix E.

[13] To contrast Gali and Monacelli[8] in our study, we venture to assume that the authority conducts its policy discretionarily.

Next, we inspect the supply side. Combining the first and second equalities in Eq.(44), we have:

$$\pi_t^W = \delta \mathrm{E}_t \pi_{P,t+1}^W + \lambda_\varphi \tilde{y}_t^W.$$

This equality implies that the union-wide output gap is always zero when the central bank conducts monetary policy, following the optimal policy rule such as Eq.(53) because the central bank seeks to stabilize perfectly the union-wide inflation rate. For instance, we suppose that $\pi_t^W = 0$ for all t is realized under the optimal policy rule. Paying attention to this equality, this means that $\tilde{y}_t^W = 0$ for all t. The same finding is reported by Gali and Monacelli[8] and Benigno[4]. The fact that inflation–output trade-offs can be resolved simultaneously holds not only at the union-wide level but also at the country level when there are no nontradables. Under Eq.(53), the PPI inflation rate is also stabilized similarly to the union-wide inflation rate. This implies that the economy does not have the disparity of the PPI inflation rate, i.e., $\pi_t^R = 0$ for all t. Subtracting the first equality from the second equality in Eq.(44), we have:

$$\pi_{P,t}^R = \delta \mathrm{E}_t \pi_{P,t+1}^R + \lambda_\varphi \tilde{y}_t^R$$

where we assume that there are no nontradables, i.e., $\gamma = 1$. When $\pi_{P,t}^R = 0$ for all t, that $\tilde{y}_t^R = 0$ for all t is guessed by this equality. Not only $\pi_t^W = 0$ but also $\pi_t^R = 0$ for all t implies that $\pi_{P,t} = \pi_{P,t}^* = 0$ for all t. Hence, under optimal monetary policy, the PPI inflation rates in countries H and F are fully stabilized, while the output gaps in countries H and F are fully stabilized, i.e., $\tilde{y}_t = \tilde{y}_t^* = 0$. The same is stated by Benigno[4].[14]

However, when nontradables exist in the economy, the state of affairs brought about by optimal monetary policy is altered. While union-wide NKPC is not affected by the share of tradables, NKPC in terms of the PPI inflation rate differential between the two countries is affected by the share of tradables. Without a restriction $\gamma = 1$ implying that there are no nontradable goods, NKPC in terms of PPI inflation rate differential is given by:

$$\pi_{P,t}^R = \delta \mathrm{E}_t \pi_{P,t+1}^R + \lambda_\varphi \tilde{y}_t^R - \psi \lambda \mathsf{n}_t.$$

In this case, although optimal monetary policy is adopted, trade-offs between inflation rate and output gap in countries H and F arise.[15] When does the NPD fluctuate? Rearranging Eq.(47), we obtain:

$$\begin{aligned} \mathsf{n}_t \;\; = \;\; & \frac{1}{\lambda}\pi_{N,t}^R - \frac{\delta}{\lambda}\mathrm{E}_t \pi_{N,t+1}^R - \varphi \tilde{y}_t^R - \varphi_\varphi \hat{g}_t^R - \varphi_\varphi \gamma a_{H,t} \\ & + \varphi_\varphi \gamma a_{F,t} + \varphi_\sigma a_{N,t} - \varphi_\sigma a_{N,t}^* \end{aligned} \tag{54}$$

implying that when any shifters without union-wide preference result, the NPD changes. This change affects NKPCs unless $\gamma = 1$, which coincides with $\psi = 0$. Thus, inflation–output trade-offs cannot be dissolved unless $\gamma = 1$ under optimal monetary policy. This can be explained by fluctuation of the CPI disparity. Using Eq.(38), NKPCs are given by:

$$\pi_{P,t}^R = \delta \mathrm{E}_t \pi_{P,t+1}^R + \lambda_\varphi \tilde{y}_t^R - \gamma(\eta - 1)\lambda \mathsf{q}_t.$$

[14] Benigno[4] gives an account of this under the assumption that the degrees of price stickiness are equivalent in the two countries.

[15] $\psi = 0$ holds only if $\gamma = 1$.

Unless $\gamma = 1$, $\mathsf{q}_t = 0$ does not hold because of NKBS Eq.(48). When union-wide economy produces an ounce of nontradables, changes in productivity result in a CPI disparity between two countries. The CPI disparity expands the disparity of the output gap between the two countries. Although union-wide inflation, output gap and the PPI inflation in each country are stabilized by the optimal monetary policy, the output gap in each country cannot be stabilized by the policy when there are nontradables.

Gali and Monacelli[7] reach a similar conclusion to ours with regard to optimal monetary policy in the currency union that consists of innumerable small open economies. In such a currency union, optimal monetary policy can stabilize union-wide inflation and output gap whereas the PPI inflation and output gap are not stabilized because the small open economy has peculiar CPI different from the union-wide CPI. This difference stems from the fact that the scale of the small open economy is infinitesimal. Our model, however, does not assume small open economies but two countries' economies and allows nontradables. Existing nontradables necessarily result in a disparity of CPIs between the two countries. On that point, we can double-check an implication derived by Gali and Monacelli[7] using a two-country economic model with nontradables.

We refer to another case in which changes in NPD do not affect NKPCs, namely, optimal monetary policy can dissolve inflation–output trade-offs. When $\eta = 1$ implying perfect substitution between tradables and nontradables, $\psi = 0$ holds. In this case, changes in any shifters without union-wide preference shifters do not affect NKPCs through NKBS although nontradables exist. Thus, it can be said that elasticity of substitution between tradable and nontradables η is related to the Balassa–Samuelson theorem in the same way as γ.

4.2 Optimal Monetary and Fiscal Policy Mix

Under a currency union with nontradables, mere monetary policy cannot stabilize both the PPI inflation and the output gap at the individual country level. Now, we abandon Eq.(29) as a constraint but allow country-level government expenditure under Eq.(28). In this optimal monetary and fiscal policy mix regime, not only the central bank but also the central government seeks to minimize Eq.(52) subject to the structural model. The optimal fiscal policy rule derived by the FONCs of Lagrangian is given by:

$$\hat{g}_t^R = \bar{g}_t^R + \theta_\sigma \pi_{P,t}^R + \frac{1}{\varphi\lambda}\pi_{N,t}^R - \frac{\lambda+\delta}{\varphi_\varphi\lambda}\mathsf{n}_t \tag{55}$$

with $\theta_\sigma \equiv (1+\varphi)\,\theta$ where $\bar{g}_t^R \equiv -\gamma a_{H,t} + \gamma a_{F,t} + \frac{1+\varphi\gamma}{\varphi} a_{N,t} - \frac{1+\varphi\gamma}{\varphi} a_{N,t}^*$ addresses real natural public expenditure disparity or real natural transfers of income. In contradistinction to the optimal monetary policy rule, Eq.(53), Eq.(55) consists of relative variables between the two countries. Plugging Eq.(55) into Eq.(47) yields:

$$\mathsf{n}_t = \mathrm{E}_t \pi_{N,t+1}^R + \frac{\lambda\varphi}{\delta}\tilde{y}_t^R + \frac{\lambda\varphi\theta}{\delta}\pi_{P,t}^R.$$

By contradistinction between this equality and Eq.(54), it is clarified that no productivity shifter can affect the NPD as long as the central government conducts optimal fiscal policy. Moreover, optimal monetary policy insulates any productivity and preference shifter from the NKIS, and the optimal monetary and fiscal policy mix insulates the PPI inflation rate and output gap not only union-wide but also at individual country level from any exogenous productivity and preference shifter regardless of the share of nontradables.

5 Numerical Analysis

In this subsection, we illustrate the equilibrium behavior of the currency union under the alternative policy regime described above. We resort to a series of dynamic simulations and adopt the following benchmark parameterization. We assume an inverse of labor supply elasticity φ, the elasticity of substitution across goods θ, price stickiness consistent with an inverse of an average period of one year between price adjustments α, the share of nontradables in the CPI γ, the elasticity of substitution between tradables and nontradables η, and the subjective discount factor δ set equal to 3, 7.88, 0.66, 0.5, 0.44 and 0.99, respectively as if the timing of the model were quarterly. Except for γ and η, these parameterizations are frequently used in DSGE literature including Benigno[4], Gali and Monacelli[7] and [8] and Rotemberg and Woodford[14]. As mentioned in the introduction, nontradables account for 50.3% of the major Euro area, thus we set $\gamma = 0.5$. Following Stockman and Tesar[16], we set $\eta = 0.44$.[16] We notice that setting $\alpha = 0.66$ and $\delta = 0.99$ implies that the slope of the NKPC λ is identical to 0.1786, $\theta = 7.88$ implies that the Taylor Principle ϕ is identical with 7.88 while constant markup ζ is identical to approximately 1.1453, and $\delta = 0.99$ implies that a riskless annual return is equal to about 4.04%. We also assume that the productivity and preference shifters are described according to the following AR (1) processes:

$$\mathsf{s}_t = \rho I \mathsf{s}_{t-1} + \xi_t$$

where $\mathsf{s}_t = \begin{bmatrix} a_{H,t} & a_{N,t} & a_{F,t} & a^*_{N,t} & d_t & z_t \end{bmatrix}'$ denotes that the vector consists of the productivity and the preference shifters, ξ denotes that the vector consists of i.i.d. shocks, I denotes the identity matrix, and ρ denotes the coefficient associated with AR (1) processes. We set ρ equal to 0.7.

5.1 Special Cases

Prior to analyzing benchmark parameterization, we consider two special cases in which all goods are tradable and the Balassa–Samuelson effect vanishes under optimal monetary policy without fiscal policy. We substitute $\gamma = 1$ in the former case and $\eta = 1$ in the latter case for benchmark parameterization. Figure 1 shows impulse responses to shocks on any shifters under optimal monetary policy without fiscal policy in the case that all goods are tradable. As mentioned above, optimal monetary policy depicted by Eq.(53) realizes not only $\pi_t^W = 0$ but also $\pi_{P,t}^R = 0$ because NKISs in the two countries are identical, and the structural parameter ψ associated with the NPD in NKPCs in the two countries remains at zero, thus inflation–output trade-offs are fully dissolved. This can be confirmed by inspecting the top four panels in Figure 1. As shown in the optimal monetary

[16]Setting $\eta = 0.44$ is widely used including Benigno and Thoenissen[2] and Selaive and Tuesta[15], who analyze a consumption-real exchange rate anomaly.

policy rule Eq.(53), nominal interest rates insulate the union-wide economy from any shocks by equalizing with the natural real interest rate. The fifth panel in Figure 1 shows this. The sixth panel in Figure 1 shows that the CPIs in the two countries are identical because nontradables do not exist. This implies that not only the current accounts but also the consumptions in the two countries are identical. Assuming that all goods are tradable in a currency union, Benigno and Benigno[4] show that union-wide inflation targeting consistent with $\pi_t^W = 0$ in our context or optimal monetary policy can fully resolve inflation–output trade-offs. It can be said that our result reconfirms the result of Benigno and Benigno[4].[17]

The result in the latter case, in which the Balassa–Samuelson effect vanishes under optimal monetary policy without fiscal policy, resembles that of the former case. Figure 2 displays the impulse responses to shocks under optimal monetary policy without fiscal policy in the case that the Balassa–Samuelson effect vanishes. Because of $\eta = 1$ implying $\psi = 0$, the NKISs in this case are identical with the former case while half of the goods are nontradable in the union in this case. As shown in the sixth panel in Figure 2, inefficient supply shocks cause a CPI disparity between the two countries because the CPIs in the two countries are not identical. The Balassa–Samuelson effect is at the vanishing point, however, current accounts are balanced. Thus, optimal monetary policy resolves the inflation–output trade-offs. The top four panels in Figure 2 show this result. In these special cases, where NKPCs are identical to those under a closed economy, optimal monetary policy or simple inflation targeting can resolve the inflation–output trade-offs.[18] Fiscal policy takes no part in resolving inflation–output trade-offs.

5.2 Optimal Monetary Policy without Fiscal Policy

In accordance with the above finding, we inspect the benchmark parameterization case under optimal monetary policy without fiscal policy. Figure 3 displays the impulse responses to shocks under optimal monetary policy without fiscal policy. Neither $\gamma = 1$ nor $\eta = 1$ hold in the benchmark case, and inflation–output trade-offs cannot be resolved by optimal monetary policy without fiscal policy. This can be confirmed by inspecting the top two panels in Figure 3. When inefficiency shocks change the productivity shifter, the output gap results in a CPI disparity between the two countries, which is shown as the sixth panel in Figure 3. For instance, we consider the occurrence of changes in the productivity shifter of tradables produced in country H. The nominal interest rate is lowered to maintain zero PPI inflation when this change occurs. When $\psi = 0$, this implies that $\gamma = \eta = 1$ does not hold, thus reducing the insulation of the output gap from this shock. In the benchmark case, however, ψ does not equal zero, hence the NKPCs are affected by changes in the NPD, namely the CPI differential. As shown in the sixth panel in Figure 3, changes in the productivity shifter of tradables produced in country H cause changes in the CPI differential through NKBS. As mentioned in the former section, increasing productivity of tradables produced in country H decreases the CPI differential

[17]See top panel in Table 1 for details on macroeconomic volatility under optimal monetary policy without fiscal policy in this case.

[18]See the last panel in Table 1 for details on macroeconomic volatility under optimal monetary policy without fiscal policy in this case.

through the Balassa–Samuelson theorem. An increase in wages in the tradables sector stems from increases in productivity of tradables, produced in country H cause an increase in the price of nontradables produced in country H. This is the cause of an increase in the CPI in country H. As shown in Eq.(41), a decrease in the CPI differential stemming from an increase in the price of nontradables produced in country H causes a current account deficit in country H, because of a rising demand for tradables including country F goods. Thus, An output gap in country H decreases while in country eventually F decreases. While monetary policy is optimal and can stabilize PPI inflation in both countries, NKBS affects NKPCs in the benchmark case implying that half of goods are nontradable. Thus, optimal monetary policy without fiscal policy is not adequate to resolve the inflation–output trade-offs when nontradables exist in the currency union.

Table 2 depicts macroeconomic volatility under optimal monetary policy without fiscal policy. Whereas the PPI inflation rate is fully stabilized to any changes in productivity shifters, output gaps in neither country are stabilized. It can be noted in Table 2 that any changes in productivity shifters cause output gap fluctuation through changes in the CPI differential and current account.

5.3 Optimal Monetary and Fiscal Policy Mix

Unlike the special cases, the output gaps in both countries are stabilized with regard to any changes in productivity shifters under the optimal monetary policy without fiscal policy. As mentioned in the former section, an optimal monetary and fiscal policy mix regime can stabilize both the PPI inflation rate and the output gap. Figure 4 shows impulse responses to shocks under an optimal monetary and fiscal policy mix with benchmark parameterization. That output gap and PPI inflation are stabilized simultaneously when productivity shifter changes can be confirmed by the first to the fourth panels in Figure 3. Needless to say, an increase in the productivity shifter associated with tradables produced in country H pressures the output gap in country H to decrease through NKBS. This is because the increase in price associated with nontradables produced in country H stemming from increase in productivity associated with tradable goods produced in country H shifts demands for goods from nontradables produced in country H to tradables including goods produced in country F. Under an optimal monetary and fiscal policy mix regime, however, decrease in the NPD caused by an increase in prices associated with nontradables produced in country H is prevented by a decrease in government expenditure in country H, namely, fiscal surplus in country H. Decrease in government expenditure in country H controls increases in the PPI in country H including nontradable prices. Thus, the CPI differential is unchanged while an increase in pressure associated with nontradable prices in country H results. Finally, the output gap in country H is unchanged because the current account is fully stabilized. In country F, adverse changes occur in response to a shock to the productivity shifter associated with tradables produced in country H. This is shown in the last panel in Figure 4. An increase in a productivity shifter associated with tradables produced in country H decreases prices associated with nontradables produced in country F relative to those in country H. This relative decrease in nontradables price in country F, namely, a decrease in the NPD, places an increased pressure on the output gap in country F. Under this regime, however, government expenditure in country F gets larger, which in turn increases the PPI including the nontradable goods price in country F.

As shown in Table 2, an optimal monetary policy regime can fully stabilize not only the PPI inflation but also the output gap whereas optimal monetary policy without a fiscal policy regime stabilizes only the PPI inflation rate. Benigno[4] asserts that the optimal monetary policy, nemely, simple union-wide inflation targeting can stabilize both the inflation rate and the output gap simultaneously. When a currency union consists of homogeneous economies, a solitary instrument, namely the interest rate of the consolidated currency, can resolve inflation–output trade-offs. When a currency union consists of heterogeneous economies, however, the interest rate cannot correct a disparity across economies, although it can stabilize the union-wide economy. Similarly to our paper, Gali and Monacelli[7] insist on the importance of fiscal policy in currency union. Gali and Monacelli[7] do not assume the existence of nontradables but rather a currency union that consists of infinitesimal countries. Because a currency union that consists of infinitesimal countries is much the same as our settings for heterogeneity, the issue associated with an optimal monetary and fiscal policy mix should concern policy administration.

5.4 Sensitivity Analysis: The Role of Share of Nontradable Goods

In this section, we investigate to what extent the welfare-based ranking of the regimes discussed above may be sensitive to the calibration of a central parameter characterizing the currency union: the share of nontradables $1-\gamma$. Prior to this sensitivity analysis, we define the expected welfare loss criterion. Taking unconditional expectations on Eq.(51) and letting $\delta \to 1$, the expected welfare loss function is given by:

$$\mathcal{V} = \frac{1}{4}\left\{\frac{\theta}{\lambda}\operatorname{var}\left(\pi_{P,t}\right) + (1+\varphi)\operatorname{var}\left(\tilde{y}_t\right) + \frac{\theta}{\lambda}\operatorname{var}\left(\pi^*_{P,t}\right) + (1+\varphi)\operatorname{var}\left(\tilde{y}^*_t\right)\right\}. \quad (56)$$

Eq.(63) shows that welfare losses can be evaluated by the variance of PPI inflation and output gap. Accordingly, we show the relationship between macroeconomic volatilities and share of nontradables for a start.

Figure 5 displays the effect on volatilities of varying share of nontradables under alternative policy regimes. Although PPI inflation in both countries is zero, which is applied in any share of nontradables, the volatility of output gap depends on the share of nontradables. The top and second panels in Figure 5 show that the volatility of output gap is zero if $\gamma = 0$ and $\gamma = 1$ under optimal monetary policy without fiscal policy whereas the output gap is zero which is applied in any share of nontradables under an optimal monetary and fiscal policy mix.

Figure 6 displays the effect on welfare of varying share of nontradables. Reflecting the relationship between volatilities and share of nontradables on output gap, welfare loss results under optimal monetary policy without a fiscal policy regime while welfare loss does not result under an optimal monetary and fiscal policy mix regime. Not only from the viewpoint of dissolving inflation–output trade-offs but also eliminating a well-defined welfare loss, it can be said that an optimal monetary and fiscal policy mix is an important policy issue in an actual currency union, namely, the Euro area.

6 Conclusion

We investigate an optimal monetary and fiscal policy mix, which has been forgotten for a long time. Some implications can be derived from our paper, the greatest contribution of this paper being that it indicates the importance of a monetary and fiscal policy mix in a currency union with nontradables. Also, perfect risk sharing in assets markets makes union-wide economy homogeneous, the existence of nontradable goods makes optimal monetary policy insufficient to eliminate welfare loss. Not only monetary policy but also fiscal policy plays a part in achieving zero welfare loss in a currency union with nontradables. In connection with the necessity of policy mix, our paper agrees with the proposal made by Gali and Monacelli[7]. From another viewpoint, this paper can justify approximately a canonical argument associated with policy mix in a currency union.

While we agree with the canonical argument, the necessity of centralized fiscal policy remains to be further examined. The welfare loss function derived by us is not country specific but union wide. When fiscal policy is conducted to maximize welfare not in a union-wide economy but in each country and it complements a completely monetary policy, a policy implication that there is no need to centralize fiscal policy to maximize welfare is derived. Further discussion is desirable.

Appendix A Details on Derivation of the Model

A.1 Households

Preference of the representative household in country H is given by Eq.(1) where E_t denotes the expectation conditional on the information set at period t, $\delta \in (0,1)$ denotes the subjective discount factor, C_t and C_t^* denote consumptions in countries H and F, respectively, $N_t \equiv \int_0^1 N_t(i)\,di$ and $N_t^* \equiv \int_0^1 N_t^*(i)\,di$ denote hours of work in countries H and F, respectively, $N_t(i)$ and $N_t^*(i)$ denote hours of work spent by generic household i in countries H and F, respectively, D_t denotes union-wide consumption preference shifter, Z_t denotes union-wide disutility of work shifter, and φ denotes the inverse of labor supply elasticity. We note that quantities and prices peculiar to country F are denoted by asterisks, while quantities and prices without asterisks are those in country H or common to both countries.

More precisely, private consumption is a composite index defined by Eq.(2) where $C_{T,t} \equiv 2C_{H,t}^{\frac{1}{2}}C_{F,t}^{\frac{1}{2}}$ denotes the consumption index for tradables in country H, $C_{T,t}^* = C_{T,t}$ denotes the consumption index for tradables in country F, $C_{N,t} \equiv \left[\int_0^1 C_{N,t}(i)^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}}$, $C_{N,t}^* \equiv \left[\int_1^2 C_{N,t}^*(i)^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}}$ $C_{H,t} \equiv \left[\int_0^1 C_{T,t}(i)^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}}$ and $C_{F,t} \equiv \left[\int_1^2 C_{T,t}(i)^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}}$ denote the consumption indices for nontradables produced in country H, nontradables produced in country F, tradables produced in country H and tradables produced in country F, respectively, and $C_{N,t}(i)$, $C_{N,t}^*(i)$ and $C_{T,t}(i)$ denote generic nontradable good i produced in H, generic nontradable good i produced in F, and generic tradable good i, respectively, γ denotes the share of tradables in the consumer price index (CPI), $\theta > 1$ denotes the elasticity of substitution across goods produced within a country, and $\eta > 0$ denotes the elasticity of substitution between tradables and nontradable goods.

A sequence of budget constraints of the form is given by:

$$
\begin{aligned}
B_t + W_tN_t + S_t &\geq \int_0^1 P_{H,t}(i)\,C_{T,t}(i)\,di + \int_1^2 P_{F,t}(i)\,C_{T,t}(i)\,di \\
&\quad + \int_0^1 P_{N,t}(i)\,C_{N,t}(i)\,di + \mathrm{E}_tQ_{t,t+1}B_{t+1}, \\
B_t + W_t^*N_t^* + S_t^* &\geq \int_0^1 P_{H,t}(i)\,C_{T,t}^*(i)\,di + \int_1^2 P_{F,t}(i)\,C_{T,t}^*(i)\,di \\
&\quad + \int_0^1 P_{N,t}^*(i)\,C_{N,t}^*(i)\,di + \mathrm{E}_tQ_{t,t+1}B_{t+1} \qquad (57)
\end{aligned}
$$

where $P_{H,t}(i)$ denotes the price of generic tradable good i produced in country H, $P_{F,t}(i)$ denotes the price of generic tradable good i produced in country F, $P_{N,t}(i)$ denotes the price of generic nontradable good i, $Q_{t,t+1}$ denotes the stochastic discount factor, B_t denotes the nominal payoff of the portfolio, W_t denotes the nominal wage, and S_t denotes the lump-sum transfers.[19]

The optimal allocation of any given expenditure within each category of

[19]Individuals in country F face a parallel constraint.

goods yields the demand functions as follows:

$$C_{T,t}(h) = \left(\frac{P_{T,t}(h)}{P_{H,t}}\right)^{-\theta} C_{H,t} \quad ; \quad C_{T,t}(f) = \left(\frac{P_{T,t}(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}$$

$$C^*_{T,t}(h) = \left(\frac{P_{T,t}(h)}{P_{H,t}}\right)^{-\theta} C^*_{H,t} \quad ; \quad C^*_{T,t}(f) = \left(\frac{P_{T,t}(f)}{P_{F,t}}\right)^{-\theta} C^*_{F,t}$$

$$C_{N,t}(i) = \left(\frac{P_{N,t}(i)}{P_{N,t}}\right)^{-\theta} C_{N,t} \quad ; \quad C^*_{N,t}(i) = \left(\frac{P^*_{N,t}(i)}{P^*_{N,t}}\right)^{-\theta} C^*_{N,t}$$

with $h \in [0,1]$ and $f \in [1,2]$ where $P_{T,t}(h)$ and $P_{T,t}(f)$ denote the prices of typical tradable goods produced in country H and country F, respectively, $P_{N,t}(h)$ denotes the price of a typical nontradable good produced in country H, $P_{H,t} \equiv \left[\int_0^1 P_{T,t}(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ denotes the goods price index of tradables produced in country H, $P_{F,t} \equiv \left[\int_1^2 P_{T,t}(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ denotes the price of tradables produced in country F, $P_{N,t} \equiv \left[\int_0^1 P_{N,t}(h)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ denotes the price index of nontradables produced in country H, with parallel demands in country F. These equalities imply that $\int_0^1 P_{H,t}(i)\, C_{T,t}(i)\, di = P_{H,t}C_{H,t}$ and $\int_1^2 P_{F,t}(i)\, C_{T,t}(i)\, di = P_{F,t}C_{F,t}$ and $\int_0^1 P_{N,t}(i)\, C_{N,t}(i)\, di = P_{N,t}C_{N,t}$.

The optimal allocation of expenditures across each typical good is given by:

$$C_{H,t} = \frac{1}{2}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C_{T,t} \quad ; \quad C_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C_{T,t}$$

$$C^*_{H,t} = \frac{1}{2}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C^*_{T,t} \quad ; \quad C^*_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C^*_{T,t}$$

$$C_{T,t} = \left(\frac{P_{T,t}}{P_t}\right)^{-\eta} C_t \quad ; \quad C^*_{T,t} = \left(\frac{P_{T,t}}{P^*_t}\right)^{-\eta} C^*_t$$

$$C_{N,t} = \left(\frac{P_{N,t}}{P_t}\right)^{-\eta} C_t \quad ; \quad C^*_{N,t} = \left(\frac{P^*_{N,t}}{P^*_t}\right)^{-\eta} C^*_t \tag{58}$$

where:

$$\begin{aligned} P_{T,t} &\equiv P_{H,t}^{\frac{1}{2}} P_{F,t}^{\frac{1}{2}} \\ P_t &\equiv \left[\gamma P_{T,t}^{1-\eta} + (1-\gamma) P_{N,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \\ P^*_t &\equiv \left[\gamma P_{T,t}^{1-\eta} + (1-\gamma)\left(P^*_{N,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \end{aligned} \tag{59}$$

denotes the TPI and the CPI in countries H and F, respectively. We also note that the PPI is defined as Eq.(6).

Total consumption expenditures by households in country H are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{N,t}C_{N,t} = P_tC_t$ while their counterparts in country F are given by $P_{F,t}C_{F,t} + P_{H,t}C_{H,t} + P^*_{N,t}C^*_{N,t} = P^*_tC^*_t$. Combining these results, Eq.(57) can be rewritten as Eq.(3).

The representative household maximizes Eq.(1) subject to Eq.(3). Optimality conditions are given as Eqs.(7) and (8). Combining and iterating both parts of Eq.(7), we have optimal risk sharing condition Eq.(9).

A.2 Firms

Each firm is classified as belonging to one of two groups: tradables producers and nontradables producers. Production functions are given by Eq.(10).

Following Calvo–Yun-style price-setting behavior, we assume that each firm resets its price with a probability of $1-\alpha$ in each period, independently of the time elapsed since the last adjustment. The pricing rule implies:

$$\begin{aligned} P_{H,t} &= \left\{\alpha\left(P_{H,t-1}\right)^{1-\theta}+(1-\alpha)\left(\tilde{P}_{H,t}\right)^{1-\theta}\right\}^{\frac{1}{1-\theta}} \\ P_{N,t} &= \left\{\alpha\left(P_{N,t-1}\right)^{1-\theta}+(1-\alpha)\left(\tilde{P}_{N,t}\right)^{1-\theta}\right\}^{\frac{1}{1-\theta}} \\ P_{F,t} &= \left\{\alpha\left(P_{F,t-1}\right)^{1-\theta}+(1-\alpha)\left(\tilde{P}_{F,t}\right)^{1-\theta}\right\}^{\frac{1}{1-\theta}} \\ P^*_{N,t} &= \left\{\alpha\left(P^*_{N,t-1}\right)^{1-\theta}+(1-\alpha)\left(\tilde{P}^*_{N,t}\right)^{1-\theta}\right\}^{\frac{1}{1-\theta}} \end{aligned} \tag{60}$$

where $\tilde{P}_{H,t}$, $\tilde{P}_{N,t}$, $\tilde{P}^H_{F,t}$ and $\tilde{P}^*_{H,t}$ denote the adjusted prices of tradables produced in country H and nontradables produced in country H, respectively.

When setting a new price in period t, producers seek to maximize the expected discounted value of profits as follows:

$$\begin{aligned} &\max_{\tilde{P}_{H,t}} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\Lambda_{t+k}\tilde{C}_{H,t+k}\left(\tilde{P}_{H,t}-P_{P,t+k}(1-\tau)MC_{H,t+k}\right)\right], \\ &\max_{\tilde{P}_{N,t}} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\Lambda_{t+k}\tilde{C}_{N,t+k}\left(\tilde{P}_{N,t}-P_{P,t+k}(1-\tau)MC_{N,t+k}\right)\right], \\ &\max_{\tilde{P}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\Lambda^*_{t+k}\tilde{C}_{F,t+k}\left(\tilde{P}_{F,t}-P^*_{P,t+k}(1-\tau)MC_{F,t+k}\right)\right], \\ &\max_{\tilde{P}^*_{N,t}} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\Lambda^*_{t+k}\tilde{C}^*_{N,t+k}\left(\tilde{P}^*_{N,t}-P^*_{P,t+k}(1-\tau)MC^*_{N,t+k}\right)\right] \end{aligned}$$

where $\Lambda_t \equiv (P_tC_t)^{-1}$ and $\Lambda^*_t \equiv (P^*_tC^*_t)^{-1}$ denote the marginal utilities of nominal income in countries H and F, respectively, $MC_{H,t} \equiv \frac{W_t}{P_{P,t}A_{H,t}}$ denotes the marginal cost associated with tradables produced in country H, $MC_{N,t} \equiv \frac{W_t}{P_{P,t}A_{N,t}}$ denotes the marginal cost associated with nontradables produced in country H, $MC_{F,t} \equiv \frac{W^*_t}{P^*_{P,t}A_{F,t}}$ denotes the marginal cost associated with tradables produced in country F, $MC^*_{N,t} \equiv \frac{W^*_t}{P^*_{P,t}A^*_{N,t}}$ denotes the marginal cost associated with nontradables produced in country F, $\tilde{C}_{H,t+k} \equiv \left(\frac{\tilde{P}^H_{H,t}}{P_{H,t+k}}\right)^{-\theta} C_{H,t+k}$ denotes the total demand of tradables produced in country H when the price are changed, $\tilde{C}_{N,t+k} \equiv \left(\frac{\tilde{P}_{N,t}}{P_{N,t+k}}\right)^{-\theta} C_{N,t+k}$ denotes the total demand of nontradables produced in country H when the prices are changed, $\tilde{C}_{F,t+k} \equiv \left(\frac{\tilde{P}_{F,t}}{P_{F,t+k}}\right)^{-\theta} C_{F,t+k}$ denotes the total demand of tradables produced in country F when the prices are changed, $\tilde{C}^{*}_{N,t+k} \equiv \left(\frac{\tilde{P}^{*}_{N,t}}{P^{*}_{N,t+k}}\right)^{-\theta} C^{*}_{N,t+k}$ denotes the total demand of nontradables produced in country F when the prices are changed, and $A_{H,t}$, $A_{N,t}$, $A_{F,t}$ and $A^{*}_{N,t}$ denote stochastic productivity shifters associated with tradables produced in country H, nontradables produced in country H, tradable goods produced in country F and nontradables produced in country F, respectively. We also note that using Eq.(8), marginal cost can be rewritten as follows.

$$
\begin{aligned}
MC_{H,t} &= \frac{C_t N_t^{\varphi} Z_t}{D_t} \frac{P_t}{P_{P,t} A_{H,t}} \\
MC_{N,t} &= \frac{C_t N_t^{\varphi} Z_t}{D_t} \frac{P_t}{P_{P,t} A_{N,t}} \\
MC_{F,t} &= \frac{C_t^{*} \left(N_t^{*}\right)^{\varphi} Z_t}{D_t} \frac{P_t^{*}}{P^{*}_{P,t} A_{F,t}} \\
MC^{*}_{N,t} &= \frac{C_t^{*} \left(N_t^{*}\right)^{\varphi} Z_t}{D_t} \frac{P_t^{*}}{P_{P,t} A^{*}_{N,t}} \qquad (61)
\end{aligned}
$$

The FONC of these optimization problems are given by Eq.(11).

A.3 Centralized Government

Government expenditure index is given by:

$$
G_t \equiv \left(\int_0^1 G_t\left(i\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} \quad ; \quad G_t^{*} \equiv \left(\int_0^1 G_t^{*}\left(i\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}
$$

where G_t and G_t^{*} denote government expenditures on goods produced in countries H and F, respectively, $G_t\left(i\right)$ denotes the quantity of good i produced in country H purchased by the government, and $G_t^{*}\left(i\right)$ denotes the quantity of good i produced in country F purchased by the government. For simplicity, we assume that government purchases are fully allocated to a domestically produced good. For any given level of public consumption, the government allocates expenditures across goods in order to minimize total cost. This yields the following set of government demand schedules, analogous to those associated with private consumption.

$$
\begin{aligned}
G_t\left(i\right) &= \left(\frac{P_{T,t}\left(h\right)}{P_{H,t}}\right)^{-\theta} = \left(\frac{P_{N,t}\left(i\right)}{P_{N,t}}\right)^{-\theta} G_t \\
G_t^{*}\left(i\right) &= \left(\frac{P_{T,t}\left(f\right)}{P_{F,t}}\right)^{-\theta} = \left(\frac{P^{*}_{N,t}\left(i\right)}{P^{*}_{N,t}}\right)^{-\theta} G_t^{*} \qquad (62)
\end{aligned}
$$

A.4 Market Clearing

The market in country H for tradables clears when domestic demand is given by Eq.(13). As for nontradables, equilibrium requires Eq.(14).

Using Eqs.(4), (9) and (62), Eq.(13) can be rewritten as:

$$
Y_{H,t}\left(i\right) = \left(\frac{P_{T,t}\left(h\right)}{P_{H,t}}\right)^{-\theta} \left\{\frac{1}{2}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C_t \left[\left(\frac{P_{T,t}}{P_t}\right)^{-\eta} + \left(\frac{P_{T,t}}{P_t^{*}}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_t\right\}
$$

$$Y_{F,t}(i) = \left(\frac{P_{T,t}(f)}{P_{F,t}}\right)^{-\theta}\left\{\frac{1}{2}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C_t\left[\left(\frac{P_{T,t}}{P_t}\right)^{-\eta}+\left(\frac{P_{T,t}}{P_t^*}\right)^{-\eta}\mathsf{Q}_t^{-1}\right]+G_t^*\right\}$$

where we use the fact that $C_t^* = \frac{C_t}{\mathsf{Q}_t}$. Let $Y_{H,t} \equiv \left(\int_0^1 Y_{H,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}}$ and $Y_{F,t} \equiv \left(\int_1^2 Y_{F,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}}$ denote the aggregate outputs of tradables produced in countries H and F, respectively. Using this definition, the above equalities can be rewritten as Eq.(15).

Using Eqs.(4), (9) and (62), Eq.(14) can be rewritten as follows.

$$\begin{aligned} Y_{N,t}(i) &= \left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta}\left\{\left(\frac{P_{N,t}}{P_t}\right)^{-\eta}C_t+G_t\right\} \\ Y_{N,t}^*(i) &= \left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta}\left\{\left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\eta}\frac{C_tD_t}{\mathsf{Q}_tD_t}+G_t^*\right\} \end{aligned}$$

Let $Y_{N,t} \equiv \left(\int_0^1 Y_{N,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}}$ and $Y_{N,t}^* \equiv \left(\int_0^1 Y_{F,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}}$ denote the aggregate outputs of tradables produced in countries H and F, respectively. Using this definition, the above equalities can be rewritten as Eq.(16).

We define aggregate domestic indices as Eq.(17). Combining Eq.(17) and the fact that:

$$N_t = \int_0^1 Y_{H,t}(i)\,di = \int_0^1 Y_{N,t}(i)\,di\ ;\ N_t^* = \int_0^1 Y_{F,t}di = \int_0^1 Y_{N,t}^*(i)\,di,$$

which is derived from Eq.(60), we have aggregate production functions as Eq.(18).

Appendix B Nonstochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which the PPI inflation rate is zero. In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor as follows:

$$R = \delta^{-1}$$

with R being the gross nominal interest rate in the steady state.

Because the LOOP always holds,

$$P_T = P_H = P_F$$

must hold.

When $\alpha \to 0$, Eq.(11) implies:

$$MC_H = \frac{1}{(1-\tau)\zeta}.$$

Thus, $MC_H = MC_N = MC_F = MC_N^*$ must hold. Also,

$$MC_H = CN^{\varphi}.$$

Thus,

$$CN^{\varphi} = C^* \left(N^*\right)^{\varphi}.$$

Eq.(9) implies that:

$$C = C^*.$$

Thus,

$$N = N^*.$$

Eqs.(15) and (63) imply that:

$$C = Y_H = Y_F = Y_N = Y_N^*.$$

Appendix C Equilibrium Determinacy in the Currency Union

To be added.

Appendix D Derivation of the NKPC

The first equality of Eq.(11) can be rewritten as:

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\tilde{\mathsf{X}}_{H,t+k}^{-(\theta-1)} \mathsf{X}_{T,t+k}^{-(\eta-1)} - \zeta (1-\tau) \tilde{\mathsf{X}}_{H,t+k}^{-\theta} \mathsf{X}_{H,t+k}^{-1} \mathsf{X}_{T,t+k}^{-\eta} \mathsf{X}_{P,t+k} MC_{H,t+k} \right] = 0$$

with $\tilde{\mathsf{X}}_{H,t+k} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t+k}}$, $\mathsf{X}_{H,t+k} \equiv \frac{P_{H,t+k}}{P_{T,t+k}}$, $\mathsf{X}_{T,t+k} \equiv \frac{P_{T,t+k}}{P_{t+k}}$ and $\mathsf{X}_{P,t+k} \equiv \frac{P_{P,t+k}}{P_{t+k}}$.

Log-linearizing this equality, we have:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{\mathsf{x}}_{H,t+k} + \mathsf{x}_{H,t+k} + \mathsf{x}_{T,t+k} - \mathsf{x}_{P,t+k} - mc_{H,t+k} \right) \right] = 0$$

with $\tilde{\mathsf{x}}_{H,t+k} \equiv \ln \tilde{\mathsf{X}}_{H,t+k}$, $\mathsf{x}_{H,t+k} \equiv \ln \mathsf{X}_{H,t+k}$, $\mathsf{x}_{T,t+k} \equiv \ln \mathsf{X}_{T,t+k}$ and $\mathsf{x}_{P,t+k} \equiv \ln \mathsf{X}_{P,t+k}$.

Using the fact that $\tilde{\mathsf{x}}_{H,t+k} = \mathsf{x}_{H,t} - \sum_{s=1}^{k} \pi_{H,t+s}$, this can be rewritten as follows.

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{\mathsf{x}}_{H,t} - \sum_{s=1}^{k} \pi_{H,t+s} + \mathsf{x}_{H,t+k} + \mathsf{x}_{T,t+k} - \mathsf{x}_{P+k,t} - mc_{H,t+k} \right) \right] = 0$$

Furthermore, using the fact that $\sum_{k=0}^{\infty} (\alpha\delta)^k \sum_{s=1}^{k} \pi_{H,t+s} = \frac{1}{1-\alpha\delta} \sum_{k=1}^{\infty} (\alpha\delta)^k \pi_{H,t+k}$, this can be rewritten as follows.

$$\frac{1}{1-\alpha\delta} \tilde{\mathsf{x}}_{H,t} - \frac{1}{1-\alpha\delta} \mathrm{E}_t \sum_{k=1}^{\infty} (\alpha\delta)^k \pi_{H,t+k} + \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{H,t+k} + \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{T,t+k}$$
$$- \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{P+k,t} - \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k mc_{H,t+k} = 0$$

Rearranging this, we have:

$$\begin{aligned}
\tilde{\mathsf{x}}_{H,t} &= \sum_{k=1}^{\infty}(\alpha\delta)^k \pi_{H,t+k} - (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k \mathsf{x}_{H,t+k} - (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k \mathsf{x}_{T,t+k} \\
&\quad + (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k \mathsf{x}_{P+k,t} + (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k mc_{H,t+k} \\
&= \alpha\delta\pi_{H,t+1} - (1-\alpha\delta)\mathsf{x}_{H,t} - (1-\alpha\delta)\mathsf{x}_{T,t} + (1-\alpha\delta)\mathsf{x}_{P,t} + (1-\alpha\delta)mc_{H,t} + \alpha\delta\tilde{\mathsf{x}}_{H,t+1}.
\end{aligned}$$

Log-linearizing Eq.(7), we have:

$$\tilde{\mathsf{x}}_{H,t} = \frac{\alpha}{1-\alpha}\pi_{H,t}.$$

Using this fact, we have:

$$\begin{aligned}
\pi_{H,t} &= \delta\pi_{H,t+1} - \lambda\mathsf{x}_{H,t} - \lambda\mathsf{x}_{T,t} + \lambda\mathsf{x}_{P,t} + \lambda mc_{H,t} \\
&= \delta\pi_{H,t+1} + (1-\gamma)\lambda p_{N,t} - (1-\gamma)\lambda p_{H,t} + \lambda mc_{H,t}.
\end{aligned}$$

Taking conditional expectation at t, the second equality in this equation is clearly the same as the first equality in Eq.(30) in the text. Other NKPCs are derived similarly.

Appendix E Welfare Criterion

Following Gali and Monacelli[8] and Woodford[17], we show derivation of the welfare criterion in the text based on the second-order approximated utility function Eq.(1) in the present appendix.

The second-order Taylor expansion of $U(C_t) \equiv \ln C_t$ and $U(C_t^*) \equiv \ln C_t$ is as follows:

$$\begin{aligned}
U(C_t) &= c_t + \text{t.i.p.} + o\left(\|\xi\|^3\right) \\
U(C_t^*) &= c_t^* + \text{t.i.p.} + o\left(\|\xi\|^3\right). \qquad (63)
\end{aligned}$$

The second-order Taylor expansion of $V(N_t) \equiv \frac{1}{1+\varphi}N_t^{1+\varphi}$ and $V(N_t^*) \equiv \frac{1}{1+\varphi}(N_t^*)^{1+\varphi}$ is as follows:

$$V(N_t) = V(N) + V_N(N)(N_t - N) + \frac{1}{2}V_{NN}(N)(N_t - N)^2 + o\left(\| a \|^3\right) \quad (64)$$

with N_t being hours of labor and N being the steady-state value of N_t. Expanding N_t with a second-order Taylor expansion, we have:

$$N_t = N + Nn_t + \frac{1}{2}Nn_t^2 + o\left(\| a \|^3\right) \quad (65)$$

with $n_t \equiv \ln N_t$.

Plugging Eq.(2) into Eq.(1), we obtain:

$$\begin{aligned}
V(N_t) &= N^{1+\varphi}\left\{n_t + \frac{1+\varphi}{2}n_t^2\right\} + \text{t.i.p.} + o\left(\| a \|^3\right) \\
V(N_t^*) &= N^{1+\varphi}\left\{n_t^* + \frac{1+\varphi}{2}(n_t^*)^2\right\} + \text{t.i.p.} + o\left(\| a \|^3\right) \qquad (66)
\end{aligned}$$

where we use the fact that $\left(N+Nn_t+\frac{1}{2}Nn_t^2\right)^2 = 2N^2n_t + 2N^2n_t^2 + \text{t.i.p.} + o\left(\| a \|^3\right)$ and $V_N(N)N = N^{1+\varphi}$.

Combining Eqs.(24), (25) and (34), we have:

$$\begin{aligned} c_t &= \frac{1}{2}n_t + \frac{1}{2}n_t^* + \frac{1-\gamma}{2}\mathsf{n}_t \\ c_t^* &= \frac{1}{2}n_t + \frac{1}{2}n_t^* - \frac{1-\gamma}{2}\mathsf{n}_t. \end{aligned} \tag{67}$$

Notice that we use the fact that $g_t^W = 0$ implying that union-wide government expenditure is always zero. When all goods are tradable, i.e. $\gamma = 1$, this equality reduces to $c_t = \frac{1}{2}y_t + \frac{1}{2}y_t^*$. Taking this equality as an exponential equation, we obtain:

$$C_t = N_t^{\frac{1}{2}}\left(N_t^*\right)^{\frac{1}{2}}\mathsf{N}_t^{\frac{1-\gamma}{2}}. \tag{68}$$

Optimal allocation must maximize $U(C_t) - V(N_t)$subject to Eq.(64) and technological constraint $Y_t = N_t$. An optimality condition is given by:

$$\frac{\partial\left(U(C_t) - V(N_t)\right)}{\partial N_t} = \frac{1}{2}N_t^{-1} - N_t^{\varphi} = 0.$$

This equality implies that:

$$N_t^{1+\varphi} = \frac{1}{2}. \tag{69}$$

Combining Eqs.(63), (66), (67) and (69), we obtain:

$$\begin{aligned} U(C_t) - V(N_t) &= \frac{1-\gamma}{2}\mathsf{n}_t - \frac{1+\varphi}{4}\tilde{n}_t^2 + \text{t.i.p} + o\left(\|\xi\|^3\right) \\ U(C_t^*) - V(N_t^*) &= -\frac{1-\gamma}{2}\mathsf{n}_t - \frac{1+\varphi}{4}\left(\tilde{n}_t^*\right)^2 + \text{t.i.p} + o\left(\|\xi\|^3\right) \end{aligned}$$

where $\tilde{n}_t$ denotes logarithmic hours of labor gap from its natural level. Summing the above equalities, we can eliminate linear terms as follows:

$$\mathcal{W} = -\frac{1+\varphi}{4}\left[\tilde{n}_t^2 + \left(\tilde{n}_t^*\right)^2\right] + \text{t.i.p} + o\left(\|\xi\|^3\right) \tag{70}$$

with $\mathcal{W} \equiv U(C_t) + U(C_t^*) - \left(V(N_t) + V(N_t^*)\right)$.

Notice that:

$$\begin{aligned} N_t &\equiv \int_0^1 N_t(i)\,di = Y_{H,t}\frac{\int_0^1 Y_{H,t}(i)\,di}{Y_{H,t}} = Y_{N,t}\frac{\int_0^1 Y_{N,t}(i)\,di}{Y_{N,t}} \\ &= Y_t\frac{\int_0^1 Y_t(i)\,di}{Y_t} \\ N_t^* &\equiv \int_0^1 N_t^*(i)\,di = Y_{F,t}\frac{\int_0^1 Y_{F,t}(i)\,di}{Y_{F,t}} = Y_{N,t}\frac{\int_0^1 Y_{N,t}^*(i)\,di}{Y_{N,t}^*} \\ &= Y_t^*\frac{\int_0^1 Y_t^*(i)\,di}{Y_t^*}. \end{aligned}$$

Furthermore:

$$\frac{\int_0^1 Y_t(i)\,di}{Y_t} = \int_0^1 \left(\frac{P_{P,t}(i)}{P_{P,t}}\right)^{-\theta} \quad ; \quad \frac{\int_0^1 Y_t^*(i)\,di}{Y_t^*} = \int_0^1 \left(\frac{P_t^*(i)}{P_t^*}\right)^{-\theta}.$$

Using these facts, we have:

$$\tilde{n}_t = \tilde{y}_t + \ln \int_0^1 \left(\frac{P_{P,t}(i)}{P_{P,t}}\right)^{-\theta} + \text{t.i.p.} \quad ; \quad \tilde{n}_t^* = \tilde{y}_t^* + \int_0^1 \left(\frac{P_{P,t}^*(i)}{P_{P,t}^*}\right)^{-\theta} + \text{t.i.p.} \tag{71}$$

Lemma 1 derived by Gali and Monacelli[8] is given by:

$$\ln \int_0^1 \left(\frac{P_{P,t}(i)}{P_{P,t}}\right)^{-\theta} = \frac{\theta}{2}\text{var}_i\left(p_{P,t}(i)\right) + o\left(\|\xi\|^3\right).$$

Lemma 2 derived by Woodford[17] is given by:

$$\sum_{t=0}^{\infty} \delta^t \text{var}_i\left(p_{P,t}(i)\right) = \frac{1}{\lambda}\sum_{t=0}^{\infty} \delta^t \pi_{P,t}^2.$$

Combining Lemma 1, Lemma 2, Eqs.(70) and (71), we have:

$$U_t^W = \frac{1}{4}\sum_{t=0}^{\infty} \delta^t \left\{ \frac{\theta}{\lambda}\pi_{P,t}^2 + (1+\varphi)\tilde{y}_t^2 + \frac{\theta}{\lambda}\left(\pi_{P,t}^*\right)^2 + (1+\varphi)\left(\tilde{y}_t^*\right)^2 \right\} + \text{t.i.p.} + o\left(\|\xi\|^3\right).$$

Appendix F Optimal Monetary Policy Rule

The central bank seeks to minimize Eq.(63) subject to Eqs.(46) and (54). The Lagrangian is given by:

$$\mathcal{L} = \text{E}_0 \sum_{t=0}^{\infty} \delta^t 2 \begin{bmatrix} L_t + \mu_{1,t}\left(\tilde{y}_t - \tilde{y}_{t+1} + 2\hat{r}_t - \pi_{P,t+1} - \pi_{P,t+1}^* - \Delta\tilde{y}_{t+1}^*\right) \\ +\mu_{2,t}\left(\tilde{y}_t^* - \tilde{y}_{t+1}^* + 2\hat{r}_t - \pi_{P,t+1}^* - \pi_{P,t+1} - \Delta\tilde{y}_{t+1}\right) \\ +\mu_{3,t}\left(\mathsf{n}_{t+1} - \mathsf{n}_t - \pi_{N,t+1}^R\right) \\ +\mu_{4,t}\left(\pi_{P,t} - \delta\pi_{P,t+1} - \lambda_\varphi\tilde{y}_t + \frac{\psi\lambda}{2}\mathsf{n}_t\right) \\ +\mu_{5,t}\left(\pi_{P,t}^* - \delta\pi_{P,t+1}^* - \lambda_\varphi\tilde{y}_t^* - \frac{\psi\lambda}{2}\mathsf{n}_t\right) \\ +\mu_{6,t}\left(\pi_{N,t}^R - \delta\pi_{N,t+1}^R + \varphi\lambda\tilde{y}_t - \varphi\lambda\tilde{y}_t^* + \lambda\mathsf{n}_t\right) \end{bmatrix}.$$

FONCs associated with this Lagrangian with respect to r_t, $\pi_{P,t}$, $\pi_{P,t}^*$, $\tilde{y}_t$, $\tilde{y}_t^*$, n_t and $\pi_{N,t}^R$ are given by:

$$\begin{aligned} \mu_{1,t} + \mu_{2,t} &= 0 \\ \frac{\theta}{\lambda}\pi_{P,t} + \mu_{4,t} &= 0 \\ \frac{\theta}{\lambda}\pi_{P,t}^* + \mu_{5,t} &= 0 \\ (1+\varphi)\tilde{y}_t + \mu_{1,t} + \mu_{2,t} - \lambda_\varphi\mu_{4,t} + \varphi\lambda\mu_{6,t} &= 0 \\ (1+\varphi)\tilde{y}_t^* + \mu_{1,t} + \mu_{2,t} - \lambda_\varphi\mu_{5,t} + \varphi\lambda\mu_{6,t} &= 0 \\ \mu_{6,t} &= 0 \\ -\mu_{3,t} + \frac{\psi\lambda}{2}\mu_{4,t} - \frac{\psi\lambda}{2}\mu_{5,t} + \lambda\mu_{6,t} &= 0. \end{aligned}$$

Rearranging FONCs, we obtain:

$$\begin{aligned} \tilde{y}_t &= -\theta\pi_{P,t} \\ \tilde{y}_t^* &= -\theta\pi_{P,t}^*. \end{aligned} \tag{72}$$

Combining both first and second equalities in Eq.(72) yields:

$$\tilde{y}_t^W = -\theta\pi_t^W. \tag{73}$$

Following Monacelli[10], let NKPCs be as follows:

$$\begin{aligned} \pi_{P,t} &= \delta \mathrm{E}_t \pi_{P,t+1} + \lambda_\varphi \tilde{y}_t - \frac{\psi\lambda}{2}\mathsf{n}_t + \varepsilon_t \\ \pi_{P,t}^* &= \delta \mathrm{E}_t \pi_{P,t+1}^* + \lambda_\varphi \tilde{y}_t^* + \frac{\psi\lambda}{2}\mathsf{n}_t + \varepsilon_t^* \end{aligned} \tag{74}$$

where ε_t and ε_t^* denote supply shocks that prevent the central bank from simultaneously stabilizing inflation and the output gap in countries' equalities H and F, respectively.[20]

Plugging Eq.(72) into Eq.(74), we have:

$$\begin{aligned} \pi_{P,t} &= \frac{\delta}{1+\lambda_\varphi\theta}\mathrm{E}_t\pi_{P,t+1} - \frac{\psi\lambda}{(1+\lambda_\varphi\theta)\,2}\mathsf{n}_t + \frac{1}{1+\lambda_\varphi\theta}\varepsilon_t \\ \pi_{P,t}^* &= \frac{\delta}{1+\lambda_\varphi\theta}\mathrm{E}_t\pi_{P,t+1}^* + \frac{\psi\lambda}{(1+\lambda_\varphi\theta)\,2}\mathsf{n}_t + \frac{1}{1+\lambda_\varphi\theta}\varepsilon_t^*. \end{aligned}$$

Iterating forward, we have:

$$\begin{aligned} \pi_{P,t} &= \frac{1}{1+\lambda_\varphi\theta}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\lambda_\varphi\theta}\right)^j \varepsilon_{t+j} - \frac{\psi\lambda}{(1+\lambda_\varphi\theta)\,2}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\lambda_\varphi\theta}\right)^j \mathsf{n}_{t+j} \\ \pi_{P,t}^* &= \frac{1}{1+\lambda_\varphi\theta}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\lambda_\varphi\theta}\right)^j \varepsilon_{t+j}^* + \frac{\psi\lambda}{(1+\lambda_\varphi\theta)\,2}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\lambda_\varphi\theta}\right)^j \mathsf{n}_{t+j} \end{aligned}$$

where we use the fact that $\frac{\delta}{1+\lambda_\varphi\theta} < 1$. Taking conditional expectations at t, this can be altered as:

$$\begin{aligned} \pi_{P,t} &= \frac{1}{1+\lambda_\varphi\theta}\varepsilon_t - \frac{\psi\lambda}{(1+\lambda_\varphi\theta)\,2}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\lambda_\varphi\theta}\right)^j \mathsf{n}_{t+j} \\ \pi_{P,t}^* &= \frac{1}{1+\lambda_\varphi\theta}\varepsilon_t^* + \frac{\psi\lambda}{(1+\lambda_\varphi\theta)\,2}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\lambda_\varphi\theta}\right)^j \mathsf{n}_{t+j}. \end{aligned} \tag{75}$$

Combining Eqs.(73) and (75), we have:

$$\pi_t^W = \frac{1}{1+\lambda_\varphi\theta}\varepsilon_t^W \ ; \ \tilde{y}_t^W = -\frac{\theta}{1+\lambda_\varphi\theta}\varepsilon_t^W$$

which implies:

$$\mathrm{E}_t\pi_{t+1}^W = \mathrm{E}_t\tilde{y}_{t+1}^W = 0. \tag{76}$$

[20]While ε_t does not appear explicitly in our model, this is introduced to derive an optimal policy rule. See Monacelli[10] for details.

Combining the first and second equalities in Eq.(46), we obtain:

$$\tilde{y}_t^W = \mathrm{E}_t \tilde{y}_{t+1}^W - 2\hat{r}_t + \mathrm{E}_t \pi_{t+1}^W + \bar{r}_t. \tag{77}$$

Plugging Eqs.(73) and (76) into Eq.(77) yields:

$$\hat{r}_t = \frac{1}{2}\bar{r}_t + \frac{\theta}{2}\pi_{P,t} + \frac{\theta}{2}\pi_{P,t}^*.$$

This equality is Eq.(53) in the text.

Appendix G Optimal Fiscal Policy Rule

The central bank seeks to minimize Eq.(63) subject to Eqs.(46) and (54). The Lagrangian is given by:

$$\mathcal{L} = \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t 2 \begin{bmatrix} L_t + \mu_{1,t}\left(\tilde{y}_t - \tilde{y}_{t+1} + 2\hat{r}_t - \pi_{P,t+1} - \pi_{P,t+1}^* - \Delta\tilde{y}_{t+1}^*\right) \\ +\mu_{2,t}\left(\tilde{y}_t^* - \tilde{y}_{t+1}^* + 2\hat{r}_t - \pi_{P,t+1}^* - \pi_{P,t+1} - \Delta\tilde{y}_{t+1}\right) \\ +\mu_{3,t}\left(\mathsf{n}_{t+1} - \mathsf{n}_t - \pi_{N,t+1}^R\right) \\ +\mu_{4,t}\left(\pi_{P,t} - \delta\pi_{P,t+1} - \lambda_\varphi \tilde{y}_t + \frac{\psi\lambda}{2}\mathsf{n}_t\right) \\ +\mu_{5,t}\left(\pi_{P,t}^* - \delta\pi_{P,t+1}^* - \lambda_\varphi \tilde{y}_t^* - \frac{\psi\lambda}{2}\mathsf{n}_t\right) \\ +\mu_{6,t}\left(\pi_{N,t}^R - \delta\pi_{N,t+1}^R + \varphi\lambda\tilde{y}_t - \varphi\lambda\tilde{y}_t^* + \lambda\mathsf{n}_t + \varphi_\varphi\lambda g_t^R\right) \end{bmatrix}.$$

FONCs associated with this Lagrangian with respect to $\pi_{P,t}$, $\pi_{P,t}^*$, $\tilde{y}_t$, $\tilde{y}_t^*$, n_t, $\pi_{N,t}^R$ and g_t^R are given by:

$$\begin{aligned} \frac{\theta}{\lambda}\pi_{P,t} + \mu_{4,t} &= 0 \\ \frac{\theta}{\lambda}\pi_{P,t}^* + \mu_{5,t} &= 0 \\ (1+\varphi)\,\tilde{y}_t + \mu_{1,t} + \mu_{2,t} - \lambda_\varphi\mu_{4,t} + \varphi\lambda\mu_{6,t} &= 0 \\ (1+\varphi)\,\tilde{y}_t^* + \mu_{1,t} + \mu_{2,t} - \lambda_\varphi\mu_{5,t} + \varphi\lambda\mu_{6,t} &= 0 \\ \mu_{6,t} &= 0 \\ -\mu_{3,t} + \frac{\psi\lambda}{2}\mu_{4,t} - \frac{\psi\lambda}{2}\mu_{5,t} + \lambda\mu_{6,t} &= 0 \\ \mu_{6,t} &= 0. \end{aligned}$$

Rearranging FONCs, we obtain:

$$\theta\pi_{P,t} + \tilde{y}_t = \theta\pi_{P,t}^* + \tilde{y}_t^*.$$

This implies that:

$$\tilde{y}_t^R = -\theta\pi_{P,t}^R. \tag{78}$$

Combining Eq.(74), we have:

$$\pi_{P,t}^R = \delta\mathrm{E}_t\pi_{P,t+1}^R + \lambda_\varphi\tilde{y}_t^R - \psi\lambda\mathsf{n}_t + \varepsilon_t^R$$

with $\varepsilon_t^R \equiv \varepsilon_t - \varepsilon_t^*$. Using Eq.(78), Eq.(79) can be rewritten as:

$$\pi_{P,t}^R = \frac{\delta}{1+\lambda_\varphi\theta}\mathrm{E}_t\pi_{P,t+1}^R - \psi\lambda\mathsf{n}_t + \varepsilon_t^R. \tag{79}$$

Iterating Eq.(79) forward yields:

$$\pi_t^R = \frac{1}{1+\lambda_\varphi \theta}\varepsilon_t^R - \frac{\psi\lambda}{1+\lambda_\varphi\theta}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\lambda_\varphi\theta}\right)^j \mathsf{n}_{t+j}. \tag{80}$$

Putting Eq.(80) forward one period and taking conditional expectation at period t, we have:

$$\mathrm{E}_t \pi_{P,t+1}^R = -\frac{\psi\lambda}{1+\lambda_\varphi\theta}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\lambda_\varphi\theta}\right)^j \mathsf{n}_{t+1+j}. \tag{81}$$

Plugging Eqs.(80) and (81) into Eq.(79) yields:

$$\mathsf{n}_t = \frac{1}{\psi\lambda}\varepsilon_t^R. \tag{82}$$

This implies:

$$\begin{aligned} \mathrm{E}_t \pi_{N,t+1}^R &= -\mathrm{E}_t \mathsf{n}_{t+1} + \mathsf{n}_t \\ &= \frac{1}{\psi\lambda}\varepsilon_t^R. \end{aligned} \tag{83}$$

Plugging Eqs.(82) and (83) into Eq.(47), we obtain:

$$\hat{g}_t^R = \bar{g}_t^R + \theta_\sigma \pi_{P,t}^R + \frac{1}{\varphi\lambda}\pi_{N,t}^R - \frac{\lambda+\delta}{\varphi_\varphi\lambda}\mathsf{n}_t.$$

This is Eq.(55) in the text.

References

[1] Aoki, Kousuke (2001), "Optimal Monetary Policy Responses to Relative-price Changes," Journal of Monetary Economics, 48, 55–80.

[2] Benigno, Gianluca and Christoph Thoenissen (2005), "On the Consumption-Real Exchange Rate Anomaly," Bank of England Working Paper, No. 254.

[3] Benigno, Pierpaolo and Gianluca Benigno (2001b), "Implementing Monetary Cooperation through Inflation Targeting," CEPR Discussion Paper, No.3226.

[4] Benigno, Pierpaolo (2004), "Optimal Monetary Policy in a Currency Area," Journal of International Economics, 63, 293–320.

[5] Canzoneri, Matthew B., Robert E. Cumby and Behzad Diba (1999), "Relative Labor Productivity and the Real Exchange Rate in the Long Run: Evidence of Panel of OECD Countries," Journal of International Economics, 47, 245–266.

[6] Chari, Varadarajan V., P. Kehoe and E. McGrattan (2002), "Can Sticky Prices Generate Volatile and Persistent Real Exchange Rates?," Review of Economic Studies, 69, 533–563.

[7] Gali, Jordi and Tommaso Monacelli (2004), "Optimal Monetary and Fiscal Policy in a Currency Union," mimeo.

[8] Gali, Jordi and Tommaso Monacelli (2005), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72, 707–734.

[9] McKinnon, Ronald I. (1963), "Optimum Currency Areas," *American Economic Review*, 53, 717–725.

[10] Monacelli, Tommaso (2004), "Principles of Optimal Monetary Policy," Lecture Notes.

[11] Mundel, Robert A. (1961). "A Theory of Optimum Currency Areas," *American Economic Review*, 51, 657–665.

[12] Obstfeld, Maurice and Kenneth Rogoff (2000), "New Directions for Stochastic Open Economy Models," *Journal of International Economics*, 50, 117–153.

[13] Okano, Eiji (2005), "The Choice of the Inflation Rate as a Target in an Economy with Pricing to Market," *Japan and the World Economy*, forthcoming.

[14] Rotemberg, Julio J. and Micael Woodford (1997), "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," In Bernanke, B. S. and Rotemberg, J. J. (Eds.), *NBER Macroeconomic Annual*, 12, 297–346.

[15] Selaive, Jorge and Vicente Tuesta (2006), "The Consumption-Real Exchange Rate Anomaly: Non tradable Goods, Incomplete Markets and Distribution Services," *Central Bank of Chile Working Papers*, No. 359.

[16] Stockman, Alan C. and Linda L. Tesar (1995), "Tastes and Technology in a Two-Country Model of the Business Cycle: Expanding International Comovements," *American Economic Review*, 85, 168–185.

[17] Woodford, Michael (2001), "Inflation Stabilization and Welfare," *NBER Working Paper*, No. 8071.

Table 1: Macroeconomic Volatility under Optimal Monetary Policy without Fiscal Policy: Special Case

$\gamma = 1$, $\eta = 0.44$

Variable	Shocks				
	$a_{H,t}$	$a_{N,t}$	$a_{F,t}$	$a^*_{N,t}$	d^W_t
$\tilde{y}_t$	0.0000	0.0000	0.0000	0.0000	0.0000
$\tilde{y}^*_t$	0.0000	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}$	0.0000	0.0000	0.0000	0.0000	0.0000
$\pi^*_{P,t}$	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{r}_t$	0.0393	0.0000	0.0393	0.0000	0.2359
n_t	0.8966	1.1955	0.8966	1.1955	0.0000
q_t	0.0000	0.0000	0.0000	0.0000	0.0000
$\widehat{ca}_t$	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{g}_t$	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{g}^*_t$	0.0000	0.0000	0.0000	0.0000	0.0000
y_t	0.0786	0.0000	0.0000	0.0000	0.0786
y^*_t	0.0000	0.0000	0.0786	0.0000	0.0786

$\gamma = 0.5$, $\eta = 1$

Variable	Shocks				
	$a_{H,t}$	$a_{N,t}$	$a_{F,t}$	$a^*_{N,t}$	d^W_t
$\tilde{y}_t$	0.0000	0.0000	0.0000	0.0000	0.0000
$\tilde{y}^*_t$	0.0000	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}$	0.0000	0.0000	0.0000	0.0000	0.0000
$\pi^*_{P,t}$	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{r}_t$	0.0197	0.0197	0.0197	0.0197	0.2359
n_t	0.4483	0.7472	0.4483	0.7472	0.0000
q_t	0.2242	0.3736	0.2242	0.3736	0.0000
$\widehat{ca}_t$	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{g}_t$	0.0786	0.1311	0.0786	0.1311	0.0000
$\hat{g}^*_t$	0.0000	0.0000	0.0000	0.0000	0.0000
y_t	0.0393	0.0393	0.0000	0.0000	0.0786
y^*_t	0.0000	0.0000	0.0393	0.0393	0.0786

Table 2: Macroeconomic Volatility under Alternative Regimes: Benchmark

Variable	Regime	Shocks				
		$a_{H,t}$	$a_{N,t}$	$a_{F,t}$	$a^*_{N,t}$	d^W_t
$\tilde{y}_t$	MP	0.0131	0.0218	0.0131	0.0218	0.0000
	Mix	0.0000	0.0000	0.0000	0.0000	0.0000
$\tilde{y}^*_t$	MP	0.0131	0.0218	0.0131	0.0218	0.0000
	Mix	0.0000	0.0000	0.0000	0.0000	0.0000
$\pi_{P,t}$	MP	0.0000	0.0000	0.0000	0.0000	0.0000
	Mix	0.0000	0.0000	0.0000	0.0000	0.0000
$\pi^*_{P,t}$	MP	0.0000	0.0000	0.0000	0.0000	0.0000
	Mix	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{r}_t$	MP	0.0197	0.0197	0.0197	0.0197	0.2359
	Mix	0.0197	0.0197	0.0197	0.0197	0.2359
n_t	MP	0.7460	1.2434	0.7460	1.2434	0.0000
	Mix	0.0000	0.0000	0.0000	0.0000	0.0000
q_t	MP	0.3730	0.6217	0.3730	0.6217	0.0000
	Mix	0.0000	0.0000	0.0000	0.0000	0.0000
$\widehat{ca}_t$	MP	0.0522	0.0870	0.0522	0.0870	0.0000
	Mix	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{g}_t$	MP	0.0000	0.0000	0.0000	0.0000	0.0000
	Mix	0.0786	0.1311	0.0786	0.1311	0.0000
$\hat{g}^*_t$	MP	0.0000	0.0000	0.0000	0.0000	0.0000
	Mix	0.0786	0.1311	0.0786	0.1311	0.0000
y_t	MP	0.0524	0.0176	0.0524	0.0176	0.0786
	Mix	0.0197	0.0721	0.0197	0.0328	0.0786
y^*_t	MP	0.0131	0.0218	0.0131	0.0218	0.0786
	Mix	0.0197	0.0328	0.0197	0.0721	0.0786

MP: Monetary Policy without Fiscal Policy
Mix: Optimal Monetary and Fiscal Policy Mix

Figure 1: Impulse Responses to Shocks under Optimal Monetary Policy without Fiscal Policy: Special Case ($\gamma = 1$, $\eta = 0.44$)

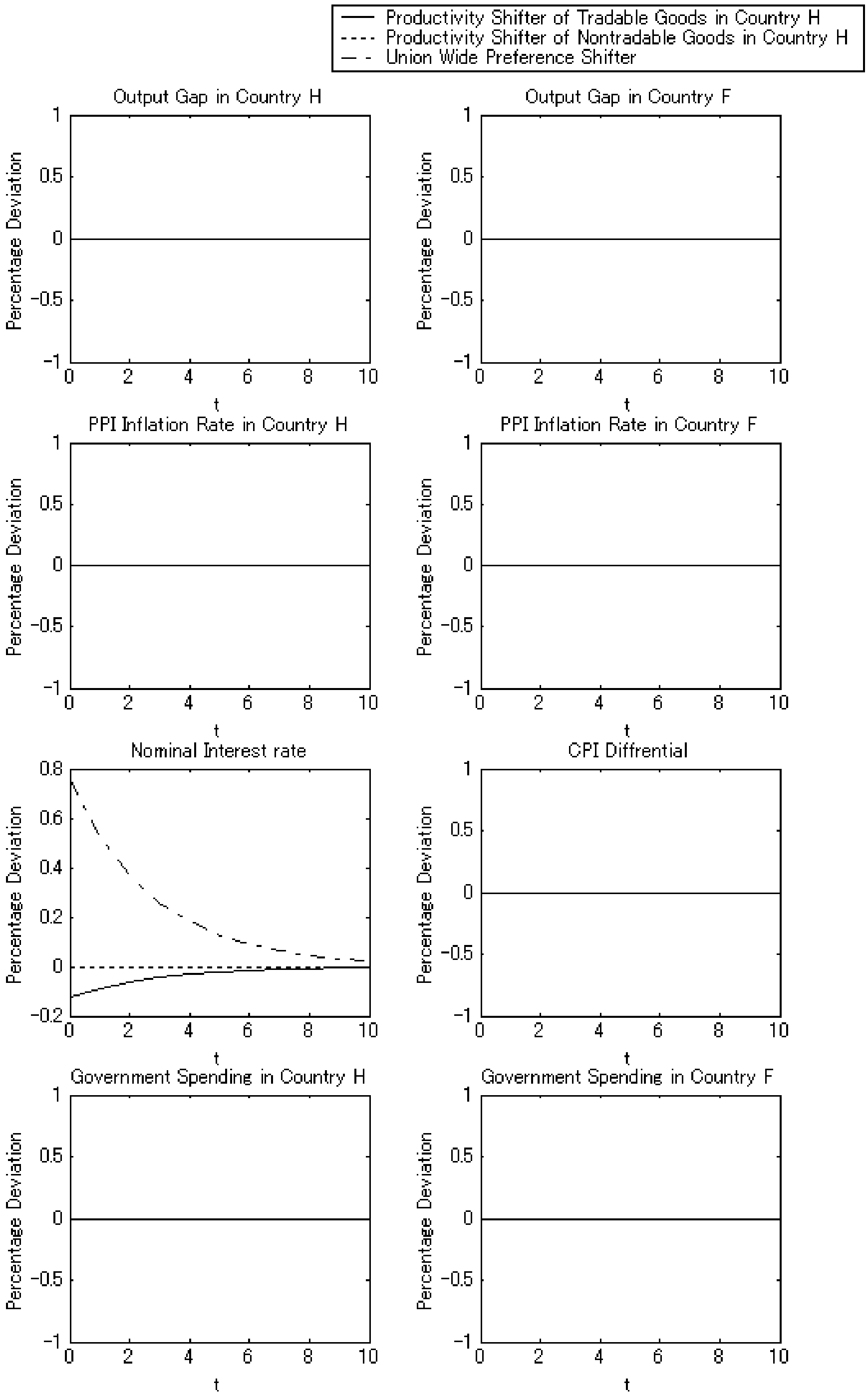

Figure 2: Impulse Responses to Shocks under Optimal Monetary Policy without Fiscal Policy: Special Case ($\gamma = 0.5$, $\eta = 1$)

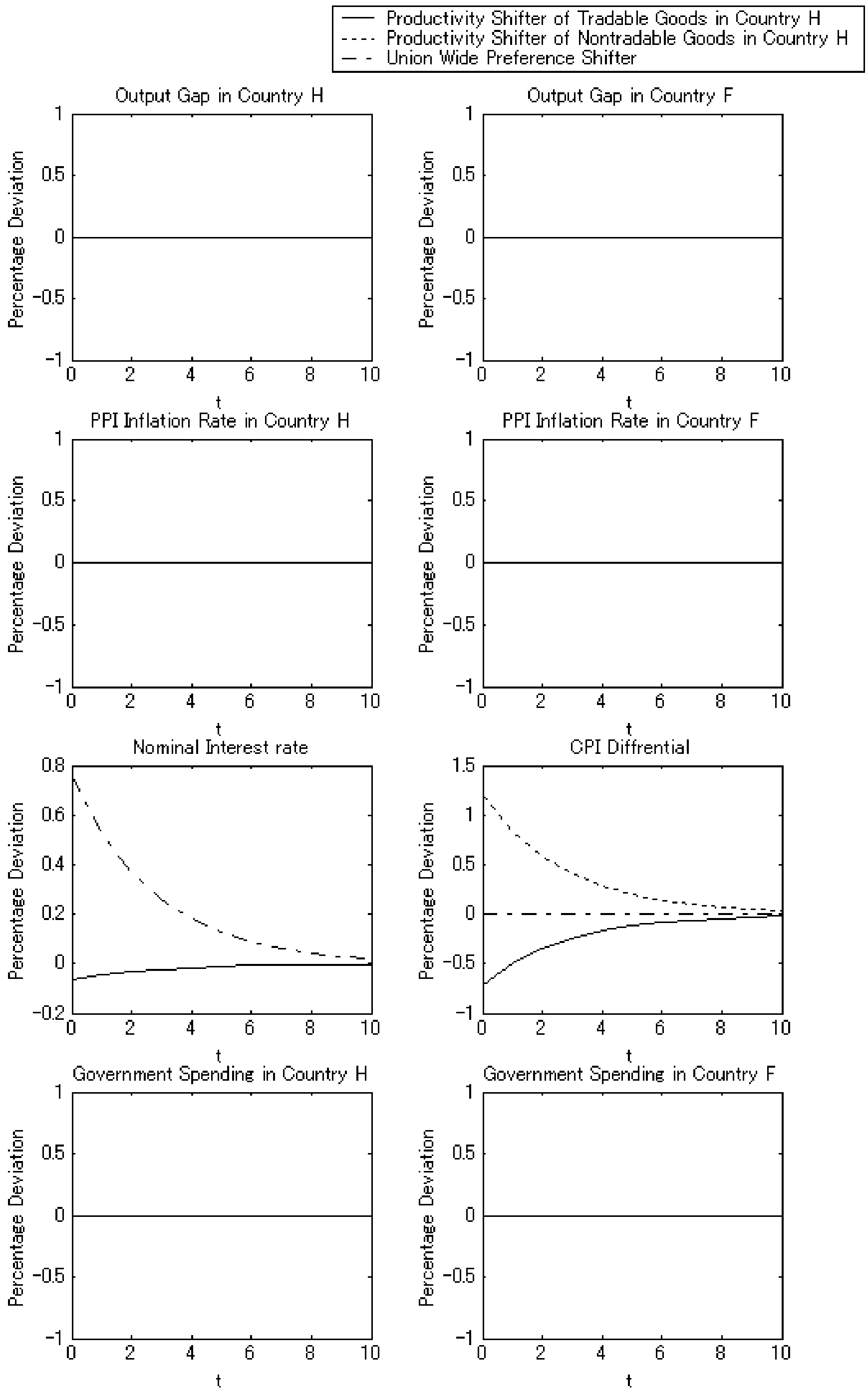

Figure 3: Impulse Responses to Shocks under Optimal Monetary Policy without Fiscal Policy: Benchmark

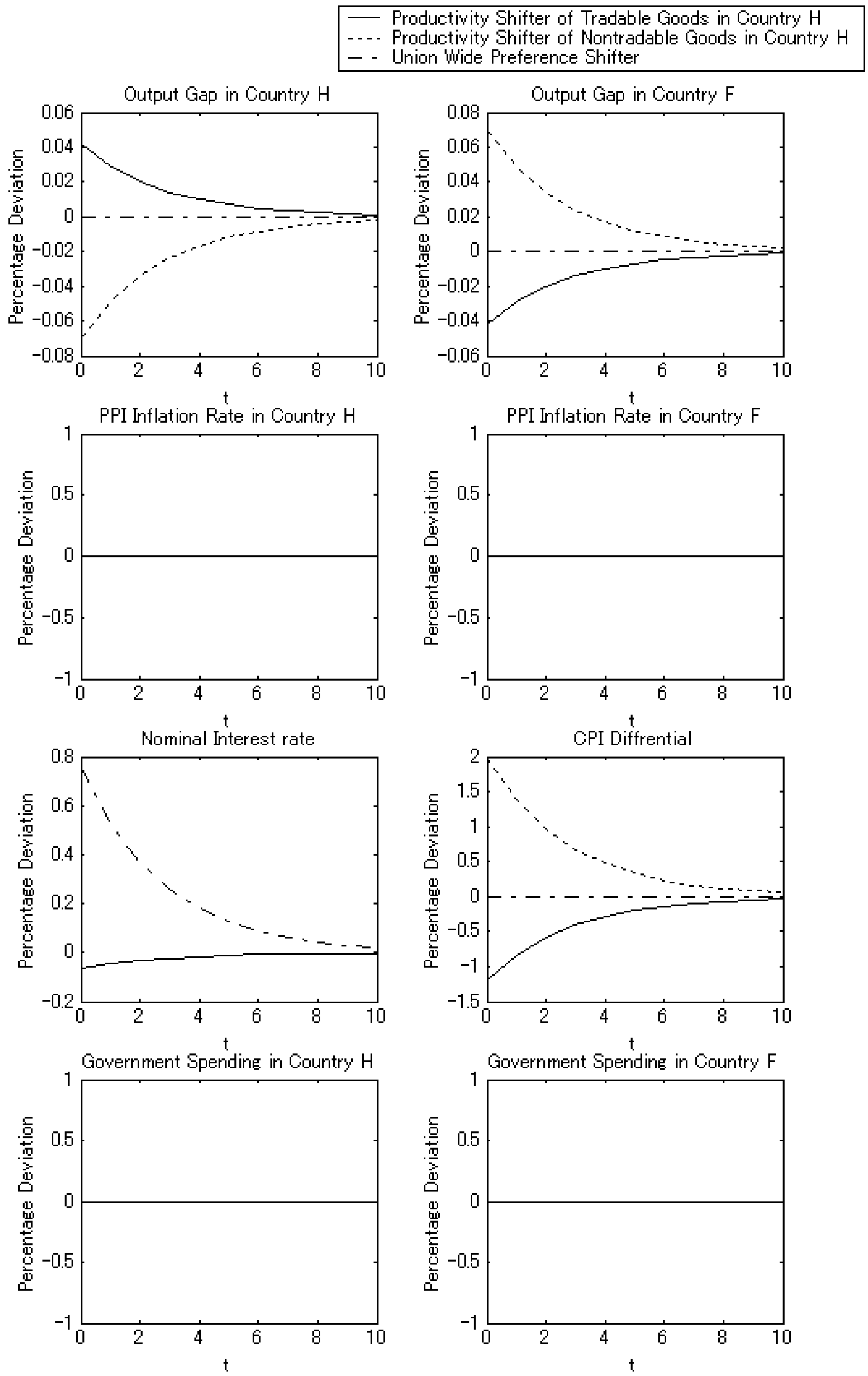

Figure 4: Impulse Responses to Shocks under Optimal Monetary and Fiscal Policy Mix: Benchmark

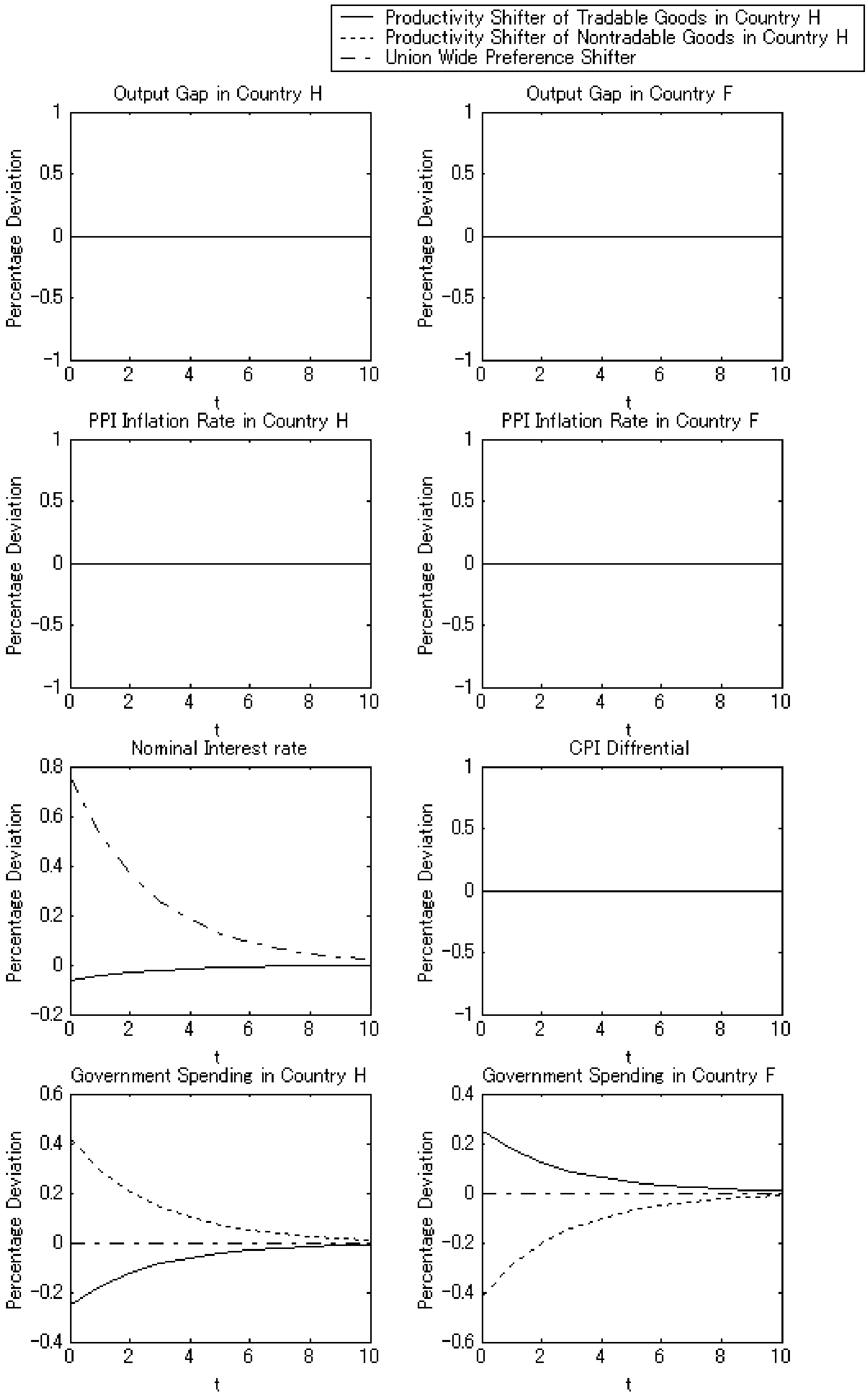

Figure 5: Effect on Volatility of Varying Share of Nontradable Goods

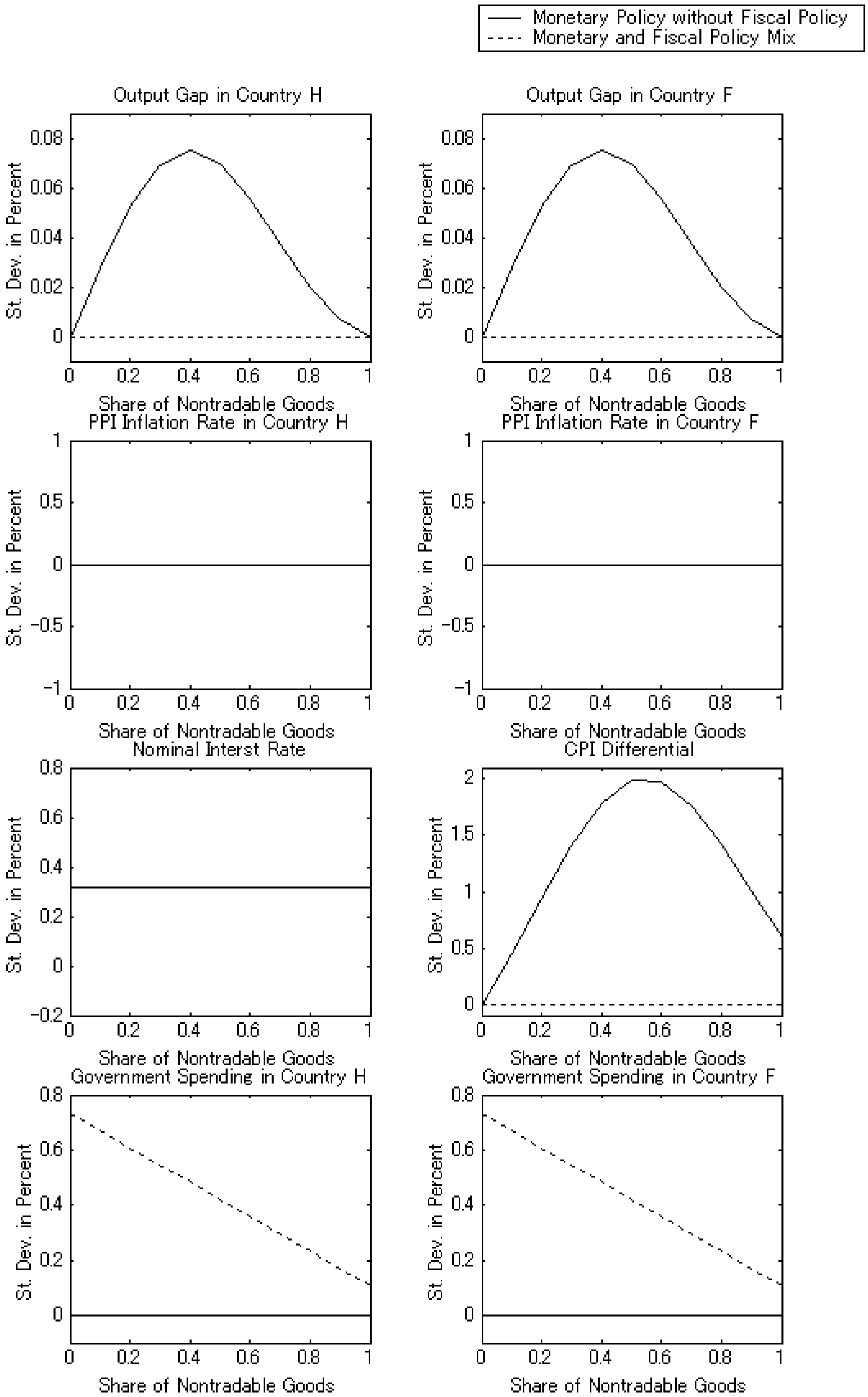

Figure 6: Effect on Welfare of Varying Share of Nontradable Goods