Optimal Monetary Policy under Heterogeneous Banks*

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Abstract

We introduce the heterogeneous stickinesses in loan interest rate adjustments, which are supported by empirical analyses in developed countries, into the standard New Keynesian model with bank sector. The welfare analysis reveals that the central bank should care about the interest rate difference, i.e. credit spread, between heterogeneous loan interest rates and the first differences of each loan interest rate. We show that the central bank should negatively respond to the credit spread shock between heterogeneous loan interest rates, i.e. idiosyncratic shock in the financial market. There the heterogeneity in staggeredness of loan interest rates rather than the credit spread is quantitatively so important to implement the optimal monetary policy when stickinesses of loan interest rates are high. Finally, we show that the central bank puts its priority to the loan interest rate with more stickiness rather than a weighted average of heterogeneous loan interest rate to achieve the optimal monetary policy when the market share of loans is not so different. But, as the share of the sticky loan decreases, the central bank should pay attention to both sticky and less sticky loans.

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1 Introduction

Conventional theory of monetary policy has been focusing on a channel through which a policy shock directly affects the demand side of the economy (e.g. Rotemberg and Woodford (1997) and Goodfriend and King (1997)). Recent empirical research by Barth and Ramey (2001), on the other hand, present ample evidence that a monetary policy shock works as a cost-push shock to the firms. They argue that when working capital is an essential component of production and the firms' cost is closely tied to a policy interest rate, a monetary policy shock affects the output through a supply side channel as well as the traditional demand side channel.

Incorporating this cost channel mechanism into the theoretical framework requires a consideration for banks, since in most of the countries, monetary authorities are not the direct loan supplier to firms. In Ravenna and Walsh (2006), where flexible cost channel is studied in the New Keynesian (NK) framework, firms borrow money from banks for the purchase of their inputs, and so their marginal costs are directly affected by the loan rates variations. Banks are playing important role in the model, as they transmit a shock of the policy interest rate to firms' cost structure, via a bank lending channel.

How banks transmit monetary policy shocks to their behavior is studied empirically in several aspects (e.g. Kashyap and Stein (2000), Hannan and Berger (1991), Slovin and Sushka (1983), and Berger and Udel (1992)).[1] In terms of loan interest rates, macroeconomist are aware of the two facts on banks' responses to a policy interest rate shock: (*i*) loan

[1] Kashyap and Stein (2000) examines the impact of the policy rate change on lending volume. Hannan and Berger (1991) investigates the impact of the policy rate change on the deposit rates. Slovin and Sushka (1983) and Berger and Udel (1992) investigate the impact of the policy rate change on the loan rates.

interest rates set by banks are sticky compared to a policy interest rate and (*ii*) responses of the loan interest rates to a policy interest rates are not uniform across banks depending on the characteristics of each banks.[2] Slovin and Sushka (1983) estimate the time series determinants of interest rates on US commercial loans and show that commercial loan rates respond less than one-for-one to changes in the market rates in the short run. Berger and Udel (1992) also indicate that bank loan interest rates are sticky compared to a policy interest rate using US panel data. Moreover, many empirical studies for Euro Area report that the heterogeneity in loan stickiness are observed across banks. For example, Gambacorta (2004) shows that the short-run heterogeneity in loan rates are present among Italian banks. He points out that well-capitalized banks respond quicker to a shock. De Graeve et al (2007) and Weth (2002) report the analogous results for Belgium and Germany, respectively.[3]

The first contribution of the paper is that we incorporate this heterogeneity into the NK model with bank sector. We assume the monopolistic competition in a bank lending market and introduce heterogeneous stickiness in loan interest rate settings. Thus, some banks adjust the loan rates more frequently than other banks do so that the responses of the loan rates to a innovation in the policy interest rate differ across banks.

The second contribution is that we think of the optimal monetary policy under a second orderly approximated welfare function. Welfare analysis reveals that the central bank should respond to the interest rate difference, i.e. credit spread, between loan interest rates and heterogeneous first differences of loan interest rates. There the heterogeneity in stag-

[2]Kashyap and Stein (2000) reports a heterogeneity in banks responses to a monetary policy shock, in terms of the lending volume. They claim that banks with less liquid balance sheet are likely to respond more to policy shocks.

[3]De Graeve et al (2007) obtains an incomplete and heterogenous pass-through in the loan market in Belgian. They report that the degree of capitalization and the size of liquidity are responsible for the diverse response of banking sectors to a change in market rates. Weth (2002) examines German markets and concludes that lending rates are stickier for small banks, banks with high savings deposits and banks with a high volume of non-bank business.

geredness of loan interest rates rather than the credit spread is quantitatively so important to implement the optimal monetary policy when stickinesses of loan interest rates are high. But, when stickinesses of loan interest rates are low enough, the credit spread quantitatively becomes more crucial element to conduct the optimal monetary policy. Provided this heterogeneity across banks, we derive an optimal monetary policy rule. The simulation results show that the central bank puts its priority to the loan interest rate with more stickiness rather than a weighted average of heterogeneous loan interest rate to achieve the optimal monetary policy when the market share of loans is not so different. But, as the share of the sticky loan decreases, the central bank should pay attention to both sticky and less sticky loans.

The third contribution is that we discuss the current issue represented by Taylor (2008) and Cúrdia and Woodford (2008). Now, many papers investigate whether the central banks should respond to the credit spread of interest rates that economic agents face (e.g. Taylor (2008), McCulley and Toloui (2008), and Cúrdia and Woodford (2008)). Taylor (2008) implies that the Federal Reserve Board have negatively reacted to the credit spread in the money market for the last few years to stimulate economy even though such an additional easing eventually induces the economic boom leading to the sub-prime mortgage loan problem from fall of 2007[4]. In contrast to Taylor (2008), Cúrdia and Woodford (2008) theoretically investigate whether a central bank should react to the credit spread between saver and borrower in consumers. They conclude that the central bank qualitatively should pay attention to the credit spread but the optimal monetary policy in the basic NK model without credit spread still quantitatively provides a good approximation to the optimal monetary policy in the NK model with credit spread. Our conclusion is that the credit

[4]The response to the credit spread is also suggested in McCulley and Toloui (2008) that show a one-for-one correspondence of the policy rate to the market credit spread in the explicit interest rate rule in US.

spread is quantitatively not so important to conduct the optimal monetary policy, which support Cúrdia and Woodford (2008). But, we also show that the standard optimal monetary policy can not be a good approximation in heterogeneous financial market. This means that the heterogeneity in staggeredness of loan interest rates is very crucial to achieve the optimal monetary policy.

The paper is organized as follows. Section 2 shows the empirical evidence. Section 3 describes our model with staggered loan rates. Section 4 analyzes the welfare implication in our model. Section 5 investigates the properties of optimal monetary policy. Section 6 gives discussion. Section 7 concludes.

2 Facts

The heterogenous stickiness of loan rates are observed in a large literature since there are variety of ways that pricing decisions can differ across banks.

A coexistence of multiple banking systems under a single currency is one source of the heterogeneity. Introduction of euro provides its clear example. In the Euro area, while a monetary policy is conducted by one institution, the pricing behaviors of banks are different across countreies, maybe reflecting the differences in the degree of competition (Sorensen and Werner (2006)). Sorensen and Werner (2006) estimate the stickiness of bank loan rates for ten countries in the euro area. They report that even controlling for the loan types (e.g., Mortgage loans, Consumer loans), the speed of loan rate adjustment across contries is different in statistically significant manner. Their results for the stickiness of "long-term loans rate to enterprises" category imply that upon the innovation in the policy rate by 100 basis, loan rates in Germany is reverted by 45% while those in Belgium is reverted only by 10% in the next period. As we will discuss below, with these observed heterogeneity in the loan interest rate dynamics across countries in the euro area, our model provides a fair description that ECB faces after the uniozation.[5]

The stickiness of loan rates may differ within a country reflecting the variety of bank types. The response of loan rates to a change in the market rate differ, depending on the size of banks, liability of banks. For each country of Euro, existing empirical studies have provided the ample evidence for the heterogeneity. Gambacorta (2004) investigates the Italian banks to find out the banks' liability structures are the important determinants of heterogenous responses of loan rates across banks. Weth (2002) reports for the German banks that large banks and banks with few saving deposits adjust their lending rates more quickely. Similarly for Japan, according to the report released from Bank of Japan (BOJ (2007, 2008)), the loan rates set by larger banks (City Banks) tend to respond quicker than those set by smaller banks (Regional Banks, Regional Banks II) do.[6] Thus the consideration for the heterogeneity in loan rate stickiness is also important for the implementation of monetary policy for one country.

Heterogeneity of loan rate stickiness arises from the differences associated with the loan type, too. In Sorensen and Werner (2006), observed loan stickiness set by banks may vary depending on the terms of the loan, such as the type of borrowers or the length of the loan

[5] Ito and Ueda (1981) shows that the prime loan interest rate adjustment speeds are very different between U.S. and Japan. They conclude that the prime loan rate adjustment in U.S. is much faster than that in Japan.

[6] BOJ (2007) reports that City Bank needs about three to four quarters and Regional Banks and Regional Banks II need about five to six quarters to adjust the loan interest rates in the first two quarters of 2006. Bank of Japan releases the loan rate series for several groups of banks. City Banks are the banks that have nation-wide brunches, whose main business activities are basing on large cities. Regional Banks and Regional Banks II are comparatively smaller size of banks and most of their brunches are limited in specific prefectures. The table below reports the summary statistics to illustrate the characteristics of each bank group.

	Number of Banks	Share in Deposit	Share in Loans/Discounts
City Banks	6	31.7%	33.6%
Regional Banks	64	23.4%	26.1%
Regional Banks II	42	6.5%	7.6%

contract. The study for Belgian banks by De Graeve et al. (2007) show that short-term pass-through for invest loan rate is larger than consumer credit or mortgage. Berger and Udel (1992), using the panel data of the U.S. banks, show that the stickiness of the loan rate is larger for secured loans and fixed-rate loans, than unsecured loans and floating-rate loans.

3 Model

We introduce the heterogeneous staggered nominal loan interest rate contracts between private banks and firms into a model based on a standard NK framework built by Woodford (2003). The model consists of four agents: consumers, firms, a central bank, and private banks.

3.1 Cost Minimization

In this model, we have two cost minimization problems. The first determines the optimal allocation of differentiated goods for the consumer. The second determines the optimal allocation of labor services, given the loan rates and wages for the firm's president.

For the consumer, we assume that the consumer's utility from consumption is increasing and concave in the consumption index, which is defined as a Dixit-Stiglitz aggregator as in Dixit and Stiglitz (1977), of bundles of differentiated goods $f \in [0,1]$ produced by firm's project groups as

$$C_t \equiv \left[\int_0^1 c_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}},$$

where C_t is aggregate consumption, $c_t(f)$ is a particular differentiated good along a continuum produced by the firm's project group f, and $\theta > 1$ is the elasticity of substitution across goods produced by project groups. For the consumption aggregator, the appropriate consumption-based price index is given by

$$P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}},$$

where P_t is aggregate price and $p_t(f)$ is the price on a particular differentiated good $c_t(f)$. As in other applications of the Dixit-Stiglitz aggregator, the consumer's allocation across differentiated goods at each time must solve a cost minimization problem. This means that the relative expenditures on a particular good is decided according to:

$$c_t(f) = C_t \left[\frac{p_t(f)}{P_t} \right]^{-\theta}. \tag{1}$$

An advantage of this consumption distribution rule is to imply that the consumer's total expenditure on consumption goods is given by $P_t C_t$. We use this demand function for differentiated goods in the firm sector.

Firms optimally hire differentiated labor as price takers. This optimal labor allocation is carried out through two-step cost minimization problems. Firm f hires all types of labor. There, each firm has to use two types of loan, sticky loans and less sticky loans. Private banks reset loan interest rates with longer interval in sticky loan and they reset with shorter interval in less sticky loan. To replicate this situation, we assume that to finance a labor cost for labor type $h \in [0, n)$, the firm has to use sticky loan, and to finance the cost for labor type $\overline{h} \in [n, 1]$, it has to use less sticky loan. We can think of this setting as a firm uses sticky loan to some project which is characterized by labor type h, but uses less sticky loan to some project which is characterized by labor type $\overline{h}$. (more and more) When hiring a labor from $h \in [0, n)$, portion of the labor cost associated with labor type h, which we denote as γ, is financed by borrowing from the bank h. Then, the first-step cost minimization problem for the allocation of differentiated labor from $h \in [0, n)$ is given by:

$$\min_{l_t(h,f)} \int_0^n \left[1 + \gamma r_t\left(h\right)\right] w_t\left(h\right) l_t(h,f) dh,$$

subject to the aggregate domestic labor supply to firm f:

$$L_t\left(f\right) \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n l_t\left(h,f\right)^{\frac{\epsilon-1}{\epsilon}} dh\right]^{\frac{\epsilon}{\epsilon-1}},$$

where $r_t(h)$ is sticky loan interest rate applied to the employment of a particular labor type h, $l_t(h,f)$ is the differentiated labor input with respect to h that is supplied to firm f, and ϵ is a preference parameter on differentiated labors. The sticky loan bank h has some monopoly power over setting loan interest rates. Thus, we assume the monopolistic competition on the loan market where the transaction between banks and firms take place. The relative demand on differentiated labor is given as follows:

$$l_t\left(h,f\right) = \frac{1}{n} L_t \left\{\frac{\left[1 + \gamma r_t\left(h\right)\right] w_t\left(h\right)}{\Omega_t}\right\}^{-\epsilon}, \tag{2}$$

where

$$\Omega_t \equiv \left\{\frac{1}{n} \int_0^n \left\{\left[1 + \gamma r_t\left(h\right)\right] w_t\left(h\right)\right\}^{1-\epsilon} dh\right\}^{\frac{1}{1-\epsilon}}. \tag{3}$$

As a result, we can derive:

$$\int_0^n \left[1 + \gamma r_t\left(h\right)\right] w_t\left(h\right) l_t\left(h,f\right) dh = \Omega_t L_t\left(f\right).$$

Through a similar cost minimization problem, we can derive the relative demand for each type of differentiated labor from $\overline{h} \in [n, 1]$ as:

$$l_t\left(\overline{h},f\right) = \frac{1}{1-n} \overline{L}_t \left\{\frac{\left[1 + \overline{\gamma} r_t^*\left(\overline{h}\right)\right] w_t\left(\overline{h}\right)}{\overline{\Omega}_t}\right\}^{-\epsilon}, \tag{4}$$

where

$$\overline{\Omega}_t \equiv \left\{\frac{1}{1-n} \int_n^1 \left\{\left[1 + \overline{\gamma} r_t^*\left(\overline{h}\right)\right] w_t\left(\overline{h}\right)\right\}^{1-\epsilon} d\overline{h}\right\}^{\frac{1}{1-\epsilon}}, \tag{5}$$

and where $r_t^*\left(\overline{h}\right)$ is the less sticky loan interest rate, and $\overline{\gamma}$ is a portion of the labor cost financed by bank $\overline{h}$. Then:

$$\int_n^1 \left[1+\overline{\gamma}r_t^*\left(\overline{h}\right)\right] w_t\left(\overline{h}\right) l_t\left(\overline{h},f\right) dh = \overline{\Omega}_t \overline{L}_t\left(f\right).$$

According to the above two optimality conditions, the firms optimally choose the allocation of differentiated workers between the two groups. Because firms have production function that hires n workers from $h \in [0,n)$ and $(1-n)$ workers from $\overline{h} \in [n,1]$, the second-step cost minimization problem describing the allocation of differentiated labor between these two groups is given by:

$$\min_{L_t,\overline{L}_t} \Omega_t L_t\left(f\right) + \overline{\Omega}_t \overline{L}_t\left(f\right),$$

subject to the labor index:

$$\widetilde{L}_t\left(f\right) \equiv \frac{\left[L_t\left(f\right)\right]^n \left[\overline{L}_t\left(f\right)\right]^{1-n}}{n^n\left(1-n\right)^{1-n}}. \tag{6}$$

Then, the relative demand functions for each differentiated type of labor are derived as follows:

$$L_t\left(f\right) = n\widetilde{L}_t\left(f\right)\left(\frac{\Omega_t}{\widetilde{\Omega}_t}\right)^{-1}, \tag{7}$$

$$\overline{L}_t\left(f\right) = (1-n)\widetilde{L}_t\left(f\right)\left(\frac{\overline{\Omega}_t}{\widetilde{\Omega}_t}\right)^{-1}, \tag{8}$$

and

$$\widetilde{\Omega}_t \equiv \Omega_t^n \overline{\Omega}_t^{1-n}.$$

Therefore, we can obtain the following equations:

$$\Omega_t L_t\left(f\right) + \overline{\Omega}_t \overline{L}_t\left(f\right) = \widetilde{\Omega}_t \widetilde{L}_t\left(f\right),$$

$$l_t\left(h,f\right) = \left\{\frac{\left[1+\gamma r_t\left(h\right)\right] w_t\left(h\right)}{\Omega_t}\right\}^{-\epsilon}\left(\frac{\Omega_t}{\widetilde{\Omega}_t}\right)^{-1}\widetilde{L}_t\left(f\right), \tag{9}$$

and

$$l_t\left(\overline{h}, f\right) = \left\{ \frac{\left[1 + \gamma r_t\left(\overline{h}\right)\right] w_t\left(\overline{h}\right)}{\overline{\Omega}_t} \right\}^{-\epsilon} \left(\frac{\overline{\Omega}_t}{\widetilde{\Omega}_t} \right)^{-1} \widetilde{L}_t\left(f\right), \tag{10}$$

from equations (2), (4), (7), and (8). We can now clearly see that the demand for each differentiated worker depends on wages and loan interest rates, given the total demand for labor.

Finally, from the assumption that the firms finance part of the labor costs by loans, we can derive:

$$\begin{aligned} q_t\left(h, f\right) &= \gamma w_t\left(h\right) l_t\left(h, f\right) \\ &= \gamma w_t\left(h\right) \left\{ \frac{\left[1 + \gamma r_t(h)\right] w_t\left(h\right)}{\Omega_t} \right\}^{-\epsilon} \left(\frac{\Omega_t}{\widetilde{\Omega}_t} \right)^{-1} \widetilde{L}_t\left(f\right), \end{aligned}$$

and

$$\begin{aligned} q_t\left(\overline{h}, f\right) &= \overline{\gamma} w_t\left(\overline{h}\right) l_t\left(\overline{h}, f\right) \\ &= \overline{\gamma} w_t\left(\overline{h}\right) \left\{ \frac{\left[1 + \overline{\gamma} r_t^*\left(\overline{h}\right)\right] w_t\left(\overline{h}\right)}{\overline{\Omega}_t} \right\}^{-\epsilon} \left(\frac{\overline{\Omega}_t}{\widetilde{\Omega}_t} \right)^{-1} \widetilde{L}_t\left(f\right). \end{aligned}$$

These conditions demonstrate that the demands for each differentiated loan also depend on the wages and loan interest rates, given the total labor demand.

For aggregate labor demand conditions, we can obtain following expression:

$$\widetilde{L}_t = \int_0^1 \widetilde{L}_t\left(f\right) df.$$

3.2 Consumer

We consider the representative consumer who derives utility from consumption and disutility from a supply of work. The consumer maximizes the following utility function:

$$UT_t = \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T) - \int_0^n V(l_T(h)) dh - \int_n^1 V(l_T(\overline{h})) d\overline{h} \right] \right\},$$

where E_t is an expectation conditional on the state of nature at data t. The function U is increasing and concave in the consumption index as shown in the last subsection. The budget constraint of the consumer is given by

$$\begin{aligned} P_t C_t + \mathrm{E}_t\left[X_{t,t+1} B_{t+1}\right] + D_t \;\; &\leq \;\; B_t + (1+i_{t-1}) D_{t-1} + \int_0^n w_t(h) l_t(h) dh \\ &\quad + \int_n^1 w_t(\overline{h}) l_t(\overline{h}) d\overline{h} + \Pi_t^B + \Pi_t^F, \qquad (11) \end{aligned}$$

where B_t is a risky asset, D_t is the amount of bank deposit, i_t is the nominal deposit rate set by the central bank from t to $t+1$, $w_t(h)$ is the nominal wage for labor supply, $l_t(h)$, to the firm's business unit of type h, $\Pi_t^B = \int_0^1 \Pi_{t-1}^B(h) dh$ is the nominal dividend stemming from the ownership of banks, $\Pi_t^F = \int_0^1 \Pi_{t-1}^F(f) df$ is the nominal dividend from the ownership of the firms, and $X_{t,t+1}$ is the stochastic discount factor. We assume a complete financial market for risky assets. Thus, we can hold a unique discount factor and can characterize the relationship between the deposit rate and the stochastic discount factor:

$$\frac{1}{1+i_t} = \mathrm{E}_t\left[X_{t,t+1}\right]. \qquad (12)$$

Given the optimal allocation of consumption expenditure across the differentiated goods, the consumer must choose the total amount of consumption, the optimal amount of risky assets to hold, and an optimal amount to deposit in each period. Necessary and sufficient conditions are given by

$$U_C(C_t, \nu_t) = \beta(1+i_t)\mathrm{E}_t\left[U_C(C_{t+1}, \nu_{t+1})\frac{P_t}{P_{t+1}}\right], \qquad (13)$$

$$\frac{U_C(C_t, \nu_t)}{U_C(C_{t+1}, \nu_{t+1})} = \frac{\beta}{X_{t,t+1}} \frac{P_t}{P_{t+1}}.$$

Together with equation (12), we can find that the condition given by equation (13) expresses the intertemporal optimal allocation on aggregate consumption. Assuming that the market clears, so that the supply of each differentiated good equals its demand, $c_t(f) = y_t(f)$ and $C_t = Y_t$, we finally obtain the standard New Keynesian IS curve by log-linearizing equation (13):

$$x_t = E_t x_{t+1} - \sigma(\widehat{i}_t - E_t \pi_{t+1} - \widehat{r}_t^n), \tag{14}$$

where we name x_t the output gap that is defined in the next section, π_{t+1} inflation, and $\widehat{r}_t^n$ the natural rate of interest. $\widehat{r}_t^n$ will be an exogenous shock. Each variable is defined as the log deviation from its steady states (except x_t and π_t. Also, the log-linearized version of variable m_t is expressed by $\widehat{m}_t = \ln(m_t/\overline{m})$, where $\overline{m}$ is steady state value of m_t.). We define $\sigma \equiv -\frac{U_Y}{U_{YY}\overline{Y}} > 0$.

In this model, the consumer provides differentiated types of labor to the firm and so holds the power to decide the wage of each type of labor as in Erceg, Henderson and Levin (2000). We assume that each project group hires all types of workers in the same proportion. The consumer sets each wage $w_t(h)$ for any h in every period to maximize its utility subject to the budget constraint given by equation (11) and the demand function of labor given by equation (2).[7] Then we have the following relation

$$\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{V_l\left[l_t(h)\right]}{U_C(C_t)}, \tag{15}$$

and

$$\frac{w_t(\overline{h})}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{V_l\left[l_t(\overline{h})\right]}{U_C(C_t)}. \tag{16}$$

In this paper, we assume that the consumer supplies its labors only for the firm, not for the private bank. We use the relations given by equations (15) and (16) in the firm side.

[7] We assume a flexible wage setting in a sense that the consumer can change wage in every period.

3.3 Firms

There exists a continuum of firms populated over unit mass $[0,1]$. Each firm plays two roles. First, each firm decides the amount of differentiated labor to be employed from both $h \in [0,n)$ and $\overline{h} \in [n,1]$, through the two-step cost minimization problem on the production cost. Part of the costs of labor must be financed by external loans from banks. For example, to finance the costs of hiring workers from $h \in [0,n)$, the firm must borrow from banks that provide sticky loan. However, to finance the costs of hiring workers from $\overline{h} \in [n,1]$, the firm must borrow from banks that provide less sticky loan. Here, we assume that firms must use all types of labor and therefore borrow both sticky and less sticky loan by the fixed proportion.[8] Second, in a monopolistically competitive goods market, where individual demand curves on differentiated consumption goods are offered by consumers, each firm sets a differentiated goods price to maximize its profit. Prices are set in a staggered manner as in the Calvo (1983) - Yun (1992) framework.

As is standard in the New Keynesian model following the Calvo (1983) - Yun (1992) framework, each firm f resets its price with probability $(1-\alpha)$ and maximizes the present discounted value of profit, which is given by:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left[p_t(f)\, c_{t,T}(f) - \widetilde{\Omega}_T \widetilde{L}_T(f) \right], \tag{17}$$

where we assume the production function as $y_t(f) = F(\widetilde{L}_T(f))$. The production function is an increasing and concave. Here, the firm sets $p_t(f)$ under the Calvo (1983) - Yun (1992) framework. The present discounted value of the profit given by equation (17) is further transformed into:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left\{ p_t(f) \left[\frac{p_t(f)}{P_T} \right]^{-\theta} C_T - \widetilde{\Omega}_T \widetilde{L}_T(f) \right\}.$$

[8] The same structure is assumed for employment in Woodford (2003).

It should be noted that price setting is independent of the loan interest rate setting of private banks.

The optimal price setting of $\overline{p}_t(f)$ under the situation in which managers can reset their prices with probability $(1-\alpha)$ is given by:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{U_C(C_T)}{P_T} y_{t,T}(f) \left[\frac{\theta-1}{\theta} \overline{p}_t(f) - \widetilde{\Omega}_T \frac{\partial \widetilde{L}_T(f)}{\partial y_{t,T}(f)} \right] = 0, \tag{18}$$

where we substitute equation (1). By further substituting equations (15) and (16) into equation (18), equation (18) can be now rewritten as:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T) y_{t,T}(f) \left[\frac{\theta-1}{\theta} \frac{\overline{p}_t(f)}{P_t} \frac{P_t}{P_T} - \frac{\epsilon}{\epsilon-1} Z_{t,T}(f) \right] = 0, \tag{19}$$

where

$$\begin{aligned} Z_{t,T}(f) \;\; = \;\; & \left\{ \left(\frac{1}{n} \right) \int_0^n \left[1 + \gamma^L r_t(h) \right]^{1-\epsilon} \left\{ \frac{V_l[l_T(h)]}{U_C(C_t)} \frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)} \right\}^{1-\epsilon} dh \right\}^{\frac{n}{1-\epsilon}} \\ & \times \left\{ \left(\frac{1}{1-n} \right) \int_n^1 \left[1 + \gamma^S r_t(\overline{h}) \right]^{1-\epsilon} \left\{ \frac{V_l\left[l_T(\overline{h})\right]}{U_C(C_t)} \frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)} \right\}^{1-\epsilon} d\overline{h} \right\}^{\frac{1-n}{1-\epsilon}}. \end{aligned}$$

By log-linearizing equation (19), we derive:

$$\frac{1}{1-\alpha\beta} \widehat{\overline{p}}_t(f) = \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\sum_{\tau=t+1}^{T} \pi_{H,\tau} + \Theta_1 \widehat{R}_{L,T} + \Theta_2 \widehat{R}_{S,T} + \widehat{mc}_{t,T}(f) \right], \tag{20}$$

where $\Theta_1 \equiv n \frac{\gamma^L \left(1+\overline{R}_L\right)}{1+\gamma^L \overline{R}_L}$ and $\Theta_2 \equiv (1-n) \frac{\gamma^S \left(1+\overline{R}_S\right)}{1+\gamma^S \overline{R}_S}$ are positive parameters, and we define the real marginal cost as:

$$\widehat{mc}_{t,T}(f) \equiv \int_0^n \widehat{mc}_{t,T}(h,f)\, dh + \int_n^1 \widehat{mc}_{t,T}\left(\overline{h}, f\right) d\overline{h},$$

where

$$mc_{t,T}(h,f) \equiv \frac{V_l[l_T(h)]}{U_Y(C_T)} \frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)},$$

and

$$mc_{t,T}\left(\overline{h},f\right)\equiv\frac{V_l\left[l_T\left(\overline{h}\right)\right]}{U_Y\left(C_T\right)}\frac{\partial\widetilde{L}_{t,T}\left(f\right)}{\partial y_{t,T}\left(f\right)}.$$

We also define:

$$R_{L,t}\equiv\frac{1}{n}\int_0^n r_t\left(h\right)dh, \tag{21}$$

$$R_{S,t}\equiv\frac{1}{1-n}\int_n^1 r_t\left(\overline{h}\right)d\overline{h}, \tag{22}$$

$$\widetilde{p}_t\left(f\right)\equiv\frac{\overline{p}_t\left(f\right)}{P_t},$$

and

$$\pi_t\equiv\frac{P_t}{P_{t-1}}.$$

Then, equation (20) can be transformed into:

$$\frac{1}{1-\alpha\beta}\widehat{\widetilde{p}}_t\left(f\right)=\mathrm{E}_t\sum_{T=t}^{\infty}\left(\alpha\beta\right)^{T-t}\left[\left(1+\eta_2\sigma\right)^{-1}\left(\widehat{mc}_T+\Theta_1\widehat{R}_{L,T}+\Theta_2\widehat{R}_{S,T}\right)+\sum_{\tau=t+1}^{T}\pi_\tau\right], \tag{23}$$

where we make use of the relationship:

$$\widehat{mc}_{t,T}\left(f\right)=\widehat{mc}_T-\eta_2\theta\left[\widehat{\widetilde{p}}_t\left(f\right)-\sum_{\tau=t+1}^{T}\pi_\tau\right],$$

where η_2 is the elasticity of $\frac{\partial\widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)}$ with respect to y. We further denote the average real marginal cost as:

$$\widehat{mc}_T\equiv\int_0^n\widehat{mc}_T\left(h\right)dh+\int_n^1\widehat{mc}_T\left(\overline{h}\right)d\overline{h},$$

where

$$mc_T\left(h\right)\equiv\frac{V_l\left[l_T\left(h\right)\right]}{U_Y\left(C_T\right)}\frac{\partial\widetilde{L}_T}{\partial Y_{H,T}},$$

and

$$mc_T\left(\overline{h}\right)\equiv\frac{V_l\left[l_T\left(\overline{h}\right)\right]}{U_Y\left(C_T\right)}\frac{\partial\widetilde{L}_T}{\partial Y_{H,T}}.$$

The point is that unit marginal cost is the same for all firms in the situation where each firm uses all types of labor and loans with the same proportion. Thus, all firms set the same price if they have a chance to reset their prices at time t.

In the Calvo (1983) - Yun (1992) setting, the evolution of the aggregate price index P is described by the following law of motion:

$$\int_0^1 p_t(f)^{1-\theta} df = \alpha \int_0^1 p_{t-1}(f)^{1-\theta} df + (1-\alpha)\int_0^1 \overline{p}_t(f)^{1-\theta} df,$$

$$\Longrightarrow P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)(\overline{p}_t)^{1-\theta}, \tag{24}$$

where

$$P_t^{1-\theta} \equiv \int_0^1 p_t(f)^{1-\theta} df,$$

and

$$\overline{p}_t^{1-\theta} \equiv \int_0^1 \overline{p}_t(f)^{1-\theta} df.$$

The current aggregate price is given by the weighted average of changed and unchanged prices. Because the chances of resetting prices are randomly assigned to each firm with equal probability, an aggregate price change at time t should be evaluated by an average of price changes by all firms. By log-linearizing equation (24), together with equation (23), we can derive the following New Keynesian Phillips curve:

$$\pi_t = \chi\left(\widehat{mc}_t + \Theta_1\widehat{R}_{L,t} + \Theta_2\widehat{R}_{S,t}\right) + \beta \mathrm{E}_t\pi_{t+1}, \tag{25}$$

where the slope coefficient $\chi \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta_2\theta)}$ is a positive parameter. This is quite similar to the standard New Keynesian Phillips curve, but contains loan interest rates as cost components.

Here, according to the discussion in Woodford (2003), we define the natural rate of output Y_t^n from equation (19) as

$$\frac{\theta-1}{\theta} = \frac{\epsilon}{\epsilon-1}\left[1+\gamma^L\overline{R}\right]^n\left[1+\gamma^S\overline{R}\right]^{1-n}\left\{\left(\frac{1}{n}\right)\int_0^n\left\{\frac{V_l\left[l_t^n\left(h\right)\right]}{U_C\left(C_t\right)}\frac{\partial\widetilde{L}_t^n\left(f\right)}{\partial Y_t^n\left(f\right)}\right\}^{1-\epsilon}dh\right\}^{\frac{n}{1-\epsilon}}$$
$$\times\left\{\left(\frac{1}{1-n}\right)\int_n^1\left\{\frac{V_l\left[l_t^n\left(\overline{h}\right)\right]}{U_C\left(Y_t^n\right)}\frac{\partial\widetilde{L}_t^n\left(f\right)}{\partial Y_t^n\left(f\right)}\right\}^{1-\epsilon}d\overline{h}\right\}^{\frac{1-n}{1-\epsilon}},$$

where, under the natural rate of output, we assume a flexible price setting, $p_t^*(f) = P_t$, and assume no impact of monetary policy, $r_t(h) = r_t(\overline{h}) = \overline{R}$, and so hold $y_t(f) = Y_t^n$. $l_t^n(h)$, $l_t^n(\overline{h})$, $\widetilde{L}_t^n\left(f\right)$, and $\widetilde{L}_t^n\left(f\right)$ are the amount of labor under Y_t^n, respectively. Then, we have

$$\widehat{mc}_t = (\omega+\sigma^{-1})(\widehat{Y}_t - \widehat{Y}_t^n),$$

where $\widehat{Y}_t \equiv \ln(Y_t/\overline{Y})$, and $\widehat{Y}_t^n \equiv \ln(Y_t^n/\overline{Y})$, and ω is a sum of the elasticity of the marginal disutility of work with respect to output increase and the elasticity of $\frac{1}{F'(F^{-1}(y))}$ with respect to output increase.[9] Then, by defining $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n$, we finally have

$$\pi_t = \kappa x_t + \chi\left(\Theta_1\widehat{R}_{L,t} + \Theta_2\widehat{R}_{S,t}\right) + \beta\mathrm{E}_t\pi_{t+1}, \tag{26}$$

where $\kappa \equiv \chi(\omega+\sigma^{-1})$.

3.4 Private banks

There exists a continuum of private banks populated over $[0,1]$. There are two types of banks; banks that provide sticky loans populate over $[0,n)$ and banks that provide the less sticky loans populate over $[n,1]$. Each private bank plays two roles: (1) to collect the deposits from consumers, and (2) under the monopolistically competitive loan market, to set differentiated nominal loan interest rates according to their individual loan demand

[9] $\omega \equiv \eta_2 + \omega_w$, where ω_w is the elasticity of marginal disutility of work with respect to output increase in $\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)}$. We can see more detailed derivation in Woodford (Ch. 3, 2003).

curves, given the amount of their deposits. We assume that each bank sets the differentiated nominal loan interest rate according to the types of labor force as examined in Teranishi (2007). Staggered loan contracts between firms and private banks produce a situation in which the private banks fix the loan interest rates for a certain period.

A sticky loan bank only provides a loan to firms when they hire labor from $h \in [0, n)$. However, a less sticky loan bank lends only to firms when they hire labor from $\overline{h} \in [n, 1]$. First, we describe the optimization problem of a bank that provide sticky loan. This bank can reset loan interest rates with probability $\left(1 - \varphi^L\right)$ following the Calvo (1983) - Yun (1992) framework. Under the segmented environment stemming from differences in labor supply, private banks can set different loan interest rates depending on the types of labor. As a consequence, the private bank holds some monopoly power over the loan interest rate to firms. Therefore, the bank h chooses the loan interest rate $r_t(h)$ to maximize the present discounted value of profit:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \left(\varphi^L\right)^{T-t} X_{t,T} q_{t,T}(h, f) \left\{[1 + r_t(h)] - (1 + i_T)\right\}.$$

The optimal loan condition is now given by:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \left(\varphi^L \beta\right)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T)}{U_C(C_t)} q_{t,T}(h) \left\{\left[1 + \gamma^L r_t(h)\right] - \epsilon \gamma^L \left\{[1 + r_t(h)] - (1 + i_T)\right\}\right\} = 0. \tag{27}$$

Because the sticky loan banks that have the opportunity to reset their loan interest rates will set the same loan interest rate, the solution of $r_t(h)$ in equation (27) is expressed only with $\overline{r}_t$. In this case, we have the following evolution of the aggregate loan interest rate index:

$$1 + R_{L,t} = \varphi^L (1 + R_{L,t-1}) + \left(1 - \varphi^L\right)(1 + \overline{r}_t). \tag{28}$$

By log-linearizing equations (27) and (28), we can determine the relationship between the loan and deposit interest rate as follows:

$$\widehat{R}_{L,t} = \lambda_1^L \mathrm{E}_t \widehat{R}_{L,t+1} + \lambda_2^L \widehat{R}_{L,t-1} + \lambda_3^L \widehat{i}_t, \tag{29}$$

where $\lambda_1^L \equiv \frac{\varphi^L \beta}{1+(\varphi^L)^2\beta}$, $\lambda_2^L \equiv \frac{\varphi^L}{1+(\varphi^L)^2\beta}$, and $\lambda_3^L \equiv \frac{1-\varphi^L}{1+(\varphi^L)^2\beta}\frac{\epsilon}{\epsilon-1}\frac{(1-\beta\overline{\varphi}^*)(1+\bar{i})}{1+\overline{R}_L}$ are positive parameters. This equation describes the sticky loan interest rate (supply) curve by the banks.

Similarly, from the optimization problem of bank $\overline{h}$ that provide less sticky loan, we can obtain the relationship between loan interest rate and deposit interest rates as follows:

$$\widehat{R}_{S,t} = \lambda_1^S \mathrm{E}_t \widehat{R}_{S,t+1} + \lambda_2^S \widehat{R}_{S,t-1} + \lambda_3^S \widehat{i}_t, \tag{30}$$

where φ is the Calvo parameter for bank $\overline{h}$. We assume $\varphi^L > \varphi^s$. $\lambda_1^S \equiv \frac{\varphi^S \beta}{1+(\varphi^S)^2\beta}$, $\lambda_2^S \equiv \frac{\varphi^S}{1+(\varphi^S)^2\beta}$, and $\lambda_3^S \equiv \frac{1-\varphi^S}{1+(\varphi^S)^2\beta}\frac{\epsilon}{\epsilon-1}\frac{(1-\beta\varphi^S)(1+\bar{i})}{1+\overline{R}_S}$ are positive parameters.

The market loan clearing conditions are expressed as:

$$q_{t,T}(h) = \int_0^1 q_{t,T}(h,f)df,$$

$$q_{t,T}\left(\overline{h}\right) = \int_0^1 q_{t,T}\left(\overline{h},f\right)df,$$

$$\int_0^n q_{t,T}(h)dh = nD_T,$$

and

$$\int_n^1 q_{t,T}\left(\overline{h}\right)d\overline{h} = (1-n)\,D_T.$$

4 Optimal Monetary Policy Analysis

4.1 Approximated Welfare Function

We derive a second order approximation to the welfare function following Woodford (2003). The detail of derivation is shown in Appendix A.

The consumer welfare in the home country is given by

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t UT_t = \mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^{T-t}\left[U(C_t) - \int_0^n V(l_t(h))dh - \int_n^1 V(l_t(\overline{h}))d\overline{h}\right]\right\}, \tag{31}$$

Then, we have a second order approximated loss function as follows:

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t UT_t \simeq -\Lambda\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left(\begin{array}{c}\lambda_\pi\pi_t^2 + \lambda_x(x_t - x^*)^2 + \lambda_L(\widehat{R}_{L,t} - \widehat{R}_{L,t-1})^2 \\ +\lambda_S(\widehat{R}_{S,t} - \widehat{R}_{S,t-1})^2 + \lambda_{LS}\left(\widehat{R}_{L,t} - \widehat{R}_{S,t}\right)^2\end{array}\right), \tag{32}$$

where λ_π, λ_x, λ_L, λ_S, and λ_{LS} are positive parameters.

It is important to note that equation (32) implies the central bank should care about the heterogeneity in the financial markets. In particular, the interest rate spread between loan interest rate with quick adjustment and one with slow adjustment as well as the heterogeneous loan interest rate changes are elements in setting the policy interest rates. This finding is not trivial because there are not such properties under homogeneous loan interest rate contracts, i.e. $\widehat{R}_t = \widehat{R}_{L,t} = \widehat{R}_{S,t}$ and $\lambda = \lambda_L = \lambda_S$ as:

$$\sum_{t=0}^{\infty}\beta^t UT_t \simeq -\Lambda\sum_{t=0}^{\infty}\beta^t\left(\lambda_\pi\pi_t^2 + \lambda_x(x_t - x^*)^2 + \lambda(\widehat{R}_t - \widehat{R}_{t-1})^2\right). \tag{33}$$

In this case, we have neither the term of the interest rate spread nor the term of heterogeneous loan interest rate changes. We only have the term of a homogeneous loan interest rate change.[10] Clearly, the heterogeneity in the financial market makes the monetary policy complicated.

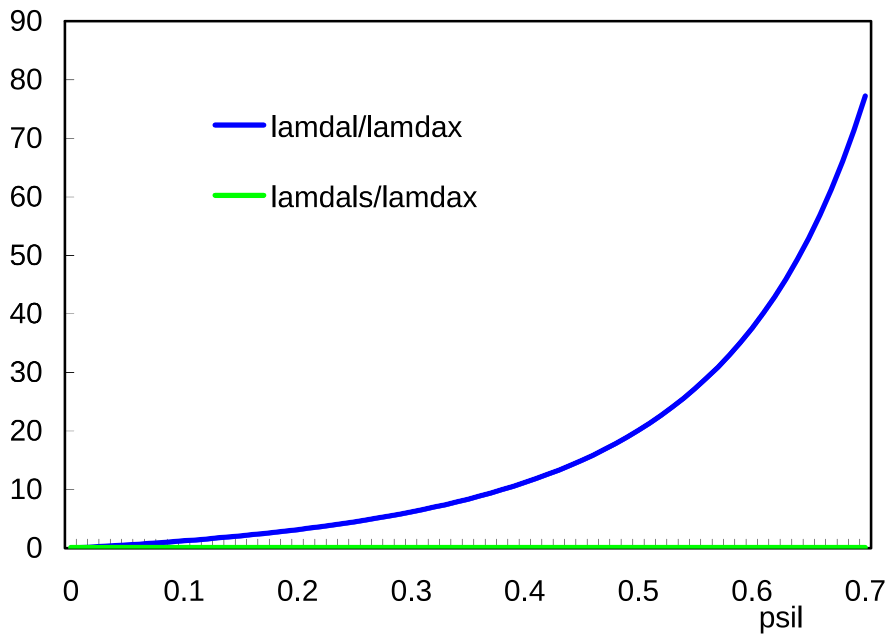
[10]When stickiness in loan rates are the same in sticky and less sticky loans, we still have the term of credit spread under a shock in either of sticky or less sticky loan or different shocks in sticky and less sticky loans as:

$$\sum_{t=0}^{\infty}\beta^t UT_t \simeq -\Lambda\sum_{t=0}^{\infty}\beta^t\left(\begin{array}{c}\lambda_\pi\pi_t^2 + \lambda_x(x_t - x^*)^2 \\ +\lambda_L(\widehat{R}^*_{L,t} - \widehat{R}^*_{L,t-1})^2 + \lambda_S(\widehat{R}^*_{S,t} - \widehat{R}^*_{S,t-1})^2 \\ +\lambda_{LS}\left(\widehat{R}^*_{L,t} - \widehat{R}^*_{S,t}\right)^2\end{array}\right),$$

where $\widehat{R}^*_{L,t}$ and $\widehat{R}^*_{S,t}$ denote sticky and less sticky loan interest rates with heterogeneous idiosyncratic shocks, respectively. There heterogeneity in loan interest rates is in idiosyncratic shocks rather than in difference of loan stickiness.

We show how the weight in the welfare function change according to the loan stickiness. Figure 1 reports the value of $\frac{\lambda_L}{\lambda_x}$ and $\frac{\lambda_{LS}}{\lambda_x}$ with changing loan interest rate stickiness from 0 to 0.7. It demonstrates that the heterogeneity in staggeredness of loan interest rates rather than the credit spread is quantitatively so important to implement the optimal monetary policy when stickinesses of loan interest rates are high. But, when stickinesses of loan interest rates are low enough, the credit spread quantitatively becomes more crucial element to conduct the optimal monetary policy.[11]

Figure 3. Welfare weight



[11] To small φ^L, we have $\frac{\lambda_L}{\lambda_x} \leq \frac{\lambda_{LS}}{\lambda_x}$. Here, we do not report $\frac{\lambda_\pi}{\lambda_x}$, but it also changes according to the stickiness of the price adjustment.

4.2 Optimal Monetary Policy Rule

We consider an optimal monetary policy scheme in which the central bank is credibly committed to a policy rule in the *Timeless Perspective*[12]. In this case, as shown in Woodford (2003), the central bank conducts monetary policy in a forward looking way by paying attention to future economic variables and by taking account of the effects of monetary policy on those future variables.

The objective of monetary policy is to minimize the expected value of the loss criterion given by Eq. (56) under the standard New Keynesian IS curve given Eq. (14), the augmented Phillips curve given by Eq. (26), the loan rate curve with slow adjustment given by Eq. (30), and the loan rate curve with quick adjustment given by Eq. (29). The optimal monetary policy is expressed by the solution of the optimization problem which is represented by the following Lagrangian:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} L_t + 2\Xi_{1t}\left[x_{t+1} - \sigma(\widehat{i}_t - \pi_{t+1} - r_t^n) - x_t\right] \\ +2\Xi_{2t}\left[\kappa x_t + \chi\left(\Theta_1 \widehat{R}_{L,t} + \Theta_2 \widehat{R}_{S,t}\right) + \beta\pi_{t+1} - \pi_t\right] \\ +2\Xi_{3t}\left[\lambda_1^L \widehat{R}_{L,t+1} + \lambda_2^L \widehat{R}_{L,t-1} + \lambda_3^L \widehat{i}_t - \widehat{R}_{L,t}\right] \\ +2\Xi_{4t}\left[\lambda_1^S \widehat{R}_{S,t+1} + \lambda_2^S \widehat{R}_{S,t-1} + \lambda_3^S \widehat{i}_t - \widehat{R}_{S,t}\right] \end{array} \right\} \right\},$$

where Ξ_1, Ξ_2, Ξ_3, and Ξ_4 are the Lagrange multipliers associated with the IS curve constraint, the Phillips curve constraint, the loan rate curve constraints, respectively. We differentiate the Lagrangian with respect to π_t, x_t, $\widehat{R}_{L,t}$, $\widehat{R}_{S,t}$, and $\widehat{i}_t$ to obtain the first-order conditions. Then, by combining the first order conditions, we have a following optimal

[12] The detailed explanations about the timeless perspective are in Woodford (2003).

monetary policy rule:

$$\begin{bmatrix} -z_3^{-1}z_4^{-1}(1-z_5L)^{-1}(1-z_6F)^{-1}\left\{\begin{array}{c}\lambda_L(\triangle\widehat{R}_{L,t}-\beta E_t\triangle\widehat{R}_{L,t+1})+\lambda_{LS}(\widehat{R}_{L,t}-\widehat{R}_{S,t})\\ -\kappa^{-1}\lambda_x\Theta_1(x_t-x^*)\end{array}\right\} \\ -(z_3^*z_4^*)^{-1}(1-z_5^*L)^{-1}(1-z_6^*F)^{-1}\left\{\begin{array}{c}\lambda_S(\triangle\widehat{R}_{S,t}-\beta E_t\triangle\widehat{R}_{S,t+1})-\lambda_{LS}(\widehat{R}_{L,t}-\widehat{R}_{S,t})\\ -\kappa^{-1}\lambda_x\Theta_2(x_t-x^*)\end{array}\right\}\end{bmatrix}$$

$$= E_t\left[(1-z_1L)^{-1}(1-z_2L)^{-1}(\kappa\lambda_\pi\pi_t+\lambda_x\triangle x_t)\right], \tag{34}$$

where z_1, z_2, z_3, z_4, z_5, z_6, z_3^*, z_4^*, z_5^*, and z_6^* are parameters, satisfying $z_1+z_2=1+\beta^{-1}+\kappa\sigma\beta^{-1}$, $z_1z_2=\beta^{-1}$ $(z_1>1,\ 0<z_2<1)$, $z_3=-\frac{\beta\lambda_2^L\sigma}{\lambda_3^L}$, $z_4z_5=\frac{1}{z_3}(\frac{\sigma}{\lambda_3^L}-\frac{\Theta_1}{\kappa})$, $z_4+z_5=-\frac{1}{z_3}(\frac{\Theta_1}{\beta\kappa}-\frac{\sigma\lambda_1^L}{\beta\lambda_3^L})$, $z_6=z_4^{-1}$, $z_3^*=-\frac{\beta\lambda_2^S\sigma}{\lambda_3^S}$, $z_4^*z_5^*=\frac{1}{z_3^*}(\frac{\sigma}{\lambda_3^S}-\frac{\Theta_2}{\kappa})$, $z_4^*+z_5^*=-\frac{1}{z_3^*}(\frac{\Theta_2}{\beta\kappa}-\frac{\sigma\lambda_1^S}{\beta\lambda_3^S})$, and $z_6^*=(z_4^*)^{-1}$.

5 Optimal Monetary Policy Reaction

5.1 Response to Credit Spread Shock

We use the parameter values listed in Table 3 borrowing from Rotemberg and Woodford (1997). It should be noted that Slovin and Sushka (1983) claim that private banks, on average, need at least two quarters and perhaps more to adjust loan interest rates. Thus, the average contract duration of sticky loan interest rates is set to be two quarters for less sticky loan and be three quarters for sticky loan. We assume unexpected one percentage credit spread shock with persistence 0.9 on u_t in the loan rate curve with more sticky adjustment as:

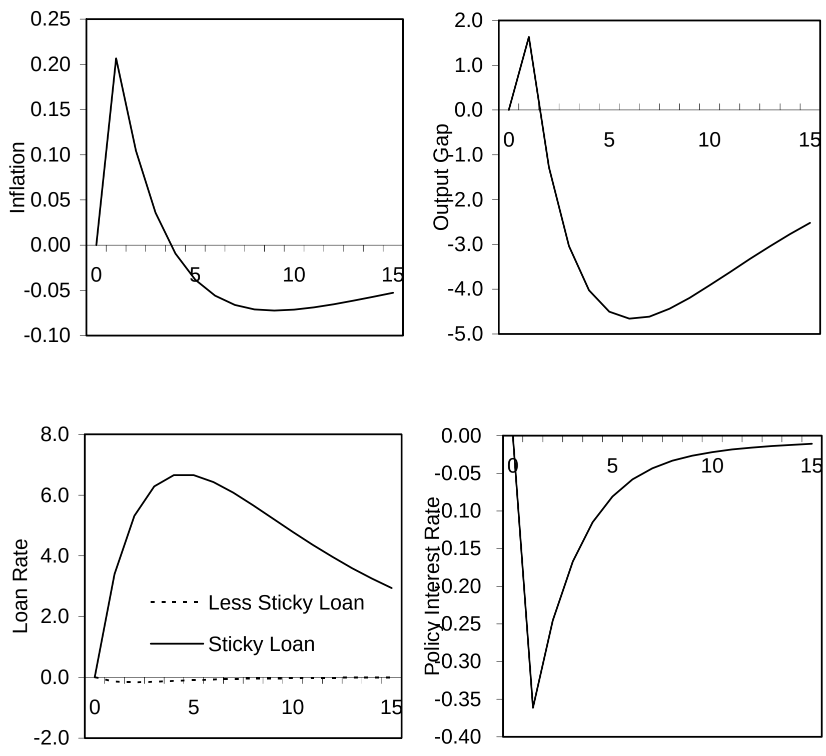
$$\widehat{R}_{L,t}=\lambda_1^L\mathrm{E}_t\widehat{R}_{L,t+1}+\lambda_2^L\widehat{R}_{L,t-1}+\lambda_3^L\widehat{i}_t+u_{L,t},$$

Table 1: Parameter values

Parameters	Values	Explanation
β	0.99	Discount factor
σ	6.25	Elasticity of output with respect to real interest rate
κ	0.032	Elasticity of inflation with respect to output
α	0.66	Probability of price change
φ	0.66	Probability of loan interest rate change in long term loan
φ^*	0.5	Probability of loan interest rate change in short term loan
θ	7.66	Substitutability of differentiated consumption goods
ϵ	7.66	Substitutability of differentiated laborers
γ, γ^*	1	Ratio of external finance to total finance
n	0.5	Preference for loan of long term

Figure 2 shows the simulation outcomes.

Figure 2. Impulse Response to Credit Spread Shock



We can confirm that a central bank decreases the policy interest rates to mitigate shock to the credit spread shock. The reason of this is that the central bank has an incentive to stabilize the loan rate fluctuation as:

$$(\widehat{R}_{L,t} - \widehat{R}_{L,t-1})^2 = \left\{ \left(1 - \lambda_1^L F - \lambda_2^L L\right) (1 - L) \left[\lambda_3^L \widehat{i}_t + u_{L,t} \right] \right\}^2 . \tag{35}$$

To decrease the loan rate fluctuation, the central bank should negatively respond to the credit spread shock u_t. Eventually, the output gaps take positive values thanks to the monetary easing. Under increasing loan interest rates, inflation rates go up due to the increase in the production cost.

This optimal monetary policy response to credit spread is consistent with the reaction by the real central banks. For example, FRB lowers the policy interest rate in the sub-prime mortgage crisis from the fall of 2007 to the current. (story. more and more)

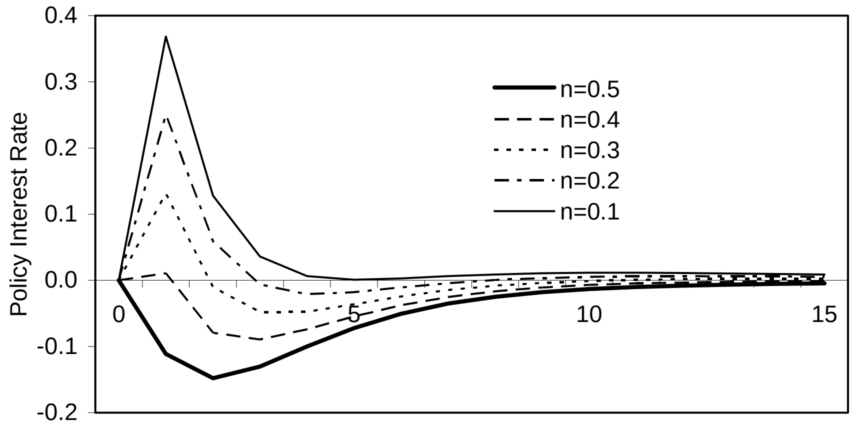
5.2 Weight on Sticky or Less Sticky?

(story. more and more)

We investigate which of the loan rates central bank should consider. We assume that the weights on sticky loan and less sticky loans in production function are equal, i.e. $n = 0.5$, and a symmetric shock in sticky and less sticky loans, i.e. $u_{L,t} = -u_{S,t}$ for any t, in the baseline case to identify the priority of the central bank.

Figure 3 shows the dynamics of policy interest rate with various shares of the sticky loan. From the outcome in Figure 2, we can confirm that the central bank has to respond more aggressively to the shock in sticky loan interest rate when the market share is equal in two types of loan. The implication, however, change according to the market share of sticky loan. As the share of the sticky loan decreases, the central bank should more care about the less sticky loan.

Figure 3. Policy Rate Dynamics to Various Share of Sticky Loan



This finding has many strong implications in particular to the real monetary policy. The central bank does not need to think of a weighted average of loan interest rates when the market share is not so different between two types of loan contracts. The central bank should put its priority to the loan interest rate with more stickiness to achieve a good policy. One reason of this is that the weight on sticky loan interest rate change in the welfare, λ_L, is larger than that on less sticky loan interest rate change, λ_S. Another reason is that the same scale shock makes larger fluctuation in more sticky loan interest rate than in less sticky loan interest rate. But, this outcome does not hold when the share of the sticly loan is enough small. In this case, the central bank should pay attention to both stiky and less sticky loans.

6 Taylor (2008) or Cúrdia and Woodford (2008) or Other?

Taylor (2008) points out that the FRB seems to deviate from the theoretical interest rate path of so called Taylor rule since the FRB have negatively reacted to the credit spread in the money market for the last few years to stimulate economy. We interpret this point in our model in particuler on whether or not the central bank should respond to such a credit spread in the financial market.

To see a clearer effect of heterogeneity of loan contracts in terms of the monetary policy, we further assume $\varphi^S = 0$ and $\varphi^L \geq 0$. In this case, the less sticky loan banks change their loan interests rate every period. The purpose of the central bank is given by

$$\sum_{t=0}^{\infty} \beta^t UT_t \simeq -\Lambda \sum_{t=0}^{\infty} \beta^t \left(\lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_L (\widehat{R}_{L,t} - \widehat{R}_{L,t-1})^2 + \lambda_{LS} \left(\widehat{R}_{L,t} - \lambda_3^S \widehat{i}_t \right)^2 \right), \tag{36}$$

where we use equation (30). This implies that a central bank has to pay attention to the spread between the policy interest rate and the interest rate which has some lags to catch up to the policy interest rate. In this case, the term of loan interest rate change in less sticky loan is zero, so the heterogeneous loan interest rate changes is all in the term of that in sticky loan. Furthermore, when we interpret the policy rate as the riskless interest rate and the loan interest rate as the risky rate with some premium shocks, a central bank has to react to the credit spread between risky rate and riskless interest rate. This implication strongly supports the idea of McCulley and Toloui (2008), Taylor (2008) and Taylor and Williams (2008), which find that the Federal Reserve Bank reacted to the credit spread.[13]

[13] Taylor (2008) and Taylor and Williams (2008) use the credit spread between three month LIBOR rate and three month OIS rate (overnight index swap rate as expected overnight federal funds rates). They also suggest that the credit spread is not enough to explain the deviation of Fed Fund rate from the Taylor rule. This lack of explanation can be compensated by the term of heterogeneous first differences of loan interest rates that are induced by heterogeneity of loan interest rates.

Under heterogeneous interest rates, it is the optimal monetary policy to actively react to heterogeneity of loan interest rates inducing the credit spread.

Cúrdia and Woodford (2008), however, suggest that a central bank eventually does not need to respond to credit spread by showing that the optimal monetary policy in the basic NK model without credit spread still quantitatively provides a good approximation to the optimal monetary policy in the NK model with credit spread. We investigate this point in our framework. We compare the difference of impulse responses in two rules. One rule is the optimal monetary policy given by equation (34) that correctly capture the heterogeneity of loan interest rates. Another rule, the near optimal rule A, is one that ignores only credit spread of loan interest rates, i.e. $\lambda_{LS} = 0$. We use the parameter values listed in Table 3 except $\varphi^S = 0$ and assume unexpected one percentage credit spread shock with persistence 0.9. Thus, the less sticky loan interest rate is flexible. Figure 4 shows the simulation outcomes. In this case, the pathes in two rules are not different each other. This implies that the rule without attention to the credit spread can be a good approximation to quantitatively achieve the optimal monetary policy, which is consistent with Cúrdia and Woodford (2008) that also point out quantitatively less importance of attention to the credit spread itself. Thus, the credit spread only that is one appearance of the financial market heterogeneity as shown above is not so important.

Finally, we compare the impulse responses in the optimal monetary policy given by equation (34) and in the rule, called the near optimal rule B, that ignore whole heteregeneity of loan interest rates in persuiting to minimize the welfare given by equations (33). In particuler, we assume that a central bank misunderstands that there is only less sticky loan interest rate in the economy. Thus, the near optimal monetary policy rule is given by the standard optimal monetary policy that keeps a linear relation between the inflation rate and change in the output gap. Figure 5 shows the simulation outcomes. We can confirm that the standard optimal monetary policy can not be a good approximation to the optimal monetary policy.

Therefore, we support the conclusions by Taylor (2008) or Cúrdia and Woodford (2008), but we insist that whole heterogeneities, not only credit spread, in financial markets are very crucial to achieve the optomal moentary policy.

Figure 4. Impulse Response in Optimal Rule and Near Optimal Rule A

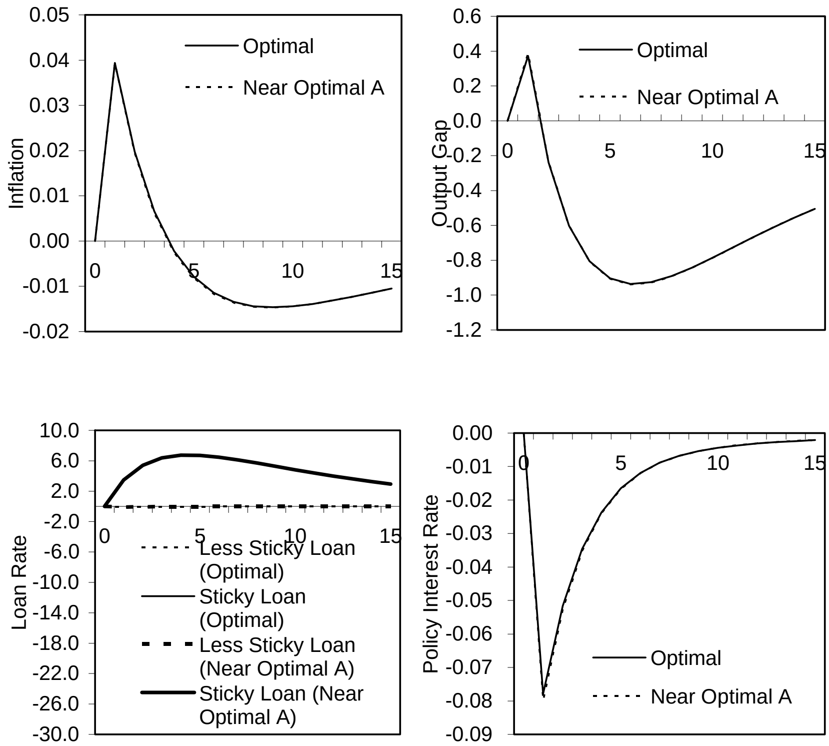
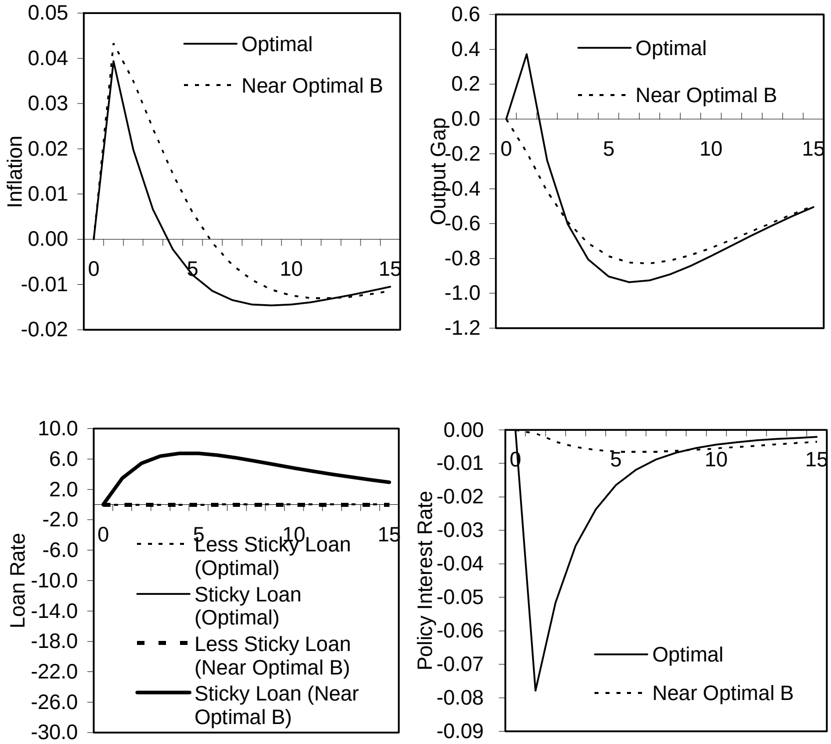


Figure 5. Impulse Response in Optimal Rule and Near Optimal Rule B



7 Concluding Remarks

TBA.

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Appendix

A Derivation of Loss Function

In this subsection, we derive a second order approximation to the welfare function (all details of these derivations and explanations are in Appendix B).

In derivation of approximated welfare function, we basically follow the way of Woodford (2003). Except x_t and π_t, log-linearized version of variable m_t is expressed by $\widehat{m}_t = \ln(m_t/\overline{m})$, where $\overline{m}$ is steady state value of m_t.[14] Under the situation in which goods supply matches goods demand in every level, $Y_t = C_t$ and $y_t(f) = c_t(f)$ for any f, the welfare criterion of consumer is given by

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t UT_t \right\},$$

where

$$UT_t = U(C_t) - \int_0^n V(l_t(h))dh - \int_n^1 V(l_t(\overline{h}))d\overline{h}, \tag{37}$$

and

$$Y_t \equiv \left[\int_0^1 y_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}.$$

We log-linearize equation (37) step by step to derive an approximated welfare function. Firstly, we log-linearize the first term of equation (37).

[14] You can see Woodford (2003) about how to log-linearize a function.

$$
\begin{aligned}
UT(Y_t;\nu_t) &= \overline{U} + U_c\widetilde{Y}_t + U_\nu\nu_t + \frac{1}{2}U_{cc}\widetilde{Y}_t^2 + U_{c\nu}\widetilde{Y}_t + \frac{1}{2}\nu_t' U_{\nu\nu}\nu_t + Order(\parallel \xi \parallel^3) \\
&= \overline{U} + \overline{Y}U_c(\widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2) + U_\nu\nu_t + \frac{1}{2}U_{cc}\overline{Y}^2\widehat{Y}_t^2 + \overline{Y}U_{c\nu}\nu_t\widehat{Y}_t + \frac{1}{2}\nu_t' U_{\nu\nu}\nu_t + Order(\parallel \xi \parallel^3) \\
&= \overline{Y}U_c\widehat{Y}_t + \frac{1}{2}\left[\overline{Y}U_c + \overline{Y}^2 U_{cc}\right]\widehat{Y}_t^2 - \overline{Y}^2 U_{cc} g_t \widehat{Y}_t + t.i.p + Order(\parallel \xi \parallel^3) \\
&= \overline{Y}U_c\left[\widehat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\widehat{Y}_t^2 + \sigma^{-1} g_t\widehat{Y}_t\right] + t.i.p + Order(\parallel \xi \parallel^3), \qquad (38)
\end{aligned}
$$

where $\overline{U} \equiv U(\overline{Y};0)$, $\widetilde{Y}_t \equiv Y_t - \overline{Y}$, $t.i.p$ means the terms that are independent from monetary policy, $Order(\parallel \xi \parallel^3)$ expresses order terms higher than the second order approximation, $\sigma^{-1} \equiv -\frac{\overline{Y}U_{cc}}{U_c} > 0$, and $g_t \equiv -\frac{U_{c\nu}\nu_t}{\overline{Y}U_{cc}}$. To replace $\widetilde{Y}_t$ by $\widehat{Y}_t \equiv \ln(Y_t/\overline{Y})$, we use the Taylor series expansion on $Y_t/\overline{Y}$ in the second line as

$$
Y_t/\overline{Y} = 1 + \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + Order(\parallel \xi \parallel^3).
$$

Secondly, we log-linearize the second term of equation (37) by a similar way.

$$
\begin{aligned}
\frac{1}{n}\int_0^n V(l_t(h);\nu_t)dh &= V_l\overline{L}(E_h\widehat{l}_t(h) + \frac{1}{2}E_h(\widehat{l}_t(h))^2) + \frac{1}{2}V_{ll}\overline{L}^2 E_h(\widehat{l}_t(h))^2 + V_{l\nu}\overline{L}\nu_t E_h\widehat{l}_t(h) \\
&\quad + t.i.p + Order(\parallel \xi \parallel^3) \\
&= \overline{L}V_l\left[\widehat{L}_t + \frac{1}{2}(1+\nu)\widehat{L}_t^2 - \nu\widetilde{\nu}_t\widehat{L}_t + \frac{1}{2}(\nu + \frac{1}{\epsilon})var_h\widehat{l}_t(h)\right] \\
&\quad + t.i.p + Order(\parallel \xi \parallel^3) \qquad (39)
\end{aligned}
$$

where $\widetilde{\nu}_t \equiv -\frac{V_{l\nu}\nu_t}{\overline{L}V_{ll}}$, $\nu \equiv \frac{\overline{L}V_{ll}}{V_l}$, $\phi_h \equiv \frac{\overline{Y}}{\overline{L}f'}$, $\omega_p \equiv \frac{ff''}{(f')^2}$, $q_t \equiv (1+\omega^{-1})a_t + \omega^{-1}\nu\widetilde{\nu}_t$, $a_t \equiv \ln A_t$, $var_h\widehat{l}_t(h)$ is the variance of $\widehat{l}_t(h)$ across all types of labor, and $var_f\widehat{p}_t(f)$ is the variance of $\widehat{p}_t(f)$ across all differentiated good prices. Here the definition of labor sub-aggregator is given by

$$
L_t \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n l_t(h)^{\frac{\epsilon-1}{\epsilon}} dh\right]^{\frac{\epsilon}{\epsilon-1}},
$$

and so we have $\widehat{L}_t = E_h \widehat{l}_t(h) + \frac{1}{2}\frac{\epsilon-1}{\epsilon} var_h \widehat{l}_t(h) + Order(\parallel \xi \parallel^3)$ in the second order approximation. We use this relation in the second line.

Thirdly, we log-linearize the third term of equation (37) by a similar way.

$$\begin{aligned}
\frac{1}{1-n}\int_n^1 V(l_t(\overline{h});\nu_t)d\overline{h} &= V_l\overline{L}(E_{\overline{h}}\widehat{l}_t(\overline{h}) + \frac{1}{2}E_{\overline{h}}(\widehat{l}_t(\overline{h}))^2) + \frac{1}{2}V_{ll}\overline{L}^2 E_{\overline{h}}(\widehat{l}_t(\overline{h}))^2 + V_{l\nu}\overline{L}\nu_t E_{\overline{h}}\widehat{l}_t(\overline{h}) \\
+t.i.p + Order(&\parallel \xi \parallel^3) \\
&= \overline{L}V_l\left[\widehat{\overline{L}}_t + \frac{1}{2}(1+\nu)\widehat{\overline{L}}_t^2 - \nu\widetilde{\nu}_t\widehat{\overline{L}}_t + \frac{1}{2}(\nu+\frac{1}{\epsilon})var_{\overline{h}}\widehat{l}_t(\overline{h})\right] \\
+t.i.p + Order(&\parallel \xi \parallel^3) \qquad (40)
\end{aligned}$$

Here the definition of labor sub-aggregator is given by

$$\overline{L}_t \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}}\int_n^1 l_t\left(\overline{h}\right)^{\frac{\epsilon-1}{\epsilon}}d\overline{h}\right]^{\frac{\epsilon}{\epsilon-1}},$$

and so we have $\widehat{\overline{L}}_t = E_{\overline{h}}\widehat{l}_t(\overline{h}) + \frac{1}{2}\frac{\epsilon-1}{\epsilon}var_{\overline{h}}\widehat{l}_t(\overline{h}) + Order(\parallel \xi \parallel^3)$ in the second order approximation. We use this relation in the second line.

Then, from equation (39) and equation (40), we have

$$\begin{aligned}
&\int_0^n V(l_t(h);\nu_t)dh + \int_n^1 V(l_t(\overline{h});\nu_t)d\overline{h} \\
&= \overline{L}V_l\left[\begin{array}{c} n\widehat{L}_t + \frac{n}{2}(1+\nu)\widehat{L}_t^2 - n\nu\widetilde{\nu}_t\widehat{L}_t + \frac{n}{2}(\nu+\frac{1}{\epsilon})var_h\widehat{l}_t(h) \\ +(1-n)\widehat{\overline{L}}_t + \frac{1-n}{2}(1+\nu)\widehat{\overline{L}}_t^2 - (1-n)\nu\widetilde{\nu}_t\widehat{\overline{L}}_t + \frac{1-n}{2}(\nu+\frac{1}{\epsilon})var_{\overline{h}}\widehat{l}_t(\overline{h})\end{array}\right] \\
+t.i.p + Order(&\parallel \xi \parallel^3) \\
&= \overline{L}V_l\left[\begin{array}{c} \widehat{\overline{\overline{L}}}_t + \frac{1+\nu}{2}\widehat{\overline{\overline{L}}}_t^2 - \nu\widetilde{\nu}_t\phi_h^{-1}\widehat{\overline{\overline{L}}}_t + n(1-n)\frac{1+\nu}{2}\left(\widehat{L}_t - \widehat{\overline{L}}_t\right)^2 \\ +\frac{n}{2}(\nu+\frac{1}{\epsilon})var_h\widehat{l}_t(h) + \frac{1-n}{2}(\nu+\frac{1}{\epsilon})var_{\overline{h}}\widehat{l}_t(\overline{h})\end{array}\right] \\
+t.i.p + Order(&\parallel \xi \parallel^3) \qquad (41)
\end{aligned}$$

where we use

$$\widehat{\overline{\overline{L}}}_t = n\widehat{L}_t + (1-n)\widehat{\overline{L}}_t,$$

from equation (6). Then, we use the condition that the demand of labor is equal to the supply of labor as

$$\widetilde{L}_t = \int_0^1 \widetilde{L}_t(f)df = \int_0^1 f^{-1}(\frac{y_t(f)}{A_t})df,$$

where the production function is given by $y_t(f) = A_t f(L_t(f))$, where $f(\cdot)$ is an increasing and concave function. By taking the second order approximation, we have

$$\widehat{\widetilde{L}}_t = \phi_h(\widehat{Y}_t - a_t) + \frac{1}{2}(1 + \omega_p - \phi_h)\phi_h(\widehat{Y}_t - a_t)^2 + \frac{1}{2}(1 + \omega_p\theta)\theta var_f \widehat{p}_t(f) + Order(\| \xi \|^3),$$

where we log-linearize the demand function on differentiated goods to derive the relation $var_f \ln y_t(f) = \theta^2 var_f \ln p_t(f)$, which can be derived from the consumer's cost minimization problem under Dixit-Stiglitz aggregator, as

$$y_t(f) = Y_t \left[\frac{p_t(f)}{P_t}\right]^{-\theta},$$

where the aggregate price index is given by $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$. Also, we use the relation of $\phi_h \nu = \omega_w$ and $\omega = \omega_p + \omega_w$, where ω_w is an elasticity of real wage under the flexible-wage labor supply with respect to aggregate output. We can transform equation

(41) as:

$$\int_0^n V(l_t(h);\nu_t)dh + \int_n^1 V(l_t(\overline{h});\nu_t)d\overline{h}$$

$$= \phi_h \overline{L} V_l \left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y}_t + n(1-n)\frac{1+\nu}{2}\left(\widehat{L}_t - \widehat{\overline{L}}_t\right)^2 \\ +\frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) + \frac{n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_h \ln l_t(h) \\ +\frac{1-n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_{\overline{h}} \ln l_t(\overline{h}) \end{array} \right]$$

$$+t.i.p + Order(\parallel \xi \parallel^3)$$

$$= \phi_h \overline{L} V_l \left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y}_t \\ +n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\eta_2}\right)^2\left(\Theta\widehat{R}_{L,t} - \Theta^*\widehat{R}_{S,t}\right)^2 + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ +\frac{n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_h \ln l_t(h) + \frac{1-n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_{\overline{h}} \ln l_t(\overline{h}) \end{array} \right]$$

$$+t.i.p + Order(\parallel \xi \parallel^3). \tag{42}$$

From the second line to the third line, we use following transformations:

$$\begin{aligned} \widehat{L}_t - \widehat{\overline{L}}_t &= \widehat{\overline{\Omega}}_t - \widehat{\Omega}_t \\ &= \left(\Theta\widehat{R}_{L,t} - \Theta^*\widehat{R}_{S,t}\right) + \frac{1}{1-n}\int_n^1 w_t(\overline{h})d\overline{h} - \frac{1}{n}\int_0^n w_t(h)dh \\ &= \left(\Theta\widehat{R}_{L,t} - \Theta^*\widehat{R}_{S,t}\right) - \eta_2\left(\widehat{L}_t - \widehat{\overline{L}}_t\right), \end{aligned}$$

where $\Theta \equiv \frac{\gamma^L(1+\overline{r}^L)}{1+\gamma^L\overline{r}^L}$ and $\Theta^* \equiv \frac{\gamma^S(1+\overline{r}^S)}{1+\gamma^S\overline{r}^S}$. There we use log-linear relations from equation (7), equation (8), equation (15), and equation (16) and the definitions from equation (2),equation (3), equation (4), equation (5), equation (21), and equation (22).

Furthermore, we can replace $\phi_h \overline{L} V_l$ by $(1-\Phi)\overline{Y}U_c$ as:

$$\int_0^n V(l_t(h);\nu_t)dh + \int_n^1 V(l_t(\overline{h});\nu_t)$$

$$= \overline{Y}U_c \left[\begin{array}{c} (1-\Phi)\widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y}_t + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ +\frac{n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_h \ln l_t(h) + \frac{1-n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_{\overline{h}} \ln l_t(\overline{h}) \\ +n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\eta_2}\right)^2\left(\Theta\widehat{R}_{L,t} - \Theta^*\widehat{R}_{S,t}\right)^2 \end{array} \right]$$

$$+t.i.p + Order(\parallel \xi \parallel^3), \tag{43}$$

Here, we use the assumption that distortion of the output level Φ, induced by firm's price mark up through

$$\left\{\left(\frac{1}{n}\right)\int_0^n \left\{\frac{V_l\left[l_T(h)\right]}{U_C(C_t)}\frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)}\right\}^{1-\epsilon} dh\right\}^{\frac{n}{1-\epsilon}} \times \left\{\left(\frac{1}{1-n}\right)\int_n^1 \left\{\frac{V_l\left[l_T(\overline{h})\right]}{U_C(C_t)}\frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)}\right\}^{1-\epsilon} d\overline{h}\right\}^{\frac{1-n}{1-\epsilon}} \tag{44}$$

which would exist under flexible price and no role of monetary policy is of order one as in Woodford (2003).[15] Thus, in terms of the natural rate of output, we actually assume that real marginal cost function of firm $Z(\cdot)$ in order to supply a good f is given by

$$\begin{aligned} Z(f)_t &= Z(y_t(f), Y_t, r_t; \nu_t) = \left\{\left(\frac{1}{n}\right)\int_0^n \left[1+\gamma^L r_t(h)\right]^{1-\epsilon}\left\{\frac{V_l\left[l_T(h)\right]}{U_C(C_t,\nu_t)}\frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)}\right\}^{1-\epsilon} dh\right\}^{\frac{n}{1-\epsilon}} \\ &\times \left\{\left(\frac{1}{1-n}\right)\int_n^1 \left[1+\gamma^S r_t(\overline{h})\right]^{1-\epsilon}\left\{\frac{V_l\left[l_T(\overline{h})\right]}{U_C(C_t,\nu_t)}\frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)}\right\}^{1-\epsilon} d\overline{h}\right\}^{\frac{1-n}{1-\epsilon}}, \end{aligned}$$

then the natural rate of output $Y_t^n = Y^n(\xi_t)$ is given by

$$Z(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta-1}{\theta} = \frac{\epsilon}{\epsilon-1}\left[1+\gamma^L\overline{R}\right]^n\left[1+\gamma^S\overline{R}\right]^{1-n}(1-\Phi) \tag{45}$$

where a parameter Φ expresses the distortion of output level and is of order one.[16] Then we can combine equation (38) and equation (43),

[15] We assume that the monetary policy has no impact on the level of the natural rate of output.

[16] By assuming a proper proportional tax on sales τ as

$$Z(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta-1}{\theta}(1-\tau) = \frac{\epsilon}{\epsilon-1}\left[1+\gamma^L\overline{R}\right]^n\left[1+\gamma^S\overline{R}\right]^{1-n}(1-\Phi),$$

we can induce $\Phi = 0$ as in Rotemberg and Woodford (1997).

$$
\begin{aligned}
U_t &= \overline{Y}U_c \left[\begin{array}{c} \Phi\widehat{Y}_t - \frac{1}{2}(\sigma^{-1}+\omega)\widehat{Y}_t^2 + (\sigma^{-1}g_t + \omega q_t)\widehat{Y}_t \\ -\frac{1}{2}\eta_\pi var_f \ln p_t(f) - \frac{n}{2}\eta_l var_h \ln l_t(h) - \frac{1-n}{2}\eta_l var_{\overline{h}} \ln l_t(\overline{h}) \\ +n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\nu}\right)^2 \left(\Theta\widehat{R}_{L,t} - \Theta^*\widehat{R}_{S,t}\right)^2 \end{array} \right] \\
+t.i.p + Order(&\parallel \xi \parallel^3) \\
&= -\frac{1}{2}\overline{Y}U_c \left[\begin{array}{c} (\sigma^{-1}+\omega)(x_t - x^*)^2 + \eta_\pi var_f \ln p_t(f) \\ +n\eta_l var_h \ln l_t(h) + (1-n)\eta_l var_h \ln l_t(\overline{h}) \\ +n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\eta_2}\right)^2 \left(\Theta\widehat{R}_{L,t} - \Theta^*\widehat{R}_{S,t}\right)^2 \end{array} \right] \\
+t.i.p + Order(&\parallel \xi \parallel^3), \qquad (46)
\end{aligned}
$$

where $\eta_\pi \equiv \theta(1+\omega_p\theta)$, $\eta_l \equiv \phi_h^{-1}(\nu+\epsilon^{-1})$, $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n$, and $x^* \equiv \ln(Y^*/\overline{Y})$. Here Y^* is a solution in equation (45) when $\Phi = 0$, which is called as an efficient level of output as defined in Woodford (2003a). In the second line, we use the log-linearization of equation (45) as

$$
\widehat{Y}_t^n \equiv \ln(Y_t^n/\overline{Y}) = \frac{\sigma^{-1}g_t + \omega q_t}{\sigma^{-1}+\omega},
$$

and the relation as

$$
\ln(Y_t^n/Y_t^*) = -(\sigma^{-1}+\omega)\Phi + Order(\parallel \xi \parallel),
$$

which is given by the relation between the efficient level of output and the natural rate of output in terms of one by equation (44). This expresses that the percentage difference between Y_t^n and Y_t^* is independent from shocks in the first order approximation. It again notes that we assume that Φ is of order one. To evaluate $var_h\widehat{l}_t(h)$ and $var_{\overline{h}}\widehat{l}_t(\overline{h})$, we use the optimal condition of labor supply and the labor demand function given by equation (9), equation (10), equation (15), and equation (16). By log-linearizing these equations, we finally have a following relation

$$
var_h \ln l_t(h) = \Xi var_h \ln(1 + r_t(h)) + Order(\parallel \xi \parallel^3),
$$

$$var_h \ln l_t(\overline{h}) = \Xi^* var_{\overline{h}} \ln(1 + r_t(\overline{h})) + Order(\| \xi \|^3),$$

where $\Xi \equiv \epsilon^2 \Theta^2 (\frac{\epsilon^2}{(\nu^{-1}+\epsilon)^2} + 1)$ and $\Xi^* \equiv \epsilon^2 (\Theta^*)^2 (\frac{\epsilon^2}{(\nu^{-1}+\epsilon)^2} + 1)$. Then, equation (45) is transformed into

$$\begin{aligned} UT_t &= -\frac{1}{2}\overline{Y}U_c \left[\begin{array}{c} (\sigma^{-1} + \omega)(x_t - x^*)^2 + \eta_\pi var_f \ln p_t(f) \\ + n\eta_r var_h \ln(1 + r_t(h)) + (1-n)\eta_r^* var_{\overline{h}} \ln(1 + r_t(\overline{h})) \\ + n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\eta_2}\right)^2 \left(\Theta \widehat{R}_{L,t} - \Theta^* \widehat{R}_{S,t}\right)^2 \end{array} \right] \\ &+ t.i.p + Order(\| \xi \|^3), \end{aligned} \tag{47}$$

where $\eta_r \equiv \Xi \eta_l = \epsilon \phi_h^{-1}(1+\nu\epsilon)\Theta^2(\frac{\epsilon^2}{(\nu^{-1}+\epsilon)^2} + 1)$ and $\eta_r^* \equiv \Xi^* \eta_l = \epsilon \phi_h^{-1}(1+\nu\epsilon)(\Theta^*)^2(\frac{\epsilon^2}{(\nu^{-1}+\epsilon)^2} + 1)$.

The remaining work to derive the approximated welfare function is to evaluate $var_h \ln p_t(f)$ and $var_h \ln(1 + r_t(h))$ in equation (47). Following Woodford (2003a), we define

$$\overline{P}_t \equiv E_f \ln p_t(f),$$

$$\triangle_t \equiv var_f \ln p_t(f).$$

Then we can make following relations

$$\begin{aligned} \overline{P}_t - \overline{P}_{t-1} &= E_f \left[\ln p_t(f) - \overline{P}_{t-1} \right] \\ &= \alpha E_f \left[\ln p_{t-1}(f) - \overline{P}_{t-1} \right] + (1-\alpha) E_f \left[\ln p_t^*(f) - \overline{P}_{t-1} \right] \\ &= (1-\alpha) E_f \left[\ln p_t^*(f) - \overline{P}_{t-1} \right], \end{aligned} \tag{48}$$

and we also have

$$
\begin{aligned}
\triangle_t &= var_f\left[\ln p_t(f) - \overline{P}_{t-1}\right] \\
&= E_f\left\{\left[\ln p_t(f) - \overline{P}_{t-1}\right]^2\right\} - (E_f \ln p_t(f) - \overline{P}_{t-1})^2 \\
&= \alpha E_f\left\{\left[\ln p_{t-1}(f) - \overline{P}_{t-1}\right]^2\right\} + (1-\alpha)E_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)E_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)(var_f(\ln p_t^*(f) - \overline{P}_{t-1}) + \left\{E_f\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]\right\}^2) - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}(\overline{P}_t - \overline{P}_{t-1}), \qquad (49)
\end{aligned}
$$

where we use equation (48) and $p_t^*(f)$ is an optimal price setting by the agent f following the Calvo (1983) - Yun (1992) framework. It notes that all project groups re-set the same price at time t when they are selected to change prices, because the unit marginal cost of production is same for all project groups. Also, we have a following relation that relates $\overline{P}_t$ with P_t

$$
\overline{P}_t = \ln P_t + Order(\parallel \xi \parallel^2),
$$

where $Order(\parallel \xi \parallel^2)$ is order terms higher than the first order approximation. Here we make use of the definition of price aggregator $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$. Then equation (49) can be transformed as

$$
\triangle_t = \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}\pi_t, \qquad (50)
$$

where $\pi_t \equiv \ln \frac{P_t}{P_{t-1}}$. From equation (50), we have

$$
\triangle_t = \alpha^{t+1}\triangle_{-1} + \sum_{s=0}^{t} \alpha^{t-s}\left(\frac{\alpha}{1-\alpha}\right)\pi_s^2,
$$

and so

$$\sum_{t=0}^{\infty}\beta^t\triangle_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p + Order(\parallel \xi \parallel^3). \tag{51}$$

To evaluate $var_h \ln(1+r_t(h))$, we define $\overline{R}_{L,t}$ and $\triangle_t^R$ as

$$\overline{R}_{L,t} \equiv E_h \ln(1+r_t(h)),$$

$$\triangle_t^R \equiv var_h \ln(1+r_t(h)).$$

Then, we can make following relations

$$\begin{aligned}
\overline{R}_{L,t} - \overline{R}_{L,t-1} &= E_h\left[\ln(1+r_t(h)) - \overline{R}_{L,t-1}\right] \\
&= \varphi^L E_h\left[\ln(1+r_{t-1}(h)) - \overline{R}_{L,t-1}\right] + (1-\varphi^L)\left[\ln(1+r_t^*) - \overline{R}_{L,t-1}\right] \\
&= (1-\varphi^L)\left[\ln(1+r_t^*(h)) - \overline{R}_{L,t-1}\right], \qquad (52)
\end{aligned}$$

and

$$\begin{aligned}
\triangle_t^R &= var_h\left[\ln(1+r_t(h)) - \overline{R}_{L,t-1}\right] \\
&= E_h\left\{\left[\ln(1+r_t(h)) - \overline{R}_{L,t-1}\right]^2\right\} - (E_h \ln(1+r_t(h)) - \overline{R}_{L,t-1})^2 \\
&= \varphi^L E_h\left\{\left[\ln(1+r_{t-1}(h)) - \overline{R}_{L,t-1}\right]^2\right\} + (1-\varphi^L)\left[\ln(1+r_t^*) - \overline{R}_{L,t-1}\right]^2 - (\overline{R}_{L,t} - \overline{R}_{L,t-1})^2 \\
&= \varphi^L\triangle_{t-1}^R + \frac{\varphi^L}{1-\varphi^L}(\overline{R}_{L,t} - \overline{R}_{L,t-1})^2, \qquad (53)
\end{aligned}$$

where we use equation (52). Also, as in the discussion on price, we have

$$\overline{R}_{L,t} = \ln(1+R_{L,t}) + Order(\parallel \xi \parallel^2), \tag{54}$$

where we make use of the definition of the aggregate loan rates $1 + R_{L,t} \equiv \int_0^1 \frac{q_t(h)}{Q_t}(1 + r_t(h))dh$. Then, from equation (53) and equation (54), we have

$$\triangle_t^R = \varphi^L \triangle_{t-1}^R + \frac{\varphi}{1-\varphi}(\widehat{R}_{L,t} - \widehat{R}_{L,t-1})^2, \tag{55}$$

where $\widehat{R}_{L,t} \equiv \ln \frac{1+R_{L,t}}{1+\bar{r}}$. From equation (55), we have

$$\triangle_t^R = \left(\varphi^L\right)^{t+1} \triangle_{-1}^R + \sum_{s=0}^{t} \varphi^{t-s} \left(\frac{\varphi}{1-\varphi}\right) (\widehat{R}_{L,s} - \widehat{R}_{L,s-1})^2.$$

Then, we have

$$\sum_{t=0}^{\infty} \beta^t \triangle_t^R = \frac{\varphi^L}{(1-\varphi^L)(1-\varphi^L\beta)} \sum_{t=0}^{\infty} \beta^t (\widehat{R}_t - \widehat{R}_{t-1})^2 + t.i.p + Order(\parallel \xi \parallel^3).$$

We have a similer relation for the less sticky loan. Then, equation (47) can finally be transformed as:

$$\sum_{t=0}^{\infty} \beta^t UT_t \simeq -\Lambda \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} \lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_L (\widehat{R}_{L,t} - \widehat{R}_{L,t-1})^2 \\ +\lambda_S (\widehat{R}_{S,t} - \widehat{R}_{S,t-1})^2 + \lambda_{LS}^* \left(\Theta \widehat{R}_{L,t} - \Theta^* \widehat{R}_{S,t}\right)^2 \end{array} \right),$$

where $\Lambda \equiv \frac{1}{2}\overline{Y}u_c$, $\lambda_\pi \equiv \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta(1+\omega_p\theta)$, $\lambda_x \equiv (\sigma^{-1}+\omega)$, $\lambda_L \equiv n\epsilon\phi_h^{-1}(1+\nu\epsilon)(\frac{\gamma^L(1+\bar{r}^L)}{1+\gamma^L\bar{r}^L})^2(\frac{\epsilon^2}{(\nu^{-1}+\epsilon)^2} + 1)\frac{\varphi^L}{(1-\varphi^L)(1-\varphi^L\beta)}$, $\lambda_S \equiv (1-n)\epsilon\phi_h^{-1}(1+\nu\epsilon)(\frac{\gamma^S(1+\bar{r}^S)}{1+\gamma^S\bar{r}^S})^2(\frac{\epsilon^2}{(\nu^{-1}+\epsilon)^2} + 1)\frac{\varphi^S}{(1-\varphi^S)(1-\varphi^S\beta)}$, and $\lambda_{LS}^* \equiv n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\eta_2}\right)^2$.

Finally, by assuming $\Theta = \Theta^*$, we have

$$\sum_{t=0}^{\infty} \beta^t UT_t \simeq -\Lambda \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} \lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_L (\widehat{R}_{L,t} - \widehat{R}_{L,t-1})^2 \\ +\lambda_S (\widehat{R}_{S,t} - \widehat{R}_{S,t-1})^2 + \lambda_{LS} \left(\widehat{R}_{L,t} - \widehat{R}_{S,t}\right)^2 \end{array} \right), \tag{56}$$

where $\lambda_{LS} \equiv \Theta\lambda_{LS}^*$.